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ABSTRACT

Should uniform pricing contraints be imposed on entrants?*

This paper analyzes the effects of universal service obligations, such as uniform pricing, coverage constraints and price caps, on markets newly opened to competition, e.g. broadband services. We show that the requirement of uniform pricing has strong repercussions on coverage decisions. Imposed on the incumbent only, it may distort his coverage decision downward to avoid duopoly entry. If also imposed on entrants it increases the likelihood that entry leads to independent monopolies rather than competition. A large enough coverage constraint on the incumbent re-establishes incentives for duopoly entry, but may lead to higher prices.

JEL Classification: L43, L51 and L52
Keywords: coverage constraints, entry, price caps, uniform pricing and universal service obligations

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1 INTRODUCTION

We analyze some of the effects of universal service obligations (USOs, see e.g. Laffont and Tirole 2000 and Crandall and Waverman 2000) on outcomes in regulated markets with new entry. Common USOs are restrictions on pricing, such as uniform pricing (UP, or non-discrimination) constraints, and coverage constraints. It has been noted in the literature that a UP constraint makes the incumbent firm less aggressive (Armstrong and Vickers 1993, Anton *et al.* 2002, Choné *et al.* 2000, Valletti *et al.* 2002), which has the direct effect of making entry more attractive. Nevertheless, since the incumbent will choose his coverage taking into account any constraint imposed on him, it is possible that the entrant's coverage and total coverage are smaller with the UP constraint than without it (Valletti *et al.* 2002). Therefore the competitive effects of UP constraints are ambiguous.

In this paper we add a further dimension by studying the imposition of a UP constraint on the entrant as well as on the incumbent. We also consider the interplay of these constraints with coverage constraints and price caps. Price discrimination arises for two reasons in our model. First, since local markets differ in service costs and demand, firms would prefer to charge a different price in each market. Typically, low-cost and high-density "urban markets" would be served at lower prices than high-cost and low-density "rural markets". Uniform pricing constraints have been introduced precisely to create more "equity" between urban and rural areas, with the urban ones subsidizing service provision in the rural ones. Second, firms would like to charge higher prices where they are monopolists, and lower prices where they compete. A uniform pricing constraint links these markets strategically and has a decisive influence on the market equilibrium.

Our findings are as follows: If the entrant has the possibility of entering vacant high-cost areas or low-cost duopoly areas, he may choose to enter in one or both types of areas. This choice depends on the incumbent's coverage, and on whether the entrant himself is subject to a uniform pricing constraint.

In particular, entry only in high-cost areas can be optimal. The incumbent may not cover these markets so as not to drive up even further his average cost and uniform price. As a result, these markets can be served by the entrant at higher uniform or non-uniform prices. In the latter case duopoly entry is unaffected since the entrant can charge localized prices and compete effectively where he enters. On the other hand, a uniform pricing constraint makes it rather less attractive to compete in duopoly markets: First, the entrant cannot choose low prices because of the losses he would inflict on his high-cost areas. Second, if he charges high prices then he will not be a very effective competitor in the duopoly markets. In other words, the uniform pricing constraint turns the entrant into a "fat cat" (Fudenberg and Tirole 1984) if he decides to acquire monopoly areas of his own.[2]

The cable TV and broadband market in Portugal serves as an illustration of this outcome. The telecom incumbent's daughter TVCabo covers mainly the large and populous areas of Lisbon and Porto. The surrounding areas and many towns in the rural interior of Portugal are covered by competitors such as Cabovisão, using their own networks. Since the overlap of cable networks

[2] A referee proposed the term "skim milk skimming" for this result, as opposed to "cream skimming". In fact there may still be enough cream left in half-skimmed milk to make cats fat!

is minimal, there seems to exist a clear decision by TVCabo's competitors to avoid duplication of infrastructure and competition with the incumbent.

A further result is that if the incumbent firm is subject to a uniform pricing constraint but free to choose its coverage, it may choose a smaller coverage to avoid subsequent duopoly entry. This is an instance of the "lean and hungry look" in Fudenberg and Tirole (1984). The necessary reduction in coverage is smaller if the entrant is subject to a uniform pricing constraint, therefore this constraint again reduces competitive entry.

The latter effect can be avoided by imposing a sufficiently large coverage constraint on the incumbent. But even though duopoly entry will follow, it is not clear whether equilibrium prices will increase or decrease since the direct effect of the incumbent's higher coverage is a price rise.

A previous version of this paper, Hoernig (2002), made use of a completely different model. Goods were homogeneous, and therefore all pricing equilibria only existed in mixed strategies. Nevertheless, the main results were quite similar, which attests to their robustness.

Valletti, Hoernig and Barros (2002) is closest to the spirit of this paper in that it also analyses the relation between the coverage choices by an incumbent firm and an entrant. In their model local markets are characterized by different fixed costs of being served, but are otherwise identical. Since the entrant does not cover monopoly areas and chooses the same price in all locations, a uniform price on the entrant does not impose any binding constraint. Therefore the questions which we ask in this paper cannot be dealt with in their framework. The same is true of Bourguignon and Yáñez (2003).

Foros and Kind (2003) consider the choice of coverage by various firms, but only construct symmetric scenarios. All firms enter and choose (identical) coverage simultaneously, and all are either subject to coverage or uniform pricing constraints or not.

In Faulhaber and Hogendorn (2000) firms enter sequentially, so that firms entering later will cover smaller areas. On the other hand, they do not assume any uniform pricing or coverage constraints. Therefore local markets are completely independent from each other, and entry occurs simply based on market size.

The structure of the paper is as follows: Section 2 introduces the model and Section 3 sets out the monopoly case. Section 4 considers pricing best responses and equilibrium outcomes, given coverage. Section 5 analyses entry decisions and coverage choice with and without uniform pricing. Section 6 concludes.

2 THE MODEL

We consider a game with three stages, between an incumbent (firm 1) and an entrant (firm 2). In the first and second stage the incumbent and the entrant choose their coverage, respectively. In the third stage both firms compete in prices. We assume that for all levels of coverage there exists a unique pure strategy Nash equilibrium in prices in this stage, which implies that services are sufficiently differentiated.[3] The equilibrium concept adopted is subgame-perfect equilibrium. Firms' choices at all three stages may be subject to regulatory constraints such as uniform pricing constraints, price caps or a minimum coverage.

[3] Only mixed equilibria exist in pricing games if goods are sufficiently homogeneous and at least one firm has captive (monopoly) clients.

There is a continuum of local markets $x \in [0, x^{\max}]$. There are sunk costs $F_i > 0$, $i = 1, 2$, of entry in each local market. We assume that the entrant will in equilibrium enter in some markets, therefore we neglect non-local entry costs. In the first stage the incumbent covers an area $[0, m_1]$, while in the second stage the entrant will cover areas $[0, l]$ and $(m_1, m_2]$, with $0 \leq l \leq m_1$ and $m_2 \geq m_1$.

For prices $p_1, p_2 \geq 0$ profits in local market x are $\pi_i(p_i, p_j, x)$, $i \neq j$. They obey the following assumptions:

1. π_i is twice continuously differentiable in (p_i, p_j, x);

2. for all (p_j, x), π_i is quasi-concave in p_i and has a unique finite maximizer (or local best response) $r_i(p_j, x)$;

3. π_i increases in p_j $(\partial \pi_i / \partial p_j > 0)$, and $\partial^2 \pi_i / \partial p_i \partial p_j > 0$;

4. π_i decreases in x $(\partial \pi_i / \partial x < 0)$, and marginal profits are increasing in x: $\partial^2 \pi_i / \partial p_i \partial x > 0$.

The first two assumptions are made for technical convenience. Assumption 3 means that firms' prices are strategic complements, and that each firm's profits increase if the other firm increases its price.

Assumption 4 implies that locations are ordered by their profitability, and that firms would like to charge higher prices in high-cost areas: $\partial r_i / \partial x > 0$. This assumption captures the effects of increasing cost of providing services in rural areas, as well as any change in demand composition.

The advantage of introducing assumptions only on profits, and not on demand and cost separately, is two-fold: On the one hand all effects, especially when they are contradictory, are neatly summarized. And on the other hand, all assumptions on demand and cost will have to be translated into properties of profits anyway. Directly postulating the properties of profits makes the economic intuition much clearer.

Profits in markets where only one firm is present are defined by $\pi_{im}(p_i, x) = \lim_{p_j \to \infty} \pi_i(p_i, p_j, x)$. Local monopoly prices are $p_i^m(x) = \lim_{p_j \to \infty} r_i(p_j, x)$. From the above assumptions it follows that for all $p_j \geq 0$ we have $\pi_{im}(p_i, x) > \pi_i(p_i, p_j, x)$, $\partial \pi_{im}/\partial p_i > \partial \pi_i/\partial p_i$ and $p_i^m(x) > r_i(p_j, x)$. We assume that $\partial \pi_{im}/\partial x < 0$ and $\partial^2 \pi_{im}/\partial p_i \partial x > 0$, the latter of which implies $\partial p_i^m/\partial x > 0$.[4]

The constraints that can be imposed by the regulator are the following:

- Uniform prices P_1 and P_2;
- minimum coverage constraints $\hat{m}_1$ and $\hat{m}_2$;
- a price cap $\hat{p}$.

We will denote uniform prices and their corresponding best responses R_1 and R_2 with capital letters in order to distinguish them easily from local prices and best responses. The coverage constraint on the incumbent can also be interpreted as historical coverage, in the sense that the incumbent is already covering these markets when the possibility of entry arises. In this case we assume that he is obliged to continue to serve these markets.

3 MONOPOLY

We first discuss the monopoly outcome without entry, which will serve as the background for the entry analysis. Furthermore, it will demonstrate how pricing and coverage choice interact.

[4] The definition of π_{im} only leads to weak inequalities.

Suppose that the incumbent is covering a area $[0, m_1]$, and is not subject to a uniform pricing constraint. If he charges price $p_1(x)$ at location x, his profits are

$$\Pi_1^{NUP}(\{p_1\}, m_1) = \int_0^{m_1} \pi_{1m}(p_1(x), x)\, dx - m_1 F_1. \tag{1}$$

Monopoly prices $p_1^m(x)$ are chosen independently at each location, and the profit-maximizing coverage $\tilde{m}_1^{NUP}(F_1)$ is determined by

$$\frac{\partial \Pi_1^{NUP}}{\partial m_1} = \pi_{1m}\left(p_1^m\left(\tilde{m}_1^{NUP}\right), \tilde{m}_1^{NUP}\right) - F_1 = 0 \tag{2}$$

Given the assumption $\partial \pi_{1m}/\partial x < 0$ there is a unique optimal coverage $\tilde{m}_1^{NUP}$ if any markets are covered at all.

If on the other hand the incumbent is subject to a uniform pricing constraint then his profits are

$$\Pi_1^{UP}(P_1, m_1) = \int_0^{m_1} \pi_{1m}(P_1, x)\, dx - m_1 F_1.$$

Given coverage m_1, the profit-maximizing uniform monopoly price $P_1^m(m_1)$ is given by

$$\frac{\partial \Pi_1^{UP}}{\partial P_1} = \int_0^{m_1} \frac{\partial \pi_{1m}(P_1^m(m_1), x)}{\partial p_1} dx = 0, \tag{3}$$

where we assume the second-order condition $\int_0^{m_1} \left(\partial^2 \pi_{1m}/\partial p_1^2\right) dx < 0$ to hold. If $m_1 > 0$ this condition implies that there is an $x(m_1) \in (0, m_1)$, increasing in m_1, such that $\partial \pi_{1m}/\partial p_1$ is negative for $x < x(m_1)$ ($P_1^m(m_1)$ is too high for these markets), and positive for $x > x(m_1)$ ($P_1^m(m_1)$ is too low). Similar results hold in all situations where uniform prices are involved. Furthermore, $P_1^m(m_1)$ is increasing in m_1 since

$$\frac{dP_1^m(m_1)}{dm_1} = -\frac{\int_0^{m_1} \left(\partial^2 \pi_{1m}/\partial p_1 \partial x\right) dx}{\int_0^{m_1} \left(\partial^2 \pi_{1m}/\partial p_1^2\right) dx} > 0, \tag{4}$$

i.e. the inclusion of further high-cost areas drives up the uniform price. Profit-maximizing coverage $\tilde{m}_1^{UP}(F_1)$ is defined by

$$\frac{\partial \Pi_1^{UP}}{\partial m_1} = \pi_{1m}\left(P_1^m\left(\tilde{m}_1^{UP}\right), \tilde{m}_1^{UP}\right) - F_1 = 0. \tag{5}$$

Since for $\tilde{m}_1^{UP}(F_1) > 0$, the price $P_1^m\left(\tilde{m}_1^{UP}\right)$ charged in the last market is too low, we have

$$F_1 = \pi_{1m}\left(P_1^m\left(\tilde{m}_1^{UP}\right), \tilde{m}_1^{UP}\right) < \pi_{1m}\left(p_1^m\left(\tilde{m}_1^{UP}\right), \tilde{m}_1^{UP}\right),$$

and profit-maximizing coverage will be smaller than under non-uniform pricing: $\tilde{m}_1^{UP}(F_1) < \tilde{m}_1^{NUP}(F_1)$.

Now let us assume that a price cap $\hat{p}$ is imposed. If it is not binding then nothing changes. On the other hand, if it is binding then the corresponding profit-maximizing coverage $\tilde{m}_1^{PC}(F_1, \hat{p})$ will be determined by the condition

$$\pi_{1m}\left(\hat{p}, \tilde{m}_1^{PC}\right) - F_1 = 0. \tag{6}$$

Since either $\hat{p} < p_1^m$ or $\hat{p} < P_1^m$, coverage $\tilde{m}_1^{PC}(F_1, \hat{p})$ is smaller than $\tilde{m}_1^{NUP}(F_1)$ or $\tilde{m}_1^{UP}(F_1)$. Clearly $\tilde{m}_1^{PC}(F_1, \hat{p})$ is increasing in $\hat{p}$ while the price cap is binding.

The effect of a binding coverage constraint $m_1 \geq \hat{m}_1$ depends on whether a uniform pricing constraint is imposed simultaneously or not. If none is imposed then prices are unaffected, since they continue to be determined separately for each local market. Under uniform pricing, though, (4) shows that the resulting uniform price will be higher the higher the coverage constraint. As a result, previously covered consumers suffer a reduction in their surplus. Naturally this effect does not exist if a binding price cap is imposed, too.

These results make clear that there is an intimate relationship between coverage and price setting: The uniform price strategically links all covered markets if it is not "cut off" by a price cap. As we will see in the following sections, this effect is magnified when entry occurs since entry in some markets affects all other markets subject to the uniform price.

4 THE PRICING EQUILIBRIUM

In this section we analyze the Nash pricing equilibria, taking both firms' coverage as given. We will first consider each firm's profit maximization problem and best response, and then determine the effects of coverage on the equilibrium prices.

Here and in the following the incumbent will always be subject to a uniform pricing constraint, while the entrant may or may not.

4.1 Price best responses

Assume that the entrant sets prices $p_2(x)$ in the markets where he is present. These are equal to a uniform price P_2 if a uniform pricing constraint is imposed. The incumbent's profits are

$$\Pi_1(P_1, \{p_2\}; l, m_1) = \int_0^l \pi_1(P_1, p_2(x), x)\, dx + \int_l^{m_1} \pi_{1m}(P_1, x)\, dx - m_1 F_1. \tag{7}$$

Maximizing Π_1 with respect to P_1 leads to the first-order condition

$$\frac{\partial \Pi_1}{\partial P_1} = \int_0^l \frac{\partial \pi_1}{\partial p_1}(P_1, p_2(x), x)\, dx + \int_l^{m_1} \frac{\partial \pi_{1m}}{\partial p_1}(P_1, x)\, dx = 0, \tag{8}$$

which defines the uniform price best response $R_1(\{p_2\}; l, m_1)$, assuming that a unique maximizer exists. We assume that the sufficient second-order condition

$$\frac{\partial^2 \Pi_1}{\partial P_1^2} = \int_0^l \frac{\partial^2 \pi_1}{\partial p_1^2}(P_1, p_2(x), x)\,dx + \int_l^{m_1} \frac{\partial^2 \pi_{1m}}{\partial p_1^2}(P_1, x)\,dx < 0 \qquad (9)$$

is fulfilled. Since prices are strategic complements, R_1 increases if the entrant's price or prices increase. For example, if the entrant is subject to a uniform price then

$$\frac{\partial^2 \Pi_1}{\partial P_1 \partial P_2} = \int_0^l \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2}(P_1, P_2, x)\,dx > 0, \qquad (10)$$

and $\partial R_1/\partial P_2 = -\left(\partial^2\Pi_1/\partial p_1 \partial p_2\right) / \left(\partial^2 \Pi_1/\partial p_1^2\right)$. The corresponding result with non-uniform pricing by the entrant follows if $p_2(x)$ is raised on a set of locations of positive mass. The comparative statics of the best response R_1 with respect to coverage are determined by the following cross-derivatives:

$$\begin{aligned} \frac{\partial^2 \Pi_1}{\partial P_1 \partial l} &= \frac{\partial \pi_1}{\partial p_1}(P_1, p_2(l), l) - \frac{\partial \pi_{1m}}{\partial p_1}(P_1, l) < 0, \qquad (11) \\ \frac{\partial^2 \Pi_1}{\partial P_1 \partial m} &= \pi_{1m}(P_1, m_1) > 0. \end{aligned}$$

Thus the incumbent's best response $R_1(P_2, l, m_1)$ is increasing in $\{p_2\}$ or P_2, and it is increasing in m_1 and decreasing in l. Clearly it is independent of m_2.

The entrant's profits are

$$\begin{aligned} \Pi_2(\{p_2\}, P_1; l, m_1, m_2) &= \int_0^l \pi_2(p_2(x), P_1, x)\,dx \qquad (12) \\ &+ \int_{m_1}^{m_2} \pi_{2m}(p_2(x), x)\,dx - (l + m_2 - m_1) F_2 \end{aligned}$$

If the entrant is not subject to a uniform pricing constraint then all his best responses are local,

$$\frac{\partial \pi_2(r_2(P_1; x), P_1, x)}{\partial p_2} = 0 \text{ if } x \in [0, l], \qquad (13)$$

$$\frac{\partial \pi_{2m}(p_2^m(x), x)}{\partial p_2} = 0 \text{ if } x \in (m_1, m_2]. \qquad (14)$$

As noted above, r_2 is increasing in P_1, and both r_2 and p_2^m are increasing in x. They do not depend on equilibrium coverage.

If the entrant is subject to a uniform pricing constraint, his best response is of the form $P_2 = R_2(P_1, l, m_1, m_2)$ and defined by the first-order condition

$$\frac{\partial \Pi_2}{\partial P_2} = \int_0^l \frac{\partial \pi_2}{\partial p_2}(P_2, P_1, x)\,dx + \int_{m_1}^{m_2} \frac{\partial \pi_{2m}}{\partial p_2}(P_2, x)\,dx = 0. \tag{15}$$

Again we assume that a unique maximizer exists and that the sufficient second-order condition

$$\frac{\partial^2 \Pi_2}{\partial P_2^2} = \int_0^l \frac{\partial^2 \pi_2}{\partial p_2^2}(P_2, P_1, x)\,dx + \int_{m_1}^{m_2} \frac{\partial^2 \pi_{2m}}{\partial p_2^2}(P_2, x)\,dx < 0 \tag{16}$$

holds. Then in "cheap" markets with $x \approx 0$ the uniform price P_2 is too high, while in "costly" markets with $x \approx m_2$ this price is too low. There are markets $x^d = x^d(P_2, P_1)$ and $x^m = x^m(P_2)$ such that this price is locally optimal, with $\partial \pi_2(P_2, P_1, x^d)/\partial p_2 = 0$ and $\partial \pi_{2m}(P_2, x^m)/\partial p_2 = 0$. We have $0 < x^m < m_2$, and $x^m < x^d$ because $\partial^2 \pi_2 / \partial p_1 \partial p_2 > 0$. As discussed above for the monopoly case then $\partial \pi_2/\partial p_2 \gtreqless 0$ if $x \gtreqless x^d$, and $\partial \pi_{2m}/\partial p_2 \gtreqless 0$ if $x \gtreqless x^m$.

The entrant's best response R_2 is increasing in P_1 and m_2:

$$\begin{aligned}
\frac{\partial^2 \Pi_2}{\partial P_1 \partial P_2} &= \int_0^l \frac{\partial^2 \pi_2}{\partial p_1 \partial p_2}(P_2, P_1, x)\,dx > 0 \qquad (\text{if } l > 0), \\
\frac{\partial^2 \Pi_2}{\partial P_2 \partial m_2} &= \frac{\partial \pi_{2m}}{\partial p_2}(P_2, m_2) > 0.
\end{aligned} \tag{17}$$

The effects of l and m_1 on the entrant's best response depend on the relative size of the duopoly and monopoly areas. The necessary information is contained in x^d and x^m. Thus

$$\frac{\partial^2 \Pi_2}{\partial P_2 \partial l} = \frac{\partial \pi_2}{\partial p_2}(P_2, P_1, l) \lesseqgtr 0 \qquad \text{if } x^d \gtreqless l \tag{18}$$

$$\frac{\partial^2 \Pi_2}{\partial P_2 \partial m_1} = -\frac{\partial \pi_{2m}}{\partial p_2}(P_2, m_1) \gtreqless 0 \qquad \text{if } x^m \gtreqless m_1 \tag{19}$$

The significance of these results is the following: If the duopoly area $[0, l]$ is very small compared to the monopoly area $(m_1, m_2]$ then P_2 is very close to the monopoly uniform price. This implies that $x^d > x^m > m_1 > l$ and therefore $\partial R_2/\partial l < 0$ and $\partial R_2/\partial m_1 > 0$. On the other hand, if the duopoly area is large as compared to the monopoly area then $x^m < x^d < l < m_1$, with $\partial R_2/\partial l > 0$ and $\partial R_2/\partial m_1 < 0$.

4.2 The effects of coverage on equilibrium prices

We make use an innovative technique to perform comparative statics on the pricing Nash equilibrium. It is equally well-suited to deal with both the cases of uniform and non-uniform pricing by the entrant. Instead of applying the inverse function theorem to the set of first-order conditions as usual, we use the first-order condition of one firm and the best response(s) of the other firm.

Under uniform pricing on both firms, the incumbent's equilibrium price P_1^* is defined by

$$T_1^{UP}\left(P_1^*, l, m_1, m_2\right) = \int_0^l \frac{\partial \pi_1}{\partial p_1}\left(P_1^*, R_2\left(P_1^*, l, m_1, m_2\right), x\right) dx + \int_l^{m_1} \frac{\partial \pi_{1m}}{\partial p_1}\left(P_1^*, x\right) dx = 0 \tag{20}$$

If no uniform pricing is imposed on the entrant then the corresponding condition is

$$T_1^{NUP}\left(P_1^*, l, m_1, m_2\right) = \int_0^l \frac{\partial \pi_1}{\partial p_1}\left(P_1^*, r_2\left(P_1^*, x\right), x\right) dx + \int_l^{m_1} \frac{\partial \pi_{1m}}{\partial p_1}\left(P_1^*, x\right) dx = 0. \tag{21a}$$

Finally, the entrant's uniform price is defined by

$$T_2\left(P_2^*, l, m_1, m_2\right) = \int_0^l \frac{\partial \pi_2}{\partial p_2}\left(P_2^*, R_1\left(P_2^*, l, m_1\right), x\right) dx + \int_{m_1}^{m_2} \frac{\partial \pi_{2m}}{\partial p_2}\left(P_2^*, x\right) dx = 0, \tag{22}$$

and by

$$T_2^{PC}\left(P_2^*, l, m_1, m_2\right) = \int_0^l \frac{\partial \pi_2}{\partial p_2}\left(P_2^*, \hat{p}, x\right) dx + \int_{m_1}^{m_2} \frac{\partial \pi_{2m}}{\partial p_2}\left(P_2^*, x\right) dx = 0 \quad (23)$$

if a price cap is imposed on the incumbent.

From these conditions we can conclude the following (The proof is relegated to the appendix):

Lemma 1 *1. If a uniform pricing constraint is imposed on the entrant, then P_1^* and P_2^* are decreasing in l and increasing in m_1 if l is small compared to $m_2 - m_1$. They are both increasing in m_2. These statements continue to be true for P_2^* if a price cap is imposed on the incumbent.*

2. If no uniform pricing constraint is imposed on the entrant then P_1^* and $r_2\left(P_1^*, x\right)$ are decreasing in l, increasing in m_1, and not affected by m_2.

These comparative statics results are as expected. They are straightforward if the entrant is not subject to a uniform price. While prices in the entrant's monopoly area simply remain unchanged, in the duopoly area prices are determined by the incumbent's uniform price and the entrant's local reactions to it. Any change in coverages l and m_1 feeds into the incumbent's uniform price, with the entrant's local prices following in the same direction.

If uniform pricing is imposed on the entrant, then an increase in his monopoly coverage m_2 decreases the competitive intensity in the duopoly markets – this is the fat-cat effect. Furthermore, the changes in l and m_1 have ambiguous effects, because it will depend on their relative size whether the competitive intensity increases or decreases. If l is small then an increase in l or decrease in m_1 makes competition fiercer. On the other hand, if l is already large then the duopoly covers relatively high-cost areas, and an increase in l or decrease in m_1 may reduce competitive intensity. This leads to higher prices because the size of the entrant's monopoly area increases relative to his duopoly area.

5 COVERAGE AND ENTRY

Now we analyze firms' coverage decisions. In a first step we take the incumbent's coverage as given, and see where the entrant will enter. In a second step we consider the incumbent's coverage choice and its implications.

5.1 The Entrant's Coverage Choice

For most of the following we assume that the entrant is free to choose his coverage. We will consider this choice with and without the imposition of a uniform pricing constraint. At the end of each of the next two sections we will deal with the effects of a price cap on the incumbent and a coverage constraint on the entrant.

5.1.1 Non-uniform pricing

Let us now consider how the entrant chooses coverage in equilibrium if he is free to do so. If no uniform pricing constraint is imposed on him, he maximizes over $l \in [0, m_1]$ and $m_2 \geq m_1$

$$\begin{aligned} \Pi_2^{NUP*}(l, m_1, m_2) &= \int_0^l \pi_2\left(r_2\left(P_1^*, x\right), P_1^*, x\right) dx \\ &+ \int_{m_1}^{m_2} \pi_{2m}\left(p_2^m(x), x\right) dx - \left(l + m_2 - m_1\right) F_2. \end{aligned} \tag{24}$$

We denote profit-maximizing coverage by $l^{NUP}(m_1; F_2)$ and $m_2^{NUP}(F_2)$. If $l^{NUP} > 0$ and $m_2^{NUP} > m_1$, the necessary first-order conditions are (here and in the following we assume that the sufficient second-order condition holds)

$$\int_0^l \frac{\partial \pi_2}{\partial p_1} dx \frac{\partial P_1^*}{\partial l} + \left[\pi_2\left(r_2\left(P_1^*, l^{NUP}\right), P_1^*, l^{NUP}\right) - F_2\right] = 0, \tag{25}$$

$$\pi_{2m}\left(p_2^m\left(m_2^{NUP}\right), m_2^{NUP}\right) - F_2 = 0 \tag{26}$$

In other words, the profits of the marginal duopoly market need not only cover the fixed cost of entry, but the decrease of equilibrium prices in the existing duopoly markets must be taken into account. The decision to enter monopoly markets is non-strategic, with entry if and only if local monopoly profits recover fixed costs. This means in particular that m_2^{NUP} does not depend on the value of m_1 if the entrant enters monopoly areas.

The extent of entry into duopoly markets is not related to the choice of entry into monopoly markets. The sign of dl^{NUP}/dm_1 equals that of

$$\frac{\partial^2 \Pi_2^{NUP*}}{\partial l \partial m_1} = \frac{\partial \pi_2}{\partial p_1} \frac{\partial P_1^*}{\partial m_1}. \tag{27}$$

By point 2 of Lemma 1, this is positive. The entrant does not enter any duopoly markets if and only if the gross profits of only entering market zero do not cover fixed cost,

$$Z_2^{NUP}(m_1) = \pi_2\left(r_2\left(P_1^m(m_1), 0\right), P_1^m(m_1), 0\right) \leq F_2. \tag{28}$$

He will then cover no other duopoly markets because $P_1^* < P_1^m(m_1)$ if $l^{NUP} > 0$ and

$$\pi_2\left(r_2\left(P_1^*, l^{NUP}\right), P_1^*, l^{NUP}\right) < \pi_2\left(r_2\left(P_1^*, 0\right), P_1^*, 0\right) < Z_2^{NUP}(m_1). \tag{29}$$

$Z_2^{NUP}(m_1)$ depends positively on the coverage of the incumbent, since

$$\frac{dZ_2^{NUP}(m_1)}{dm_1} = \frac{\partial \pi_2}{\partial p_1} \frac{\partial P_1^m}{\partial m_1} > 0. \tag{30}$$

The higher the incumbent's coverage the more profitable duopoly entry will be under the price umbrella created by the uniform pricing constraint on the incumbent. There will be no duopoly entry at all if $F_2 \geq Z_2^{NUP}(\infty) = \pi_{2m}(p_2^m(0), 0)$, and there will always be entry if $F_2 \leq Z_2^{NUP}(0)$. This shows that the interesting cases lie between these limits.

Lemma 2 *If no uniform pricing constraint is imposed on the entrant, then:*

1. *The entrant covers all monopoly markets between m_1 and $m_2^{NUP}(F_2)$. The latter does not depend on m_1.*

2. *If $F_2 \in \left(Z_2^{NUP}(0), Z_2^{NUP}(\infty)\right)$, then there is duopoly entry if and only if $m_1 > M_1^{NUP}(F_2) = \left(Z_2^{NUP}\right)^{-1}(F_2)$. The optimal coverage $l^{NUP}(m_1; F_2)$ increases in m_1, and is chosen independently of m_2.*

Proof. Z_2^{NUP} is a continuous increasing function, the rest follows from the above discussion. ■

If a price cap $\hat{p}$ on the incumbent is binding at the monopoly price P_1^m, then $\hat{p} < P_1^m$ and $\pi_2(r_2(\hat{p}, 0), \hat{p}, 0) < Z_2^{NUP}(m_1)$: the entrant's local profits at $x = 0$ are smaller. That is, a price cap on the incumbent makes duopoly entry less attractive. On the other hand, while the price cap is binding the incumbent's coverage m_1 cannot affect the entrant's profits at zero. It therefore loses its strategic role.

One can imagine two possible types of coverage constraints on the entrant. The first is to make him cover a minimum number of monopoly areas: $m_2 - m_1 \geq \hat{m}_2$. In this case duopoly coverage remains free, and by Lemma 2 is not affected by the coverage constraint. The second option is to force the entrant to cover a minimum number of all markets: $l + m_2 - m_1 \geq \hat{m}_2$.

Corollary 1 *If no uniform pricing constraint is imposed on the entrant, then:*

1. *If the entrant's coverage constraint applies only to monopoly areas then equilibrium duopoly coverage and prices remain unaffected.*

2. *If the entrant's coverage constraint applies to all markets, and is binding, then equilibrium duopoly prices decrease in $\hat{m}_2$. In this case the entrant's coverage $\left(l^{C,NUP}, m_2^{C,NUP}\right)$ is defined by the condition*

$$\begin{aligned} & \pi_2\left(r_2\left(P_1^*, l^{C,NUP}\right), P_1^*, l^{C,NUP}\right) - F_2 \qquad (31) \\ = \; & \pi_{2m}\left(p_2^m\left(m_2^{C,NUP}\right), m_2^{C,NUP}\right) - F_2 < 0 \end{aligned}$$

Proof. The first statement follows from Lemma 2. As concerns the second statement, equation (31) is the first-order condition resulting from the maximization of profits subject to the constraint $l + m_2 - m_1 = \hat{m}_2$. Prices decrease because $l^{C,NUP}$ is increasing in $\hat{m}_2$. ■

5.1.2 Uniform pricing

Under a uniform pricing constraint, the entrant chooses duopoly coverage $l \in [0, m_1]$ and monopoly coverage $m_2 \geq m_1$ such as to maximize

$$\begin{aligned} \Pi_2^{UP*}(l, m_1, m_2) = & \int_0^l \pi_2\left(P_2^*, P_1^*, x\right) dx \qquad (32) \\ & + \int_{m_1}^{m_2} \pi_{2m}\left(P_2^*, x\right) dx - \left(l + m_2 - m_1\right) F_2. \end{aligned}$$

The first-order conditions for profit maximization over $m_2^{UP}(m_1; F_2) > m_1$ and $l^{UP}(m_1; F_2) > 0$, respectively, are

$$\frac{\partial \Pi_2^{UP*}}{\partial m_2} = \int_0^{l^{UP}} \frac{\partial \pi_2}{\partial p_1} dx \frac{\partial P_1^*}{\partial m_2} + \left[\pi_{2m}\left(P_2^*, m_2^{UP}\right) - F_2\right] = 0 \qquad (33)$$

$$\frac{\partial \Pi_2^{UP*}}{\partial l} = \int_0^{l^{UP}} \frac{\partial \pi_2}{\partial p_1} dx \frac{\partial P_1^*}{\partial l} + \left[\pi_2\left(P_2^*, P_1^*, l^{UP}\right) - F_2\right] = 0 \qquad (34)$$

A non-trivial complication is that the interaction between l and m_2 is not clear. The cross-derivative $\partial^2 \Pi_2^{UP*}/\partial l \partial m_2$ cannot be signed, therefore l and m_2 can change in the same or opposite directions if m_1 increases. Luckily our results do not depend on pinning down the precise nature of this interaction.

The first term in (33) is positive and accounts for the lessening of competitive intensity caused by the entrant's higher uniform price. Therefore, if duopoly areas are covered, the entrant makes (local) losses in his highest-cost monopoly markets. The direction of causality is interesting: It is not the case that the entrant "must" charge higher prices to cover monopoly losses, but by covering high-cost areas he commits himself to charging higher prices where he competes.

Because of the presence of the strategic effect the comparison between m_2^{UP} and m_2^{NUP} does not allow for any clear conclusion. On the one hand, the gross profits in the last monopoly market are smaller, indicating a lower monopoly coverage. On the other hand, the strategic "fat cat" effect works in the opposite direction. Without further assumption it is not clear which effect dominates.

Furthermore, the dependence of m_2^{UP} on m_1 in general is ambiguous because of the effect that m_1 has on equilibrium prices:

$$\frac{\partial^2 \Pi_2^{UP*}}{\partial m_1 \partial m_2} = \frac{\partial}{\partial m_1}\left[\int_0^{l^{UP}} \frac{\partial \pi_2}{\partial p_1} dx \frac{\partial P_1^*}{\partial m_2}\right] + \frac{\partial \pi_{2m}\left(P_2^*, m_2^{UP}\right)}{\partial p_2} \frac{\partial P_2^*}{\partial m_1} \qquad (35)$$

It is not possible to sign the first term without imposing additional assumptions on payoffs, while the second term is positive if l^{UP} is small compared to $\left(m_2^{UP} - m_1\right)$. Still, at $l^{UP} = 0$ the first term disappears, and we obtain $dm_2^{UP}/dm_1 > 0$: An increase in m_1 increases the average cost of monopoly markets covered by the entrant because some of the "cheapest" are occupied by the entrant. Therefore his uniform monopoly price increases, allowing him to cover additional monopoly markets.

As concerns duopoly coverage, the first term in (34) is negative if l^{UP} is small. This term incorporates the increase in competitive intensity if the entrant's duopoly coverage is enlarged. As a result, the profitability of covering at least the first few duopoly markets is decreased by a strategic effect, and fewer markets will be covered. In general we cannot decide how l^{UP} depends on m_1, given that complicated price effects are involved.

On the other hand, if $l^{UP} = 0$ the first term disappears. As in the case of non-uniform pricing by the entrant, the incumbent's equilibrium price will then be $P_1^* = P_1^m(m_1)$, while the entrant's will be the monopoly uniform price $P_2^m(m_1, m_2)$ on $(m_1, m_2]$. At $l^{UP} = 0$ the dependence of l^{UP} on m_1 is described by

$$\frac{\partial^2 \Pi_2^{UP*}}{\partial m_1 \partial l} = \frac{\partial \pi_2 \left(P_2^*, P_1^*, 0\right)}{\partial p_1} \frac{\partial P_1^*}{\partial m_1} > 0. \tag{36}$$

By continuity this continues to hold for sufficiently small l^{UP}, i.e. if l^{UP} is small then it is increasing in m_1.

No duopoly entry is an equilibrium choice if and only if

$$Z_2^{UP}(m_1; F_2) = \pi_2 \left(P_2^m \left(m_1, m_2^{UP}(m_1; F_2)\right), P_1^m(m_1), 0\right) \leq F_2. \tag{37}$$

Contrary to Z_2^{NUP}, now Z_2^{UP} also depends on monopoly coverage m_2^{UP} through the entrant's uniform price P_2^m. Covering high-cost monopoly markets distorts the uniform price upwards and reduces the profitability of duopoly entry. Therefore for all (m_1, F_2) with $m_2^{UP} > m_1$ we have

$$Z_2^{UP}(m_1; F_2) < Z_2^{NUP}(m_1). \tag{38}$$

The comparison between Z_2^{UP} is complicated by the facts that Z_2^{UP} itself depends on F_2 and that it may not be everywhere increasing in m_1. We can get around this problem by defining

$$M_1^{UP}(F_2) = \min\left\{m_1 \geq 0 \,|\, Z_2^{UP}(m_1; F_2) = F_2\right\} \tag{39}$$

as the minimum incumbent's coverage below which the entrant cannot recover fixed cost in market zero. The function M_1^{UP} may not be continuous, but this is irrelevant in the following analysis.

Finally, if $m_2^{UP} = m_1$ then $P_2^* = r_2(P_1^m(m_1), 0)$ and $Z_2^{UP} = Z_2^{NUP}$.[5] This implies that the limits on fixed cost F_2 which delineate the cases where coverage affects entry are the same as above.

We thus arrive at the following results:

Lemma 3 *If a uniform pricing constraint is imposed on the entrant, then:*

1. *The entrant covers all monopoly markets between m_1 and $m_2^{UP}(m_1; F_2)$. The latter limit increases with m_1 if duopoly coverage is small.*

2. *If $F_2 \in \left(Z_2^{NUP}(0), Z_2^{NUP}(\infty)\right)$, then there is no duopoly entry if $m_1 < M_1^{UP}(F_2)$. The optimal coverage $l^{UP}(m_1; F_2)$ increases in m_1 if l^{UP} is small.*

[5] Strictly speaking P_2^* is discontinuous at $(l, m_2) = (0, m_1)$, with values between the duopoly best response and a monopoly price. Here we presume that the entrant is only considering duopoly entry.

Now we consider a binding price cap on the incumbent. The entrant's first-order conditions for the optimal choice of coverage (33) and (34) simplify to

$$\frac{\partial \Pi_2^{PC*}}{\partial m_2} = \pi_{2m}\left(P_2^*, m_2^{PC}\right) - F_2 = 0,\ \frac{\partial \Pi_2^{PC*}}{\partial l} = \pi_2\left(P_2^*, \hat{p}, l^{PC}\right) - F_2 = 0. \quad (40)$$

If $m_2^{PC} > m_1$ then again $\pi_2\left(P_2^m, \hat{p}, 0\right) < Z_2^{UP}(m_1)$, and the price cap reduces the profitability of duopoly entry. This time, though, the incumbent's coverage m_1 continues to decrease the profitability of duopoly entry, through the cost effect on the entrant's monopoly price P_2^m: While $m_2^{PC} > m_1$,

$$\frac{\partial^2 \Pi_2^{PC*}}{\partial l \partial m_1} = \frac{\partial \pi_2\left(P_2^m, \hat{p}, 0\right)}{\partial p_2}\left(\frac{\partial P_2^m}{\partial m_1} + \frac{\partial P_2^m}{\partial m_2}\frac{dm_2^{PC}}{dm_1}\right) < 0. \quad (41)$$

Finally we return to the effects of a coverage constraint on the entrant. Under uniform pricing the effects are more involved since strategic effects are spread out over both duopoly and monopoly areas. Moreover, the entrant's choice of duopoly coverage now is intertwined with monopoly coverage.

Corollary 2 *If a uniform pricing constraint is imposed on the entrant, then:*

1. *If the entrant's coverage constraint applies only to monopoly areas then the effect on equilibrium duopoly coverage and prices is ambiguous.*

2. *If the entrant's coverage constraint applies to all markets, and is binding, then equilibrium duopoly prices decrease in* $\hat{m}_2$ *if the entrant's duopoly coverage is small. In this case the entrant's coverage* $\left(l^{C,UP}, m_2^{C,UP}\right)$ *is defined by the condition* $\partial \Pi_2^{UP*}/\partial m_2 = \partial \Pi_2^{UP*}/\partial l < 0$.

Proof. The first statement follows from the observation that we cannot determine how duopoly coverage $l^{C,UP}$ changes with m_2 under uniform pricing. The proof of the second statement is the same as in the previous corollary. The only difference is that we can only state with certainty that duopoly prices are decreasing while l^{UP} is small enough. ■

5.2 The Incumbent's Coverage Choice

The main result we can derive from the previous discussion concerns duopoly entry with or without uniform pricing on the entrant. It highlights the strategic impact of the incumbent's coverage:

Proposition 3 *If the entrant covers monopoly markets, the critical value of the incumbent's coverage below which there is no duopoly entry is higher if a uniform pricing constraint is imposed on the entrant:* $M_1^{UP}(F_2) > M_1^{NUP}(F_2)$.

Proof. The statement follows directly from (38). ■

That is, if the incumbent's coverage is between $M_1^{NUP}(F_2)$ and $M_1^{UP}(F_2)$, and if the entrant finds it profitable to enter monopoly markets in equilibrium, then he would enter duopoly areas under non-uniform pricing, but would not do so if subject to uniform pricing. This does not only mean that duopoly entry is less likely, but also that the incumbent may have even have strategic incentives to choose a low coverage to avoid subsequent duopoly entry. We will show that both is indeed the case.

First we consider the incumbent's choice of coverage m_1, taking into account that the entrant's coverage will depend on it. Denote the entrant's equilibrium prices by $p_2^*(m_1, x)$. This is equal to $r_2(P_1^*, x)$ with non-uniform pricing, and equal to P_2^* otherwise. The entrant's coverage levels are $m_2^*(m_1; F_2)$ and $l^*(m_1; F_2)$. For now assume that there is duopoly entry, i.e. $l^* > 0$. If no coverage constraint is imposed on him, the incumbent maximizes over $m_1 \geq 0$

$$\begin{aligned}\Pi_1^{**}(m_1) &= \Pi_1^*(l^*(m_1), m_1, m_2^*(m_1)) \\ &= \int_0^{l^*} \pi_1(P_1^*, p_2^*(m_1, x), x)\,dx + \int_{l^*}^{m_1} \pi_{1m}(P_1^*, x)\,dx - m_1 F_1.\end{aligned} \tag{42}$$

The first-order condition for profit-maximizing coverage choice is

$$\begin{aligned}\frac{d\Pi_1^{**}}{dm_1} &= [\pi_{1m}(P_1^*, m_1) - F_1] + \int_0^{l^*} \frac{\partial \pi_1}{\partial p_2} \frac{\partial p_2^*}{\partial m_1} dx \\ &\quad + [\pi_1(P_1^*, p_2^*, l^*) - \pi_{1m}(P_1^*, l^*)] \frac{dl^*}{dm_1}\end{aligned} \tag{43}$$

Only the first term corresponds to the profits of the marginal monopoly market in the monopolist's first-order condition (5). The other two are strategic effects, both of which disappear if m_1 is so small that there is no duopoly entry.

The second term indicates the effect of coverage on competitive intensity. It is ambiguous because it is not clear whether the total effect of m_1 on the entrant's price is positive or negative. This price decreases in $l^*(m_1)$ and increases in m_1 under non-uniform pricing for the entrant, or uniform pricing if $l^*(m_1)$ is small. At $l^* = 0$ this term disappears, though.

The third term takes account of the shift in the entrant's duopoly coverage. Generically, the direction of this shift cannot be determined without making further assumptions. Still, if l^* is small then $dl^*/dm_1 > 0$ under both uniform and non-uniform pricing, so this term is negative: Some monopoly markets are lost, leading to lower optimal coverage by the incumbent.

Our aim is to determine conditions under which the optimal choice of the incumbent's coverage implies that the entrant does not enter duopoly areas. Remember that $\tilde{m}_1^{UP}(F_1)$ is the monopolist's coverage choice under uniform pricing, which is decreasing in F_1. Let $\tilde{F}_1^{NUP} = \left(\tilde{m}_1^{UP}\right)^{-1} \circ M_1^{NUP}$ and $\tilde{F}_1^{UP} =$

$\left(\tilde{m}_1^{UP}\right)^{-1} \circ M_1^{UP}$. Both expressions define limits on the incumbent's fixed costs, and by definition we have $\tilde{F}_1^{NUP} \geq \tilde{F}_1^{UP}$. Our main result is the following:

Proposition 4 *Assume that no coverage constraint is imposed on the incumbent.*

1. *If $F_1 \geq \tilde{F}_1^{NUP}(F_2)$ then there is no duopoly entry under both uniform or non-uniform pricing on the entrant.*

2. *If $F_1 \in \left[\tilde{F}_1^{UP}(F_2), \tilde{F}_1^{NUP}(F_2)\right)$ then there is no duopoly entry if uniform pricing is imposed on the entrant.*

3. *The incumbent may reduce his coverage, as compared to the absence of entry, in order to avoid duopoly entry:*

   (a) *Under non-uniform pricing on the entrant, there is $F_1^{NUP} < \tilde{F}_1^{NUP}(F_2)$ such that for $F_1 \in \left[F_1^{NUP}, \tilde{F}_1^{NUP}(F_2)\right)$ the incumbent's optimal coverage is $\tilde{m}_1^{UP}(F_1) = M_1^{NUP}(F_2)$.*

   (b) *Under uniform pricing on the entrant, there is $F_1^{UP} < \tilde{F}_1^{UP}(F_2)$ such that for $F_1 \in \left[F_1^{UP}, \tilde{F}_1^{UP}(F_2)\right)$ the incumbent's optimal coverage is $\tilde{m}_1^{UP}(F_1) = M_1^{UP}(F_2)$.*

4. *If $F_1 < F_1^{UP}$ then there will always be duopoly entry.*

Proof. The first statement follows from the definition of $M_1^{NUP}(F_2)$, because $F_1 \geq \tilde{F}_1^{NUP}(F_2)$ is equivalent to $\tilde{m}_1^{UP}(F_1) \leq M_1^{NUP}(F_2)$. In a similar vein the second statement follows from the definition of $M_1^{UP}(F_2)$.

Now consider the third statement, using the first-order condition (43). At $m_1 = M_1^{NUP}(F_2)$ and $m_1 = M_1^{UP}(F_2)$ the entrant's choice is $l^* = 0$, under non-uniform or uniform pricing, respectively. The limit of the coverage first-order condition at $l^* \searrow 0$, i.e. either $F_1 \nearrow \tilde{F}_1^{NUP}(F_2)$ or $F_1 \nearrow \tilde{F}_1^{UP}(F_2)$, becomes

$$\lim_{l^* \searrow 0} \frac{d\Pi_1^{**}}{dm_1} = [\pi_{1m}(P_1^m, m_1) - F_1] + [\pi_1(P_1^m, p_2^*, 0) - \pi_{1m}(P_1^m, 0)] \frac{dl^*}{dm_1} \quad (44)$$

The first term is zero at $\tilde{m}_1^{UP}(F_1)$, while the second term is negative. Therefore $\lim_{l^* \searrow 0} d\Pi_1^{**}/dm_1$ is negative, which by continuity continues to hold for slightly smaller $F_1 \geq F_1^{NUP}$ and $F_1 \geq F_1^{UP}$. Note, though, that $d\Pi_1^{**}/dm_1$ is discontinuous at $l^* = 0$ itself since the second term does not exist for lower m_1. As a result, the incumbent's objective function has a kink at $l^* = 0$ where its slope decreases. For $F_1 \geq F_1^{NUP}$ and $F_1 \geq F_1^{UP}$ the maximum remains at the kink, while for smaller values of F_1 it continues to move towards larger m_1.

Finally, the last statement follows from the definition of F_1^{UP}. ■

Naturally, a sufficiently high coverage constraint on the incumbent makes the entrant enter in low-cost areas:

Proposition 5 *If a coverage constraint $m_1 \geq \hat{m}_1$ is imposed on the incumbent, then it guarantees duopoly entry only if*

1. *$\hat{m}_1 > M_1^{NUP}(F_2)$ if the entrant is not subject to a uniform pricing constraint;*

2. *$\hat{m}_1 > M_1^{UP}(F_2) > M_1^{NUP}(F_2)$ if the entrant is subject to a uniform pricing constraint.*

That is, a coverage constraint on the incumbent that is meant to encourage duopoly entry must be larger if a uniform pricing constraint is imposed on the entrant.[6]

The effect of this coverage constraint on equilibrium prices depends on how it is. If $\hat{m}_1$ just allows for entry in duopoly markets then equilibrium prices will decrease. On the other hand, if $\hat{m}_1$ largely exceeds this value then two effects contribute to a rise in prices: First, the incumbent's average cost increases. Second, as the entrant's duopoly coverage becomes large competitive intensity in the duopoly markets decreases. Therefore welfare are generically ambiguous.

6 CONCLUSIONS

We have considered a model of entry into a series of markets with decreasing profitability. Firms in these markets are subject to regulatory intervention such as uniform pricing constraints, coverage constraints and price caps. Their effects cannot be judged in isolation since constraints on prices have effects on coverage and *vice versa*. In particular uniform pricing constraints strongly affect market outcomes since they strategically link otherwise independent local markets.

In emerging markets such as broadband services in telecommunications there is a real possibility that entry into these markets is not "competitive". Rather, entrants install their capacity in areas not yet served by any of their competitors. The result is that the opening of the market to competition gives rise to a series of neighboring monopolies rather than competition for consumers. It is even possible that the incumbent strategically reduces his coverage to remain "lean and hungry" and discourage competitive entry.

[6] The same is true if the incumbent's network already has significant historical coverage. In this case the regulator should make sure that the incumbent effectively serves all covered local markets, in order to reproduce the effects of a coverage constraint.

In this paper we have concentrated on the imposition of a uniform pricing constraint on the entrant, and how it shapes equilibrium prices and coverage decisions. We have found that imposing a uniform pricing constraint on the entrant increases the likelihood of this phenomenon. This can be counterbalanced by a sufficiently high coverage constraint on the incumbent, but its welfare effects are unclear.

A strength of our approach is that we have used a very generic formulation of firms' profits, and did not assume any kind of symmetry on demand or cost. The assumptions imposed on these profits simply translate the intuitions of differentiated goods price competition and cost differences between urban and rural markets. This means that any more specific results will be due to more specific assumptions, and maybe not true in general.

References

[1] Anton, James J., James H. Vander Weide, and Nikolaos Vettas. 2002. "Entry Auctions and Strategic Behavior under Cross-Market Price Constraints." *International Journal of Industrial Organization* 20(5):611–629.

[2] Armstrong, Mark and John Vickers. 1993. "Price Discrimination, Competition and Regulation." *The Journal of Industrial Economics* 41(4):335–359.

[3] Bourguignon, Hélène and Jorge Yáñez. 2003. "Skimming the others' cream: competitive effects of an asymmetric universal service obligation." CREST Working Paper 2003-43.

[4] Choné, Philippe, Laurent Flochel, and Anne Perrot. 2000. "Universal Service Obligations and Competition." *Information Economics and Policy* 12(3):249–259.

[5] Crandall, Robert W. and Leonard Waverman. 2000. *Who Pays for Universal Service: When Telephone Subsidies Become Transparent*. Washington DC: Brookings Institution Press.

[6] Faulhaber, Gerald R. and Christiaan Hogendorn. 2000. "The market structure of broadband telecommunications." *Journal of Industrial Economics* 48(3):305–329.

[7] Foros, Øystein and Hans J. Kind. 2003. "The Broadband Access Market: Competition, Uniform Pricing and Geographical Coverage." *Journal of Regulatory Economics* 23(3):215–235.

[8] Fudenberg, Drew and Jean Tirole. 1984. "The Fat-Cat Effect, The Puppy-Dog Ploy, and the Lean and Hungry Look." *American Economic Review*, Papers and Proceedings, 74(2):361-366.

[9] Hoernig, Steffen H. 2002. "Should Uniform Pricing Constraints be Imposed on Entrants?" Universidade Nova de Lisboa Economics Working Paper.

[10] Laffont, Jean-Jacques and Jean Tirole. 2000. *Competition in Telecommunications*. Cambridge, MA: MIT Press.

[11] Valletti, Tommaso M., Steffen H. Hoernig, and Pedro P. Barros. 2002. "Universal Service and Entry: The Role of Uniform Pricing and Coverage Constraints." *Journal of Regulatory Economics* 21(2):169–190.

Appendix

Proof of Lemma 1:

Proof. For any coverage parameter $k \in \{l, m_1, m_2\}$ we have

$$\frac{\partial P_i^*}{\partial k} = -\frac{\partial T_i/\partial k}{\partial T_i/\partial P_i^*},$$

where T_i was defined by (20) – (22). The equilibrium prices $p_2^*(x) = r_2(P_1^*, x)$ of the entrant when no uniform pricing constraint is imposed on him increase if and only if P_1^* increases. Therefore they are not mentioned below.

The equilibrium with uniform pricing on both firms is stable if and only if

$$\begin{aligned}\frac{\partial T_1^{UP}}{\partial P_1^*} &= \int_0^l \left[\frac{\partial^2 \pi_1}{\partial p_1^2} + \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2}\frac{\partial R_2}{\partial P_1}\right] dx + \int_l^{m_1} \frac{\partial^2 \pi_{1m}}{\partial p_1^2} dx \\ &= \frac{\partial^2 \Pi_1}{\partial P_1^2} + \int_0^l \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} dx \frac{\partial R_2}{\partial P_1} < 0\end{aligned}$$

Since the last term is positive, this condition is stricter than the second-order condition, as usual. The same holds for T_1^{NUP} and T_2, while for T_2^{PC} stability is equivalent to the second-order condition:

$$\begin{aligned}\frac{\partial T_1^{NUP}}{\partial P_1^*} &= \frac{\partial^2 \Pi_1}{\partial P_1^2} + \int_0^l \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2}\frac{\partial r_2}{\partial P_1} dx < 0, \\ \frac{\partial T_2}{\partial P_2^*} &= \frac{\partial^2 \Pi_2}{\partial P_2^2} + \int_0^l \frac{\partial^2 \pi_2}{\partial p_2 \partial p_1} dx \frac{\partial R_1}{\partial P_2} < 0, \\ \frac{\partial T_2^{PC}}{\partial P_2^*} &= \frac{\partial^2 \Pi_2}{\partial P_2^2} < 0.\end{aligned}$$

Therefore in all cases the sign of $\partial P_i^*/\partial k$ equals that of the denominator $\partial T_i/\partial k$.

Now we will consider the effects of l on equilibrium uniform prices:

$$\frac{\partial T_1^{UP}}{\partial l} = \int_0^l \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} dx \frac{\partial R_2}{\partial l} + \left[\frac{\partial \pi_1}{\partial p_1}(P_1^*, P_2^*, l) - \frac{\partial \pi_{1m}}{\partial p_1}(P_1^*, l)\right]$$

The term in brackets is negative, while the first term is negative if $x^d > l$, i.e. if l is small compared to $(m_2 - m_1)$. Without the uniform pricing constraint,

$$\frac{\partial T_1^{NUP}}{\partial l} = \frac{\partial \pi_1}{\partial p_1}(P_1^*, p_2^*(l), l) - \frac{\partial \pi_{1m}}{\partial p_1}(P_1^*, l) < 0.$$

As concerns the entrant,

$$\begin{aligned} \frac{\partial T_2}{\partial l} &= \int_0^l \frac{\partial^2 \pi_2}{\partial p_2 \partial p_1} dx \frac{\partial R_1}{\partial l} + \frac{\partial \pi_2}{\partial p_2} \left(P_2^*, P_1^*, l \right), \\ \frac{\partial T_2^{PC}}{\partial l} &= \frac{\partial \pi_2}{\partial p_2} \left(P_2^*, \hat{p}, l \right) \end{aligned}$$

The first term above is always negative, while $\partial \pi_2 / \partial p_2$ is again negative if $x^d > l$.

The effects of the incumbent's coverage are the following:

$$\begin{aligned} \frac{\partial T_1^{UP}}{\partial m_1} &= \int_0^l \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} dx \frac{\partial R_2}{\partial m_1} + \frac{\partial \pi_{1m}}{\partial p_1} \left(P_1^*, m_1 \right), \ \frac{\partial T_1^{NUP}}{\partial m_1} = \frac{\partial \pi_{1m}}{\partial p_1} \left(P_1^*, m_1 \right) > 0 \\ \frac{\partial T_2}{\partial m_1} &= \int_0^l \frac{\partial^2 \pi_2}{\partial p_2 \partial p_1} dx \frac{\partial R_1}{\partial m_1} - \frac{\partial \pi_{2m}}{\partial p_2} \left(P_2^*, m_1 \right), \ \frac{\partial T_2^{PC}}{\partial m_1} = -\frac{\partial \pi_{2m}}{\partial p_2} \left(P_2^*, m_1 \right) \end{aligned}$$

The expressions $\partial R_2 / \partial m_1$ and $(-\partial \pi_{2m} / \partial p_2)$ are positive if $x^m > m_1$, i.e. if l is small as compared to $(m_2 - m_1)$, while all the other terms are positive.

Finally, a higher m_2 unambiguously increases all prices or leaves them unchanged:

$$\begin{aligned} \frac{\partial T_1^{UP}}{\partial m_2} &= \int_0^l \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} \frac{\partial R_2}{\partial m_2} dx > 0, \ \frac{\partial T_1^{NUP}}{\partial m_2} = 0, \\ \frac{\partial T_2}{\partial m_2} &= \frac{\partial T_2^{PC}}{\partial m_2} = \frac{\partial \pi_{2m}}{\partial p_2} \left(P_2^*, m_2 \right) > 0. \ \blacksquare \end{aligned}$$