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ABSTRACT

International Interdependencies in Fiscal Stabilization Policies*

Trade links imply that business cycle fluctuations are transmitted to trade partners. To the extent that fiscal policy can mitigate business cycle fluctuations this implies that there are international interdependencies in stabilization policies. We analyse the role of fiscal policy in mitigating risk or providing implicit insurance in the presence of capital market imperfections, and how this is affected by adjustment failures (rigid wages). It is shown that there is a welfare case for an active stabilization policy and that it is larger in the presence of adjustment failures (rigid wages). The international interdependency may, in the absence of adjustment failures, imply that non-cooperative stabilization policies entail excessive stabilization, whereas there is always insufficient stabilization in the presence of adjustment failures.

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1 Introduction

International recessions are often accompanied by calls for active stabilization policy being coordinated across countries. The academic debate on stabilization policy also seems to have its own cycle, and after a period in which stabilization policy has been less in vogue there is an increasing interest in the need and scope for stabilization policy (see e.g. Seidman (2001)). The debate has in particular focussed on international aspects of stabilization policy raising questions on the scope for active stabilization policies in increasingly open economies, and the interdependencies in policies across countries.

Most of the recent literature within the so-called "New Open Macroeconomics" has focussed on the role of monetary policy (see e.g. Obstfeld and Rogoff (2002)). Likewise there is a large literature on the design of monetary policy rules (see e.g. Svensson (1997) and Gali (2002)). However, very little research has been made on fiscal policy and its potential stabilizing role. This is surprising given that it is commonly accepted that fiscal policy – in particular through so-called automatic stabilizers – can play an important role for business cycle fluctuations (see van der Noord (2000)). In policy debates reference is also often made to the importance of automatic stabilizers (see e.g. ECB (2001)).

The aim of this paper is to address the issue of fiscal stabilization policies in open economies from the following perspective. First, a welfare case for active stabilization must rest on the fact that fiscal policy can diversify or cope with risks in a way which mitigates market failures in risk diversification (within or across countries). Second, shocks which are country specific in origin[1] are via trade transmitted to trading partners, which also implies that there are potential spillover effects of stabilization policies. Finally, adjustment failures in the form of rigidities in wage or price adjustment may have crucial effects for how the economy responds to shocks, and therefore also on the need and scope for an active stabilization policy.

We pursue this agenda in a setting of a two-country model with specialized production which gives a basic reason for trade and thus a potential mechanism for transmission of shocks. The specific shocks considered are country-specific productivity (supply) shocks. To focus on the basic effects of policy in relation to risk we use a static model since absence of financial markets automatically precludes an important mechanism for risk diversification (a similar approach is adopted by Obstfeld and Rogoff (2002)) Although taking the case of imperfect capital markets to the limit, it has the advantage of simplifying an analysis which even in the static case turns out to be fairly complicated[2]. The crucial assumption is that capital markets do not make it possible to completely diversify consumption risk via trade in equities or state contingent securities. That

[1] There may also be a need to provide risk diversification to aggregate or common global shocks. We focus here on country-specific shocks to address the issue of international interdependencies in stabilization policy.

[2] Actually, this assumption increases the costs of an active stabilization policy since public consumption cannot be smoothed over time. In general different contingencies in taxation and public consumption would be optimal.

risk diversification falls short of being complete is supported by both explicit intertemporal models with incomplete capital markets (see e.g. Obstfeld and Rogoff (1996) and empirical analysis (see e.g. Lewis (1999)). With insufficient risk diversification via private capital markets it is a nontrivial question whether an active stabilization policy can improve upon risk diversification[3], especially when policy makers cannot directly overcome capital market failures (that is, offer financial intermediation which private markets cannot).

The analysis is made in two steps. First we consider the case of competitive markets in which case the only rationale for an active stabilization policy is the ability to improve on risk diversification, since there are no adjustment failures in the response to shocks. Second, we introduce adjustment failures by assuming rigid wages. Specifically, this is done by introducing market power in the labour market in the form of labour unions, and rigidities arise under a contract structure stipulating that wages are determined prior to the realization of shocks. In this case an active stabilization policy involves a concern both for the adjustment to shocks and the diversification of shocks.

The issue of international interdependencies in fiscal policy has a long tradition within economic theory. According to simple Keynesian reasoning there would be a tendency that countries choose insufficiently expansionary fiscal policies since the demand leakage reduces the expansionary domestic effects of fiscal policies (see e.g. Cooper (1985) and Hamada (1985)). This line of reasoning has often motivated proposals for coordinated fiscal expansions intended to overcome free rider problems in policies oriented towards output and employment. This view has been contested on two related grounds, namely, the usual problems associated with the theoretical foundation of Keynesian models and the fact that policy evaluations are not based on an explicit welfare analysis but instead rely on arbitrary policy objective function.

A number of authors have considered international interdependencies in fiscal policy in explicitly formulated general equilibrium models, and have addressed the issue of cooperative vs non-cooperative policy making from an explicit welfare approach. The standard set-up has featured specialized production, that is, countries specialize in production of specific commodities which they trade with each other. One surprising finding in these models is that fiscal policies tend to be too expansionary when comparing the non-cooperative to the cooperative policy outcome (see e.g. Chari and Kehoe (1990), Devereux (1991), Turnovsky (1988) and van der Ploeg (1987, 1988)). The reason is a term-of-trade or "beggar thy neighbor" effect. Fiscal policy in the form of demand for domestically produced goods tends to shift demand from foreign to domestic products, which in turn improves the terms of trade and thus the real income of the home country. No such terms-of-trade effect arises in the cooperative case, and therefore non-cooperative policies tend to be too expansionary[4]. This result

[3] One example of how the public sector can provide risk diversification better than the market arises in an overlapping generations model where the horizon of agents constrained risk diversification via markets is limited, see Andersen and Dogonowski (2002). See also Thomas (1995) for an analysis of fiscal policy in a setting with an incomplete market structure.

[4] Irrespective of whether the policy in absolute terms is expansionary or contractionary.

holds even when the employment level is inefficiently low due to e.g. imperfectly competitive markets (Andersen, Rasmussen and Sørensen (1996)). Likewise the terms-of-trade effect implies that the optimal setting of tax rates should aim at twisting demand towards domestically produced goods (See Holmlund and Kolm (2002), Lockwood (2000)).

However, the above-mentioned references deal with the level of public consumption and the optimal tax structure but not stabilization policy in the sense of responses to shocks. The relevance of the findings for stabilization policies is therefore an open question. We show that the explicit introduction of shocks and a consideration of fiscal stabilization policy as a way to diversify shocks or provide implicit (social) insurance are important. Under the same trade structure[5] (induced by specialized production) as in the references given above we find that there are interdependencies in stabilization policy which under flexible wages may imply that non-cooperative policies may entail excessive stabilization, whereas they always imply insufficient stabilization under rigid wages.

The paper is organized as follows: Section 2 develops the stochastic two-country model, and characterizes the equilibrium. Section 3 develops the welfare rationale for an active stabilization policy, and introduces state contingencies in fiscal policy. Section 4 analyses the effects of an active fiscal stabilization policy in the case of competitive product and labour markets, and considers optimal policies and international interdependencies. Section 5 introduces rigidities in wage adjustment, and considers the implications for policy making and policy spillovers. Some numerical illustrations are provided in section 6 to shed some light on the quantitative importance of the mechanisms analysed. Section 7 offers some concluding comments.

2 Two-country model

Consider a static two country model. The two countries specialize in production of different commodities which are tradeables. To focus on the interaction in fiscal stabilization policy, monetary aspects are disregarded[6]. The two countries are symmetric and all foreign variables are denoted by a *.

Households

The representative household has a utility function depending positively on the private consumption bundle (B), negatively on labour supply (L), and positively

[5] In the short-run perspective relevant for discussing stabilization policies it is reasonable to take the trade structure as given as in the specialized production model.

[6] One can think of countries in a monetary union, for which trade with the outside world is relatively unimportant. To a first approximation this characterizes e.g. the European Monetary Union which has substantial trade among european countries, but a consolidated trade share of GDP, which has remained steady at approximately 10% the last decades, see e.g. Coppel and Durand (1999).

on publicly provided goods and services (G), i.e.

$$U(B,L,G)=\frac{1}{1-\epsilon}B^{1-\epsilon}-\lambda L+\frac{\rho}{1-\gamma}G^{1-\gamma}\quad ;\quad \epsilon>0,\gamma>0 \tag{1}$$

where the private consumption bundle (B) is defined over the consumption of the domestic (C) and the foreign (C^*) commodity, respectively, i.e.

$$B=\frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}C^{\alpha}\left(C^*\right)^{1-\alpha}\quad ;\quad \alpha>\frac{1}{2}$$

The assumption that $\alpha>\frac{1}{2}$ captures a home-bias in preferences, that is, the preference for domestically produced goods is (in the aggregate) always stronger than for foreign produced goods[7]. If I denotes disposable income it follows that demands are given as

$$\begin{aligned} C &= \alpha\frac{I}{P} \\ C^* &= \left(1-\alpha\right)\frac{I}{P^*} \end{aligned}$$

The private consumption bundle can thus be written

$$B=\frac{I}{Q}$$

where Q denotes the consumer price index defined as[8]

$$Q=P^{\alpha}\left(P^*\right)^{1-\alpha}$$

and disposable private income is

$$I=\left(WL+\Pi\right)\left(1-t\right)$$

Finally, labour supply is found to be determined by the first order condition

$$B^{-\epsilon}\frac{W}{Q}\left(1-t\right)=\lambda \tag{2}$$

Firms

The representative domestic firm produces subject to a production function

$$Y=Z\frac{1}{\beta}L^{\beta}\quad ,\quad \beta<1$$

[7] If this assumption is not fulfilled, a domestic productivity shock reduces the terms of trade, and vice versa for domestic productivity shocks. This case is implausible and therefore ruled out.

[8] The foreign price index is given as $Q^*=\left(P^*\right)^{\alpha}P^{1-\alpha}$.

where L denotes labour input, and Z is the productivity variable. It is assumed that $\frac{Z}{\overline{Z}}$ (current productivity Z relative to its steady state or trend value $\overline{Z}$) is log normally distributed $N(0, \sigma^2)$ (productivity shocks are uncorrelated across countries). Firms are price and wage takers, and hence, labour demand is given by

$$ZL^{\beta-1} = \frac{W}{P} \tag{3}$$

Public sector

The public sector provides public goods and services to households which are produced by use of the domestic good as an input. This captures the stylized fact that the larger part of public consumption is directed towards domestic markets[9]. For simplicity the production function is assumed linear (output = input = G). Public activities are financed by an income tax. i.e.

$$G = tY$$

where t is the tax rate. In real terms we have that public consumption equals the revenue ($G = R \equiv tY$).

Equilibrium

In equilibrium we have that disposable income is given as

$$I = (WL + \Pi) - T = P(Y - G)$$

Hence, the equilibrium condition for the domestic product market or the aggregate demand relation reads

$$Y = \alpha\,(Y - G) + (1 - \alpha)\,\Gamma\,(Y^* - G^*) + G$$

where $\Gamma \equiv \frac{P^*}{P}$ denotes the terms of trade. The aggregate demand relation can be written

$$\Gamma = \frac{Y - G}{Y^* - G^*} \tag{4}$$

Combining the labour demand (3) and the labour supply relation (2) we have the following aggregate supply relation

$$\Gamma^{(\alpha-1)(1-\epsilon)} Z^{1+\frac{1-\beta}{\beta}} Y^{\frac{-1}{\beta}} \left(Y - G\right)^{1-\epsilon} = \lambda\beta^{\left(\frac{1-\beta}{\beta}\right)} \tag{5}$$

[9] For all OECD countries the larger part of public spending arises via public employment which by definition is biased towards domestic markets. Formulation of public demand either as demand for products or labour does not change anything qualitatively.

For later reference it is useful to formulate the model in logs, and defining $x \equiv \log(\frac{X}{\overline{X}})$[10] we have that[11]

$$y = \eta_{sz} z + \eta_{s\tau} \tau + \eta_{sg} g$$

where

$$\begin{aligned} \eta_{sz} &\equiv (1 - \beta\mu(1-\epsilon))^{-1} > 0 \\ \eta_{s\tau} &\equiv (1 - \beta\mu(1-\epsilon))^{-1} \beta(\alpha - 1)(1-\epsilon) \lesseqgtr 0 \\ \eta_{sg} &\equiv (1 - \beta\mu(1-\epsilon))^{-1} \beta(1-\mu)(1-\epsilon) \lesseqgtr 0 \end{aligned}$$

Considering the partial effects of the variables determining supply, we find that higher productivity always increases supply. An increase in the terms of trade increases (decreases) supply if $\epsilon > 1$ ($\epsilon < 1$) since the implied fall in private consumption induces an increase (decrease) in labour supply. Finally, an increase in public consumption and thus taxation induces a larger (smaller) supply if $\epsilon > 1$ ($\epsilon < 1$) because the income effect is dominating (dominated by) the substitution effect of a tax increase (see Baxter and King (1993) and Dixon (1987))[12]. Similarly, the foreign supply relation can be written

$$y^* = \eta^*_{sz} z^* + \eta^*_{s\tau} \tau^* + \eta^*_{sg} g^*$$

where

$$\begin{aligned} \eta^*_{sz} &\equiv (1 - \beta\mu^*(1-\epsilon))^{-1} > 0 \\ \eta^*_{s\tau} &\equiv (1 - \beta\mu^*(1-\epsilon))^{-1} \beta(1-\alpha)(1-\epsilon) \lesseqgtr 0 \\ \eta^*_{sg} &\equiv (1 - \beta\mu^*(1-\epsilon))^{-1} \beta(1-\epsilon)(1-\mu^*) \lesseqgtr 0 \end{aligned}$$

Note that the coefficient to the terms of trade has the opposite sign to that of the home country, for the obvious reason that this is a relative price.

Finally, the aggregate demand relation (4) can be written (in log deviations from steady state)

$$\tau = \mu y + (1-\mu) g - \mu^* y^* - (1-\mu^*) g^* \tag{6}$$

3 Stabilization policy

Two issues are of importance here, namely, why there might be a welfare rationale for pursuing an active stabilization policy, and how this can be achieved via fiscal policy.

[10] For the terms of trade we define $\tau \equiv \log\left(\frac{\Gamma}{\overline{\Gamma}}\right)$

[11] Use has been made of the approximation

$$\log(Y - G) = \mu y + (1-\mu) g$$

where $\mu = \frac{\overline{Y}}{\overline{Y} - \overline{G}} > 1$.

[12] For $\epsilon > 1$ the income effect dominates the substitution effect in labour supply, and therefore a worsening of relative prices or an increase in distortionary taxation leads to an increase in labour supply, and therefore in turn output supply.

Risk and welfare

To address the rationale for an active stabilization policy consider the expected utility of households. Observe that the first order condition characterizing the labour supply decision (2), the labour demand (3) and the production function implies that the disutility of labour can be written

$$\lambda L = B^{-\epsilon}\frac{PY}{Q}\beta\left(1-t\right) = \beta B^{1-\epsilon}$$

Hence utility can be written

$$U(B,L,G) = \left(\frac{1}{1-\epsilon}-\beta\right)B^{1-\epsilon} + \frac{\rho}{1-\gamma}G^{1-\gamma} \quad ; \quad \epsilon > \beta^{-1}-1$$

Expected utility can now be written (see appendix A)

$$\begin{aligned} E[U(B,L,G)] \quad \simeq \quad & \left(\frac{1}{1-\epsilon}-\beta\right)\left(E(B)^{1-\epsilon}\exp\left(-\frac{\left(1-\epsilon\right)\epsilon}{2}Var(\log B)\right)\right) + \\ & \frac{\rho}{1-\gamma}\left(E(G)^{1-\gamma}\exp\left(-\frac{\left(1-\gamma\right)\gamma}{2}Var(\log G)\right)\right) \end{aligned}$$

Hence, there is aversion wrt to variations in both private and public consumption, i.e.

$$\frac{\partial E[U(B,L,G)]}{\partial Var(\log B)} < 0; \frac{\partial E[U(B,L,G)]}{\partial Var(\log G)} < 0$$

The aversion to variations in both private and public consumption is a straightforward implication of risk-aversion. In the present setting the assumed capital market structure precludes any possibility of insuring income risk, i.e. income fluctuations translate directly into variations in private consumption. Since households are averse to such fluctuations it follows that an active stabilization policy is potentially welfare improving if it can stabilize private consumption. However, such stabilization is not costless to the extent that it transfers risk from the private sector into the public sector. In the present case this arises via induced variations in public consumption, to which households are also averse. Hence, there is a potential welfare case to be made for an active stabilization policy, but the issue of optimal policy design is not trivial since there is a trade-off involved between risk in private and public consumption. Note that the case for an active stabilization policy does not arise because the public sector can access the capital market on better terms than the private sector.

Introducing contingencies in fiscal policies may in general affect both mean levels and variances of the endogenous variables (here of B and G), since these are non-linear functions of the stochastic state variables (see Appendix A). Since the main interest of this paper is to explore the effects on risk, and since it is difficult to obtain transparent analytical results in the general case it is assumed throughout that changes in the contingencies in fiscal policy are accompanied by changes in other tax instruments so as to keep mean levels of private and public consumption unchanged (see Appendix A for details). This corresponds to the procedure often used to eliminate level effects of imperfect competition (see e.g. Obstfeld and Rogoff (2002)).

It follows (see Appendix A) that maximizing expected utility, under the constraint that policies which leave expected levels of private and public consumption unchanged, is equivalent to minimizing a loss function of the form

$$\Omega = Var(b) + \varphi Var(g) \quad , \quad \varphi \geq 0 \tag{7}$$

where φ has the interpretation of giving the relative weight attached to fluctuations in consumption of public goods relative to fluctuations in the consumption of private goods.

Contingencies in fiscal policy

Turn next to how an active fiscal stabilization policy should be designed. For fiscal policy to have a stabilizing role in mitigating risk or affecting how risk is distributed between private and public consumption it is necessary that fiscal policy is state contingent, that is, variations in public sector activities (public consumption, taxation etc.) should be contingent on the state of nature. In practice these contingencies arise via a variety of items on the expenditure and financing side of the public budget. In the present context we are addressing the basic question whether there is a welfare rationale for such contingencies, and we therefore adopt a very simple and stylized formulation by assuming a state-contingency for public (real) revenue of the form[13]

$$g = \kappa y \tag{8}$$

Policies in this (log) linear class are considered, and the policy problem is the choice of κ. Note that the parameter κ can be interpreted as capturing automatic stabilizers running via the public budget. Empirical evidence shows that the strongest contingencies are found on the revenue side (see e.g. van den Noord (2000)), and the formulation adopted above implies that tax revenue moves pro-cyclically with output (provided $\kappa > 0$). It is an implication of the static formulation adopted here that public consumption and revenues have to move synchronously over the business cycle.[14] Note for later reference that the variability in public consumption becomes

$$Var(g) = \kappa^2 Var(y)$$

[13] To ensure that public consumption (taxation) does not become negative it is required that $\kappa < \mu(1-\mu)^{-1}$.

[14] This biases the gains from active stabilization policy in a downward direction since it precludes smoothing of public consumption, that is, the formulation forces a variation in public consumption which is lowering expected utility, cf below. See e.g. Andersen and Dogonowski (2002) for an analysis where fiscal policy can improve welfare by diversifying shocks over time.

4 Flexible wages

In the case of flexible wages and the policy reaction function stipulated in (8) we have that there exists equilibrium of the form (neglecting constants unimportant for the adjustment to shocks, see appendix B)

$$\begin{aligned} \tau &= \phi_{\tau z} z + \phi_{\tau z^*} z^* \\ y &= \phi_{yz} z + \phi_{yz^*} z^* \\ b &= \phi_{bz} z + \phi_{bz^*} z^* \end{aligned}$$

Trade implies that both domestic and foreign supply shocks affect the equilibrium allocation. Table 1 summarizes information on the coefficients in the equilibrium relations given above, and how they depend on the domestic and foreign policy parameters.

Table 1: Coefficient signs and partial derivatives: Flexible wages

	$\epsilon < 1$			$\epsilon > 1$		
coefficient	sign	$\frac{\partial \phi_{ij}}{\partial \kappa}$	$\frac{\partial \phi_{ij}}{\partial \kappa^*}$	sign	$\frac{\partial \phi_{ij}}{\partial \kappa}$	$\frac{\partial \phi_{ij}}{\partial \kappa^*}$
$\phi_{\tau z}$	> 0	< 0	> 0	> 0	< 0	< 0
$\phi_{\tau z^*}$	< 0	< 0	> 0	< 0	> 0	> 0
ϕ_{yz}	> 0	< 0	< 0	> 0	> 0	< 0
ϕ_{yz^*}	> 0	< 0	< 0	< 0	< 0	> 0
ϕ_{bz}	> 0	< 0	< 0	> 0	< 0	> 0
ϕ_{bz^*}	> 0	< 0	< 0	> 0	> 0	< 0

The parameter ϵ capturing labour supply responses (see (5)) turns out to be critical for the way in which the two economies interact. Consider a domestic supply shock ($z > 0$). On impact this tends to increase domestic production and consumption and to lower the terms of trade. The transmission to foreign runs via the terms-of-trade effect, which induces a higher level of foreign consumption. This gain affects foreign labour supply. If the income effect dominates the substitution effect ($\epsilon > 1$) foreign labour supply falls, and therefore foreign output falls, and oppositely if the income effect is dominated by the substitution effect ($\epsilon < 1$) labour supply expands, and foreign output increases.

The interdependence in the response to shocks obviously translates into the transmission of stabilization policies. If the domestic economy pursues an aggressive stabilization policy (κ high) it follows that variations in domestic output release a stronger fiscal policy response. In the case $\epsilon > 1$ this implies that domestic production becomes more sensitive to both domestic and foreign shocks. The reason is that for $\epsilon > 1$ the elasticity of output wrt taxation is positive, so a procyclical fiscal policy amplifies the fluctuations in output through its effect on labour supply. If instead $\epsilon < 1$ an increase in the tax rate will reduce labour supply, implying that state contingent fiscal policy will have a stabilizing effect on output. However, a more aggressive fiscal policy in foreign affects the domestic economy in the sense of reducing the sensitivity of output to both domestic and foreign shocks. The intuition for this result is that with a higher κ^* there is less feedback from the foreign country in the adjustment process to the shock.

Crucial for welfare is how private consumption is affected. By letting the public sector absorb more resources in good than in bad states ($\kappa > 0$), an aggressive domestic stabilization policy cushions the effect of domestic shocks on domestic consumption. However, the international transmission of stabilization policies depends critically on the labour supply responses. In the case $\epsilon < 1$ more domestic stabilization (κ higher) unambiguously stabilizes consumption to both domestic and foreign shocks, and likewise more foreign stabilization (κ^* higher) contributes stability to both types of shocks. For $\epsilon > 1$ more domestic stabilization (κ higher) unambiguously stabilizes consumption to domestic shocks but increases the sensitivity to foreign shocks, and likewise more foreign stabilization (κ^* higher) contributes stability to foreign shocks, but adds to the sensitivity to domestic shocks.

Stabilization policies

As noted above stabilization policies should be evaluated in terms of their effects on the variability of private and public consumption. The key variance terms are thus

$$\begin{aligned} VAR(y) &= \left[(\phi_{yz})^2 + \left(\phi_{yz^*}\right)^2 \right] \sigma^2 \\ VAR(b) &= \left[(\phi_{bz})^2 + (\phi_{bz^*})^2 \right] \sigma^2 \end{aligned}$$

It follows directly from table 1 that the choice of the policy parameter (κ) affects the equilibrium distribution for the endogenous variables, e.g. terms of trade, output and private consumption. It is a straightforward implication that stabilization policy by affecting the risk profile of income and private consumption can affect welfare. Table 2 summarizes how the variability of output and private consumption depends on the policy parameters.

Table 2: Variability and policy parameters: Flexible wages

	$\epsilon < 1$		$\epsilon > 1$	
	κ	κ^*	κ	κ^*
$Var(y)$	< 0	< 0	> 0	< 0
$Var(b)$	< 0	< 0	< 0	$\lesseqgtr 0$

Domestic stabilization may decrease ($\epsilon < 1$) or increase ($\epsilon > 1$) output variability, whereas foreign stabilization unambiguously reduces output variability. Domestic stabilization unambiguously reduces consumption variability, which is also the case for foreign stabilization if $\epsilon < 1$, while the effect is ambiguous for $\epsilon > 1$. In the latter case we have

$$sign\left(\frac{\partial Var\,(b)}{\partial \kappa^*}\right) = sign\left(1 + \left(\frac{1}{\widehat{\mu}\beta\,(1-\epsilon)}\right)^2 - 2\alpha\right)$$

Hence, there exists an $\underline{\epsilon}$ such that $\frac{\partial Var(b)}{\partial \kappa^*} < 0$ for $\epsilon > \underline{\epsilon}$.

Since the choice of the fiscal stabilization parameter affects the risk profile of both private and public consumption, it is of interest to work out how the optimal policy should be designed. Moreover, with the risk profile of domestic private and public consumption being affected by the foreign policy choice there is an international interdependence in fiscal stabilization policies, which leads us to consider possible gains from fiscal policy cooperation.

Optimal Non-cooperative policies

The approach taken here is to consider optimal fiscal policies within the class specified by (8), that is, what value of the stabilization parameter κ should be chosen to maximize expected utility of the representative household? In choosing its policy, the domestic government takes foreign policies as given (the non-cooperative Nash case)

The problem facing the domestic government is thus to choose the stabilization parameter κ so as to minimize (7), implying that the optimal policy is characterized by the first order condition

$$\frac{\partial Var(b)}{\partial \kappa} + \varphi \left(\kappa^2 \frac{\partial Var(y)}{\partial \kappa} + 2\kappa Var(y) \right) = 0$$

An increase in the contingency parameter κ reduces the variability of the private consumption bundle, but at the costs of making public activities more variable. The latter effect is made up both of the direct consequence of a stronger contingency and the indirect effect arising because the variance of income and thus the tax base is affected.

The optimal stabilization policy implies that $\kappa > 0$. This is easily seen by observing that for $\kappa = 0$ we have a corner solution where all risk is absorbed by the private consumption bundle and none by public consumption, which is not optimal given that there is risk aversion wrt variations in both private and public consumption. Since an increase in κ shifts risk from the private consumption bundle to public consumption it follows that the optimal policy has $\kappa > 0$. Note, that this policy implies a procyclical variation in taxation - in good states of nature tax revenues are raised, and vice versa in bad states of nature. It is interesting that this "Keynesian"-type policy response arises as the optimal response to fluctuations induced by supply shocks[15].

Policy cooperation

Comparing the non-cooperative policies to the cooperative policies, it follows straightforwardly that the non-cooperative policies are inefficient. The reason is that the domestic stabilization policy has a spillover effect on foreign households,

[15] In the special case where $\varphi = 0$ we have that the optimal policy removes variability in the private consumption bundle by choosing $\kappa = 1 - \beta\mu(1 - \epsilon)$.

and this is not taken into account by non-cooperative policy makers. Specifically, we have

$$\frac{\partial L}{\partial \kappa^*} = \frac{\partial Var(b)}{\partial \kappa^*} + \varphi \kappa^2 \frac{\partial Var(y)}{\partial \kappa^*} \gtreqless 0$$

that is, the spillover effect involves both the effect on volatility of private consumption and the effect on the volatility of output (and therefore public consumption). Since for some parameter values the two are oppositely signed the net-effect is in general ambiguous (see e.g. Cooper and John (1988)), i.e.

$$\kappa^{non-cooperative} \lesseqgtr \kappa^{cooperative}$$

For $\epsilon < 1$ the spillover on the foreign loss function is negative, implying that non-cooperative policies entail too little stabilization ($\kappa^{non-cooperative} < \kappa^{cooperative}$). However, with the value of ϵ being sufficiently large and the weight to the variability in public consumption sufficiently small, the spillover effect on the foreign loss function is positive, implying that non-cooperative policies entail too much stabilization ($\kappa^{non-cooperative} > \kappa^{cooperative}$). This shows that if policy makers are sufficiently concerned (φ low) about stabilizing utility from private consumption (goods and leisure) it is possible that non-cooperative stabilization policies entail too much stabilization. This resembles the finding in deterministic models that public consumption in non-cooperative equilibrium is too high, cf the introduction.

5 Rigid wages and adjustment failures

The preceding analysis considered the role of an active stabilization policy in the absence of any adjustment failures to highlight the basic mechanisms through which stabilization policies may affect the risk profile of the endogenous variables and thus welfare. In this section we introduce failures in the market adjustment to shocks to address how this affects the need for stabilization policy and the international transmission of shocks. Specifically, we assume the labour market to be imperfectly competitive having unions determining wages under a contract structure stipulating one period wage contract under a right to manage structure (see e.g. Blanchard and Fischer (1989)). It is thus an implication that equilibrium employment is determined by labour demand at the pre-set real wage. Since the model is real, it is assumed that the real wage is predetermined and denotes the real wage by $R(= \frac{W}{Q})$. The crucial property here is that the real wage is not contingent on the shocks affecting the economy, and therefore wages adjustments do not contribute to cushion the economy to shocks. Appendix C shows how to derive the optimal real wage pre-set by utilitarian unions respecting the utility function of workers, and the trade-off they face between wages and employment, cf (3). The wage equation can be written[16]

$$R = \frac{\lambda \theta}{\theta - 1} \frac{E\{L\}}{E\{B^{-\epsilon}(1-t)L\}} \tag{9}$$

[16] As usual unions are assumed to be "atomised" perceiving that they cannot influence and aggregate variables.

Using that $B=(1-t)\frac{PY}{Q}$, and $\frac{WL}{PY}=\beta$ we have that this expression can be rewritten

$$\lambda E\{L\}=\frac{\beta(\theta-1)}{\theta}E\left\{B^{1-\epsilon}\right\}$$

Accordingly, expected utility can be written

$$E\left[\left(\frac{1}{1-\epsilon}-\frac{\beta(\theta-1)}{\theta}\right)B^{1-\epsilon}+\frac{\rho}{1-\gamma}G^{1-\gamma}\right] \tag{10}$$

Note that the preset wage level depends on the risk faced by wage setters, and hence an active fiscal stabilization policy that changes the risk profile may potentially have effects running via a change in wage demands and therefore in turn the levels of output, employment etc. around which fluctuations are taking place (for an analysis of the risk to wage channel, see e.g. Andersen and Sørensen (1988) and Obstfeld and Rogoff (2002)). As in the case of flexible wages we assume tax policies removing such level effects since they do not affect how the economy responds to shocks

With rigid real wages the aggregate supply relation becomes (compare to (5)),

$$Y=\frac{1}{\beta}Z^{1+\frac{\beta}{1-\beta}}R^{\frac{-\beta}{1-\beta}}\Gamma^{\frac{\beta(\alpha-1)}{1-\beta}}$$

The important difference is that labour supply responses induced by consumption changes do not affect the adjustment process. Accordingly the parameter ϵ does not affect the adjustment process, which removes the ambiguities arising under flexible wages depending on whether income or substitution effects dominate in labour supply.

We find (see appendix D) that equilibrium variables can be written

$$\begin{aligned}\tau &= \phi^f_{\tau z}z+\phi^f_{\tau z^*}z^* \\ y &= \phi^f_{yz}z+\phi^f_{yz^*}z^* \\ b &= \phi^f_{bz}z+\phi^f_{bz^*}z^*\end{aligned}$$

where the sup-scripts refer to the fixed (non contingent) wage equilibrium. Table 3 summarizes how the coefficients depend on the domestic and foreign policy parameter. With wages being preset labour supply is independent of the terms of trade. Hence, the improvement in the terms of trade that the domestic country achieves after a foreign productivity shock will unambiguously lead to an increase in domestic output. The absence of labour supply responses also implies that fiscal policy only affects output through its effect on the terms of trade. Increased demand for public consumption improves the terms of trade and consequently output expands. Therefore, a procyclical fiscal policy will magnify the responsiveness of domestic output to both domestic and foreign shocks. As with flexible wages an aggressive stabilization policy in the foreign country stabilizes domestic output. Considering how fiscal policy affects the link between productivity shocks and consumption we note that state contingent fiscal policy in both countries makes domestic consumption less responsive to both domestic and foreign productivity shocks.

Table 3: Coefficient signs and partial derivatives: Rigid wages

coefficient	sign	$\frac{\partial \phi_{ij}^f}{\partial \tau}$	$\frac{\partial \phi_{ij}^f}{\partial \tau^*}$
$\phi_{\omega z}^f$	> 0	< 0	> 0
$\phi_{\omega z^*}^f$	< 0	< 0	> 0
ϕ_{yz}^f	> 0	> 0	< 0
$\phi_{yz^*}^f$	> 0	> 0	< 0
ϕ_{bz}^f	> 0	< 0	< 0
$\phi_{bz^*}^f$	> 0	< 0	< 0

The effects of policy parameters on variability are given in table 4. Domestic stabilization adds to output variability whereas foreign stability reduces output variability. Both domestic and foreign stabilization unambiguously reduce consumption variability.

Table 4: Variability and policy parameters: Rigid Wages

	κ^f	κ^{f*}
$Var(y^f)$	> 0	< 0
$Var(b^f)$	< 0	< 0

Wage adjustment and fluctuations

Rigid wages means that the burden of adjustment is shifted from relative prices to quantities, but how does this affect the risk profile of the shocks? To see the direct effects of rigid wages on business cycle fluctuations it is illuminating to consider the case in which there is no public sector ($\mu = 1, \kappa = 0$). We show in appendix F that

$$Var(b) < Var(b^f) \qquad \text{for } \mu = 1, \kappa = 0$$

This confirms the usual presumption that rigidities in wage (or price) adjustment tends to imply more volatility in employment and activity since quantities take on a larger burden of adjustment as compared to the case with flexible wages (and prices). This points out that the potential welfare gains from an active stabilization policy are larger under rigid than flexible wages.

Optimal non-cooperative policies

With preset wages the policy maker's problem is

$$\min_{\kappa^f} \; L^f = Var(b^f) + \varphi^f Var(g^f)$$

The index f marks the case of fixed (unconditional) wages relative to the previous case of flexible wages[17]. It follows straightforwardly that the optimal policy is characterized by the first order condition

$$\frac{\partial Var(b^f)}{\partial \kappa^f} + \varphi \left(\kappa^{f2} \frac{\partial Var(y^f)}{\partial \kappa^f} + 2\kappa^f Var(y^f) \right) = 0$$

International spillovers

Turning to the spillover effects we have that

$$\frac{\partial L^f}{\partial \kappa^{f*}} = \frac{\partial Var(b^f)}{\partial \kappa^{f*}} + \varphi \kappa^{f2} \frac{\partial Var(y^f)}{\partial \kappa^{f*}} < 0$$

that is, there is unambiguously a negative spillover from foreign stabilization to the domestic loss. This is interesting since the effect under flexible wages under plausible parameter values runs in the opposite direction. The reason for this difference is that with preset wages an increase in κ^* stabilizes both b and also g through stabilization of y, whereas with flexible wages, foreign stabilization policy had an ambiguous effect on the variance of b.

Since the spillover effect is such that foreign stabilization policy contributes to increase expected domestic utility it follows (see e.g. Cooper and John (1988)) that in a comparison of cooperative and non-cooperative policies that we have

$$\kappa^{f,non-coop} < \kappa^{f,coop}$$

Non-cooperative policies imply insufficient stabilization relative to the cooperative policies.

6 Numerical illustrations

To assess the quantitative importance of the possible gains from stabilization, we simulate the model numerically. We only do this for the model with preset wages, as we find this the most realistic case. We assume that $\sigma = 1$, and to obtain the standard wage share we set $\beta = \frac{2}{3}$. As a starting point we choose $\alpha = 0.7$ to match the average degree of openness for countries within the EU. To calibrate the weight φ^f in the objective function determining the relative weight given to stabilization of public consumption relative to private consumption we choose to take the relative size of the public sector and the automatic stabilizers as given. We then interpret the initial situation as a non-cooperative Nash equilibrium allowing us to determine the implicit weight φ^f. Note that this procedure allows us to surpass a problem of relating our model to the data, since the model assumes a balanced budget tying expenditures and

[17] Note that $\varphi^f \neq \varphi$ due to the fact that the weights to private consumption in the utility function depends on parameters related to the market power of unions (see above and appendix C).

taxes, whereas in practice budget variations imply that expenditures and taxes are not closely tied in the short run. This leaves the question of whether to relate automatic stabilizers in the model to observed stabilizers in expenditures or in taxes. The approach chosen implies that we side step this issue. The theoretical analysis shows that the potential gains from stabilization may depend on the level of G, so we calibrate the model for two different policy regimes. In the first regime we assume a relatively large public sector, i.e. $\frac{G}{Y} = \frac{1}{2}$, whereas in the second regime we set $\frac{G}{Y} = \frac{1}{3}$. Note that since we view stabilization as running primarily through the revenue side, the relevant interpretation of G is that it includes all revenues raised by the public sector, including those passed back to the private sector as transfers. Since large public sectors are often associated with relatively large public stabilizers, we let the country with the large public sector be associated with a value of $\kappa = 0.7$, whereas we set $\kappa = 0.5$ for the country with the smaller public sector[18]. For these policy rules to be optimal from the perspectives of the individual countries we find the relative weights to variability of public consumption in the objective function (φ^f) to be given 0.85 and 0.62, respectively.

Table 5 shows the optimal policy response or state contingency in fiscal policy in the non-cooperative and cooperative case, as well as the reduction welfare loss due to risk attained by stabilization policy.

Table 5: Gains from active stabilization policy and international policy coordination

Regime	$\frac{G}{Y}$	policy $\kappa^{non-coop}$	parameter κ^{coop}	loss reduction $non-coop$	active policy $coop$	gains from cooperation
Large public sector	$\frac{1}{2}$	0.7	0.96	0.46	0.49	0.04
Small public sector	$\frac{1}{3}$	0.5	0.75	0.22	0.25	0.03

The table reports for each policy regime the fraction of the utility loss due to risk that is removed in the case of an active stabilization policy relative to a passive policy, for the case of both non-cooperative and cooperative policies. The last column gives the utility gain implied by moving from the non-cooperative to the cooperative case, i.e. this measures the gains from cooperation. Note that the cooperative outcome is found as the utilitarian solution to the policy problem, that is utility in the two countries weighs equally. It is seen that the gains from an active stabilization policy are fairly large in the sense that the utility loss due to risk is reduced significantly. The gains from an active stabilization policy are larger, the larger the public sector and thus the optimal stabilization parameter. However, the gains from cooperation are small relative to the gains from an active policy in the first place.

The importance of international integration for both the gains from an active stabilization policy, and the gains from policy coordination can in this framework

[18] See van der Noord (2000) for empirical evidence on automatic stabilizers for OECD countries, and the correlation between public sector size and automatic stabilizers.

be assessed by changing the openness of the economy measured by the home bias in consumption, that is, the import share in private consumption (α). The base case reported above had the import share to be 30%, and in table 6 below this case is compared to one with an import share of 45%. The relative weights to utility of private and public consumption are unchanged across the two regimes of, respectively, a large and small public sector. Note that a change in openness induces a change in the optimal policy parameter, this reflects that with more integration risk is transmitted more strongly between trading partners, and the cooperative policy therefore calls for more policy-activism.

Table 6: Gains from active stabilization policy and increased openness

Regime	Import share	policy $\kappa^{non-coop}$	parameter κ^{coop}	loss reduction non-coop	active policy coop	gains from cooperation
Large public sector	0.3	0.7	0.96	0.46	0.49	0.04
	0.45	0.65	0.99	0.45	0.51	0.06
Small public sector	0.3	0.5	0.75	0.22	0.25	0.03
	0.45	0.45	0.78	0.21	0.26	0.05

Qualitatively, it is seen that more openness induces policy makers acting non-cooperatively to reduce the size of the stabilization parameter. The reason is that with more openness the spillover effect is larger, and therefore single policy makers perceive that the gains are smaller from an active stabilization policy than in a more closed economy. This captures the often-made argument that more integration reduces the incentive of single governments to pursue an active stabilization policy due to free rider problems. The cooperative policy calls for a more active stabilization policy due to the increase in risk following tighter trade links. As a consequence the gains from policy cooperation are larger in more open economies. However, quantitatively none of the effects on the gains from policy cooperation reported here could be argued to be large[19]. Relatively, the gains from policy coordination almost double, but the initial levels are small, and therefore the overall effect remains modest, especially compared to the gains from an active stabilization policy in the first place.

7 Concluding remarks

Contingencies in public sector consumption and revenues can be a way to mitigate the consequences of risk for private households. To the extent that agents are risk averse and capital markets incomplete this leaves a welfare case for an active stabilization policy, which can be interpreted as a form of implicit or social insurance. The contingencies considered here can be interpreted as automatic budget reactions or stabilizers well known from standard macro models.

[19] Similar results are found for international monetary policy coordination, see Obstfeld and Rogoff (2001).

This paper has taken a first step in analysing the welfare implications of such stabilizers.

Since shocks are transmitted to trading partners it follows that "policies" are also transmitted. There are therefore in general international interdependencies in stabilization policies. Interestingly we find that the direction of the externality depends on the structural aspects. With flexible wages and prices, a welfare rationale for an active stabilization policy relies entirely on capital market imperfections. In this case a more aggressive domestic stabilization policy may imply more risk for trading partners. As a consequence, non-cooperative stabilization policies may imply excessive stabilization relative to the cooperative policies. The reason is that the domestic country in this case absorbs less of the foreign shock. However, with adjustment failures in the form of inflexible wages, quantities are assuming a larger adjustment burden, and in this case domestic stabilization policies have beneficial effects for trading partners. Accordingly, non-cooperative stabilization policies always imply insufficient stabilization relative to the cooperative policies.

The paper has considered business cycles driven by real (productivity) shocks. Interestingly, it turns out that there even in the case of supply shocks, is a welfare case for policies of a very "Keynesian" nature in the form of pro-cyclical variations in taxation. In future research it would be interesting to consider more general shocks structures, including the covariance between shocks, and different types of shocks. It would also be interesting to build the model into an explicit intertemporal model, to allow for a more detailed modeling of capital markets.

The finding that the direction in which coordination of stabilization policies should go depending on adjustment process suggests that countries may have different views on the need and form of policy cooperation. An interesting issue for further research would be to allow for country asymmetries with respect to e.g. adjustment mechanisms in the labour market.

It is a common finding in the literature that the gains from policy cooperation is small (see Obstfeld and Rogoff (2002) for the case of monetary policy, and McKibbin (1997) for a survey of the empirical analysis)). An open question is whether this result is robust to the introduction of heterogeneity implying that different groups are differently affected by international shocks. In closed economy models it has been shown that the welfare costs of business cycle fluctuations may increase substantially when heterogeneity is allowed for (see e.g. Storesletten, Telmer and Yaron (2000a,b)).

References

Andersen, T.M., and R. Dogonowski, 2002, Social Insurance and the Public Budget, Economica, 69, 415-431.

Andersen, T.M., B.S. Rasmussen, and J.R. Sørensen, 1996, Optimal Fiscal Policies in Open Economies with Labour Market Distortions, Journal of Public Economics, 63, 103-117.

Andersen, T.M. and J.R. Sørensen, 1988, Exchange rate variability and wage formation in open economies, Economics Letters, 28, 263-268.

Baxter, M., and R.G. King, 1993, Fiscal Policy in General Equilibrium, American Economic Review, 83, 315-334.

Blanchard, O. and S. Fischer, 1989, Lectures on Macroeconomics, MIT Press.

Chari, V.V., and P.J. Kehoe, 1990, International coordination of fiscal policy in limiting economies, Journal of Political Economy, 98, 617-636.

Coppel, J. and M. Durand, 1999, Trends in Market Openness, OECD Working paper 221

Cooper, R., 1985, Economic Interdependence and Coordination of Economic Policies, in Jones, R.W, and P. B. Kenen (eds), Handbook of International Economics, Vol II, North-Holland (Amsterdam).

Cooper, R., and A. John, 1988, Coordinating Coordination Failures in Keynesian Models, Quarterly Journal of Economics, 103, 441-463.

Devereux, M.B., 1991, The terms of trade and the international coordination of fiscal policy, Economic Inquiry, 29, 720-736.

Dixon, H.D., 1987, A Simple Model of Imperfect Competition with Walrasian Features, Oxford Economic Papers, 39,134-60.

European Central Bank, 2001, Fiscal Policies and Economic Growth, Monthly Bulletin August.

Gali, 2002, New Perspectives on Monetary Policy, Inflation and the Business Cycle, WP 3210.

Hamada, K., 1985, The political economy of international monetary interdependence, MIT Press.

Holmlund, B., and A.-S. Kolm, 1999, Economic Integration, Imperfect Competition, and International Policy Coordination, Working paper.

Lewis, K.K., 1999, Trying to Explain Home Bias in Equities and Consumption, Journal of Economic Literature, vol. XXXVII, 571-608.

Lockwood, B., 2000, Commodity Taxation and Tax Coordination under Destination and Origin Principle, CEPR DP 2556.

McKibbin, W.J., 1997, Empirical Evidence on International Policy Coordination, in M.U. Fratianni, D. Salvatore, and J. von Hagen (eds), Handbook of Comparative Economic Policies, Greenwood Publishing House (Westwood).

Obstfeld, M. and K. Rogoff, 1996, Foundations of International Macroeconomics, MIT Press.

Obstfeld, M. and K. Rogoff, 2002, Global Implications of Self-Oriented National Monetary Rules, Quarterly Journal of Economics, CXVII, 503-35.

Seidman, L, 2001, Reviving Fiscal Policy, Challenge, 44, 17-42.

Storesletten, K., C.I. Telmer, and A. Yaron, 2000a, Consumption and Risk Sharing over the Life Cycle, NBER Working Paper 7995.

Storesletten, K., C.I. Telmer, and A. Yaron, 2000b, The Welfare Cost of Business Cycles Revisited: Finite Lives and Cyclical Variations in Idiosyncratic Shocks, NBER Working Paper 8040.

Svensson, L.E, 1997, Optimal Inflation Targets, "Conservative" Central Banks, and Linear Inflation Contracts, American Economic Review, 87, 98-114.

Thomas, C.P., 1995, The role of fiscal policy in an incomplete markets framework, Review of Economic Studies, 52, 449-68.

Turnovsky, S.J., 1988, The gains from fiscal cooperation in the two-commodity real trade model, 25, 111-127, Journal of International Economics.

van der Noord, P., 2000, The size and role of automatic stabilizers in the 1990's and beyond, OECD Working Paper 230.

van der Ploeg, R., 1987, Coordination of Optimal Taxation in a Two-Country Equilibrium Model, Economics Letters, 24, 279-285.

van der Ploeg, R., 1988, International policy coordination in interdependent monetary economies, Journal of International Economics, 25, 1-23.

Appendix A: Equilibrium with flexible wages

This appendix solves the model in levels, and develops the details of how to design tax policy such that it is possible to focus only on the risk effects of state contingencies in fiscal policy.

Using the definition of the private consumption bundle B and the expression for the terms of trade we have

$$\begin{aligned} B &= \frac{I}{Q} = \frac{P(Y-G)}{Q} = \left(\frac{P}{P^*}\right)^{1-\alpha}(Y-G) \\ &= (Y-G)^{\alpha}(Y^*-G^*)^{1-\alpha} \end{aligned}$$

Assume that there is an employment subsidy s to the use of labour implying that the first order condition for labour demand can be written

$$ZL^{\beta-1} = \frac{W(1-s)}{P}$$

The subsidy is financed by a lump-sum tax levied on all firms (implying that profits $= PY - WL$).

>From the labour market clearing condition we have

$$(1-s)\lambda L = B^{-\epsilon}\frac{PY}{Q}\beta\,(1-t) = \beta B^{1-\epsilon}$$

We make the following approximation

$$Y - G = Y^{\mu}G^{1-\mu} \tag{11}$$

where

$$\mu = \frac{\overline{Y}}{\overline{Y}-\overline{G}} \geq 1$$

and an upper bar denotes the steady-state value around which the approximation is made. Note that this approximation has the same elasticities as $(Y-G)$ wrt Y and G in the steady-state equilibrium. We shall consider fiscal rules belonging to the following class of reaction functions, where nominal revenue is given as

$$T = P\overline{T}Y^{\kappa}$$

implying a tax rate

$$t = \frac{T}{PY} = \overline{T}Y^{\kappa-1}$$

that is, the reaction function stipulates that tax revenue (and therefore public consumption) is contingent on the level of activity, if $\kappa > 0$ it follows that tax revenue is procyclical, and if $\kappa > 1$ the tax rate is pro-cyclical. Using (11) and the contingent rule for public taxation and thus consumption ($G = \overline{T}Y^{\kappa}$) we have

$$Y - G = (\overline{T})^{1-\mu}Y^{\widehat{\mu}}$$

where we make use of the definitions

$$\widehat{\mu} \equiv (\mu + (1-\mu)\,\kappa) \quad ; \quad \widehat{\mu}^* \equiv (\mu^* + (1-\mu^*)\,\kappa^*)$$

Inserted in the equilibrium relation between employment and the private consumption bundle we have

$$\begin{aligned}
(1-s)\lambda L &= \beta\left[\left[(\overline{T})^{1-\mu}Y^{\widehat{\mu}}\right]^{\alpha}\left[(\overline{T}^*)^{1-\mu^*}Y^{*\widehat{\mu}^*}\right]^{1-\alpha}\right]^{1-\epsilon} \\
(1-s)\lambda\left[\frac{\beta Y}{Z}\right]^{\frac{1}{\beta}} &= \beta\left[\left[(\overline{T})^{1-\mu}Y^{\widehat{\mu}}\right]^{\alpha}\left[(\overline{T}^*)^{1-\mu*}Y^{*\widehat{\mu}^*}\right]^{1-\alpha}\right]^{1-\epsilon}
\end{aligned}$$

Hence, in logs

$$\pi_y y = \pi + z + \pi_{y^*} y^*$$

$$\begin{aligned}
\pi &\equiv \beta\left[\begin{array}{c} \log\beta - \log((1-s)\lambda\beta^{\frac{1}{\beta}}) + \alpha(1-\epsilon)(1-\mu)\log\overline{T} \\ (1-\alpha)(1-\epsilon)(1-\mu^*)\log\overline{T}^* \end{array}\right] \\
\pi_y &\equiv 1-\alpha\beta(1-\epsilon)\widehat{\mu} \\
\pi_{y^*} &\equiv \beta(1-\epsilon)(1-\alpha)\widehat{\mu}^*
\end{aligned}$$

Similarly,

$$\pi^*_{y*}y^* = \pi^* + z^* + \pi^*_y y$$

$$\begin{aligned}
\pi^* &\equiv \beta\left[\begin{array}{c} \log\beta - \ln(\lambda(1-s)\beta^{\frac{1}{\beta}}) + \alpha(1-\epsilon)(1-\mu^*)\log\overline{T}^* \\ (1-\alpha)(1-\epsilon)(1-\mu)\log\overline{T} \end{array}\right] \\
\pi^*_{y^*} &\equiv 1-\alpha\beta(1-\epsilon)\widehat{\mu}^* \\
\pi^*_y &\equiv \beta(1-\epsilon)(1-\alpha)\widehat{\mu}
\end{aligned}$$

>From which it follows that

$$y = \phi + \phi_{yz}z + \phi_{yz^*}z^*$$

where

$$\phi \equiv \frac{\pi^*_{y^*}\pi + \pi_{y^*}\pi^*}{\pi_y\pi^*_{y^*} - \pi_{y^*}\pi^*_y} \quad ; \quad \phi_{yz} \equiv \frac{\pi^*_{y^*}}{\pi_y\pi^*_{y^*} - \pi_{y^*}\pi^*_y} \quad ; \quad \phi_{yz^*} \equiv \frac{\pi_{y^*}}{\pi_y\pi^*_{y^*} - \pi_{y^*}\pi^*_y}$$

Similarly

$$y^* = \phi^* + \phi_{y^*z}z + \phi_{y^*z^*}z^*$$

where

$$\phi^* \equiv \frac{\pi^*_y\pi + \pi_y\pi^*}{\pi_y\pi^*_{y^*} - \pi_{y^*}\pi^*_y} \quad ; \quad \phi_{y^*z} \equiv \frac{\pi^*_y}{\pi_y\pi^*_{y^*} - \pi_{y^*}\pi^*_y} \quad ; \quad \phi_{y^*z^*} \equiv \frac{\pi_y}{\pi_y\pi^*_{y^*} - \pi_{y^*}\pi^*_y}$$

By inserting the equilibrium values for private consumption (B), the terms of trade (Ω) follows straightforwardly

Turning next to the expected utility for the representative household. First note that if X is a log-normally distributed random variable, then

$$
\begin{aligned}
E(\frac{1}{1-\epsilon}X^{1-\epsilon}) &= \exp\log\left(E\left(\frac{1}{1-\epsilon}X^{1-\epsilon}\right)\right) \\
&= \exp\left(\log\frac{1}{1-\epsilon}+(1-\epsilon)\,E(\log X)+\frac{(1-\epsilon)^2}{2}Var(\log X)\right) \\
&= \exp\left(\begin{array}{c}\log\frac{1}{1-\epsilon}+(1-\epsilon)\left(E(\log X)+\frac{1}{2}Var(\log X)\right) \\ +\left(\frac{(1-\epsilon)^2}{2}-\frac{1-\epsilon}{2}\right)Var(\log X)\end{array}\right)
\end{aligned}
$$

Using that

$$
E(X)=\exp\left(E(\log X)+\frac{1}{2}Var(\log X)\right)
$$

we have

$$
E(\frac{1}{1-\epsilon}X^{1-\epsilon})=\frac{1}{1-\epsilon}\left(E(X)^{1-\epsilon}\exp\left(-\frac{(1-\epsilon)\,\epsilon}{2}Var(\log X)\right)\right) \tag{12}
$$

We have that expected utility for the representative household can be written

$$
E\left[\left(\frac{1}{1-\epsilon}-\beta\right)B^{1-\epsilon}+\frac{1}{1-\gamma}G^{1-\gamma}\right]
$$

Using (12) we get that expected utility can be written

$$
\begin{array}{l}
\left(\frac{1}{1-\epsilon}-\beta\right)\left(E(B)^{1-\epsilon}\exp\left(-\frac{(1-\epsilon)\,\epsilon}{2}Var(\log B)\right)\right)+ \\
\frac{\rho}{1-\gamma}\left(E(G)^{1-\gamma}\exp\left(-\frac{(1-\gamma)\,\gamma}{2}Var(\log G)\right)\right)
\end{array}
$$

Assuming that the employment subsidy s and the tax level $\overline{T}$ are chosen such that $E(B)$ and $E(G)$ are constant or invariant to changes in the stabilization/contingency parameter κ, it follows that a first order approximation of the objective function implies that maximization of expected utility is equivalent to minimizing the following loss function

$$
Var(b)+\varphi Var(g)
$$

where

$$
\varphi=\frac{\frac{(1-\gamma)\gamma}{2}\frac{\rho}{1-\gamma}\left(E(G)^{1-\gamma}\exp\left(-\frac{(1-\gamma)\gamma}{2}Var(\log G)\right)\right)}{\frac{(1-\epsilon)\epsilon}{2}\left(\frac{1}{1-\epsilon}-\beta\right)\left(E(B)^{1-\epsilon}\exp\left(-\frac{(1-\epsilon)\epsilon}{2}Var(\log B)\right)\right)}
$$

Appendix B: Coefficients and equilibrium relations

Throughout the paper we make the assumption that

$$(1-\beta\mu(1-\epsilon))>0 \Longleftrightarrow \epsilon>\frac{\beta\mu-1}{\beta\mu}$$

We make the following definitions

$$\begin{array}{ll} \psi \equiv (1-\beta\mu(1-\epsilon)) & \widehat{\psi} \equiv (1-\beta\widehat{\mu}(1-\epsilon)) \\ \psi^* \equiv (1-\beta\mu^*(1-\epsilon)) & \widehat{\psi}^* \equiv (1-\beta\widehat{\mu}^*(1-\epsilon)) \end{array}$$

For the coefficients in the supply relation we have

$$\begin{array}{lll} \eta_{sz} \equiv \psi^{-1} & \eta_{s\tau} \equiv \psi^{-1}\beta(\alpha-1)(1-\epsilon) & \eta_{sg} \equiv \psi^{-1}\beta(1-\epsilon)(1-\mu) \\ \eta_{sz}^* \equiv (\psi^*)^{-1} & \eta_{s\tau}^* \equiv (\psi^*)^{-1}\beta(1-\alpha)(1-\epsilon) & \eta_{sg}^* \equiv (\psi^*)^{-1}\beta(1-\epsilon)(1-\mu^*) \end{array}$$

implying that

$$\begin{array}{lll} \eta_{sz}>0 & sign(\eta_{s\tau})=-sign(1-\epsilon) & sign\left(\eta_{sg}\right)=-sign(1-\epsilon) \\ \eta_{sz}^*>0 & sign(\eta_{s\tau}^*)=sign(1-\epsilon) & sign\left(\eta_{sg}^*\right)=-sign(1-\epsilon) \end{array}$$

(ii) Equilibrium relations

The aggregate demand relation implies that the terms of trade can be written

$$\begin{aligned} \tau &= \mu y+(1-\mu)g-\mu^* y^*-(1-\mu^*)g^* \\ &= \widehat{\mu}y-\widehat{\mu}^* y^* \end{aligned} \tag{13}$$

where the last equality follows from inserting the fiscal policy rules. Also using the fiscal policy rules we can rewrite output as

$$y = \frac{1}{1-\eta_{sg}\kappa}(\eta_{sz}z+\eta_{s\kappa}\tau) \tag{14}$$

$$y^* = \frac{1}{1-\eta_{sg}^*\kappa^*}(\eta_{sz}^* z^*+\eta_{s\kappa}^*\tau) \tag{15}$$

implying that the terms of trade is given as

$$\tau=\phi_{\tau y}(\eta_{sz}z+\eta_{s\tau}\tau)+\phi_{\tau y^*}(\eta_{sz}^* z^*+\eta_{s\tau}^*\tau) \tag{16}$$

where

$$\begin{aligned} \phi_{\tau y} &\equiv \frac{\widehat{\mu}}{1-\eta_{sg}\kappa}>0 \quad ; \frac{\partial\phi_{\tau y}}{\partial\kappa}=\frac{1-\mu\left(1-\eta_{sg}\right)}{\left(1-\eta_{sg}\kappa\right)^2}<0 \\ \phi_{\tau y^*} &\equiv -\frac{\widehat{\mu}^*}{1-\eta_{sg}^*\kappa^*}<0 \;\; ; \frac{\partial\phi_{\tau y^*}}{\partial\kappa^*}=-\frac{1-\mu^*\left(1-\eta_{sg}^*\right)}{\left(1-\eta_{sg}^*\kappa^*\right)^2}>0 \end{aligned}$$

The signs follow from noting that

$$1-\eta_{sg}\kappa=\psi^{-1}\widehat{\psi}>0 \quad ; \quad 1-\mu\left(1-\eta_{sg}\right)=\psi^{-1}(1-\mu)<0$$

Now consider the coefficients in the terms of trade. Solving for τ in (16) enables us to write

$$\tau = \phi_{\tau z} z + \phi_{\tau z^*} z^*$$

where

$$\phi_{\tau z} \equiv \frac{\phi_{\tau y}\eta_{sz}}{1-\phi_{\tau y}\eta_{s\tau}-\phi_{\tau y^*}\eta^*_{s\tau}} > 0 \qquad (17)$$

$$\phi_{\tau z^*} \equiv \frac{\phi_{\tau y^*}\eta^*_{sz}}{1-\phi_{\tau y}\eta_{s\tau}-\phi_{\tau y^*}\eta^*_{s\tau}} < 0 \qquad (18)$$

The signs follow from noting that a sufficient condition for the denominator to be positive is that

$$\phi_{\tau y}\eta_{s\tau} < \frac{1}{2} \quad ; \quad \phi_{\tau y^*}\eta^*_{s\tau} < \frac{1}{2}$$

These conditions are fulfilled since

$$\phi_{\tau y}\eta_{s\tau} < \frac{1}{2} \Longleftrightarrow \widehat{\psi}^{-1}\left(\left(\alpha-\frac{1}{2}\right)\beta\left(1-\epsilon\right)\widehat{\mu}-\frac{1}{2}\right) < 0$$

and by a symmetric argument it follows that $\phi_{\tau y^*}\eta^*_{s\tau} < \frac{1}{2}$. Now we check what happens as we vary the policy parameters. We have that

$$\frac{\partial\phi_{\tau z}}{\partial\kappa} = \frac{\left(1-\phi_{\tau y^*}\eta^*_{s\tau}\right)\eta_{sz}\frac{\partial\phi_{\tau y}}{\partial\kappa}}{\left(1-\phi_{\tau y}\eta_{s\tau}-\phi_{\tau y^*}\eta^*_{s\tau}\right)^2}$$

$$\frac{\partial\phi_{\tau z}}{\partial\kappa^*} = \frac{\eta^*_{s\tau}\phi_{\tau y}\eta_{sz}\frac{\partial\phi_{\tau y^*}}{\partial\kappa^*}}{\left(1-\phi_{\tau y}\eta_{s\tau}-\phi_{\tau y^*}\eta^*_{s\tau}\right)^2}$$

First note that

$$1-\phi_{\tau y^*}\eta^*_{s\tau} = \left(\widehat{\psi}^*\right)^{-1}\left(1-\alpha\beta\left(1-\epsilon\right)\widehat{\mu}^*\right) > 0$$

Then it is easy to check that

$$\frac{\partial\phi_{\tau z}}{\partial\kappa} < 0 \quad ; \quad sign\left(\frac{\partial\phi_{\tau z}}{\partial\kappa^*}\right) = sign\left(1-\epsilon\right)$$

By use of symmetry arguments it follows that in a symmetric equilibrium

$$\phi_{\tau z} > 0 \quad ; \quad \phi_{\tau z^*} = -\phi_{\tau z} < 0$$

$$\frac{\partial\phi_{\tau z}}{\partial\kappa} < 0 \quad ; \quad sign\left(\frac{\partial\phi_{\tau z}}{\partial\kappa^*}\right) = sign\left(1-\epsilon\right)$$

$$\frac{\partial\phi_{\tau z^*}}{\partial\kappa^*} = -\frac{\partial\phi_{\tau z}}{\partial\kappa} > 0$$

$$sign\left(\frac{\partial\phi_{\tau z^*}}{\partial\kappa}\right) = -sign\left(\frac{\partial\phi_{\tau z}}{\partial\kappa^*}\right) = -sign\left(1-\epsilon\right)$$

Considering the consumption bundle we can write

$$B = \Gamma^{\alpha-1}(Y-G)$$

Taking logs and inserting the fiscal policy rule this can be rewritten as

$$\begin{aligned} b &= (\alpha-1)\tau + \widehat{\mu} y \\ &= \phi_{bz} z + \phi_{bz^*} z^* \end{aligned}$$

where

$$\begin{aligned} \phi_{bz} &\equiv \frac{\alpha - \phi_{\tau y^*}\eta^*_{s\tau}}{1-\phi_{\tau y}\eta_{s\tau} - \phi_{\tau y^*}\eta^*_{s\tau}} \phi_{\tau y}\eta_{sz} \\ \phi_{bz^*} &\equiv \frac{\left(\alpha - 1 + \eta_{s\tau}\phi_{\tau y}\right)}{1-\phi_{\tau y}\eta_{s\tau} - \phi_{\tau y^*}\eta^*_{s\tau}} \phi_{\tau y^*}\eta^*_{sz} \end{aligned}$$

We know that the demonimator is positive. Now let us consider the numerator. We have

$$\begin{aligned} \alpha - \phi_{\tau y^*}\eta^*_{s\tau} &= \left(\widehat{\psi}^*\right)^{-1}\left(\alpha + \beta(1-\epsilon)(1-2\alpha)\widehat{\mu}^*\right) > 0 \\ \left(\alpha - 1 + \eta_{s\tau}\phi_{\tau y}\right) &= -\widehat{\psi}^{-1}(1-\alpha) < 0 \end{aligned}$$

Moreover since $\phi_{\tau y} > 0$, $\phi_{\tau y^*} < 0$, $\eta_{sz} > 0$, $\eta^*_{sz} > 0$ we conclude that

$$\phi_{bz} > 0 \quad ; \quad \phi_{bz^*} > 0$$

Now consider the derivatives

$$\begin{aligned} \frac{\partial \phi_{bz}}{\partial \kappa} &= \frac{\left(\alpha - \phi_{\tau y^*}\eta^*_{sz}\right)\eta_{sz}[1-\phi_{\tau y^*}\eta^*_{s\tau}]}{\left(1-\phi_{\tau y}\eta_{s\tau} - \phi_{\tau y^*}\eta^*_{s\tau}\right)^2}\frac{\partial \phi_{\tau y}}{\partial \kappa} \\ \frac{\partial \phi_{bz}}{\partial \kappa^*} &= -\frac{\phi_{\tau y}\eta_{sz}\eta^*_{s\tau}\left[1-\phi_{\tau y}\eta_{s\tau} - \alpha\right]}{\left(1-\phi_{\tau y}\eta_{s\tau} - \phi_{\tau y^*}\eta^*_{s\tau}\right)^2}\frac{\partial \phi_{\tau y^*}}{\partial \kappa^*} \\ \frac{\partial \phi_{bz^*}}{\partial \kappa} &= \frac{\eta^*_{sz}\phi_{\tau y^*}\eta_{s\tau}\left[\alpha - \phi_{\tau y^*}\eta^*_{s\tau}\right]}{\left(1-\phi_{\tau y}\eta_{s\tau} - \phi_{\tau y^*}\eta^*_{s\tau}\right)^2}\frac{\partial \phi_{\tau y}}{\partial \kappa} \\ \frac{\partial \phi_{bz^*}}{\partial \kappa^*} &= \frac{\eta^*_{sz}\left(\alpha - 1 + \eta_{s\tau}\phi_{\tau y}\right)\left[1-\phi_{\tau y}\eta_{s\tau}\right]}{\left(1-\phi_{\tau y}\eta_{s\tau} - \phi_{\tau y^*}\eta^*_{s\tau}\right)^2}\frac{\partial \phi_{\tau y^*}}{\partial \kappa^*} \end{aligned}$$

To evaluate these we first show that

$$1-\phi_{\tau y}\eta_{s\tau} = \frac{1-\alpha(1-\epsilon)\beta\widehat{\mu}}{\widehat{\psi}} > 0$$

This together with signs that we already know leads us to conclude that

$$\begin{aligned} \frac{\partial \phi_{bz}}{\partial \kappa} &< 0 \quad ; \quad sign\left(\frac{\partial \phi_{bz}}{\partial \kappa^*}\right) = -sign(1-\epsilon) \\ sign\left(\frac{\partial \phi_{bz^*}}{\partial \kappa}\right) &= -sign(1-\epsilon) \quad ; \quad \frac{\partial \phi_{bz^*}}{\partial \kappa^*} < 0 \end{aligned}$$

Now output. From (14) we can write

$$\phi_{yz} \equiv \frac{\eta_{sz} + \eta_{s\tau}\phi_{\tau z}}{1 - \eta_{sg}\kappa}$$

To sign this use (17) to rewrite it as

$$\phi_{yz} = \frac{\eta_{sz}\left(1 - \phi_{\tau y^*}\eta^*_{s\tau}\right)}{\left(1 - \eta_{sg}\right)\left(1 - \phi_{\tau y}\eta_{s\tau} - \phi_{\tau y^*}\eta^*_{s\tau}\right)} > 0$$

Now consider derivatives

$$\frac{\partial \phi_{yz}}{\partial \kappa} = -\frac{\eta_{sz}(1 - \phi_{\tau y^*}\eta^*_{s\tau})\left(-\eta_{sg}(1 - \phi_{\tau y^*}\eta^*_{s\tau}) - (1-\mu)\eta_{s\tau}\right)}{\left(1 - \eta_{sg}\kappa\right)(1 - \phi_{\tau y}\eta_{s\tau} - \phi_{\tau y^*}\eta^*_{s\tau})^2} > 0$$

First note that

$$\begin{aligned}
& sign\left[-\eta_{sg}(1 + \phi_{\tau y^*}\eta^*_{s\tau}) - (1-\mu)\eta_{s\tau}\right] \\
= \; & sign\left[-\frac{\beta\,(1-\epsilon)\,(1-\mu)}{\psi}\left(\frac{\alpha + (1-2\alpha)\beta\,(1-\epsilon)\,\widehat{\mu}^*}{\widehat{\psi}^*}\right)\right] \\
= \; & sign\,(1-\epsilon)
\end{aligned}$$

so

$$sign\left(\frac{\partial \phi_{yz}}{\partial \kappa}\right) = -sign\,(1-\epsilon)$$

And

$$\frac{\partial \phi_{yz}}{\partial \kappa^*} = \frac{\eta_{s\tau}}{1 - \eta_{sg}\kappa}\frac{\partial \phi_{\tau z}}{\partial \kappa^*} < 0$$

Now consider the response to the foreign shock

$$\phi_{yz^*} \equiv \frac{\eta_{s\tau}\phi_{\tau z^*}}{1 - \eta_{sg}\kappa}$$

so

$$sign\left(\phi_{yz^*}\right) = sign\,(1-\epsilon)$$

To sign $\frac{\partial \phi_{yz^*}}{\partial \kappa}$ it is useful to note that

$$\begin{aligned}
\phi_{yz^*} &= \frac{\eta_{\tau s}\phi_{\tau z^*}}{1 - \eta_{sg}\kappa} \\
&= \frac{1}{1 - \eta_{sg}\kappa}\left[\eta_{s\tau}\frac{\phi_{\tau y^*}\eta^*_{sz}}{1 - \phi_{\tau y}\eta_{s\tau} - \phi_{\tau y^*}\eta^*_{s\tau}}\right] \\
&= \frac{\eta_{s\tau}\phi_{\tau y^*}\eta^*_{sz}}{\left(1 - \eta_{sg}\kappa\right)\left(1 - \phi_{\tau y^*}\eta^*_{s\tau}\right) - \widehat{\mu}\eta_{s\tau}}
\end{aligned}$$

Hence

$$\frac{\partial \phi_{yz^*}}{\partial \kappa} = -\frac{\eta_{s\tau}\phi_{\tau y^*}\eta_{sz}^*\left(-\eta_{sg}\left(1+\phi_{\tau y^*}\eta_{s\tau}^*\right)-(1-\mu)\,\eta_{s\tau}\right)}{\left[\left(1-\eta_{sg}\kappa\right)\left(1-\phi_{\tau y^*}\eta_{s\tau}^*\right)-\widehat{\mu}\eta_{s\tau}\right]^2} < 0$$

And finally

$$\frac{\partial \phi_{yz^*}}{\partial \kappa^*} = \frac{\eta_{s\tau}}{1-\eta_{sg}\kappa}\frac{\partial \phi_{\tau z^*}}{\partial \kappa^*}$$

implying that

$$sign\left(\frac{\partial \phi_{yz^*}}{\partial \kappa^*}\right) = -sign\,(1-\epsilon)$$

Now we examine how the variances of production and consumption are affected by the choice of policy parameters. We have that

$$\begin{aligned} Var\,(b) &= \left(\phi_{bz}^2+\phi_{bz^*}^2\right)\sigma^2 \\ Var\,(y) &= \left(\phi_{yz}^2+\phi_{yz^*}^2\right)\sigma^2 \end{aligned}$$

so

$$\begin{aligned} \frac{\partial Var\,(b)}{\partial \kappa} &= 2\sigma^2\left(\phi_{bz}\frac{\partial \phi_{bz}}{\partial \kappa}+\phi_{bz^*}\frac{\partial \phi_{bz^*}}{\partial \kappa}\right) \\ \frac{\partial Var\,(b)}{\partial \kappa^*} &= 2\sigma^2\left(\phi_{bz}\frac{\partial \phi_{bz}}{\partial \kappa^*}+\phi_{bz^*}\frac{\partial \phi_{bz^*}}{\partial \kappa^*}\right) \\ \frac{\partial Var\,(y)}{\partial \kappa} &= 2\sigma^2\left(\phi_{yz}\frac{\partial \phi_{yz}}{\partial \kappa}+\phi_{yz^*}\frac{\partial \phi_{yz^*}}{\partial \kappa}\right) \\ \frac{\partial Var\,(y)}{\partial \kappa^*} &= 2\sigma^2\left(\phi_{yz}\frac{\partial \phi_{yz}}{\partial \kappa^*}+\phi_{yz^*}\frac{\partial \phi_{yz^*}}{\partial \kappa^*}\right) \end{aligned}$$

Using the signs of the coefficients and their derivatives derived earlier it is easy to check that

$$sign\left(\frac{\partial Var\,(y)}{\partial \kappa}\right) = -sign\,(1-\epsilon) \quad ; \quad \frac{\partial Var\,(y)}{\partial \kappa^*} < 0$$

With respect to the consumption bundle we insert the expression for the coefficients and their derivatives, ie.

$$\frac{\partial Var\,(b)}{\partial \kappa} = 2\sigma^2\left[\begin{array}{c} \frac{\left(\alpha-\phi_{\tau y^*}\eta_{s\tau}^*\right)\phi_{\tau y}\eta_{sz}}{1-\phi_{\tau y}\eta_{s\tau}-\phi_{\tau y^*}\eta_{s\tau}^*}\frac{\left(\alpha-\phi_{\tau y^*}\eta_{s\tau}^*\right)\eta_{sz}\left(1-\phi_{\tau y^*}\eta_{s\tau}^*\right)}{\left(1-\phi_{\tau y}\eta_{s\tau}-\phi_{\tau y^*}\eta_{s\tau}^*\right)^2}\frac{\partial \phi_{\tau y}}{\partial \kappa} \\ +\frac{\left(\alpha-1+\eta_{s\tau}\phi_{\tau y}\right)\phi_{\tau y^*}\eta_{sz}^*}{1-\phi_{\tau y}\eta_{s\tau}-\phi_{\tau y^*}\eta_{s\tau}^*}\frac{\eta_{sz}^*\phi_{\tau y^*}\eta_{s\tau}\left(\alpha-\phi_{\tau y^*}\eta_{s\tau}^*\right)}{\left(1-\phi_{\tau y}\eta_{s\tau}-\phi_{\tau y^*}\eta_{s\tau}^*\right)^2}\frac{\partial \phi_{\tau y}}{\partial \kappa} \end{array}\right]$$

This derivative is negative if and only if

$$\left(\alpha-\phi_{\tau y^*}\eta_{s\tau}^*\right)^2\phi_{\tau y}\eta_{sz}\eta_{sz}\left(1-\phi_{\tau y^*}\eta_{s\tau}^*\right)+\left(\alpha-1+\eta_{s\tau}\phi_{\tau y}\right)\phi_{\tau y^*}\eta_{sz}^*\eta_{sz}^*\phi_{\tau y^*}\eta_{s\tau}\left(\alpha-\phi_{\tau y^*}\eta_{s\tau}^*\right) > 0$$

In a symmetric equilibrium the condition reduces to

$$\left(\alpha - \phi_{\tau y}\eta_{s\tau}\right)\left(1 - \phi_{\tau y}\eta_{s\tau}\right) + \left(\alpha - 1 + \eta_{s\tau}\phi_{\tau y}\right)\phi_{\tau y}\eta_{s\tau} > 0$$

Sufficient conditions for this to be satisfied are

$$\left(\alpha - \phi_{\tau y}\eta_{s\tau}\right) > \left(1 - \alpha - \eta_{s\tau}\phi_{\tau y}\right) \quad ; \quad \left(1 - \phi_{\tau y}\eta_{s\tau}\right) > \phi_{\tau y}\eta_{s\tau}$$

Since we are assuming $\alpha > \frac{1}{2}$ these conditions are obviously satisfied, so

$$\frac{\partial Var\left(b\right)}{\partial \kappa^{*}} < 0$$

Similarly it can be shown that in a symmetric equilibrium

$$\begin{aligned} \frac{\partial Var\left(b\right)}{\partial \kappa^{*}} &< 0 \Longleftrightarrow \\ \left(\alpha - \phi_{\tau y}\eta_{s\tau}\right)\eta_{s\tau}\phi_{\tau y} &< \left(1 - \alpha - \eta_{s\tau}\phi_{\tau y}\right)\left(1 - \phi_{\tau y}\eta_{s\tau}\right) \end{aligned}$$

It holds that

$$\phi_{\tau y}\eta_{s\tau} = \frac{1-\alpha}{1+\Xi} \text{ where } \Xi \equiv -\frac{1}{1-\widehat{\psi}}$$

implying that the condition can be written as

$$\frac{\partial Var\left(b\right)}{\partial \kappa^{*}} < 0 \Longleftrightarrow 1 - 2\alpha + \Xi^{2} < 0$$

so

$$sign\left(\frac{\partial Var\left(b\right)}{\partial \kappa^{*}}\right) = sign\left(1 + \left(\frac{1}{1-\widehat{\psi}}\right)^{2} - 2\alpha\right)$$

Appendix C: Wage setting

To analyse the model with rigid wages we define the labour aggregate

$$L = \left[\int_{0}^{1} L\left(i\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1$$

Each type of labour has a representative agent with a utility function as given in (1).

Demand for labour of a given type i is given as

$$L\left(i\right) = \left(\frac{W\left(i\right)}{W}\right)^{-\theta} L = \left(\frac{W\left(i\right)}{W}\right)^{-\theta} Z^{\frac{-1}{\beta}}\beta^{\frac{1}{\beta}}Y^{\frac{1}{\beta}}$$

and the costs of acquiring one unit of the composite labour input can be written

$$W = \left[\int_{0}^{1} W(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$$

The specification of the production side is otherwise unchanged.

Wage setting
Wage earners pre-set their wage, and since the model is real there is no issue in respect to whether the wage rigidity is nominal or real. Denote the pre-set real wage by R, where

$$R \equiv \frac{W}{Q}$$

The objective of wage setters is to set a wage R so as to solve the following problem,

$$\begin{aligned} & \max_{R} E\left\{\frac{1}{1-\epsilon}B\left(i\right)^{1-\epsilon}-\lambda\left(\frac{R\left(i\right)}{R}\right)^{-\theta}L+\frac{1}{1-\gamma}G^{1-\gamma}\right\} \\ & s.t. \\ B\left(i\right) &= Q^{-1}\left(R\left(i\right)Q\left(\frac{R\left(i\right)}{R}\right)^{-\theta}L+\Pi\right)\left(1-t\right) \end{aligned}$$

the solution is

$$R=\frac{\lambda\theta}{\theta-1}\frac{E\left\{L\right\}}{E\left\{B^{-\epsilon}\left(1-t\right)L\right\}} \tag{19}$$

Appendix D: Equilibrium with Rigid Wages

With sticky wages the coefficients of the supply relations are given as

$$\begin{array}{lll} \eta_{sz}^{f}\equiv\frac{1}{1-\beta}>0 & \eta_{s\tau}^{f}\equiv\frac{\beta(\alpha-1)}{1-\beta}<0 & \eta_{sg}^{f}\equiv 0 \\ \eta_{sz}^{f*}\equiv\frac{1}{1-\beta}>0 & \eta_{s\tau}^{f*}\equiv\frac{\beta(1-\alpha)}{1-\beta}>0 & \eta_{sg}^{f*}\equiv 0 \end{array}$$

(i) Terms of trade
It follows directly that

$$\begin{aligned} \phi_{\tau y}^{f} &\equiv \widehat{\mu}>0 \quad ; \quad \frac{\partial\phi_{\tau y}^{f}}{\partial\kappa}=(1-\mu)<0 \\ \phi_{\tau y}^{f*} &\equiv -\widehat{\mu}^{*}<0 \quad ; \quad \frac{\partial\phi_{\tau y}^{f*}}{\partial\kappa}=-\left(1-\mu^{*}\right)>0 \end{aligned}$$

Moreover, we have

$$\begin{aligned} \phi_{\tau z}^{f} &\equiv \frac{\phi_{\tau y}^{f}\eta_{sz}^{f}}{1-\phi_{\tau y}^{f}\eta_{s\tau}^{f}-\phi_{\tau y^{*}}^{f}\eta_{s\tau}^{f*}}=\frac{\widehat{\mu}}{1-\beta-\beta(\alpha-1)\left(\widehat{\mu}+\widehat{\mu}^{*}\right)}>0 \\ \phi_{\tau z^{*}}^{f} &\equiv \frac{\phi_{\tau y^{*}}^{f}\eta_{sz}^{f*}}{1-\phi_{\tau y}^{f}\eta_{s\tau}^{f}-\phi_{\tau y^{*}}^{f}\eta_{s\tau}^{f*}}=\frac{-\mu^{*}}{1-\beta-\beta(\alpha-1)(\widehat{\mu}+\widehat{\mu}^{*})}<0 \end{aligned}$$

Implying that

$$\begin{aligned}
\frac{\partial \phi^f_{\tau z}}{\partial \kappa} &= \frac{(1-\mu)[(1-\beta)-\beta(\alpha-1)\widehat{\mu}^*]}{\left(1-\beta-\beta(\alpha-1)\left(\widehat{\mu}+\widehat{\mu}^*\right)\right)^2} < 0 \\
\frac{\partial \phi^f_{\tau z}}{\partial \kappa^*} &= \frac{\widehat{\mu}\beta(\alpha-1)\left(1-\mu^*\right)}{\left(1-\beta-\beta(\alpha-1)(\widehat{\mu}+\widehat{\mu}^*)\right)^2} > 0
\end{aligned}$$

Of course symmetric results hold for foreign

ii) Consumption
As is the case with flexible wages we have

$$\begin{aligned}
b &= (\alpha-1)\,\tau+\widehat{\mu}y \\
&= \phi^f_{bz}z+\phi^f_{bz^*}z^*
\end{aligned}$$

where

$$\begin{aligned}
\phi^f_{bz} &\equiv \frac{\alpha-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}}{1-\phi^f_{\tau y}\eta^f_{s\tau}-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}}\phi^f_{\tau y}\eta^f_{sz} > 0 \\
\phi^f_{bz^*} &\equiv \frac{\left(\alpha-1+\eta^f_{s\tau}\phi^f_{\tau y}\right)}{1-\phi^f_{\tau y}\eta^f_{s\tau}-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}}\phi^f_{\tau y^*}\eta^{f*}_{sz} > 0
\end{aligned}$$

To see how these depend on the policy rules we have

$$\begin{aligned}
\frac{\partial \phi^f_{bz}}{\partial \kappa} &= \frac{\left(\alpha-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}\right)\eta^f_{sz}[1-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}]}{\left(1-\phi^f_{\tau y}\eta^f_{s\tau}-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}\right)^2}\frac{\partial \phi^f_{\tau y}}{\partial \kappa} < 0 \\
\frac{\partial \phi^f_{bz}}{\partial \kappa^*} &= -\frac{\phi^f_{\tau y}\eta^f_{sz}\eta^{f*}_{s\tau}\left[1-\phi^f_{\tau y}\eta^f_{s\tau}-\alpha\right]}{\left(1-\phi^f_{\tau y}\eta^f_{s\tau}-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}\right)^2}\frac{\partial \phi^f_{\tau y^*}}{\partial \kappa^*} < 0 \\
\frac{\partial \phi^f_{bz^*}}{\partial \kappa} &= \frac{\eta^{f*}_{sz}\phi^f_{\tau y^*}\eta^f_{s\tau}\left[\alpha-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}\right]}{\left(1-\phi^f_{\tau y}\eta^f_{s\tau}-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}\right)^2}\frac{\partial \phi^f_{\tau y}}{\partial \kappa} < 0 \\
\frac{\partial \phi^f_{bz^*}}{\partial \kappa^*} &= \frac{\eta^{f*}_{sz}\left(\alpha-1+\eta^f_{s\tau}\phi^f_{\tau y}\right)\left[1-\phi^f_{\tau y}\eta^f_{s\tau}\right]}{\left(1-\phi^f_{\tau y}\eta^f_{s\tau}-\phi^f_{\tau y^*}\eta^{f*}_{s\tau}\right)^2}\frac{\partial \phi^f_{\tau y^*}}{\partial \kappa^*} < 0
\end{aligned}$$

iii) Output
We have

$$y=\phi^f_{yz}z+\phi^f_{yz^*}z^*$$

where

$$\phi^f_{yz} \equiv \eta^f_{sz} + \eta^f_{s\tau}\phi^f_{\tau z} > 0 \quad ; \quad \phi^f_{yz^*} \equiv \eta^f_{s\tau}\phi^f_{\tau z^*} > 0$$

Derivatives:

$$\begin{aligned}
\frac{\partial \phi^f_{yz}}{\partial \kappa} &= \eta^f_{s\tau}\frac{\partial \phi^f_{\tau z}}{\partial \kappa} > 0 \quad ; \quad \frac{\partial \phi^f_{yz}}{\partial \kappa^*} = \eta^f_{s\tau}\frac{\partial \phi^f_{\tau z}}{\partial \kappa^*} < 0 \\
\frac{\partial \phi^f_{yz^*}}{\partial \kappa} &= \eta^f_{s\tau}\frac{\partial \phi^f_{\tau z^*}}{\partial \kappa} > 0 \quad ; \quad \frac{\partial \phi_{yz^*}}{\partial \kappa^*} = \eta^f_{s\tau}\frac{\partial \phi_{\tau z^*}}{\partial \kappa^*} < 0
\end{aligned}$$

iv) Interdependencies in stabilization policy

We have

$$Var\,(b) = \left(\left(\phi^f_{bz}\right)^2 + \left(\phi^f_{bz^*}\right)^2\right)\sigma^2$$

Implying that

$$\begin{aligned}
\frac{\partial Var\,(b)}{\partial \kappa} &= 2\sigma^2\left(\phi^f_{bz}\frac{\partial \phi^f_{bz}}{\partial \kappa} + \phi^f_{bz^*}\frac{\partial \phi^f_{bz^*}}{\partial \kappa}\right) < 0 \\
\frac{\partial Var\,(b)}{\partial \kappa^*} &= 2\sigma^2\left(\phi^f_{bz}\frac{\partial \phi^f_{bz}}{\partial \kappa^*} + \phi^f_{bz^*}\frac{\partial \phi^f_{bz^*}}{\partial \kappa^*}\right) < 0
\end{aligned}$$

And wrt output

$$Var\,(y) = \left(\left(\phi^f_{yz}\right)^2 + \left(\phi^f_{yz^*}\right)^2\right)\sigma^2$$

implying that

$$\begin{aligned}
\frac{\partial Var\,(y)}{\partial \kappa} &= 2\sigma^2\left(\phi^f_{yz}\frac{\partial \phi^f_{yz}}{\partial \kappa} + \phi^f_{yz^*}\frac{\partial \phi^f_{yz^*}}{\partial \kappa}\right) \\
\frac{\partial Var\,(y)}{\partial \kappa^*} &= 2\sigma^2\left(\phi^f_{yz}\frac{\partial \phi^f_{yz}}{\partial \kappa^*} + \phi^f_{yz^*}\frac{\partial \phi^f_{yz^*}}{\partial \kappa^*}\right)
\end{aligned}$$

Note that in a symmetric equilibrium

$$\phi^f_{yz} > \phi^f_{yz^*}$$

and

$$\left|\frac{\partial \phi^f_{yz}}{\partial \kappa}\right| = \left|\frac{\partial \phi^f_{yz^*}}{\partial \kappa^*}\right| > \left|\frac{\partial \phi^f_{yz}}{\partial \kappa^*}\right| = \left|\frac{\partial \phi^f_{yz^*}}{\partial \kappa}\right|$$

Use this to show that

$$\frac{\partial Var\,(y)}{\partial \kappa} > 0 \quad ; \quad \frac{\partial Var\,(y)}{\partial \kappa^*} < 0$$

Appendix E: Cooperative policies

We assign equal weights to both countries and consider policy rule that solves

$$\min_{\kappa,\kappa^*} L^c = \frac{1}{2}\left[Var\left(b\right)+\varphi Var\left(g\right)\right]+\frac{1}{2}\left[Var\left(b^*\right)+\varphi^* Var\left(g^*\right)\right]$$

Assuming that the countries are symmetric in steady state, the optimal cooperative policy rules will be identical. Hence the problem can be reformulated as

$$\min_{\kappa,\kappa^*} L^c|_{\kappa=\kappa^*} = \left[Var\left(b\right)+\varphi Var\left(g\right)\right]|_{\kappa=\kappa^*}$$

We now impose the cooperation-condition $\kappa = \kappa^*$ as well as symmetry in steady state on the coefficients. We have

$$\begin{aligned}
\phi^c_{\tau y} &= -\phi^c_{\tau y^*} = \frac{\widehat{\mu}}{1-\eta_{sg}\kappa} \\
\frac{\partial \phi^c_{\tau y}}{\partial \kappa} &= -\frac{\partial \phi^c_{\tau y^*}}{\partial \kappa} = \frac{1-\mu\left(1-\eta_{sg}\right)}{\left(1-\eta_{sg}\kappa\right)^2}
\end{aligned}$$

Moreover

$$\begin{aligned}
\phi^c_{bz} &= \frac{\alpha\phi^c_{\tau y}\eta_{sz}-\eta_{sz}\eta_{s\tau}\left(\phi^c_{\tau y}\right)^2}{1-2\phi^c_{\tau y}\eta_{s\tau}} \\
\phi^c_{bz^*} &= -\frac{\left(\alpha-1\right)\phi^c_{\tau y}\eta_{sz}+\eta_{s\tau}\eta_{sz}\left(\phi^c_{\tau y}\right)^2}{1-2\phi^c_{\tau y}\eta_{s\tau}}
\end{aligned}$$

So the derivatives are

$$\begin{aligned}
\frac{\partial \phi^c_{bz}}{\partial \kappa} &= \frac{\alpha\eta_{sz}-2\eta_{sz}\eta_{s\tau}\phi^c_{\tau y}+2\eta_{sz}\eta^2_{s\tau}\left(\phi^c_{\tau y}\right)^2}{\left(1-2\phi^c_{\tau y}\eta_{s\tau}\right)^2}\frac{\partial \phi^c_{\tau y}}{\partial \kappa} \\
\frac{\partial \phi^c_{bz^*}}{\partial \kappa} &= \frac{-\left(\alpha-1\right)\eta_{sz}-2\eta_{s\tau}\eta_{sz}\phi^c_{\tau y}+2\eta_{sz}\eta^2_{s\tau}\left(\phi^c_{\tau y}\right)^2}{\left(1-2\phi^c_{\tau y}\eta_{s\tau}\right)^2}\frac{\partial \phi^c_{\tau y}}{\partial \kappa}
\end{aligned}$$

Now, with respect to y we have

$$\phi^c_{yz} = \frac{\eta_{sz}+\eta_{s\tau}\phi^c_{\tau z}}{1-\eta_{sg}\kappa} \quad ; \quad \phi^c_{yz^*} = -\frac{\eta_{s\tau}\phi^c_{\tau z}}{1-\eta_{sg}\kappa}$$

Where

$$\phi^c_{\tau z} = \frac{\phi^c_{\tau y}\eta_{sz}}{1-2\eta_{s\tau}\phi^c_{\tau y}} \quad ; \quad \frac{\partial \phi^c_{\tau z}}{\partial \kappa} = \frac{\eta_{sz}\frac{\partial \phi^c_{\tau y}}{\partial \kappa}}{\left(1-2\eta_{s\tau}\phi^c_{\tau y}\right)^2}$$

Derivatives

$$\begin{aligned}
\frac{\partial \phi^c_{yz}}{\partial \kappa} &= \frac{\left(1-\eta_{sg}\kappa\right)\eta_{s\tau}\frac{\partial \phi^c_{\tau z}}{\partial \kappa}+\eta_{sg}\left(\eta_{sz}+\eta_{s\tau}\phi^c_{\tau z}\right)}{\left(1-\eta_{sg}\kappa\right)^2} \\
\frac{\partial \phi^c_{yz^*}}{\partial \kappa} &= -\frac{\left(1-\eta_{sg}\kappa\right)\eta_{s\tau}\frac{\partial \phi^c_{\tau z}}{\partial \kappa}+\eta_{sg}\eta_{s\tau}\phi^c_{\tau z}}{\left(1-\eta_{sg}\kappa\right)^2}
\end{aligned}$$

In a coperative equilibrium we have

$$
\begin{aligned}
b &= \phi^c_{bz} z + \phi^c_{bz^*} z^* \quad ; \quad Var\,[b] = \left[(\phi^c_{bz})^2 + (\phi^c_{bz^*})^2 \right] \sigma^2 \\
y &= \phi^c_{yz} z + \phi^c_{yz^*} z^* \quad ; \quad Var\,[y] = \left[(\phi^c_{yz})^2 + \left(\phi^c_{yz^*}\right)^2 \right] \sigma^2
\end{aligned}
$$

Imposing the policy rule on the loss function we obtain

$$\min \left. L^c \right|_{\kappa=\kappa^*} = \left. \left[Var\,(b) + \varphi \kappa^2 Var\,(y) \right] \right|_{\kappa=\kappa^*}$$

with the corresponding first order condition

$$\left. \frac{\partial Var(b)}{\partial \kappa} \right|_{\kappa=\kappa^*} + \varphi \left(\kappa^2 \left. \frac{\partial Var(y)}{\partial \kappa} \right|_{\kappa=\kappa^*} + 2\kappa Var(y) \right) = 0$$

All of the above can be used to determine the optimal cooperative policy rule numerically.

Appendix F: Comparison of volatility under the two wage-setting regimes

Terms of trade variability
We have

$$\phi^f_{\tau z} = \frac{\phi^f_{\tau y} \eta^f_{sz}}{1 - 2\phi^f_{\tau y} \eta^f_{s\tau}} \quad ; \quad \phi_{\tau z} = \frac{\phi_{\tau y} \eta_{sz}}{1 - 2\phi_{\tau y} \eta_{s\tau}}$$

so

$$
\begin{aligned}
\frac{\phi^f_{\tau z}}{\phi_{\tau z}} &= \frac{\phi^f_{\tau y} \eta^f_{sz}}{\phi_{\tau y} \eta_{sz}} \frac{1 - 2\phi_{\tau y} \eta_{s\tau}}{1 - 2\phi^f_{\tau y} \eta^f_{s\tau}} \\
&= \frac{1 - \eta_{sg}\kappa}{1} \frac{1 - \beta\mu(1-\epsilon)}{1-\beta} \frac{1 - 2\frac{\widehat{\mu}}{1-\eta_{sg}\kappa} \frac{\beta(\alpha-1)(1-\epsilon)}{1-\beta\mu(1-\epsilon)}}{1 - 2\frac{\widehat{\mu}}{1} \frac{\beta(\alpha-1)}{1-\beta}} \\
&= \frac{\left(1 - \eta_{sg}\kappa\right)(1 - \beta\mu(1-\epsilon)) - 2\widehat{\mu}\beta(\alpha-1)(1-\epsilon)}{1 - \beta - 2\widehat{\mu}\beta(\alpha-1)} \\
&= \frac{1 - \beta\widehat{\mu}(1-\epsilon) - 2\widehat{\mu}\beta(\alpha-1)(1-\epsilon)}{1 - \beta - 2\widehat{\mu}\beta(\alpha-1)} \\
&= \frac{1 - \beta\widehat{\mu}(1-\epsilon)(2\alpha-1)}{1 - \beta\left(1 + 2\widehat{\mu}\left(\alpha-1\right)\right)}
\end{aligned}
$$

Now turn to the volatility of the private consumption bundle. We have

$$\begin{aligned}
\phi_{bz} &= \frac{\alpha - \phi_{\tau y}\eta_{s\tau}}{1-2\phi_{\tau y}\eta_{s\tau}}\phi_{\tau y}\eta_{sz} > 0 \\
\phi_{bz^*} &= -\frac{\left(\alpha - 1 + \eta_{s\tau}\phi_{\tau y}\right)}{1-2\phi_{\tau y}\eta_{s\tau}}\phi_{\tau y}\eta_{sz} > 0 \\
\phi^f_{bz} &= \frac{\alpha - \phi^f_{\tau y}\eta^f_{s\tau}}{1-2\phi^f_{\tau y}\eta^f_{s\tau}}\phi^f_{\tau y}\eta^f_{sz} > 0 \\
\phi^f_{bz^*} &= -\frac{\left(\alpha - 1 + \eta^f_{s\tau}\phi^f_{\tau y}\right)}{1-2\phi^f_{\tau y}\eta^f_{s\tau}}\phi^f_{\tau y}\eta^f_{sz} > 0
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{\phi^f_{bz}}{\phi_{bz}} &= \frac{\left(\alpha - \phi^f_{\tau y}\eta^f_{s\tau}\right)\phi^f_{\tau y}\eta^f_{sz}}{\left(\alpha - \phi_{\tau y}\eta_{s\tau}\right)\phi_{\tau y}\eta_{sz}}\frac{1-2\phi_{\tau y}\eta_{s\tau}}{1-2\phi^f_{\tau y}\eta^f_{s\tau}} = \frac{\left(\alpha - \phi^f_{\tau y}\eta^f_{s\tau}\right)}{\left(\alpha - \phi_{\tau y}\eta_{s\tau}\right)}\frac{\phi^f_{\tau z}}{\phi_{\tau z}} \\
&= \frac{\left(\widehat{\mu}\beta\left(\alpha-1\right)+\alpha\left(\beta-1\right)\right)\left(1-\widehat{\mu}\beta\left(1-\epsilon\right)\right)\left(1-\beta\widehat{\mu}\left(2\alpha-1\right)\left(1-\epsilon\right)\right)}{\left(1-\beta\right)\left(\alpha-\beta\widehat{\mu}\left(2\alpha-1\right)\left(1-\epsilon\right)\right)\left(2\widehat{\mu}\beta\left(\alpha-1\right)+\left(\beta-1\right)\right)}
\end{aligned}$$

It is not clear whether in general this is larger than 1. However, for $\mu = 1$ and $\kappa = 0$ the expression becomes

$$\left.\frac{\phi^f_{bz}}{\phi_{bz}}\right|_{\mu=1,\kappa=0} = \frac{\left(1-\beta\left(1-\epsilon\right)\right)\left(\alpha+\beta\left(1-2\alpha\right)\right)\left(1+\beta\left(1-2\alpha\right)\left(1-\epsilon\right)\right)}{\left(1-\beta\right)\left(1+\beta\left(1-2\alpha\right)\right)\left(\alpha+\beta\left(1-2\alpha\right)\left(1-\epsilon\right)\right)} > 1 \text{ for } \epsilon > 0$$

Now consider the response to the foreign shock. We have

$$\begin{aligned}
\frac{\phi^f_{bz^*}}{\phi_{bz^*}} &= \frac{\left(1-\alpha-\eta^f_{s\tau}\phi^f_{\tau y}\right)\phi^f_{\tau y}\eta^f_{sz}}{\left(1-\alpha-\eta_{s\tau}\phi_{\tau y}\right)\phi_{\tau y}\eta_{sz}}\frac{1-2\phi_{\tau y}\eta_{s\tau}}{1-2\phi^f_{\tau y}\eta^f_{s\tau}} = \frac{\left(1-\alpha-\eta^f_{s\tau}\phi^f_{\tau y}\right)}{\left(1-\alpha-\eta_{s\tau}\phi_{\tau y}\right)}\frac{\phi^f_{\tau z}}{\phi_{\tau z}} \\
&= \frac{\left(1+\beta\widehat{\mu}\left(1-\epsilon\right)\left(1-2\alpha\right)\right)\left(1-\widehat{\mu}\beta\left(1-\epsilon\right)\right)\left(1-\beta\left(1-\widehat{\mu}\right)\right)}{\left(1+\beta\left(2\widehat{\mu}\left(1-\alpha\right)-1\right)\right)\left(1-\beta\right)}
\end{aligned}$$

Again we consider the case where $\mu = 1$ and $\kappa = 0$

$$\left.\frac{\phi^f_{bz^*}}{\phi_{bz^*}}\right|_{\mu=1,\kappa=0} = \frac{\left(1+\beta\left(1-\epsilon\right)\left(1-2\alpha\right)\right)\left(1-\beta\left(1-\epsilon\right)\right)}{\left(1+\beta\left(1-2\alpha\right)\right)\left(1-\beta\right)} > 1 \text{ for } \epsilon > 0$$

Since $\phi^f_{bz} > \phi_{bz}$ and $\phi^f_{bz^*} > \phi_{bz^*}$ for $\mu = 1$ and $\kappa = 0$ we can conclude that

$$Var(b) < Var(b^f) \qquad \text{for } \mu = 1, \kappa = 0$$