DISCUSSION PAPER SERIES

No. 3546

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FINANCIAL ECONOMICS and INDUSTRIAL ORGANIZATION

Centre for Economic Policy Research

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ISSN 0265-8003

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Discussion Paper No. 3546
September 2002

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CEPR Discussion Paper No. 3546

September 2002

ABSTRACT

The Performance of Optimally Diversified Firms: Reconciling Theory and Evidence

We construct an equilibrium model of firm diversification to show that the main empirical findings about firm diversification and performance are consistent with the maximization of shareholder value. In our model, diversification allows a firm to explore better productive opportunities while taking advantage of synergies. By explicitly linking the diversification strategies of the firm to differences in size and productivity, our model provides a natural laboratory to quantitatively investigate several aspects of the relationship between diversification and performance. Specifically, we show that our model is able to rationalize both the evidence on the diversification discount (Lang and Stulz (1994)) and the observed relation between diversification and firm productivity (Schoar (2002)).

JEL Classification: D21, G32 and G34
Keywords: diversification discount, firm diversification, Tobin's Q and total factor productivity

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Submitted 15 August 2002

1 Introduction

Empirical work on firm diversification has often been interpreted as supporting the view that conglomerates are inefficient. Findings such as the fact that conglomerates trade at a discount, relative to a portfolio of comparable stand-alone firms, have led researchers to believe that diversification destroys value.[1] Popular explanations for this "diversification discount" have generally emphasized the agency and behavioral problems associated with the existence of conglomerates.[2] Unfortunately, this view of diversification creates at least two difficulties for researchers. First, while addressing the effects of diversification on performance, agency models often fail to answer the more fundamental economic question of why diversified firms exist at all, as diversification usually is ex-ante inefficient. Second, the empirical predictions of these agency-based models are usually very hard to quantify and thus quite difficult to test. As a consequence, direct evidence supporting this agency view is quite limited. Instead, their support typically comes from the perceived failures of the competing theories.

In this paper we show that the main empirical regularities about firm diversification are broadly consistent with the neoclassical view of efficient firm diversification. In our model, firms diversify for two reasons. First, diversification allows firms to take advantage of economies of scope by eliminating redundancies across different activities and lowering fixed costs of production. Second, diversification allows a mature, slow growing firm to explore attractive new productive opportunities. We formalize this concept by assuming that production activities exhibit decreasing returns to scale. As scale grows returns decrease, eventually leading the firm to search for profit opportunities in new activities.

In contrast to standard agency arguments, the structure of our model provides a natural environment to investigate *quantitatively* the role of firm diversification on performance. Since the model generates an artificial cross-sectional distribution of firms, we are able to directly compare our results with the available empirical evidence.

[1]See Wernefelt and Montgomery (1988), Lang and Stulz (1994), Berger and Ofek (1995), Campa and Kedia (1999), Rajan, Servaes, and Zingales (2000), Graham, Lemmon, and Wolf (2001), Whited (2001), and Lamont and Polk (2001, 2002) among others.

[2]See Jensen (1986), Amihud and Levy (1981), Jensen and Murphy (1990), Shleifer and Vishny (1989), and Stulz (1990), Denis, Denis and Sarin (1997), and Scharfestein and Stein (2000) among others.

We have two main sets of findings. First, the model predicts that diversified firms have, on average, a lower value of Tobin's Q than focused firms, as documented by Lang and Stulz (1994). This happens despite the fact that diversification is optimal and there is no source of inefficiency in our model. The intuition, however, is simple. In our model, firms diversify only when they become relatively unproductive in their current activities. It is this *endogenous selection* mechanism that accounts for the lower valuation of diversified firms. Second, because our model explicitly links productivity with corporate diversification, we can also address recent evidence on the effects of diversification on productivity (Schoar (2002)). We find that, just as in the data, our model predicts that firms following diversification strategies also experience empirically plausible productivity losses.

This emphasis on the importance of firm selection in accounting for the performance of conglomerates effectively presents a theoretical foundation for the recent empirical findings by Chevalier (1999), Villalonga (2001), Graham, Lemmon and Wolf (2002), and Campa and Kedia (2002). Although their exact sources and methodologies differ, all of these papers are part of a growing empirical literature suggesting that sample selection accounts for most, if not all, of the ex-post differences between conglomerates and specialized firms. More broadly, our work is also part of a recent strand of literature that emphasizes a neoclassical view of optimal resource allocation in determining the observed pattern of diversification.[3]

This paper builds on work by Gomes and Livdan (2002), who first formalize a dynamic model of optimal firm diversification, capturing the same ideas of economies of scope and decreasing returns. That paper was focused on providing a detailed characterization of the optimal diversification decision of the firm under very general conditions. By embedding our original environment into an industry equilibrium environment, we are able to provide a complete characterization of the behavior of the cross-sectional distribution of firms, thus creating a framework naturally suited to address most of available evidence.[4]

Finally, our work also offers a useful framework to study the natural boundaries of the

[3] See Matsusaka (2001), Maksimovic and Phillips (2001), and Bernardo and Chowdhry (2002) for example.

[4] Hopenhayn (1992) introduces most of the necessary theroretical concepts for this class of models. Recently, industry equilibrium models have been used to investigate properties of the cross-sectional distribution of firms such as the relationship between Tobin's Q, investment and cash flow (Gomes (2001)), the role of financial variables in determining firm size and growth (Cooley and Quadrini (2001)), and the value premium in stock returns (Zhang (2002)).

firm in the context of a neoclassical environment. While our model is silent about the exact micro-foundations for the interactions between (and within) firms (for example, internal capital markets, incomplete contracts, and power relationships within contracts), it provides something of a reduced form approach that is well suited for detailed empirical study, again a serious difficulty in this field of research.

The rest of this paper is organized as follows. Section 2 details the basic economic environment and discusses our main assumptions. Section 3 establishes our main empirical implications. Section 4 concludes.

2 Model

The economy consists of 2 sectors: households and firms. The core of the analysis is our description of the production sector, where a large number of firms is engaged in the production of the consumption good. The role of households is limited and summarized by a single representative household making optimal consumption and portfolio decisions.

2.1 Firms

The production side of the economy consists of a large number of firms and two separate industries or sectors. While the model can be augmented to include more sectors, this would make the analysis unnecessarily complicated. Empirically, the effects of diversification on performance are most notable when firms first expand from one to two segments, with additional expansions have only marginal effects on performance (Lang and Stulz (1994)).

Description

We assume that time is discrete and the horizon is infinite. In each time period t, a firm can either be focused in sector $s_t = 1, 2$ or operate in both sectors simultaneously, in which case we will say that a firm is diversified and set $s_t = 1 + 2 = 3$. We assume that sectoral mobility is costly so that specialized firms cannot simply move all resources from sector 1 to sector 2 (say). Formally, we assume that:

$$s_t \in \begin{cases} \{s_{t-1}, 3\}, & s_{t-1} = 1, 2 \\ \{1, 2, 3\}, & s_{t-1} = 3 \end{cases} . \tag{1}$$

In other words, a firm that has previously been focused in sector s can only choose to remain in sector s ($s_t = s_{t-1}$), or to expand to both sectors ($s_t = 3$). Diversified firms, however, face no restrictions: they can either remain diversified, or they can contract and focus on just one industry. This costly mobility ensures that a firm must diversify before focusing on entirely new activities, a pattern that is consistent with the data.[5]

The outcome of production in sector s, during period t, is the final good y_t^s. For simplicity, we assume that the goods are perfect substitutes so that the relative price between y_t^1 and y_t^2 is always equal to 1. Production in either sector requires two inputs: capital or productive capacity, k_t, and labor, l_t, and is subject to a firm, and sector, specific technology shock z_t^s. Labor is hired at the competitive wage rate $W_t > 0$, but capacity is owned by the firm. Production possibilities for an individual firm operating in sector s are described by a Cobb-Douglas production function:

$$y_t^s = e^{z_t^s} k_t^{\alpha_k} l_t^{\alpha_l}, \qquad 0 < \alpha_k + \alpha_l < 1, \tag{2}$$

where α_k and α_l are the output elasticities of capital and labor, respectively. The restrictions on these coefficients guarantee that production in each sector exhibits decreasing returns to scale, so that returns fall as the firm grows.

Productivity levels for the firm are described by the vector $z_t = (z_t^1, z_t^2)$, and are assumed to follow the joint stationary VAR(1) process:

$$\begin{bmatrix} z_t^1 \\ z_t^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} z_{t-1}^1 \\ z_{t-1}^2 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{bmatrix}, \tag{3}$$

where ε_t^1 and ε_t^2 are firm-specific (uncorrelated) normal random variables with mean zero and variance σ^2. For the sake of simplicity we assume that there is no cross-correlation between the shocks in the two sectors.

Finally, total firm capacity is described by the law-of-motion

$$k_{t+1} = (1 - \delta) k_t + i_t, \tag{4}$$

where i_t denotes gross investment spending, and δ is the depreciation rate of capital. Thus, new investment, i_t, only becomes productive at the beginning of next period.

[5] This assumption seems plausible but it is not crucial. It will, however, make it easier to construct Figure 1 below and to gain some intuition for our results.

The timing of events is as follows. Every firm arrives at period t with a pre-chosen level of capacity k_t. Before any activity takes place the firm observes the (firm-specific) vector of productivity levels in both sectors, z_t. With this information at hand, each firm makes the following choices during the period t:

- the optimal sectoral decision for the current period, s_t, by choosing whether to operate one ($s_t = 1$ or 2) or both ($s_t = 3$) production units in period t;
- the optimal allocation of capital and labor across its activities;
- how much to invest for the future, i_t, and, as a consequence, the total amount of capacity to install at the beginning of the next period, k_{t+1}.

A firm that chooses to focus its activities in sector s_t alone generates the following profits during period t:

$$\pi(s_t, k_t, z_t; W_t) = \max_{l_t} \left\{ e^{z_t^s} k_t^{\alpha_k} l_t^{\alpha_l} - W_t l_t - f \right\}, \qquad s_t = 1, 2 \tag{5}$$

where $f \geq 0$ is a fixed cost of production that must be paid if the firm is active in sector s. Conversely, if the firm chooses to be diversified (so that $s_t = 3$), profits are described by:

$$\begin{aligned} \pi(3, k_t, z_t; W_t) &= \max_{l_t, \theta_t} \Big\{ e^{z_t^1} (\theta_t k_t)^{\alpha_k} (\theta_t l_t)^{\alpha_l} + e^{z_t^2} ((1-\theta_t) k_t)^{\alpha_k} ((1-\theta_t) l_t)^{\alpha_l} \\ &\quad -W_t l_t - (1-\lambda) 2f \Big\}, \\ s.t. \qquad 0 &\leq \theta_t \leq 1, \end{aligned} \tag{6}$$

where θ_t denotes the fraction of resources (capital and labor) that the diversified firm allocates to sector 1 in period t.[6] Because diversified firms operate in both sectors, they face larger fixed costs of production. However, equation (6) embeds our assumption that they can eliminate redundancies and thus save a fraction λ of the combined costs. Thus, a conglomerate pays only fixed costs in the amount $(1-\lambda) \times 2f$.

The solution to these static optimization problems yields optimal decision rules for total firm employment, $l_t = \mathbf{l}(s_t, k_t, z_t; W_t)$, the size of each segment, $\theta_t = \boldsymbol{\theta}(s_t, k_t, z_t; W_t)$, as well as total production, $y_t = \mathbf{y}(s_t, k_t, z_t; W_t)$.

[6] Since wages and prices do not differ across sectors, capital-labor ratios must also be identical. It follows that the conglomerate must allocate the same share of capital and labor inputs to each sector.

Discussion

Our environment is constructed to incorporate the basic incentives for the creation of conglomerates identified by the literature on firm diversification. Somewhat loosely our model emphasizes some of the most popular advantages of firm diversification: "synergies" and the exploration of "free" cash flows.

Synergies are created through the elimination of redundancies across business lines, such as overhead. In our model, this feature is captured by the savings parameter λ. Such dilution of costs generates a form of economies of scope and creates an incentive for diversification.

Decreasing returns to scale in each activity generate something like a "free cash flow" effect: as the firm grows in size, marginal productivities fall and it becomes unprofitable for the firm to invest additional resources in on-going activities. Instead, the firm can better use resources by exploring new production possibilities. Thus, diversification is more likely to be optimal for large firms, since it enables them to overcome the decreasing returns nature of the single sector technology. This feature is also consistent with the empirical observation that large firms are much more likely to become diversified.

In addition to these two core advantages, conglomerates also benefit from two additional features of our environment. They have more options than stand-alone firms (the mobility restriction (1)). Although this is not a crucial feature of our model, it stands in contrast to Bernardo and Chowdry (2002), who rationalize the diversification discount by assuming that focused firms have more options than conglomerates. Finally, since the productivity shocks z_t^1 and z_t^2 are not perfectly correlated (as in equation (3) above), firm diversification also lowers cash flow risk. In the absence of trading frictions, however, this risk pooling is easy to replicate with a portfolio of stand-alone firms and is not valued by investors.

Synergies and overcoming decreasing returns, however, generate value to shareholders. In each of these cases production is more efficient and resources are saved, when operations are combined in a conglomerate. Hence, unlike much of the literature, our model captures some of the most plausible benefits to corporate diversification while abstracting from any of its potential drawbacks, such as those induced by agency or behavioral problems.

We believe that emphasizing these advantages of the conglomerates is important because it ensures that a model does not deliver a diversification discount "by assumption". Since conglomerates have generally more resources and better opportunities in our model, their low valuation can only be the *endogenous* outcome of self-selection and not the obvious consequence of assuming that focused firms are, a priori, better. As a number of recent studies suggest, this explanation seems to consistent with the available evidence.

Optimality

Let (s, k, z) denote the state for a firm that was active in sector s in period $t-1$, has k units of installed capacity at the beginning of period t, and faces a vector of productivity shocks z. The optimal behavior of this firm can be summarized by the value function $v(s, k, z; W)$, that solves the dynamic programming problem:

$$v(s, k, z; W) = \max_{\{k', s'\}} \left\{ \pi(s', k, z; W) + (1-\delta)k - k' + \beta \int v(s', k', z'; W') N(dz'|z) \right\} \tag{7}$$

subject to equation (1).[7] Here $0 < \beta < 1$ is the intertemporal discount factor and $N(dz'|z)$ is the cumulative (Gaussian) distribution of z', conditional on z. Note that current cash flows (dividends) are given by current profits net of investment spending, which is described by (4). Equations (5) and (6) show that profits depend on the current sectoral decision of the firm, s', the beginning of period capital stock, k, and the current period shocks z. The (discounted) expected continuation value, depends on the *current* decisions about capital accumulation, k' and sectoral decision, s', as well as the future productivity shocks z'.

The solution to the dynamic programming problem (7) produces a set of policy functions, $\mathbf{k}(s, k, z; W)$ and $\mathbf{s}(s, k, z; W)$, associated with the optimal accumulation of capital and the sectoral choices of the firm. It is straightforward to show that all these functions are well defined. It is also immediate to show that the value function $v(s, k, z; W)$ is: (i) bounded, (ii) continuous, and (iii) increasing in both k and z. While these properties are standard, it is worth noting that the value function is always increasing in both technology shocks. Intuitively, even a focused firm benefits from a high level of productivity in the other sector since this increases the option value associated with a possible future diversification.

[7] We use the convention s', k', z', etc. to denote the value of the state variables that are relevant at the beggining of the next period.

Optimal Firm Diversification

Although our ultimate goal is to explicitly compare the quantitative implications of the model with existing empirical evidence, it is useful to briefly explore some of the inner workings of our model, and thus gain some intuition for our numerical results below. Accordingly, this section attempts to shed some light on the optimal diversification decision of an individual firm.

The optimal industrial decision, $s' = \mathbf{s}(s,k,z)$, can be computed as follows. First, define the function

$$p(s',k,z) \equiv \pi(s',k,z) + (1-\delta)k + \max_{k'} \left\{ \beta \int v(s',k',z')N(dz'|z) - k' \right\} \tag{8}$$

as the value of the firm, *conditional* on having adopted sectoral decision s' in the current period. Since focused firms are not allowed to simply switch sectors, a firm that was previously specialized in sector $s \in \{1,2\}$, will find corporate diversification optimal if, and only if:[8]

$$p(3,k,z) \geq p(s',k,z) \mid_{s'=s} = p(s,k,z) \tag{10}$$

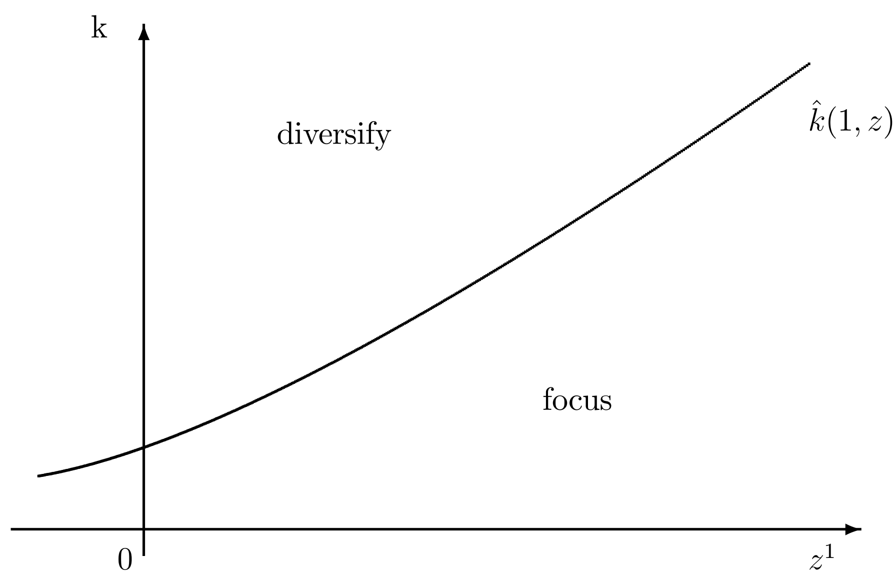
Figure 1 illustrates the shape of the optimal sectoral decision, $\mathbf{s}(1,k,z)$, for a firm that was previously focused in sector 1. Remember that a firm entering the current period as focused in sector 1 has only two choices: it can either remain in sector 1, or diversify, and operate in both sectors simultaneously. Figure 1 shows the contour line of the optimal sectoral decision, holding the level of z^2 fixed. Points along this line correspond to combinations of productivity, z, and size, k, for which the firm is indifferent between focusing and diversifying. Effectively, this line is something like an "indifference" curve for the firm. We will refer to it simply as the optimal diversification threshold.

[8] For a firm that was diversified in the previous period ($s = 3$) diversification remains optimal provided that:

$$p(3,k,z) \geq \max\{p(1,k,z), p(2,k,z)\} \tag{9}$$

Figure 1: The Diversification Threshold

This Figure illustrates the shape of the optimal sectoral decision, $s(1, k, z)$, for a firm that was previously focused in sector 1. The horizontal axis shows capacity, k, and the vertical axis shows the level of productivity in on-going activities, z^1. Since the firm was previously focused in sector 1, it has only two choices: it can either remain in sector 1 in the current period, or it can diversify and operate in both sectors simultaneously. The Figure shows the contour line of the optimal sectoral decision, holding the level of productivity in the other sector, z^2, fixed. Points along this line correspond to combinations of productivity, z, and size, k, for which the firm is indifferent between focusing and diversifying.



Gomes and Livdan (2002) show that, under very general conditions on the form of the production function and the conditional distribution of shocks, this threshold is always upward slopping for a previously focused firm. Intuitively this implies that, given size, firms are more likely to remain focused when productivity is high in their incumbent sector, z^1. Conversely, diversification becomes optimal only when on-going activities are so unproductive that z^1 falls below its threshold value.

It is this *endogenous* selection feature of our model that drives several of our results below, in particular our findings of a diversification discount in the cross-section of firms. Hence, our model effectively formalizes the argument proposed in several empirical studies (see Chevalier (1999), Villalonga (2001), Graham, Lemmon and Wolf (2002), and Campa and Kedia (2002)), that conglomerates are not simply a random subsample of the cross-sectional distribution of firms. Instead, because the decision to diversify is endogenous, it is often associated with ex-ante differences in firm-specific features such as productivity and size. It is those ex-ante features that account for the ex-post findings about the performance and valuation of conglomerates.

2.2 Aggregation

To compute the equilibrium in our model, we need to aggregate the individual decisions of every firm in the economy. Since each firm can be described by its current individual state (s, k, z), the cross-sectional distribution of firms is completely summarized by a measure, $\mu(s, k, z)$, defined over this state space. Simply put, $\mu(s, k, z)$ denotes the mass of firms in the state (s, k, z). It follows that the law of motion for μ is given by:

$$\mu'(s', k', z') = \int \mathbf{1}_{\{k'=\mathbf{k}(s,k,z;W)\}} \times \mathbf{1}_{\{s'=\mathbf{s}(s,k,z;W)\}} N(z'|dz)\mu(s, k, z), \tag{11}$$

where $\mathbf{1}_{\{.\}}$ is an indicator function that equals 1 if the argument is satisfied and 0 otherwise. Intuitively, next period's cross-sectional distribution of firms is determined by combining the exogenous transition probabilities implied by $N(\cdot)$ with the endogenous ones, prescribed by the optimal policies for capacity, $\mathbf{k}(s, k, z)$, and sectoral choices, $\mathbf{s}(s, k, z)$. Naturally we will be interested in computing a *stationary* equilibrium, where $\mu' = \mu$.

Given information on the cross-sectional distribution of firms, we can compute aggregate quantities by directly integrating the relevant individual firm decisions. Accordingly, we can define the aggregate demand for labor, total profits, and the aggregate investment, as[9]

$$\mathbf{L}(\mu; W) = \sum_s \int \mathbf{l}(s, k, z; W)\mu(s, dk, dz), \tag{12}$$

$$\mathbf{I}(\mu; w) = \sum_s \int (\mathbf{k}(s, k, z; W) - (1 - \delta)k)\, \mu(s, dk, dz) \tag{13}$$

$$\mathbf{\Pi}(\mu; W) = \sum_s \int \boldsymbol{\pi}(s, k, z; W)\mu(s, dk, dz). \tag{14}$$

Aggregate dividends (or aggregate cash flows) are then given by

$$\mathbf{D}(\mu; W) = \mathbf{\Pi}(\mu; W) - \mathbf{I}(\mu; w). \tag{15}$$

2.3 Households

To close the model, we must offer a description of market demand for the final goods produced, as well as the supply of labor input. It is easy to provide simple reduced form expression for these functions. However, it is more interesting and rather straightforward to show how this can be done in a general equilibrium context by adding a very stylized description of household/shareholder behavior. Specifically, we summarize the household sector with a single representative agent deriving utility from leisure and consumption and income from wages and dividends. Without aggregate uncertainty, all aggregate quantities and prices are constant and the consumer problem collapses to a simple static representation:

$$\max_{C,H} U = \ln(C - AH) \tag{16}$$
$$s.t. \qquad C = WH + \mathbf{D}(\mu; W).$$

The optimality conditions for this problem yield a demand for final goods given by $C = \mathbf{C}(\mu; W)$ and an infinitely elastic labor supply which pins down the wage rate at $W = A$.[10]

[9] Note that $\pi(s', k, z; W) = \pi(\mathbf{s}(s, k, z), k, z; W)$, or simply $\pi(s, k, z; W)$. Similarly, we can also write $\mathbf{l}(s, k, z; W)$.

[10] It is easy to show that this problem is formally equivalent to that of a shareholder investing in the stocks of individual firms (see Gomes (2001) for details), provided that her discount factor is also β.

2.4 Equilibrium

The following definition characterizes the stationary equilibrium for this economy.

Definition 1 (Stationary Equilibrium) *A stationary competitive equilibrium is: (i) a set of decision rules* $\mathbf{k}(s,k,z)$, $\mathbf{l}(s,k,z)$ *and* $\mathbf{s}(s,k,z)$ *as well as a value function* $v(s,k,z;W)$ *for each firm; (ii) an allocation rule* $\mathbf{C}(\mu;W)$ *for the representative household; (iii) aggregate quantities* $\mathbf{L}(\mu;W)$, $\mathbf{I}(\mu;W)$, $\mathbf{\Pi}(\mu;W)$ *and* $\mathbf{D}(\mu;W)$*;(iv) a wage rate* W *and (v) a measure* μ *of firms such that:*

(a) the firm decision rules and value function solve (7) for each firm;

(b) the consumer decision rules solve (16);

(c) conditions (12)-(15) are satisfied and the distribution μ *obeys the law of motion (11), with* $\mu = \mu'$.

(d) market clearing

$$\mathbf{C}(\mu;A) = A\mathbf{L}(\mu;A) + \mathbf{D}(\mu;A). \tag{17}$$

Equation (17) also summarizes labor market equilibrium, by imposing $W = A$. Given our assumptions, establishing the existence of a stationary competitive equilibrium is immediate.[11] Although the definition seems abstract and its computation is non-trivial, this equilibrium concept is the key to our analysis. It delivers a non-degenerate cross-sectional distribution of firms, μ, which provides us with an artificial dataset of firms of different size, productivities, and more importantly, diversification strategies. With this information at hand we are ready to address the key empirical findings in this area.

3 Quantitative Results

Computing the equilibrium described in Definition 1 involves two main steps. First, we must specify values for all the relevant parameters. These must be selected to be consistent with either long run properties of the data (usually unconditional first moments) or with prior empirical evidence. Second, we need to develop and implement a numerical algorithm capable

[11] The proof follows the arguments provided in Hopenhayn (1992) and Gomes (2001).

of approximating the stationary equilibrium up to an arbitrarily small error. Appendix A describes this procedure in detail. With the equilibrium computed, we focus on two key empirical issues. Section 3.2 investigates the model's implications for the so-called "diversification discount", by comparing our predictions with the results from Lang and Stulz (1994). Since our model implies that diversification is driven by productivity differentials, it is important to investigate its predictions for the relation between firm diversification and productivity. Section 3.3 explores this issue by comparing our results with recent empirical evidence in Schoar (2002).

3.1 Calibration and Summary Statistics

Since most data is available at an annual frequency, we assume that a time period in the model corresponds to one year. The calibration exercise is divided in two parts. First, we use independent evidence on the degree of returns to scale (Burnside's (1996)) to set the output elasticities $\alpha_l = 0.65$ and $\alpha_k = 0.3$. The rate of depreciation in the capital stock is set to 0.1, a value close to that found in the data by Gomes (2001).

The four remaining parameters, f, λ, σ, and a, cannot be individually identified from the available data. Instead, they are chosen so that the model is able to approximate the *unconditional* moments on the panel studied by Lang and Stulz (1994) for Compustat. Since the main stylized facts are, in effect, *conditional* moments, or regressions, from this panel, this seems appropriate. Accordingly, we select these parameters so that the model approximates the cross-sectional mean and dispersion of Tobin's Q, the number (percentage) of diversified firms in the sample, and the average level of Q for conglomerates.[12]

Table 1 summarizes our calibration procedure while Table 2 compares the key summary statistics generated by the stationary equilibrium of the model with those of the Compustat dataset used by Lang and Stulz (1994). Although our model calibration does not reproduce these four statistics exactly, the artificial sample is reasonably similar to its empirical counterpart, particularly in terms of cross-sectional dispersion and the relative weight of conglomerates in the sample, the two crucial elements for statistical inference.

[12] The preference parameters are not important and we simply use $\beta = 1/1.065$ and $A = W = 0.5$.

Table 1: **: Parameter Choices**

This table reports our parameter choices. The time period is one year. Output elasticities, α_k and α_l are set using evidence from Burnside (1996). The rate of depreciation for the capital stock, δ, is set close to the value found by Gomes (2001). The four remaining parameters, f, λ, σ, and a are choosen so that the model approximates four unconditional moments from the COMPUSTAT panel studied by Lang and Stulz (1994). The moments are the mean and standard deviation of Tobin's Q, the number (percentage) of diversified firms in the sample and the average level of Tobin's Q for conglomerates.

Parameter	Benchmark Value
Technology	
α_k	0.3
α_l	0.65
δ	0.1
f	0.002
λ	0.6
Shocks	
σ	0.025
a	0.95

Table 2: **: Summary Statistic**

This Table compares the summary statistics generated by the stationary equilibrium of the model, given the parameter choices in Table 1, with those of the COMPUSTAT panel studied by Lang and Stulz (1994) and reported in Table 1 of their paper.

Statistics	Data	Model
Fraction Focused Firms	0.40	0.33
Tobin's Q		
Average	1.11	1.87
Standard Deviation	1.22	1.11
Average (Conglomerates)	0.91	1.56

3.2 Diversification Discount

Most empirical studies on the efficiency of conglomerates examine the relation between diversification and firm value, as measure by Tobin's (average) Q. Specifically, this is often done by estimating linear reduced form equations:

$$Q_{it} = b_0 + b_1 DIV_{it} + b_2 \ln(k_{it}) + \xi_{it}, \tag{18}$$

where Q_{it} is the value of Tobin's Q for firm i at the beginning of period t, k_{it} is the beginning of period size of the firm, and DIV_{it} is a dummy variable that takes a value of one if firm is diversified in period t and zero otherwise.

In the context of our model, it is straightforward to estimate equation (18) for our artificial panel of firms by defining the variables:

$$Q = \frac{p(s', k, z)}{k},$$

and

$$DIV = \begin{cases} 1, & \text{if } s' = 3 \\ 0, & \text{else} \end{cases},$$

where $p(s', k, z)$ denotes the value of the firm of size k that chooses to operate in sector s in period t.

Table 3 compares the results of estimating (18) in our model with the empirical findings in Lang and Stulz (1994). In all cases we report the means across 100 simulations, for both the coefficients and the corresponding t-statistics. As in Lang and Stulz (1994), Table 3 reports results for both the full panel and a subset that includes only those firms with a value of Q below 5.[13]

Overall the model performs very well. As in the data, we consistently find that diversified firms are discounted, that this discount is statistically significant, and that this is only partially accounted for by differences in firm size (the coefficient on $\ln(k)$). Moreover, the model also predicts a diversification discount that is *quantitatively* similar to that found in the real data.

[13] When possible we focus on the numbers reported by Lang and Stulz (1994) for "industry-adjusted" q's, since these control for the fact that diversified firms are generally concentrated in low-q industries.

Table 3: **: The Diversification Discount**

This Table reports the results of estimating the following regression:

$$Q_{it} = b_0 + b_1 DIV_{it} + b_2 \ln(k_{it}) + \xi_{it},$$

on our artificial panel of firms. Here Q_{it} is the value of Tobin's Q for firm i at the beginning of period t, k_{it} is the beginning of period size of the firm, and DIV_{it} is a dummy variable that takes value one if firm is diversified in period t and zero otherwise. The results of this estimation are then compared with the empirical findings from Table 6 in Lang and Stulz (1994). In all cases we report the means across 100 simulations, for both the coefficients and the corresponding t-statistics. The Table also reports our findings for the subset of firms for which the value of Q is below 5, and compares those with the results in Lang and Stulz (1994).

	All Firms		**Q < 5**	
Variable	**Data**	**Model**	**Data**	**Model**
DIV	-0.34	-0.20	-0.29	-0.07
(t-stat)	(-3.77)	(-5.39)	(-4.53)	(-3.71)
$\log(k)$	-0.12	-0.70	-0.13	-0.31
(t-stat)	(-3.48)	(-5.26)	(-5.22)	(-5.29)

Looking solely at the subset of firms with a value of Q below 5 shows that the observed diversification discount is not due to a small number of outliers. Table 3 confirms that in the model, as in the data, eliminating outliers does decrease the discount's magnitude but it does not eliminate it. Although smaller, the coefficient on the diversification dummy is significant, both statistically and economically.

Thus, despite the fact that conglomerates operate efficiently and that diversification clearly adds value to the firm, our model is able to rationalize the documented diversification "discount". Moreover, this discount also seems to possess the same robustness properties that are observed in the actual data. Since diversification is optimal however, the explanation cannot be that conglomerates destroy value. Instead, the success of the model hinges on the endogenous selection mechanism identified in section 2.1.

It is important to note that our results accord with the view that conglomerates are indeed less efficient firms. Crucially however, they are not *inefficient*. In particular, and as long as $\lambda > 0$, separation of their units destroys shareholder value.

Finally, the exact magnitude of the discount depends on synergies created by the conglomerate, measured by the parameter λ. Indeed, if these synergies are too large, the discount may disappear altogether. We view this dependence as an important strength of the model and a useful direction for future research. For instance, allowing λ to vary across firms could rationalize recent evidence suggesting that the magnitude of the discount seems to vary with the level of synergies created by diversification (for example Chevalier (2000)).

Source of the Diversification Discount

Following Lang and Stulz (1994), Tables 4 and 5 attempt to shed light on the source of the discount. We focus on two subsamples of the full panel of firms: on-going conglomerates and newly diversified firms. Table 4 reports the results of estimating (18) for the subsample of firms that do not change the number of segments in which they operate. Specifically, we consider only the set of firms for which $s_t = s_{t-1} = ... = s_{t-4}$, thus excluding all newly diversified (as well as refocused) firms from the sample. As Table 4 documents, however, excluding these newly diversified firms does not eliminate the observed discount both in the data and in the model. Moreover, the actual value of the discount in our model is again very close to that observed by Lang and Stulz (1994).

By contrast, Table 5 looks at the behavior of firms that change the numbers of segments of activity across adjacent years. Specifically, these firms are classified as "diversifying", if they change the number of sectors they operate in from one to two (formally $s_{t-1} = 1$ or 2 and $s_t = 3$) and "focusing" firms if they reduce the number of activities from 2 to 1 ($s_{t-1} = 3$ and $s_t = 1$ or 2). These firms are then compared with those that maintained the number of activities constant during the same period. For "diversifying" firms, the comparison group is the set of other previously focused firms that chose not to become diversified in the current period. Similarly, focusing firms are compared with other diversified firms that chose to remain diversified.

Following Lang and Stulz (1994) we report two alternative results. First, we look at the average differences in Q at the time that the firms choose to expand (or contract). Next, we also look at the dynamic effects of the decision, by comparing the effects of diversification (refocusing) on ΔQ.

The findings are somewhat inconclusive, both in the model and in the data. Whether we look at levels or changes in Q, no coefficient is statistically significant. Although there is some suggestion, again both in the model and the data, that diversifying firms seem to experience drops in Q (while the opposite happens for focusing firms) the evidence is just not strong enough. The model's implications for the level of Q are somewhat less successful, but again not statistically significant.

Overall, Tables 4 and 5 broadly confirm the empirical success of our model. It is not only capable of generating a diversification discount, but also provides quantitatively realistic results for the subsets of existing and newly diversified firms.

Table 4: **: Firms With Constant Segments**

This Table reports the results estimating the regression:

$$Q_{it} = b_0 + b_1 DIV_{it} + b_2 \ln(k_{it}) + \xi_{it},$$

on our artificial panel of firms. Here Q_{it} is the value of Tobin's Q for firm i at the beginning of period t, k_{it} is the beginning of period size of the firm, and DIV_{it} is a dummy variable that takes value one if firm is diversified in period t and zero otherwise. The regression is performed only on the subsample of firms that do not change the number of segments in which they operate for a number of years. Specifically, we consider only firms for which $s_t = s_{t-1} = ... = s_{t-4}$. In all cases we report the means across 100 simulations, for both the coefficients and the corresponding t-statistics. The results of this estimation are then compared with the empirical findings from Table 8 in Lang and Stulz (1994).

Variable	**Data**	**Model**
DIV	-0.20	-0.17
(t-stat)	(-2.05)	(-3.14)
$\ln(k)$	-0.03	-0.66
(t-stat)	(-0.64)	(-3.48)

Table 5: **: Firms Changing Segments**

This Table compares firms that change the numbers of segments of activity across adjacent years with those firms that maintain the number of activities constant. Specifically, firms are classified as "diversifying" if they change the number of sectors they operate from one to two (formally $s_{t-1} = 1$ or 2 and $s_t = 3$). We provide two separate results. First, we look at the average differences in Q at the time of the diversification takes place by estimating the regression:

$$Q_{it} = b_0 + b_1 DIV_{it} + \xi_{it},$$

for the subset of previouslly focused firms ($s_{t-1} < 3$). Here Q_{it} is the value of Tobin's Q for firm i at the beginning of period t, and DIV_{it} is a dummy variable that takes value one if firm has been focused at $t-1$ and becomes diversified in period t and zero otherwise. Next, we look at the dynamic effects of diversification, by comparing the effects of diversification on ΔQ. We accomplish that by estimating the following regression:

$$\Delta Q_{it} = b_0 + b_1 DIV_{it} + \xi_{it},$$

again only for the subset of previouslly focused firms. Here $\Delta Q_{it} = Q_{it} - Q_{it-1}$. This Table also reports the effects of refocusing on firm value, by estimating the same regressions as above, and letting DIV_{it} equal one if firm has been diversified at $t-1$ and becomes focused in period t and zero otherwise. These regressions are only estimated for the subset of previouslly diversified firms. In all cases we report the means across 100 simulations, for both the coefficients and the corresponding t-statistics. The results of this estimation are then compared with the empirical findings from Table 8 in Lang and Stulz (1994).

	Regression on Q_t		Regression on $\Delta Q_t = Q_{t+1} - Q_t$	
Variable	**Data**	**Model**	**Data**	**Model**
		Diversifying Firms		
DIV	-0.163	0.045	-0.204	-0.038
(t-stat)	(-1.23)	(0.37)	(-1.60)	(-1.46)
		Focusing Firms		
DIV	-0.016	0.035	0.024	0.020
(t-stat)	(-0.70)	(1.40)	(1.39)	(1.22)

3.3 Diversification and Productivity

In our model productivity differentials play a key role in determining firm behavior and the observed link between diversification and firm valuation. In this section, we investigate whether the implied movements in firm and sectoral productivity are also consistent with existing empirical evidence. In a recent study, Schoar (2002) carefully documents the productivity patterns in manufacturing using the LRD database. Specifically, she computes Total Factor Productivity (TFP) for each plant, j, in each firm, i, and every period, t, by estimating the residual, ε_{ijt}, in the following log-linear Cobb-Douglas production function:

$$\ln(y_{ijt}) = a_{jt} + b_{jt} \ln(k_{ijt}) + c_{jt} \ln(l_{ijt}) + \varepsilon_{ijt}, \tag{19}$$

Given this measure of productivity, we can examine the relation between firm diversification and firm productivity. Schoar (2002) focuses on two measures. First, she seeks to capture *static* differences in *average* productivity across firms by estimating the following equation:

$$TFP_{ijt} = a_1 + b_1 \times SEG_{it} + \mu_{ijt}. \tag{20}$$

where SEG_{it} is the logarithm of the number of segments in which firm i operates in period t. Thus, estimating $b_1 > 0$ implies that diversified (multi-segment) firms are, on average, more productive than focused firms. In addition, she also examines the *dynamic* effects of diversification on *future* productivity. This is accomplished by estimating the equation:

$$TFP_{ijt} = a_2 + b_2 \times AFTER_{it} + \nu_{ijt}. \tag{21}$$

where $AFTER_{it}$ is defined as a dummy variable that equals one in the period after the firm diversifies and it is equal to zero otherwise.[14] Thus, a finding of $b_2 > 0$ implies that diversification *improves* plant productivity.

It is again relatively straightforward to use the artificial panel of firms generated by our model to replicate Shoar's (2002) procedures and compare the results. Given our measures of capital, labor and output and assuming that each activity corresponds to one plant we

[14] Schoar (2002) also adds variables such as age and the number of segments the firm operates. In the context of our model, however, age is not defined and the number of segments is redundant.

can easily estimate (19-21). Table 6 compares our findings with the results in Tables II and IV from Schoar (2002).

While Schoar (2002) finds a significant productivity *premium* of more than 3% for diversified firms, our model implies that focused firms are, on average, 2.3% more productive. However, this result depends on the magnitude of the diversification discount, since lower productivity leads to lower valuations. This is important since in Schoar's LRD sample the average market discount for diversified firms is only about 10%, while our model, which is calibrated to replicate the Lang and Stulz's (1994) results, implies a discount of about 20%. The last column of Table 6 reconciles these findings by focusing on the subsample of firms with Q below 5, where the observed discount is of only 7% (see Table 3). In this case, we find that our model can also match the observed productivity *premium* for conglomerates.

Table 6 also shows that our model successfully reproduces the observed *losses* of productivity *after* the firm diversifies.[15] As Schoar (2002) argues, these findings reinforce the importance of distinguishing between the static effect of *being* diversified and the dynamic effect of *becoming* diversified. From a static, or cross-sectional, point-of-view, diversified firms are, on average, more productive than focused firms. However, as Figure 1 illustrates, diversification in our model is often the result of bad productivity shocks in on-going activities. Thus, it is not surprising to find that, on average, diversification is associated with productivity losses in incumbent sectors, just as Schoar (2002) finds.[16]

These results suggest that our basic argument that diversification decisions are driven by efficient responses to productivity differentials is not the result of assuming unrealistic patterns for productivity. The fact that our model is consistent with much of the evidence also suggests a possible alternative interpretation to the more popular "new toy effect" that emphasizes a shift in focus by managers towards the newly acquired segments at the expense of incumbent ones. Our findings show that this evidence can also be rationalized in the context of a value maximizing model.

[15] Here, our results can only be compared with Schoar's (2002) estimates for incumbent plants since, in our model, new plants have no prior history.

[16] Firms may also diversify if the diversification threshold moves because outside opportunities inprove. In this case productivity in the incumbent sector need not fall for these firms.

Table 6: **: Diversification and Productivity**

This Table compares of our findings with the results in Tables II and IV from Schoar (2002). First, we capture *static* differences in *average* productivity across firms by estimating the equation:

$$TFP_{ijt} = a_1 + b_1 \times SEG_{it} + \mu_{ijt}.$$

where TFP_{ij} and SEG_{it} denote, respectively, total factor productivity in segment j and the logarithm of the number of segments in which firm i operates in period t. Second, the *dynamic* effects of diversification on *future* productivity are summarized with the regression:

$$TFP_{ijt} = a_2 + b_2 \times AFTER_{it} + \nu_{ijt}.$$

where $AFTER_{it}$ defined as a dummy variable that equals one in the period after the firm diversifies and it is equal to zero otherwise. Results are reported for the full sample and for the subset of firms with $Q < 5$. In both cases we report the means across 100 simulations, for both the coefficients and the corresponding t-statistics.

Variable	**Data**	**Model**	
		All Firms	$Q < 5$
SEG	0.034	−0.023	0.018
(t-stat)	(2.13)	(−22.38)	(17.45)
$AFTER$	−0.026	−0.013	−0.007
(t-stat)	(−6.50)	(−7.81)	(−0.61)

4 Conclusion

In this paper we argue that the main empirical regularities about firm diversification are broadly consistent with the neoclassical view of efficient firm diversification. In our model, firms diversify for two reasons. First, diversification allows firms to take advantage of economies of scope, by eliminating redundancies across different activities and lowering fixed costs of production. Second, diversification allows a mature, slow growing, firm to explore attractive new productive opportunities. We formalize this concept by assuming that production activities exhibit decreasing returns to scale. As scale grows returns decrease eventually leading the firm to search for opportunities in new activities.

The paper addresses two key empirical findings about the performance of diversified firms - the "diversification discount" and the "productivity premium". The model proves to be extremely successful in explaining both of them, as well as a number of complementary findings.

Moreover, the source of this empirical success lies in the *endogenous selection* of conglomerates. Accordingly, our model effectively provides a theoretical foundation for several recent empirical studies (for example Chevalier (1999), Villalonga (2001), Graham, Lemmon and Wolf (2002), and Campa and Kedia (2002)) that emphasized this property of the data.

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A Appendix

The computational strategy involves the following steps

1. Solving the Bellman Equation (7) and computing the optimal firm decision rules;

2. Using the optimal decision rules to iterate on (11) and compute the stationary measure $\mu = \mu' = \mu^*$

3. Computing aggregate quantities and using the market clearing condition (17) to determine the equilibrium levels of consumption and labor.

Given the properties of our problem, the first step is better implemented with the less efficient but more robust method of value function iteration on a discrete state space. We specify a grid with a finite number of points for the capital stock as well as a finite approximation to the normal random vector z. The later task is accomplished using in Tauchen and Hussey's (1991) method for optimal discrete state space approximations to normal random variables. We use 15×15 grid points for this procedure. The space for the capital stock is divided in 201 equally spaced elements. In either case the results were relatively unchanged when we use finer grids. The upper bound for capacity, k, was chosen to be non-binding at all times.

To compute μ^*, we take the optimal value function $v(s, k, z)$ and the decision rules $\mathbf{k}(s, k, z)$ and $\mathbf{s}(s, k, z)$, as well as the stochastic process for the technology shocks z and proceed as follows:

- Define the size of the panel data, by specifying the number of firms M and the length of time T.

- Simulate a sequence of exogenous technology shocks $z_{it} = (z_{it}^1, z_{it}^2)$ for each firm i in every period t.

- For the initial period

(i) Initiate each firm's capital stock at $k = k_0$.

(ii) Start the simulation by using draws from a uniform distribution to randomly allocating firms to either sector 1 or 2.

- For all other periods

(i) Given the current state for each firm i, $(s_{it-1}, k_{it}, z_{it})$ use the optimal policy functions to determine next period's capital stock, k_{it+1}, and sectoral decision, s_{it}.

(ii) Using the value function, compute the current market value of the firm i, v_{it}.

(iii) Using the stochastic process for z, compute next period's shock z_{it+1}.

(v) Construct the cross-sectional distribution of firms $\mu_{it} = \mu(s_{it}, k_{it}, z_{it})$.

- Continue the simulation until $\left|\left|\mu_{it} - \mu_{it+1}\right|\right| < \varepsilon$.

Using the stationary distribution, μ, it is then straightforward to compute the aggregate (constant) quantities for labor demand, L, profits, Π, investment, I, and dividends, D from the equilibrium functions (12)-(15). With these numbers at hand it is also immediate to use the goods market condition to obtain aggregate consumption.