# DISCUSSION PAPER SERIES

No. 3542

**REGIONAL SPECIALIZATION AND TRANSPORT COSTS**

Takatoshi Tabuchi and
Jacques-François Thisse

***INTERNATIONAL MACROECONOMICS***

*ISSN 0265-8003*

# REGIONAL SPECIALIZATION AND TRANSPORT COSTS

**Takatoshi Tabuchi,** University of Tokyo
**Jacques-François Thisse,** CERAS, Paris, CORE, Université Catholique de Louvain and CEPR

Discussion Paper No. 3542
September 2002

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 3542

September 2002

# ABSTRACT

## Regional Specialization and Transport Costs*

We consider an economic geography model in which all firms and workers are mobile, but the agglomeration of firms and workers within a region generates urban costs. We show that industries with high transport costs tend to be more agglomerated than industries with low transport costs. This is to be contrasted to the result obtained in the one-industry case in which agglomeration arises for low transport costs. We also show that firms supplying non-tradable consumer services are more agglomerated than firms belonging to light industries. In this case, the equilibrium involves an urban hierarchy: for each good, a larger array of varieties is produced within the same city.

JEL Classification: F12, F16, J60, L13 and R12
Keywords: agglomeration, interregional mobility, intersectoral mobility, transport costs and urban costs

Takatoshi Tabuchi
University of Tokyo
Hongo 7-3-1
Bunkyo-ku
Tokyo 113-0033
JAPAN
Tel: (81 3) 5841 5603
Fax: (81 3) 5841 5521
Email: ttabuchi@e.u-tokyo.ac.jp

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=156606

Jacques-François Thisse
CORE
Université Catholique de Louvain
34 Voie du Roman Pays
B-1348 Louvain-la-Neuve
BELGIUM
Tel: (32 10) 474312
Fax: (32 10) 474301
Email: thisse@core.ucl.ac.be

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=104425

*This Paper is produced as part of a CEPR Research Network on ‘Specialization Versus Diversification: The Microeconomics of Regional Development and the Spatial Propagation of Macroeconomic Shocks in Europe’, funded by the European Commission under the Research Training Network Programme (Contract No: HPRN-CT-2000-00072).

Submitted 14 August 2002

# 1 Introduction

One of the most unsatisfactory aspects of the new economic geography is the way the agricultural (or immobile) sector is described (Krugman, 1991; Ottaviano, Tabuchi and Thisse, 2002). Farmers are not allowed to move between regions and sectors, whereas the good they produce can be shipped at zero cost.[1] More importantly, this sector must be sufficiently large for dispersion to arise as an equilibrium outcome; otherwise there is always agglomeration. This makes the core-periphery model somewhat awkward to deal with the fast-growing mobility of production factors. The relevance of an immobile sector becomes, indeed, more and more questionable in a world experiencing an increasing mobility of all factors. In this respect, the reader should keep in mind that the reason for the agricultural sector in the core-periphery model is to act as a dispersion force, while permitting trade imbalances in the industrial good. The role of such a sector is clearly declining in industrialized countries and another dispersion force, more in line with the evolution of modern economies, should be considered. This is the main purpose we want to achieve in this paper.

To this effect, we disregard the agricultural sector and assume that *all workers and firms are mobile*. As in Tabuchi (1998), the dispersion force rests on urban costs that rise with the size of the population established within the same region. Such a setting, which combines the mobility of industry and the existence of urban costs, strikes us as being more suitable to study modern economies of the 21st century than is the core-periphery model. It is worth mentioning right away that this setting leads to results that significantly differ from those obtained in the standard core-periphery model. Even though the general pattern of one industry against declining transport and commuting costs remains the same, it appears that *heavy industries are likely to be more concentrated than light industries*. In addition, product differentiation acts here as a dispersion force because price competition is no longer a dispersion force once all consumers are mobile. Quite the opposite: *price competition fosters agglomeration* because workers are to be compensated for the high urban costs associated with the emergence of a cluster.

Another drawback of the core-periphery model is the systematic focus on the industry as a whole.[2] This implies that this model is not able to cope

[1]See, however, Fujita, Krugman and Venables (1999, Chapter 7) as well as Puga (1999) for some extensions along these lines.

[2]See, however, the noticeable exceptions of Fujita, Krugman and Mori (1999), Venables

with specific industries displaying different spatial patterns. The second contribution of this paper is precisely to study *the location of several industries once it is recognized that all workers can change places*. Specifically, we consider two industries that differ only in the cost of shipping their output and show how such a difference may affect the location of the two industries. We want to stress the fact that this modeling strategy, though restrictive, is very much in the tradition of Weber's (1909) and Lösch's (1940) location theory in which different transport costs may explain why firms belonging to different industries obey different locational patterns (see Hoover (1948) for more details). Therefore, it is fair to say that our paper also aims at connecting "classical" location theory and "new" economic geography.

By allowing for several industries to seek location, we open the door to a new and richer sets of spatial configurations in which the industrial composition of regions is endogenous. In particular, we will see that commodity-specific transport costs may lead to equilibria in which one sector is agglomerated, whereas the other is partially dispersed. Besides the extreme cases of agglomeration and dispersion, we show that *industries with high transport costs tend to be more agglomerated than industries with low transport costs*. This seemingly counterintuitive result is illustrated in the special case of an industry producing varieties whose transport costs are negligible, whereas the other industry produces a nontradeable good. Spatial patterns involving some degree of regional specialization typically emerge as equilibrium outcomes, depending on the relative value of transport and commuting costs. However, complete regional specialization occurs only in very special cases. Instead, regions are diversified in that they involve firms belonging to each industry.

Finally, our analysis of the two-industry case shows that results obtained in the case of one industry do not necessarily carry over to several industries. As a consequence, policy recommendations based on standard economic geography models have to be applied with extreme caution.

The remainder of the paper is organized as follows. The model is introduced in section 2. Instead of using the Dixit-Stiglitz-iceberg framework, we retain the alternative model developed by Ottaviano *et al.* (2002) because it leads to analytical results. In section 3, we consider the case of a single sector and show how the prediction of our model can be compared with the ones derived from the standard core-periphery model. Even though the gen-

(1999), and Laussel and Paul (2002).

eral pattern against declining transport costs is the same, it turns out that changes in the key-parameters of the core-periphery model, namely the fixed cost level and the degree of product differentiation, lead to very different predictions. The case of two industries with perfect mobility between sectors and regions is considered in section 4. We show that different locational patterns involving full dispersion, partial regional specialization, or full agglomeration may emerge as equilibrium outcomes. However, allowing for both regional and sectorial mobility of workers renders the analysis especially complex and prevents a full analysis of the equilibrium configurations. This leads us to investigate, in section 5, the special but meaningful case of a light industry with negligible transport costs and of a business-to-consumer industry with prohibitive transport costs. Section 6 concludes.

# 2 The model

Consider an economy formed by a population of $L$ mobile workers and by two regions, denoted $H$ and $F$. There are $n+2$ goods. The first one is homogenous and available as an endowment; it is chosen as the numéraire. There are $n$ differentiated goods made available under the form of a continuum of varieties, each variety being supplied by a single firm producing under increasing returns. The common fixed cost is denoted by $\phi$ whereas the marginal cost is set equal to zero.

In the case of one differentiated good ($n=1$), the utility function of a worker is as follows:[3]

$$\begin{aligned} U(q_0; q(j), j \in [0,N]) &= \alpha \int_0^N q(j)dj - \frac{\beta-\gamma}{2}\int_0^N [q(j)]^2 dj \\ &\quad -\frac{\gamma}{2N}\left[\int_0^N q(j)dj\right]^2 + q_0 \end{aligned} \tag{1}$$

in which $q(i)$ stands for quantity of variety $i$ and $q_0$ for the quantity of the numéraire, whereas $\alpha$, $\beta$ and $\gamma$ are three positive parameters such that $\beta > \gamma$.

In the case of $n$ differentiated goods, the utility (1) is extended as follows:[4]

[3]This utility is the one proposed by Vives (1990), which slightly differs from that used by Ottaviano *et al.* (2002). It has been chosen for analytical convenience.

[4]Note that (2) is equivalent to (1) when $n=1$.

$$U(q_0; q_i(j), j \in [0, N_i], i = 1, \ldots, n) = \sum_{i=1}^{n} \left[ \alpha \int_0^{N_i} q_i(j) dj \right.$$
$$\left. - \frac{(\beta - \gamma) N_i}{2 \sum_{k=1}^{n} N_k} \int_0^{N_i} [q_i(j)]^2 dj - \frac{\gamma}{2 \sum_{k=1}^{n} N_k} \left( \int_0^{N_i} q_i(j) dj \right)^2 \right] + q_0 \quad (2)$$

In the bracketed terms of (2), the nonlinear terms are weighted by the relative size of each sector $N_i / \sum_{k=1}^{n} N_k$. This assumption is made to capture the idea that, everything else being equal, an industry with a small range of varieties has less impact on the consumer well-being than an industry with large array of varieties.

When $\beta > \gamma$, (2) encapsulates both a *preference for diversity* between the different goods as well as a *preference for variety* across varieties of the same good. Assume, first, that an individual consumes a given mass of $Q_i$ units of good $i$ $(= 1, \ldots, n)$ and that consumption of good $i$ is uniform and equal to $Q_i/x_i$ on $[0, x_i]$ and zero on $(x_i, N_i]$. Evaluating (2) at this consumption pattern yields

$$\begin{aligned} U &= \sum_{i=1}^{n} \left[ \alpha \int_0^{x_i} \frac{Q_i}{x_i} dj - \frac{(\beta - \gamma) x_i}{2 \sum_{k=1}^{n} x_k} \int_0^{x_i} \left( \frac{Q_i}{x_i} \right)^2 dj \right. \\ &\qquad \left. - \frac{\gamma}{2 \sum_{k=1}^{n} x_k} \left( \int_0^{x_i} \frac{Q_i}{x_i} dj \right)^2 \right] + q_0 \\ &= \sum_{i=1}^{n} \left[ \alpha Q_i - \frac{\beta Q_i^2}{2 \sum_{k=1}^{n} x_k} \right] + q_0 \end{aligned} \quad (3)$$

which is strictly increasing in $x_i$ for each $i = 1, ..., n$ when $\beta > \gamma$. Hence, $x_i = N_i$ must hold for each good $i$, that is, each consumer prefers to consume all the varieties of each of the $n$ available goods. Assume now that the total consumption $Q = \sum_{i=1}^{n} Q_i$ is fixed. Then, maximizing (3) with respect to $Q_i$ subject to $Q = \sum_{i=1}^{n} Q_i$ yields $Q_i = Q/n$ for $i = 1, ..., n$. In words, each good is equally consumed, and hence each variety of any good is equally consumed.

The last good is land. As in standard urban economics, firms established within the same region are located in the Central Business District (CBD) of a linear city. Each worker living in the region uses one unit of land and commutes to the regional CBD. As shown in Ottaviano *et al.* (2002), the urban costs born by a worker living in region $H$ (resp. $F$) after redistribution of the total land rent are equal to $\theta \sum_{i=1}^{n} \lambda_i L_i/4$ (resp. $\theta \sum_{i=1}^{n}(1-\lambda_i)L_i/4$), where $\theta > 0$ is the commuting cost per unit of distance. Denote by $L_i$ the number of workers in industry $i$, with $\sum_{i=1}^{n} L_i = L$. In such a context, the budget constraint of a worker residing in region $H$, say, is given by

$$\sum_{i=1}^{n}\left[\int_0^{N_i} p(j)q(j)dj + \frac{\theta}{4}\lambda_i L_i\right] + q_0 = \overline{q}_0 + w_H$$

where $N_i = L_i/\phi$ is the number of varieties in industry $i$ $(= 1, \ldots, n)$, $p(j)$ the full price of variety $j$ in region $H$, $\lambda_i$ the fraction of the work force in industry $i$ located in region $H$, $\overline{q}_0$ the worker's initial endowment, and $w_H$ her wage. The initial endowment $\overline{q}_0$ is supposed to be sufficiently large for the equilibrium consumption of the numéraire to be positive for each individual.

Markets are segmented, that is, each firm is able to set a price specific to the market in which its output is sold. This assumption is supported by empirical investigations (Greenhut 1981; Head and Mayer, 2000) and can be given some theoretical justification (Thisse and Vives, 1988). Hence, the profits made by a sector $i$-firm located in region $r = H, F$ are defined as follows:

$$\Pi_{ir} = p_{irr}q_{irr}(p_{irr})\sum_{k=1}^{n}\lambda_k L_k + (p_{irs} - \tau_i)q_{irs}(p_{irs})\sum_{k=1}^{n}(1-\lambda_k)L_k - \phi w_{ir}$$

where $q_{irr}$ (resp. $q_{irs}$) represents the individual demand for good $i$ of a worker living in region $r$ (resp. $s$) and $w_{ir}$ the wage prevailing in sector $i$ and region $r$. As in Krugman (1991), entry and exit are free so that profits are zero in equilibrium. The equilibrium wage in sector $i$ and region $r$ is then obtained from the zero profit condition evaluated at the equilibrium prices.

Clearly, all prices and wages depend on *the distribution of the labor force across sectors* $(L_1, ..., L_n)$ as well as on *the interregional distribution of workers within each industry* ($\lambda_i$ and $1 - \lambda_i$). These distributions depend themselves on the transport costs $\tau_i$ and the commuting costs $\theta$. It is a well-documented fact that both these types of costs have dramatically decreased since the beginning of the Industrial Revolution (Bairoch, 1985), although the commuting costs are often neglected in the literature. Ideally, each of them should be treated independently. However, in order to keep things tractable, we state our results in terms of the ratio between the transport costs $\tau_i$ and the commuting costs $\theta$ and study how the evolution of the spatial economy changes with this ratio.

# 3 The one-sector economy

## 3.1 Market equilibrium

It is worth examining the case of a single industry ($n = 1$) in order to identify its (dis)similarities with the standard core-periphery model. Since the labor market clearing condition is given by $\lambda L = \lambda N/\phi$, any change in the population of workers located in one region must be accompanied by a corresponding change in the number of firms.

Let $a \equiv \alpha/\beta$, $b \equiv 1/(\beta - \gamma)$ and $c \equiv \gamma/[\beta(\beta - \gamma)]$ which are independent of the number $N$ of firms; since $\beta > \gamma$, it must be that $b > c$. It is readily verified that the equilibrium prices and wages corresponding to $\lambda$ are as follows:

$$\begin{aligned}
p^*_{HH} &= \frac{2a + \tau c(1-\lambda)}{2(2b-c)} \qquad p^*_{FF} = \frac{2a + \tau c\lambda}{2(2b-c)} \\
p^*_{HF} &= p^*_{FF} + \frac{\tau}{2} \qquad\qquad p^*_{FH} = p^*_{HH} + \frac{\tau}{2} \\
w^*_H &= \frac{b}{\phi}[(p^*_{HH})^2 \lambda L + (p^*_{HF} - \tau)^2 (1-\lambda) L] \\
w^*_F &= \frac{b}{\phi}[(p^*_{FF})^2 (1-\lambda) L + (p^*_{FH} - \tau)^2 \lambda L]
\end{aligned} \tag{4}$$

Interregional trade occurs if and only if

$$\tau < \tau_{trade} \equiv \frac{2a}{2b-c}$$

Let $\rho \equiv \tau/\theta > 0$ be the ratio of the transport and commuting costs. Observe that $\rho = 0$ implies transport costs are negligible, as in the case of IT-related industries; by contrast, $\rho = \infty$ means that goods are nontradeable, as in the case of service industries.

Since

$$V_H - V_F = \frac{[\phi\,(2b-c)^2/\rho - 4ab\,(3b-c) + b\tau(6b^2 - 6bc + c^2)]\tau L}{2\,(2b-c)^2\,\phi}\,(1/2 - \lambda)$$

$\lambda^* = 1/2$ is always a spatial equilibrium. Furthermore, the symmetric equilibrium is stable if the term in brackets is positive. Otherwise, the industry is agglomerated into a single region, with $\lambda^* = 0, 1$ according to the initial distribution. It is straightforward to compute the corresponding threshold at which the spatial structure changes:

$$\tau^* = \frac{4ab\,(3b-c)\,\rho - \phi(2b-c)^2}{b(6b^2-6bc+c^2)\rho} = \frac{4ab\,(3b-c)\,\tau - \phi(2b-c)^2\theta}{b(6b^2-6bc+c^2)\tau}$$

Because of the existence of urban costs ($\theta > 0$), $\tau^*$ may be negative. It may also exceed $\tau_{trade}$. The unique solution of the equation $\tau^* = 0$ is given by

$$\rho^a \equiv \frac{(2b-c)^2\phi}{4ab(3b-c)}$$

whereas the unique solution of $\tau^* = \tau_{trade}$ is

$$\rho^b \equiv \frac{(2b-c)^3\phi}{2ab(6b^2-4bc+c^2)}$$

which are both independent of $\tau$ and $\theta$. The following result thus holds:

**Proposition 1** *Assume that $\tau < \tau_{trade}$.*

*(i) If $\rho < \rho^a$, then the symmetric configuration is the only stable spatial equilibrium.*

*(ii) If $\rho^a < \rho < \rho^b$, the following configurations may arise: if $\tau > \tau^*$, then the symmetric configuration is the only stable spatial equilibrium; if $\tau < \tau^*$, there are two stable spatial equilibria corresponding to the agglomerated configurations; if $\tau = \tau^*$, then any configuration is a spatial equilibrium.*

*(iii) If $\rho^b < \rho$, then there are two stable spatial equilibria corresponding to the agglomerated configurations.*

Cases (i) and (iii) are obvious: when transport costs are very small (resp. large) relative to commuting costs, the only stable equilibrium involves dispersion (resp. agglomeration). Case (iii) corresponds to the black hole condition in the core-periphery model but case (i) has no counterpart. As expected, equilibrium always involves full dispersion (resp. agglomeration) when $\tau = 0$ and $\theta > 0$ (resp. $\tau > 0$ and $\theta = 0$) because the agglomeration (resp. dispersion) force vanishes. More interesting is case (ii), which exhibits the evolutionary process from dispersion to agglomeration as $\tau$ decreases (together with $\theta$). Thus, we have a model qualitatively similar to Krugman (1991) and Ottaviano *et al.* (2002): improvements in transport technologies induces the spatial transition from dispersion to agglomeration. Those results are illustrated in Figure 1.

Figure 1: The equilibrium pattern in the one-industry case

Next, it is readily verified that

$$\partial\tau^*/\partial c > 0$$

which does not agree with Krugman (1991) and Ottaviano *et al.* (2002). Stated differently, in the absence of an agricultural sector but in the presence of urban costs, the closer the substitutes, the more likely the agglomeration of the industry. Such a seemingly counterintuitive result may be explained as follows. On the one hand, when there is no immobile demand, firms supplying close substitutes have no reason to be dispersed because their demand may be geographically concentrated. That is, *price competition is an agglomeration force when all consumers are mobile*, whereas it is a dispersion force when demand is immobile (d'Aspremont, Gabszewicz and Thisse, 1979). On the other hand, when there are urban costs, workers - hence firms - always have an incentive to relax congestion by moving to the periphery. Accordingly, agglomeration arises only when firms sell close substitutes because the price competition effect is sufficiently strong to compensate the workers for the high urban costs associated with an agglomeration. By contrast, when substitutes are bad, the price competition effect is too weak to make up for the urban costs and dispersion prevails.

The impact of $\phi$ on $\tau^*$ is similar. More precisely, when fixed costs are zero, the industry is always agglomerated in a single region because $\rho^a = \rho^b = 0$. Consequently, we may conclude that, *in the absence of an agricultural sector but in the presence of urban costs, agglomeration is more likely when the degree of product differentiation is low and when fixed costs are low.* Hence, our model predicts that light industries, such as IT-related industries, should move to less populated regions before heavy industries. This is the opposite of what has been obtained in the core-periphery model (Fujita *et al.*, 1999; Ottaviano *et al.*, 2002) and shows how different the various modeling strategies regarding the dispersion force may be.

Specifically, *the agricultural sector and the urban costs play very different roles as dispersion forces.* Immobile demand in the periphery attract firms when the home market effect in the core region is not sufficiently strong. Hence, firms with higher transport costs are then likely to move to the periphery because this allows them to relax price competition. On the other hand, high urban costs in the core region induce firms to move to the periphery. Hence firms with low transport costs are likely to move away from the core because, in doing so, they avoid paying high wages to compensate workers for high urban costs.

Finally, we find it useful to reinterpret our results to deal with the case where $\theta$ is constant whereas $\tau$ steadily decreases in order to make our setting comparable to Helpman (1998). We know that the sign of $d(V_H - V_F)/d\lambda$ changes at $\tau = \tau^*$ or, equivalently, at the solution $\theta(\tau)$ of this equation solved with respect to $\theta$. It is readily verified that $\theta(\tau)$ is a concave parabola passing through the origin and intersecting the $\tau$-axis at a value $\widehat{\tau}$ that exceeds $\tau_{trade}$ (see Figure 1). Clearly, the domain of admissible $(\tau, \theta)$-values inside (resp. outside) this parabola is such that the stable equilibrium involves agglomeration (resp. dispersion) because $d(V_H - V_F)/d\lambda > 0$ (resp. $< 0$). As seen in Figure 1, when urban costs are sufficiently large ($\theta = \theta_H$), the economy always involves dispersion. However, for lower values of $\theta$ such as $\theta = \theta_L$ in Figure 1, *as $\tau$ steadily decreases from $\tau_{trade}$, the economy moves from agglomeration to dispersion at $\tau = \tau^*(\theta)$*, thus confirming the numerical results obtained by Helpman (1998).

## 3.2 Welfare

We now compare the market outcome with the optimal allocation. As usual, in the first best the planner is able to use lump sum transfers (i) in order to assign any number of workers (or, equivalently, of firms) to a specific region and (ii) in order to pay for the loss firms may incur while pricing at marginal cost. Because our setting assumes transferable utility, the planner chooses $\lambda$ in order to maximize the sum of individual indirect utilities:

$$W(\lambda) \equiv V_H(\lambda)\lambda L + V_F(\lambda)(1-\lambda)L \tag{5}$$

in which all prices have been set equal to marginal cost:

$$p^o_{HH} = p^o_{FF} = 0 \qquad \text{and} \qquad p^o_{HF} = p^o_{FH} = \tau \tag{6}$$

thus implying that operating profits and, hence, wages are zero. Maximizing (5) subject to (6) yields $\lambda = 1/2$ as a candidate for the optimum allocation of workers. Examining the second order condition, we see that $W(\lambda)$ is concave (convex) when $\tau$ is larger (smaller) than the threshold

$$\tau^o \equiv \frac{4a\rho - \phi}{(2b - c)\,\rho}$$

Hence, *the first best optimum involves dispersion for $\tau > \tau^o$ and agglomeration for $\tau < \tau^o$.*

Comparing $\tau^*$ and $\tau^o$ does lead to straightforward conclusions. Indeed, when $\rho \in (\rho^a, \rho^b)$,[5] the sign of

$$\tau^* - \tau^o = \frac{4ab^2c\rho - (b-c)\,(2b^2 - 4bc + c^2)\,\phi}{b\,(2b-c)\,(6b^2 - 6bc + c^2)\,\rho}$$

in undetermined and varies with the parameter values. For example, if the varieties are sufficiently differentiated (resp. good substitutes), then $\tau^* < \tau^o$ (resp. $\tau^* > \tau^o$) holds. In the present setting, besides the standard market distortion generated by monopolistic competition, there is something like a congestion externality because workers underpay for the social cost of land. Indeed, the (average) urban cost that enters the indirect utility after redistribution of the aggregate land rent is equal to $\theta\lambda L/4$ whereas the marginal cost is equal to $\theta\lambda L/2$. As a result, it is hard to compare the market and the social outcomes.

In order to identify the impact of these two distortions, we consider the situation in which the total land rent is not redistributed among workers but goes to absentee landlords. In this case, the congestion problem is fixed because the land rent reflects the social cost of land (see Proposition 5.1 in Fujita (1989)). This is easy to understand: the average urban cost is now equal to $\theta\lambda L/2$, i.e. to the marginal cost. The equilibrium threshold $\tau^*$ is no longer valid and the new threshold $\tau^{**}$is obtained from $\tau^*$ by replacing $\rho$ by $\rho/2$ so that

$$\tau^{**} = \frac{4ab\,(3b-c)\,\rho - 2\phi(2b-c)^2}{b(6b^2 - 6bc + c^2)\rho}$$

[5]Note that both the market outcome and the social optimum involve symmetry when $\rho \in (0, \rho^a)$, whereas they involve agglomeration when $\rho \in (\rho^b, \infty)$.

This threshold is still distorted by monopolistically competitive pricing only but not by urban congestion. It is then readily verified that

$$\tau^{**} < \tau^{o}$$

for all $\rho \in (\rho^a, \rho^b)$. This inequality implies that, for intermediate values of the trade costs ($\tau^{**} < \tau < \tau^o$), *the market provides insufficient agglomeration.* Monopolistic competition generates various pecuniary externalities, such as the home market effect, which is a centripetal force, and the price competition effect, which is a centrifugal force. The foregoing inequality shows that the second effect tends to overcome the first one for intermediate values of the trade costs. Thus, we have:

**Proposition 2** *If the aggregate land rent is given to absentee landlords, the market outcome tends to be more dispersed than the socially desirable outcome.*

This is just the opposite of what Ottaviano and Thisse (2002) have obtained. For these authors (like for Krugman (1991)), the main dispersion force lies in the existence of immobile workers, whereas here urban costs explain why dispersion may arise. Furthermore, we also have $\tau^{**} < \tau^{*}$. In other words, as expected, the congestion effect induces the market to provides excessive agglomeration. These results are sufficient to show that the desirability of agglomeration may completely vary with the nature of the dispersion force, thus inviting to be very careful in policy recommendations.

# 4 The two-sector economy

Consider now the case of two industries. There are several meaningful ways to make them asymmetric: $c_1 \neq c_2$ (different degrees of product differentiation), $\phi_1 \neq \phi_2$ (different levels of fixed cost), and $\tau_1 \neq \tau_2$ (different transport costs). In this paper, we focus on the impact of different transport costs with $\tau_1 < \tau_2$, the role of which is strongly emphasized in classical location theory (Beckmann and Thisse, 1986). In other words, the good in industry 1 (resp. 2) has low (resp. high) transport costs. Without loss of generality, we set $\rho_1 \equiv \tau_1/\theta$ and $\rho_2 \equiv \tau_2/\theta$ with $\rho_2 > \rho_1 > 0$. Thus, as the commuting costs $\theta$ goes down, the transport costs $\tau_1$ and $\tau_2$ also decrease according to some given ratios $\rho_1$ and $\rho_2$.

Since workers can change both places and jobs, the labor market clearing conditions imply that

$$N_i = \frac{L_i}{\phi} = \frac{\mu_i L}{\phi} \qquad i = 1, 2$$

where $\mu_i$ is the global, but variable, labor share of industry $i$. Given $L$, $\mu_2 = 1 - \mu_1$ must hold and, hence, there are three endogenous variables to be considered: $\lambda_1$, $\lambda_2$ and $\mu_1$.

An individual working in sector $i$ and residing in region $H$ maximizes (2) under her budget constraint

$$\int_0^{N_1} p_1(j)q_1(j)dj + \int_0^{N_2} p_2(j)q_2(j)dj + \frac{\theta}{4}(\lambda_1 L_1 + \lambda_2 L_2) + q_0 = w_{iH} + \overline{q}_0$$

thus yielding the individual demands for the variety $j$ of good $i$:

$$q_i(j) = \left[a - bp_i(j) + c\frac{P_i}{N_i}\right]\frac{N_1 + N_2}{N_i} \tag{7}$$

where the price index $P_i$ is defined as follows:

$$P_i \equiv \int_0^{N_i} p_i(k)dk$$

It follows from (7) that the demand $q_i(j)$ is affected by $P_i$, which consists of prices of varieties belonging to industry $i$ only. As a result, the cross-elasticity of demand between any two varieties belonging to different industries is zero, but is positive when they belong to the same industry. This is consistent with the classical definition of an industry given by Triffin (1940), even though we focus on the polar case in which there is no direct interaction between the two industries. However, the demand for each variety of good $i$ is negatively affected by the share of the corresponding industry because consumers distribute their consumption over a larger range of varieties. We will see how this connection leads the industries to interact through workers' sectorial mobility, which affects the size of each industry.

Since we allow for both regional and sectorial mobility, there is no obvious way to model the adjustment process of workers. One intuitive dynamics is as follows:

$$\begin{cases} \dot{\lambda}_1 = V_{1H} - V_{1F} \equiv f_1 \\ \dot{\lambda}_2 = V_{2H} - V_{2F} \equiv f_2 \\ \dot{\mu}_1 = [\lambda_1 V_{1H} + (1-\lambda_1)V_{1F}] - [\lambda_2 V_{2H} + (1-\lambda_2)V_{2F}] \equiv f_3 \end{cases} \tag{8}$$

In words, when workers choose a job, they compare the expected interindustry across the two regions. In other words, the intersectional mobility ($f_3$) depends on the expected interindustry utility differential since $\lambda_i V_{iH} + (1 - \lambda_i)V_{iF}$ is the average utility in sector $i$. However, the interregional mobility of a worker ($f_1$ and $f_2$) is driven by the interregional utility differential within each industry (1 and 2) only. Although our dynamics is simultaneous, this difference in modeling the two types of mobility aims at capturing the idea that the choice of a job often precedes the choice of a place. It also allows for some interaction between the two types of mobility. In particular, a change in the population of industry $i$'s workers in one region is no longer accompanied by a corresponding change in the number of firms of that industry.

Some long calculations show that the first two equations may be rewritten as follows:

$$f_1 = C_{11}\,(\theta_{11} - \theta)\,\theta(\lambda_1 - 1/2) + C_{12}\,(\theta_{12} - \theta)\,\theta(\lambda_2 - 1/2)$$

$$f_2 = C_{21}\,(\theta_{21} - \theta)\,\theta(\lambda_1 - 1/2) + C_{22}\,(\theta_{22} - \theta)\,\theta(\lambda_2 - 1/2)$$

where the $C_{ij}$'s and $\theta_{ij}$'s $(i, j = 1, 2)$ are functions of $\mu_1$ given in Appendix 1. Regarding our third equation, we have

$$f_3 = \frac{d_3\mu_1^3 + d_2\mu_1^2 + d_1\mu_1 + d_0}{4(2b-c)^2\mu_1(1-\mu_1)\phi}$$

where the coefficients $d_i$ are defined in Appendix 2.

Finally, for trade to occur in both industries regardless of the distribution of firms, it must be that

$$\theta < \theta_{trade} \equiv \frac{2a}{\rho_2\,(2b-c)}$$

holds for all $\rho_2 > \rho_1 > 0$.

Because the system $f_1 = f_2 = 0$ is linear, it has at least one solution given by $\lambda_1^* = \lambda_2^* = 1/2$, so that there is always an interior equilibrium.[6] Furthermore, there are also equilibria involving corner solutions, the set of which is much richer than what we get in the one-industry case. More precisely, the candidate equilibria we want to discuss are as follows:[7]

($I$) both industries are evenly dispersed: $\lambda_1^* = \lambda_2^* = 1/2$;

($II$) the industry with high transport costs is more agglomerated: $\lambda_1^* < \lambda_2^*$;

($III$) the industry with low transport costs is more agglomerated: $\lambda_1^* > \lambda_2^*$;

($IV$) both industries are agglomerated within the same region: $\lambda_1^* = \lambda_2^* = 1$.

Intuitively, we expect the ($I$)-configuration to be a spatial equilibrium when $\theta$, hence $\tau_1$ and $\tau_2$, is large enough and the ($IV$)-configuration to arise for sufficiently low values of $\theta$ (as in Proposition 1(ii)). However, in the case of two industries, one sector may be agglomerated whereas the other is not for intermediate values of $\theta$ (this corresponds to the ($II$)- and ($III$)-configurations). In this case, we will show below that the market outcome is the ($II$)-configuration.

The nature of trade vastly differs according to the type of configuration that emerges. In the full dispersion case, there is intraindustry as well as interindustry trade, whereas there is no trade in the full agglomeration case, region $F$ being empty. In the remaining cases, there are intraindustry trade as well as interindustry trade.

## 4.1 Spatial and industrial equilibria

In what follows, we focus on the most interesting case in which $\rho_1$ and $\rho_2$ belong to the interval $[\rho_a, \rho_b]$ in which both fully dispersed and agglomerated equilibria exist (see Proposition 1).

Recall that $\rho_a$ is the smallest value of $\rho$ for which there exists a stable

[6]When the linear system is degenerate, there exists other interior equilibria, which are asymmetric.

[7]Without loss of generality, we disregard the candidate equilibria $\lambda_1^* = \lambda_2^* = 0$ and $\lambda_i^* \in (0, 1)$, $\lambda_j^* = 0$, which are just the mirror images of $\lambda_1^* = \lambda_2^* = 1$ and of $\lambda_i^* \in (0, 1)$, $\lambda_j^* = 1$, respectively.

fully dispersed equilibrium

$$(\lambda_1^*, \lambda_2^*, \mu_1^*) = (1/2, 1/2, \widehat{\mu}_1)$$

where

$$\widehat{\mu}_1 \equiv \frac{16a^2 - 16a(b-c)\tau_1 + (8b^2 - 12bc + 5c^2)\tau_1^2}{32a^2 - 16a(b-c)(\tau_1+\tau_2) + (8b^2 - 12bc + 5c^2)(\tau_1^2 + \tau_2^2)} \in [1/2, 1)$$

is the unique solution of the equation $V_{1r} = V_{2r}$ or, equivalently, $w_{1r} = w_{2r}$, with $\lambda_1 = \lambda_2 = 1/2$, for $r = H, F$ and for all $\tau_1 < \tau_2$. Hence, when both industries are fully dispersed, the low transport cost industry attracts a larger share of the work force (when $\tau_1 = \tau_2$, we have $\widehat{\mu}_1 = 1/2$).

Likewise, $\rho_b$ is the largest value of $\rho$ for which agglomeration is an equilibrium

$$(\lambda_1^*, \lambda_2^*, \mu_1^*) = (1, 1, 1/2)$$

When both industries are agglomerated within the same region, the work force is equally split between the two industries because $\mu_1^* = 1/2$ is the unique solution of the equation $f_3 = V_{1H} - V_{2H} = 0$ in which we have plugged $\lambda_1 = \lambda_2 = 1$. This is because the differences in transport costs no longer matter once the two industries are together.

After the fully dispersed and agglomerated equilibria, we analyze asymmetric equilibria involving different forms of regional specialization.

### 4.1.1 Full dispersion

When both industries are fully dispersed such as $(\lambda_1^*, \lambda_2^*) = (1/2, 1/2)$, we know that the equilibrium share of the labor force in industry 1 is given by $\widehat{\mu}_1$. This dispersed equilibrium becomes unstable at the symmetry breaking threshold, which is obtained by computing the Jacobian of (8). For this purpose, let $\theta_1^{**}$ be the larger solution of the second order equation:

$$C_{11}C_{22}\left(\theta_{11} - \theta\right)\left(\theta_{22} - \theta\right) - C_{12}C_{21}\left(\theta_{12} - \theta\right)\left(\theta_{21} - \theta\right) = 0$$

evaluated at $\mu_1 = \widehat{\mu}_1$ and set

$$\theta_2^{**} \equiv \left.\frac{C_{11}\theta_{11} + C_{22}\theta_{22}}{C_{11} + C_{22}}\right|_{\mu_1 = \widehat{\mu}_1}$$

We restrict our attention to the domain of parameters for which

$$0 < \max\{\theta_1^{**}, \theta_2^{**}\} < \theta_{trade}$$

holds so that the symmetry breaking point always exists. In this case, we have the following result.

**Proposition 3** *If* $\theta > \max\{\theta_1^{**}, \theta_2^{**}\}$*, then* $(1/2, 1/2, \widehat{\mu}_1)$ *is a stable equilibrium.*

**Proof:** Computing the Jacobian of (8) and evaluating them at $(1/2, 1/2, \widehat{\mu}_1)$, we see that $\partial f_3/\partial\lambda_1 = \partial f_3/\partial\lambda_2 = 0$. Consequently, the stability conditions become:

$$\begin{aligned} \frac{\partial f_3}{\partial \mu_1} &< 0 \\ \frac{\partial f_1}{\partial \lambda_1} + \frac{\partial f_2}{\partial \lambda_2} &< 0 \\ \frac{\partial f_1}{\partial \lambda_1}\frac{\partial f_2}{\partial \lambda_2} - \frac{\partial f_1}{\partial \lambda_2}\frac{\partial f_2}{\partial \lambda_1} &> 0 \end{aligned}$$

when they are evaluated at $(1/2, 1/2, \widehat{\mu}_1)$. First, it is readily verified that $\partial f_3/\partial \mu_1 < 0$ always holds. Furthermore, we have

$$\begin{aligned} \text{sgn}\left(\frac{\partial f_1}{\partial \lambda_1} + \frac{\partial f_2}{\partial \lambda_2}\right) &= \text{sgn}\left(\theta_2^{**} - \theta\right) \\ \text{sgn}\left(\frac{\partial f_1}{\partial \lambda_1}\frac{\partial f_2}{\partial \lambda_2} - \frac{\partial f_1}{\partial \lambda_2}\frac{\partial f_2}{\partial \lambda_1}\right) &= \text{sgn}\left(\theta - \theta_1^{**}\right) \end{aligned}$$

by definition of $\theta_1^{**}$ and $\theta_2^{**}$. □

In words, full dispersion of the two industries arises when commuting and transport costs are high.

### 4.1.2 Full agglomeration

When both industries are agglomerated such as $(\lambda_1^*, \lambda_2^*) = (1, 1)$, it must be that $\mu_1^* = 1/2$. This is an equilibrium if and only if the following three conditions

$$f_1 \geq 0 \qquad f_2 \geq 0 \qquad f_3 = 0 \tag{9}$$

hold. The stability of this equilibrium is guaranteed because we also have

$$\left.\frac{\partial f_3}{\partial \mu_1}\right|_{\lambda_1=\ \lambda_2=1, \mu_1=1/2} = -\frac{8a^2bL}{(2b-c)^2\,\phi} < 0$$

Let the sustain point associated with full agglomeration be defined as

$$\theta_1^* \equiv \left.\frac{C_{11}\theta_{11} + C_{12}\theta_{12}}{C_{11} + C_{12}}\right|_{\mu_1=1/2}$$

that is, the unique solution of $f_1 = 0$ evaluated at $\lambda_1 = 1$, $\lambda_2 = 1$ and $\mu_1 = 1/2$. From now on, we assume that all parameters take values such that $0 < \theta_1^* < \theta_{trade}$ for the sustain point to exist. At $\theta = \theta_1^*$, the three conditions (9) are satisfied since $f_1 = f_3 = 0$ hold by definition, whereas $f_2 > 0$ holds because we have:

$$f_1 - f_2|_{\lambda_1=\ \lambda_2=1, \mu_1=1/2} = -\frac{bL\left[4a - (2b-c)(\tau_1+\tau_2)\right](\tau_1-\tau_2)}{2(2b-c)\,\phi} < 0$$

Therefore, we have shown the following result.

**Proposition 4** *If* $0 < \theta \leq \theta_1^*$, *then* $(1, 1, 1/2)$ *is a stable equilibrium.*

Hence, full agglomeration is a stable equilibrium when commuting and transport costs are sufficiently low.

### 4.1.3 Regional specialization

It remains to consider the case of intermediate values of $\theta \in (\theta_1^*, \max\{\theta_1^{**}, \theta_2^{**}\})$. In this case, we expect one industry to be agglomerated and the other to be dispersed for some values of $\theta$ (i.e., there is regional specialization of type $(II)$ or type $(III)$). Because of the nonlinearity of (8), we have not been able to characterize all the asymmetric equilibria arising in the above domain of $\theta$-values. However, we can study the behavior of a stable equilibrium in the neighborhood of the sustain point $\theta_1^*$.

To this end, consider the agglomerated equilibrium $(1, 1, 1/2)$ at $\theta_1^*$, in which $\lambda_2^* = 1$ is a corner solution ($f_2 > 0$) whereas $\lambda_1^* = 1$ and $\mu_1^* = 1/2$ are interior solutions ($f_1 = f_3 = 0$). This equilibrium is stable because

$$f_1 = f_3 = 0 \qquad \text{and} \qquad f_2 > 0$$

Since $\lambda_2^* = 1$ is an interior solution at $\theta_1^*$, $\lambda_2 = 1$ still holds when $\theta$ is slightly perturbed. Then, we have $\partial f_2/\partial\lambda_2 = 0$ so that

$$\left.\frac{\partial\lambda_1^*}{\partial\theta}\right|_{\theta=\theta_1^*,\lambda_1=1,\mu_1=1/2} = -\left.\frac{\frac{\partial f_1}{\partial\theta}\frac{\partial f_3}{\partial\mu_1} - \frac{\partial f_1}{\partial\mu_1}\frac{\partial f_3}{\partial\theta}}{\frac{\partial f_1}{\partial\lambda_1}\frac{\partial f_3}{\partial\mu_1} - \frac{\partial f_1}{\partial\mu_1}\frac{\partial f_3}{\partial\lambda_1}}\right|_{\theta=\theta_1^*,\lambda_1=1,\mu_1=1/2}$$

The numerator of the RHS of this expression can be shown to be positive, thus implying that

$$\operatorname{sgn}\left(\left.\frac{\partial\lambda_1^*}{\partial\theta}\right|_{\theta=\theta_1^*,\lambda_1=1,\mu_1=1/2}\right) = \operatorname{sgn}\left(\left.-\frac{\partial f_1}{\partial\lambda_1}\frac{\partial f_3}{\partial\mu_1} + \frac{\partial f_1}{\partial\mu_1}\frac{\partial f_3}{\partial\lambda_1}\right|_{\theta=\theta_1^*,\lambda_1=1,\mu_1=1/2}\right) \tag{10}$$

The equilibrium $(\lambda_1^*, \mu_1^*) = (1, 1/2)$ with $\lambda_2 = 1$ evaluated at $\theta = \theta_1^*$ is stable when the RHS of (10) is negative. Thus, for any stable equilibrium in the neighborhood of $\theta_1^*$, it must be that $\partial\lambda_1^*/\partial\theta < 0$. In other words, if the equilibrium is stable when $\theta$ is slightly above $\theta_1^*$, industry 1 is no longer agglomerated but industry 2 remains agglomerated. Hence, we have shown the following result.

**Proposition 5** *For $\theta$ just above the sustain point $\theta_1^*$, industry 2 is agglomerated but industry 1 is not.*

Since $\lambda_1^* < \lambda_2^* = 1$, Proposition 5 states *industries with high (resp. low) transport costs tend to be more concentrated (resp. dispersed).* This result can be explained as follows: when urban costs are large, some low-transport-cost firms move to the periphery where they can afford to pay lower wages without loosing much sales in the core region. By contrast, all high-transport-cost firms remain in the core region because they would lose a substantial fraction of the demand prevailing there by moving to the periphery.[8] As seen above, the opposite occurs in the case of one sector in that industries with high (resp. low) transport costs are likely to be dispersed (resp. concentrated). This suggests that *results obtained with one industry do not necessarily carry over to the case of several industries.*

[8] Again, it should be noted that the reverse is true in the absence of urban costs but in the presence of an immobile demand: high-transport-cost firms follow the immobile consumers in the periphery, and low-transport-cost firms agglomerate in the core.

The proposition above also invites us to consider the case in which one industry is agglomerated because it produces a good that can be trade at prohibitive costs, whereas the other industry is footloose because shipping its output is almost costless. This is what we do in the next section.

### 4.2 Spatial equilibrium without intersectral mobility

In order to gain some additional insights, we consider the case in which workers can change places but cannot shift jobs. That is, we set the industrial employment share at $\mu_1 = 1/2$ and (8) boils down to the system:

$$\begin{cases} \dot{\lambda}_1 = V_{1H} - V_{1F} \\ \dot{\lambda}_2 = V_{2H} - V_{2F} \end{cases}$$

Some tedious calculations yield the following results: (i) if commuting costs are large, then the symmetric configuration is the only stable spatial equilibrium; (ii) when $\theta$ takes intermediate values, there is a stable spatial equilibrium involving the agglomeration of the industry with higher transport costs, whereas the size of the low-transport cost industry in the core region rises as $\theta$ keeps decreasing; (iii) finally, when commuting costs are sufficiently low, there is one stable spatial equilibrium corresponding to the agglomerated configuration.[9] This is reminiscent of Proposition 6: in both cases, the economy evolves from dispersion to agglomeration, through the (partial) agglomeration of the high-transport cost firms. It also implies that the market outcome emerging after symmetry is always of type $(II)$ since $\lambda_1^* < \lambda_2^* = 1$, implying that *city $H$ is diversified whereas city $F$ is specialized.* This agrees with Henderson (1988, ch.1) who observes that large cities (here $H$ whose labor share exceeds 1/2) tend to be more diversified than small cities (here $F$ whose labor share is lower than 1/2).

## 5 IT-related industry vs. b-to-c industry

In this section, we deal with the limiting, but meaningful, case in which good 1 can be traded at negligible transport costs (formally, $\tau_1 = 0$) - think of an IT-related industry, whereas good 2 is nontradeable ($\tau_2 > a/(b-c)$) -

[9] Proofs may be obtained from the authors upon request.

think of a business-to-consumer service industry. Thus, we now have two nontradeable goods: consumer services provided by industry 2 and land. Since good 1 can be shipped costlessly, it would seem that the location of the corresponding industry is undetermined. We will show below that the presence of the nontradeable good 2 is sufficient to get rid of this indeterminacy. More importantly, we are able to provide here a full characterization of the equilibrium configurations. In the absence of good 2, the market equilibrium always involves dispersion because workers seek to minimize urban costs. By contrast, we will see that *the existence of a nontradeable consumption good (other than land) is sufficient to generate an urban hierarchy once commuting costs are not too high.*[10]

Consider a distribution of firms across regions and sectors such as the number of firms in region $H$ (resp. $F$) is $N_H \equiv N_1 + \lambda_2 N_2$ (resp. $N_F \equiv N_1 + (1-\lambda_2)N_2$). It is readily verified that the price of good 1 is the same regardless of the region where the varieties are sold and equal to $p^* \equiv a/(2b-c)$ (set $\tau = 0$ in $p^*_{HH}$ of (4)). Similarly, since good 2 cannot be shipped, its price in each region is also equal to $p^* \equiv a/(2b-c)$ (set $\lambda = 1$ in $p^*_{HH}$ of (4)). The fact that this price is independent of the number of firms located in the region where the corresponding varieties are produced simplifies the analysis. As will be seen, it does not prevent us, however, from studying diversity as an centripetal force.

Since transport costs of good 1 are zero the equilibrium wages in sector 1 are equal across regions. By contrast, wages in sector 2 vary with the total number of firms in each region. Specifically, for a given distribution of firms and workers across regions, equilibrium wages are given by (see (4))

$$w^*_{1H} = w^*_{1F} = \frac{bp^{*2}\left[N_H L_H + N_F\left(L - L_H\right)\right]}{L_1}$$

and

$$w^*_{2H} = \frac{bp^{*2} N_H L_H}{\lambda_2 L_2} \qquad w^*_{2F} = \frac{bp^{*2} N_F\left(L - L_H\right)}{(1-\lambda_2)L_2}$$

where $L_H \equiv \lambda_1 L_1 + \lambda_2 L_2$ is the total number of workers in region $H$. Furthermore, the mobility of workers across sectors ensures that the factor price

[10]This idea has already been put foward by several authors. See Abdel-Rahman (2000) for a recent survey of city systems.

equalization

$$w^* \equiv w_{1H}^* = w_{1F}^* = w_{2H}^* = w_{2F}^*$$

holds in equilibrium. Consequently, whatever the equilibrium distribution $(\lambda_1^*, \lambda_2^*, \mu_1^*)$, there is no wage differential and no price index differential between regions. However, since the indirect utility of an individual working in industry $i$ and residing in region $r$ is given by

$$V_{ir}^* = \frac{\left[a - (b-c)\, p^*\right]^2 N_r^*}{b-c} + w^* - \frac{\theta L_r^*}{4} \qquad i = 1, 2, \quad r = H, F \tag{11}$$

an uneven distribution of workers implies the existence of an urban cost differential (the third term in (11)). In equilibrium, this one is just compensated by the differential in the number of varieties available in each region (the first term in (11)). In other words, *workers may choose to live in a larger city in which they bear higher urban costs because they enjoy there a larger variety of differentiated services*. This is consistent with the empirical fact that, in cities of different sizes, the price index differential and/or the nominal wage differential is lower than the differential in housing rent or land price (Tabuchi, 2001).

Solving $f_1 = f_2 = f_3 = 0$ with respect to $\lambda_1$, $\lambda_2$ and $\mu_1$ simultaneously, we obtain the following two interior equilibria:[11]

$$(\lambda_1^*, \lambda_2^*, \mu_1^*) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \tag{12}$$

$$(\lambda_1^*, \lambda_2^*, \mu_1^*) = \left(\frac{1}{2} + \frac{(4-T)\sqrt{3-T}}{2T}, \frac{1}{2} + \frac{\sqrt{3-T}}{2}, \frac{1}{2}\right) \tag{13}$$

where

$$T \equiv \frac{(b-c)\phi}{b^2 p^{*2}}\theta \in [2, 3]$$

Since transport costs are zero, the sole relevant parameter is the unit commuting cost $\theta$, which is positively related to $T$ since $b > c$: the larger $\theta$, the larger $T$.

[11]The mirror image of the asymmetric equilibrium is disregarded.

The first equilibrium (12) involves full dispersion. This is always an equilibrium, whatever the value of the commuting cost $\theta$. However, (13) is an equilibrium if and only if $T \in [2,3]$ because each variable must be real and must take its values in $[0,1]$. Note that, for all $T \in (2,3)$, $1/2 < \lambda_1^* < \lambda_2^* < 1$ holds, thus implying that the industry producing the nontradeable varieties is more agglomerated than the light industry. The two equilibria coincide with full dispersion when $T = 3$. On the other hand, (13) involves full agglomeration with $(1,1,1/2)$ at $T = 2$. Thus, $T = 3$ is the symmetry breaking point and $T = 2$ is agglomeration sustain point.

Since all possible equilibria have been accounted for, we may conclude that *the labor force is equally split between the two industries despite the fact that workers can shift jobs.* Indeed, when compared to firms producing costlessly tradeable goods, firms producing nontradeable varieties have fewer customers, but each individual demand is higher. Because these two effects just cancel out, the labor share is never higher in one sector than in the other.

The fact that $\mu_1^* = 1/2$ holds vastly eases the stability analysis when compared to the case with finite and positive values of $\tau_1$ and $\tau_2$. The following result is then proven in Appendix 3.

**Proposition 6** *Assume that good* 1 *can be traded at zero cost whereas good* 2 *is nontradeable. As* $T$ *decreases, there is only one path of stable equilibria. Furthermore, this path is as follows: (i) if* $T \geq 3$*, then each industry is fully dispersed; (ii) if* $2 < T < 3$*, then both industries are partially agglomerated within the same region and the industry producing the nontradeable good is more agglomerated than the industry producing the costlessly tradeable good; (iii) if* $T \leq 2$*, then both industries agglomerate in one region.*

The following remarks are in order. First, although varieties of good 1 can be shipped at zero cost, the existence of a nontradeable good allows for a well-defined distribution of industry 1 between regions. More precisely, except for fairly high commuting and fixed costs ($T \geq 3$), *the nontradeable sector acts as a centripetal force that yields partial or full agglomeration of the other industry.* When commuting and fixed costs are sufficiently low ($T \leq 2$), the two industries are located into a single region while the other region is empty. As discussed above, the availability of more differentiated services compensates workers for the higher urban costs they bear within the agglomeration.

Second, *the locational pattern chosen by each of the two industries is in general different.* This is because the IT-related industry does not care a priori about its location, whereas the service industry cares about the spatial distribution of its demand. As a result, the former industry tends to be more dispersed than the latter industry. This agrees with empirical facts: within modern cities, the share of the manufacturing industries tends to be smaller than the share of the service sectors.[12] However, one region accommodates more than half of each industry. As a result, *the market outcome involves an urban hierarchy* in that, for each good, a larger array of varieties is produced within the same city. In the limit, once commuting costs are sufficiently low, the economy involves a single city. State differently, the number of cities depends on how large are the commuting costs.

Third, according to export-base theory (Richardson, 1978), a strong export sector, such as the low transport costs industries, is a powerful engine of regional development in that it attracts the service sectors. Proposition 6 just says the opposite. When the agglomeration process begins (i.e. when $T$ takes intermediate values), the service sector is always more agglomerated than the export sector, and hence the market outcome is not of type $(III)$ but of type $(II)$ as in the previous section). In other words, in the present setting *it is the agglomeration of the service industry that causes the subsequent agglomeration of the export industry.* This difference in results is due to the fact that the export-base theory relies on intermediate services whereas we focus here on consumer services.

Fourth, when fixed costs are very low ($\phi \simeq 0$), or when varieties are little differentiated ($c$ is close to $b$), or both, $T \simeq 0$ and there is always agglomeration. This agrees with what we have seen in the one-sector case, as well as with the result obtained in section 4: industries with high transport costs tend to be more agglomerated than industries with low transport costs.

Fifth, and last, we can compute the labor share of industry 1 in each region as follows:

$$\mu_{1H}^* \equiv \frac{\lambda_1^* L_1^*}{\lambda_1^* L_1^* + \lambda_2^* L_2^*} \qquad \mu_{1F}^* \equiv \frac{(1-\lambda_1^*) L_1^*}{(1-\lambda_1^*) L_1^* + (1-\lambda_2^*) L_2^*}$$

[12]For example, the Tokyo Metropolitan Area consists of Tokyo, Kanagawa, Chiba and Saitama prefectures, whose population share is 25.8% in 1996. Its employment share is 23.0% in construction industry and 23.8% in manufacturing industry, which is slightly less than population. However, the service sectors are more concentrated than population: the employment share is 30.2% in transport and communications, 31.6% in wholesale, 32.5% in restaurants and bars, 35.2% in finance and insurance, 40.1% in real estate, and 29.1% in other services.

Using the equilibrium values (13), it follows that

$$\sqrt{6} - 2 \leq \mu_{1H}^{*} \leq 1/2 \leq \mu_{1F}^{*} \leq 3/4 \tag{14}$$

where the inequalities are strict for all $T \in (2, 3)$.

There are two important implications. First, since $\lambda_2^* \geq \lambda_1^* \geq 1/2$ from (13), the large region $H$ has a larger share of each industry than the small region $F$, and the large region $H$ has more varieties of good 2 than the small region. This agrees with Christaller's (1933) *central place theory* in which large cities have more firms and varieties. Second, since $\mu_{1H}^* \leq 1/2 \leq \mu_{1F}^*$ from (14), the large region $H$ has a larger labor share in industry 2 whereas the small region a larger labor share in industry 1. This is consistent with Ricardo's *comparative advantage* when each region is partially specialized in different industries: the large region has here a comparative advantage in the nontradeables because it has a larger market whereas the small region has a comparative advantage in terms of urban costs.

# 6 Concluding remarks

In the advanced societies of our new century, workers are likely to become more and more mobile, while industries will become freer from immobile production factors. Thus, although a model with an immobile sector but without urban costs, such as the standard core-periphery model, is applicable to the time of the Industrial Revolution, a model with no immobile sector but with urban costs, such as the one considered in this paper, seems to be more suitable to our time. Having said that, our analysis reveals that low-transport cost firms tend to be less spatially concentrated than high-transport cost firms.[13] This is to be contrasted to the main result obtained in the one-industry model in which agglomeration arises for low transport costs. In addition, we have seen that firms supplying nontradeable consumer services (restaurants, theaters, cultural facilities) are always more agglomerated than firms belonging to IT-related industries, a result which is confirmed by armchair evidence. Observe also that our model does not seem to support a pattern of full regional specialization in which industry 1 (resp. 2) would be located in region $H$ (resp. $F$). Such a configuration may be sustained

[13]Note that a similar result has been obtained by Laussel and Paul (2002) in a Krugman-like model with two industries.

as a stable equilibrium only for small domains of parameters such as, for example, when the labor share and transport costs of industry 1 are small whereas those of industry 2 are large. Of course, for this case to arise, intersectional mobility must be forbidden. Finally, we have chosen to focus on industries that differ only in terms of transport costs. The next line of research to be addressed is the location of several industries which differ in several structural parameters, for example different market sizes as well as different fixed production costs and transport costs. Such an analysis would accomplish what Lösch aimed at doing but did not succeed to do.

# Appendix

**1. Definition of the parameters of $f_1$ and $f_2$:**

$$
\begin{aligned}
C_{11} &\equiv \frac{b\rho_1^2\left[(2b-c)c+2(b-c)(3b-c)\mu_1\right]L}{2\,(2b-c)^2\,\phi\mu_1} > 0 \\
C_{12} &\equiv \frac{b(b-c)\left[(2b-c)\,\rho_1^2(1-\mu_1)+b\rho_2^2\mu_1\right]L}{(2b-c)^2\,\phi\mu_1} > 0 \\
C_{21} &\equiv \frac{b(b-c)\left[b\rho_1^2(1-\mu_1)+(2b-c)\rho_2^2\mu_1\right]L}{(2b-c)^2\,\phi(1-\mu_1)} > 0 \\
C_{22} &\equiv \frac{b^2\rho_2^2\left[(6b^2-6bc+c^2)-2(b-c)(3b-c)\mu_1\right]L}{2\,(2b-c)^2\,\phi(1-\mu_1)} > 0 \\
\theta_{11} &\equiv \frac{[4ab\rho_1\,(3b-c)-(2b-c)^2\,\phi\mu_1]\mu_1}{b\rho_1^2\left[(2b-c)c+2(b-c)(3b-c)\mu_1\right]} \\
\theta_{12} &\equiv \frac{4ab\left[(2b-c)\,\rho_1(1-\mu_1)+b\rho_2\mu_1\right]-(2b-c)^2\,\phi(1-\mu_1)\mu_1}{2b(b-c)\left[(2b-c)\,\rho_1^2(1-\mu_1)+b\rho_2^2\mu_1\right]} \\
\theta_{21} &\equiv \frac{4ab\left[b\rho_1(1-\mu_1)+(2b-c)\,\rho_2\mu_1\right]-(2b-c)^2\,\phi(1-\mu_1)\mu_1}{2b(b-c)\left[b\rho_1^2(1-\mu_1)+(2b-c)\rho_2^2\mu_1\right]} \\
\theta_{22} &\equiv \frac{[4ab\rho_2\,(3b-c)-(2b-c)^2\,\phi(1-\mu_1)](1-\mu_1)}{b^2\rho_2^2\left[(6b^2-6bc+c^2)-2(b-c)(3b-c)\mu_1\right]}
\end{aligned}
$$

**2. Definition of the coefficients of $f_3$:**

$$d_3 \equiv 2(2b-c)^2L(\lambda_1-\lambda_2)^2\tau\phi$$

$$
\begin{aligned}
d_2 \equiv\ & L(\lambda_2-\lambda_1)\tau\{4ab(3b-2c)[\rho_1\,(2\lambda_1-1)+\rho_2(2\lambda_2-1)]-6b^3[\rho_1^2\,(2\lambda_1-1) \\
&+\rho_2^2(2\lambda_2-1)]\tau+c^2(1+2\lambda_1-4\lambda_2)\phi-4bc[c(\rho_1^2\,(2\lambda_1-1)+\rho_2^2(2\lambda_2-1))\tau \\
&+(1+2\lambda_1-4\lambda_2)\phi]+2b^2[5c(\rho_1^2\,(2\lambda_1-1)+\rho_2^2(2\lambda_2-1))\tau+2(1+2\lambda_1-4\lambda_2)\phi]\}
\end{aligned}
$$

$$
\begin{aligned}
d_1 \equiv\ & L\{-8a^2b+[4ab(4c[\rho_2(\lambda_2-1)\lambda_2-\rho_1(\lambda_1^2-2\lambda_1\lambda_2+\lambda_2)]+b\{\rho_2[(5-6\lambda_2)\lambda_2 \\
&+\lambda_1(2\lambda_2-1)]+\rho_1[6\lambda_1^2+5\lambda_2-\lambda_1-10\lambda_1\lambda_2)]\})-(2b-c)^2(\lambda_1-\lambda_2)(2\lambda_2-1)\phi]\tau \\
&-b\{c^2[5\rho_2^2(1-\lambda_2)\lambda_2+\rho_1^2(-3\lambda_1+11\lambda_1^2+8\lambda_2-16\lambda_1\lambda_2)]\} \\
&+2bc[3\rho_1^2(\lambda_1+6\lambda_1\lambda_2-4\lambda_1^2-3\lambda_2)+\rho_2^2(\lambda_1-2\lambda_1\lambda_2-7\lambda_2+8\lambda_2^2)] \\
&+2b^2[\rho_1^2(6\lambda_1^2+5\lambda_2-\lambda_1-10\lambda_1\lambda_2)+\rho_2^2(-\lambda_1+2\lambda_1\lambda_2+5\lambda_2-6\lambda_2^2)]\}\tau^2\}
\end{aligned}
$$

$$
\begin{aligned}
d_0 \equiv\ & bL\{4a^2 - 8a\rho_1(-b+c)(\lambda_2+\lambda_1-2\lambda_1\lambda_2)\tau + \rho_1^2[c^2(\lambda_1+3\lambda_1^2+4\lambda_2-8\lambda_1\lambda_2) \\
& -4bc(\lambda_1+\lambda_1^2+2\lambda_2-4\lambda_1\lambda_2)+4b^2(\lambda_1+\lambda_2-2\lambda_1\lambda_2)]\tau^2\}
\end{aligned}
$$

## 3. Proof of Proposition 6

(i) Let $T > 3$. Evaluating the derivatives of $f_i$ at the fully dispersed equilibrium $(\lambda_1^*, \lambda_2^*, \mu_1^*) = (1/2, 1/2, 1/2)$, we see that $\partial f_3/\partial\lambda_1 = \partial f_3/\partial\lambda_2 = 0$, $\partial f_3/\partial\mu_1 < 0$, whereas

$$
\begin{aligned}
\operatorname{sgn}\left(\frac{\partial f_1}{\partial\lambda_1} + \frac{\partial f_2}{\partial\lambda_2}\right) &= \operatorname{sgn}(2-T) < 0 \\
\operatorname{sgn}\left(\frac{\partial f_1}{\partial\lambda_1}\frac{\partial f_2}{\partial\lambda_2} - \frac{\partial f_1}{\partial\lambda_2}\frac{\partial f_2}{\partial\lambda_1}\right) &= \operatorname{sgn}(T-3) > 0 \qquad (15)
\end{aligned}
$$

Hence, the fully dispersed equilibrium is stable for all $T > 3$.

(ii) For $T = 3$, the stability of the fully dispersed equilibrium $(\lambda_1^*, \lambda_2^*, \mu_1^*) = (1/2, 1/2, 1/2)$ cannot be studied by computing eigenvalues of the Jacobian because (15) is zero at $(1/2, 1/2, 1/2)$. From Taylor's theorem, it follows that the signs of the first-order approximations of (8) in the neighborhood of $(1/2, 1/2, 1/2)$ are given by

$$
\begin{cases}
\operatorname{sgn}(\dot{\lambda}_1) \simeq \operatorname{sgn}(-3\lambda_1+\lambda_2+1) \\
\operatorname{sgn}(\dot{\lambda}_2) \simeq \operatorname{sgn}[(3b-4c)(3\lambda_1-\lambda_2-1)] \\
\operatorname{sgn}(\dot{\lambda}_3) \simeq \operatorname{sgn}(1/2-\mu_1)
\end{cases} \qquad (16)
$$

These signs change at when $\lambda_2 = 3\lambda_1 - 1$ or when $\mu_1 = 1/2$. Let $\mathbb{L}$ be the intersection of these two planes. Consider an initial point such that $\lambda_2 \neq 3\lambda_1 - 1$ or $\mu_1 \neq 1/2$ in the 3-dimensional space $(\lambda_1, \lambda_2, \mu_1)$. Two subcases may arise.

(a) When $3b \geq 4c$, any trajectory moves toward $\mathbb{L}$. Indeed, since the third-order approximation of (8) on the line $\mathbb{L}$ is such as

$$
\dot{\lambda}_2 \simeq \frac{144a^2bL\,(1/2-\lambda_2)^3}{(2b-c)^2\phi}
$$

any trajectory moves toward $(1/2, 1/2, 1/2)$.

(b) When $3b < 4c$, trajectories move either toward the line $\mathbb{L}$ or toward one of the planes $\lambda_2 = 0$ or $\lambda_2 = 1$. When the trajectories reach either of these planes, they move on the corresponding plane toward $(1/3, 0, 1/2)$ or $(2/3, 1, 1/2)$ by the first equation in (16). Since these two points are also on the line $\mathbb{L}$, the same argument as in (a) applies.

(iii) Let $2 < T < 3$. Evaluating the derivatives of $f_i$ at the asymmetric equilibrium

$$(\lambda_1^*, \lambda_2^*, \mu_1^*) = \left(\frac{1}{2} + \frac{(4-T)\sqrt{3-T}}{2T}, \frac{1}{2} + \frac{\sqrt{3-T}}{2}, \frac{1}{2}\right) \qquad \text{for } T \in [2, 3]$$

we have

$$\begin{aligned} \operatorname{sgn}\left(\frac{\partial f_1}{\partial \lambda_1} + \frac{\partial f_2}{\partial \lambda_2}\right) &= \operatorname{sgn}\left(\frac{S}{T-2}\right) \\ \operatorname{sgn}\left(\frac{\partial f_1}{\partial \lambda_1}\frac{\partial f_2}{\partial \lambda_2} - \frac{\partial f_1}{\partial \lambda_2}\frac{\partial f_2}{\partial \lambda_1}\right) &= \operatorname{sgn}\left(\frac{3-T}{T-2}\right) \end{aligned}$$

where $S \equiv b(T^3 - 6T^2 - 4T + 48) + 2c(T^2 + 4T - 24)$, which is convex in the interval $[2, 3]$. The local minimizer of $S$ is given by $\hat{T} = 2(3b - c + \sqrt{12b^2 - 12b + c^2})/3b$. Since $S|_{T=\hat{T}} > 0$, $S$ must be positive for all $T \in [2, 3]$, and hence the above stability conditions always hold for all $T \in (2, 3)$.

(iv) For $T \leq 2$, $\operatorname{sgn}(f_2) = \operatorname{sgn}[b(2-T) + 6(b-c)] > 0$ always holds at the agglomerated equilibrium $(\lambda_1^*, \lambda_2^*, \mu_1^*) = (1, 1, 1/2)$. This implies that any trajectory moves toward $\lambda_2 = 1$, and hence it suffices to consider the two-variable dynamics of $(\lambda_1, \mu_1)$ with $\lambda_2 = 1$. Computing the eigenvalues of the Jacobian matrix of the two-variable dynamics at $(\lambda_1^*, \lambda_2^*, \mu_1^*) = (1, 1, 1/2)$ yields solution whose real parts are readily shown to be negative. Hence, the agglomerated equilibrium is stable for all $T \leq 2$. $\square$

# References


[1] Abdel-Rahman H. (2000) City systems: general equilibrium approaches. In: J.-M. Huriot and J.-F. Thisse (eds.). *Economics of Cities. Theoretical Perspectives.* Cambridge: Cambridge University Press, 109-137.

[2] Bairoch P. (1985) *De Jéricho à Mexico. Villes et économie dans l'histoire*. Paris: Gallimard. English translation: *Cities and Economic Development: From the Dawn of History to the Present*. Chicago: University of Chicago Press (1988).

[3] Beckmann M.J. and J.-F. Thisse (1986) The location of production activities. In: P. Nijkamp (ed.). *Handbook of Regional and Urban Economics, Volume 1.* Amsterdam: North-Holland, 21-95.

[4] Christaller W. (1933) *Die Zentralen Orte in Süddeutschland*. Jena: Gustav Fischer Verlag. English translation: *The Central Places of Southern Germany*. Englewood Cliffs (N.J.): Prentice-Hall, 1966.

[5] d'Aspremont C., J.J. Gabszewicz and J.-F. Thisse (1979) On Hotelling's "Stability in Competition". *Econometrica* 47, 1045-1150.

[6] Fujita M. (1989) *Urban Economic Theory. Land Use and City Size*, Cambridge: Cambridge University Press.

[7] Fujita M., P. Krugman and T. Mori (1999) On the evolution of hierarchical urban systems, *European Economic Review* 43, 209-251.

[8] Fujita M., P. Krugman and A.J. Venables (1999) *The Spatial Economy. Cities, Regions and International Trade*. Cambridge (Mass.): MIT Press.

[9] Greenhut M.L. (1981) Spatial pricing in the United States, West Germany and Japan, *Economica* 48, 79-86.

[10] Head K. and T. Mayer (2000) Non-Europe. The magnitude and causes of market fragmentation in the EU, *Weltwirtschaftliches Archiv* 136, 284-314.

[11] Helpman E. (1998) The size of regions, in: D. Pines, E. Sadka and I. Zilcha, eds., *Topics in Public Economics. Theoretical and Applied Analysis*, Cambridge, Cambridge University Press, 33-54.

[12] Henderson J.V. (1974) The sizes and types of cities, *American Economic Review* 64, 640-56.

[13] Henderson J.V. (1988) *Urban Development. Theory, Fact and Illusion.* Oxford: Oxford University Press.

[14] Hoover E.M. (1948) *The Location of Economic Activity.* New York: McGraw-Hill.

[15] Krugman P. (1991) Increasing returns and economic geography, *Journal of Political Economy* 99, 483-499.

[16] Laussel D. and T. Paul (2002) Trade and the location of industries: some clarifications on the role of country size, GREQAM, Université de la Méditéranée, mimeo.

[17] Lösch A. (1940) *Die Räumliche Ordnung der Wirtschaft.* Jena: Gustav Fischer. English translation: *The Economics of Location.* New Haven, CN: Yale University Press (1954).

[18] Ottaviano G., T. Tabuchi and J.-F. Thisse (2002) Agglomeration and trade revisited, *International Economic Review* 43, 409-436.

[19] Ottaviano G. and J.-F. Thisse (2002) Integration, agglomeration and the political economics of factor mobility, *Journal of Public Economics* 83, 429-456.

[20] Puga D. (1999) The rise and fall of regional inequalities, *European Economic Review* 43, 303-334.

[21] Richardson H.W. (1978) *Regional Economics.* Urbana: University of Illinois Press.

[22] Tabuchi T. (1998) Urban agglomeration and dispersion: a synthesis of Alonso and Krugman, *Journal of Urban Economics* 44, 333-351.

[23] Tabuchi T. (2001) On interregional price differentials, *Japanese Economic Review* 52, 104-115.

[24] Thisse J.-F. and X. Vives (1988) On the strategic choice of a spatial price policy. *American Economic Review* 78, 122-37.

[25] Triffin R. (1940) *Monopolistic Competition and General Equilibrium Theory.* Cambridge (Mass.): Harvard University Press.

[26] Venables A.J. (1999) The industrial division of industries: Clustering and comparative advantage in multi-industry model, *Scandinavian Journal of Economics* 101, 495-513.

[27] Vives X. (1990) Trade association disclosure rules, incentives to share information, and welfare, *Rand Journal of Economics* 21, 409-430.

[28] Weber A. (1909) *Über den Standort der Industrien.* Tübingen: J.C.B. Mohr. English translation: *The Theory of the Location of Industries.* Chicago: Chicago University Press (1929).

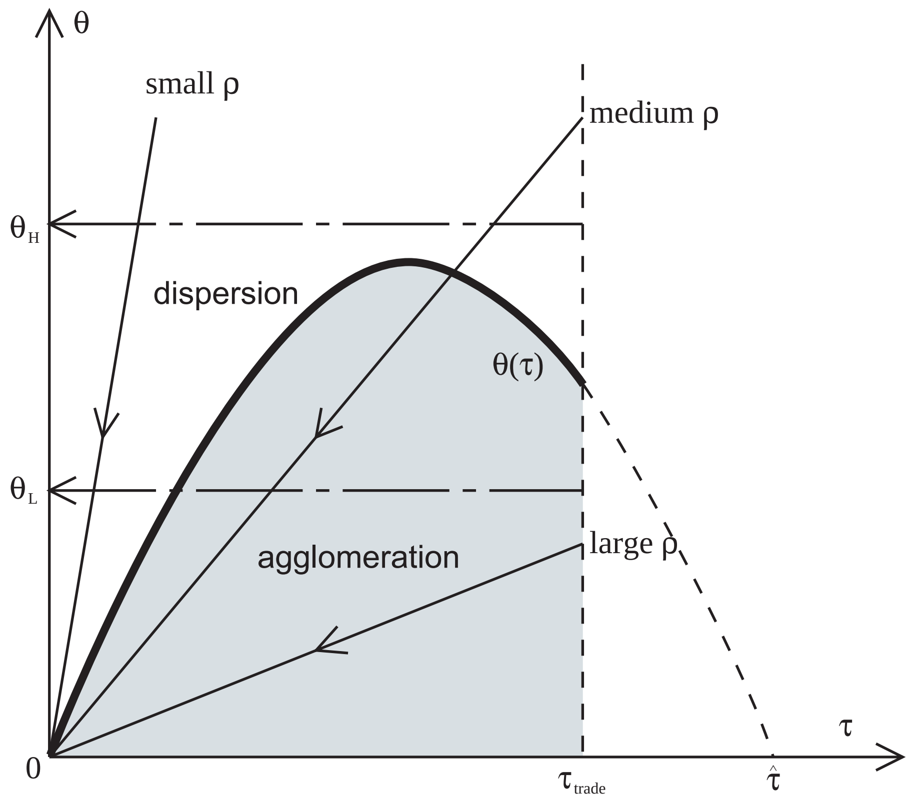


Figure 1: The equilibrium pattern in the one-industry case