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ABSTRACT

Relative Performance, Risk and Entry in the Mutual Fund Industry*

This Paper constitutes a first attempt to analyse the impact of the emergence of new funds on portfolio decisions of mutual fund managers who are evaluated on the basis of relative performance. Recent theoretical literature has pointed to the inefficiencies in portfolio selection caused by relative performance evaluation of fund managers. We find that the ongoing process of creation of new funds, by posing an entry threat to the incumbent fund managers, greatly alleviates these inefficiencies. Hence the transitory market structure that characterizes the mutual fund industry could explain why relative performance evaluation is widely in use.

JEL Classification: G11, G24 and L10

Keywords: family of funds, fund management industry, ranking objectives and relative performance evaluation

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“It does not matter if you’re plus 200%, if the market’s plus 210%. What I have to do is to ensure that this fund can act as a window for our sales force. It’s got to be median or above, otherwise we can’t sell it.” (interview with a fund manager reported in Ashton et al [2]).

1 Introduction

1.1 Motivation

This paper constitutes a first attempt to analyse the impact of the emergence of new funds on portfolio decisions of mutual fund managers who are evaluated on the basis of relative performance.

Among practitioners there is no dispute on the fact that competition among fund managers is based on relative, rather than absolute performance. From the early 90ies a revived research interest on the relationship between performance, compensation and portfolio decisions of money managers has developed alongside with the more and more important role played by mutual fund groups and institutional investors in financial markets.

Recent empirical studies have tested the hypothesis that risk-taking decisions by fund managers can indeed be explained by a tournament model where each fund aims at market leadership. Brown et al [3] and Chevalier and Ellison [6] find that riskiness profiles of funds’ portfolios are altered during the course of the calendar year as if fund managers would respond to relative performance incentives. Managers with very good mid-year performances tend to lock in their gains in more conservative positions; worse performing funds take higher risks in order to gamble their way up the yearly rankings. They attribute this result to the strong relationship between the inflow of new investment in the fund and the fund’s past relative performance. Sirri and Tufano [16] study

the flow of funds into and out of equity mutual funds and find that clients invest disproportionately more in funds that performed very well in the prior period. Similarly Chevalier and Ellison [6], looking at a sample of growth and income funds, find significant non-linearities in the shape of the relationship between fund flow and past performance. Thus mutual fund managers with asset based compensation schemes would like to maximise the ranking of the fund in order to achieve maximal investment inflow. The flow-performance relationship works as an implicit incentive contract for the fund manager. In most of these studies it is argued that the “end of the year” effect results from the diffuse system of assessing and reporting fund performance on an annual basis. Calendar year data appear to be the most generally available to investors: listings of mutual funds, accompanied by calendar year returns, are published on an annual basis in many newspapers and business and financial publications. Information presented to the public often refers to rankings as a measure of relative performance.

Why do investors select funds on the basis of their rank, rather than looking at their absolute performance? Agency theory provides a rationale for relative performance evaluation. In a static multi-agent setting Holmström [8] and Mookherjee [13] consider the situation where the agent’s output depends not only on effort and idiosyncratic noise, but also on a common shock experienced by the other agents. In these circumstances the optimal contract may entail relative performance evaluation. By applying this reasoning to delegated portfolio management, the theory would predict that an investor is able to elicit superior performance by evaluating fund managers against each other.

However the nature of the mutual fund industry is such that delegated portfolio management differs from a standard principal-agent relationship at least in two ways: it is typically dynamic and it involves not only effort exertion but also decisions about the risks to be taken.

Meyer and Vickers [12] show that, in a dynamic setting, comparative performance evaluation has an ambiguous impact, and that it is not guaranteed that it enhances efficiency. Moreover, Cabral [4], Hvide [9] and Goriaev et al. [7] point out that relative performance evaluation might be undesirable when agents not only choose the level of effort (expected return) but also the riskiness of their actions (variance). Further, Sciubba [15] shows that relative performance incentives serve as a coordination device for fund managers to herd on inefficient portfolios.

Does this imply that investors are necessarily wrong when they base their fund selection on relative performance? Should we conclude that asset based compensation schemes necessarily lead fund managers to inefficient portfolio choices? In this paper we argue that the models that regard relative performance evaluation in delegated portfolio management as inefficient do not take into account the fact that the mutual fund industry is characterised by an extremely uncertain market structure. The new insight that our paper provides with respect to the existing literature is that relative performance evaluation need not be inefficient when the market where the fund managers operate is contestable.

The mutual fund industry is characterised by an on-going process of emergence of new funds, often launched by the same mutual fund groups as part of a larger *family of funds*. In any given year during the 80ies and early 90s, 6% of all fund assets were held by new funds with existence of less than 1 year (and 25% in funds with existence of less than 2 years)¹. Assets in the US-based mutual funds increased 24% in 1999: investment performance accounted only for two thirds of the growth; the remainder was attributable to net new cash flow and emergence of new funds².

Khorana and Servaes [11] study the determinants of mutual fund starts and

¹Data reported in Khorana and Servaes [11].

²Data are from the Mutual Fund Fact Book 2000 [10].

argue that the launch of a new fund is often related to past performances: if a fund has underperformed with respect to its peers, the family may open a new fund to attract funds under management. Also, if a family has built a reputation for good performance, it may capitalise on the brand name to open a new fund.

Such a rapid industry growth has a significant impact on the persistence of mutual fund performance. Carhart [5] tests the consistency in fund rankings and finds that, although “winners are somewhat more likely to remain winners and losers more likely to remain losers or perish [...], the funds in the top decile differ substantially each year, with more than 80 percent annual turnover in composition. [...] The year to year rankings on most funds appear largely random” [in [5], p. 71]. Carhart [5] also examines the determinants in the persistence of a fund in the top ranks and finds that the probability of disappearing from the chart decreases monotonically with past performance.

Building on the insights in Carhart, in this paper we argue that the presence of an entry threat alleviates the inefficiencies caused by relative performance incentives. A poorly performing fund, even when ahead of its current competitors, faces a higher probability of losing its leadership versus a potential entrant. In presence of contestability, fund managers who compete on the basis of rank, still aim at a high absolute performance in order to deter entry.

We model the dynamic game played by two incumbent fund managers who at the beginning of each time period choose the riskiness profile of their portfolios. Fund managers are rewarded at the end of the game on the basis of their cumulative performance: the top performing fund receives a strictly positive bonus; the worse performing fund receives nothing. We first analyse the game with no entry and find, in accordance with the existing literature³, that fund

³Among the empirical literature: Chevalier and Ellison [6] and Brown et al [3]. In the theoretical literature: Cabral [4], Goriaev et al [7], Sciubba [15].

managers who compete for rank condition their portfolio choices in the second stage on their past performance: the interim leader will pursue strategies which are more conservative than those that the follower will instead implement. We provide necessary and sufficient conditions so that the effect described above occurs. Our setup allows us to find a very clear indication of the fact that the amount of risk that a fund will be willing to take only depends on the performance gap between close competitors. Hence expected values of investment alternatives are entirely irrelevant for risk-taking decisions. This clearly leads fund managers to take portfolio decisions which are unlikely to be in the best interest of the investors.

We then ask how the presence of entry threats affect fund managers' behaviour. In particular, we assume that there is a third fund that might decide to enter the market at the end of the second stage and also competes for the leadership. The crucial assumption is that the two incumbent funds are uncertain about the performance of the entrant, which we believe to be in line with the transitory market structure of the mutual fund industry. We assume that there is a cost in entering the market, which is lower than the bonus that is awarded to the best performer. As a result, the third fund will enter if he is guaranteed to be the best performer.

An interim leader in a contestable market knows that, even if its current competitor might not succeed in catching up, there is still some positive probability of losing the leadership position vis-a-vis a new emerging fund. In such a market, the leading incumbent would not want to see entirely foregone the possibility of growing: higher absolute performance preempts entry with higher probability. When the leadership is contestable, a top-ranked fund will need to trade off conservatism against growth. As a result, portfolio choices will be sensitive to expected returns of investment alternatives and hence more aligned with investors' objectives.

1.2 Related Literature

The novelty of this paper is in its attempt to analyse the impact of contestability on risk-taking decisions of fund managers who compete on rank. The existing literature has examined the rank-based competition between fund managers for a given market structure, along the same lines as our benchmark case with no entry.

The results that we obtain in the benchmark setting could be compared to Cabral [4], Goriaev et al [7], and Sciubba [15]. Cabral considers an infinite-period race where players choose between projects characterised by low gains with high probability and projects characterised by high gains with low probability. He provides sufficient conditions under which, in equilibrium, the leader chooses a safe project (technology) and the laggard a risky one; and conditions under which the laggard prefers to differentiate from the leader whereas the leader prefers to imitate the follower. Goriaev et al [7] analyse competition over two investment periods between two money managers that have ranking-based objectives. They derive conditions on intermediate performances under which managers play conservative and excessively risky strategies and find that, if the difference in performances in the first period is large, the interim winner has incentives to minimise the level of risk undertaken in the second period to lock in his gains of the first period. Sciubba [15] develops a market model where two assets are traded over two periods by two fund managers with ranking objectives. She shows that rank-based competition may result in inefficient herding.

The benchmark case with no entry in our paper shares the intuition for its results with [4], [7] and [15]. However, with respect to Cabral [4], we deal with a finite horizon game where strategy choices are also influenced by an end of the game effect. The simpler setup allows us to completely characterise necessary and sufficient conditions in order to obtain a unique SPE that displays the risk-

taking features pointed out in [4]. Moreover Cabral [4] assumes that players receive a payoff at every stage, which in our setup is ruled out by the requirement that fund managers make portfolio decisions more often than rankings are made available to investors; hence they take actions more often than they receive compensation.

With respect to Gorjaev et al [7] and Sciubba [15], we allow for correlation between investment outcomes of the two competing fund managers: this differs from Gorjaev et al [7], where no correlation is assumed and from Sciubba [15], where perfect correlation is assumed⁴. We believe that this makes our framework more suitable at describing the mutual fund industry.

Most importantly, our main contribution, which is the study of the effect of contestability, is entirely novel.

1.3 Overview

In section 2 we present the model. In section 3 we derive our results for the benchmark case with no entry. Section 4 constitutes the main contribution of the paper, where we characterise the equilibria in the model with entry. Section 5 concludes the paper and provides policy implications. For ease of exposition all proofs are in the appendix.

2 The Model

Consider a two-period model of a financial market where two fund managers make portfolio decisions at the beginning of each time period (at $t = 0$ and $t = 1$). The two funds are ex-ante symmetric⁵: they manage identical amounts at $t = 0$, both normalised to be equal to 1. For simplicity we assume that

⁴Cabral [4] also assumes perfect correlation.

⁵The same results would obtain if one had to assume that the incumbents in the markets are: two better performing funds and $n > 0$ worse performing funds that are active in the market but do not indeed compete for the leadership as, given their rank, they would have no chance to outperform the top fund.

returns from the first period are entirely reinvested, so that final returns are cumulative.

Portfolio Choice: We assume that fund managers can form their portfolios from two alternative asset classes, with different expected return and risk characteristics. The portfolio selection in this paper can be interpreted as the choice on the weights to be given to specific industries, say traditional sectors and new technologies. Fund managers might decide to invest their endowments into innovative sectors, hence facing higher risks and higher returns; alternatively, they might decide to invest into traditional sectors, taking little risk and enjoying relatively smaller returns. To keep our analysis as simple as possible we assume that it is too costly for fund managers to invest in both sectors at the same time, so that in each time period they specialise in one or the other sector⁶.

In particular, investment in traditional sectors results in a (safer) portfolio S and investment in new technologies results in a (riskier) portfolio R . More in detail, portfolio S yields a positive rate of return $s > 0$ with a known probability $p > 0$, and 0 otherwise; portfolio R yields a positive rate of return $r > 0$ with probability $q > 0$, and 0 otherwise⁷. We assume that $p > q$. Moreover we assume that the rate of return that the fund invested in new technologies displays in case of success is higher than the rate of return of a portfolio of traditional assets in case of success, so that $r > s$. We pose however a bound on the latter inequality by assuming that

$$(1 + s)^2 > (1 + r) \tag{1}$$

so that the cumulative performance of a traditional fund which is successful for two periods in a row is higher than the performance that a fund invested in new technologies can achieve in a single period.

⁶From the theoretical perspective this could be explained within a model where acquiring information about available assets is costly.

⁷The assumption of positive rates of return is not crucial: allowing for negative and bounded rates of return would not affect our results from a qualitative point of view.

For the analysis that follows it is convenient to define $\alpha \equiv \frac{q}{p}$ and $\beta \equiv \frac{s}{r}$. We can represent the parameter space on a diagram where β is on the horizontal axis, and α on the vertical axis, as in figure 1.

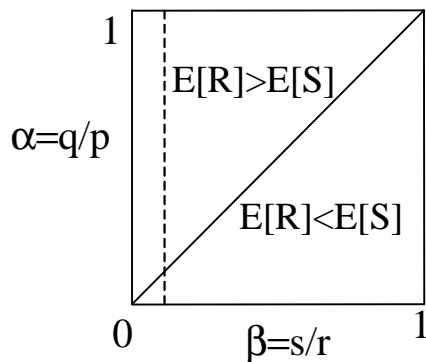


Figure 1 - Parameter Space.

Notice that $E[S] = E[R]$ can be rewritten as $\alpha = \beta$. In fact

$$qr = ps \Leftrightarrow \alpha = \beta$$

Hence along the diagonal in figure 1, the two portfolios display the same expected rate of return. In the area above the diagonal we have that $\alpha > \beta$ which implies that the portfolio in new technologies displays a higher expected rate of return than the traditional portfolio. Symmetrically, in the area below the diagonal we have that $\alpha < \beta$ which implies that the portfolio in new technologies displays a lower expected rate of return than the traditional one. Finally, the dotted line in figure 1 corresponds to our parametric restriction (1) which requires $\beta > \frac{1}{2+s}$.

Correlation: We assume that there are no direct links between the two sectors⁸, so that the success of the traditional portfolio is not correlated to the success of the portfolio in new technologies. On the contrary we assume that

⁸This assumption is not crucial: the introduction of market risk (for example, in the form of a common shock to both sectors) would not affect our results.

when fund managers choose to invest in the same asset class, the likelihood of their success displays some positive correlation. As a result, the joint probability of success is determined by the risk that each fund is taking as well as by some degree of sectorial risk. In particular, when both funds are investing in the traditional sector, the joint probability of success is equal to $p^2\rho \geq p^2$, where $\rho \geq 1$ denotes positive correlation. Similarly, when both funds are investing in new technologies, the joint probability of success is equal to $q^2\chi$, with $\chi \geq 1$. Clearly the case $\rho = \chi = 1$ corresponds to independence and the case $\rho = 1/p$ and $\chi = 1/q$ corresponds to perfect (positive) correlation⁹.

For our results it is important that sectorial risk is not too substantial, so that we rule out very high correlation¹⁰. In order to assess how restrictive a requirement this assumption poses, we need to establish how comprehensive we believe each asset class to be. If the definition of sector is broad enough to allow for reasonably diverse portfolios within the sector itself, then our assumption does not impose significant restrictions. In the present paper fund managers choose the riskiness profiles of their portfolios rather than a specific industry; hence we believe that a broad definition of asset class is precisely in the spirit of our model. In what follows, we assume that:

Assumption 1 $\rho \leq \frac{1}{p} \left[1 - \frac{q}{p} \right]$ and $\chi \leq \frac{p}{q}$

Incentives and Compensation: The choice on the riskiness profile of the portfolio for each of the two incumbent funds is taken by a fund manager (a different one for each fund) who is indeed acting as an agent for the fund owners, rather than managing his own funds. Hence we assume that fund managers do not derive utility directly from the returns that they obtain on behalf of the

⁹The full joint probability matrices are in the appendix.

¹⁰From an empirical point of view, this is consistent with some recent evidence on style investing (see, for example, Ahmed [1]): equity mutual funds tend to specialise in narrowly defined market segments. Ahmed [1] also shows that correlation among equity funds is not too high. Hence a multi-fund portfolio is far less risky than its single-fund counterpart, which also explains why investors often prefer to pursue a multi-fund strategy.

investors. Managers in our model only care about their compensation. We consider a very simple compensation scheme for fund managers which takes the form of a bonus contract: in particular we assume that managers are not compensated period by period, but rather at a final assessment stage on the basis of their cumulative performance. The model would extend to a compensation scheme whereby managers are paid a wage period by period, provided that some part of their compensation is sensitive to performance and that performance assessment is less frequent than portfolio decisions.

Following the empirical evidence on fund managers' compensation¹¹, we further assume that performance is measured in relative terms and that "leaders" are more than proportionately compensated with respect to "followers", so that the incentive scheme is convex. For the purpose of this analysis, we consider the simplest incentive scheme that displays these characteristics, i.e. a bonus contract based on final rank.

More in detail, managers are ranked according to their performance at the end of each period; the rank achieved at the end of period 1 (*interim stage*) is observable but has no impact on the manager's compensation; the rank achieved at the end of period 2 (*bonus stage*) fully determines the manager's compensation. The fund manager that achieves first rank obtains a strictly positive bonus, which we normalise to 1; the manager who achieves second rank obtains a smaller bonus, which we normalise to 0. Convexity of the compensation scheme requires that when both fund managers achieve the same (top) performance, their compensation is sufficiently smaller than if they were first rank; for simplicity we assume that in case of tie, both fund managers are compensated as if they achieve second rank.

Contestability: We think of the market structure described above as intrinsically transitory. In the mutual fund industry low performances can often lead to

¹¹See for example Brown et al [3], Chevalier and Ellison [6], Sirri and Tufano [16].

further entry, which can take the form of creation of new funds, or take place by the emergence of new players through the restructuring of existing ones. In this paper, rather than concentrating on the particular entry mechanism, we focus on the effects of an entry threat on the portfolio decisions of the two incumbents. In the presence of entry threats, fund managers evaluate their expected compensation by taking into account the possibility of losing their bonuses vis-a-vis a successful entrant.

We assume that there is a third fund that might decide to enter the market at the end of the second stage. For example, we could think of the entrant as of a fund previously distributed in another country that becomes suddenly available as a consequence of regulatory changes. Alternatively, the entrant could appear as the result of the decision to launch a new fund by some of the families of runner up funds, in the light of the empirical evidence in Carhart [5]. The entrant also competes for leadership, so that if the new competitor displays a performance which is superior to the two incumbent funds, then the entrant gets the bonus. Until bonuses are distributed the two incumbent funds are uncertain about the performance of the entrant: however a low performing incumbent faces higher probability of losing his leadership than a high performing incumbent. Here we require that the entry threat is effective also for the most successful incumbent. We model this by assuming that the performance of the entrant is random and uniformly distributed between 1 and $E \geq (1 + r)^2$. For simplicity, in what follows we assume $E = (1 + r)^2$.

What is crucial here is not our parametric specification, but rather the requirement on the information structure: there is less information available about the entrant than there is about each of the two incumbents. In particular the expectation over the entrant's performance cannot be based on his track record of interim performance(s). In this paper we take the extreme view that, while the entrant can perfectly observe the cumulative performance of each of the two

incumbents, the two incumbent funds can only assess the performance of their potential competitor on the basis of prior information in the form of a given probability distribution. We believe that the intuition behind our results would still hold in a richer model where some information over the interim performance track record of the entrant is available, provided that this information is not as accurate as the information which is available for each of the two incumbents¹².

We assume that there is a cost in entering the market, which is lower than the bonus that is awarded to the best performer. The entrant observes the performance of the two incumbents and, as a result, he will only come into the picture if he is guaranteed to be the best performer.

In the next section, in order to assess what is the impact of entry on risk-taking decisions by fund managers, we first solve the benchmark case with no entry.

3 The Benchmark Case: No Entry

Suppose that there is no entrant at the end of the second stage. As a result, the two incumbent fund managers play a two-stage game, where payoffs (the bonus) are distributed only at the end of the second stage. The best performer between the two at the end of the second stage wins the bonus. We can solve the game between fund managers backwards: call the two stages of play interim stage and bonus stage respectively, and focus on the bonus stage first.

3.1 The Bonus Stage

We need to fix a history for the interim stage which consists of the strategy pair played and of the realisations for the risky and safe portfolios. A convenient

¹²For example, in a richer model where fund managers are of different abilities, the longer track record that incumbent fund managers display with respect to new entrants, provides a much better assessment over the fund manager's ability and likelihood to succeed in the future. On the contrary, the shorter track record of newly established funds provides poorer information on ability and future success.

feature of this setting is that the bonus subgame will only depend on the interim relative performances and not on the interim absolute performances and the actual strategies played. The following definition provides us with a useful notion of interim relative performance between the two bank managers.

Definition 1 *If, at the end of the first stage, players 1 and 2 display performances $(1 + x)$ and $(1 + y)$ respectively, we call $|x - y|$ the performance gap between the two players.*

One can easily verify the following

Remark 1 *The subgame in the bonus stage only depends on the performance gap between the two players.*

At the end of the interim stage each fund has an endowment equal to either 1, or $(1 + s)$ or $(1 + r)$. According to the realisation of the performance gap we can distinguish four different scenarios after the interim stage: performance gap equal to 0, s , $(r - s)$ and r .

To begin with, assume that the two funds reach the bonus stage with a 0 performance gap. We can construct the payoff matrix for the continuation game as follows. If both funds invest in the safe portfolio S , the bonus is distributed only if asset realisations turn out to be different, i.e. in cases when for one fund the returns are high while for the other are low. This happens with probability $p(1 - p\rho)$ for either fund. Therefore, normalising the size of the bonus to 1, the expected payoffs of playing (S, S) are $p(1 - p\rho)$ for both funds. Similarly, when both funds invest in the risky portfolio R , the bonus is distributed with probability $q(1 - q\chi)$ to each of the two funds. Finally, when funds invest in different sectors, bad realisations for both funds lead to tie, good realisations for the risky portfolio result in winning the bonus with probability one, and bad realisation for the risky and good realization for the safe reward the holder of the

safe portfolio. Hence the fund that invested in the safe portfolio wins the bonus with probability $p(1 - q)$ and the fund that invested in the risky portfolio wins the bonus with probability q . The payoffs in the 0 performance gap subgame can be summarised by the following payoff matrix:

	S	R
S	$p(1 - p\rho) ; p(1 - p\rho)$	$p(1 - q) ; q$
R	$q ; p(1 - q)$	$q(1 - q\chi) ; q(1 - q\chi)$

Similarly we can construct the payoff matrices for the other 3 scenarios. Here and in what follows we adopt the convention of denoting the interim leader as first player (in the payoff matrices as row player), and the interim follower as second player (in the payoff matrices as column player).

We are now ready to state a full characterisation of the equilibrium in the second stage of the game. First, we define a useful notion of “size” for the performance gap:

Definition 2 *We call the performance gap large if it is larger than $(r - s)$; and small otherwise.*

Notice that, given the discrete nature of returns that we consider here, the performance gap is small only if it is equal either to 0 or to $(r - s)$; it is large¹³ only if it is equal either to s or to r .

Lemma 1 (Bonus subgame - No entry) *Under assumption 1, when the performance gap is small the unique NE of the game is (S, S) ; when the performance gap is large (S, R) is the unique NE of the game.*

Proof. See appendix.

When the performance gap is large, S is a dominant strategy for the leader. Intuitively, given that he is sufficiently ahead with respect to the follower, he prefers to maximise the probability of not losing ground, rather than investing

¹³Notice that our assumption on returns implies that $(r - s) < s$.

in the riskier portfolio that would further increase the performance gap. When the performance gap is equal to s , playing risky is a dominant strategy for the interim follower. In fact, in this case the interim follower has still some chance to outperform the leader but he needs to invest in the risky portfolio to do so. When the performance gap is equal to r , there is no chance for the interim follower to outperform the interim leader. Hence the follower is indifferent between the two portfolios: we assume¹⁴ that he will invest in the risky portfolio. Summarising, when the performance gap is large the unique NE of the game is to play (S, R) ; when the gap is zero, both players want to maximise the probability to become leader, hence both of them invest in the portfolio which yields a good realisation with a higher probability, i.e. the safe one. The same occurs when the performance gap is $(r - s)$. In this case even if one of the funds is ahead, the gap is small enough, so that the laggard can in principle outperform the leader with any of the two strategies. By choosing the safe portfolio the follower maximises the probability of jumping ahead. Similar reasoning applies to the leader: given that he can lose his initial advantage with both strategies, he prefers to invest in the safe portfolio that insures good performance with higher probability.

These results are in line with the empirical findings by Brown et al [3] and Chevalier and Ellison [6]. In the pre-assessment period (here the bonus stage) funds have an incentive to alter their riskiness profiles. In particular, top-ranked funds lock in their gains in conservative positions, while worse performing funds attempt to gamble their way up the rankings.

¹⁴The reason why the follower is indifferent between safe and risky portfolios is that the latter is not “risky” enough and does not allow him to catch up with the leader. In such a circumstance it seemed reasonable to assume that the follower, when indifferent, would still go for the risky portfolio. This would certainly happen in a richer model with a larger choice set of risky options.

3.2 The Interim Stage

In this game, funds are ex-ante identical and stand the same chances of winning the bonus, so that we can expect the game in the interim stage to be symmetric.

Suppose that both fund managers choose a safe portfolio in the first stage. If the investments are both successful or both unsuccessful (which occurs with probability $1 - 2p + 2p^2\rho$), the two fund managers reach the end of the first stage with equal interim performance. As a result, the 0 performance gap subgame follows and in equilibrium the expected payoff for each of the two players is equal to $p(1 - p\rho)$. If only one fund is successful (which occurs with probability $2p(1 - p\rho)$), the leading fund manager reaches the end of the first stage with performance equal to $(1 + s)$, and the laggard with performance equal to 1, so that the s performance gap subgame follows and in equilibrium the expected payoffs are equal to $1 - q(1 - p)$ and $q(1 - p)$ respectively for the interim leader and the interim follower.

Hence expected payoffs after (S, S) for each of the two players, are

$$\begin{aligned} & [1 - 2p + 2p^2\rho] p(1 - p\rho) + p(1 - p\rho) [1 - q(1 - p)] + p(1 - p\rho) q(1 - p) = \\ & = p(1 - p\rho) (2 - 2p + 2p^2\rho) \end{aligned}$$

Similarly we can compute expected payoffs for each of the two players after (R, R) , (S, R) and (R, S) . The payoff matrix for the interim stage is as follows:

	S	R
S	$p(1-p\rho)(2-2p+2p^2\rho);$ $p(1-p\rho)(2-2p+2p^2\rho)$	$(1-p)(1-p\rho)(1-q)p+$ $p(1-q)[1-(1-p)q]+$ $p^2q(1-p\rho);$ $(1-p)(1-p\rho)(1-q)p+$ $pq(1-q)(1-p)+$ $+q(1-p)[1-q(1-p)]+$ $pq[1-p+p^2\rho]$
R	$(1-p)(1-p\rho)(1-q)p+$ $pq(1-q)(1-p)+$ $+q(1-p)[1-q(1-p)]+$ $pq[1-p+p^2\rho]$ $(1-p)(1-p\rho)(1-q)p+$ $p(1-q)[1-(1-p)q]+$ $p^2q(1-p\rho);$	$[1-2q+2q^2\chi]p(1-p\rho)+$ $q(1-q\chi)[1-q(1-p)];$ $[1-2q+2q^2\chi]p(1-p\rho)+$ $q(1-q\chi)[1-q(1-p)]$

We can show that the interim stage of the game with no entry has (S, S) as unique equilibrium.

Lemma 2 (Interim stage - No entry) *Under assumption 1, the unique NE of the interim stage of the game with no entry is (S, S) .*

Proof. See appendix.

In the bonus stage the expected payoff of the interim leader is always higher than that of the interim follower. As a result, in the interim stage of the game both players aim at reaching the interim leadership. Since funds are ex-ante identical, they stand the same chances of becoming interim leaders, and consequently they will invest in the safe portfolio that maximises this chance.

3.3 Equilibrium Paths

We can now fully characterise the equilibrium path of the two-stage game with no entry. The equilibrium play in the second stage will be contingent on the realisation of the performance gap between the two players. The two-stage game with no entry has a unique SPE. Funds play safe in the interim stage; in the

bonus stage they keep on playing safe if the performance gap is null; when the performance gap is large (here equal to s), the interim leader plays safe and the interim follower plays risky. Hence we can claim the following

Proposition 1 (SPE - No entry) *Under assumption 1, the game with no entry has a unique SPE. The equilibrium path is: (S, S) in the first stage; (S, S) in the second stage if the performance gap in the first stage is null, and (S, R) otherwise.*

Proof. By lemma 1 and 2. ■

Remark 2 (Sub-optimality) *Notice that the outcome of the game with no entry is not sensitive to the expected return of either investment type: given the incentive structure, expected returns of the investment opportunities are entirely irrelevant for risk-taking choices. Hence rank-based competition easily results in sub-optimal risk-taking behaviour.*

4 Entry

Suppose that there is a third fund that might decide to enter at the end of the second stage and compete for market leadership alongside with the two incumbents. We assume that the entrant has a clear informative advantage in that he can perfectly observe the cumulative performance of the two incumbent funds. If there is a (small) cost to entry, the new fund will only come into the picture when it displays a performance that guarantees market leadership. The two incumbent funds have an informative disadvantage in that they are uncertain about the performance of the entrant fund. However they can anticipate that the higher their performance, the lower the probability of entry.

The effect of entry on expected payoffs is that each of the two incumbents, say of performance $(1 + x)$, will now win the bonus if: their opponent displays

interim performance inferior to $(1 + x)$ and if the entrant displays a performance less than $(1 + x)$, which occurs with probability $\frac{x}{(1+r)^2-1}$, given our assumption on the distribution of the performance of the entrant.

4.1 The Bonus Stage

Unlike the game with no entry, where the only variable that determined which continuation game funds would play in the bonus stage was the interim performance gap between the two players, here the actual strategies played by the managers in the interim stage matter since the absolute performance of both funds (and not only their relative performance) is important in determining what is the chance that an outperforming entrant might arise. Hence we need to distinguish more cases than in the benchmark.

Take for example the case with interim performance gap equal to zero. Note that funds can tie at three different performance levels, in particular at 1, $(1 + s)$ and $(1 + r)$. However the game managers play in the bonus stage at different performance levels will not be the same, as the probability to lose the bonus vis-a-vis a potential entrant will be a decreasing function of absolute performance¹⁵.

Albeit different, the solutions for the three subgames follow the same qualitative structure. In particular: if the probability that the risky strategy pays a high return is very small, then the unique NE is (S, S) ; if it is very high then the unique NE is (R, R) ; for intermediate values both (S, R) and (R, S) are NE. More in detail, let $i = 0, s, r$ identify the cases 1, $(1 + s)$, $(1 + r)$ for the interim performance level in the 0 performance gap subgame with entry. We are now ready to state the following:

Lemma 3 (Bonus stage - zero gap) *In the 0 performance gap bonus subgame with entry, there exist $\alpha_i \in (0, 1)$ for $i = 0, s, r$, such that*

¹⁵This feature of our model fits the empirical evidence in Carhart [5]: the probability that a fund disappears from the list of the top-ranked funds is monotonically decreasing in its past performance.

- (a) iff $\alpha \leq (1 - p\rho)\alpha_i$ then (S, S) is the unique NE;
- (b) iff $(1 - p\rho)\alpha_i \leq \alpha \leq \frac{1-q}{1-q\chi}\alpha_i$ then (S, R) and (R, S) are NE;
- (c) iff $\alpha \geq \frac{1-q}{1-q\chi}\alpha_i$ then (R, R) is the unique NE.

where α_i for $i = 0, s, r$ are defined in the appendix.

Proof. See appendix.

In figure 2 we represent the different equilibria in the parameter space.

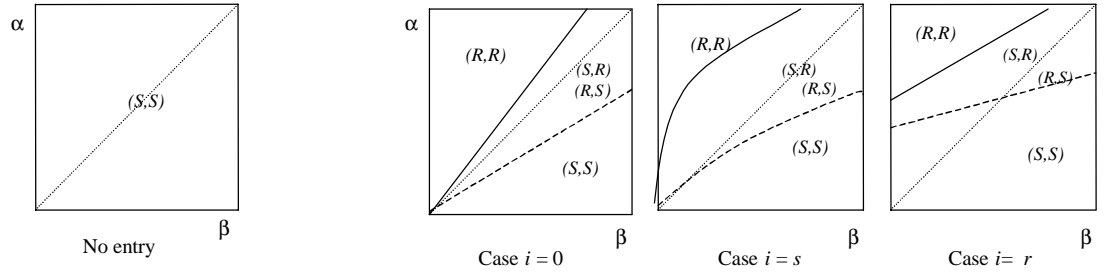


Figure 2 - Equilibria in the 0 performance gap subgame.

With respect to the benchmark case, one first conclusion we can draw is that, while in the case with no entry the unique equilibrium in the 0 performance gap subgame entailed (S, S) for the whole parameter space, here we have parameter regions where either one fund or both funds choose the risky portfolio in equilibrium.

Efficiency of Portfolio Selection: Rank based competition in absence of entry makes portfolio decisions entirely insensitive to the expected values of investment opportunities; uncertainty about new competitors restores sensitivity of risk-taking decisions to expected values of available assets. In presence of contestability the two incumbent funds have two main objectives: outperforming their opponent and deterring entry. Risk-taking becomes appealing to

fund managers because high performance reached through a riskier portfolio preempts entry with higher probability.

Costs and Benefits of Entry Deterrence: It is interesting to assess how, given the level of absolute performance achieved, the costs and benefits of exerting entry deterrence are related to the parameters α and β . The benefit of risk-taking in order to deter entry is decreasing in β : if the ratio $\frac{r}{s}(= \frac{1}{\beta})$ is large, then a successful risky portfolio can lead to much higher performance than a safer portfolio and as a result entry deterrence obtained through risk-taking is highly effective. The cost of entry deterrence is linked to the missed opportunity of outperforming the current opponent with higher probability through a safer portfolio. This opportunity cost is negatively related to the parameter α : when $\alpha(= \frac{q}{p})$ is high, the probability with which the risky portfolio pays out is not much lower than the probability with which the safe portfolio pays out, hence entry deterrence becomes cheaper. As a result, given a value for β , as α increases, it becomes convenient at first for one fund, then for both of them, to undertake entry deterrence. The same effect is observed, given a value for α , as β decreases. Hence for high values of α and low values of β both fund managers choose a risky portfolio; for low values of α and high values of β both funds invest in safer alternatives.

Free Riding on Entry Deterrence: Intermediate values of α and β represent situations where the balance between costs and benefits of risk-taking is such that it is optimal for one of the two funds, but not both, to exercise entry deterrence. In the asymmetric equilibria the fund manager who tilts his portfolio towards riskier alternatives is worse off with respect to the fund manager who stays on safer positions. The latter nevertheless benefits from the entry deterrence exerted by his competitor. Thus we can interpret this situation as *free riding* on entry deterrence.

Effects of Sectorial Risk: In this setting a higher sectorial risk (higher values for χ and ρ) makes symmetric outcomes less appealing. Graphically we could represent higher levels of correlation by an increased wedge between the (S, S) and (R, R) areas. However asymmetric equilibria - and hence the effect that we have interpreted as free riding on entry deterrence - would persist for intermediate values of α and β , even in the absence of sectorial risk.

It is interesting to assess what is the impact of absolute interim performance on risk-taking decisions. We can compare the parameter regions in the three different cases analysed above (cases $i = 0, s, r$) and show that worse performing funds are more prone to risk-taking behaviour. In particular, worse performing funds tend to be more “eager” to make use of growth opportunities and invest in riskier strategies that give higher wealth when they succeed. Entry clearly poses a higher threat to low performing funds than to high performing ones: hence worse performing funds tilt their portfolios towards risky alternatives for a larger set of parameter values. Formally we can state the following:

Proposition 2 (Performance and Risk) *In the 0 performance gap subgame worse performing funds are more risk prone than better performing funds. Formally $\alpha_0 \leq \alpha_s \leq \alpha_r$.*

Proof. See appendix.

An intriguing parallel in the case with no entry is the result that for large enough performance gap worse performing funds (i.e. followers) tend to be riskier than better performing funds, i.e. leaders. This phenomenon is independent of the absolute performance of the fund. In the case with entry, we find that absolute performance matters: even when funds are of equal performance, the probability that they choose the risky strategy in the second period decreases with performance.

Let us now consider the subgame that follows an interim performance gap equal to $(r - s)$ in the game with entry. Recall that in the benchmark case the unique equilibrium entails (S, S) . When entry is considered, similarly as in the previous case examined, we find that equilibria entail more risk-taking and, under some parametric conditions, either one fund or both choose a risky portfolio in equilibrium. More in detail we can claim the following:

Lemma 4 (Bonus stage - $(r - s)$ gap) *In the $(r - s)$ performance gap subgame, there exist $\alpha_1 \in (0, 1)$ and $\alpha_2 \in (0, 1)$ such that*

- (a) (S, S) is the unique NE iff $\alpha \leq \min\{\alpha_1; \frac{1-p\rho}{1-p}\alpha_s\}$
- (b) (R, S) is the unique NE iff $\alpha_1 \leq \alpha \leq \frac{1-q}{1-q\chi}\alpha_s$
- (c) (S, R) is the unique NE iff $\frac{1-p\rho}{1-p}\alpha_s \leq \alpha \leq \alpha_2$
- (d) (R, R) is the unique NE iff $\alpha \geq \max\left\{\frac{1-q}{1-q\chi}\alpha_s, \alpha_2\right\}$

where $\alpha_1 > \alpha_2$ are defined in the appendix.

Proof. See appendix.

The equilibria in the continuation game with $(r - s)$ performance gap have a similar structure to the one given for the 0 performance gap subgame: with respect to our benchmark case, entry threats make portfolio decisions of fund managers more sensitive to the expected values of the investment opportunities. In particular entry deterrence through risk-taking becomes convenient for one, and then for both funds, for high values of α and low values of β , i.e. whenever the benefits of entry deterrence are high enough and its opportunity costs are not too severe. Unlike the 0-performance gap subgame, the continuation game we have here is not symmetric: although the interim performance gap is small, there is an interim leader and an interim follower. This asymmetry leads to the outcome that, for intermediate values of α and β , the asymmetric equilibria (S, R) and (R, S) do not coexist¹⁶. The intuition behind this result lies in the

¹⁶The fact that $\alpha_1 > \alpha_2$ implies that there is no value for α such that (b) and (c) in

fact that as soon as the game becomes asymmetric, the incentives to undertake entry deterrence differ between the two funds. In particular, the benefit of entry deterrence is higher for the leader, who has more at stake; while the opportunity cost of entry deterrence is lower for the follower, for whom risk-taking works also towards the objective of outperforming his current opponent. The balance between these two effects is different across parameter regions, so that (S, R) and (R, S) never coexist for the same parameters.

Finally we need to consider the case of large interim performance gap. Recall that in the benchmark case with no entry the unique equilibrium entails (S, R) . We can show that also in this case entry implies that fund managers will choose riskier portfolios. In fact, a parameter region exists such that both funds, and not only the interim follower, play risky.

Why is it so? The reason is that although choosing a safe strategy may allow the leader to keep his leadership with a higher probability and get the bonus in the end, when there is also entry, he may lose it vis-a-vis a potential entrant, if the performance of the new entrant is sufficiently high. To choose the equilibrium strategy he should trade-off the positive effect of choosing the riskier strategy which amounts to higher performance whenever it succeeds (entry preemption with higher probability), with its negative effect that is its lower success probability compared to the safe strategy. If the benefits of entry deterrence are not too low (i.e. α large) it becomes optimal also for the leader to choose the risky portfolio.

As in the benchmark case, the subgames that follow a performance gap equal to s or equal to r are different. However, the solution for these two subgames follows the same qualitative structure. In particular: if the probability that the risky strategy pays a high return is small, then the unique NE is (S, R) ; if it is high then the unique NE is (R, R) . More in detail, let $j = s, r$ identify the proposition 4 are both satisfied.

cases with performance gap equal to s and r respectively.

Lemma 5 (Bonus stage - large gap) *In the bonus subgame with large performance gap, there exists $\hat{\alpha}_j \in (0, 1)$ for $j = s, r$ such that*

- (a) (S, R) is the unique NE iff $\alpha \leq \hat{\alpha}_j$
- (b) (R, R) is the unique NE iff $\alpha \geq \hat{\alpha}_j$

where $\hat{\alpha}_j$ for $j = s, r$ is defined in the appendix.

Proof. See appendix.

In the diagram in figure 3, we represent parameter regions for the two different equilibria in the two cases $j = s, r$.

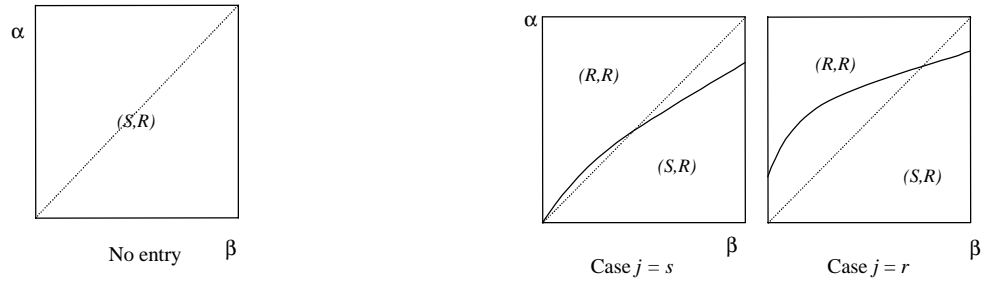


Figure 3 - Equilibria in the large performance gap subgame.

Comparing our results to the benchmark case, we can conclude that, while in the case with no entry the unique equilibrium in the large performance gap subgame entailed (S, R) for the whole parameter space, here we have parameter regions where both funds choose the risky portfolio in equilibrium. In particular, there always exists a parameter region where it is dominant for the interim leader to play risky. Once again, entry threats align portfolio choices with expected values of investment opportunities. The distinctive feature of the equilibria in the large performance gap subgame is that, even when parameters are such that the benefit of entry deterrence is very small (β large), the interim follower still

finds it optimal to play risky. The intuition behind this result lies in the fact that, when the interim performance gap is sufficiently large, for the follower there is no longer conflict between deterring entry and aiming at outperforming the current opponent: both objectives require higher risk-taking.

4.2 The Interim Stage and Equilibrium Paths

We can compute the expected payoff for each of the players in the interim stage in the same fashion as for the case with no entry: we need to consider with which probability each subgame is reached and then substitute for the expected payoffs that each player obtains in equilibrium in the second stage. As analysed in the previous section, equilibria in the bonus subgames depend on parametric conditions. Moreover the parametric conditions needed for different equilibria to arise vary across subgames, which makes a complete characterisation of the game in the interim stage quite tedious and - we believe - not particularly illuminating.

In what follows we concentrate on two polar cases. We first consider the top-left region of the parameter space, where the risky portfolio displays a much higher expected return than the safe portfolio; then we consider the bottom-right region of the parameter space, where the risky portfolio displays a much lower expected return than the safe portfolio. We define the two polar regions by taking the most stringent parametric conditions over all subgames: in the first case, we take parameter values such that in all subgames fund managers play (R, R) ; in the second case, we take parameter values such that in all small performance gap subgames fund managers play (S, S) and in all large performance gap subgames they play (S, R) .

We find that the presence of a potential competitor affects fund managers' behaviour in the first period of play as well: in particular, also in the first stage entry threats restore sensitivity of portfolio decisions to expected values

of available assets.

We start our analysis with the case where the risky portfolio has a significantly higher expected return than the safe. Formally we can state the following result

Proposition 3 (SPE - $E[R] \gg E[S]$) *When $\alpha \geq \alpha^*$, the game with entry has a unique SPE. The equilibrium path is: (R, R) in the first stage and (R, R) in the second stage.*

where α^* is defined in the appendix.

Proof. See appendix.

On the opposite side of the spectrum, when playing safe yields a much higher expected return than playing risky, the unique SPE of the game with entry entails playing (S, S) in the first stage and for the second stage (S, S) if the performance gap in the first stage is null, and (S, R) otherwise. Formally we can state the following:

Proposition 4 (SPE - $E[S] \gg E[R]$) *When $\alpha \leq \alpha^{**}$, the game with entry has a unique SPE with equilibrium paths as follows: (S, S) in the first stage; (S, S) in the second stage if the performance gap in the first stage is small, and (S, R) otherwise.*

where α^{**} is defined in the appendix.

Proof. See appendix.

Remark 3 (Optimality) *Notice that equilibria are here such that portfolio choices are sensitive to the expected returns of the investment opportunities. In particular, they entail more risk-taking whenever the expected return of the risky portfolio is sufficiently high with respect to the expected return of its safer*

alternative. Rank-based incentives lead to sub-optimal decision making in absence of entry threats. In presence of a potential competitor, as long as there is uncertainty over his performance, rank-based competition can induce efficient risk-taking behaviour.

5 Policy Implications and Concluding Remarks

The main objective of our analysis was to assess whether contestability improves on efficiency in our setting. There are at least two different perspectives one might want to look at. First of all, one might be interested in assessing whether entry restores fund managers' incentives and aligns them with the best interest of the original investors. Secondly, one might want to ask whether contestability in the fund management industry should be favoured by a benevolent regulator, who may or may not lift, for example, entry restrictions in the market.

The fact that the objectives of fund managers who care about their rank are not aligned with the investors' best interests is not a surprising result; fund managers who are compensated on the basis of their relative performance take decisions which are detrimental for the investors from an expected value point of view. In spite of this finding, relative performance evaluation seems to be widely in use in the mutual fund industry. Empirical evidence shows that even when fund managers are not explicitly rewarded on the basis of their relative performance, the flow-performance relationship still works as an implicit incentive device. We ask whether rank based objectives lead to more efficient outcomes when competition in the market increases and in particular leadership in the sector can be at stake because of new competitors. Our findings suggest that the entry threats posed by the on-going process of creation of new funds make portfolio decisions of fund managers who compete for rank, more sensitive to the expected values of investment alternatives.

Clearly this does not yet explain why evaluating fund managers according to relative performance should be preferred to absolute performance evaluation. In our framework the inefficiencies of relative performance evaluation are alleviated but not eliminated. However, in a richer model where investment outcomes also depend on an unobservable managerial effort, relative performance evaluation could be used to elicit superior performances. The main implication of our analysis, which we believe will be useful to future research, is the consideration that in a market where relative performance evaluation has a role in reducing the moral hazard problem, the potential distortion in the form of inefficient risk-taking behaviour can be greatly alleviated by uncertainty over potential new competitors.

From a policy perspective, academics and practitioners alike have often expressed concerns about the effects of relative performance evaluation of fund managers: in particular rank-based competition has frequently been pointed at as one of the main causes of excessive conservatism and herding behaviour among institutional investors¹⁷. The policy implication that one could draw from our analysis is that regulators need not be too concerned about the unwanted effects of relative performance evaluation, provided that the mutual fund industry is believed to be open to competitive pressure enough so that fund managers face a realistic threat of being displaced by their competitors.

¹⁷See for example the concerns expressed in the recent Report on Institutional Investment by P. Myners [14].

APPENDIX

Joint Probability Matrices

	s	0	Marg. prob.
s	$p^2\rho$	$p(1-p\rho)$	p
0	$p(1-p\rho)$	$1-2p+p^2\rho$	$(1-p)$
Marg. prob.	p	$(1-p)$	

	r	0	Marg. prob.
r	$q^2\chi$	$q(1-q\chi)$	q
0	$q(1-q\chi)$	$1-2q+q^2\chi$	$(1-q)$
Marg. prob.	q	$(1-q)$	

Proof of Lemma 1 (Bonus subgame - No entry). By remark 1, we only need to distinguish between four subgames.

(1) 0 gap subgame:

	S	R
S	$p(1-p\rho) ; p(1-p\rho)$	$p(1-q) ; q$
R	$q ; p(1-q)$	$q(1-q\chi) ; q(1-q\chi)$

(2) $(r-s)$ gap subgame:

	S	R
S	$1-p+p^2\rho ; p(1-p\rho)$	$1-q ; q(1-p)$
R	$1-p(1-q) ; p(1-q)$	$1-q+q^2\chi ; q(1-q\chi)$

(3) s gap subgame:

	S	R
S	$1-p(1-p\rho) ; 0$	$1-q(1-p) ; q(1-p)$
R	$1-p(1-q) ; 0$	$1-q+q^2\chi ; q(1-q\chi)$

(4) r gap subgame:

	S	R
S	$1 ; 0$	$1-q(1-p) ; 0$
R	$1 ; 0$	$1-q(1-q\chi) ; 0$

Under assumption 1, it is easy to verify the following: in game 1 safe is a dominant strategy for both players, so that unique NE is (S, S) ; in game 2,

the unique NE is (S, S) ; in game 3, playing safe is a dominant strategy for the interim leader and playing risky is a dominant strategy for the interim follower, so that unique NE is (S, R) ; finally in game 4, S is a dominant strategy for the interim leader and, as for the interim follower, he is indifferent between S and R . By our assumption for the case of indifference, the follower prefers to hold a risky portfolio. Hence the unique NE in game 4 is (S, R) . ■

Proof of Lemma 2 (Interim stage - No entry) When the second player is doing safe, the payoff to the first player when he plays safe is:

$$\pi_S = p(1 - p\rho)(2 - 2p + 2p^2\rho)$$

Payoff to the first player when he plays risky is:

$$\pi_R = (1 - p)(1 - p\rho)(1 - q)p + pq(1 - q)(1 - p) + q(1 - p)[1 - q(1 - p)] + pq[1 - p + p^2\rho]$$

The difference $(\pi_S - \pi_R)$ is a linear and decreasing function of ρ . Hence for the upper bound on ρ it reaches its minimum. When $\rho = \frac{1}{p} \left[1 - \frac{q}{p}\right]$, $(\pi_S - \pi_R)$ reduces to $pq(p - q) > 0$. As a result $\pi_S > \pi_R, \forall \rho \leq \frac{1}{p} \left[1 - \frac{q}{p}\right]$.

When the second player is doing risky, the payoff for first player when he plays safe is:

$$\pi_S = (1 - p)(1 - q)p(1 - p\rho) + p(1 - q)[1 - q(1 - p)] + pqp(1 - p\rho)$$

and when he plays risky:

$$\pi_R = (1 - 2q + 2q^2\chi)p(1 - p\rho) + q(1 - q\chi)[1 - q(1 - p)]$$

$$\frac{\partial (\pi_S - \pi_R)}{\partial \rho} = p^3(1 - 2q) + p^2(-q + 2\chi q^2) \geq p^2(p - q)(1 - 2q)$$

By assumption 1

$$q < p(1 - p\rho) < p(1 - p) \Rightarrow 1 - 2q > 0$$

Hence $\frac{\partial(\pi_S - \pi_R)}{\partial \rho} > 0$. Similarly we can compute

$$\frac{\partial(\pi_S - \pi_R)}{\partial \chi} = q^2 ((1 - q) + p(q - 2) + 2p^2\rho) \geq q^2(1 - p - q)^2 \geq 0$$

It follows that $(\pi_S - \pi_R) \geq (\pi_S - \pi_R)_{\rho=1, \chi=1}$. After some manipulation

$$(\pi_S - \pi_R)_{\rho=1, \chi=1} = (p - q) [2pq(1 - p) + p(p - q^2) + (1 - q)((1 - p) - q)] > 0$$

A fortiori $(\pi_S - \pi_R) \geq 0$ for $\rho, \chi > 1$. Hence (S, S) is the unique NE. ■

Proof of Lemma 3 (Bonus stage - zero gap). Consider the case $i = 0$ first:

	S	R
S	$p(1 - p\rho) \frac{s}{(1+r)^2 - 1} ; p(1 - p\rho) \frac{s}{(1+r)^2 - 1}$	$p(1 - q) \frac{s}{(1+r)^2 - 1} ; q \frac{r}{(1+r)^2 - 1}$
R	$q \frac{r}{(1+r)^2 - 1} ; p(1 - q) \frac{s}{(1+r)^2 - 1}$	$q(1 - q\chi) \frac{r}{(1+r)^2 - 1} ; q(1 - q\chi) \frac{r}{(1+r)^2 - 1}$

It is easy to verify that S is a best response to S iff:

$$(1 - p\rho)\beta \geq \alpha$$

which proves (a) if we let $\alpha_0 \equiv \beta$. Clearly risky is a best response to safe iff the opposite occurs:

$$\alpha \geq (1 - p\rho)\beta$$

On the other hand, S is a best response to R iff:

$$\alpha \geq \beta \frac{1 - q}{1 - q\chi}$$

Hence for $\beta(1 - p\rho) \leq \alpha \leq \frac{1 - q}{1 - q\chi}\beta$, (S, R) and (R, S) are NE, which proves (b)

if we let, as above, $\alpha_0 \equiv \beta$. Finally R is a best response to R iff:

$$\alpha \geq \frac{1 - q}{1 - q\chi}\beta$$

which proves (c).

Consider now the case $i = s$:

	S	R
S	$p(1 - p\rho) \frac{(1+s)^2 - 1}{(1+r)^2 - 1} ; p(1 - p\rho) \frac{(1+s)^2 - 1}{(1+r)^2 - 1}$	$p(1 - q) \frac{(1+s)^2 - 1}{(1+r)^2 - 1} ; q \frac{(1+s)(1+r) - 1}{(1+r)^2 - 1}$
R	$q \frac{(1+s)(1+r) - 1}{(1+r)^2 - 1} ; p(1 - q) \frac{(1+s)^2 - 1}{(1+r)^2 - 1}$	$q(1 - q\chi) \frac{(1+s)(1+r) - 1}{(1+r)^2 - 1} ; q(1 - q\chi) \frac{(1+s)(1+r) - 1}{(1+r)^2 - 1}$

It is easy to verify that S is a best response to S iff:

$$\alpha \leq (1 - p\rho) \frac{2\beta + \beta^2 r}{1 + \beta + \beta r}$$

which proves (a), if we let

$$\alpha_s \equiv \frac{2\beta + \beta^2 r}{1 + \beta + \beta r}$$

Clearly R is a best response to S iff the opposite occurs, i.e. iff:

$$\alpha \geq (1 - p\rho)\alpha_s$$

On the other hand, S is a best response to R iff:

$$\alpha \leq \frac{1 - q}{1 - q\chi} \cdot \frac{2\beta + \beta^2 r}{1 + \beta + \beta r} = \frac{1 - q}{1 - q\chi} \alpha_s$$

Hence (S, R) and (R, S) are NE when

$$(1 - p\rho)\alpha_s \leq \alpha \leq \frac{1 - q}{1 - q\chi} \alpha_s$$

which proves (b). Finally R is a best response to R iff:

$$\alpha \geq \frac{1 - q}{1 - q\chi} \alpha_s$$

which proves (c).

Thirdly consider the case $i = r$:

	S	R
S	$p(1 - p\rho) \frac{(1+s)(1+r)-1}{(1+r)^2-1} ; p(1 - p\rho) \frac{(1+s)(1+r)-1}{(1+r)^2-1}$	$p(1 - q) \frac{(1+s)(1+r)-1}{(1+r)^2-1} ; q \frac{(1+r)^2-1}{(1+r)^2-1}$
R	$q \frac{(1+r)^2-1}{(1+r)^2-1} ; p(1 - q) \frac{(1+s)(1+r)-1}{(1+r)^2-1}$	$q(1 - q\chi) \frac{(1+r)^2-1}{(1+r)^2-1} ; q(1 - q\chi) \frac{(1+r)^2-1}{(1+r)^2-1}$

Safe is a best response to safe iff:

$$\alpha \leq (1 - p\rho) \frac{1 + \beta + \beta r}{2 + r}$$

which proves (a), if we let

$$\alpha_r \equiv \frac{1 + \beta + \beta r}{2 + r}$$

Safe is a best response to risky iff

$$p[s + r + sr] \geq q \frac{1-q}{1-q\chi} [2r + r^2]$$

Risky is a best response to safe iff:

$$q[2r + r^2] \geq p(1-p\rho)[s + r + sr]$$

With same manipulation one finds that (S, R) and (R, S) are NE iff:

$$(1-p\rho)\alpha_r \leq \alpha \leq \frac{1-q}{1-q\chi}\alpha_r$$

which proves (b). Finally, R is a best response to R iff:

$$\alpha \geq \frac{1-q}{1-q\chi} \cdot \frac{1+\beta+\beta r}{2+r} \equiv \frac{1-q}{1-q\chi}\alpha_r$$

which proves (c). ■

Proof of Proposition 2 (Performance and Risk). Recall that

$$\begin{aligned} \alpha_0 &= \beta \\ \alpha_s &= \frac{2\beta + \beta^2 r}{1 + \beta + \beta r} \\ \alpha_r &= \frac{1 + \beta + \beta r}{2 + r} \end{aligned}$$

Notice that $\alpha_0 \leq \alpha_s$ requires

$$\beta \leq \beta \frac{2 + \beta r}{1 + \beta + \beta r}$$

which is clearly true because $2 \geq (1 + \beta)$. Moreover

$$\alpha_r - \alpha_s = \frac{(-1 + \beta)^2}{(1 + \beta + \beta r)(2 + r)} \geq 0$$

which completes the proof. ■

Proof of Lemma 4 (Bonus stage - $(r - s)$ gap). After an interim perfor-

mance gap equal to $(r - s)$ the continuation game is as follows

	S	R
S	$(1 - 2p + p^2\rho) \frac{r}{(1+r)^2-1} + p \frac{(1+s)(1+r)-1}{(1+r)^2-1} ;$ $p(1 - p\rho) \frac{(1+s)^2-1}{(1+r)^2-1}$	$(1 - p)(1 - q) \frac{r}{(1+r)^2-1} +$ $+p(1 - q) \frac{(1+s)(1+r)-1}{(1+r)^2-1} ;$ $q(1 - p) \frac{(1+s)(1+r)-1}{(1+r)^2-1}$
R	$(1 - p)(1 - q) \frac{r}{(1+r)^2-1} + q \frac{(1+r)^2-1}{(1+r)^2-1} ;$ $p(1 - q) \frac{(1+s)^2-1}{(1+r)^2-1}$	$(1 - 2q + q^2\chi) \frac{r}{(1+r)^2-1} + q \frac{(1+r)^2-1}{(1+r)^2-1} ;$ $q(1 - q\chi) \frac{(1+r)(1+s)-1}{(1+r)^2-1}$

First, let us check the conditions under which (S, S) is a NE of this game. It can be easily verified that the following two inequalities need to hold:

$$(1 - 2p + p^2\rho) r + p[s + r + sr] \geq (1 - p)(1 - q)r + q[2r + r^2] \quad (2)$$

$$p(1 - p\rho) [2s + s^2] \geq q(1 - p) [s + r + sr] \quad (3)$$

After some manipulation one finds that inequality (2) requires:

$$\alpha \leq \frac{\beta + \beta r + p\rho}{1 + r + p} \equiv \alpha_1$$

Similarly inequality (3) requires:

$$\alpha \leq \frac{1 - p\rho}{1 - p} \cdot \frac{2\beta + \beta^2 r}{1 + \beta + \beta r} = \frac{1 - p\rho}{1 - p} \alpha_s$$

Hence (S, S) is a NE iff

$$\alpha \leq \min \left\{ \alpha_1; \frac{1 - p\rho}{1 - p} \alpha_s \right\}$$

which proves (a). Let us now consider the conditions under which (R, S) is a NE of this game. When the interim follower plays safe, for the leader is optimal to play risky iff:

$$(1 - p)(1 - q)r + q[2r + r^2] \geq (1 - 2p + p^2\rho) r + p[s + r + sr] \quad (4)$$

When the interim leader plays risky, for the follower is optimal to play safe iff:

$$p(1 - q) [2s + s^2] \geq q(1 - q\chi) [s + r + rs] \quad (5)$$

Inequality (4) holds whenever inequality (2) does not, hence for

$$\alpha \geq \alpha_1$$

Inequality (5) holds iff

$$\alpha \leq \frac{1-q}{1-q\chi} \alpha_s$$

Hence (R, S) is a NE iff

$$\alpha_1 \leq \alpha \leq \frac{1-q}{1-q\chi} \alpha_s$$

which proves (b). Let us now check the conditions under which (S, R) is a NE of this game. When the interim leader is playing safe, the interim follower finds it optimal to play risky iff:

$$q(1-p)[s+r+sr] \geq p(1-p\rho)[2s+s^2] \quad (6)$$

When the interim follower is playing risky, the interim leader finds it optimal to respond with safe iff:

$$(1-p)(1-q)r + p(1-q)[s+r+sr] \geq (1-2q+q^2\chi)r + q[2r+r^2] \quad (7)$$

Inequality (6) is the same as inequality (3), but with the opposite sign. Hence it holds iff

$$\alpha \geq \frac{1-p\rho}{1-p} \alpha_s$$

Inequality (7) can be rewritten as follows

$$\alpha \leq \frac{\beta(1+r)(1-q)-q}{1+r+q\chi} \equiv \alpha_2$$

Hence (S, R) is the NE of the game when

$$\frac{1-p\rho}{1-p} \alpha_s \leq \alpha \leq \alpha_2$$

which proves (c). Let us finally consider the conditions under which (R, R) is the NE of this game. When the interim leader plays risky, it is optimal for the follower to choose risky iff:

$$q(1 - q\chi)[s + r + sr] \geq p(1 - q)[2s + s^2] \quad (8)$$

When the interim follower is playing risky, it is optimal for the leader to respond with risky iff:

$$(1 - 2q + q^2\chi)r + q[r^2 + 2r] \geq (1 - p)(1 - q)r + p(1 - q)[s + r + sr] \quad (9)$$

Notice now that inequality (8) is the same as inequality (5), but with the opposite sign, so that it holds iff

$$\alpha \geq \frac{1 - q}{1 - q\chi} \alpha_s$$

Similarly, inequality (9) is the same as inequality (7), but with the opposite sign, so that it holds iff:

$$\alpha \geq \alpha_2$$

As a result, (R, R) is the NE iff

$$\alpha \geq \max \left\{ \frac{1 - q}{1 - q\chi} \alpha_s; \alpha_2 \right\}$$

which proves (d). ■

Proof of Lemma 5 (Bonus stage - large gap). First consider the case $j = s$. After an interim performance gap equal to s , the bonus subgame is as follows

	S	R
S	$(1 - 2p + p^2\rho) \frac{s}{(1+r)^2-1} + p \frac{(1+s)^2-1}{(1+r)^2-1};$ 0	$(1 - p)(1 - q) \frac{s}{(1+r)^2-1} +$ $+ p \frac{(1+s)^2-1}{(1+r)^2-1}; 0$
R	$(1 - p)(1 - q) \frac{s}{(1+r)^2-1} +$ $+ q \frac{(1+s)(1+r)-1}{(1+r)^2-1}; 0$	$(1 - 2q + q^2\chi) \frac{s}{(1+r)^2-1} + q \frac{(1+s)(1+r)-1}{(1+r)^2-1};$ $q(1 - q\chi) \frac{r}{(1+r)^2-1}$

For the interim follower it is weakly dominant to play risky. One can easily verify that when the interim follower is playing risky it is optimal for the leader to play risky iff:

$$\alpha \geq \beta \frac{1 + q + \beta r}{1 + \beta q \chi + \beta r} \equiv \hat{\alpha}_s$$

Hence, (R, R) is NE for

$$\alpha \geq \hat{\alpha}_s$$

which proves (b). Finally notice that (S, R) is a NE whenever (R, R) is not, which proves (a).

Consider now the case $j = r$. After an interim performance gap equal to r , the bonus subgame is as follows

	S	R
S	$(1 - p) \frac{r}{(1+r)^2 - 1} + p \frac{(1+s)(1+r) - 1}{(1+r)^2 - 1}; 0$	$(1 - p)(1 - q) \frac{r}{(1+r)^2 - 1} + p \frac{(1+s)(1+r) - 1}{(1+r)^2 - 1}; 0$
R	$(1 - q) \frac{r}{(1+r)^2 - 1} + q \frac{(1+r)^2 - 1}{(1+r)^2 - 1}; 0$	$(1 - 2q + q^2 \chi) \frac{r}{(1+r)^2 - 1} + q \frac{(1+r)^2 - 1}{(1+r)^2 - 1}; 0$

We have assumed that the follower, when indifferent, plays risky, so that we only have two candidate equilibria: (S, R) and (R, R) . Let us check first the conditions under which (S, R) is a NE. When the interim follower is playing risky, for the interim leader it is optimal to play safe iff:

$$(1 - p)(1 - q)r + p[s + r + sr] \geq (1 - 2q + q^2 \chi)r + q[2r + r^2]$$

which can be rewritten as

$$\alpha \leq \frac{q + \beta + \beta r}{1 + r + q \chi} \equiv \hat{\alpha}_r$$

which proves (a). Clearly (R, R) is the NE when the opposite holds, which proves (b) ■

Proof of Proposition 3 (SPE - $E[R] \gg E[S]$). In the second stage (R, R) is the unique equilibrium in every subgame by construction, as we take

$\alpha^* \equiv \max \left\{ \frac{1-q}{1-q\chi} \alpha_r, \alpha_2, \hat{\alpha}_s, \hat{\alpha}_r \right\}$. Let us now consider the equilibrium in the interim stage. We can summarise payoffs as follows

	S	R
S	$\frac{A}{(1+r)^2-1}; \frac{A}{(1+r)^2-1}$	$\frac{C}{(1+r)^2-1}; \frac{D}{(1+r)^2-1}$
R	$\frac{D}{(1+r)^2-1}; \frac{C}{(1+r)^2-1}$	$\frac{B}{(1+r)^2-1}; \frac{B}{(1+r)^2-1}$

where

$$A \equiv (1 - 2p + p^2\rho)(1 - q\chi)qr + p(1 - p\rho)[q(1 - q\chi)r + q(s + r + sr) + s(1 - 2q + q^2\chi)] + p^2\rho q(1 - q\chi)(s + r + sr)$$

$$B \equiv q(1 - 2q + q^2\chi)(1 - q\chi)r + q(1 - q\chi)[q(2r + r^2) + r(1 - 2q + q^2\chi)] + q^3\chi(1 - q\chi)(2r + r^2)$$

$$C \equiv q(1 - q)(1 - q\chi)(1 - p)r + p(1 - q)[q(s + r + rs) + s(1 - 2q + q^2\chi)] + pq^2(1 - q\chi)(r + s + rs)$$

$$D \equiv (1 - q)(1 - q\chi)q(1 - p)r + q(1 - p)[q(2r + r^2) + r(1 - 2q + q^2\chi)] + p(1 - q)q(1 - q\chi)r + pq[p(r + s + rs) + (1 - p)(1 - q)r]$$

Strategy R is a best response to strategy S , as long as : $D \geq A$. After some manipulation one obtains

$$\begin{aligned} D - A &= q \{ (p - q)(1 - q\chi) + (1 - p)[1 - 2q + q^2\chi + p(1 - q)] \} r + \\ &\quad - p(1 - p\rho)(1 - 2q + q^2\chi)s + \\ &\quad + q^2(1 - p)(2r + r^2) + \\ &\quad + pq[p - (1 - p\rho) - p\rho(1 - q\chi)](s + r + sr) \end{aligned}$$

Notice that by assumption 1

$$1 - q\chi \geq 1 - p$$

hence, substituting $(1 - p)$ for $(1 - q\chi)$ in the first term and rearranging, we obtain

$$\begin{aligned}
D - A &\geq qr(1 - p)(1 + 2p - 3q - pq + q^2\chi) + \\
&\quad -ps(1 - p\rho)(1 - 2q + q^2\chi) + \\
&\quad +q^2(1 - p)(2r + r^2) + \\
&\quad -qp(1 - p - pq\rho\chi)(s + r + sr)
\end{aligned}$$

Notice now that the sum between the first two terms is always positive. In fact in the parameter space that we are considering

$$qr > ps$$

and moreover

$$\begin{aligned}
1 - p &\geq 1 - p\rho \\
1 + 2p - 3q - pq + q^2\chi &\geq 1 - 2q + q^2\chi
\end{aligned}$$

The sum of the third and fourth terms is also positive. In fact

$$q(1 - p)(2r + r^2) \geq p(1 - p - pq\rho\chi)(s + r + sr)$$

requires

$$\alpha \geq \frac{1 - p - pq\rho\chi}{1 - p} \alpha_r$$

which holds true in the parameter space that we are considering, where

$$\alpha \geq \frac{1 - q}{1 - q\chi} \alpha_r$$

Hence, as $(D - A) \geq 0$, risky is a best response to safe. We can show that risky is a best response to risky as well, so that (R, R) is the unique equilibrium of the first-period game. R is a best response to R iff $(B - C) \geq 0$. After some algebra

we can rewrite the difference $(B - C)$ as the sum of four terms as follows:

$$\begin{aligned}
B - C &= q(1 - q\chi)r(p - q + q^2\chi - pq) + \\
&\quad + q[(1 - q\chi)q(2r + r^2) - p(1 - q)(s + r + sr)] + \\
&\quad + q(1 - q\chi)[q^2\chi(2r + r^2) - pq(s + r + sr)] + \\
&\quad + (1 - 2q + q^2\chi)[qr(1 - q\chi) - p(1 - q)s]
\end{aligned}$$

The first term is increasing in χ ; it reaches its minimum for $\chi = 1$, where it is equal to

$$(p - q)(1 - q) > 0$$

Hence the first term is always positive. The second and third terms are also positive. In particular, the second term is positive as long as

$$\alpha \geq \frac{1 - q}{1 - q\chi} \left(\frac{s + r + sr}{2r + r^2} \right) = \frac{1 - q}{1 - q\chi} \alpha_r$$

which holds true by assumption in the parameter space that we are considering.

The third term is positive as long as

$$\alpha \geq \frac{1}{\chi} \left(\frac{s + r + sr}{2r + r^2} \right)$$

Notice that this is a less stringent condition than the one required for the previous term. Hence the third term is also positive. Finally the fourth term is positive for

$$\alpha \geq \frac{1 - q}{1 - q\chi} \beta$$

which, again, holds true by assumption in the parameter space that we are considering. As a result $B > C$ and risky is a best reply to risky. Therefore the unique equilibrium is (R, R) . ■

Proof of Proposition 4 (SPE - $E[S] \gg E[R]$). In the second stage, the unique NE in every subgame is (S, S) when the performance gap is small and

(S, R) when the performance gap is large, by construction. In fact we define $\alpha^{**} \equiv \min \{(1 - p\rho)\alpha_0, \alpha_1, \hat{\alpha}_s, \hat{\alpha}_r\}$. Let us now consider the equilibrium in the interim stage. The interim stage payoff matrix is as follows

	S	R
S	$\frac{A}{(1+r)^2-1}; \frac{A}{(1+r)^2-1}$	$\frac{C}{(1+r)^2-1}; \frac{D}{(1+r)^2-1}$
R	$\frac{D}{(1+r)^2-1}; \frac{C}{(1+r)^2-1}$	$\frac{B}{(1+r)^2-1}; \frac{B}{(1+r)^2-1}$

where

$$\begin{aligned} A &\equiv p(1 - p\rho)(1 - 2p + p^2\rho)s + pq(1 - p\rho)(1 - p)r + \\ &\quad p^2(1 - p\rho)(s^2 + 2s) + p(1 - p)(1 - p\rho)(1 - q)s + \\ &\quad p^3\rho(1 - p\rho)(s^2 + 2s) \end{aligned}$$

$$\begin{aligned} B &\equiv (1 - 2q + q^2\chi)p(1 - p\rho)s + pq(1 - q\chi)(s + r + sr) + \\ &\quad q(1 - q\chi)(1 - p)(1 - q)r + q^2\chi p(1 - p\rho)(s + r + sr) \end{aligned}$$

$$\begin{aligned} C &\equiv p(1 - p)(1 - p\rho)(1 - q)s + p(1 - q)^2(1 - p)s + \\ &\quad p^2(1 - q)(2s + s^2) + p^2q(1 - p\rho)(2s + s^2) \end{aligned}$$

$$\begin{aligned} D &= (1 - p)(1 - p\rho)(1 - q)ps + (1 - p)(1 - q)pqr + \\ &\quad (1 - p)qp(s + r + sr) + q(1 - p)^2(1 - q)r + \\ &\quad p^2q(s + r + sr) + pq(1 - 2p + p^2\rho)r \end{aligned}$$

We can show that $C \geq B$ as follows. First, notice that $(C - B)$ is increasing in χ . In fact

$$\begin{aligned} \frac{\partial (C - B)}{\partial \chi} &= -q^2p(1 - p\rho)s - q^2p(1 - p\rho)(s + r + sr) + \\ &\quad + q^2p(s + r + sr) + q^2(1 - p)(1 - q)r \end{aligned}$$

As

$$q^2p(s + r + sr) > q^2p(1 - p\rho)(s + r + sr)$$

and, by assumption 1:

$$q^2(1-p)(1-q)r > q^2p(1-p\rho)s$$

it follows that

$$\frac{\partial(C-B)}{\partial\chi} > 0$$

Hence $(C-B)$ is increasing in χ and reaches its minimum for $\chi = 1$. We can now show that, even when $\chi = 1$, $(C-B) > 0$, so that playing safe is always the best reply when the opponent plays risky.

$$\begin{aligned} (C-B)_{\chi=1} &= p(1-p\rho)(1-q)(q-p)s + \\ &\quad + p(1-p\rho)q[p(s^2+2s) - q(r+s+sr)] + \\ &\quad + (1-q)p[p(s^2+2s) - q(r+s+rs)] + \\ &\quad + (1-p)(1-q)^2(sp-rq) \end{aligned}$$

Notice now that the second and the third terms are positive, because

$$p(s^2+2s) - q(r+s+sr) \geq 0$$

holds true in the parameter space that we are considering, where

$$\frac{q}{p} = \alpha \leq \alpha_s = \frac{s^2+2s}{r+s+rs}$$

Hence we only need to prove that the sum of the first and the fourth terms is positive. Notice that, since $(q-p) < 0$:

$$\begin{aligned} &p(1-p\rho)(1-q)(q-p)s + (1-p)(1-q)^2(sp-rq) \\ &> p(1-p)(1-q)(q-p)s + (1-p)(1-q)^2(sp-rq) \\ &= (1-p)(1-q)[sp(1-p) - (1-q)rq] \end{aligned}$$

The latter expression is positive in the parameter space that we are considering, where $\alpha \leq (1-p\rho)\beta$. Now we need to prove that playing safe is a best response

also when the opponent plays safe, i.e. we need to show that $D < A$. After some manipulation, we obtain

$$\begin{aligned} D - A &= p(1 - 2p + p^2\rho) [rq - (1 - p\rho)s] + \\ &\quad + qr \left[(1 - p)^2 (1 - q) + (1 - p)p(p\rho - q) \right] + \\ &\quad + p \left[q(s + r + sr) - p(1 - p\rho)(1 + p\rho)(2s + s^2) \right] \end{aligned}$$

Recall that we are working under the assumption $\alpha \leq \beta(1 - p\rho)$, which is equivalent to

$$rq \leq ps(1 - p\rho)$$

It is easy to verify that $(D - A)$ is increasing in rq . Substituting the upper bound for rq (i.e. $ps(1 - p\rho)$) in the expression above, we find

$$\begin{aligned} (D - A) &\leq (D - A)_{rq=ps(1-p\rho)} = \\ &= p(1 - p\rho)s [(1 - p)(p - q) + p - p\rho(s + 2)] - qps \leq \\ &\leq p(1 - p)s [(1 - p)(p - q) - p(s + 1)] < 0 \end{aligned}$$

In the interim stage $C > B$ and $A > D$ imply that playing safe is a dominant strategy. Hence (S, S) is the unique NE in the interim stage. ■

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