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ABSTRACT

The Effect of Monetary Unification on Public Debt and its Real Return

We explore the implications of monetary unification for real interest rates and (relative) public debt levels. The adoption of a common monetary policy renders the risk-return characteristics of the participating countries more similar, so that the substitutability of their public debt increases after unification. This implies that the average expected real return on this debt increases. Also, the share of the debt issued by relatively short-sighted governments, or by countries that initially have a relatively dependent central bank, increases after unification. A transfer scheme that penalizes debt increases beyond the union average is able to undo the interest rate effect of unification, but further magnifies the spread of the relative debt levels.

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Keywords: (relative) public debt, central bank independence, externalities, monetary union, real interest rates and substitutability

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1 Introduction

The implications of the European Monetary Union (EMU) for public debt and, in particular, the real returns on this debt, have so far received only limited attention.[1] The dominant view is that the spillovers of a change in an individual country's stock of debt on other countries' real debt returns is negligible, because the real debt return is tied down at the world market level.

In this paper, we demonstrate that this view may not be correct. Monetary unification makes the risk-characteristics of the public debt of union members more similar. Hence, it increases the substitutability of public debt from the various member countries in investors' portfolios. Using a simple theoretical model, we show that as a result the average expected real debt return is higher after monetary unification than before. Such an increase in the interest costs of issuing debt is likely to be harmful for an economy, for example, because it raises the government's need for potentially distortionary tax revenues.

Because of indirect effects running via the consumption pattern, the general equilibrium implications for the debt level are ambiguous. However, we find that the share of the union area's debt issued by countries with relatively undisciplined governments or relatively dependent central banks before unification increases after unification. We believe that this is an important implication of the model, because relative debt levels seem politically more relevant than absolute debt levels, as it is hard for governments to blame other governments for their high public debt when their own debt is large. Relative debt levels will be a much bigger source of tension in a monetary union and indicate the relative importance of the interest rate externalities that countries exert on each other. It seems likely that low-debt countries become worried about the debt policies followed by high-debt countries, if the former realize that the policies of the latter drive up their own debt-servicing costs. A transfer scheme that punishes debt increases beyond the union average is able to undo the expected interest rate effect of unification. However, it magnifies the spread of the relative debt levels.

[1] Van Aarle et al. (1997), Beetsma and Bovenberg (1999) and Illing (1999) study the implications of monetary unification for public debt, while Beetsma and Uhlig (1999) and Leith and Wren-Lewis (2000, 2002) investigate public debt in a monetary union when fiscal rules are present. Missale (1999) provides an extensive discussion of public debt policies in Europe before EMU. Canzoneri and Diba (1991) and Restoy (1996) analyse real interest rates in a monetary union, while Chang (1990) explores the international externalities of national fiscal policies for the real interest rate. For empirical work on the relation between public debt and real interest rates, see e.g. Ford and Laxton (1999).

The remainder of this paper is structured as follows. Section 2 presents the model. This section also derives the outcomes of the model under national monetary policymaking and a monetary union. Section 3 discusses the main results. Section 4 explores a simple transfer scheme that induces governments to internalize the interest rate externalities caused by their debt policies. Finally, Section 5 concludes the paper.

2 The model

2.1 Set-up

We consider a world with $N > 1$ countries. The countries $1, .., N^u$, with $2 \leq N^u \leq N$, may form a monetary union, and we will compare the situation before and after monetary unification. The demographic structure consists of overlapping generations of a constant size. The countries are of equal size, with the size of a generation in a country normalized to 1. Each generation lives for two periods and is represented by a representative agent per country. In this subsection, we describe the behavior of the representative agents, the governments and the central banks in the different countries. In the next subsection, we will solve the model and consider separately the case before and after monetary unification.

2.1.1 The representative agents

In each country i, there is a representative agent per generation. In the first period of her life, the representative agent of the generation born in period t receives an endowment Y, pays taxes $T_{i,t}$, consumes part of her after-tax endowment, and saves the remainder for the second period of her life. In the second period of her life, she is retired and consumes her gross savings. The agent can save by holding public debt of each of the N countries, or by investing in a saving technology that yields an exogenously determined risk-free real rate of return r^*. Therefore, her budget constraint is given by:

$$C^i_{2,t+1} = \left(Y - T_{i,t} - C^i_{1,t}\right)\left[\sum_{j=1}^{N} \omega^i_{j,t}\left(1 + r_{j,t+1}\right) + \left(1 - \sum_{j=1}^{N} \omega^i_{j,t}\right)\left(1 + r^*\right)\right], \tag{1}$$

where $C^i_{1,t}$ and $C^i_{2,t+1}$ are her consumption in periods t and $t+1$, respectively, while $\omega^i_{j,t}$ is the share of her savings in period t invested in public debt of country j (such that $1 - \sum_{j=1}^{N} \omega^i_{j,t}$ is the share of her savings invested using

the risk-free savings technology) and $r_{j,t+1}$ is the (ex-post) real interest rate paid in period $t+1$ on the debt issued in period t.

Subject to (1), the representative agent maximizes her expected lifetime utility:

$$U_t^i = C_{1,t}^i - \tfrac{1}{2}\gamma\left(C_{1,t}^i\right)^2 + \frac{1}{1+\rho}\mathrm{E}_t\left[C_{2,t+1}^i - \frac{1}{2}\gamma\left(C_{2,t+1}^i\right)^2\right], \quad \text{with } \gamma > 0, \tag{2}$$

where ρ is the agent's discount rate and $\mathrm{E}_t[.]$ is the expectations operator conditional on the available information (see below). In what follows, we will assume for simplicity that the risk-free interest rate r^* is equal to the discount rate ρ. We also assume that γ is such that the marginal utility of consumption is always positive in the situations that are relevant for the ensuing analyis.

2.1.2 The governments

Each country i has a government (called "government i") which spends an exogenously determined amount $G_{i,t}$ in period t. Government i finances its spending by raising taxes and issuing nominal public debt with a maturity of one period. Hence, its dynamic budget constraint for period t can be written as:

$$B_{i,t} = G_{i,t} + B_{i,t-1}(1 + r_{i,t}) - T_{i,t}, \tag{3}$$

where $B_{i,t}$ is the amount of public debt issued in period t, while $B_{i,t-1}$ is the debt carried over from period $t-1$ and to be paid off in period t against an interest rate $r_{i,t}$. We exclude the possibility of debt default.

Subject to (3), the government has to make a trade-off in period t between higher taxes or issuing more public debt. On the one hand, higher taxes in period t lower the lifetime utility of the generation that has to pay the taxes in period t. On the other hand, higher public debt leads to higher taxes in the future, and lowers the lifetime utility of the generations that have to pay taxes then. Let us capture this idea by assuming that government i minimizes the following loss-function:

$$L_{i,t}^{gov} = T_{i,t} + \frac{1}{1+\beta_i}\mathrm{E}_t\left[B_{i,t}(1 + r_{i,t+1})\right], \quad \text{with } \beta_i > 0. \tag{4}$$

Parameter β_i is the rate at which government i discounts the expected real debt burden for the future generations. This discount rate may differ across countries. In reality, differences in the government's discount rate will be related to political factors, such as political polarization and the government

turnover.[2] The expected future debt burden includes the real interest which government i has to pay at that time. As public debt is issued at a nominal interest rate, uncertainty about the inflation rate in country i between periods t and $t+1$ causes uncertainty about the *ex-post* real interest rate $r_{i,t+1}$ (see below). As a result, the real value of the debt burden in period $t+1$ is in general not known with certainty as of period t.

2.1.3 The central banks

Before monetary unification takes place, each country i has its own central bank. Every period t, the central bank determines the growth rate of the money supply from period t to period $t+1$, which we denote by $m_{i,t}$. The growth rate of the money supply determines the inflation rate, but the link between money growth and inflation is stochastic (for instance, because of velocity shocks):

$$\pi_{i,t} = m_{i,t}\left(1+\varepsilon_{i,t}\right), \tag{5}$$

where $\pi_{i,t}$ is the inflation rate in country i between periods t and $t+1$, and $\varepsilon_{i,t}$ is a stochastic shock with mean zero, variance σ^2 and i.i.d. over time and across countries (at all time lags). We observe that with perfectly substitutable goods and perfect arbitrage across countries, the nominal exchange rate adjusts perfectly and instantaneously to ensure that the real return is the same for each investor, no matter in which country this investor is residing. Hence, unexpected movements in relative inflation rates as a result of the ε-shocks are offset by nominal exchange rate changes.

The primary objective of the central bank is to maintain price stability, that is, to stabilize the inflation rate around zero. However, the central bank also cares about the public debt which the government has to repay (possibly because of pressure from the fiscal authorities), and therefore has an incentive to create surprise inflation. Let us model this trade-off by assuming that the central bank minimizes the following loss-function:

$$L^{cb}_{i,t} = \tfrac{1}{2}\mathrm{E}_t\left(\pi_{i,t}\right)^2 + \lambda_i \mathrm{E}_t\left[B_{i,t}\left(1+r_{i,t+1}\right)\right], \qquad \text{with } \lambda_i > 0, \tag{6}$$

where $r_{i,t+1} \simeq r^e_{i,t+1} + \pi^e_{i,t} - \pi_{i,t}$. Here, and in the sequel, we use a superscript "e" as short-hand notation for $\mathrm{E}_t[.]$. One can interpret the parameter λ_i as

[2] One would expect governments to discount the future more heavily when preferences of political parties differ more or when the government has a higher chance of losing office. In both cases, there is a stronger incentive to spend more now, because any remaining resources might be spent by another government with potentially different preferences. Cukierman et al. (1992) in their model derive the government's "effective" discount rate as an explicit function of the abovementioned political factors.

a measure of the central bank's dependence on the goverment. The higher is λ_i, the more dependent is the central bank, because it has a larger interest in softening the budgetary situation of the government, as measured by $B_{i,t}$.

When the countries $1, .., N^u$ form a monetary union, their central banks are replaced by a common central bank for the whole union. This common central bank determines m_t^u, the common nominal money growth rate in each country of the union. This, in turn, determines π_t^u, the common inflation rate in each country of the union. The relation between m_t^u and π_t^u is as in equation (5), where the stochastic shocks are now perfectly correlated among the countries that participate in the monetary union, but are still independent otherwise.[3] The other properties of the shocks are unaffected. The loss function of the common central bank now depends on the union-wide inflation rate π_t^u and on the average real public debt burden which the different governments in the member countries will have to repay:

$$L_t^{cb,u} = \tfrac{1}{2}\mathrm{E}_t\left(\pi_t^u\right)^2 + \lambda^u \mathrm{E}_t\left[\frac{1}{N^u}\sum_{i=1}^{N^u} B_{i,t}\left(1 + r_{i,t+1}\right)\right], \qquad \text{with } \lambda^u > 0. \quad (7)$$

Equation (5) implies that higher money growth, and therefore also higher expected inflation, goes hand in hand with a higher variance of inflation. Furthermore, as inflation uncertainty is the only source of uncertainty in the real debt returns, the variance of the inflation rate $\pi_{i,t}$ immediately yields the variance of $r_{i,t+1}$:

$$\mathrm{Var}_t\left(r_{i,t+1}\right) = \mathrm{Var}_t\left(\pi_{i,t}\right) = m_{i,t}^2\sigma^2, \quad (8)$$

which for $i \in \{1, .., N^u\}$ thus equals $\left(m_t^u\right)^2\sigma^2$ in the case of monetary unification. Given that inflation is perfectly correlated across countries if they both participate in the monetary union but independent across countries otherwise, we have the following expressions for the covariances between the real debt returns:

$$\begin{aligned} \mathrm{Cov}_t\left(r_{i,t+1}, r_{j,t+1}\right) &= \mathrm{Cov}_t\left(\pi_{i,t}, \pi_{j,t}\right) = \left(m_t^u\right)^2\sigma^2, \quad \text{if } i \text{ and } j \text{ both} \\ &\qquad \text{participate in a monetary union,} \\ \mathrm{Cov}_t\left(r_{i,t+1}, r_{j,t+1}\right) &= \mathrm{Cov}_t\left(\pi_{i,t}, \pi_{j,t}\right) = 0, \quad \text{otherwise.} \end{aligned} \quad (9)$$

[3] We can allow for imperfect correlations in the ε-shocks across union members, if they represent velocity shocks. However, given that the local inflation rates are equalized across countries (because of the perfect arbitrage and perfect substitutability of the goods), differences in these shocks are offset by differences in the growth rates of national holdings of the union-wide currency. Eventually, it is only the cross-country average money growth rate that matters, because it determines the common inflation rate, which, in turn, causes the real returns on all the debt issued in the union to be perfectly correlated (see below).

2.1.4 Equilibrium conditions

Equilibrium in the goods market requires that:

$$NY = C_{1,t} + C_{2,t} + I_t + G_t, \tag{10}$$

where $C_{1,t} \equiv \sum_{i=1}^{N} C_{1,t}^i$ is the aggregate consumption of all young agents, $C_{2,t} \equiv \sum_{i=1}^{N} C_{2,t}^i$ is the aggregate consumption of all old representative agents, $I_t \equiv \sum_{i=1}^{N} \left(Y - T_{i,t} - C_{1,t}^i\right) \left(1 - \sum_{j=1}^{N} \omega_{j,t}^i\right)$ is the aggregate investment using the risk-free rate savings technology, and $G_t \equiv \sum_{i=1}^{N} G_{i,t}$ is aggregate government consumption.

Equilibrium in the market for each country's public debt requires:

$$B_{i,t} = \sum_{j=1}^{N} \left(Y - T_{j,t} - C_{1,t}^j\right) \omega_{i,t}^j, \quad \text{for all } i. \tag{11}$$

2.2 Solution of the model

The timing within an arbitrary period t is as follows. First, each policymaker sets its policy instrument, taking as given the instrument choices of all the other policymakers. Hence, the policymakers are involved in a Nash game against each other. Government i selects $B_{i,t}$, the central bank of country i chooses $m_{i,t}$, if i does not participate in a monetary union, and a union central bank chooses m_t^u for each union member. Simultaneously with the instrument choices, conditional expectations $\mathrm{E}_t[.]$ are formed and private sector agents take their investment decisions. In selecting their public debt, the governments take into account the private sector demand schedule for public debt. Finally, the shocks $\varepsilon_{i,t}$ materialize. This determines the realized inflation rate and the realized rate of return on the various countries' public debt. Hence, $\varepsilon_{i,t}$ is *not* included in the information set on which the conditional expectation $\mathrm{E}_t[.]$ is based.

We first derive the representative agents' demand for public debt. Then, we derive the solutions to the problems faced by the governments and the central banks.

The first-order conditions of the representative young agent in country i with respect to her decisions to hold country j's public debt and to invest with the risk-free saving technology, are:

$$\begin{aligned} 1 - \gamma C_{1,t}^i &= \frac{1}{1+\rho} \mathrm{E}_t \left[(1 + r_{j,t+1}) \left(1 - \gamma C_{2,t+1}^i\right) \right], \quad \text{for all } j, \\ C_{1,t}^i &= C_{2,t+1}^{i,e}. \end{aligned}$$

Substitute the budget constraint (1) into the first first-order-condition and work out the argument of the expectations operator. One then finds:

$$\begin{aligned} 1-\gamma C_{1,t}^{i} &= \frac{1}{1+\rho}\left(1+r_{j,t+1}^{e}\right)\left(1-\gamma C_{2,t+1}^{i,e}\right) \\ &\quad -\frac{\gamma\left(Y-T_{i,t}-C_{1,t}^{i}\right)}{1+\rho}\left[\omega_{j,t}^{i}\mathrm{Var}_{t}\left(r_{j,t+1}\right)+\sum_{k=1,k\neq j}^{N}\omega_{k,t}^{i}\mathrm{Cov}_{t}\left(r_{j,t+1},r_{k,t+1}\right)\right], \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for all } j. \end{aligned}$$

Aggregating across the representative agents of the different countries, taking into account the equilibrium conditions (11) and using the assumption that $r^{*}=\rho$, we obtain the following "mean-variance"-expression for the *demand for country i's debt* as a function of its expected real interest rate:

$$\begin{aligned} r_{i,t+1}^{e} &= r^{*}+\mu_{t}\left[B_{i,t}\mathrm{Var}_{t}(r_{i,t+1})+\sum_{k=1,k\neq i}^{N}B_{k,t}\mathrm{Cov}_{t}\left(r_{i,t+1},r_{k,t+1}\right)\right], \\ \text{with } \mu_{t} &= \frac{\gamma}{N\left(1-\gamma\bar{C}_{1,t}\right)}. \end{aligned} \tag{12}$$

Here, $\bar{C}_{1,t}=C_{1,t}/N$ is the average consumption of the representative agents of the young generation across the different countries, so that $1-\gamma\bar{C}_{1,t}$ is the average marginal utility of consumption across agents.

The problem of government i amounts to the minimization of its loss function (4), subject to its budget constraint (3) and expression (12), taking the public debt of the other countries and monetary policy (and, thus, the (co)variances of the returns) as given. This yields the following expression for the *supply of country i's debt* as a function of its expected real interest rate:

$$r_{i,t+1}^{e}=\beta_{i}-\mu_{t}B_{i,t}\mathrm{Var}_{t}\left(r_{i,t+1}\right). \tag{13}$$

The intuition for this expression is as follows. Government i issues debt until the real interest rate which it expects to pay on its public debt equals its discount rate *minus* a term that takes into account that a marginal increase of its indebtedness drives up the interest rate and, therefore, makes all of its outstanding debt more expensive. In the following, we will confine the analysis to the case of $\beta_{i}>r_{i,t+1}^{e}$(such that $B_{i,t}>0$) for all countries i.

We shall now consider the central banks' optimization problem. To this end, we will distinguish the situation before and after monetary unification.

2.2.1 Solution without a monetary union

In the absence of a monetary union each country has its own central bank. The problem of the central bank of country i is then to choose the money

growth rate $m_{i,t}$ such that it minimizes its loss function (6), subject to equation (5) and taking the country's public debt level $B_{i,t}$ and the expected values $r^e_{i,t+1}$ and $\pi^e_{i,t}$ as given. This leads to:

$$m_{i,t} = \pi^e_{i,t} = \frac{\lambda_i B_{i,t}}{1+\sigma^2}. \tag{14}$$

Hence, the money growth rate is increasing in the amount of debt and the degree of central bank dependence, λ_i.

From equations (8) and (9) we then obtain the second moments of the debt returns:

$$\begin{aligned} \text{Var}_t\left(r_{i,t+1}\right) &= \left(\frac{\lambda_i B_{i,t}}{1+\sigma^2}\right)^2 \sigma^2 \equiv \tilde{\sigma}_i^2, & (15)\\ \text{Cov}_t\left(r_{i,t+1}, r_{j,t+1}\right) &= 0, \qquad \text{for all } j \neq i. & (16) \end{aligned}$$

Substitute these expressions into equations (12) and (13), equate the resulting right-hand sides and solve to give:

$$B_{i,t} = \frac{\beta_i - r^*}{2\mu_t \tilde{\sigma}_i^2}, \tag{17}$$

After substituting (15) into (17), we solve the resulting equation to give:

$$B^{nu}_{i,t} = \left[\frac{\beta_i - r^*}{2\mu^{nu}_t \lambda_i^2}\left(\frac{1+\sigma^2}{\sigma}\right)^2\right]^{\frac{1}{3}}, \tag{18}$$

where the superscript "nu" refers to the case without a monetary union. For future use, we derive the public debt level of country $i \in \{1, .., N^u\}$ relative to the average public debt for this group of countries, which we denote by $\bar{B}^{nu}_t$:

$$b^{nu}_{i,t} \equiv \frac{B^{nu}_{i,t}}{\bar{B}^{nu}_t} = \frac{\left[\left(\beta_i - r^*\right)/\lambda_i^2\right]^{\frac{1}{3}}}{\frac{1}{N^u}\sum_{j=1}^{N^u}\left[\left(\beta_j - r^*\right)/\lambda_j^2\right]^{\frac{1}{3}}}. \tag{19}$$

Finally, the equilibrium expected real debt return follows upon combining (17) and (13):

$$r^{e,nu}_{i,t+1} = \frac{1}{2}\left(\beta_i + r^*\right). \tag{20}$$

2.2.2 Solution with a monetary union

Let us now consider the case where the countries $1, .., N^u$ form a monetary union. The problem of the common central bank is to choose the union-wide money growth rate m_t^u such that its loss function (7) is minimized subject to equation (5), taking the public debt levels in its member countries as well as the expected values $r_{i,t+1}^e$ and $\pi_t^{e,u}$ as given. This leads to:

$$\begin{aligned} m_t^u &= m_{i,t} = \pi_t^{e,u} = \pi_{i,t}^e = \frac{\lambda^u \bar{B}_t^u}{1+\sigma^2}, \qquad \text{for all countries } i \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{that participate in the union,} \end{aligned}$$

where $\bar{B}_t^u = \frac{1}{N^u}\sum_{i=1}^{N^u} B_{i,t}^u$ is the average public debt level in the union's member countries. The second moments of the debt returns then follow from equations (8) and (9):

$$\begin{aligned} \mathrm{Var}_t\left(r_{i,t+1}\right) &= \left(\frac{\lambda^u \bar{B}_t^u}{1+\sigma^2}\right)^2 \sigma^2 \equiv \hat{\sigma}^2, \qquad \text{for all countries } i \quad (21) \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{that participate in the union,} \end{aligned}$$

$$\begin{aligned} \mathrm{Cov}_t\left(r_{i,t+1}, r_{j,t+1}\right) &= \hat{\sigma}^2, \qquad\qquad \text{if countries } i \text{ and } j \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{both participate in the union,} \end{aligned}$$

$$= 0, \qquad \text{otherwise.} \qquad (22)$$

If we substitute the above expressions into equations (12) and (13), we obtain, respectively,

$$r_{i,t+1}^e = r_{t+1}^e = r^* + N^u \mu_t \bar{B}_t^u \hat{\sigma}^2, \qquad (23)$$

$$r_{i,t+1}^e = \beta_i - \mu_t B_{it} \hat{\sigma}^2, \qquad (24)$$

Take the union average of (24):

$$r_{t+1}^e = \bar{\beta} - \mu_t \bar{B}_t^u \hat{\sigma}^2, \qquad (25)$$

where $\bar{\beta} = \frac{1}{N^u}\sum_{i=1}^{N^u} \beta_i$ is the average value of β_i across the union members. Combining (25) with (23) yields

$$\bar{B}_t^u = \frac{1}{\mu_t \hat{\sigma}^2} \frac{\bar{\beta} - r^*}{1+N^u}.$$

Substitute (21) into this expression and solve to obtain:

$$\bar{B}_t^u = \left[\frac{\bar{\beta} - r^*}{(1+N^u)\,\mu_t^u\,(\lambda^u)^2} \left(\frac{1+\sigma^2}{\sigma} \right)^2 \right]^{\frac{1}{3}}, \tag{26}$$

where the superscript "u" in μ_t^u refers to the case of a monetary union. For future use, we also derive the relative debt level:

$$\begin{aligned} b_{i,t}^u &\equiv \frac{B_{i,t}^u}{\bar{B}_t^u} = 1 + \frac{\beta_i - \bar{\beta}}{\bar{B}_t^u \mu_t^u \hat{\sigma}^2} \\ &= 1 + (1+N^u)\,\frac{\beta_i - \bar{\beta}}{\bar{\beta} - r^*}, \end{aligned} \tag{27}$$

which we obtain by combining (23), (24) and (25). Finally, the equilibrium expected real debt return follows upon combining (23) and (25):

$$r_{i,t+1}^{e,u} = \frac{N^u}{1+N^u}\bar{\beta} + \frac{1}{1+N^u} r^*, \tag{28}$$

The expected debt returns and the public debt levels of the other countries are still given by (20) and (19), respectively, with μ_t^{nu} replaced by μ_t^u.

3 The effect of a monetary union on debt returns and public debt

In this section, we analyze how monetary unification affects the public debt levels and their returns in the union's member countries.

Let us first consider the expected real debt returns. In the derivation of equations (20) and (28) the variance of the real return ($\tilde{\sigma}^2$, respectively $\hat{\sigma}^2$) drops out and, thus, in equilibrium does not affect the expected real return. The reason for this is as follows. According to equation (12), the higher the variance $\mathrm{Var}(r_{i,t+1})$, the less public debt investors are willing to hold in their portfolio for a given expected rate of return. However, according to equation (13), the higher $\mathrm{Var}(r_{i,t+1})$, the less public debt the government will want to supply for a given expected rate of return (because the more the interest rate will rise and the more expensive the outstanding debt will become when the government tries to supply an extra unit of public debt). As a result, any change in the demand for debt due to a change in $\mathrm{Var}(r_{i,t+1})$ will be accompanied by a change in the supply of debt of equal magnitude. This implies that in equilibrium, a change in $\mathrm{Var}(r_{i,t+1})$ only affects the level of the

public debt, but not its expected rate of return.[4] This leads us to Proposition 1, which follows immediately from the preceding discussion:

Proposition 1 *In equilibrium, uncertainty about the inflation rate does not affect the expected real debt returns. Consequently, whether inflation becomes more or less predictable when countries form a monetary union, does not matter for the expected real debt returns.*

However, when countries form a monetary union, the returns on their public debt become perfectly correlated with each other. As a result, holding public debt of these countries increases the risk of the investors' overall portfolio, even when the variance of the debt returns does not change. Individuals will therefore demand a higher risk premium, which drives up the average expected rate of return. This yields Proposition 2:

Proposition 2 *(a) The average expected real return on the public debt of the potential union members increases as a result of monetary unification. (b) Given that countries form a monetary union, the average expected real return on their public debt is increasing in the union size.*

Proof. (a) The average expected real rate of return on the public debt of countries 1 until N^u before they form a monetary union follows from equation (20):

$$\frac{1}{N^u}\sum_{i=1}^{N^u} r_{i,t+1}^{e,nu} = \frac{1}{2}\left(\bar{\beta} + r^*\right).$$

Equation (28) immediately yields the average expected real rate of return when the countries do form a monetary union:

$$\frac{1}{N^u}\sum_{i=1}^{N^u} r_{i,t+1}^{e,u} = \frac{N^u}{1+N^u}\bar{\beta} + \frac{1}{1+N^u}r^*.$$

As we assume throughout the analysis that $B_{i,t} > 0$ for all countries i, it follows from equation (26) that $\bar{\beta} > r^*$. This implies that $\frac{N^u}{1+N^u}\bar{\beta} + \frac{1}{1+N^u}r^* > \frac{1}{2}\left(\bar{\beta} + r^*\right)$, such that $\frac{1}{N^u}\sum_{i=1}^{N^u} r_{i,t+1}^{e,u} > \frac{1}{N^u}\sum_{i=1}^{N^u} r_{i,t+1}^{e,nu}$.

[4] Clearly, this result depends on the linear and quadratic specifications of the objective functions in the model. But the main message may well survive more nonlinear specifications: the higher the inflation variance and the variance of the debt returns, the less debt investors will want to hold in their portfolio for a given expected rate of return, but also the less debt governments will want to supply. This implies that the effect of inflation uncertainty on the equilibrium real rate of return may be relatively small.

(b) Immediate from (28) and the fact that $\bar{\beta} > r^*$. ■

To see the intuition for part (b) of Proposition 2, observe that an increase in the number of union members means that a larger amount of perfectly substitutable debt is issued. Hence, for investors to be willing to buy this debt, the required expected return must be higher for any given level of $\bar{B}_t^u$. This is captured by the term N^u in (23). In addition, the return variance, $\hat{\sigma}^2$, rises because of the increase in $\bar{B}_t^u$. This further raises the required expected return for any given level of $\bar{B}_t^u$. However, in equilibrium, this effect is offset by the fall in the supply of debt resulting from the higher return variance – see (25). Figure 1 depicts (23) and (25) as functions of $\bar{B}_t^u$. An increase in N^u means that (23) becomes steeper and, hence, the equilibrium expected average return rises, while the average union debt level falls.

Now, we further analyze the outcomes for public debt. Equation (18) shows that $B_{i,t}^{nu}$ depends on μ_t^{nu}. Similarly, $B_{i,t}^u$ depends on μ_t^u. In turn, the values of μ_t^{nu} and μ_t^u are functions of the consumption of young agents without and with a monetary union, respectively, while consumption itself is determined jointly with the public debt levels. Unfortunately, there does not seem to exist a closed-form solution for consumption and the public debt levels. This prevents us from formally comparing the public debt levels before and after monetary unification.

However, we can circumvent this problem by looking at relative debt levels. These are also politically more relevant, as it is hard for governments to blame other governments for their indebtedness when their own debt is large. Relative debt levels will be a much bigger source of tension in a monetary union, because they may lead to differences in the potential externalities that countries exert on each other. Here, the relevant externality is the effect of a country's debt on another country's expected real debt return. As equation (12) shows, the size of this effect of country j's debt on country i's expected real debt return is $\mu_t B_{j,t} \mathrm{Cov}_t \left(r_{i,t+1}, r_{j,t+1} \right)$. Hence, the relative size of the effect that two countries i and j exert on each other's expected required debt return is the inverse of the ratio of their debt levels, $B_{j,t}/B_{i,t}$.

We are now ready to present Proposition 3:

Proposition 3 *(a) If country i's government ($i = 1, .., N^u$) before monetary unification is sufficiently impatient compared to the governments of the other prospective members, then i's relative public debt level will increase after unification. (b) For countries that already form a union, the relative public debt is increasing (decreasing) in the union size N^u if $\beta_i > \bar{\beta}$ ($\beta_i < \bar{\beta}$).*

Proof. (a) From equations (19) and (27) it follows that the relative public debt level of country i increases after monetary unification if and only

if

$$1+(1+N^u)\frac{\beta_i-\bar{\beta}}{\bar{\beta}-r^*} > \frac{\left[\left(\beta_i-r^*\right)/\lambda_i^2\right]^{\frac{1}{3}}}{\frac{1}{N^u}\sum_{j=1}^{N^u}\left[\left(\beta_j-r^*\right)/\lambda_j^2\right]^{\frac{1}{3}}} \tag{29}$$

$$= \frac{1}{\frac{1}{N^u}\sum_{j=1}^{N^u}\left[\left(\frac{\beta_j-r^*}{\beta_i-r^*}\right)\left(\frac{\lambda_i}{\lambda_j}\right)^2\right]^{\frac{1}{3}}}.$$

As we assume throughout the analysis that $B_{i,t}>0$ for all countries i, equation (27) implies that the left-hand-side of the inequality above is always positive. If β_i goes to ∞ (holding β_j $(j\neq i)$ fixed), then the left-hand-side of this inequality goes to $1+(1+N^u)(N^u-1)$, while the right-hand-side goes to N^u. Hence, the inequality will hold if government i is sufficiently impatient relative to the other governments.

(b) Immediate from (27). ■

We can explain part (a) of Proposition 3 as follows. A country with an very impatient government will tend to have a high public debt level for a given variance of real debt returns – see (18). Suppose now that this country has its own central bank. The high public debt level will then induce the central bank to try to create surprise inflation, which leads to a high money growth rate in equilibrium – see (14). However, this has a mitigating feedback effect on the incentive of the government to issue public debt: a high money growth rate causes high inflation uncertainty, and therefore also a high variance of the real debt returns, such that the government will want to limit its public debt somewhat. Suppose now that the country joins a monetary union, so that its central bank is replaced by a union-wide central bank. This union-wide central bank takes into account the public debt levels of all its member countries when deciding to what extent it will try to create inflation. It will therefore be less tempted to try to create high inflation and maintain a high money growth rate in response to the high public debt level in one of its member countries. But if the money growth rate decreases in a country with a relatively high public debt level when it joins a monetary union, the inflation uncertainty, and therefore also the variance of real debt returns, will decrease as well. As a result, the mitigating feedback-effect on the public debt decision of the government will weaken, and the government will feel less restrained to issue debt. The inverse reasoning holds for countries with a very patient government. So, the indebtedness of countries with a sufficiently impatient government will increase after monetary unification compared to the average indebtedness in the union's member countries.

Part (b) of Proposition 3 can be understood as follows. First, recall that the expected real debt return is increasing in the number of member countries of the monetary union (see Proposition 2(b)). Now, note from equation (13) that each member country will reduce its public debt by the same amount when the expected real debt return rises. Hence, when the union expands, countries with a relatively patient government that already start with a relatively low public debt level will decrease their public debt in absolute terms as much as countries with a relatively impatient government that start with a relatively high public debt level. As a result, countries with a relatively impatient government will face an increase of their relative indebtedness compared with the average indebtedness in the union when the number of member countries increases.

Proposition 4 *If the central bank of a country before monetary unification is sufficiently dependent relative to the other prospective members' central banks, then the relative public debt level of the country increases after monetary unification (i.e.,* $b_{i,t}^{u} > b_{i,t}^{nu}$*).*

Proof. Observe that the right-hand of (29) converges to 0 when λ_i goes to infinity (holding λ_j $(j \neq i)$ fixed). Hence, (29) holds if the central bank of country i is sufficiently dependent relative to the other central banks. ■

The intuition behind Proposition 4 is as follows. For a given level of public debt, a country with a very dependent central bank will have a strong desire to try to create surprise inflation. In equilibrium, this leads to a high money growth rate. As explained above, this has a mitigating effect on the amount of debt issued by the government. However, this mitigating effect weakens when the country joins a monetary union with a more independent central bank. For a given (average) debt level this central bank produces a lower money growth rate and lower inflation uncertainty. As a result, the government will feel less restrained to increase its public debt after monetary unification. The inverse reasoning holds for countries with a very independent central bank before joining the monetary union. Hence, after monetary unification, the public debt level in a country with a relatively dependent central bank will increase compared to the average public debt level in the union's member countries.

4 A simple transfer scheme to avoid the interest rate hike

In the previous section, we showed that the average expected real debt return increases when countries form a monetary union. In fact, this is due to a pecuniary externality which the governments of the individual countries do not internalize when they decide how much debt to supply. As equation (12) shows, the externality arises, because monetary unification raises the correlation of the returns on the public debt issued by the member countries. Although our model does not provide us with a framework to formally investigate the broader economic consequences of the increase in the expected real debt return, in reality there is a variety of reasons why this increase is likely to be harmful. For example, it raises the government's need for potentially distortionary tax revenues. Hence, it is of interest to explore whether the increase in the expected real debt return as a result of monetary unification can be avoided.

It turns out that the upward pressure on the expected real debt return can be offset with a simple transfer scheme that transfers resources to/from other countries when a country's public debt deviates from the average public debt level of the union countries.[5] Hence, with the transfer scheme the budget constraint of the government of country i becomes:

$$B_{i,t} = G_i + B_{i,t-1}(1 + r_{i,t}) + \tau \left(B_{i,t} - \bar{B}_t^u\right) - T_{i,t},\ i = 1, .., N^u, \tag{30}$$

where $\tau \left(B_{i,t} - \bar{B}_t^u\right)$ is the transfer, with τ a constant which will be chosen in such a way that the average expected interest rate is restored to the pre-monetary union level. Observe that the transfer scheme is budgetary neutral at the union level, because the sum of the transfers of all countries is zero.

The demand for country i's debt as a function of its expected real interest rate, equation (12), remains unchanged. However, the supply of country i's debt as a function of its expected real interest rate is now found by minimizing the loss function (4), subject to equations (30) and (12):

$$r_{i,t+1}^e = \beta_i - \mu_t B_{i,t} \mathrm{Var}_t \left(r_{i,t+1}\right) - \tau \left(\frac{N^u - 1}{N^u}\right)(1 + \beta_i). \tag{31}$$

[5] Transfer schemes and fiscal rules aimed at disciplining profligate governments have received attention in the recent literature, especially in connection with the Stability and Growth Pact, which is intended to penalize excessive deficits in EMU. For example, see Beetsma and Uhlig (1999), Beetsma and Bovenberg (2001), Debrun (2001), Dixit (2001) and Milesi-Ferretti (1998). More informal discussions of the issues involved can be found in Artis and Winkler (1998), Brunila et al. (2001) and Uhlig (2002).

Substituting (21) and (22) into equations (12) and (31) and solving for $r^e_{i,t+1}$ then yields the expected real debt return in the monetary union with the transfer scheme:

$$r^{e,u}_{i,t+1} = \frac{N^u}{1+N^u}\bar{\beta} + \frac{1}{1+N^u}r^* - \left(\frac{N^u-1}{N^u+1}\right)\tau\left(1+\bar{\beta}\right), \tag{32}$$

which is, of course, constant across countries. The value of τ which restores the pre-unification expected average real interest rate for the region follows from setting (32) equal to the union average of (20), $\frac{1}{2}\left(\bar{\beta}+r^*\right)$. The solution is:

$$\tau^u = \frac{1}{2}\left(\frac{\bar{\beta}-r^*}{1+\bar{\beta}}\right) > 0. \tag{33}$$

Hence, the transfer scheme results in payments from member countries with a higher-than-average public debt to member countries with a lower-than-average public debt. The amount which a member country pays or receives is proportional to the difference between its public debt and the average public debt level in the union's member countries. Note also that the proportionality factor is an increasing function of $\bar{\beta}$, the average discount rate of the governments.

Now, substitute (33) for τ, $\frac{1}{2}\left(\bar{\beta}+r^*\right)$ for $r^e_{i,t+1}$ and (21) for $\mathrm{Var}_t\left(r_{i,t+1}\right)$ into (31). Upon rewriting the resulting expression, we obtain:

$$B_{i,t} = \frac{\left(1+\sigma^2\right)^2}{\mu_t\left(\lambda^u\bar{B}^u_t\right)^2\sigma^2}\left\{\beta_i - r^* - \frac{1}{2}\left[1+\left(\frac{1+\beta_i}{1+\bar{\beta}}\right)\left(\frac{N^u-1}{N^u}\right)\right]\left(\bar{\beta}-r^*\right)\right\}. \tag{34}$$

The final term in the curly brackets of this expression arises from the presence of the transfer scheme, which provides an incentive to government i to reduce its public debt. With the use of (34), we compute the average debt level in the union:

$$\bar{B}^{u,\tau}_t = \left[\frac{\bar{\beta}-r^*}{2N^u\mu^{u,\tau}_t\left(\lambda^u\right)^2}\left(\frac{1+\sigma^2}{\sigma}\right)^2\right]^{\frac{1}{3}},$$

using superscript "τ" to denote the dependence on the presence of the transfer scheme. Abstracting from possible general equilibrium effects on μ_t, the average debt level falls compared to the situation without the transfer scheme. This is not surprising, given that the transfer scheme makes it more costly for governments to issue debt.

The relative debt levels can be written as:

$$b_{i,t}^{u,\tau} \equiv \frac{B_{i,t}^{u,\tau}}{\bar{B}_t^{u,\tau}} = 1 + 2N^u \frac{\beta_i - \bar{\beta}}{\bar{\beta} - r^*} + (1 - N^u) \frac{\beta_i - \bar{\beta}}{1 + \bar{\beta}}. \tag{35}$$

Now, compare (35) with (27). It is easy to check that $b_{i,t}^{u,\tau} > b_{i,t}^u$ iff $\beta_i > \bar{\beta}$ and $b_{i,t}^{u,\tau} < b_{i,t}^u$ iff $\beta_i < \bar{\beta}$. The fall in the average debt level caused by the introduction of the transfer scheme "blows up" the relative debt levels, despite the fact that the transfer scheme provides a more impatient government with a stronger incentive to reduce its debt – see (34). Hence, with the proposed transfer scheme, the least disciplined governments issue an even larger share of the union's debt.

We can now summarize the results of this section in the following proposition:

Proposition 5 *We have that (a) a linear transfer scheme which depends on the difference between the individual and average union debt is able to restore the pre-unification average expected debt return and (b) the dispersion in relative debt levels increases further with the introduction of this transfer scheme.*

5 Conclusion

While the effects of monetary unification on real interest rates and public debt accumulation have to a large extent been ignored both in policy discussions and in the literature, in the preceding analysis we have demonstrated that this may be unjustified. The implications for the real interest rate become important when the risk-return characteristics of the public debt differ across countries. Monetary unification increases the substitutability of public debt. As a result, the expected interest payment on the debt rises. In addition, the share of the union debt issued by relatively undisciplined governments increases. Similarly, the share of the union debt issued by a country initially characterized by a sufficiently dependent (relative to others) central bank increases. These effects of monetary unification on the spread of the relative debt levels may be a source of political tension and provide some governments with an incentive to put pressure on other governments to reduce their debt.

In the preceding analysis we have also explored the implications of a transfer scheme which punishes increases of the national debt beyond the average debt level in the union. Although such a transfer scheme can reduce the average expected real debt return to the pre-unification level, it leads to a further spread of relative debt levels in the union.

While the purpose of the paper was to develop a simple framework to shed light on the effects of monetary unification on public debt and the expected real debt returns, a number of extensions might make the framework more realistic. One such extension would be to allow for the possibility that, even though the correlation of the national inflation rates increases with monetary unification, it does not become perfect because goods produced in different countries are no longer perfectly substitutable. This adjustment would preclude closed-form solutions and one would be forced to resort to numerical explorations. However, we conjecture that our results would go through in qualitative terms. Another extension could be to endogenize the differences in the discount rates of the governments by explicitly taking into account political factors such as the degree of polarization and the government turnover rate. This would link differences in relative debt and in the externalities that countries exert on each other via the interest rate to the political processes in the union members.[6]

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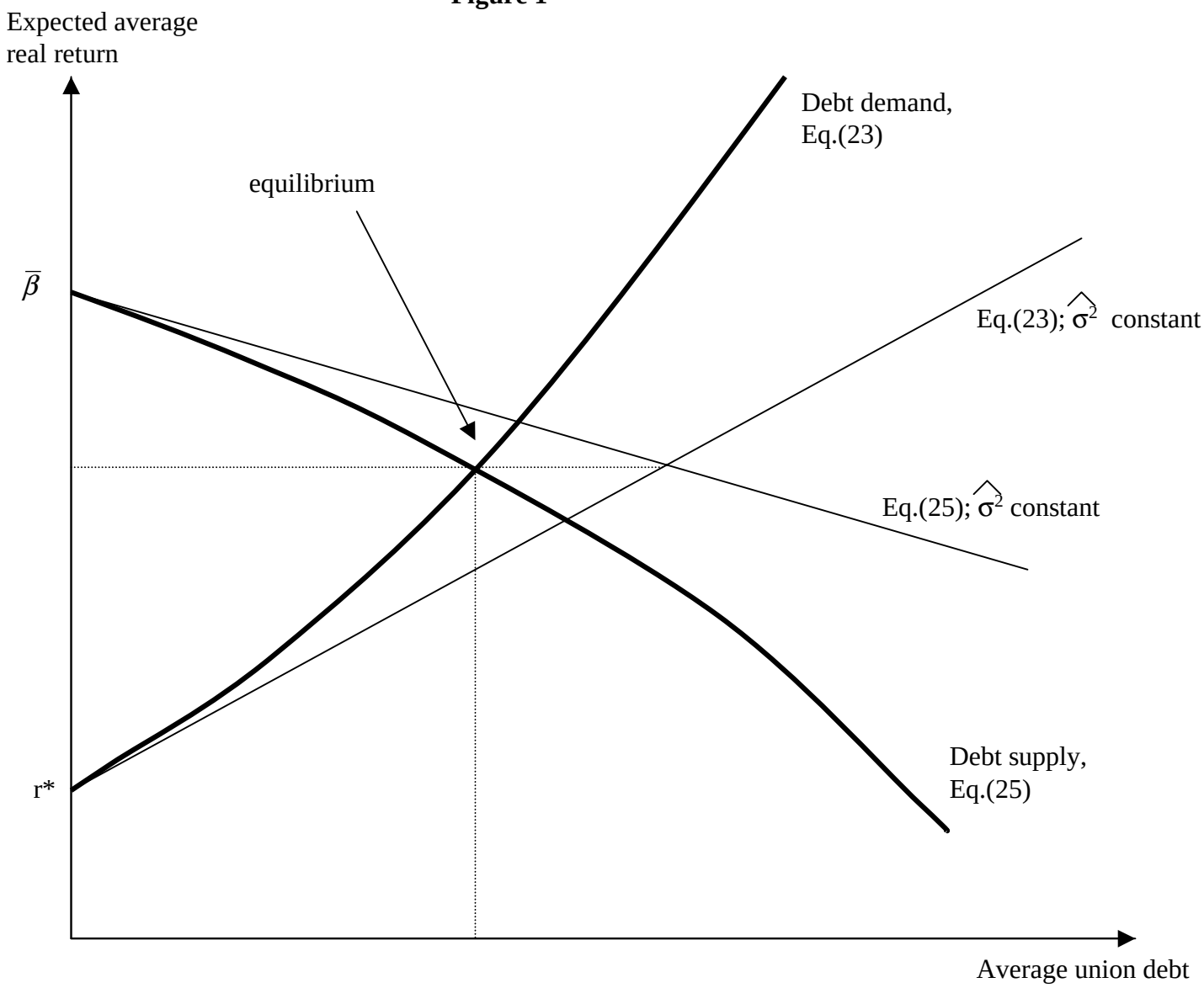
Figure 1


Expected average
real return
Debt demand,
Eq.(23)
equilibrium
$\overline{\beta}$
Eq.(23); $\widehat{\sigma^2}$ constant
Eq.(25); $\widehat{\sigma^2}$ constant
Debt supply,
Eq.(25)
r*
Average union debt