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ABSTRACT

Audit Competition in Insurance Oligopolies*

We provide a simple framework for analysing how competition affects the choice of audit structures in an oligopolistic insurance industry. When the degree of competition increases, fraud increases but the response of the industry in terms of investment in audit quality follows a U-shaped pattern. Following increases in competition, the investment in audit quality will decrease if the industry is initially in a low competition regime, while it will increase when the industry is in a high competition regime. We use these results to show that firms will benefit from forming a joint audit agency only when the degree of competition is intermediate and that cooperation might improve total welfare; we also analyse the effects of contract innovation on the performance of the industry.

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1 Introduction

Insurance fraud is a universal and *costly* phenomenon. The cost of fraud in the US is estimated by the Insurance Information Institute to be between 10% to 20% of either claims, reimbursements or premiums. The appearance of new insurance products like wage loss or medical coverage both for physical and psychological traumas have inflated the total bill for insurers. It has also opened the door to “soft” fraud, namely the exaggeration of existing damages to the car, body or mind. Indeed, the Coalition Against Insurance Fraud points at health care, workers compensation, and auto insurance to be the sectors most vulnerable to fraud.

The insurance sector has in recent years increased efforts to fight fraud: by sharing information,¹ by investing in the training of special investigation units,² and by advertising “toughness” with respect to fraud.³ The sector also successfully lobbied for more stringent laws with the US Insurance Fraud Act of 1994 turning several kinds of fraud into federal crimes (cf. also the online review of state bills). Furthermore the NICB successfully lobbied in 1999 against a federal bill aiming at protecting personal privacy (and restraining the use of nationwide databases). State Insurance Fraud Bureaus have grown from 8 in 1990 to 46 in 2000 (for a total of 51 states) and their budget is over \$100 million in 2000, up 21% from 1999. However, these efforts vary significantly across countries. At one extreme, like in the US or in South Africa, there are coordinated efforts in the industry to prevent and fight fraud, as well as active advertising by individual firms. At another extreme, like in most European countries, there are little attempts to coordinate the fight against fraudulent activities and often individual firms are reluctant to formally acknowledge the fraud problem. Nevertheless, even in these countries, individual firms develop contractual and organizational responses to the problem.

Differences in regulatory and legal systems could explain such differences: strong punishments make the deterrent effect of investments in fraud detection more effective; privacy laws prevent the sharing of data in the industry and limit industry coordination.⁴ In this paper we leave aside differences in legal or regulatory regimes, and rather focus on a factor that has received until now little attention: competition. At an abstract level, competition must indeed matter since it affects directly the return

¹Nearly all insurers use public databases like the “all claims database”, 70% use the National Insurance Crime Bureau automobile database, 60% a database on claims (CLUE).

²According to the 1996 report of the Insurance Research Council (IRC), insurers have tripled their fraud control spending in 4 years and virtually all companies now have a special investigation unit to investigate fraud.

³According to the 1996 IRC report, insurers now place public awareness as the number one deterrent of fraud.

⁴For instance, European firms find it difficult to establish a common data base for insurance violations partly because of the privacy laws. In the US, insurance fraud is now considered a criminal offense, which makes private investments against fraud activity more likely to have a deterrent effect.

on contracts and organizational innovations – whether private or as part of a cooperative effort – and therefore the incentives to undertake such innovations.

The recent history of the insurance industry is evidence that contracts have become more complex; contracts not only include premium and deductible but also a range of conditions and obligations related to the quality of service and fraud prevention or detection. Generally, measures to control fraud impose direct costs on consumers. For instance, taking pictures of a car when signing a contract reduces the possibility of a consumer to make a claim for a pre-existing default but increases the transaction costs of the contract. Inspection of a damaged car via “authorized experts” limits the possibilities of collusion with the repairer and of false claims but increases the delay for repairs or forces the consumer to free time to go to the inspection. As soon as 1989, Allstate, a major US player, introduced a Priority Repair Option to accelerate repairing and reduce the loss adjustment expenses.

Hence, contracts in the insurance industry have two dimensions: a “price” or pecuniary dimension – basically the premium and the reimbursement – and a “quality” or non-pecuniary dimension – additional services and the thoroughness and speed of the audit of claims. Thoroughness of the audit is important for the insurance company in order to credibly deter fraud; speed is important for the customers since it reduces the cost he will have to bear in case of a loss, hence increases his ex-ante utility from the contract. One question we ask in this paper is how equilibrium contracts will be located on these two dimensions (price and quality) and how these two dimensions will be modified when the degree of competition changes on the market.

The exact combination of price and quality is important since it modifies the incentives of consumers to fraud and therefore the expected cost of the contract to the firm. Market equilibrium dictates that customers obtain a certain level of expected utility – typically a non-trivial function of the degree of competition on the market – and therefore firms will readjust their contracts in order to provide this level of utility at lowest cost to them. This is how competition affects contracts and the substitution between price and quality. This is also how changes in the degree of competition can explain shifts from price competition to quality competition.⁵

We analyze an horizontal differentiation oligopoly model of insurance provision in which firms compete for consumers with two instruments: contracts (premium, reimbursement) and the quality of their audit technology. Both contracting terms (in particular the level of reimbursement) and the quality of audit affect the incentives of the policyholders to fraud. Later on, we allow firms to create cooperatively an external audit agency that provides the audit service to the industry; cooperation must be self-enforced however, and we still allow individual firms to privately invest in audit (and

⁵For instance, deregulation of the insurance supervision has resulted in appreciable price decreases in Western Europe since 1994 and in Japan since 1998, while contracts include now more non-pecuniary clauses than before.

refrain from using the common agency).

We will show that the effect of competition on the level of private or cooperative audit quality is ambiguous because there is a shift from price competition to quality competition below a certain degree of competition. This shift implies a U-shaped response of the equilibrium to the degree of competition: first, quality decreases and then increases again. This shift also implies that coordination via a common audit agency is an equilibrium outcome only for lower levels of competition. Indeed, the audit quality in the common agency must be at a level that prevents individual firms to privately invest instead of using the cooperative agency, which implies that the audit quality in the common agency is higher than the equilibrium audit that would prevail in the oligopoly equilibrium.

If competition is strong, firms have private incentives to use their own lower quality rather than the highest possible one provided by the common agency. Hence, it is only for low levels of competition that the firms cannot commit to privately invest in high audit quality; in this case, the common agency enables the industry to solve its free-rider problem and to commit to a higher level of audit quality.

As this suggests, most of the interesting effects arise when the degree of competition is intermediate. This is why comparing only perfect competition and monopoly – the two market structures usually assumed in the insurance fraud literature – would not be very useful. This is the theoretical reason for assuming an oligopoly structure. More importantly, there are obvious empirical reasons for making the assumption of an oligopolistic market. First, there are significant barriers to entry in the industry (reputation, anticipated cost of bankruptcy,⁶ regulatory barriers). Second, as the table below shows, in the market for non-life insurance (including car insurance), the first ten firms have more than 50% of the market, except for the US where the first ten firms have 44% of the market. Finally, deregulation of the insurance supervision has coincided with – if not triggered – a series of mergers and acquisition that have dramatically altered the market share figures (in particular outside the US.). This suggests that individual firms have market power in the industry, and that this market power is changing.⁷

⁶In 1998, 55 US non-life companies experienced an insolvency above \$ 1 million for a total of \$ 474 millions.

⁷First two columns: 1998 OECD report on insurance. Last column: National Association of Insurance Commissioners (NAIC) for the US and SwissRe for other countries. These data are aggregated and a better measure for the US would be the state level data; especially since legislation and regulation vary across states.

1998	# of non life insurers	premiums in \$bn	market share of the first 10
US	3100	594	44%
UK	490	68	55%
D	328	109	58%
France	307	48	75%
Japan	59	80	78%

2 The Insurance Market

Owners of an insurable items (e.g., car, health or a real estate) are risk-neutral with respect to money. They value the use of the item at V and have initial wealth w . There is a probability β that the item will be damaged,⁸ in which case the cost of repair is L . We assume that

$$V > L > w \quad (1)$$

Limited liability prevents the agents to borrow ex-post to repair the damaged item, however, as long as the probability of the loss is small enough, the agent can transfer the risk of the loss on an insurance company in exchange for the payment of a premium. This formulation of the insurance motive makes the model of oligopoly competition more tractable than with the usual assumption of concave utility for money. Modelling the motive for insurance from a limited liability constraint is pertinent for risks where the probability of a loss is small but the damage can be large.

To focus on the organizational problem of the firms in the industry, we assume that wealth w is observable. A contract specifies a premium P and a reimbursement R . A contract is *feasible* if the agent is able to pay the premium and if the reimbursement net of the premium covers the cost of damage, i.e., if

$$w \geq P \quad (2)$$

$$R \geq L - (w - P) \quad (3)$$

Obviously, (3) combined with (1) require that the reimbursement is larger than the premium

$$R > P \geq 0. \quad (4)$$

If the loss can be verified at no cost to the firm, feasibility of contracts is reduced to these conditions. However, we are interested in situations where losses can be verified only if there is an audit. The

⁸Our set-up applies equally for build-up fraud and the exaggeration of damages if there are only two states of nature like (loss,no-loss) or (small loss,big loss) one of which is not directly observable for the insurer.

probability to detect a fraudulent claim is a function of the experience of the claim adjusters, ex-ante documentation about the state of the item, the time spent on the audit. The *quality* of audit is modelled by the probability q with which fraud is detected by the audit. If the claim is *honest*, audit shows that there is no fraud with probability one while if the claim is *dishonest*, the audit detects the fraud with probability q . The agent is reimbursed unless fraud is established, in which case the agent pays a fine F to the state.⁹

While a firm might indeed hire highly experienced claim adjusters, there is a decision to be made ex-post of whether or not to audit a claim. We assume that the insurance firms can commit ex-ante to a *quality* q of audit but not to its frequency,¹⁰ and that there is a variable cost of audit $C^f(q)$ that is borne ex-post, only if the audit is actually made.¹¹ We assume that $C^f(q)$ is an increasing and convex function of q , with $C^f(0) > 0$.

Fraud control, or audit, imposes costs on consumers; these costs can be due to the increased delay for repair or for reimbursement, or can be indirect due to the perception of “mistrust” for the policyholder. The overall opportunity cost is denoted $C^a(q)$ and is assumed to be increasing and convex in audit quality, with $C^a(0) > 0$.

We define the *ideal* audit quality of firms q^f (resp. consumers q^a) as the minimum of the U-shaped average cost $C^f(q)/q$ (resp. $C^a(q)/q$) over $[0; 1]$. Stylized facts¹² lead us to assume that insurers desire a better audit quality, relative to policyholders i.e., $q^a < q^f$.

We consider the following timing of events describing the insurance market:

- Insurer $i = 1$ to n chooses an audit quality q^i and offers insurance contracts with premium P^i and reimbursement R^i .
- Agents observe $(q^i, P^i, R^i)_{i=1}^n$ and decide to purchase or not an insurance contract.
- Agents incur a loss with an exogenous probability β , we assume that β is “small”.¹³
- An agent who purchased contract (P^i, R^i) can claim a loss (ask for a payment R^i).

⁹The penalty could include an amount F_0 awarded to the company. We set $F_0 = 0$ to simplify exposition. F is the product of a fine (or disutility of jail term) by the probability of being found guilty in court. We may interpret the recent criminalization of insurance fraud in the US as an attempt to keep F at a reasonable level despite the increased congestion of the judicial system, that is despite the lower probability of being found guilty in court in “reasonable time”.

¹⁰See Khalil (1997) and Picard (1996).

¹¹Introducing set-up costs will not modify our results.

¹²It is well documented - e.g., the white paper (2000) – that insurance salesmen (the demand side) are calling for low audit quality contrariwise to claim payers (the supply side).

¹³For instance, typical risks like car theft or road accident have a probability below 5% (in a given year).

- The insurance company decides to audit or not the claim and pays the agent according to the result of the audit.

Our two stage game is solved by backward induction using the concept of Perfect Bayesian Equilibrium (PBE). In the next section we analyze the fraud and control game between a policyholder and its insurer; it depends on the audit quality q^i and the contract (P^i, R^i) . Then, we compute the expected utility of agents and firms conditional on choosing (q^i, P^i, R^i) and we derive the symmetric equilibrium of the quality and contract competition between insurance companies.

To analyze how the intensity of competition (whatever its origin) affects the equilibrium design of contracts and the choice of audit quality, we assume that consumers are horizontally differentiated. The *residual demand* facing an individual firm offering consumers a level of expected utility v when the other firms in the industry offer the level v^* is $D(v - v^*)$ where $D(0) = \frac{1}{n}$ and $D'(0) = t$.¹⁴ The parameter $\theta \equiv nt$ is the *index* of competition that we will use in our comparative statics. By varying the mobility cost t or the number of firms we will be able to vary exogenously the intensity of competition. The ongoing deregulation in many countries or the greater visibility of entrants permitted by the IT revolution can then be the exogenous factors affecting t and n .

3 Equilibrium and Audit Response to Competition

In this section we first solve for the game of fraud and audit played by an insurer and one of its policyholder (Propositions 1 and 2). We then show that the equilibrium analysis and the comparative statics can be simply captured in an Edgeworth box in which the “commodities” are the reimbursement and the quality of audit. It is then possible to show the existence of two regimes: one in which the reimbursement is compatible with a premium level that does not make the liability constraint of the consumers binding, the other where the liability constraint of the consumer is binding; these two regimes correspond also to equilibrium regimes when the degree of competition is high or low.

¹⁴This indirect form can be obtained, for instance, in the circular city setting introduced by Salop (1979). Consumers are uniformly distributed over the unit circle and bear a mobility cost $\frac{2}{t}$ per unit of distance while the n firms are located at an equal distance one from another (i.e., $\frac{1}{n}$). Agents buy insurance contracts from the company that offers them the highest expected utility. Hence, the demand addressed to a company offering an expected utility of v while all other firms offer v^* is $D(v - v^*) = t(v - v^*) + \frac{1}{n}$.

3.1 The Fraud and Audit Game

Since a damage is private information to the policyholder, iff an insurer were to pay all claims without audit all policy holders would fraud.¹⁵ However systematic audit is not credible in our model and we therefore have a double moral hazard problem: that of inducing insurance companies to audit with a high probability and that of inducing policyholders to fraud with a low probability. Clearly, the equilibrium is in mixed strategies, and the following proposition characterizes the equilibrium levels of fraud and audit. (All proofs missing from the text have been relegated in the Appendix.)

Proposition 1 *In a PBE, the game of fraud and control following a contract (π, R) with a quality q (second stage) has a unique Nash equilibrium; it is in mixed strategies with*

$$\sigma^* \equiv \frac{\beta C^f(q)}{(1-\beta)(qR - C^f(q))} \text{ and } \tau^* \equiv \frac{R}{q(R+F)} \quad (5)$$

Observe that while the audit quality q generates future costs for the agent and the firm, it brings benefits too since it influences the desire of the agent to fraud and of the firm to audit. Its effect on utilities for insurers and consumers can be summarized by the average cost functions $c^f(q) \equiv \frac{C^f(q)}{q}$ and $c^a(q) \equiv \frac{C^a(q)}{q}$. Using the equilibrium levels of fraud σ^* and control τ^* , we are able to compute the expected utility levels of an agent u and the expected profit of the firm π as

$$u(q, P, R) \equiv V + w - P - \beta(L - R) - \beta \frac{Rc^a(q)}{R+F} \quad (6)$$

$$\pi(q, P, R) \equiv P - \beta \frac{R^2}{R - c^f(q)}. \quad (7)$$

The ratios $\frac{Rc^a(q)}{R+F} > 0$ in (6) and $\frac{R^2}{R - c^f(q)} > 0$ in (7) reflects the cost of fraud for insurers and consumers respectively (without fraud both c^a and c^f would be nil).

Two simple facts follow from (6) and (7). First, the convexity of c^a and c^f together with $q^a < q^f$ imply that both $\pi(q, P, R)$ and $u(q, P, R)$ are increasing with q on $[0; q^a]$ and decreasing on $[q^f; 1]$. Hence offering (q, R, P) with $q < q^a$ or $q > q^f$ is strictly dominated by offering either (q^a, R, P) or (q^f, R, P) . In a PBE, audit is in the interval $[q^a, q^f]$.

Second, if $R > P + L - w$, the insurer can decrease both R and P in order to keep its market share but increase his per-consumer profit.¹⁶ Therefore, in equilibrium, constraint (3) binds and we

¹⁵Agent have no morality in this model. A recent study of the Coalition Against Insurance Fraud reveals that people fraud to save money or reduce costs, to get expensive work done they would not otherwise be able to afford and to “get back” at insurance companies.

¹⁶If $R > P + L - w$ consider $\Delta R < 0$ and $\Delta P = \frac{\partial u}{\partial R} \Delta R = \beta \Delta R \left(1 - \frac{Fc^a(q)}{(R+F)^2}\right) > \beta \Delta R$. As $u(q, P + \Delta P, R + \Delta R) = u(q, P, R)$ and the firm has the same market share. Since $\pi(q, R, P) = P - \beta R \frac{R}{R - c^f(q)}$ and $R(R - 2c^f(q)) < (R - c^f(q))^2$, we have $\frac{\partial \pi}{\partial R} = -\beta R \frac{R - 2c^f(q)}{(R - c^f(q))^2} > -\beta < 0$. Hence, the total per-capita profit variation $\Delta \pi$ is greater than $\Delta P - \beta \Delta R > 0$.

can reduce competition on the “pecuniary dimension” to the single reimbursement R which is a proxy of the premium. These observations are summarized in the next proposition.

Proposition 2 *In a PBE, an insurer chooses a quality between the ideal of firms (q^f) and of consumers (q^a) and offers minimal reimbursement with $R = P + L - w$.*

Using the equality in the feasibility constraint (3), the liquidity constraint (2) becomes

$$R \leq L \quad (8)$$

while the positiveness of the premium (4) yields

$$L - w \leq R \quad (9)$$

Hence in a PBE the reimbursement varies in the interval $[L - w, L]$ and the premium varies in the interval $[0, w]$. The indirect utility function of a policyholder and the per-capita profit of an insurer can then be written:

$$u(q, R) \equiv V + (1 - \beta)L - R \left(1 - \beta + \frac{\beta c^a(q)}{R + F} \right) \quad (10)$$

$$\pi(q, R) \equiv R \left(1 - \beta \frac{R}{R - c^f(q)} \right) - L + w \quad (11)$$

Note that a zero premium ($R = L - w$) generates losses for an insurer, hence the only relevant constraint is the liquidity one (8). To avoid trivialities, we assume

$$\text{For all } q \in [q^a, q^f], \frac{V}{L} \geq \frac{L}{L - c^f(q)} + \frac{c^a(q)}{L + F} \quad (\text{H1})$$

With this condition the sum of payoffs $u(q, R) + \pi(q, R)$ is larger than the agent’s default payoff $\underline{v} \equiv (1 - \beta)V + w$ when contract variables q and R vary in their PBE range. This guarantees that the insurance market is indeed active.

3.2 Competition

A symmetric equilibrium is denoted (q^θ, R^θ) ; it yields an equilibrium utility level $v^\theta \equiv u(q^\theta, R^\theta)$ to consumers. The default utility for a consumer is $\underline{v} \equiv (1 - \beta)V + w$ (no insurance contract). Hence, when insurer i chooses the pair (q^i, R^i) such that $R^i \leq L$ (feasibility) and $u(q^i, R^i) \geq \underline{v}$ (participation) while all other firms choose (q^θ, R^θ) , its profit is

$$\Pi_i(q^i, R^i) \equiv D \left(u(q^i, R^i) - v^\theta \right) \pi(q^i, R^i)$$

A symmetric equilibrium is a pair (q^θ, R^θ) solving the program

$$P(L, v^\theta) : \begin{cases} \max_{q, R} \Pi_i(q, R) \\ \text{s.t. } u(q, R) \geq v^\theta \equiv u(q^\theta, R^\theta) \\ \text{and } R \leq L. \end{cases}$$

The two FOCs for an interior solution are

$$\frac{\partial \Pi_i(q, R)}{\partial R} = 0 \Leftrightarrow D' u_R \pi + D \pi_R = 0 \quad (12)$$

$$\frac{\partial \Pi_i(q, R)}{\partial q} = 0 \Leftrightarrow D' u_q \pi + D \pi_q = 0 \quad (13)$$

which are equivalent to

$$\frac{u_R}{u_q} = \frac{\pi_R}{\pi_q} \quad (14)$$

$$-u_q \theta \pi = \pi_q \quad (15)$$

Equation (14) is the equation of the contract curve. Indeed, if the marginal rates of substitution of policyholders and insurers $\frac{u_R}{u_q}$ and $\frac{\pi_R}{\pi_q}$ are not equalized, the insurer can offer a better contract (\tilde{q}, \tilde{R}) in the sense that it leaves its clients indifferent with respect to (q, R) (market share does not change) but increase its per-capita profit.

Equation (15) illustrates the traditional trade-off between the quantity effect $-u_q \theta$ (multiplied by the per-capita profit π) and the profit effect π_q of a change in the strategic variable q . It characterizes the position of the equilibrium of the contract competition on the contract curve. The analysis of the latter is therefore crucial to understand how changes in the exogenous parameter θ transmit into changes of the equilibrium contract variables. Remember that we assume that the risk of loss is small.

3.2.1 The Edgeworth box

When the liquidity constraint $R \leq L$ is not binding, a Pareto optimum is characterized by the equality of the (absolute) marginal rates of substitution $MRS^a = \frac{u_R}{u_q}$ and $MRS^f = \frac{\pi_R}{\pi_q}$ as depicted on the Edgeworth box of Figure 1 below. The iso-profit curve (solid line) is tangent to the iso-utility curve (dashed line) at point x and the slope is equal to the marginal rate of substitution. The arrows indicate the direction of increasing utility for each type of agent. With assumption (H2), reimbursement and audit quality are substitute for the insurer and also for the agent.¹⁷

¹⁷Indeed, for small β , $u_R = -1 + \beta - \beta \frac{F c^a(q)}{(R+F)^2} < 0$ and $\pi_R = 1 - \beta \frac{R(R-2c^f(q))}{(R-c^f(q))^2} > 0$. Since $q \in [q^a, q^f]$, c^a is increasing and c^f is decreasing in q over the relevant range, hence $u_q = -\beta R \frac{c_q^a}{(R+F)^2} < 0$ and $\pi_q = -\beta R^2 \frac{c_q^f}{(R-c^f(q))^2} > 0$, which yields positive MRS for the policy holder and the firm.

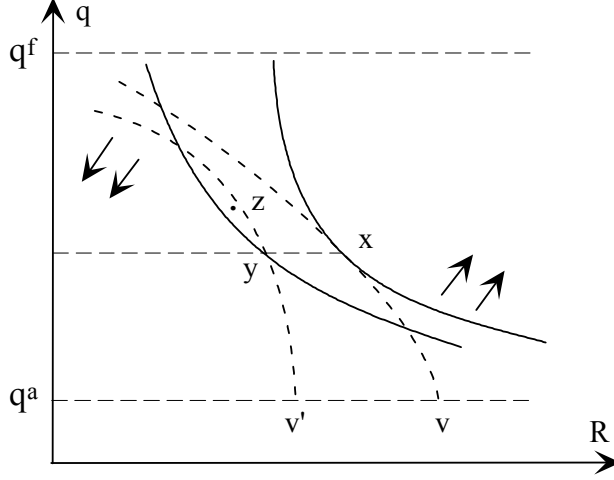


Figure 1: The Edgeworth box

Starting from point of tangency x on Figure 1, imagine that the reimbursement R decreases and that we move to point y . Note that audit matters for the insurer only when a damage occurs, which is a low probability event, even taking into account the inflating effect of fraud; reimbursement will thus be the main channel of profit, hence of utility changes. This suggests that the insurer values less R at point y than at point x , which is confirmed by the fact that the MRS is increasing in R for the firm, or formally that $\left. \frac{\partial MRS^f}{\partial R} \right|_{q=cte} > 0$.¹⁸

Since the feasibility condition binds, R is a proxy for the level of premium, hence policyholders dislike R . However because R also affects the return to fraud, policyholders actually benefit from a second order gain when R decreases; thus they value more R at y than at x and formally, $\left. \frac{\partial MRS^a}{\partial R} \right|_{q=cte} < 0$.¹⁹

Combining our findings on marginal rates of substitution, we deduce that, at point y , the insurer can successfully offer the policyholder a Pareto improving contract modification with a further reduction of R coupled with an increase of audit quality q . The new contract where MRSs are equalized is therefore point z on Figure 1. The contract curve linking all Pareto optima is decreasing in the (R, q) space. We may conclude that in equilibrium the two instruments, premium and audit quality, are substitutes.

¹⁸Indeed, using $\pi_{RR} = -2\beta \frac{(c^f(q))^2}{(R-c^f(q))^2}$ and $\pi_{qR} = 2\beta R \frac{c_q^f c^f}{(R-c^f)^3} < 0$, we have for β small, $\left. \frac{\partial MRS^a}{\partial R} \right|_{q=cte} = \frac{\pi_{RR}\pi_q - \pi_R\pi_{qR}}{(\pi_q)^2} > 0$ since the π_R effect dominates the π_q effect.

¹⁹First, $u_R = -1 + \beta - \beta \frac{Fc^a(q)}{(R+F)^2} < 0$ as $\beta \ll 1$, and $u_q = -\beta \frac{Rc_q^a(q)}{R+F} < 0$ as $c_q^a > 0$. Taking further derivatives, we obtain $u_{RR} = \beta \frac{Fc^a(q)}{(R+F)^3} > 0$ and $u_{qR} = -\beta \frac{Fc_q^a(q)}{(R+F)^2} < 0$, thus $MRS_R^a = \frac{u_{RR}u_q - u_R u_{qR}}{(u_q)^2} < 0$.

3.2.2 Comparative Statics

The previous section shows that as long as the industry equilibrium involves an interior solution in the (R, q) space, the contract curve is decreasing. We now study how changes in the degree of competition affect contracts. First, we confirm in proposition 3 the intuition according to which consumer utility falls when competitiveness falls. Next, we demonstrate that the premium (linked to the reimbursement) is the main channel of transmission when focusing on small risks ($H2$). This implies that utility losses are associated with premium increases; since the contract curve is downward slopping, there must be a simultaneous decrease in the quality of audit (see Figure 2).

However, at some value θ^L , the liquidity constraint starts to bind and the only way for insurers to increase their profits is to increase audit quality i.e., move up vertically towards their ideal quality q^f . The position of the equilibrium on the vertical line still obeys equation (15) but computed at $R = L$ instead of being computed on the contract curve. These findings lead to Proposition 3 below.

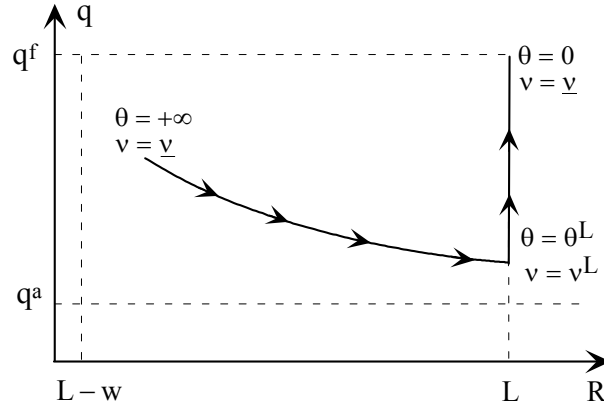


Figure 2: The Equilibrium path

Proposition 3 *There exists two wealth levels \underline{w} and \bar{w} such that*

(i) *for an intermediate wealth $w \in]\underline{w}; \bar{w}[$, the symmetric equilibrium between insurers has two regimes*

a **weak** competition regime $\theta \in [0; \theta^L[$: *equilibrium contracts feature full insurance, maximal premium while quality is decreasing with θ .*

a) **strong** competition regime $\theta \in [\theta^L; +\infty[$: *equilibrium reimbursement and premium decreases with θ while quality increases with θ .*

(ii) *If $w < \underline{w}$, only the weak regime applies*

(iii) *If $w > \bar{w}$, only the strong regime applies.*

Another issue is whether fraud increases or decreases with the degree of competition. The answer is immediate in the weak competition regime since the premium is fixed while the audit quality tends

decreases, i.e., the *average cost* of audit increases making insurers *less credible* auditors. Consumers then fraud more in the equilibrium of the last stage game. In the strong competition regime, the increase in quality reduces the cost of control; this effect makes firms *more credible* auditors and fraud falls. Yet competition also reduces the price *and* the benefit of audit: this effect makes firms less credible auditors. Since the price dimension is the main channel of competition, the net effect is a reduction in the incentives of the firm to audit and an increase in the equilibrium level of fraud.

Corollary 1 *Fraud increases with the degree of competition.*

4 Two Extensions

4.1 Centralized Audit Agency

A major source of cost for insurers is their inability to commit to an audit frequency. They sometimes launch campaigns of systematic claim checking but this conduct never last very long. One alternative is to have an intermediary offering high quality audit services and leave insurance companies decide if they want to use the intermediary rather than their own control structure.²⁰ For instance many US states or European countries have a public auditing agency of high quality.

To simplify we assume that the agency is created in a cooperative fashion by the industry (for instance by selecting the best offer in an auction for auditing services). The costs of creation (if any) are shared between the firms and each agrees to pay the cost of using the agency. Note that this set-up serves as a commitment device only if the consumers expect that the agency will be used (firms still trade-off the costs and benefits of audit). The timing previously considered is altered as follows:

- The industry creates cooperatively an audit agency with quality \bar{q} .
- Insurers choose their control structure q^i and offer insurance contracts (P^i, R^i) .
- Consumers observe $\{\bar{q}, (q^i, P^i, R^i)_{i=1}^n\}$ and purchase an insurance contract or not.
- A policyholder incurs a loss or does not incur a loss; he then can make a claim for reimbursement.
- Insurers decide to audit or not claims and if so to use the agency or not (use \bar{q} or q^i).

We show that the intermediary is a valuable commitment device on quality only if competition is centered on the quality, that is for low levels of competitiveness of the market. When $\theta \leq \theta^L$, the

²⁰We ignore commitment on contracts because it is more difficult to enforce and would be probably illegal under current antitrust legislation.

reimbursement is constrained in equilibrium with $R = L$ and competition is on q only. A commitment to high quality $\bar{q} = q^f$ is self-enforcing since consumers anticipate that a firm choosing $q_i < \bar{q}$ will never use q_i because it is more expensive than the technology of the agency. Consequently with cooperation, consumers derive the utility level $u(q^f, L)$ which is less than their equilibrium utility level v^θ (since $\bar{q} > q^\theta$).²¹ Hence there is a benefit to insurers of setting up the intermediary.

This does not work anymore for $\theta > \theta^L$ because competition also involves the pecuniary dimension. The set of technologies that an individual firm can deviate to is $[\bar{q}; q^f]$ for the reason explained before. The new game has therefore a smaller strategy set (with respect to the original model). If $\bar{q} < q^\theta$, the latter quality remains a Nash equilibrium i.e., the external audit agency does not affect competition and is inactive in equilibrium. If there is a positive fixed cost of creating the agency, the insurers will not create it. If

$q > q^\theta$, a new equilibrium emerges. We show in the appendix that the profit of insurers is lower with than without the external audit agency because the quality used in this new equilibrium is excessively large. Hence insurers will not create the agency.

Proposition 4 *The audit agency is created only in the weak competition regime ($\theta \leq \theta^L$). If created the preferred quality of the industry q^f is chosen.*

To address the welfare issue we suppose that the government puts as much weight on consumers as on firms. Since the total mass of consumers is unity, the government maximizes $\pi(q, R) + u(q, R)$ under the constraints $\pi(q, R) \geq 0$, $u(q, R) \geq \underline{v}$ and $R \leq L$. Let \hat{v} be the utility achieved at this optimum. By continuity, there exists an index $\hat{\theta}$ such that $v^{\hat{\theta}} = \hat{v}$.²² Since the FOCs of the government program are identical to those of $P(L, \hat{\theta})$ (whether $R \leq L$ is binding or not) we conclude that the social optimum is the equilibrium outcome corresponding to $\hat{\theta}$. The social optimum also solves the FOC

$$\pi_q = -u_q \Leftrightarrow \frac{c_q^f}{-c_q^a} = \left(\frac{R - c^f(q)}{R} \right)^2 \quad (16)$$

thus if $c^f(\cdot)$ has a large curvature (highly convex) then the solution of (16) is close to q^f while if $c^a(\cdot)$ has the largest curvature, the solution of (16) is close to q^a .

Although fears of increased market power for the insurance companies are legitimate, coordination on a common auditing agency can be welfare increasing. Recall indeed that it is a commitment to set a control structure that *reduces* the cost of audit, thus *increases* the desire of firms to audit and ultimately *reduces* fraud in the economy. Consumers might then value high quality audit structures

²¹If this level is lower than \underline{v} then q^A can be adjusted to meet the participation constraint.

²²This is because u, q^θ and R^θ are all continuous functions (cf. proof of Proposition 3).

that give them lower incentives to fraud since they can then capture (via the competition between firms) some of the additional surplus linked to the lower cost imposed by fraud to the economy.

The external audit agency is created only for $\theta \leq \theta^L$. In this case the premium and the reimbursement does not change and the welfare comparison depends only on audit quality. Consider a degree of competition θ close to θ^L . When the agency is created, audit quality jumps from a low q^θ to the highest level q^f . This change is a Pareto improvement if the collective optimum $q^{\hat{\theta}}$ is closer to q^f than to q^{θ^L} . This happens when $\hat{\theta}$ is either small or large (cf. Figure 2 above) i.e., if the average cost $c^f(\cdot)$ and $c^a(\cdot)$ have very different curvatures.

Corollary 2 *The external audit agency can be welfare improving when there is an intermediate degree of competition.*

4.2 Replacement Clauses

Insurance contracts often include clauses that decrease the opportunity cost of audit for policyholders (e.g., Allstate’s priority repair option). A relevant example is when the policyholder of a damaged car is provided free of charge a replacement vehicle while its car is examined by a company expert and then fixed in a garage under covenant with the insurance company (generally farther away from the consumer’s home).

The effect of such a clause can be captured in our model by a reduction $\delta + \epsilon$ of the opportunity cost $C^a(q)$ together with an increase δ of the audit cost $C^f(q)$. It is clear that such a substitution from money to in-kind transfer has to satisfy $\epsilon > 0$ in order to appear in equilibrium, for otherwise a premium reduction would dominate quality improvement for the firm. We show that, within our model, the insurer must benefit from significant increasing returns to scale to be able to introduce the clause. Indeed, the sum of equilibrium payoffs $u + \pi$ given by equations (10) and (11) increases with the inclusion of the replacement clause only if the cost terms satisfy

$$\epsilon > \underline{\epsilon} \equiv \delta \left(\frac{R}{R - c^f(q)} \frac{R + F}{R - c^f(q) - \delta} - 1 \right) \quad (17)$$

We can now study how the introduction of the replacement clause affects the equilibrium contracts. Since the clause cannot be patented and is a Pareto improvement, it is immediately adopted by all insurers as sit is “discovered”.

We first consider the strong competition regime. The equilibrium is characterized by the *no-arbitrage* equation (14) linking premium and quality on the one hand and the *market* equation (15) $-u_q \theta \pi = \pi_q$ linking a *quantity* effect (LHS) and a *revenue* effect (RHS) on the other hand.

The introduction of a replacement clause reduces the per-capita profit π i.e., the *quantity* side of (15) but at the same time increases the marginal per-capita profit π_q i.e., the *revenue* side (15).²³ Hence, all insurers have an incentive to put more weight on their revenue improving strategies i.e., to increase either premium or quality. Since the two variables are substitute along the equilibrium path, intuition would suggest that one variable increases while the other variable decreases. Furthermore when β is small, we know that the main variable of competition is R so that one would expect the reimbursement to increase and audit quality to decrease. However the effect of the replacement clause is to move the whole contract curve upward. We show in the Appendix that reimbursement and quality are increased as a result of the availability of the new clause.

If the wealth constraint is binding in equilibrium ($R^\theta = L$), the audit quality is the only instrument available to restore equality in (15) when the replacement clause is introduced, hence audit quality increases. As it is a Pareto improvement, the replacement clause benefits to everybody whatever the competition regime because anyone could refuse to use it.²⁴ We summarize our finding in the following proposition.

Proposition 5 *The introduction of the replacement clause increases the audit quality and the premium (whenever not already binding). Furthermore insurers profits increase and consumer utility does not fall.*

5 Conclusion

Our approach offers a fresh look at the question of insurance fraud by developing a tractable equilibrium model of imperfection competition. Key to our analysis is the fact that firms compete both on prices (premium and reimbursement) and on their quality of audit (or some additional non pecuniary variables like in our last section). If a firm increases audit quality, it affects the residual demands of all other firms. Therefore audit quality has the flavor of a public good, leading to standard free riding and possibly under-investment problems. One of our results is an inverted U shaped relationship between competition and audit quality: competition does not necessarily make this free riding problem worse since *more competition* (starting from a low level) can translate into *better quality audit structures*. Our framework enabled us to analyze additional issues, like the effects on industry and society welfares of the creation of a cooperative audit agency or of “contractual innovations” – like replacement clauses educe the cost of audit for policy holders.

²³The same occurs if one considers the equation $-a_R\theta\pi = \pi_R$.

²⁴The only caveat is that if ϵ is small the new equilibrium may be constrained by the individual rationality constraint $u^\delta(q, R) \geq u(q^\theta, R^\theta)$ where u^δ denotes consumer utility in the presence of the clause.

We believe that our approach will prove useful for empirical work on insurance fraud; especially since it links contracts, fraud and industrial organization. Waiting for such empirical analysis, we find comfort in that stylized facts seem broadly consistent with our theoretical findings. As already noted in the introduction, in the US where there is a high degree of competition, firms advertise aggressively about the quality of their control structure (web sites). In France there is a lower degree of competition and firms are viewed as “soft” on the question of fraud (no awareness campaign, no appearance of the word “fraud” on web sites). Consistent with our findings, French firms have cooperated to create a control agency of high quality, ALFA.

Appendix

Proof of Proposition 1

Characterization of the equilibrium of the fraud and control game in a perfect Bayesian equilibrium.

Step 1 Candidates equilibria

A customer’s strategy is to defraud with probability σ while an insurer’s strategy is to control claims with probability τ . The rate of claims is the sum of truthful ones and fraudulent ones: $\beta + (1 - \beta)\sigma$. Because a verification is successful only with probability q (and cost F) and defrauders don’t suffer being audited, the utility function of the consumer and the insurer are

$$\begin{aligned} u(\sigma, \tau) &= V + w - P - \beta(L - R) + (1 - \beta)\sigma((1 - \tau q)R - \tau q F) - \beta \tau C^a(q) \\ \pi(\sigma, \tau) &= P - \beta R - C^f(q)\tau(\beta + (1 - \beta)\sigma) - (1 - \beta)\sigma(1 - \tau q)R \end{aligned}$$

The agent’s best reply is given by the frequency τ that makes the coefficients of σ nil in $u(\sigma, \tau)$.

We obtain

$$\tau \begin{matrix} \leq \\ > \end{matrix} \tau^* \equiv \frac{R}{q(R + F)} \Rightarrow \sigma \in \begin{matrix} \{1\} \\ [0; 1] \\ \{0\} \end{matrix} \quad (18)$$

The slope coefficient of τ in π is $S \equiv \sigma(1 - \beta)(qR - C^f(q)) - \beta C^f(q)$. Three cases can occur:

- If $\tau < \tau^*$ then $\sigma = 1$ so that $S = (1 - \beta)qR - C^f(q)$. Hence the optimal behavior is to play $\tau = 1$ if $S > 0$. We have a contradiction if $\tau^* \leq 1$. Otherwise the equilibrium is $\sigma = 1, \tau = 1$. The last possibility is $S < 0$ so that $\tau = 0$ is optimal; in that case the equilibrium is $\sigma = 1, \tau = 0$.
- If $\tau > \tau^*$ then $\sigma = 0$ so that $S = -C^f(q)\beta < 0$ implying that τ should optimally be set equal to 0, a contradiction.
- If $\tau = \tau^*$ then any $\sigma \in [0; 1]$ is optimal and to make τ^* optimal as well we look for the value of σ

that equalizes S to zero. Formally

$$\sigma \begin{matrix} \geq \\ < \end{matrix} \sigma^* \equiv \frac{\beta C^f(q)}{(1-\beta)(qR - C^f(q))} \Rightarrow \tau \in \begin{matrix} \{1\} \\ [0; 1] \\ \{0\} \end{matrix} \quad (19)$$

If $\sigma^* < 0 \Leftrightarrow R < C^f(q)/q$ or $\sigma^* > 1 \Leftrightarrow C^f(q)/q > (1-\beta)R$ then $S < 0$ thus $\tau = 0$ is optimal and $\sigma = 1$ is the optimal response. We have a pure strategy equilibrium. When $0 \leq \sigma^* \leq 1$ and $\tau^* \leq 1$ the Nash equilibrium is (σ^*, τ^*) . Since the agent is indifferent between fraud and honesty, his equilibrium utility is $u(q, R, p) = V + w - P - \beta \left(L - R + \frac{RC^a(q)}{(R+F)q} \right)$ while the equilibrium per-capita profit of the insurer is $\pi(q, R, p) = P - (\beta + (1-\beta)\sigma^*)R = p - \beta R \frac{R+F}{R-C^f(q)/q}$.

Step 2 Elimination of the pure strategy equilibria

The pure strategy equilibrium $(\sigma = 1, \tau = 0)$ leads to payoffs $\hat{u} = V + w - \beta L - P + R$ and $\hat{\pi} = P - R$. Thus, the contract is producing no economic surplus. Since the no-losses condition of the insurer yields $P \geq R$, the agent is better off not signing this contract. Hence this situation will not appear in a PBE.

The pure strategy equilibrium $(1, 1)$ exists only when $\tau^* > 1 \Leftrightarrow R > q(R + F)$. Payoff are $\hat{\pi} = P - R + (1-\beta)qR - \phi_\pi(q)$ and $\hat{u} = V + w - \beta(L + C^a(q)) - (1-\beta)q(R + F) + R - P$. We claim that offering (q, R) is a dominated strategy. Indeed, reducing R and P so as to keep $P - R + (1-\beta)qR$ constant maintain \hat{u} and $\hat{\pi}$ constant. The insurer can therefore bring the equality $R = q(R + F)$. With this new contract, the threshold is $\tau^* = 1$ and $\hat{u} = u(\sigma^*, \tau^*)$ because $\tau^* = 1$ makes the coefficients of σ nil in $u(\sigma, \tau)$. Hence, by switching from the pure strategy equilibrium to the mixed one, the insurer keep its customers (their utility level remains constant). Still, there is a benefit of inducing the mixed strategy equilibrium with $\sigma^* < 1$ because $\frac{\partial \pi}{\partial \sigma} \Big|_{\tau=\tau^*} = -C^f(q)\tau^*(1-\beta) < 0$ means that $\pi(\sigma^*, \tau^*) > \hat{\pi}$ when $R = q(R + F)$.

Proof of Proposition 3

There exists two wealth levels \underline{w} and \bar{w} such that for an intermediate wealth $w \in]\underline{w}; \bar{w}[$, the symmetric equilibrium between insurers has two regimes

(i) weak competition regime $\theta \in [0; \theta^L[$: equilibrium contracts feature full insurance, maximal premium while quality is decreasing with θ .

(ii) strong competition regime $\theta \in [\theta^L; +\infty[$: equilibrium reimbursement and premium decreases with θ while quality increases with θ .

If $w < \underline{w}$, only the weak regime applies while if $w > \bar{w}$, only the strong regime applies.

We use a series of lemmas. In the first, we show that the contract curve is decreasing in the (R, q) space and that consumer utility decreases as we move down on the contract curve (when $R \nearrow$

and $q \searrow$). Next we use this property to characterize the solution of $\hat{P}(L, v) \equiv \max_{q, R} \pi(q, R)$ under constraints $u(q, R) \geq v$ and $R \leq L$. Lastly, we relate the equilibrium of the insurer competition for a given θ to $\hat{P}(L, v^\theta)$ where v^θ is the equilibrium utility of consumers.

Lemma 1 *The contract curve is decreasing. When moving on this curve towards a lower R and a greater q , the consumer utility increases.*

Proof Observe that $u(q, R)$ is bounded over $[q^a; q^f] \times [L - w; L]$ and spans an interval $[v_1; v_2]$. Assumption H2 ($\beta \ll 1$) implies $u_R < 0$ thus the solution $R = \rho(q, v)$ to the equation $v = u(q, R)$ is also bounded over $[q^a; q^f] \times [v_1; v_2]$. Let us now introduce

$$\Phi^q(q, R) \equiv -\frac{\pi_q}{u_q} = \frac{-R(R+F)c_q^f(q)}{(R-c^f(q))^2 c_q^a(q)} \quad (20)$$

$$\Phi^R(q, R) \equiv -\frac{\pi_R}{u_R} = \frac{1 - \beta R \frac{R-2c^f(q)}{(R-c^f(q))^2}}{1 - \beta + \beta \frac{Fc^a(q)}{(R+F)^2}} \quad (21)$$

The solution $q^*(v)$ to $\frac{\pi_R}{\pi_q} = \frac{u_R}{u_q}$ evaluated at $R = \rho(q, v)$ also solves $\Phi^q(q, \rho(q, v)) = \Phi^R(q, \rho(q, v))$. We want to show that $q^*(v)$ is increasing. Then it will be clear that $R^*(v) \equiv \rho(q^*(v), v)$ is decreasing since ρ_q and ρ_v are negative. This will indirectly prove that the relationship between R and q on the contract curve is negative and also that the consumer utility v increases when R decreases (as $R^v \searrow$).

Observe that for any v in $[\underline{v}; \bar{v}]$, $\Phi^R(q, \rho(q, v))$ is bounded positive over $[q^a; q^f]$ with a lower bound $\underline{\Phi}^R$ while $\Phi^q(q, \rho(q, v))$ varies from $+\infty$ to 0 over $[q^a; q^f]$ hence the equation has a solution. We assume that it is unique. To obtain q_v^* we differentiate the equation $\Phi^q - \Phi^R = 0$ using $\rho_q = -\frac{u_q}{u_R} < 0$ and $\rho_v = \frac{1}{u_R} < 0$.

$$\begin{aligned} 0 &= (\Phi_q^q - \Phi_q^R) q_v^* + (\Phi_R^q - \Phi_R^R) (\rho_q q_v^* + \rho_v) = (\Phi_q^q - \Phi_q^R) q_v^* + (\Phi_R^q - \Phi_R^R) \frac{1 - q_v^* u_q}{u_R} \\ \Rightarrow q_v^* \left((\Phi_q^q - \Phi_q^R) - (\Phi_R^q - \Phi_R^R) \frac{u_q}{u_R} \right) &= \frac{\Phi_R^R - \Phi_R^q}{u_R} \Rightarrow q_v^* = \frac{\Phi_R^R - \Phi_R^q}{(\Phi_q^q - \Phi_q^R) u_R - (\Phi_R^q - \Phi_R^R) u_q} \quad (22) \end{aligned}$$

Defining $X \equiv R \frac{R-2c^f}{(R-c^f)^2}$ and $Y \equiv \frac{Fc^a}{(R+F)^2} - 1$, we have $\Phi^R = \frac{1-\beta X}{1-\beta Y}$ and derive $\Phi_R^R = \beta \frac{X_R(1-\beta Y) - Y_R(1-\beta X)}{(1-\beta Y)^2}$ and $\Phi_q^R = \beta \frac{X_q(1-\beta Y) - Y_q(1-\beta X)}{(1-\beta Y)^2} < 0$. Observe that $\Phi_q^q = \Phi^q \left(\frac{c_{qq}^f}{c_q^f} - \frac{c_{qq}^a}{c_q^a} + \frac{2c_q^f}{R-c^f} \right) < 0$ and $\Phi_R^q = \frac{2R+F}{(R-c^f)^3} \frac{c_q^f c_q^a}{c_q^a} < 0$. Hence Φ_R^R and Φ_q^R vanish in front of Φ_R^q and Φ_q^q respectively when $\beta \ll 1$. As $u_R \gg u_q$ we obtain

$$\lim_{\beta \ll 1} q_v^* = \frac{-\Phi_R^q}{\Phi_q^q u_R} = \frac{-\Phi_R^q}{u_R \Phi^q \left(\frac{c_{qq}^f}{c_q^f} - \frac{c_{qq}^a}{c_q^a} + \frac{2c_q^f}{R-c^f} \right)} = \frac{(2R+F)c^f c_q^f}{(R-c^f)^3 \pi_R c_q^a} \frac{1}{\frac{c_{qq}^f}{c_q^f} - \frac{c_{qq}^a}{c_q^a} + \frac{2c_q^f}{R-c^f}} > 0 \quad \blacksquare \quad (23)$$

Lemma 2 *There exists \underline{w} and \bar{w} such that when $w \in]\underline{w}; \bar{w}[$, $\exists v^L \in]\underline{v}; \bar{v}[$ and the solution of*

$$\hat{P}(L, v) \equiv \begin{cases} \max_{q, R} \pi(q, R) \\ \text{s.t. } u(q, R) \geq v, R \leq L \end{cases} \text{ is } \begin{cases} R^v = L \text{ and } q^v \searrow & \text{for } v \in [\underline{v}; v^L] \\ R^v \searrow \text{ and } q^v \nearrow & \text{for } v \in [v^L; \bar{v}] \end{cases}.$$

Proof As $u_q < 0$, the equation $u(q, R) = v$ has a unique solution $q = Q(R, v)$ satisfying $Q_R = -\frac{u_R}{u_q} < 0$ and $Q_v = \frac{1}{u_q} < 0$. Solving $\hat{P}(L, v)$ is equivalent to maximize $\pi(Q(R, v), R)$ over $[L - w; L]$. The FOC for an interior solution is

$$\frac{u_q}{u_R} \Big|_{q=Q(R, v)} = \frac{\pi_q}{\pi_R} \Big|_{q=Q(R, v)} \quad (24)$$

By lemma 1, the solution $R^*(v)$ of (24) is decreasing and $Q(R^*(v), v)$ is increasing. Furthermore the properties $\pi_q(q^f) = 0$ and $u_q(q^a) = 0$ imply that $Q(R^*(v), v)$ is always interior to $[q^a; q^f]$. Lastly, the inequality $\pi(\cdot, L - w) < 0$ implies that only the constraint $R \leq L$ can be binding.

The solution to $\hat{P}(L, v)$ is now very simple to obtain: starting from the largest level \bar{v} , one decreases v . As long as $v \geq v^L \equiv R^{*-1}(L)$, the contract $(R^v, q^v) \equiv (R^*(v), Q(R^*(v), v))$ is optimal i.e., the price increases while the quality decreases. For $v = v^L$, the optimal pair is simply $(L, Q(L, v))$ and as v decreases further the quality now increases (direct effect only).²⁵ To sum up, the solution of $\hat{P}(L, v)$ is $(L, \min\{q^\pi, Q(L, v)\})$ if $v < v^L$ and (R^v, q^v) otherwise.

We need to assess the conditions under which $v^L \in]\underline{v}; \bar{v}[$ i.e., when the liquidity constraint binds for the monopoly but not at the fully competitive outcome. The monopoly maximizes π under the constraint $u \geq \underline{v}$. He start from the utility level \bar{v} and moves down along the contract curve (R^v, q^v) . If $u(q^L, L) > \underline{v}$ then the monopoly chooses L and increases quality up to $\underline{q} = \min\{q^\pi, Q(L, \underline{v})\} > q^L$ and this means $v^L > \underline{v}$. At the other extreme of the competition spectrum, if $\pi(q^L, L) > 0$ then the solution of $\hat{P}(L, \bar{v})$ features $R < L$ i.e., $v^L < \bar{v}$. The two conditions we have characterized are $u(q^L, L) = V - \beta \frac{L}{L+F} c^a(q^L) > \underline{v} = (1 - \beta)V + w \Leftrightarrow w < \bar{w} \equiv \beta \left(V - \frac{L}{L+F} c^a(q^L) \right)$ and $\pi(q^L, L) > 0 \Leftrightarrow w > \underline{w} \equiv \beta L \frac{L}{L - c^f(q^L)}$. Thanks to H1, $\underline{w} < \bar{w}$ holds, thus $v^L \in]\underline{v}; \bar{v}[$ whenever $w \in]\underline{w}; \bar{w}[$. Obviously if $w \notin]\underline{w}; \bar{w}[$, only one regime applies. Equivalently we may define $v^L \equiv \min\{\bar{v}, \max\{R^{*-1}(L), \underline{v}\}\}$.

■

Lemma 3 $\exists \theta^L$ such that the equilibrium strategies are $\begin{cases} R^\theta = L \text{ and } q^\theta \searrow & \text{for } \theta \in [0; \theta^L] \\ R^\theta \searrow \text{ and } q^\theta \nearrow & \text{for } \theta \in]\theta^L; +\infty[\end{cases}$.

As in the previous lemma, we solve the problem without constraints and later introduce them.

Step 1 Solve the program $\tilde{P}(\theta) \equiv \max_{v, q} \Pi_i(q, \rho(q, v))$

²⁵The candidate value $Q(L, v)$ hits the upper bound q^f if $(V - \underline{v}) \frac{L+F}{\beta L} \geq c^a(q^f) \Leftrightarrow V \geq V^* \equiv c^a(q^f) \frac{L}{L+F} + \frac{w}{\beta}$.

Letting v^θ denote the equilibrium utility level, we may write $\tilde{P}(\theta) = \max_{v,q} D(v - v^\theta) \pi(q, \rho(q, v))$. The FOC of $\tilde{P}(\theta)$ with respect to q for an interior solution is $\frac{\pi_R}{\pi_q} = \frac{u_R}{u_q}$, thus the optimal quality is $q^*(v)$. The FOC of $\tilde{P}(\theta)$ with respect to v is $0 = \rho_v \frac{\partial \Pi_i}{\partial R} = \frac{1}{u_R} \left(t\pi + \frac{1}{n} \frac{\pi_R}{u_R} \right)$ and replacing q with the optimal value $q^*(v)$ we obtain a unique equation²⁶

$$\theta = H(v) \equiv \frac{-\pi_R}{\pi \cdot u_R} \Bigg|_{\substack{R=R^*(v) \\ q=Q(R^*(v),v)}} = \frac{\Phi^R(q^*(v), \rho(q^*(v), v))}{\pi(q^*(v), \rho(q^*(v), v))} \quad (25)$$

To show that v^θ is increasing we manipulate (25) to get

$$H' > 0 \Leftrightarrow (1 - u_q q^*) \left(\frac{\pi}{u_R} \Phi_R^R + (\Phi^R)^2 \right) > (\Phi^R \pi_q - \pi \Phi_q^R) q^* \quad (26)$$

As Φ_R^R and u_q are multiple of β while Φ^R is not (cf. Lemma 1), the LHS of (26) tends to $(\Phi^R)^2$ for $\beta \ll 1$, which is bounded away from zero (see definition (21)) while the RHS of (26) tends to zero because both π_q and Φ_q^R are multiple of β .

Step 2 Solve $P(L, \theta) = \max_{q,R} \Pi_i(q, R)$ such that $u(q, R) \geq \underline{v}$, $R \leq L$.

We define $\theta^L \equiv H(v^L)$ for $v^L = R^{*-1}(L)$, $\theta^L = 0$ if $v^L = \underline{v}$ and $\theta^L = +\infty$ if $v^L = \bar{v}$. When $\theta \geq \theta^L$, $v^\theta = H^{-1}(\theta) \geq v^L \geq \underline{v}$, thus $R^\theta = R^*(v^\theta) \leq L$ and $q^\theta = Q(R^\theta, v^\theta)$ solve $P(L, \theta)$. Yet when $\theta < \theta^L$, $R^*(H^{-1}(\theta)) > L$ means that all firms want to saturate the liquidity constraint $R \leq L$. The competition then takes place over a single variable, the audit quality varying in $[Q(R^*(v^L), v^L); q^f]$. Since the participation constraint $u(q, L) \geq \underline{v}$ is equivalent to $q \leq Q(L, \underline{v})$, the correct upper bound is thus $\min \{Q(L, \underline{v}), q^f\}$.

The profit function is $\hat{\Pi}_i(q_i) \equiv D(u(q_i, L) - u(\hat{q}, L)) \pi(q_i, L)$ where \hat{q} is the equilibrium quality. The unique FOC to be satisfied at the symmetric equilibrium is similar but different from (25):

$$\theta = G(q) \equiv \frac{-\pi_q}{\pi \cdot u_q} \Bigg|_{R=L} \quad (27)$$

Since $\Phi_q^q < 0$ and $\pi_q > 0$ we have $\Phi_q^q \pi > \Phi^q \pi_q \Leftrightarrow G'(v) < 0$. The candidate Nash equilibrium quality is $G^{-1}(\theta)$; it varies from $G^{-1}(0) = q^f$ to $G^{-1}(+\infty) = q^a$. The equilibrium is thus $\min \{G^{-1}(\theta), Q(L, \underline{v})\}$. To summarize, the symmetric equilibrium is $(R^\theta, q^\theta) = (L, \min \{G^{-1}(\theta), Q(L, \underline{v})\})$ if $\theta < \theta^L$ and $(R^\theta, q^\theta) = (R^*(H^{-1}(\theta)), Q(R^*(H^{-1}(\theta)), H^{-1}(\theta)))$ otherwise.

Proof of Corollary 1

The frequency of fraud increases with competition.

²⁶We do not need a fixed point argument to derive the equilibrium because the solution of $\tilde{P}(\theta)$ does not depend on the level v^θ . This in turn is due to the linearity of D .

The frequency of fraud in the symmetric PBE is $\sigma^\theta = \frac{\beta c^f(q^\theta)}{(1-\beta)(R^\theta - c^f(q^\theta))}$ thus $\frac{\partial \sigma^\theta}{\partial \theta} > 0 \Leftrightarrow \frac{c_q^f(q^\theta)}{c^f(q^\theta)} \frac{\partial q^\theta}{\partial \theta} R^\theta > \frac{\partial R^\theta}{\partial \theta}$.

► When $\theta < \theta^L$, the equality $R^\theta = L$ implies that $\frac{\partial \sigma^\theta}{\partial \theta} > 0 \Leftrightarrow \frac{c_q^f(q^\theta)}{c^f(q^\theta)} \frac{\partial q^\theta}{\partial \theta} R^\theta > 0$ which holds true since $\text{sign}\left(\frac{\partial q^\theta}{\partial \theta}\right) = \text{sign}(G') = \text{sign}(c_q^f) = -1$.

► When $\theta \geq \theta^L$, $v^\theta = H^{-1}(\theta)$, $R^\theta = R^*(v^\theta)$ and $q^\theta = Q(R^\theta, v^\theta)$ thus $\frac{\partial q^\theta}{\partial \theta} = Q_v \frac{\partial v^\theta}{\partial \theta} + Q_R \frac{\partial R^\theta}{\partial \theta} = \frac{1}{u_q} \left(\frac{1}{H'} - u_R R_v^* \right)$ and $\frac{\partial \sigma^\theta}{\partial \theta} > 0 \Leftrightarrow \frac{c_q^f(q^\theta)}{c^f(q^\theta)} \frac{R^\theta}{u_q} \left(\frac{1}{H'} - u_R R_v^* \right) > R_v^*$. Using $\lim_{\beta \ll 1} q_v^*$ (equation (23)) we finally obtain $\frac{\partial \sigma^\theta}{\partial \theta} > 0 \Leftrightarrow \frac{c_q^\pi}{\frac{c_q^f}{c^f} - \frac{c_q^a}{c_q^a} + \frac{2c_q^f}{R^\theta - c^f}} < \frac{1}{R^\theta - c^f}$ which holds true as cost functions are convex i.e., $\frac{c_{qq}^f}{c_q^f} - \frac{c_{qq}^a}{c_q^a} > 0$.

Proof of Proposition 4

The auditing agency does enters the insurance market when $\theta \leq \theta^L$ and $\bar{q} > q^\theta$.

Over $[L - w; L] \times [\bar{q}; q^f]$ the contract competition drives insurers choices towards (q^θ, R^θ) thus the constraint $q \geq \bar{q}$ will bind in equilibrium of the new game and firms will compete on R only. The FOC characterizing the equilibrium is thus $\theta \cdot \pi(\bar{q}, R) = \Phi^R(\bar{q}, R)$. The solution $\bar{R}(\bar{q})$ satisfies $\frac{\partial R^A}{\partial \bar{q}} = \frac{\Phi_q^R - \theta \pi_q}{\theta \pi_R - \Phi_R^R}$ and individual profits are $\frac{1}{n} \pi(\bar{q}, \bar{R}(\bar{q}))$.

If (q^θ, R^θ) is the equilibrium without the agency, the entry of the intermediary with a superior audit quality \bar{q} implies the following payoff variation between the two equilibria: $\Delta = \left(\pi_q + \pi_R \frac{\partial \bar{R}}{\partial \bar{q}} \right) (\bar{q} - q^\theta) = \frac{\pi_R \Phi_q^R - \pi_q \Phi_R^R}{\theta \pi_R - \Phi_R^R} (\bar{q} - q^\theta) = \beta \frac{(X - \beta Y)}{\theta \pi_R - \beta Z} (\bar{q} - q^\theta)$ where none of X, Y and Z are multiple of β (cf. Lemma 1); hence $\lim_{\beta \ll 1} \text{sign}(\Delta) = \text{sign}(\Phi_q^R) < 0$. ■

Proof of Proposition 5

The introduction of a replacement clause increases audit quality and premiums.

We shall neglect the difference ϵ between the avoided opportunity cost of consumers and the additional cost of insurers and simply show that $dq/d\delta$ and $dR/d\delta$ are positive.

Observe that $\pi_\delta < 0, \pi_{R\delta} > 0, u_{R\delta} > 0, \pi_{q\delta} > 0$ and $u_{q\delta} = 0$ hence $\Phi^R = -\frac{\pi_R}{u_R}$ and $\Phi^q = -\frac{\pi_q}{u_q}$ increase with δ . Differentiating $\Phi^R = \Phi^q$ with respect to δ keeping R constant we get $q_\delta|_{R=cte} = \frac{\Phi_\delta^q - \Phi_\delta^R}{\Phi_R^R - \Phi_R^q}$. It is then easy to obtain $\lim_{\beta \ll 1} q_\delta|_{R=cte} = -\Phi_\delta^q / \Phi_R^q > 0$ as $\Phi_\delta^q > 0$. Likewise $R_\delta|_{q=cte} = \frac{\Phi_\delta^q - \Phi_\delta^R}{\Phi_R^R - \Phi_R^q}$ and $\lim_{\beta \ll 1} R_\delta|_{q=cte} = -\Phi_\delta^q / \Phi_R^q > 0$. However these are local variations that do not account for the global change in the equilibrium contract induced by the replacement clause. The system characterizing the equilibrium is $\Phi^R = \Phi^q = \theta \pi$ hence

$$\begin{cases} \Phi_q^R q_\delta + \Phi_R^R R_\delta + \Phi_\delta^R = \Phi_q^q q_\delta + \Phi_R^q R_\delta + \Phi_\delta^q \\ \Phi_q^R q_\delta + \Phi_R^R R_\delta + \Phi_\delta^R = \theta (\pi_q q_\delta + \pi_R R_\delta + \pi_\delta) \end{cases} \Rightarrow \begin{cases} (\Phi_q^R - \Phi_q^q) q_\delta + (\Phi_R^R - \Phi_R^q) R_\delta = \Phi_\delta^q - \Phi_\delta^R \\ (\Phi_q^R - \theta \pi_q) q_\delta + (\Phi_R^R - \theta \pi_R) R_\delta = \theta \pi_\delta - \Phi_\delta^R \end{cases}$$

$$\Rightarrow R_\delta = \frac{(\Phi_\delta^q - \Phi_\delta^R)(\Phi_q^q - \theta\pi_q) - (\theta\pi_\delta - \Phi_\delta^q)(\Phi_q^R - \Phi_q^q)}{(\Phi_q^q - \theta\pi_q)(\Phi_R^R - \Phi_R^q) - (\Phi_q^R - \Phi_q^q)(\Phi_R^q - \theta\pi_R)} \quad \text{and} \quad q_\delta = \frac{(\Phi_R^R - \theta\pi_R)(\Phi_\delta^q - \Phi_\delta^R) - (\Phi_R^R - \Phi_R^q)(\theta\pi_\delta - \Phi_\delta^R)}{(\Phi_R^R - \theta\pi_R)(\Phi_q^R - \Phi_q^q) - (\Phi_R^R - \Phi_R^q)(\Phi_q^R - \theta\pi_q)}.$$

We now use the fact that Φ_R^R , Φ_q^R , π_δ , π_q and Φ_δ^R are multiple of β to get $\lim_{\beta \ll 1} q_\delta = -\Phi_\delta^q / \Phi_q^q$ which is positive and $\lim_{\beta \ll 1} R_\delta = \frac{\Phi_q^q \pi_\delta - \Phi_\delta^q \pi_q}{-\Phi_q^q \pi_R}$; thus $\lim_{\beta \ll 1} R_\delta > 0 \Leftrightarrow \Phi_q^q \pi_\delta > \Phi_\delta^q \pi_q$

$$\Leftrightarrow \Phi^q \left(\frac{c_{qq}^f}{c_q^f} - \frac{c_{qq}^a}{c_q^a} + \frac{2c_q^f}{R - c^f} \right) \frac{-\beta R^2}{(R - c^f)^2} > \Phi^q \left(\frac{2}{R - c^f} \right) \frac{-\beta R^2 c_q^f}{(R - c^f)^2} \Leftrightarrow \frac{c_{qq}^a}{c_q^a} - \frac{c_{qq}^f}{c_q^f} > 0$$

which is true. ■

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