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ABSTRACT

Relational Contracts and Property Rights*

We propose a general framework for analysing and comparing ownership structures with respect to creating incentives for cooperative behaviour (e.g. efficient investment) in long-run relationships. We generalize models by Garvey (1995), Halonen (2002), and Baker, Gibbons and Murphy (2002) and compare their results in the light of our theory, going in depth into the issue of renegotiation of ownership and strategies. We show that when agents are not restricted in their strategy choice, the short–term efficient ownership structure identified by Hart and Moore (1990) is not relational efficient — i.e. does not maximize the set of discount factors under which efficient investment can be supported in equilibrium of the repeated game. Moreover, the relational efficient ownership structure is independent of what can be renegotiated: ownership, strategies, both or none.

JEL Classification: D23 and L22
Keywords: theory of the firm, implicit contracts, incomplete contracts, vertical integration, non-contractual relations, ownership structures, and supply relations

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1 Introduction

Many economic transactions, perhaps most of them are not isolated exchanges between anonymous parties but rather episodes of a history of exchanges, of a relationship.[1] This is the case for almost all transactions occurring within organizations – from employment to interactions between organizations' units and employees – and for many of those between organizations, in particular supply relations, including financial ones.

When these transactions are complex and there is scope for opportunism (hold-up), they often take place within the general terms determined by an explicit contract. But complete contracts, covering all conceivable contingencies, are absent from our world.[2] Relational contracts, flexible self-enforcing implicit arrangements that complete the rigid, incomplete explicit contracts are indispensable to "have things running smoothly". A sign of this is that in developed countries with detailed employment contracts and efficient law enforcement, "working to rule" – i.e. following literally what prescribed by the explicit contract – is one of the tougher traditional weapons in the hands of employees when bargaining for higher wages. Relational contracts are also very important between organizations, in particular for the governance of specific supply relations, where crucial aspects of the transactions are hard to contract upon.[3] And they are becoming even more important than before in light of the current trend towards outsourcing previously integrated vertical stages of production, the so called "Japanesization" of many industries, that is dramatically increasing the number of long-term transactions to be governed without the help of authority (see e.g. Taylor and Wiggins, 1997; Holmström and Roberts, 1998).

Understanding the interaction between the rigid and incomplete explicit contracts enforced by courts, in particular of property rights on assets instrumental for production, and the relational contracts that complete them is crucial for the theory of organizations. The aim of this paper is to provide a general framework for analyzing and comparing different ownership structures (and potentially other explicit contracts) with respect to

[1] The classical reference is Macaulay (1963).

[2] We do not enter the debate on why contracts are incomplete. The interested reader may e.g. look at the special issue of the Review of Economic Studies (66, 1999); we just note that, independent of complexity considerations, the mere presence of (typically very high) costs of court enforcement justify the limited use of detailed explicit contracts often observed in reality.

[3] Fehr, Brown, and Falk (2001) provide striking experimental evidence of the overwhelming importance of relational contracts when there are unobservable/uncontractable aspects involved in the transactions.

their effects on parties' ability to sustain the relational contracts necessary to achieve productive efficiency in long-run business relationships.

The property rights theory of the firm, as pioneered by Coase (1937) and developed by Williamson (1975, 1986), Klein, Crawford, and Alchian (1978), and Grossmann and Hart (1986) and Hart and Moore (1990), has been a tremendous breakthrough in our understanding of the potential effects of ownership and analogous contractual rights on parties' incentives to invest and on the economic process in general. This literature, however, has focused mainly on individual, isolated transactions. Klein and Leffler (1981) and Telser (1981) have stressed relatively early that one cannot build a complete theory of the firm without taking into account the effects of formal contracts on the accompanying informal agreements. The formal theory of implicit contracts is well developed since Bull (1987) and MacLeod and Malcomson (1989) and has been extended to the case of imperfect monitoring by Levin (forthcoming). But only recently authors like Garvey (1995), Halonen (2002) and Baker et al. (2001, 2002) have began to formally analyze how asset ownership and implicit, relational contracts interact within full-fledged dynamic models of ongoing organizations.[4] The results of these recent contributions, however, are not easy to compare and evaluate, since they focus on slightly different issues and use differently specified models.

We propose a general framework where these recent models are contained as subcases, which allows us to highlight strengths and weaknesses of the respective results. In particular, we formulate a language that is able to describe and compare all the ownership structures that have raised interest in the previous literature and others that have been overlooked. We introduce efficiency measures for short term (one-shot) behavior and for long run (relational) behavior with and without the possibility to renegotiate both property rights and strategies. In particular, a "short term efficient" ownership structure maximizes joint payoffs in the static equilibrium. An ownership structure is called "relational efficient" if it supports a maximal range of discount parameters such that co-operation is an equilibrium path of the repeated game. We then compare different constellations of property rights on assets with respect to their relative "relational efficiency", that is, with how easily can parties sustain efficient behavior in equilibrium under each constellation. We follow the tradition in the theory of repeated and dynamic games by adopting the minimum discount factor $\underline{\delta}$ at which efficiency can be achieved

[4] See also Bragelien (2001). The work of Bernheim and Whinston (1998) and of Rosenkrantz and Schmitz (2001) should also be mentioned, although their two-stage models fall somewhat short of depicing a long-term relation.

in subgame perfect Nash equilibrium (with and without renegotiation) as an index of relational efficiency.[5]

Our main results are characterisations of relational efficient ownership with respect to the various renegotiation possibilities. The most intriguing observations are the following. If one restricts agents to use grim-trigger strategies, as done in all previous work on this subject, and allows the initial ownership structure to be freely renegotiated during and after a breakdown of the relation, then ownership does not affect the punishment phase after a defection, and short-term efficient ownership structures identified by Hart and Moore (1990) tend to be also relational efficient. However, if agents are free to choose and to renegotiate both ownership and strategies, as it seems natural, and take the possibility of renegotiation into account in their initial choice of strategies, then we reach the opposite conclusion. For any initial ownership structure, agents that anticipate the temptation to renegotiate punishments strategies after a defection may agree on simple asymmetric strategies that are optimal (in the sense of Abreu (1988)), robust versus mistakes (in the sense of Segerström (1988)), and implement efficient investments in a strong perfect equilibrium (as defined by Rubinstein (1980)). Besides being intuitive, optimal, and robust with respect to renegotiation and mistakes, these strategies are available whether or not ownership structure can be renegotiated, and when it can they prevent ownership renegotiation alltoghether. This implies that the relational efficient ownership structure is independent of renegotiation and that it differs generally from the short term ownership structure. This reinforces and generalizes Halonen's (2002) conclusion that joint ownership may be optimal in a dynamic environment although it is never so in Grossman-Hart-Moore's one-shot contracting models.[6]

The paper unfolds as follows. Section 2, describes the investment stage game. Section 3 describes the dynamic/repeated game, defines relational efficiency and discusses renegotiation of ownership. In section 4 we analyze the possibility to renegotiate ownership and strategies and introduce restitution strategies. Section 5 defines ownership structures and applies the general framework to the model specifications of Garvey (1995), Halonen (2002), and Baker et al. (2002). The last section concludes. All proofs are in

[5]In future work we plan to introduce other measures for the stability of co-operative agreements in dynamic games, those related to risk dominance (see Blonski and Spagnolo, 2001).

[6]Halonen (2002) has been criticized for assuming away the possibility to (costlessly) renegotiate ownership after cooperation breaks down, a possibility that in her framework destroy her result (see e.g. Bragelien (2001)). We show here that this objection, although justified in itself, does not change the results when agents are free to choose the strategies by which to support efficient investments.

the Appendix.

2 Investment stage game

Two parties $i = 1, 2$ play the following investment stage game. In the first substage both agents decide simultaneously on a costly action $e_i \in E_i$ (which can be interpreted as investment) with cost $C_i(e_i)$. In the second substage agents bargain over the jointly created "cake" $Q(e_1, e_2) \geq 0$. The size of this cake depends on both actions. Let

$$e = (e_1, e_2) \in E := E_1 \times E_2$$

denote an action profile. We assume for simplicity that there exists a unique action profile $e^c = (e_1^c, e_2^c) \in E$ that maximizes the size of the joint surplus given as

$$\pi(e) = Q(e) - C_1(e_1) - C_2(e_2)$$

and call $e_i^c \in E_i$ the "co-operative" or "first-best" action of agent i. In the tradition of Grossmann-Hart-Moore agents' actions not only determine the joint surplus but also bargaining positions. More precisely, introduce the threat point or disagreement (or status quo) payoffs

$$P(e, \omega) = (P_1(e, \omega), P_2(e, \omega)) \in \left\{(x_1, x_2) \in R^2 \,|x_1 + x_2 \leq Q(e)\right\}.$$

Agent i's disagreement payoff $P_i(e, \omega)$ depends on parameter $\omega \in \Omega$ – interpreted as "ownership structure" – and on action profile e.

We stick to the original assumption that agents split equally the "net-cake" $Q(e) - P_1(e, \omega) - P_2(e, \omega) \geq 0$, i.e. agree on the Nash-Bargaining-Solution (NBS) yielding payoff

$$u_i(e, \omega) = \frac{1}{2}\left[Q(e) + P_i(e, \omega) - P_{-i}(e, \omega)\right] - C_i(e_i) \tag{1}$$

to agent i.[7] This implies that $\pi(e) = u_1(e, \omega) + u_2(e, \omega)$ and in particular that the first best joint surplus

$$\pi^* \equiv u_i(e^c, \omega) + u_{-i}(e^c, \omega) = Q(e_1^c, e_2^c) - C_1(e_1^c) - C_2(e_2^c) \tag{2}$$

[7]It is well known that allowing for "outside options" may change the conclusions of the Grossmann-Hart-Moore approach (De Meza and Lookwood, 1998; Chiu, 1998). Extending the present model to outside option bargaining is potentially interesting but left to future work.

does not depend on ω. For any $\omega \in \Omega$ these payoffs define a game with simultaneous choice of actions in the first substage denoted by $\Gamma(\omega)$ and called subsequently the "reduced form stage game".

Obviously maximization of individual payoffs $u_i(e,\omega)$ and of joint payoffs $\pi(e) = u_i(e,\omega) + u_{-i}(e,\omega)$ create different incentives. Following Hart and Moore (1990) we are particularly interested in a structure such that independent of parameter $\omega \in \Omega$ it is not in agents' individual short term interest to act co-operatively.

Definition 1 *A family of stage games $\{\Gamma(\omega)\}_{\omega\in\Omega}$ is called a "simple holdup structure" iff for every $\omega \in \Omega$ each agent $i = 1,2$ has a strictly dominant stage game strategy denoted by $e_i^d(\omega)$ and $e^d(\omega) \neq e^c$ for all $\omega \in \Omega$.[8] This implies that $\Gamma(\omega)$ has a unique (pure strategy) Nash equilibrium called "holdup equilibrium" and denoted by $e^d(\omega) = \left(e_1^d(\omega), e_2^d(\omega)\right)$.*

In contrast to the "first best" co-operative action profile e^c the holdup equilibrium $e^d(\omega)$ depends on ω. Therefore, we can compare different ownership structures with respect to the sum of the generated (transferable) equilibrium utilities.

Definition 2 *Call $\omega^* \in \Omega$ "short term efficient" if it maximizes the sum of equilibrium payoffs*

$$u_1\left(e^d(\omega^*),\omega^*\right) + u_2\left(e^d(\omega^*),\omega^*\right) \geq u_1\left(e^d(\omega),\omega\right) + u_2\left(e^d(\omega),\omega\right) \quad \forall \omega \in \Omega. \tag{3}$$

The corresponding set of short term efficient ownership structures is denoted by Ω^*.

To keep notation suggestive and simple we introduce shortcut variables for $i = 1,2$

$$\begin{aligned}
c_i(\omega) &= u_i((e^c),\omega) \text{ for "Co-operation payoff"}, \qquad (4)\\
d_i(\omega) &= u_i\left(e^d(\omega),\omega\right) \text{ for "Defection payoff", holdup equilibrium payoff},\\
b_i(\omega) &= u_i\left(\left(e_i^d(\omega), e_{-i}^c\right),\omega\right) \text{ for "Betray payoff"},\\
a_i(\omega) &= u_i\left(\left(e_i^c(\omega), e_{-i}^d\right),\omega\right) \text{ for "Afflicted payoff"},\\
d_i^*(\omega) &= u_i\left(e^d(\omega^*),\omega^*\right) \text{ short term efficient holdup equilibrium payoff},\\
\pi^* &= c_1(\omega) + c_2(\omega) \text{ joint first best surplus},\\
D(\omega) &= d_1(\omega) + d_2(\omega) = u_1\left(e^d(\omega),\omega\right) + u_2\left(e^d(\omega),\omega\right),\\
B(\omega) &= b_1(\omega) + b_2(\omega) = u_1\left(\left(e_1^d(\omega), e_2^c\right),\omega\right) + u_2\left(\left(e_1^c(\omega), e_2^d\right),\omega\right),\\
D^* &= d_1^* + d_2^* \text{ joint short term efficient holdup equilibrium payoffs.}
\end{aligned}$$

[8] The "d" in e_i^d stands for "defect" since the resulting game contains a Prisoner's Dilemma.

Capital letters D, B stand for aggregates. It turns out to be important for all results that $\pi^* = c_1(\omega) + c_2(\omega)$ and $D^* = d_1^* + d_2^*$ do not depend on ownership structure in contrast to aggregate betray payoffs $B(\omega) = b_1(\omega) + b_2(\omega)$ and aggregate defective equilibrium payoffs $D(\omega) = d_1(\omega) + d_2(\omega)$.

Strict dominance of $e_i^d(\omega)$ and the definition of e^c imply

$$b_i(\omega) > c_i(\omega) > d_i(\omega) > a_i(\omega). \tag{5}$$

>From here we suppose agents who restrict their attention to the two strategies "co-operate" and "defect", the two most salient modes of behavior. For these agents any given ownership structure resembles a Prisoner's Dilemma $\Gamma(\omega)$ with payoff bi-matrix

$\Gamma(\omega)$	e_2^c	$e_2^d(\omega)$
e_1^c	$c_1(\omega)$, $c_2(\omega)$	$a_1(\omega)$, $b_2(\omega)$
$e_1^d(\omega)$	$b_1(\omega)$, $a_2(\omega)$	$d_1(\omega)$, $d_2(\omega)$

(6)

If agents are unable to commit to co-operate (because of limited contractability) it might still be feasible to renegotiate on ownership. Agents who recognize that co-operation is not sustainable indeed have an incentive to renegotiate ownership whenever the initial ownership structure ω was inefficient and the cost of renegotiation is negligible. If the total cost of renegotiating/reallocating ownership is $z \geq 0$, agents' payoff increases by

$$S = \frac{1}{2}\left(D^* - D(\omega) - z\right)$$

if agents again apply Nash bargaining (split the pie).

3 Relational Efficiency

Suppose parties play repeatedly (with positive probability) the investment game described in the previous section. Moreover, suppose that at the beginning of each period, agents can renegotiate and change the ownership structure before playing the investment game. Call $\Gamma(\delta, \Omega)$ the dynamic game with joint discount factor δ generated by the repeated play of a game $\Gamma(\omega)$ with $\omega \in \Omega$, so that ω is a state variable for $\Gamma(\delta, \Omega)$.

To support the efficient level of investment on the equilibrium path of this dynamic game – to maximize joint payoffs, "the pie" – agents may need to split the pie in a different way than specified by the payoffs of $\Gamma(\omega)$. To optimally adjust the shares of the pie, agents may agree on a transfer $\theta_1 = \theta = -\theta_2$, so that $u_i(e^c, \omega) + \theta_i$ goes to agent i if both agents co-operate. In many cases an appropriate reallocation of payoffs by such a transfer may induce to co-operate also those agents with the strongest incentives to defect.

The generic period-τ stage of the dynamic game $\Gamma(\delta, \Omega)$ has the following three-step structure

1	2	3
ownership structure ω^τ is determined/modified	$\Gamma(\omega^\tau)$ is played (investments are undertaken)	θ^τ is paid (profits are split)

.

In this section we want to identify ownership structures that are most supportive for efficient, co-operative behavior in this dynamic game. In particular, we adopt the following efficiency criterium:

Definition 3 *Ownership structure ω is called "relational efficient" if it minimizes the lower bound $\underline{\delta}$ on discount factors such that for all $\delta \geq \underline{\delta}$ there exists a subgame-perfect equilibrium supporting indefinite co-operation with ownership structure ω on its equilibrium path for $\Gamma(\delta, \Omega)$. Similarly, call the respective equilibrium and equilibrium strategies "relational efficient".*

Negotiations on ownership structure at the beginning of the dynamic game and in any other period may involve transfers between agents and take place at some cost $z \geq 0$. When $z = 0$, we would expect agents to renegotiate ownership structure to one that is relational efficient for the continuation game, both at the beginning of the dynamic game and at each of the nodes reached thereafter.

Which ownership structure is relational efficient may depend on the strategies agents are allowed to use to support co-operative behavior (efficient investment). Previous work has focused on two cases.

On the one hand, we mentioned that Garvey (1995) and Baker et al. (2001, 2002) assume agents to support co-operation through grim trigger strategies (play co-operatively; if a defection takes place, revert for ever to non-co-operative play) and that $z = 0$. In their framework, after a defection co-operation breaks down, but agents are still able to renegotiate ownership. Hence, at the beginning of the infinite punishment phase ownership structure is renegotiated to the short-term efficient one, the gains from renegotiation being split according to Nash bargaining.

On the other hand, Halonen (2002) also assumes agents support efficient investments through grim-trigger strategies, but she assumes z large enough, so that ownership structure is not renegotiated when co-operation breaks down.

While both assumptions $z = 0$ and z large can be defended and are worth being studied, the exclusive focus on grim-trigger strategies appears restrictive. This is particularly so when allowing for renegotiation of ownership, since then grim-trigger strategies are not optimal (in the sense of Abreu (1988)), besides not being robust with respect to mistakes (Segerström (1988)) and to renegotiation (Farrell and Maskin (1989)). And it is not clear what would prevent agents from renegotiating strategies while they are renegotiating ownership. For this reason, in this paper we will focus on simple asymmetric strategies, that are natural (intuitive), optimal, and robust with respect to both mistakes and renegotiation.

3.1 No Renegotiation

To establish a first benchmark, let us follow Halonen (2002) in assuming the following.

Assumption 1. *(a) Renegotiation of ownership ω is not possible (for example, because it is too costly, i.e. $z > S = \frac{\delta}{1-\delta}\left(D^{*} - D(\omega)\right)$); (b) Renegotiation of strategies is not possible.*

Under Assumption 1a the dynamic game $\Gamma(\delta, \Omega)$ described above degenerates into a standard discounted repeated game with modified (with respect to (6)) reduced form stage game Prisoner's Dilemma

$$\begin{array}{|c|c|c|} \hline \Gamma^{N}(\omega) & e_2^c & e_2^d(\omega) \\ \hline e_1^c & c_1(\omega)+\theta \quad,\quad c_2(\omega)-\theta & a_1(\omega) \quad,\quad b_2(\omega) \\ \hline e_1^d(\omega) & b_1(\omega) \quad,\quad a_2(\omega) & d_1(\omega) \quad,\quad d_2(\omega) \\ \hline \end{array} \quad . \tag{7}$$

We can then state the following.

Proposition 1 *Under Assumption 1, if agents use support co-operation by means of grim-trigger strategies, then:*

1. *The range of supporting discount factors is maximal by paying the transfer*

$$\tilde{\theta}^N = \frac{(b_1(\omega) - c_1(\omega))(b_2(\omega) - d_2(\omega)) - (b_2(\omega) - c_2(\omega))(b_1(\omega) - d_1(\omega))}{b_2(\omega) - d_2(\omega) + b_1(\omega) - d_1(\omega)};$$

2. *An ownership structure ω is relational efficient iff*

$$\omega \in \Omega^N := \left\{ \omega \left| \frac{B(\omega) - \pi^*}{B(\omega) - D(\omega)} \leq \frac{B(\omega') - \pi^*}{B(\omega') - D(\omega')} \; \forall \omega' \in \Omega \right. \right\}$$

Note that ownership structures $\omega \in \Omega^N$ are relational efficient because under Assumption 1 grim-trigger strategies are optimal (in the sense of Abreu (1988)). Also, from statement 2 of the proposition one sees immediately that minimizing $\frac{B(\omega)-\pi^*}{B(\omega)-D(\omega)}$ may entail choosing ω to decrease $D(\omega)$. That is, in this environment the relational efficient ownership structures $\omega \in \Omega^N$ will generically differ from the short-term efficient structures $\omega^* \in \Omega$.

3.2 Renegotiation of Ownership

As a second benchmark, let us follow Garvey (1995) and Baker et al. (2001, 2002) in assuming the following.

Assumption 2. (*a*) *Renegotiation of ownership structure is possible and costless* ($z = 0$)*; (b) Renegotiation of strategies is not possible.*

When agents use grim-trigger strategies to support efficient investment, the possibility to renegotiate ownership (at zero or low cost) to the Pareto-optimal level at each node down the game tree makes the efficient investment harder to sustain in equilibrium. This is of course because punishments are weaker, since renegotiation of ownership implies a lower bound on time-average payoffs during the punishment phase, namely the payoffs obtained from non-co-operative investment under the "short-term efficient" ownership structure ω^*.

When agents use grim-trigger strategies and in each period can costlessly renegotiate ownership to the Pareto-optimal level, the efficient investment can be supported in equilibrium in the dynamic game $\Gamma(\delta, \Omega)$ for the same range of discount factors at which it can be supported in equilibrium in a corresponding discounted repeated game. For this latter game the incentive compatibility condition to co-operate supported by grim-trigger-strategies is given by

$$c_i(\omega) + \theta_i \geq (1 - \delta) b_i(\omega) + \delta(d_i(\omega) + S).$$

where $c_i(\omega) + \theta_i$ goes to agent i if both agents co-operate. The new sharing rule θ_1 depends now on per-period gains from renegotiation denoted by S. One can state the following.

Proposition 2 *Under Assumption 2, if agents support co-operation by means of grim-trigger strategies, then:*

1. *The range of supporting discount factors is maximal by paying the transfer*

$$\tilde{\theta}^R = \frac{(b_1(\omega) - c_1(\omega))(b_2(\omega) - d_2(\omega) - S) - (b_2(\omega) - c_2(\omega))(b_1(\omega) - d_1(\omega) - S)}{b_1(\omega) - d_1(\omega) + b_2(\omega) - d_2(\omega) - 2S}.$$

   where $S = \frac{1}{2}(D^* - D(\omega))$;

2. *Ownership structure ω is "*constrained *relational efficient" for the repeated interaction iff it minimizes aggregated betray payoffs, that is*

$$\omega \in \Omega^R = \{\omega \,|B(\omega) \leq B(\omega') \;\; \forall \omega' \in \Omega\}.$$

We use the lable "constrained relational efficient" – the constraint being grim trigger strategies – because *under Assumption 2 grim-trigger strategies are not optimal* (in the sense of Abreu (1988)). Stronger punishments can easily be build that enlarge the set of discount factors at which the co-operative level of investment can be supported in equilibrium. At this point a remark is in order.

Remark 1 *Aggregate betray payoffs $B(\omega) = u_1\left(\left(e_1^d(\omega), e_2^c\right), \omega\right) + u_2\left(\left(e_1^c(\omega), e_2^d\right), \omega\right)$ are small when the differences between the efficient and short term equilibrium investments $e_i^c - e_i^d(\omega)$ are small. This is also what happens when an ownership structure is short term efficient: satisfying* $\max_\omega \left(u_1\left(e^d(\omega), \omega\right) + u_2\left(e^d(\omega), \omega\right)\right)$ *implies choosing ω so that $e_i^d(\omega)$ are as high as possible, hence as close as possible to e_i^c.*

That is, the combination of Assumption 2 and grim-trigger strategies sterilizes all potential effects of ownership on out of equilibrium play (the most intresting part of the dynamic framework, in our view). Then, only the size of the hold-up problem matters. A short term efficient ownership structure minimizes the hold-up problem ($e_i^c - e_i^d(\omega)$), and a small hold-up problem implies small incentives to defect in the corresponding relation ($u_i\left(\left(e_i^d(\omega), e_j^c\right), \omega\right) - u_i(e_i^c, e_j^c)$). Because of this we will see that when ownership can be renegotiated and agents use grim-trigger strategies in all the specific examples in Section 5 short term efficient ownership structures are also relational efficient.[9]

4 Renegotiation of Ownership and Strategies

As already mentioned, we find the combination of Assumption 2 and grim trigger strategies rather unsatisfactory. First, under Assumption 2 grim trigger strategies are not optimal, hence do not allow to characterize (unconstrained) relational efficient ownership structures. Second, grim trigger strategies as any other strategies that prescribe inefficient play along the punishment path are not robust to agents' mistakes or trembles. As convincingly argued by Segerström (1988), if agents commit mistakes with positive probability, they will not choose risky strategies by which co-operation breaks down forever when a mistake occurs. Third, grim trigger strategies are not robust with respect to renegotiation, and under Assumption 2 it is not clear what could prevent agents from renegotiating strategies besides ownership. Once agents are allowed to renegotiate the ownership structure after co-operation breaks down, it becomes natural to think that they could try to renegotiate strategies as well (modifying agents' strategies should arguably be cheaper than renegotiating the ownership structure). Therefore, we now leave agents free to choose strategies other than grim trigger ones, and to renegotiate them along the game tree.

Assumption 3. *(a) Renegotiation of ownership structure is possible and costless ($z = 0$); (b) Renegotiation of strategies is possible and costless.*

In our formulation agents can pay transfers to each other. It is then natural to think that, when an agent defects unilaterally from agreed strategies, the other agent will require some form of compensation for the damage he incurred before agreeing to undertake any further co-operative action, such as a change of ownership structure.

Suppose agents adopt the following, natural class of asymmetric "restitution" strategies:[10]

[9] This also led Bragelien (2001) – who extends Baker et al. (2002) maintaining their assumption of ownership renegotiation and grim trigger strategies – to conclude that short term optimal asset ownership tends to be optimal for relational contracts (Proposition 5).

[10] The strategies are inspired to - but slightly different (due to monetary transfers) from those discussed in van Damme (1989), Farrell and Maskin (1989) and Segerström (1988). "Asymmetry" refers to the different behavior of the "defector" and the "afflicted party" off equilibrium, in contrast to grim trigger

Definition 4 ***(Restitution Strategies)****. Start playing from Phase 1.*

- Phase 1:

  Invest efficiently e_i^c *and pay the equilibrium transfer* θ*; if agent* i *defects, start Phase 2.*

- Phase 2:

  Agent $j \neq i$: *If at the beginning of the period you receive restitution* R^{ij} *from agent* i*, go back to Phase 1; set* $e_j = e_j^d$ *and* $\theta = 0$ *otherwise.*

  Agent i : *Pay transfer* R^{ij} *to agent* j *and go back to Phase 1.*

Restitution strategies are not only intuitively appealing. They are robust against mistakes since they prescribe a transfer between agents to compensate for the damages caused by an eventual mistake, but do not switch to inefficient play.

As for strategy renegotiation, the literature on renegotiation in repeated games did not reach a definitive agreement, and the issue remains open to debate.[11] However, the *strong perfect equilibrium* proposed early by Rubinstein (1980) satisfies all weaker properties – if it exists.[12] Let agents anticipate the problem that both strategies and ownership could be renegotiated after a defection, and ex ante agree on "maximal restitution strategies", that is on restitution strategies with a maximal transfer $\overline{R}$ (from the agent that defected to the agent damaged by the defection). The upper bound on R is the discounted payoff difference between indefinite co-operation and defection for the agent that defected. Clearly, the agent entitled to this transfer can never gain anything by renegotiating, since if ownership was chosen efficiently with respect to these strategies this is 100% of the remaining cake. That is, with maximal transfer restitution strategies

strategies where both parties (also the defector) punish. Asymmetry does not mean that players in the same role (e.g. defector) behave differently.

[11] The debate includes work by Farrell and Maskin (1989), Bernheim and Rey (1989), van Damme (1989), Pearce (1987), Asheim (1992), Bergin and MacLeod (1993), and Abreu, Pearce and Stacchetti (1993).

[12] An equilibrium is strong if no coalition of players can gain by a joint deviation. An equilibrium is a strong perfect equilibrium if this is the case for all subgames. A strong perfect equilibrium is weakly and strongly renegotiation proof since these concepts only compare to a subset of outcomes – i.e. to subgame equilibria of the given equilibrium (for weak renegotiation proofness) or to other renegotiation proof equilibria (for strong renegotiation proofness). While the weak renegotiation proof equilibrium always exists, the strong renegotiation proof equilibrium and thereby also the strong perfect equilibrium may fail to exist.

specify subgame payoffs "on the Pareto frontier" of the subgame. They imply that no single agent nor the coalition of both agents can gain anything by further (joint) deviations. Hence, these strategies establish a strong perfect equilibrium in the sense of Rubinstein (1980). We can then state the following.

Proposition 3 *Suppose Assumption 3 applies (ownership and strategies can be renegotiated). Then restitution strategies with* $R^{ij} = \overline{R}^{ij} = \frac{c_i - d_i(\omega) + \widetilde{\theta}}{1-\delta}$ *are optimal (in the sense of Abreu, 1988) and implement the co-operative investment levels in strong perfect equilibrium (Rubinstein 1980) for the same range of discount factors at which grim trigger strategies support co-operative investments in subgame perfect equilibrium under Assumption 1 (without any renegotiation).*

The proposition immediately implies the following.

Corollary 1 *Under Assumption 3 the optimal equilibrium transfer and the relational efficient ownership structures are those defined in Proposition 1 (*$\tilde{\theta}^N$ *and* Ω^N*).*

In other words, when agents can use these natural, asymmetric punishment strategies, the possibility to renegotiate ownership does not constrain agents' ability to punish defections, as it does under Garvey's and Baker et al.'s restriction to grim-trigger strategies.[13] Besides constituting a strong perfect equilibrium, maximal restitution strategies are payoff equivalent to grim-trigger or minmax punishments without renegotiation of ownership, and hence are optimal punishments in the sense of Abreu (1988). This last property guarantees, that the lower bound on discount factors supporting co-operation on the equilibrium path is the same as for grim-trigger strategies.

Consider now the case where renegotiation of strategies or ownership is not possible (Assumptions 2 and 3). Obviously, a "strong restitution" punishment phase as described in the proof of Proposition 3 can be built independently of whether renegotiation of ownership or strategies is possible. Hence Proposition 3 also implies the following.

Corollary 2 *The optimal equilibrium transfer and ownership structures identified by Proposition 1 (*$\tilde{\theta}^N$ *and* Ω^N*) are relational efficient also under Assumptions 1 and 2.*

[13] A related result is obtained within a specific two-stage model by Rosenkranz and Schmitz (2001), using the multiplicity of stage-game Nash equilibria generated by the assumption of perfectly substitutable investments.

One might worry about the robustness of maximal restitution strategies, since – as other two-phase optimal punishments – they induce a weak equilibrium (for the defecting agent) in the subgame following a defection (see Abreu 1986). But of course, agents concerned with the robustness of the punishment phase may make the equilibrium strict at all subgames by slightly reducing the size of the restitution. And this would not change the above conclusions on the relational efficient ownership, since such a reduction would have to be accorded whatever ownership structure was chosen at the beginning.

Even though these results point at the efficiency criterium identified by Proposition 1 as the most relevant to real world agents, we mentioned that there is no general consensus yet in the profession on the renegotiation of ownership and/or strategies in repeated games. And one could defend Garvey's and Baker et al.'s approach of allowing only for renegotiation of ownership and restricting agents to grim trigger strategies by noting that renegotiating away from the punishment phase of grim trigger strategies to a new co-operation equilibrium would entail building again trust between parties after trust has been unilaterally broken, and that this may be harder to achieve than a reallocation of ownership (which requires no trust given that ownership changes can be implemented contractually).[14] For this reason, in the remainder of the paper we will keep track of the consequences of both criteria for relational efficiency (and related optimal transfer): that identified in Proposition 1, relevant for cases where agents can optimally choose strategies; and that identified in Proposition 2, relevant for cases where only ownership can be renegotiated and agents are restricted to trigger strategies.

5 Ownership Rights

Our main object of interest in this article are ownership structures. So far we have not assumed any structure on ownership rights. Therefore, the basic structure and the results of this theory hold for any exogenous parameter that influences bargaining positions (or the threat point if bargaining breaks down). Besides ownership this could be any other part of the legal framework or environmental conditions etc. In this section, however, we want to be more explicit about ownership rights, since in many real situations "ownership rights" can be split up into "asset ownership" as promoted by Hart and Moore (1990).

Let A denote a set of nonhuman assets (machines, buildings, land, client lists, patents,

[14] Although it is hard to believe that real players would bargain from perfectly symmetric positions right after one has been cheated upon and the other has cheated unilaterally.

copy rights, etc.).

Definition 5 *A partition* $\omega = (A_1, A_2, A_{12})$ *is called* "two-party-ownership-structure"*. The subset* A_i *are privately owned assets of party i and* A_{12} *are jointly owned assets.*

Our interpretation of ownership is within the tradition of Hart and Moore. Ownership of an asset is defined as veto power over the asset. Joint ownership means that every owner has veto power, i.e. a jointly owned asset can only be used by consent of all owners. In contrast to agents' actions that are observable but not verifiable the ownership structure ω is observable and verifiable in court. The following definition differentiates several cases.

Definition 6 *We will call a two party ownership structure* $\omega = (A_1, A_2, A_{12})$:

1. Joint Ownership (J), *if all assets are owned jointly* $\omega^J = (\emptyset, \emptyset, A)$ *or* $A_{12} = A$ *and* $A_i = \emptyset$ *for* $i = 1, 2$;
2. Integration (I), *if one party owns all assets* $\omega^I = (A, \emptyset, \emptyset)$ *or* $A_1 = A$ *and* $A_2 = A_{12} = \emptyset$;
3. Outsourcing (O), *if there are no jointly owned assets and both parties own assets* $\omega^O = (A_1, A_2, \emptyset)$ *or* $A_{12} = \emptyset, A_i \neq \emptyset$ *for* $i = 1, 2$;
4. Mixed Ownership (M), *if there are privately owned assets for at least one party, say 1, and jointly owned assets* $\omega^M = (A_1, A_2, A_{12})$, *and* $A_1, A_{12} \neq \emptyset$.

The remainder of this article will be devoted to the question which of the previously defined ownership structures are relational efficient under different specifications of the model. At this stage we will follow the existing literature on relational contracts.

5.1 Application to Garvey and Halonen's Models

Garvey (1995) and Halonen (2002) are subspecifications of the following structure. The functions $Q(e), P_i(e, \omega)$ are linear in both agents' actions. More specifically, $E_i = R_+$, $Q(e_1, e_2) = q_1 e_1 + q_2 e_2$ and $P_i(e, \omega) = p_i e_i + r_{-i} e_{-i}$ with $p_i + r_i \leq q_1 + q_2$ where $p_i, q_i, r_i \in R_+$. Thereby

$$u_i(e, (\lambda_1, \lambda_2)) = \frac{1}{2}\left((q_i + \lambda_i) e_i + (q_{-i} - \lambda_{-i}) e_{-i}\right) - C_i(e_i)$$

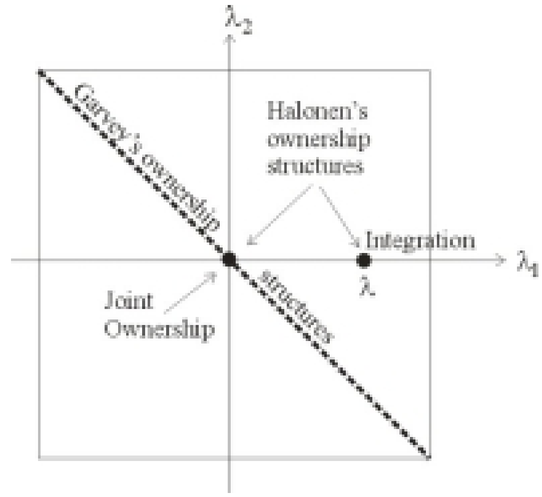

Figure 1: Garvey's and Halonen's ownership structures in a λ_1, λ_2-diagram.

where $\lambda_i \equiv p_i - r_i \in [-q_i, q_i]$ for $i = 1, 2$ and cost functions are power functions given as $C_i(e_i) = k_i e_i^\gamma$. Hence, ownership structures Ω can be parametrized as subsets of 2-vectors

$$\omega = (\lambda_1, \lambda_2) \in \Omega \subset [-q_i, q_i] \subset R^n.$$

The investigated set of ownership structures is in these examples only a finite or one-dimensional subset of $[-q_i, q_i]$. Figure 1 illuminates Garvey's and Halonen's ownership structures.

5.1.1 Garvey's Model

Garvey specifies $Q(e_1, e_2) = e_1 + e_2,\ P_i(e, \omega) = \rho_i(e_1 + e_2)$ with $\rho_1 = \rho, \rho_2 = 1 - \rho$ and $C_i(e_i) = \frac{1}{2\alpha_i} e_i^2$ with $\alpha_1 = \alpha, \alpha_2 = 1 - \alpha$ and thereby investigates a continuum of ownership structures (see figure 1) parametrized by ρ. Applying Propositions 1 and 2 and Corrolary 2 immediately yields that Garvey's main result breaks down.

Proposition 4 *Suppose agents are free to chgoose equilibrium strategies. Then in Garvey's specification, independent of what can be renegotiated, ownership structure is irrelevant for relational efficiency: $\Omega^N = \Omega$.*

If instead agents are restricted to support co-operation by grim trigger strategies and ownership can be renegotiated (Assumption 2), then the short term efficient ownership *structure and the constrained relational efficient ownership structure coincide:*

$$\begin{aligned}\rho^* &= \rho^R = \alpha \\ \Omega^* &= \Omega^R = \{\alpha\}.\end{aligned}$$

A remark is here in order.

Remark 2 *Why does our reformulation contradict the original paper by Garvey? In Garvey's formulation the transfer θ is exogenously defined by*

$$\begin{aligned}c_1 + \theta &= \rho(c_1 + c_2) \;\; \textit{or} \\ \theta &= \rho c_2 - (1-\rho) c_1 \neq \tilde{\theta}.\end{aligned}$$

This exogenous restriction is the reason behind Garvey's mistaken conclusion: the corresponding transfer payment is not relational efficient.

The next example, however, will show that ownership may matter in a relation unless cost functions are quadratic (which is the only case considered by Garvey).

5.1.2 Halonen's Model

In contrast to Garvey, Halonen only compares joint ownership ω^J with full integration ω^I (see figure 1).

Lemma 1 *The short term efficient ownership structure defined by (3) is given by full integration: $\omega^* = \omega^I$.*

Applying propositions 1–3 to her specification yields the following results, which confirm and generalize Halonen's observation that joint ownership may be optimal in a dynamic investment relation.

Proposition 5 *Suppose agents can choose optimally equilibrium strategies. Then independent of what can be renegotiated, in Halonen's specification the relational efficient ownership structure is*

$$\Omega^N = \begin{cases} \omega^I & \textit{for} \;\; \gamma \in (1,2) \\ \{\omega^J, \omega^I\} & \textit{for} \;\; \gamma = 2 \\ \omega^J & \textit{for} \;\; \gamma > 2 \end{cases},$$

where $C_i(e_i) = e_i^\gamma$ with $\gamma > 1$.

If instead agents are restricted to use grim trigger strategies and ownership can be renegotiated (Assumption 2), then the constrained relational efficient ownership structure is full integration: $\omega^R = \omega^I$.

Note that for quadratic cost functions ($\gamma = 2$) ownership is irrelevant in the general case, which is in line with our reformulation of Garvey's (1995) model.

5.2 Application to Baker, Gibbons and Murphy's Model

Baker, Gibbons and Murphy only compare outsourcing ω^O with full integration ω^I (there named "employment") that is $\Omega = \left\{\omega^O, \omega^I\right\} = \{1, 0\}$. In their specification only one party (interpreted as upstream party) can invest in the joint project.[15] Halonen (2002) and proposition 5 of this paper tought us that quadratic cost functions are special in the sense that among power functions only for quadratic cost functions the ownership structure is irrelevant if renegotiation is possible. Unfortunately, Baker, Gibbons and Murphy (as Garvey) restrict much of their analysis to this case. The corresponding specifications in our formulation are

$$\begin{aligned}
e_1 &= \left\{ \begin{array}{c} a = (a_1, a_2) \in R_+^2, \text{ a vector} \\ \text{interpreted as"multi-task actions"} \end{array} \right\} \text{upstream party} \\
e_2 &= \emptyset \text{ downstream party has no action in the investment stage} \\
C_1(a) &= \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 \\
C_2 &= 0 \\
Q(a) &= Q(a) = Q_L + (q_1 a_1 + q_2 a_2)\,\Delta Q \text{ with } \Delta Q = Q_H - Q_L > 0 \\
P_1(a, \omega) &= \omega\left(P_L + (p_1 a_1 + p_2 a_2)\,\Delta P\right) \text{ with } \Delta P = P_H - P_L > 0 \\
&= \left\{ \begin{array}{cl} P_L + (p_1 a_1 + p_2 a_2)\,\Delta P & \text{for } \omega = \omega^O = 1 \\ 0 & \text{for } \omega = \omega^I = 0 \end{array} \right.
\end{aligned}$$

[15] However, this agent may have multi-task actions available. For example one task may be beneficial for the joint project and another task may mainly promote this agent's bargaining position. As Baker Gibbons and Murphy we concentrate on the case $n = 2$ tasks.

and utility functions (1) become

$$\begin{aligned}
u_1(a,\omega) &= \frac{1}{2}\left(Q(a)+P_1(a,\omega)\right)-C_1(a) \\
&= \frac{1}{2}\left(Q_L+(q_1a_1+q_2a_2)\,\Delta Q+\omega\left(P_L+(p_1a_1+p_2a_2)\,\Delta P\right)\right)-\frac{1}{2}a_1^2-\frac{1}{2}a_2^2 \\
u_2(a,\omega) &= \frac{1}{2}\left(Q(a)-P_1(a,\omega)\right) \\
&= \frac{1}{2}\left(Q_L+(q_1a_1+q_2a_2)\,\Delta Q-\omega\left(P_L+(p_1a_1+p_2a_2)\,\Delta P\right)\right).
\end{aligned}$$

This implies

$$\begin{aligned}
e_1^c &= (q_1\Delta Q, q_2\Delta Q)\,, \\
e_1^d(\omega) &= \left(\frac{1}{2}q_1\Delta Q+\frac{1}{2}\omega p_1\Delta P, \frac{1}{2}q_2\Delta Q+\frac{1}{2}\omega p_2\Delta P\right).
\end{aligned}$$

Obviously, with no transfer payment there is no incentive to invest co-operatively for the upstream party. The downstream party can co-operate or defect by paying the transfer or not paying it. It is helpful to consider the reduced stage game payoff matrix (7) in this modified "investment-bonus-payment game". It is given as

$\Gamma^R(\omega)$	pay bonus θ	no bonus payment
e_1^c	$c_2-\theta$ $c_1+\theta$	$b_2=c_2$ $a_1=c_1$
$e_1^d(\omega)$	a_2 b_1	$d_2=a_2$ $d_1=b_1$

We can confirm Baker, Gibbons and Murphy's result (result 1 in their section IV, B) on what they call "spot outsourcing" versus "spot employment".

Lemma 2 *The short term efficient ownership structure is given by*

$$\omega^*=\begin{cases}\omega^I & \textit{for} \quad \Delta P>\Delta P^*:=\frac{2(p_1q_1+p_2q_2)}{p_1^2+p_2^2}\Delta Q \\ \omega^O & \textit{for} \quad \Delta P<\Delta P^*\end{cases}.$$

Considering relational efficiency with quadratic cost functions we are not surprised anymore by the following proposition, which does not confirm Baker, Gibbons and Murphy's claims once the restriction on grim-trigger strategies is removed. In particular, if parties are free in their strategy choice their "main proposition" (at the end of their section III, p.56) stating that asset ownership affects reneging temptations does not hold for quadratic cost functions.

Proposition 6 *As long as agents can choose optimal equilibrium strategies, independent of what can be renegotiated, ownership structure is irrelevant for relational efficiency:* $\Omega^N = \{\omega^O, \omega^I\}$.

When agents are restricted to use grim trigger strategies and ownership can be renegotiated (Assumption 2), the relational efficient ownership structure coincides with the short term efficient one:

$$\omega^R = \omega^I = \begin{cases} \omega^I & \text{for} \quad \Delta P > \Delta P^* := \frac{2(p_1 q_1 + p_2 q_2)\Delta Q}{p_1^2 + p_2^2} \\ \omega^O & \text{for} \quad \Delta P < \Delta P^* \end{cases}.$$

While being qualitatively similar the second part of our proposition also differs from Baker, Gibbons and Murphy's results (see their figure II, p. 64) because their bonus payment assumption on their p. 62 differs from the relational efficient transfer payments derived in our proposition 2. Also, Baker, Gibbons and Murphy assume that for full integration (employment) the upstream party does not invest, which amounts to assuming that the upstream party has no bargaining power in the subsequent division of the pie. We consider it more coherent to use everywhere the same bargaining rule (we always assume the Nash-solution or equal split).

6 Conclusion

We developed a general model for the analysis of optimal allocations of property rights in long term relations, where investment levels are non contractible and must be sustained in equilibrium. Applying more sophisticated elements of repeated game analysis to our framework revealed two main weaknesses of the previous literature on the subject. First, the restriction to grim trigger strategies, while technically convenient, is not only objectionable from a theoretical viewpoint since such strategies are not robust against mistakes and renegotiation. Rather, our analysis shows that this assumption turns many of the main conclusions upside down. Second, in line with the prevailing tradition in contract theory previous research has devoted much attention to renegotiation of ownership, but no attention to the possibility to renegotiate strategies. Apparently, this point has been neglected since there has not emerged yet a game-theoretical consensus on a concept that always exists. However, in the present context the "strong perfect equilibrium" proposed by Rubinstein (1980) exists and entails all other desirable properties. Opening the analysis to optimal punishment strategies robust against mistakes and renegotiation reinvigorates results of models that exclude renegotiation (e.g. Halonen

2002), since we show that the same ownership structures are relational efficient without any renegotiation and with renegotiation of both, ownership and strategies. Our results suggest that in a dynamic world the optimal allocation of property rights is generically different from the one identified by the Grossman-Hart-Moore paradigm.

7 Appendix: Proofs

Proposition 1: Proof. Incentive compatibility for trigger strategies is given by

$$\frac{1}{1-\delta}\left(c_i(\omega)+\theta_i\right) \geq b_i(\omega)+\frac{\delta}{1-\delta}d_i(\omega)$$

or

$$\underline{\delta}_i = \frac{b_i(\omega)-c_i(\omega)-\theta_i}{b_i(\omega)-d_i(\omega)}$$

and

$$\underline{\delta} = \max\left\{\underline{\delta}_1, \underline{\delta}_2\right\}.$$

The "optimal sharing rule" $\tilde{\theta}$ minimizes $\underline{\delta}$ and hence satisfies

$$\frac{b_1(\omega)-c_1(\omega)-\tilde{\theta}}{b_1(\omega)-d_1(\omega)} = \frac{b_2(\omega)-c_2(\omega)+\tilde{\theta}}{b_2(\omega)-d_2(\omega)}$$

this yields

$$\tilde{\theta} = \frac{\left(b_1(\omega)-c_1(\omega)\right)\left(b_2(\omega)-d_2(\omega)\right)-\left(b_2(\omega)-c_2(\omega)\right)\left(b_1(\omega)-d_1(\omega)\right)}{b_2(\omega)-d_2(\omega)+b_1(\omega)-d_1(\omega)} \tag{8}$$

and

$$\underline{\delta} = \frac{b_1(\omega)-c_1(\omega)+b_2(\omega)-c_2(\omega)}{b_1(\omega)-d_1(\omega)+b_2(\omega)-d_2(\omega)} \tag{9}$$

or

$$\underline{\delta}^N(\omega) = \frac{B(\omega)-\pi^*}{B(\omega)-D(\omega)} \tag{10}$$

if $\underline{\delta} \leq 1$. ■

Proposition 2: Proof. Incentive compatibility for trigger strategies

$$\frac{1}{1-\delta}\left(c_i\left(\omega\right)+\theta_i\right)\geq b_i\left(\omega\right)+\frac{\delta}{1-\delta}\left(d_i\left(\omega\right)+S\right)$$

yields

$$\underline{\delta}_i=\frac{b_i\left(\omega\right)-c_i\left(e^c,\omega\right)-\theta_i}{b_i\left(\omega\right)-d_i\left(\omega\right)-S}$$

and

$$\underline{\delta}=\max\left\{\underline{\delta}_1,\underline{\delta}_2\right\}$$

The "optimal sharing rule" $\tilde{\theta}$ minimizes $\underline{\delta}$ and hence satisfies

$$\frac{b_1\left(\omega\right)-c_1\left(\omega\right)-\tilde{\theta}}{b_1\left(\omega\right)-d_1\left(\omega\right)-S}=\frac{b_2\left(\omega\right)-c_2\left(\omega\right)+\tilde{\theta}}{b_2\left(\omega\right)-d_2\left(\omega\right)-S}$$

this yields

$$\tilde{\theta}=\frac{\left(b_1\left(\omega\right)-c_1\left(\omega\right)\right)\left(b_2\left(\omega\right)-d_2\left(\omega\right)-S\right)-\left(b_2\left(\omega\right)-c_2\left(\omega\right)\right)\left(b_1\left(\omega\right)-d_1\left(\omega\right)-S\right)}{b_1\left(\omega\right)-d_1\left(\omega\right)+b_2\left(\omega\right)-d_2\left(\omega\right)-2S}$$

and

$$\begin{aligned}
\underline{\delta}^R &= \frac{b_1\left(\omega\right)-c_1\left(\omega\right)-\tilde{\theta}}{b_1\left(\omega\right)-d_1\left(\omega\right)-S}\\
&= \frac{b_1\left(\omega\right)-c_1\left(\omega\right)-\frac{\left(b_1(\omega)-c_1(\omega)\right)\left(b_2(\omega)-d_2(\omega)-S\right)-\left(b_2(\omega)-c_2(\omega)\right)\left(b_1(\omega)-d_1(\omega)-S\right)}{b_1(\omega)-d_1(\omega)+b_2(\omega)-d_2(\omega)-2S}}{b_1\left(\omega\right)-d_1\left(\omega\right)-S}\\
&= \frac{b_1\left(\omega\right)-c_1\left(\omega\right)+b_2\left(\omega\right)-c_2\left(\omega\right)}{b_1\left(\omega\right)-d_1\left(\omega\right)+b_2\left(\omega\right)-d_2\left(\omega\right)-2S}\\
\underline{\delta}^R &= \frac{b_1\left(\omega\right)-c_1\left(\omega\right)+b_2\left(\omega\right)-c_2\left(\omega\right)}{b_1\left(\omega\right)-d_1^*+b_2\left(\omega\right)-d_2^*}
\end{aligned}$$

or

$$\underline{\delta}^R\left(\omega\right)=\frac{B\left(\omega\right)-\pi^*}{B\left(\omega\right)-D^*}. \tag{11}$$

$\underline{\delta}^R\left(\omega\right)$ strictly increases with $B\left(\omega\right)$ since $\pi^*>D^*$. This proves the proposition. ■

Proposition 3: Proof. We first show that restitution strategies with maximal R^{ij} constitute a subgame perfect Nash equilibrium for the same discount factors at which co-operation supported by grim-trigger strategies is an equilibrium. First note that defecting during Phase 2 does not increase agent $i's$ continuation payoff, hence when restitution strategies constitute an equilibrium, the equilibrium is subgame perfect. Now note that for any given equilibrium transfer θ, if agent i defects unilaterally from restitution strategies with $R^{ij} = \frac{c_i - d_i(\omega) + \widetilde{\theta}^{ij}}{1-\delta}$, he expects $u_i(e_i^d(\omega), e_j^c)$ the period of the defection and $\frac{\delta}{1-\delta} u_i(e_i^d(\omega), e_j^d(\omega))$ from the rest of the game. Then a defection is deterred when $u_i(e_i^c, e_j^c) + \theta \geq \delta u_i(e_i^d(\omega), e_j^c) + (1-\delta) u_i(e_i^d(\omega), e_j^d(\omega))$, which is exactly the condition relevant with grim trigger strategies and no renegotiation of ownership.

We now show that the equilibria in restitution strategies with maximal R^{ij} are strong perfect equilibria. Suppose $\omega \in \Omega^N$ (defined in proposition 1). After a defection by i, before the punishment phase starts i can propose j either to play alternative equilibrium strategies with the same ownership structure $\omega \in \Omega^N$, or to modify both ownership structure and equilibrium strategies. However, in the subgame after agent i defects from the given equilibrium, agent j's subgame equilibrium payoff at the beginning of phase 2 is the entire remaining net surplus from the relation with efficient investment. Since $\omega \in \Omega^N$, this is strictly greater than any payoff he could obtain by renegotiating strategies and/or ownership using Nash bargaining, hence – even though $z = 0$ – renegotiation of ownership or/and of strategies cannot occur. ■

Proposition 4: **Proof.** The utility function (1) becomes

$$u_i(e, \rho) = \rho_i (e_1 + e_2) - \frac{1}{2\alpha_i} e_i^2$$

This yields

$$\begin{aligned} e_i^c &= \alpha_i \\ e_i^d &= \rho_i \alpha_i \end{aligned}$$

and for Garvey's specification the crucial parameters (4) of the stage game are given by

$$\begin{aligned} b_i &= \frac{1}{2}\rho_i^2 \alpha_i + \rho_i (1 - \alpha_i) \\ c_i &= \rho_i - \frac{1}{2}\alpha_i \\ d_i &= \frac{1}{2}\rho_i^2 \alpha_i + \rho_i (1 - \rho_i)(1 - \alpha_i) \\ \pi^* &= \frac{1}{2} \\ B(\rho) &= \frac{1}{2}\left(1 + \alpha + \rho^2\right) - \rho\alpha \\ D(\rho) &= \frac{1}{2}\left(1 - \alpha - \rho^2\right) + \rho\alpha \end{aligned} \tag{12}$$

By plugging in parameters (12) equation (10) becomes

$$\begin{aligned}\underline{\delta}^N(\rho) &= \frac{B(\rho)-\pi^*}{B(\rho)-D(\rho)} \\ &= \frac{1}{2}\end{aligned}$$

which implies the first claim, $\Omega^N = \Omega$. Apply definition (3) and proposition 2 to parameters (12), and the second claim obtains. ■

Lemma 1: Proof. The corresponding specifications within our framework are $n = 1$, $q_1 = q_2 = 1$, $p_2 = r_1 = r_2 = 0$, $p_1 = \omega$

$$\begin{aligned}\omega &\in \Omega = \left\{\omega^J, \omega^I\right\} = \{0, \lambda\} \\ \omega^J &= 0 = \text{Joint Ownership} \\ \omega^I &= \lambda = \text{Integration with } \lambda \in [0,1] \\ E_i &= R_+ \\ Q(e_1, e_2) &= e_1 + e_2 \\ P_1(e, \omega) &= \omega e_1 \\ P_2(e, \omega) &= 0\end{aligned}$$

and for the cost structure $k_1 = 1, \gamma > 1$ or

$$C_i(e_i) = e_i^\gamma \text{ with } \gamma > 1$$

and thereby

$$\begin{aligned}u_1(e, \omega) &= \frac{1}{2}\left((1+\omega)e_1 + e_2\right) - e_1^\gamma, \\ u_2(e, \omega) &= \frac{1}{2}\left((1-\omega)e_1 + e_2\right) - e_2^\gamma.\end{aligned}$$

This yields

$$\begin{aligned}e_i^c &= \gamma^{\frac{1}{\gamma-1}} \text{ for } i = 1, 2, \\ e_1^d &= \left(\left(\frac{1+\omega}{2}\right)\gamma\right)^{\frac{1}{\gamma-1}}, \\ e_2^d &= \left(\frac{\gamma}{2}\right)^{\frac{1}{\gamma-1}},\end{aligned}$$

and the stage game parameters (4) are

$$\begin{aligned}
b_1 &= \frac{\gamma-1}{\gamma}\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}}+\frac{1}{2}\gamma^{\frac{1}{\gamma-1}}, &(13)\\
b_2 &= \frac{\gamma-1}{\gamma}\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}}+\frac{1-\omega}{2}\gamma^{\frac{1}{\gamma-1}}, &(14)\\
c_1 &= \left(\frac{1+\omega}{2}-\frac{1}{\gamma}\right)\gamma^{\frac{1}{\gamma-1}}+\frac{1}{2}\gamma^{\frac{1}{\gamma-1}}, &(15)\\
c_2 &= \left(\frac{1}{2}-\frac{1}{\gamma}\right)\gamma^{\frac{1}{\gamma-1}}+\frac{1-\omega}{2}\gamma^{\frac{1}{\gamma-1}}, &(16)\\
d_1 &= \frac{\gamma-1}{\gamma}\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}}+\frac{1}{2}^{\frac{\gamma}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}}, &(17)\\
d_2 &= \frac{\gamma-1}{\gamma}\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}}+\frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}\gamma^{\frac{1}{\gamma-1}},\\
\pi^* &= \frac{2(\gamma-1)}{\gamma}\gamma^{\frac{1}{\gamma-1}},\\
B(\omega) &= \left(\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}}+\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right)+\frac{2-\omega}{2}\right)\gamma^{\frac{1}{\gamma-1}},\\
D(\omega) &= \left(\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}}+\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right)+\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}+\frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}\right)\gamma^{\frac{1}{\gamma-1}}.
\end{aligned}$$

The lemma follows from $D(\omega)$ being increasing with ω:

$$\begin{aligned}
D'(\omega) &= \frac{d}{d\omega}\left(\begin{array}{c}\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}}+\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right)\\ +\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}+\frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}\end{array}\right)\gamma^{\frac{1}{\gamma-1}}\\
&= \gamma^{\frac{1}{\gamma-1}}\left(\begin{array}{c}\frac{1}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}-\frac{1}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}\\ +\frac{1}{2}\frac{1-\omega}{2}\frac{1}{\gamma-1}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}-1}\end{array}\right)\\
&= \gamma^{\frac{1}{\gamma-1}}\left(\frac{1}{2}\frac{1-\omega}{2}\frac{1}{\gamma-1}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}-1}\right)\\
&> 0 \text{ for } \omega<1
\end{aligned}$$

■

Proposition 5: Proof. For the parameters (13) equation (10) becomes

$$\begin{aligned}
\underline{\delta}^N(\omega) &= \frac{B(\omega)-\pi^*}{B(\omega)-D(\omega)}\\
&= \frac{\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}}+\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right)+\frac{2-\omega}{2}-\frac{2(\gamma-1)}{\gamma}}{\frac{2-\omega}{2}-\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}-\frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}}
\end{aligned}$$

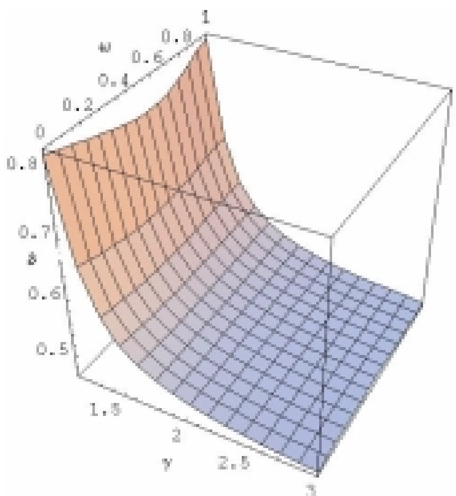

Figure 2:

Figure 2 (and the corresponding projections for $\gamma < 2, \gamma = 2, \gamma > 2$) show $\underline{\delta}^N(\omega, \gamma)$ and confirm the results as stated in Halonen. For the second statement, apply proposition 2 and verify that $B(\omega)$ decreases with ω:

$$\begin{aligned} B'(\omega) &= \frac{d}{d\omega}\left(\frac{\gamma - 1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}} + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right) + \frac{2-\omega}{2}\right)\gamma^{\frac{1}{\gamma-1}} \\ &= \frac{1}{2}\gamma^{\frac{1}{\gamma-1}}\left(\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}} - 1\right) \\ &< 0 \text{ for } \omega < 1. \end{aligned}$$

■

Lemma 2: Proof. Remember that $\omega \in \{0,1\}$, hence $\omega = \omega^2$. Hence, stage game parameters (4) are

$$\begin{aligned}
b_1 &= d_1 = \frac{1}{2}\left(\begin{array}{c} Q_L + \frac{1}{4}\left(\left(q_1^2+q_2^2\right)\Delta Q^2\right) \\ +\omega\left[P_L + \frac{1}{4}\left(p_1^2+p_2^2\right)\Delta P^2 + \frac{1}{2}\left(p_1q_1+p_2q_2\right)\Delta Q \Delta P\right] \end{array}\right), \\
c_1 &= \frac{1}{2}\left[Q_L + \omega\left(P_L + \left(p_1q_1+p_2q_2\right)\Delta Q \Delta P\right)\right], \\
c_2 &= b_2 = \frac{1}{2}\left[Q_L + \left(q_1^2+q_2^2\right)\Delta Q^2 - \omega\left(P_L + \left(p_1q_1+p_2q_2\right)\Delta Q\Delta P\right)\right], \\
d_2 &= \frac{1}{2}\left(\left(Q_L + \frac{1}{2}\left(q_1^2+q_2^2\right)\Delta Q^2\right) - \omega\left(P_L + \frac{1}{2}\left(p_1^2+p_2^2\right)\Delta P^2\right)\right), \\
\pi^* &= Q_L + \frac{1}{2}\left(q_1^2+q_2^2\right)\Delta Q^2, \\
B(\omega) &= Q_L + \frac{1}{8}\left[5\left(\left(q_1^2+q_2^2\right)\Delta Q^2\right) + \omega\left(\left(p_1^2+p_2^2\right)\Delta P^2 - 2\left(p_1q_1+p_2q_2\right)\Delta Q\Delta P\right)\right], \\
D(\omega) &= Q_L + \frac{1}{8}\left[3\left(\left(q_1^2+q_2^2\right)\Delta Q^2\right) - \omega\left(\left(p_1^2+p_2^2\right)\Delta P^2 - 2\left(p_1q_1+p_2q_2\right)\Delta Q\Delta P\right)\right].
\end{aligned}$$

By definition the short term efficient ownership structure maximizes $D(\omega)$ and therefore depends on the sign of $\left(p_1^2+p_2^2\right)\Delta P - 2\left(p_1q_1+p_2q_2\right)\Delta Q$ which yields the lemma. ■

Proposition 6: Proof. By applying proposition 1 to the present specification we obtain

$$\begin{aligned}
\underline{\delta}^N(\omega) &= \frac{B(\omega)-\pi^*}{B(\omega)-D(\omega)} \\
&= \frac{\left(q_1^2+q_2^2\right)\Delta Q^2 + \omega\left(\left(p_1^2+p_2^2\right)\Delta P^2 - 2\left(p_1q_1+p_2q_2\right)\Delta Q\Delta P\right)}{2\left(\left(q_1^2+q_2^2\right)\Delta Q^2\right) + 2\omega\left(\left(p_1^2+p_2^2\right)\Delta P^2 - 2\left(p_1q_1+p_2q_2\right)\Delta Q\Delta P\right)} \\
&= \frac{1}{2}
\end{aligned}$$

which does not depend on ω. For the second claim, to minimize $B(\omega)$ again depends on the sign of $\left(p_1^2+p_2^2\right)\Delta P - 2\left(p_1q_1+p_2q_2\right)\Delta Q$. ■

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