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Heiko Gerlach, Thomas Roende
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Heiko Gerlach, CERAS-ENPC
Thomas Roende, Universität Mannheim and CEPR
Konrad O Stahl, Universität Mannheim and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Market and Technical Risk in R&D*

We endogenize the market risk (at given technical risk) in firms' R&D decisions by introducing stochastic R&D in the Hotelling model. It is shown that if the technical risk is sufficiently high, the market risk remains low even if firms pursue similar projects. This leads firms to focus on the most valuable market segment. We then also endogenize technical risk by allowing firms to choose their R&D technology. In equilibrium, firms either pursue similar (different) R&D projects with risky (safe) technologies or they choose the same project but apply different R&D technologies. We show that R&D spillovers lead to more differentiated R&D efforts and patent protection to less. Project coordination within a RJV implies more differentiation, and may, or may not be welfare-improving.

JEL Classification: D81, L13 and O32
Keywords: market risk, R&D project choice and technical risk

Heiko A. Gerlach
CERAS-ENPC
28, Rue des Saints-Peres
75343 Paris Cedex 07
FRANCE
Tel: (33 1) 44 58 28 81
Fax: (33 1) 44 58 28 80
Email: heiko.gerlach@enpc.fr

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Thomas Roende
Department of Economics
Universität Mannheim
D-68131 Mannheim
GERMANY
Tel: (49 621) 181 1871
Fax: (49 621) 181 1874
Email: roende@econ.uni-mannheim.de

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Konrad O Stahl
Department of Economics
Universität Mannheim
D-68131 Mannheim
GERMANY
Tel: (49 621) 181 1875
Fax: (49 621) 181 1874
Email: konrad@econ.uni-mannheim.de

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=103825

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1 Introduction

It is reported from the pharmaceutical industry that the next ten years could yield an astonishing 4000 to 5000 different types of new drugs due to new research techniques. Nevertheless, according to the Financial Times, the top 50 companies in that industry concentrate over 70 per cent of their R&D expenditure in as few as 20 so-called blockbuster drugs.[1,2] In this paper we study the strategic forces that lead firms in R&D intensive industries to concentrate in such profitable market segments, and thereby forego opportunities in less competitive niche segments. We argue that the focus on blockbuster innovations stems from the stochasticity inherent in R&D activities. That stochasticity is particularly important in the pharmaceutical industry: only one or two compounds out of 10,000 evaluated ever get as far as getting a licence.[3]

In the business literature on R&D portfolio theory, it is common to distinguish between two dimensions of risks. *Technical risk* referring to the probability that a development project eventually turns into a marketable product, and *market risk* referring to the probability of meeting a competitor in a targeted market segment once the product is developed. Analytically, a firm's market risk consists of two elements: the probability that a competitor develops a product (which is in fact the technical risk taken by him), and the probability that this product is in a similar market segment. We show that if the technical risk of the firms in an industry is sufficiently high, the market risk remains low even if the firms are pursuing similar projects, and this leads firms to focus on the most valuable market segment. The notions of market and technical risks are useful for analyzing R&D choices. Yet these risks should be considered as endogenous. To the best of our knowledge, ours is the first attempt to endogenize both of these risk dimensions in one model.

Towards motivating our approach by a more specific example, consider the ongoing race to bring a new type of cancer treatment to the market.

[1] see *The Financial Times*, January 3, 1998, page 18 (London Edition).

[2] This is confirmed in a more recent report by Ansell and Sparker (1999). In the report, R&D in the pharmaceutical industry is divided into 14 therapeutic areas. On average, the top 50 firms are active in 5 of the 6 most popular therapeutic areas, but in less than 2 of the remaining 8 areas. Furthermore, 96 percent of the 50 biggest pharmaceutical companies concentrate on research in cardiovascular treatment.

[3] *BBC News Online*, Nov 21*, 2000:* "Drugs - a high-risk business" (available at *http://news.bbc.co.uk*).

The drugs, EGFR inhibitors, do not cure cancer, but stop cancer cells from growing and spreading to other parts of the body.[4] Three drugs have made it to the final trial phase and are potentially valuable weapons in the battle against cancer. Interestingly, the firms involved have chosen different drug designs. AstraZeneca and OSI Pharmaceuticals have developed drugs (Iressa and Tarceva, respectively) that are more convenient to use than ImClone Systems' Erbitux, as they can be taken in pill-form instead of being injected. This is likely to make them the first choice for patients with lung cancer, the most common cancer form (indeed, AstraZeneca and OSI Pharmaceuticals are primarily targeting this group in their trials). Erbitux, on the other hand, is less likely to cause diarrhea as a side effect, which makes it more suitable against colon cancer, a considerably less valuable market segment. This example is supposed to highlight two points: First, product differentiation matters; here, products differ by side effects. Second, there is a tendency for firms to agglomerate in the most profitable market segment; here, that for the treatment of lung cancer.

We now sketch our modelling approach to analyze strategic behavior in such a market. Towards replicating the choice between market risks, we use Hotelling's (1929) product differentiation framework. For identical known product qualities and quadratic transportation costs, we know from d'Aspremont et al. (1979) that duopolists always choose to offer niche products at the opposite end of the market. In the above terminology, firms minimize their market risk. We amend the Hotelling framework and allow for technical risk by assuming that market entry depends on the stochastic outcome of firms' R&D activities. In our benchmark model with exogenous and symmetric R&D success probabilities, we show that this may lead to clustering around, or in the center of the market. From a welfare point of view, the duopolists excessively concentrate (disperse) if innovation probabilities are low (high).

In the second part of the paper, we take the analysis one step further and endogenize the technical risk as well. In addition to the choice of the market segment, firms can now also select a R&D technology: they can adopt either a safe project yielding a low-quality product; or a risky technology yielding, if successful, a high quality one. We show that this leads to a feedback

[4] For more information about EGFR inhibitors and the firms involved in the market see: *The Scientist*, April 1, 2002: "Closing in on Multiple Cancer Targets" (available at *http://www.the-scientist.com*); *Reuters Health Information*, October 18, 2001: " "Smart Bomb" Drugs Offer Hope in Cancer Battle" (available at *http://www.cancerpage.com*).

loop between the two dimensions of risks. More technical risk leads to more clustering, and this endogenous increase in market risk makes risky projects more profitable. Three types of equilibria may emerge. Either firms cluster and both adopt the risky R&D technology, or one firm does so whilst the other chooses the safe technology; or the firms maximally differentiate and both adopt the safe technology.

A couple of straightforward extensions of the benchmark model yield further interesting insights. In contrast to standard reasoning, R&D spillovers exercise in our model a centrifugal force on the location equilibrium. The reason is that spillovers increase the probability that firms end up in a duopoly. By contrast, patent protection fosters monopolistic outcomes and therefore constitutes an additional agglomerative force. Finally, we introduce research joint ventures in a particular and, in our view, interesting way, by allowing firms to coordinate the location of their R&D effort. Relative to uncoordinated location decisions, this leads firms to cluster less, which may, or may not contribute to welfare. While, as we will argue in detail in the concluding section of our paper, the elements of strategic behavior are *in principle* difficult to observe, the outcomes sketched above for the pharmaceutical industry are certainly in consonance with our analytic ones.

The related theoretical literature is rich in papers on clustering in product space. Our work is most closely related to Harter (1993). Like us, Harter introduces R&D into the Hotelling model. However, he only derives sufficient conditions for a clustering equilibrium to arise, whence in our benchmark model, we are able to fully characterize and evaluate all equilibrium outcomes. Furthermore, we extend our model to cover a number of issues not addressed by him. Most importantly, we analyze the effects of an endogenous choice of R&D technology. We also discuss the effects of spillovers, patent protection, and research joint ventures.

Bester (1998) and Vettas (1999) look at a situation where firms signal the quality of their products to imperfectly informed consumers, and show how this might lead to agglomeration. The authors use a Hotelling set-up, with horizontal product as well as vertical quality differentiation. In our model, consumers can perfectly observe the quality of the products, so signalling does not play a role. The effects leading to agglomeration in our model are thus quite different from those demonstrated by these authors.

Within a two dimensional Hotelling framework, Neven and Thisse (1993) and Irmen and Thisse (1998) demonstrate clustering effects in one of two dimensions of product differentiation, where one is vertical and the other horizontal, and both are horizontal, respectively. Their central point is on sufficient conditions under which (maximal) product differentiation in one dimension suffices for firms to agglomerate in the centre of the market in another dimension.

There are also models on R&D portfolio choices by authors such as Bhattacharya and Mokherjee (1986), Dasgupta and Maskin (1987), and more recently Cabral (2002). The R&D technologies considered in these papers are more general than the one we use. However, these authors do not consider product differentiation, which is our main focus. An exception is Cardon and Sasaki (1998) who consider a simple, binary choice of product characteristics in a patent race. In Cardon and Sasaki, the incentive to choose the same characteristics stems from patent protection. Our basic argument does not rely on patent protection, but we show that such protection reinforces the tendency towards clustering.

The decision to make products compatible in a network industry resembles the choice of agglomerating in product space, as the products become closer substitutes. Our work is therefore related to an early paper by Katz and Shapiro (1986) who consider compatibility choices when technological progress is stochastic. However, making the products closer substitutes leads to opposite effects in the two models. In our model, it makes product market competition tougher whereas it softens competition in Katz and Shapiro, because firms do not compete in building up a customer base.

The remainder of the paper is organized as follows: In the next section, we present our benchmark model. We determine and characterize the price equilibrium, and the location equilibrium and welfare outcomes. In Section 3, we endogenize R&D decisions by allowing firms to choose the riskiness of their R&D project. In section 4, we consider a number of smaller extensions. The concluding section serves to summarize our results and to comment on their empirical verifiability. Proofs are relegated to the appendix.

2 The Benchmark Model

We employ the standard Hotelling (1929) duopoly model in the version of d'Aspremont et al. (1979). The market area is described by the unit interval $M = [0, 1]$. There is a unit mass of consumers with locations that are uniformly distributed on M. Two firms $i = A, B$ are potentially active in this market at locations of supply denoted $a, b \in M$, $a \leq b$. Consumers buy at most one unit of the product and incur quadratic distance costs of overcoming space. Thus a consumer located at y derives net utility

$$U_A(a, q_A, p_A, y) = q_A - p_A - t(a - y)^2$$

when buying good A and a corresponding utility when buying B. q_i and p_i are the quality and the price of firm i's product, respectively. $t > 0$ reflects the degree of consumer heterogeneity or horizontal product differentiation. All variables are common knowledge.

Firms costlessly invest in R&D.[5] If firm i's project succeeds, it sells a product of quality q_i, $i = A, B$. In the benchmark model, firms succeed with the probability ρ and produce the same quality if successful, $q_A = q_B = q$. Thus, firms face the same technical risk. The firms' success probabilities are uncorrelated. Firms have no fall-back quality. If the project is unsuccessful, the firm remains inactively in the market.[6]

As to the timing, firms first simultaneously choose their locations (a, b), which together with ρ determine the market risk.[7] Then the outcomes of their R&D are realized. Finally, firms set prices simultaneously, consumers buy one of the available products, and profits are realized.

2.1 Price Equilibrium for given qualities and locations

Depending on the outcome of the R&D process after the location is fixed, the typical firm may produce and sell a product at positive quality, or it may remain inactive if unsuccessful. If successful, it may either be a monopolist or a duopolist, depending on the success or failure of its competitor.

In the present stage, the firms' locations and product qualities are known and taken as given when they set their prices. It is entirely standard to derive the monopoly and the duopoly prices, so we short cut our presentation. For future reference to be used in the extensions, we calculate equilibrium prices and profits allowing for quality differences between the firms' products.

[5] Introducing a small fixed cost of R&D would not change the results.

[6] Thus, we think of a situation where firms active in other markets explore the possibility, via engaging in R&D, of becoming active in the market under scrutiny. The possibility of a fall-back product if R&D fails is discussed in section 4.4.

[7] In a duopoly model, the concept of market risk is easy to define for independent products. It is the product of the probability that the competitor targets the same product and the probability that its R&D is successful. Once products are substitutes, it is less obvious how to define market risk as it depends on how close substitutes products are. Here, we will simply refer to market risk as being higher if for given technical risks products are closer substitutes.

2.1.1 Monopoly Pricing

Let without loss of generality only firm A located at $a \leq \frac{1}{2}$ succeed in developing a product of quality q_A. The then monopolist maximizes its profits, $pD^M(q_A, a, p)$. Under the assumption $q_A \geq 3t$ used henceforth, the monopolist covers the market by charging the price $p_A^M = q_A - t(1-a)^2$. Since under this assumption all consumers are served, the monopoly profit $\Pi^M(q_A, a)$ is equivalent to the monopoly price.

2.1.2 Duopoly Pricing

Here both firms conduct successful R&D, resulting in qualities q_A and q_B, respectively. By our assumption, the market is covered. The firms' Bertrand-Nash equilibrium prices are given in Lemma 1.

Lemma 1 *(i) For $q_A - q_B < -t(b-a)(2+a+b)$, firm B is the only firm with positive market share. The unique price equilibrium is*

$$p_A^D(a,b,q_A,q_B) = 0 \textit{ and } p_B^D(a,b,q_A,q_B) = q_B - q_A - t(b^2 - a^2). \tag{1}$$

(ii) For $-t(b-a)(2+a+b) < q_A - q_B < t(b-a)(4-a-b)$, the firms share the market. The unique price equilibrium is

$$\begin{aligned} p_A^D(a,b,q_A,q_B) &= \frac{1}{3}(q_A - q_B + t(b-a)(2+a+b)) \textit{ and} \\ p_B^D(a,b,q_A,q_B) &= \frac{1}{3}(q_B - q_A + t(b-a)(4-a-b)). \end{aligned} \tag{2}$$

(iii) For $q_A - q_B > t(b-a)(4-a-b)$, firm A is the only firm with positive market share. The unique price equilibrium is

$$p_A^D(a,b,q_A,q_B) = q_A - q_B - t(b-a)(2-a-b) \textit{ and } p_B^D(a,b,q_A,q_B) = 0. \tag{3}$$

The proof is entirely standard. Not unexpectedly, Lemma 1 demonstrates that if there are substantial quality differences relative to the transportation cost t, the low quality firm is driven out of the market by the high quality one. Using Lemma 1, we can derive the equilibrium profits. In case (i) where firm A is inactive, the profits are given as

$$\Pi_B^D(a,b,q_A,q_B) = q_B - q_A - t(b^2 - a^2) \text{ and } \Pi_A^D(a,b,q_A,q_B) = 0. \tag{4}$$

Similarly, if firm B is inactive as in case (iii), the profits are

$$\Pi_A^D(a,b,q_A,q_B) = q_A - q_B - t(b-a)(2-a-b) \text{ and } \Pi_B^D(a,b,q_A,q_B) = 0. \tag{5}$$

Finally, in the case where the firms share the market, they earn profits

$$\begin{aligned} \Pi_A^D(a,b,q_A,q_B) &= \frac{(q_A - q_B + t(b-a)(2+a+b))^2}{18t(b-a)} \text{ and} \\ \Pi_B^D(a,b,q_A,q_B) &= \frac{(q_B - q_A + t(b-a)(4-a-b))^2}{18t(b-a)}. \end{aligned} \tag{6}$$

This completes the analysis of price competition in the market place.

2.2 Location under Stochastic R&D Outcomes

2.2.1 The Equilibrium

The firms' location decisions are taken before the outcomes of their R&D efforts become known. Each of the firms innovates with probability ρ. Expected profits are

$$\begin{aligned} E(\Pi_A(a,b,q_A,q_B,\rho)) &= \rho(\rho\Pi_A^D(a,b,q,q) + (1-\rho)\Pi^M(q,a)) \text{ and} \\ E(\Pi_B(a,b,q_A,q_B,\rho)) &= \rho(\rho\Pi_B^D(a,b,q,q) + (1-\rho)\Pi^M(q,b)). \end{aligned}$$

Using p_i^M and equation (4)-(6), we obtain the first order condition for firm A when choosing its location a. It can be shown that firm A's problem is concave, so solving the first-order condition, we find the optimal location of firm A as a function of the location of firm B. The equilibrium is derived formally in Proposition 1.

Proposition 1 *Consider the choice of location in the first stage of the game.*
i) For $\rho \leq \frac{2}{3}$, the unique equilibrium locations are $a^* = b^* = \frac{1}{2}$.
ii) For $\frac{2}{3} < \rho \leq 1$, the unique equilibrium locations for $a \leq b$ are

$$a^* = Max\left\{0, \frac{1}{2} - \Gamma\right\} \text{ and } b^* = 1 - a^*,$$

where $\Gamma \equiv \frac{3(3\rho-2)}{4(3-2\rho)}$.

The equilibrium outcome can be interpreted in terms of the well-known trade off between the *demand* and the *competition* effect. The former provides incentives to move to the center of the market, and the latter away from it. d'Aspremont et al. (1979) show that if both firms are active with certainty, the competition effect dominates, so firms locate as far as possible from each other. However, in our model, the firms foresee that if they enter the market, they will only meet an active competitor with probability ρ. This weakens the competition effect but not the demand effect, as a firm benefits from a central location as an *ex-post* monopolist. Therefore, firms tend to cluster more in equilibrium.

They locate in the center of the market if the technical risk is high, so the probability of ending up in a duopoly situation is low. If the technical risk is low, the duopoly outcome becomes so likely that the competition effect starts to dominate and firms fragment in equilibrium. In that sense, an increase (decrease) in technical risk leads to an endogenous decrease (increase) in market risk. As in d'Aspremont et al., that outcome does not depend on t.

2.2.2 Welfare

Is there too much, or too little clustering in equilibrium, as compared to the locational choice of a surplus maximizing social planner? The following two observations greatly simplify the calculation of the latter. First, as the market is covered by assumption, we do not need to worry about how many consumers buy the product. Second, as consumers exercise unit demand, there is no deadweight loss due to monopoly pricing. Therefore, the welfare optimal locations are simply those that minimize consumers' expected transportation costs.

Proposition 2 *The welfare maximizing locations are:*

$$a^W = \frac{1}{2} - \Omega \textit{ and } b^W = \frac{1}{2} + \Omega \textit{ where } \Omega \equiv \frac{\rho}{4(2-\rho)}.$$

Ex-post, that is, after the R&D outcomes are realized, the optimal locations depend on whether there are one or two firms active in the market. The monopolist's welfare maximizing location is $1/2$, and the duopolists' ones are $a = \frac{1}{4}$ and $b = \frac{3}{4}$, respectively. What does this imply for the firms' *ex-ante* welfare maximizing locations, i.e., before the outcomes of the R&D efforts are revealed? Let ρ, the probability that a firm innovates, become very small. Then, if there is any innovation, the innovator will tend to be a monopolist, resulting in an optimal location close to $\frac{1}{2}$. As ρ increases, the probability that a duopoly arises increases. Therefore, the distance between the welfare maximizing locations increases up to $\frac{1}{2}$ for $\rho = 1$.

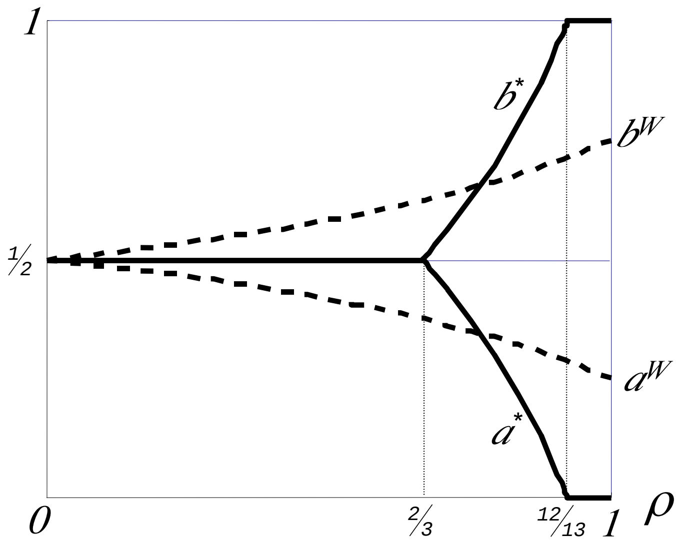

Figure 1: *Equilibrium and welfare maximising locations at varying technical risk.*

Comparing Proposition 1 and Proposition 2, we obtain immediately

Corollary 1 *There exists a unique value* $\widetilde{\rho} = 3/2 - \sqrt{15/28} \approx 0.7681$ *such that*
i) $\rho < \widetilde{\rho}$ *implies* $a^* < a^W$ $(b^* > b^W)$
ii) $\rho > \widetilde{\rho}$ *implies* $a^* > a^W$ $(b^* < b^W)$.

Figure 1 illustrates the relationship between welfare maximizing (dashed) and equilibrium (solid) locations as a function of the exogenous technical risk, ρ. There is excessive product clustering for low ρ (i.e. high technical risk) and excessive dispersion for high ρ. From a welfare point of view, firms disregard the surplus of infra-marginal consumers. It is easy to check that, *ex post*, consumer surplus is maximized if the monopolist is located at one end of the line and if duopolists are completely clustered in the center. Thus, *ex ante*, if the technical risk is high (low) and a monopoly (duopoly) outcome is more likely, the firms' marginal incentives to cluster are too strong (weak) from a social point of view.

3 Endogenous Risk-Taking in R&D

In the previous section, the firms' technical risk was assumed to be exogenous and symmetric. In the following we endogenize it and allow it to be asymmetric by considering firms to costlessly follow alternatively a 'safe' research path ('S'), or a risky and innovative path ('R'). Following the safe path, the firm develops with certainty a product of quality q and thereby minimizes its own technical risk. Instead, following the risky path, it develops with probability ρ a product of higher quality $q + \Delta$. The outcomes of risky R&D efforts are again uncorrelated. In order to keep the model manageable, we assume that the firms can only follow one of the paths. Hence, if a firm tries to develop the high quality product but fails, it cannot switch to the low risk strategy, and thus must stay inactive *ex-post*.

The firms simultaneously choose their location and R&D strategies. We already have determined the equilibrium prices for given location and qualities, so we only have to look for a Nash-equilibrium jointly in locations and R&D choices. We denote firm A's strategy by $s_A = (a, z_A)$, where a is the location and $z_A \in \{S, R\}$ is the R&D project. Firm B's strategy is denoted in the same manner.

We find the equilibrium of the game in two steps. First, we determine the reaction functions in location and the equilibrium locations for given R&D technologies. Afterwards, we determine the equilibrium of the overall game where R&D technologies and locations are chosen simultaneously.

If both firms choose the safe strategy, we know from d'Aspremont et al. that their equilibrium locations are at the opposite ends of the Hotelling line. Proposition 1 specifies the equilibrium locations conditional upon both firms choosing the risky R&D path. However, if one firm chooses the safe technology and the other the risky one, the reaction functions are highly non-linear. Towards solving the model in closed form we need to assume that $\rho \leq \frac{2}{3} \equiv \overline{\rho}$. This excludes technologies that are successful with a very high probability. Hence we concentrate our analysis on sufficiently risky R&D projects, which are anyway the more interesting ones in the present context. In the next lemma, we specify the best location that a firm with the risky technology would choose given the other firm's location and (safe) technology.

Lemma 2 *Consider a candidate equilibrium, in which one firm, say, A chooses the safe technology and locates at $a \leq \frac{1}{2}$ while B chooses the risky* *one. Then, firm B's optimal location is*

$$b^* = \begin{cases} a \text{ } if \, \Delta \geq (1-a)(6+a-3\sqrt{3+2a})t \\ 1 \text{ } if \, \Delta < (1-a)(6+a-3\sqrt{3+2a})t. \end{cases}$$

The best response for the safe technology firm is given by

Lemma 3 *Consider a candidate equilibrium, in which one firm (say, A) chooses the safe technology while the other (B) chooses the risky one. Then, firm A's optimal location is 1/2.*

The optimal locations of the two firms are substantially different for asymmetric R&D choices. The reaction function of the high quality firm exhibits an interesting discontinuity. For low quality improvements, it behaves as 'soft' as possible, and goes to the end of the line to enjoy some local monopoly power. However, once a certain threshold level of Δ is reached, it chooses the opposite strategy and behaves as 'tough' as possible. It locates then at the same place as the low quality firm, which is the most profitable way of driving the competitor out of the market. The firm with the safe technology earns much higher profits when it is alone in the market. Therefore, it chooses the location that maximizes the monopoly profits for all locations of the (potential) high quality competitor.

The next lemma summarizes the two possible candidate equilibria when firms have asymmetric R&D technologies.

Lemma 4 *The unique candidate equilibrium, in which one firm (say, A) chooses the safe technology while the other (B) chooses the risky one is* $\{(\frac{1}{2}, S), (1, R)\}$ *if* $\Delta \leq \frac{t}{4}$, *and* $\{(\frac{1}{2}, S), (\frac{1}{2}, R)\}$ *otherwise.*

We are now in the position to solve for the equilibria of the full game. For given technology choices, we have identified four candidate equilibria. These equilibria qualify as an equilibrium of the overall game if no firm has an incentive to switch unilaterally its technology (and optimally adjusts its location). Denote the possible equilibria by $\{s_A, s_B\}$ where the two entries refer to firm A's and firm B's strategies, respectively. For notational convenience, define the following threshold values

$$\begin{aligned} \Delta_1^*(\rho, t) &\equiv \frac{25t}{144\rho}, \; \Delta_2^*(\rho, t) \equiv Min\{\frac{t}{2\rho}, 3t(\frac{1}{\sqrt{\rho}} - 1)\} \text{ and} \\ \Delta_3^*(\rho, t) &\equiv \frac{(1-\rho)(4q-t)}{4\rho}. \end{aligned}$$

Proposition 3 *There exist three different types of Nash equilibria of the game:*
i) $\{(\frac{1}{2}, R), (\frac{1}{2}, R)\}$ *is the unique equilibrium if and only if* $\Delta \geq \Delta_3^*(\rho, t)$.
ii) $\{(\frac{1}{2}, S), (\frac{1}{2}, R)\}$ *and* $\{(\frac{1}{2}, R), (\frac{1}{2}, S)\}$ *are equilibria if and only if* $\Delta_1^*(\rho, t) \leq \Delta \leq \Delta_3^*(\rho, t)$.
iii) $\{(0, S), (1, S)\}$ *is an equilibrium if and only if* $\Delta \leq \Delta_2^*(\rho, t)$.

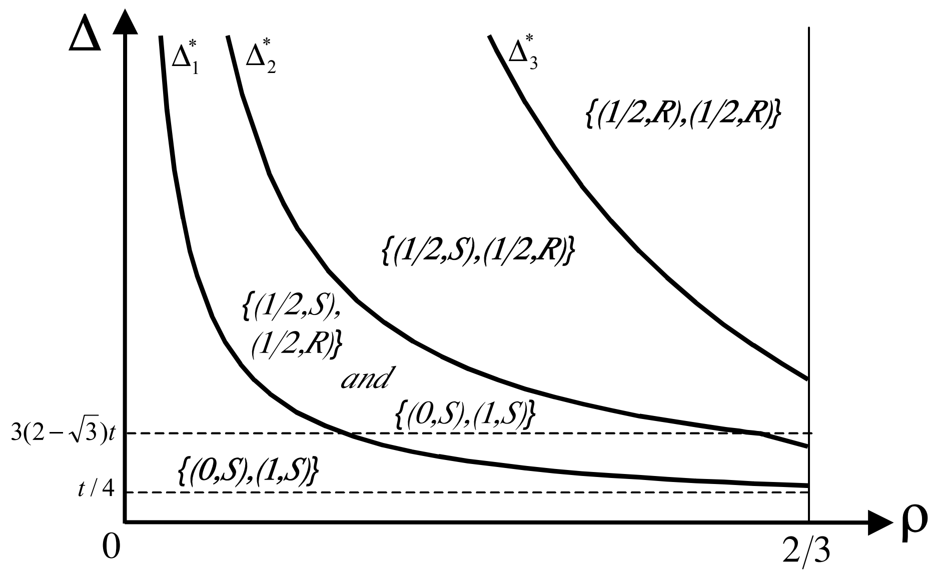

Figure 2: *Equilibrium Strategies in* $\rho - \Delta$ *Space*

Figure 2 summarizes the equilibrium outcomes. With low values of both Δ and ρ, the expected return on the risky technology is so low that the firms prefer choosing the safe technology, in which case the firms locate as far as possible from each other. However, if the innovation step is larger than $\Delta_2^*(\rho, t)$, firms have an incentive to switch unilaterally to the risky technology and differentiate maximally (minimally) for $\Delta \leq (\geq) 3(2 - \sqrt{3})t$. By contrast, if both Δ and ρ exhibit high values, the risky technology has a high expected return and is chosen by both firms. The probability of being monopolist in the market is sufficiently high that a firm only deviates to the safe technology (and staying at 1/2) if $\Delta \leq \Delta_3^*(\rho, t)$. Most interestingly, for intermediate values of Δ and ρ, we find equilibria where the firms choose different technologies and locate in the center of the market. The firm with the safe technology does not deviate to the risky technology (with optimal location at 1/2) as long as $\Delta \leq \Delta_3^*(\rho, t)$. The optimal deviation of the firm with the risky technology should be accompanied by a relocation to the end of the line. This is not beneficial for all $\Delta \geq \Delta_1^*(\rho, t)$. Finally, note that for $\Delta \leq t/4$, the fourth candidate equilibrium $\{(1/2, S), (1, R)\}$ is not an equilibrium of the overall game since the risky technology firm always deviates to the safe technology.

From this it follows that there is a non-empty subset of the parameter space consisting of $\Delta_1^*(\rho, t) \leq \Delta \leq \Delta_3^*(\rho, t)$ with two possible equilibria: $\{(0, S), (1, S)\}$ and $\{(1/2, S), (1/2, R)\}$. This multiplicity is due to the inherent interdependence between risk-taking and clustering. The more technical risk there is in the market (it suffices that one firm is taking the risk), the stronger is the incentive to cluster in the most profitable segment. And the closer both firms' R&D projects are, the more it pays to take risk, as a low quality competitor is easier to drive out of the market. This feedback loop leads to the coexistence of an equilibrium with niche projects and safe research paths and an equilibrium where firms cluster and one is taking a risky research path. It is easy to check that there is no ranking of these two equilibria in terms of firms' profits since the firm with the safe technology in the center prefers the clustering equilibrium $\{(1/2, S), (1/2, R)\}$ while the firm with the risky technology prefers the dispersion equilibrium $\{(0, S), (1, S)\}$. However, industry profits are higher in the clustering equilibrium.

These results provide an interesting link to the literature on multi-dimensional horizontal differentiation. Here, it has been shown (e.g. by Irmen and Thisse, 1998) that firms tend to differentiate maximally in one dimension and minimally in all other. This is similar to what happens in our model for low and intermediate values of Δ and ρ. For low values, firms differ maximally in locations but choose the same R&D strategy, whilst for intermediate values, locational differentiation is minimal but the R&D strategies differ. Yet this cannot happen for high values: Since consumers are willing to pay more for higher quality, both firms differentiate minimally in space, and prefer the risky technology as it has a high expected pay-off.

The risk-return trade-off in R&D has also been studied by Dasgupta and Maskin (1987) and Bhattacharya and Mookherjee (1986), among others. They used general risk-return functions. However, to make the analysis tractable, the authors restrict their attention to symmetric R&D choices by the firms in the industry. In a less general model, we demonstrate that imposing symmetry might be quite restrictive. Indeed, our results suggest that firms have incentives to differentiate themselves along the R&D dimension in order to relax *expected* competition.

The following corollary looks at the equilibrium outcome under a specific subset of the risky innovation technologies, namely all mean-preserving spreads of the safe innovation technology.

Corollary 2 *If the firms can choose between two technologies with the same mean but a different spread, i.e.* $q = \rho(q + \Delta)$*, the equilibrium outcome is* $\{(\frac{1}{2}, R), (\frac{1}{2}, R)\}$.

Hence, if the risky innovation is a mean-preserving spread of the safe innovation, the possibility of ending up as a monopolist with the high quality product is so attractive that no firm wishes to pursue the safe R&D strategy.

Unfortunately, extending the welfare evaluation does not lead to results as clear cut as those illustrated in the benchmark model. Referring to Proposition 3, there is a $\Delta_1^W(\rho, t) < \Delta_1^*(\rho, t)$ and a $\Delta_2^W(\rho, t) > \Delta_2^*(\rho, t)$ such that below $\Delta_1^W(\rho, t)$ and above $\Delta_2^W(\rho, t)$, firms both adopt the safe and the risky technology, respectively, which are the welfare maximizing R&D technology choices. However, below $\Delta_1^W(\rho, t)$ the firms excessively differentiate in product space, and above $\Delta_2^W(\rho, t)$ they excessively concentrate. In the regime between $\Delta_1^W(\rho, t)$ and $\Delta_2^W(\rho, t)$, the firms should optimally cluster in the middle and choose different R&D technologies. Thus, the interval $(\Delta_2^*(\rho, t), \Delta_3^*(\rho, t))$ contains welfare maximizing equilibrium allocations. Below this, one of the firms adopts too safe a technology and products are excessively differentiated. Above this, firms choose the welfare maximizing location in the middle, but one of the firms chooses too risky a R&D technology.[8]

4 Other Extensions

4.1 Patent Protection and Technological Spillovers

As discussed earlier, our model most appropriately reflects a situation in which the firms think of introducing a brand new type of product and creating a new market. Products of this type, such as a first effective drug against a disease, are often protected by patent laws. It is thus interesting

[8] The complete welfare analysis is contained in Gerlach et al. (2002)

to know how the above results are affected by patent protection. We assume that patents introduce an element of 'winner-takes-all' into the pay-offs. In particular, if both firms are successful in R&D, there is a probability λ that one of the firms will be granted a patent that excludes the other firm from the market. The firms are equally likely to be the lucky one awarded the patent.[9] The expected profits of firm A (and firm B symmetrically) are:

$$\rho^2(1-\lambda)\Pi_A^D(a,b) + \rho^2\lambda\Pi_A^M(a)/2 + \rho(1-\rho)\Pi_A^M(a).$$

Proposition 4 summarizes the analysis of this modified game with patent protection.

Proposition 4 *Patent protection leads to (weakly) less dispersed locations.*

The intuition behind this result is straightforward: Patent protection makes it more likely that the firms end up in a monopoly. This weakens the competition effect even more (and with it, the market risk), with the demand effect again unaffected. Thus, clustering becomes more attractive.

Following the same logic, it is clear that any force preventing the monopoly outcome to arise increases dispersion incentives. The most prominent example is technological spillovers. In the literature, spillovers are typically considered a centripetal force in models with product differentiation (e.g., Duranton, 2000; Mai and Peng, 1985). It thus seems worth highlighting the effect of spillovers on firms' locations in our framework.

Assume that if one of the firms innovates, but the other one does not, the unsuccessful firm receives a spillover that allows it to become active with probability σ.[10]

Proposition 5 *Technological spillovers lead to (weakly) more dispersed locations.*

The formal argument is analogous to the patent protection one. Spillovers decrease both firms' technical risk and increase the market risk. Clustering

[9] Under a 'first-to-file' patent system, one could imagine a sequential structure where the firms were equally likely to file the patent first. λ would be the probability that the firm holding the patent would win a patent infringement case.

[10] Such spillovers may occur due to the use of common suppliers or to employees switching job or talking informally; see Mansfield (1985).

becomes less attractive.[11] The model thus provides an interesting contrasting view to the standard argument suggesting that knowledge spillovers lead to clustering in R&D intensive industries.

Finally, notice that positive or negative correlation in R&D outcomes would have an effect similar to spillovers and patent protection, respectively.[12] Hence, a positive correlation between R&D outcomes would lead to less agglomeration in equilibrium compared to the benchmark model, and negative correlation to more agglomeration.

4.2 Research Joint Venture

Returning to the benchmark model, we assume that the firms, by forming a research joint venture (RJV), can coordinate their choice of location, but then compete in prices.[13] Calculating the locations that maximize the joint expected profits of firms yields

Proposition 6 *Consider a situation in which firms coordinate their location choices but compete in prices.*
i) For $\rho \leq \frac{1}{2}$, the unique equilibrium locations are $a^{RJV} = b^{RJV} = \frac{1}{2}$.
ii) For $\frac{1}{2} < \rho \leq 1$, the unique equilibrium locations for $a \leq b$ are

$$a^{RJV} = Max\left\{0, \frac{1}{2} - \Psi\right\} \text{ and } b^{RJV} = 1 - a^{rjv} \text{ where } \Psi \equiv \frac{2\rho - 1}{2(1-\rho)}.$$

iii) An RJV is welfare improving for $\rho \in (\frac{1}{2}, 2 - \sqrt{2})$ and welfare reducing for $\rho \in (2 - \sqrt{2}, \frac{12}{13})$.

In this semi-collusive arrangement, firms internalize the loss incurred by the competitor as they move closer together in product space. Consequently, firms always locate (weakly) further from each when they collude in locations. This explains why a RJV may be welfare improving for low values of ρ where firms are clustering too much in the non-collusive equilibrium.

[11] Actually, while the reduction in the rival's technical risk fosters differentiation, the reduction in a firm's own technical risk has no effect on project choice since this decision is taken conditional upon success in development.

[12] Yet the effects are not identical. Take the case of spillovers. With spillovers, the fact that a firm fails to innovate does not contain information about the other firm's likelihood of failing, as it would if projects were positively correlated. A positive correlation in R&D outcomes might occur for exogenous reasons (for example, the most promising research path is evident to participants in the industry) or might be a strategic choice as in Cardon and Sasaki (1998).

[13] The idea that firms collude in R&D through a research joint venture goes back to Kamien et al. (1992).

For large values of ρ, on the other hand, there is too much dispersion in equilibrium and a RJV exacerbates this tendency. Observe that this result is in sharp contrast to results à la Kamien et al. (1992), according to which RJV's always welfare dominate less co-operative equilibrium outcomes.

4.3 Endogenous Costly R&D Intensity

As a third extension, consider now the continuous choice of R&D intensity. Instead of choosing the risk profile of the R&D activities, firms decide on the technical risk by investing in the probability of success, ρ_i, for an innovation of given size q.[14] For quadratic R&D costs, $\frac{1}{2}\gamma\rho_i{}^2$, we find a symmetric equilibrium where firms choose the same R&D intensity, i.e. $\rho_A = \rho_B = \rho$. This corresponds to the equilibrium in the benchmark model, since, for a given equilibrium ρ, the location is as described in *Proposition 1*. However, the crucial parameter is now γ. If γ is high, R&D is relatively costly. Therefore, the firms choose ρ low and locate together at the center of the line. On the other hand, if γ is low, the firms choose ρ high and locate away from each other. However, unlike the benchmark model, there is the possibility of asymmetric equilibria.[15]

4.4 A Fallback Product

In the benchmark model, we have assumed that a firm can not enter the market if its R&D efforts fail. We interpret this as a situation where the firms open a new market. We have considered a variation of the benchmark model with a fallback product. It is not possible to solve the model with continuous location choice. Assuming instead that the firms can locate either at the center or at the ends of the line, the firms agglomerate at the center of the market if ρ is neither close to 0 nor to 1 and the quality improvement resulting from successful R&D is sufficiently high.

The firms prefer to locate as far as possible from each other if the products *ex-post* are of the same quality.[16] Therefore, there can only be agglom-

[14] Ideally, one would include both R&D intensity and risk choice in the same model. Unfortunately, this analysis is not tractable in our set-up.

[15] The general analysis of asymmetric equilibria is difficult because of the functional forms involved. We have, however, been able to find sufficient conditions for the existence of an asymmetric equilibrium where one firm locates at 1/2 and invests a lot in R&D whereas the other firm locates at the end of the line and invests little in R&D.

[16] Of course, a low quality firm would prefer to locate as far as possible from a high quality firm *ex-post*. However, *ex-ante* the firms are equally likely to be the high and the low quality firm. Agglomeration may thus be profitable from an *ex-ante* point of view if

eration for intermediate values of ρ where the probability of producing the same quality *ex-post* is low (first condition). The fact that the firms no longer cluster for high technical risks, i.e. low values of ρ, is the main difference to the model with no fallback option. The firms benefit from the central location, because it is easier to expand the market share when producing a higher quality than the competitor. However, competition is tougher if the firms are at the same location. Hence, it is profitable to agglomerate at 1/2 only if the quality difference between a high and a low quality firm is large enough to ensure the high quality firm a healthy profit margin (second condition).

5 Conclusions

In this paper, we analyze the interaction between firms' R&D decisions and their location choices in product space. In a benchmark model, we consider the technical risk involved in firms' R&D efforts as exogenous and endogenize the component of the market risk due to differentiation between the targeted projects. We model this by introducing stochastic R&D in the classical Hotelling model. We show that this might restore the principle of minimum differentiation even with quadratic transportation costs of consumers. The reason is that if R&D success is stochastic, a firm only meets a competitor in the product market with a certain probability and this weakens the centrifugal *competition* effect that normally dominates the centripetal *demand* effect. Comparing equilibria to welfare allocations, we obtain excessive spatial concentration if R&D success is very risky, and excessive spatial dispersion otherwise.

We then endogenize both market and technical risks by allowing firms to choose both the product characteristics and the degree of riskiness of R&D projects. We show that this leads to a feedback loop between the two dimensions of risk. More technical risk leads to more clustering, which again makes a risky project more profitable. We find three types of equilibria: (i) firms stay away from each other and choose the riskless low quality option, (ii) they cluster in the center and choose the risky technology, or (iii) they cluster in the center and differentiate themselves along the R&D dimension.

Further extensions of our benchmark model include the incorporation of patenting, R&D spillovers, and coordination of location decisions within a

the high quality firm wins more from locating together than the low quality firm loses.

research joint venture. Patent protection leads one firm to win at the cost of the other, which decreases the competitive pressure and increases clustering. In sharp contrast to extant arguments, spillovers align the firms' R&D successes and lead to more dispersion. Finally, allowing firms to coordinate their object of R&D efforts within a research joint venture brings them to differentiate more than in the uncoordinated equilibrium. Unexpectedly, this may lessen welfare rather than contribute to it.

As usual, some interesting issues must be left for future work. In particular, we have only shown how patents affect the targeted product varieties through the strategic interaction of firms. It would be interesting to analyze socially optimal patent design (in particular, patent length and breadth) taking into account the effect on the variety in the market.

It remains to comment on the observability of our results. The object of R&D is proprietary information to the firm. Before its outcome is revealed, it becomes known, at best, to its competitor(s). Most certainly it never surfaces in any publicly available statistics. In these statistics, only the outcomes of research surface, and in many R&D prone areas with heavy patenting, only the patented ones. Therefore, the strategic choices analyzed in our model are very hard to observe by outsiders. Yet we are convinced that we have modeled important aspects involved in firms' R&D choices. This conviction is reaffirmed by informal conversations with insiders into the pharmaceutical industry. After all, the outcome related data reported in the introduction of this paper are certainly not in conflict with our results.

References

[1] Ansell, J. and K. Sparkes, 1999, *R&D Portfolio Analysis: Therapeutic Focus of the Top 50 Pharmaceutical Companies*, Decision Resources, Inc.

[2] d'Aspremont, C., Gabszewicz, J. J., and J.-F. Thisse, 1979, "On Hotelling's Stability in Competition," *Econometrica*, 47, 1145-1150.

[3] Bester, H., 1998, "Quality uncertainty mitigates product differentiation," *Rand Journal of Economics*, 29, 828-844.

[4] Bhattacharya, S. and D. Mookherjee, 1986, "Portfolio choice in research and development," *Rand Journal of Economics*, 17, 594-606.

[5] Cabral, L., 2002, "Football, Sailing and R&D: Dynamic Competition with Strategic Choice of Variance and Covariance," mimeo.

[6] Cardon, J.H. and D. Sasaki, 1998, "Preemptive search and R&D clustering," *Rand Journal of Economics*, 29, 324-338.

[7] Dasgupta, P. and E. Maskin, 1987, "The Simple Economics of Research Portfolios," *The Economic Journal*, 97, 581-595.

[8] Duranton, G., 2000, "Cumulative Investment and Spillovers in the Formation of Technological Landscapes," *Journal of Industrial Economics*, 48, 205-13.

[9] Gerlach, H., T. Rønde and K. Stahl, 2002, "Market and Technical Risk", University of Mannheim, mimeo (available at *http://www.vwl.uni-mannheim.de/stahl/!/pps/papers_roende.php3*).

[10] Harter, J. F. R., 1991, "Differentiated Products with R&D," *Journal of Industrial Economics*, 41, 9-28.

[11] Hotelling, H, 1929, "Stability in Competition," *The Economic Journal*, 39, 41-57.

[12] Irmen, A. and J.-F. Thisse, 1998, "Competition in Multi-Characteristic Spaces: Hotelling was almost Right," *Journal of Economic Theory*, 78, 76-102.

[13] Kamien, M., E. Mueller, and I. Zang, 1992, "Research Joint Ventures and R&D Cartels," *American Economic Review*, 82, 1293-1306

[14] Katz, M. L. and C. Shapiro, 1986, "Product Compatibility Choice in a Market with Technological Progress," in D.J. Morris, P. J. N. Sinclair, M. D. E. Slater and J. S. Vickers, eds., *Strategic Behaviour and Industrial Competition*, New York: Oxford University Press, 146-165.

[15] Mai, C.-C. and S.-K. Peng, 1985, "Cooperation vs. Competition in a Spatial Model," *Regional Science and Urban Economics*, 29, 463-472.

[16] Mansfield, E., 1985, "How Rapidly Does New Industrial Technology Leak Out?," *Journal of Industrial Economics*, 34, 217-23.

[17] Neven, D. and J.F.Thisse, 1990, "On Quality and Variety Competition", in: Gabszewicz, J.J., Richard, R.F., and L. Wolsey, *Games, Econometrics, and Optimization. Contributions in the Honour of Jacques Dreze*, Amsterdam: North Holland, 175-199.

[18] Vettas, N., 1999, "Location as a Signal of Quality", *CEPR Discussion Paper Series (Industrial Organization)*, No. 2165.

A Proof of Proposition 1

Assume for now that $b \geq \frac{1}{2}$. We verify later that this is satisfied in equilibrium. The reaction function of firm i, which indicates the optimal location as a response to the competitor's location j, is denoted $R_i(j, \rho)$, $i = A, B$ and $j = a, b$. The expected profits of firm A as a function of (a, b) are given as:

$$E(\Pi_A(.)) = \begin{cases} \rho\left(\rho\frac{t(b-a)(2+a+b)^2}{18} + (1-\rho)(q - t(1-a)^2)\right) \text{ for } a < \frac{1}{2}, \\ \rho\left(\rho\frac{t(b-a)(2+a+b)^2}{18} + (1-\rho)(q - t(a)^2)\right) \text{ for } b \geq a \geq \frac{1}{2}, \\ \rho\left(\rho\frac{t(a-b)(4-a-b)^2}{18} + (1-\rho)(q - t(a)^2)\right) \text{ for } 1 \geq a > b. \end{cases} \tag{7}$$

From (7), we derive the following two results: 1) any location $a \leq 1$ st. $a > b$ is dominated by the location $1 - a$, 2) $\frac{\partial E(\Pi_A(a,b,q,q,\rho)}{\partial a} < 0$ for $b \geq a \geq \frac{1}{2}$. These results imply that $R_A(b, \rho) \in [0, \frac{1}{2}]$. We have

$$\frac{\partial E(\Pi_A)}{\partial a} = \frac{\rho t}{18}\left(36(1-a) - (40 + 3a^2 - 2a(14-b) - b^2)\rho\right) \text{ for } a \leq \frac{1}{2}. \tag{8}$$

As $\frac{\partial^2 E(\Pi_A(a,b,q,q,\rho))}{\partial a^2} = -\rho(18 - (14 - b - 3a)\rho)t < 0$, solving the first order condition finds the optimal location. From (8), and using $b \geq \frac{1}{2}$, we obtain the following reaction function of firm A, $R_A(b, \rho) =$

$$\begin{cases} \frac{1}{2} \text{ for } \rho \leq \frac{2}{3} \\ \frac{1}{2} \text{ for } \frac{2}{3} < \rho \leq \frac{72}{107} \text{ and } \frac{1}{2} + 3\frac{\sqrt{3\rho-2}}{\sqrt{\rho}} \leq b \\ \frac{1}{3\rho}\left(-18 + (14-b)\rho + \sqrt{X}\right) \text{ for } \frac{2}{3} < \rho \leq \frac{72}{107} \text{ and } \frac{1}{2} \leq b < \frac{1}{2} + 3\frac{\sqrt{3\rho-2}}{\sqrt{\rho}} \\ \frac{1}{3\rho}\left(-18 + (14-b)\rho + \sqrt{X}\right) \text{ for } \frac{72}{107} < \rho \leq \frac{48}{53} \\ \frac{1}{3\rho}\left(-18 + (14-b)\rho + \sqrt{X}\right) \text{ for} \frac{48}{53} < \rho \leq \frac{12}{13} \text{ and } 2\sqrt{10 - \frac{9}{\rho}} < b \\ 0 \text{ for } \frac{48}{53} < \rho \leq \frac{12}{13} \text{ and } 2\sqrt{10 - \frac{9}{\rho}} \geq b \\ 0 \text{ for } \rho > \frac{12}{13} \end{cases}$$

where $X \equiv 324 - 36(11 - b)\rho + 4(19 - (7 - b)b)\rho^2$. It can be verified that $R_A(b, \rho) \leq \frac{1}{2}$.

Deriving the reaction function of firm B in the same way as above, and using $a \leq \frac{1}{2}$, we obtain $R_B(a,\rho) =$

$$\begin{cases} \frac{1}{2} \text{ for } \rho \leq \frac{2}{3} \\ \frac{1}{2} \text{ for } \frac{2}{3} < \rho \leq \frac{72}{107} \text{ and } \frac{1}{2} - 3\frac{\sqrt{3\rho-2}}{\sqrt{\rho}} \geq a \\ \frac{1}{3\rho}\left(18 - (10+a)\rho - \frac{1}{2}\sqrt{Y}\right) \text{ for } \frac{2}{3} < \rho \leq \frac{72}{107} \text{ and } \frac{1}{2} \geq a > \frac{1}{2} - 3\frac{\sqrt{3\rho-2}}{\sqrt{\rho}} \\ \frac{1}{3\rho}\left(18 - (10+a)\rho - \frac{1}{2}\sqrt{Y}\right) \text{ for } \frac{72}{107} < \rho \leq \frac{48}{53} \\ \frac{1}{3\rho}\left(18 - (10+a)\rho - \frac{1}{2}\sqrt{Y}\right) \text{ for} \frac{48}{53} < \rho \leq \frac{12}{13} \text{ and } a \leq 1 - 2\sqrt{10 - \frac{9}{\rho}} \\ 0 \text{ for } \frac{48}{53} < \rho \leq \frac{12}{13} \text{ and } \frac{1}{2} \geq a > 1 - 2\sqrt{10 - \frac{9}{\rho}} \\ 0 \text{ for } \rho > \frac{12}{13} \end{cases}$$

where $Y \equiv (36-2(10+a)r)^2 - 12(16-a^2)r$. It can be verified that $R_B(a,\rho) \geq \frac{1}{2}$, so the initial assumption $b \geq \frac{1}{2}$ is satisfied.

The candidate equilibrium locations, as a function of ρ, are found by solving the system of equations: $R_A(b^*,\rho) = a^*$ and $R_B(a^*,\rho) = b^*$. It can be shown that the equilibrium locations described in the proposition are the only candidate locations satisfying $a^* \in [0, \frac{1}{2}]$ and $b^* \in [\frac{1}{2}, 1]$. Finally notice that for $b \leq \frac{1}{2}$, the resulting equilibrium is identical but with the roles of firm A and B switched. The equilibrium is thus unique modulo symmetry. ■

B Proof of Proposition 2

Total welfare in a monopoly where a firm offers a product of quality q at location a, $0 \leq a \leq 1$ is given by

$$W^M \equiv \int_0^1 (q - t(y-a)^2)dy = q - \frac{t}{2} + at(1-a).$$

In a duopoly with firms located at a and b, welfare is

$$\begin{aligned} W^D &\equiv \int_0^{\frac{2+a+b}{6}} (q - t(a-y)^2)dy + \int_{\frac{2+a+b}{6}}^1 (q - t(b-y)^2)dy \\ &= q - \frac{t}{3} + bt(1-b) + \frac{t}{36}(b-a)(2+a+b)(5b+5a-2). \end{aligned}$$

Expected welfare in the economy, $EW(a,b)$, is then defined as

$$EW(a,b) \equiv \rho^2 W^D + \rho(1-\rho)W^M(a) + (1-\rho)\rho W^M(b).$$

Taking the first derivatives of this function with respect to the locations a and b yields the following two necessary conditions:

$$\frac{\left(4-a\;(16+15\,a)-10\,a\,b+5\,b^2\right)\;\rho^2\,t}{36}+(1-2\,a)\;(1-\rho)\,\rho q=0$$

and

$$\frac{(32-56\,b+5\;(a+b)\;(-a+3\,b))\,\rho^2\,t}{36}+(1-2\,b)\;(1-\rho)\,\rho q=0.$$

Since $\partial EW(a,b)/\partial a^2<0$ and $\partial EW(a,b)/\partial a^2 \partial EW(a,b)/\partial b^2 - (\partial EW(a,b)/\partial a \partial b)^2 > 0$, these conditions are also sufficient. Thus, solving the two first order conditions above for (a,b) yields the welfare maximizing locations given in the proposition. ■

C Proof of Lemma 2

If firm B innovates, it drives firm A out the market iff. $t(b-a)(2+a+b)-\Delta \leq 0$. Therefore, there exists $\overline{b}$ such that Firm A stays in the market iff. $b \geq \overline{b}$. $\overline{b}$ is given by

$$\overline{b} \equiv Min\left\{1, -1+\sqrt{(1+a)^2 t+\Delta}/\sqrt{t}\right\}$$

Using equation (6), the expected profits of firm B can be written as:

$$E(\Pi_B\left((a,S),(b,R)\right)) = \begin{cases} \rho(\Delta - t(b^2-a^2)) & \text{for } b \leq \overline{b} \\ \rho\frac{(t(b-a)(4-a-b)+\Delta)^2}{18t(b-a)} & \text{for } b > \overline{b}. \end{cases}$$

Let $b \leq \overline{b}$. Then, $\partial E(\Pi_B(\cdot,\cdot))/\partial b < (>)0$ for $b > (<)a$. Hence, there is a local maximum at $b=a$.

Let $b > \overline{b}$. Then,

$$\partial E(\Pi_B((\frac{1}{2},S),(b,R)))/\partial b = -\frac{\rho(\Delta-(b-a)(4+a-3b)t)(\Delta+(b-a)(4-a-b)t)}{18(b-a)^2}.$$

This expression is negative if $\Delta > (b-a)(4+a-3b)t$. It can be shown that $\Delta-(b-a)(4+a-3b)t$ is decreasing in b. We have thus two cases to deal with. First, if $\Delta \geq 4/3t$, $\frac{\partial E(\Pi_B(a,S),(b,R)))}{\partial b} < 0$ for all a and $b \geq \overline{b}$. Using the result for $b \leq \overline{b}$, we have that the optimal location is $b^* = a$.

Consider now $\Delta < \frac{4}{3}t$. Solving $\Delta = (b-a)(4+a-3b)t$ for b, we obtain $b_L \equiv \frac{2(1+a)}{3} - \frac{1}{3\sqrt{t}}\sqrt{(2-a)^2 t - 3\Delta}$ and $b_H \equiv \frac{2}{3} + \frac{1}{3\sqrt{t}}\sqrt{(2-a)^2 t - 3\Delta}$. It can be shown that $b_H > \overline{b}$ for the parameters considered and that $b_H \leq 1$ for $\Delta \geq (1-a^2)t$. Thus there exists an interval $[Max\{b_L, \overline{b}\}, Min\{b_H, 1\}] \subseteq [\overline{b}, 1]$ for which $\frac{\partial E(\Pi_B((a,S),(b,R)))}{\partial b} \geq 0$. It follows that there is a local maximum at $b = Min\{b_H, 1\}$. Again using the result for $b \leq \overline{b}$, there are two local maxima that we need to compare to find the optimal location, $b = a$ and $b = Min\{b_H, 1\}$.

Consider first $(1-a^2)t \leq \Delta < \frac{4}{3}t$ where $b_H < 1$. Straightforward calculations show that $E(\Pi_B((a,S),(a,R))) - E(\Pi_B((a,S),(b_H,R))) \geq 0$ iff. $\Phi(t,a,\Delta) \geq 0$ where

$$\Phi(t,a,\Delta) \equiv \Delta - \frac{8(((2-a)^2 t - 3\Delta)^{3/2} + (2-a)\sqrt{t}((2-a)^2 t + 9\Delta))}{243\sqrt{t}}.$$

$\Phi(t,a,\Delta)$ is positive for all $\Delta \geq \frac{t}{128}\left(-107 - 784a - 128a^2 + 3(17+32a)^{3/2}\right)$. Calculations show that $(1-a^2)t \geq \frac{t}{128}\left(-107 - 784a - 128a^2 + 3(17+32a)^{3/2}\right)$. Therefore, $E(\Pi_B((a,S),(a,R))) - E(\Pi_B((a,S),(b_H,R))) \geq 0$ for all $(1-a^2)t \leq \Delta < \frac{4}{3}t$, so $b^* = a$. Consider now $(1-a^2)t > \Delta$. Here, we have that $E(\Pi_B((a,S),(a,R))) - E(\Pi_B((a,S),(1,R))) \geq 0$ iff. $\Delta \geq (1-a)(6+a-3\sqrt{3+2a})t$ where $(1-a^2)t \geq (1-a)(6+a-3\sqrt{3+2a})t$. Proof follows. ■

D Proof of Lemma 3

From Lemma 1 it follows that firm A is driven out the market when firm B innovates iff. $t(b-a)(2+a+b) - \Delta \leq 0$. Therefore, there exists an $\overline{a}$ such that firm A stays in the market with firm B innovating iff. $a \leq \overline{a}$. $\overline{a}$ is given by

$$\overline{a} \equiv Max\left\{0, -1 + \sqrt{(1+b)^2 - \Delta/t}\right\}.$$

Using equation (5)-(6), the profit function of firm A is $E(\Pi_A((a,S),(b,R))) =$

$$\begin{cases} \rho\frac{(t(b-a)(2+a+b)-\Delta)^2}{18t(b-a)} + (1-\rho)(q_L - t(Max\{a, 1-a\})^2) & \text{for } a \leq \overline{a} \\ (1-\rho)(q - t(Max\{a, 1-a\})^2) & \text{for } a > \overline{a}. \end{cases}$$

If $\overline{a} = 0$, $\partial E(\Pi_A((a,S),(b,R)))/\partial a > (<)0$ for $a < (>)1/2$. It follows that the optimal location is $a^* = \frac{1}{2}$.

If $0 < \overline{a} \leq \frac{1}{2}$, $E(\Pi_A((a,S),(b,R)))$ is increasing for $\frac{1}{2} \geq a > \overline{a}$ and decreasing for $a > \frac{1}{2}$. We now look at $a \leq \overline{a}$. Maximizing the profits wrt. a yields $\frac{\partial E(\Pi_A)}{\partial a} =$

$$\frac{1}{18}\left(-2(\Delta)\rho + \frac{(\Delta)^2\rho}{(a-b)^2 t} - \left(36(-1+a) + \left(40 + 3a^2 + 2a(-14+b) - b^2\right)\rho\right)t\right).$$

We have that

$$\frac{\partial^2 E(\Pi_A((a,S),(b,R)))}{\partial a \partial b} = \frac{\rho}{9}\left((b-a)t - \frac{(\Delta)^2}{(b-a)^3 t}\right).$$

From this follows that $\frac{\partial^2 E(\Pi_A((a,S),(b,R)))}{\partial a \partial b} \leq (>)\ 0$ for $b \leq (>) a + \sqrt{\frac{\Delta}{t}}$. Hence, $b = a + \sqrt{\frac{\Delta}{t}}$ is the b that minimizes $\frac{\partial^2 E(\Pi_A((a,S),(b,R)))}{\partial a \partial b}$. It follows that

$$\frac{\partial E(\Pi_A((a,S),(b,R)))}{\partial a} \geq \frac{\partial E(\Pi_A((a,S),(a+\sqrt{\frac{\Delta}{t}},R)))}{\partial a} = \frac{2t}{9}(9(1-a)-(5-a)(2-a)\rho)$$

This expression is decreasing in a. Hence, for all $a \leq \overline{a} \leq \frac{1}{2}$ we have

$$\frac{\partial E(\Pi_A((a,S),(b,R)))}{\partial a} > \frac{\partial E(\Pi_A((\frac{1}{2},S),(\frac{1}{2}+\sqrt{\frac{\Delta}{t}},R)))}{\partial a} = t(1-\frac{3}{2}\rho).$$

From $\rho \leq \frac{2}{3}$ it then follows that $\frac{\partial E(\Pi_A((a,S),(b,R))}{\partial a} > 0$ for all $a \leq \overline{a}$. The optimal location is thus $a^* = \frac{1}{2}$.

Finally, if $\overline{a} > \frac{1}{2}$ it follows from the argument above that $\frac{\partial E(\Pi_A((a,S),(b,R)))}{\partial a} \geq 0$ for $a \leq \frac{1}{2}$. Consider now $\frac{1}{2} < a \leq \overline{a} < b$. Here, the first order condition is given as $\frac{\partial E(\Pi_A((a,S),(b,R))}{\partial a} =$

$$\frac{\rho(\Delta - t(b-a)(2+a+b)t)(\Delta + t(b-a)(2+3a-b)t)}{18(b-a)^2} + (-2(1-\rho)at) < 0.$$

As above, $\frac{\partial \Pi_A^D((a,S),(1,R))}{\partial a} < 0$ for $\overline{a} < a \leq b$. Hence, $a^* = \frac{1}{2}$. ■

E Proof of Proposition 3

We denote by $E(\Pi_i(s_A, s_B))$, firm i's expected profit as a function of the two firms' strategies.

Part i) Consider the strategy choice $\{(\frac{1}{2},R),(\frac{1}{2},R)\}$. We know from Proposition 1 that if both firms choose the risky technology, the unique equilibrium locations are $a = b = \frac{1}{2}$. Therefore, we only need to check for deviations involving the safe technology. It follows from Lemma 3 that the optimal deviation for firm A (B, respectively) would be to the strategy $(\frac{1}{2},S)$. Therefore, that strategy choice is an equilibrium iff. $E(\Pi_A((\frac{1}{2},R),(\frac{1}{2},R))) \geq E(\Pi_A((\frac{1}{2},S),(\frac{1}{2},R)))$, which reduces to equation $\Delta \geq \Delta_3^*(\rho,t)$.

Part ii) It follows from Lemma 4 that the strategy choice $\{(\frac{1}{2}, S), (\frac{1}{2}, R)\}$ (the analysis for $\{(\frac{1}{2}, R), (\frac{1}{2}, S)\}$ is analogous) is the unique candidate equilibrium for asymmetric technology choices and $\Delta > \frac{t}{4}$. Thus, we only need to consider deviations to a different technology. Proposition 1 implies that the optimal deviation for firm A is $(\frac{1}{2}, R)$, while it follows from d'Aspremont et al. that the optimal deviation for firm B is $(1, S)$. Hence, no deviation takes place iff. $E(\Pi_A((\frac{1}{2}, S), (\frac{1}{2}, R))) \geq E(\Pi_A((\frac{1}{2}, R), (\frac{1}{2}, R)))$ and if $E(\Pi_B((\frac{1}{2}, S), (\frac{1}{2}, R))) \geq E(\Pi_B((\frac{1}{2}, S), (1, S)))$, which reduce to $\Delta_1^*(\rho, t) \leq \Delta \leq \Delta_3^*(\rho, t)$. Now consider $\Delta \leq \frac{t}{4}$, where the candidate equilibrium is $\{(\frac{1}{2}, S), (1, R)\}$. From d'Aspremont et al. we know that the best deviation for firm B is $(1, S)$. Firm B deviates if and only if $\Delta \leq \frac{5t}{4}(\frac{1}{\sqrt{\rho}} - 1)$ which always holds for $\rho \leq \frac{2}{3}$ and $\Delta \leq \frac{t}{4}$. Thus, there exists no equilibrium with asymmetric R&D choices for $\Delta \leq \frac{t}{4}$.

Part iii) Consider the strategy choices $\{(0, S), (1, S)\}$. From d'Aspremont et al. it follows that this is the only candidate equilibrium given the technology choice. Suppose that firm B (or, alternatively, A) would choose the technology R. From Lemma 2, it follows that the best deviation is $(0, R)$ if $\Delta \geq 3(2 - \sqrt{3})t$. Therefore, the strategy choices constitute an equilibrium iff. $\Pi_A^D((0, S), (1, S)) \geq \Pi_A^D((0, S), (0, R))$, which reduces to $\Delta \leq \frac{t}{2\rho}$. Consider now $\Delta \leq 3(2 - \sqrt{3})t$ where the optimal deviation is $(1, R)$. For these parameters, $\{(0, S), (1, S)\}$ is an equilibrium iff. $\Pi_A^D((0, S), (1, S)) \geq \Pi_A^D((0, S), (1, R)) \Leftrightarrow \Delta \leq 3t(\frac{1}{\sqrt{\rho}} - 1)$. Taken together, we get $\Delta \leq \Delta_2^*(\rho, t)$. ■

F Proof of Corollary 2

Using $q = \rho(q + \Delta)$, we can express the quality difference as $\Delta = \frac{1-\rho}{\rho}q$, which is strictly greater than $\frac{(1-\rho)(4q-t)}{4\rho}$. It follows from Proposition 3 that the unique equilibrium under Assumption 2 is $\{(\frac{1}{2}, R), (\frac{1}{2}, R)\}$. ■

G Proof of Proposition 4 and 5

We prove that patent protection is leading to more clustering. In the following we need that $\frac{\partial \Pi_A^D(.)}{\partial a} < 0$, $\frac{\partial \Pi_A^M(.)}{\partial a} > 0$ (for $a \leq \frac{1}{2}$), $\frac{\partial^2 \Pi_A^D(.)}{(\partial a)^2} = -\frac{t(4+3a+b)}{9} < 0$ and $\frac{\partial^2 \Pi_A^M(.)}{(\partial a)^2} = -2t < 0$. The first order condition for firm A is given by

$$\rho^2(1-\lambda)\frac{\partial \Pi_A^D(.)}{\partial a} + (\rho - \rho^2 + \frac{1}{2}\rho^2\lambda)\frac{\partial \Pi_A^M(.)}{\partial a} = 0.$$

Note that the two first terms are strictly decreasing in a. Thus, if an interior solution exists, it is unique. Moreover, note that any increase in λ leads to an increase of the absolute value of the two first terms. But this implies that the optimal a increases with λ, i.e. the reaction function of A shifts (weakly) upwards for any b and (symmetrically and with omitted proof) the reaction function of B shifts downwards for any a. It follows that in the new equilibrium firms are (weakly) more clustered.

With the same line of reasoning, we now show that spillovers are a centrifugal force. The expected profits of firm A, $E[\Pi_A(.)]$ are

$$\rho^2\Pi_A^D(.) + \rho(1-\rho)(1-\sigma)\Pi_A^M(.) + 2\rho(1-\rho)\Pi_A^D(.)$$

and the resulting first order condition is

$$\rho^2\frac{\partial\Pi_A^D(.)}{\partial a} + \rho(1-\rho)((1-\sigma)\frac{\partial\Pi_A^M(.)}{\partial a} + 2\sigma\frac{\partial\Pi_A^D(.)}{\partial a}) = 0.$$

With the same arguments as above, increasing σ decreases the whole expression which implies that spillovers lead to equilibria with more differentiated products. ■