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ACCUMULATION IN A MONETARY UNION**

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ABSTRACT

Inflation Targets and Debt Accumulation in a Monetary Union*

This Paper explores the interaction between centralized monetary policy and decentralized fiscal policy in a monetary union. Discretionary monetary policy suffers from a failure to commit. Moreover, decentralized fiscal policy-makers impose externalities on each other through the influence of their debt policies on the common monetary policy. These imperfections can be alleviated by adopting state-contingent inflation targets (to combat the monetary policy commitment problem) and shock-contingent debt targets (to internalize the externalities due to decentralized fiscal policy).

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NON-TECHNICAL SUMMARY

In the European Monetary Union (EMU) monetary policy is decided at the central level by the European Central Bank (ECB), while fiscal policy is decentralized and conducted at the national level. This Paper explores the interaction between a centralized monetary policy and decentralized fiscal policies in a monetary union. If the union central bank were completely independent, one would expect little interaction between fiscal and monetary policy-making. Irrespective of the fiscal policies pursued by the national authorities, a completely independent central bank would only be concerned with price stability. However, we are sceptical that this will be the case in reality. Even though the Maastricht Treaty endows the ECB with a high degree of independence, it is hard to imagine the ECB standing by completely unmoved, no matter what happens on the fiscal front. The analysis in this Paper therefore takes the perspective that the common, union central bank can indeed be influenced by the national fiscal policies.

We allow for the possibility that the common central bank pursues a strategy of state-contingent inflation targeting. Although the ECB has announced a two-tier monetary policy strategy based on a reference value for money growth and an indicator that is composed of a large number of other measures (such as output gaps, inflation expectations, etc.), it may well shift over time to implicit inflation targeting (i.e. inflation targeting in practice, although not in name). A number of economists have argued that this is what the Bundesbank has actually done in the past. Indeed, the limited amount of evidence so far suggests that the reference value for M3 money growth, 4.5% on a yearly basis, has been taken less seriously than one might have expected in advance. During the first ten months of EMU existence, broad money growth has rather substantially exceeded its reference value, without the ECB responding by raising the nominal interest rate (in April 1999 the ECB even lowered the interest rate). Apparently, given the situation on the labour markets and the low inflation rate in the Euro area, the ECB saw no reason to restrict monetary policy to the extent needed to keep money growth close to its reference value. Of course, in assessing the performance of the ECB, one needs to take into account that the common monetary policy has been in place for a short period only and that the ECB may still be in search of the appropriate reference value for money growth.

This Paper builds on our earlier work in Beetsma and Bovenberg (1999). There we show that monetary unification raises debt accumulation, because in a monetary union countries only partly internalize the effects of their debt policies on future monetary policy. We showed that, in the absence of shocks, making the central bank sufficiently conservative (by imposing on the central

bank a loss function that attaches a sufficiently high weight to price stability) can lead the economy to become second best. We also demonstrate that this result no longer holds in the presence of common shocks, as economies are then confronted with a trade-off between credibility and flexibility.

While Beetsma and Bovenberg (1999) emphasizes the effects of a lack of commitment in monetary policy, this Paper introduces another complication in the form of strategic interactions between decentralized fiscal policy-makers who hold different views on the preferred stance of the common monetary policy. These different views originate in differences among the economies in the monetary union. In particular, we allow for systematic differences in labour and product market distortions, public spending requirements and initial public debt levels. The incorporation of such systematic differences seems particularly realistic in the context of EMU, because the Euro area features large cross-country differences in public debt levels, unemployment and price levels for similar services or goods. Differences in unemployment and price levels may be attributable to different degrees of distortions in the relevant markets. We allow not only for systematic differences among countries but also for idiosyncratic stochastic shocks hitting the economies.

The combination of cross-country differences and the decentralization of fiscal policy produces conflicts between the countries about the future stance of the common monetary policy. As a direct consequence, countries employ their debt policy strategically to induce the union's central bank to move the common monetary policy into the direction they prefer. In particular, countries that suffer from severe distortions in labor and product markets, feature large public spending requirements or high initial debt levels, or are hit by bad idiosyncratic shocks accumulate additional public debt so as to encourage the central bank to make monetary policy more expansionary. This strategic behaviour imposes negative externalities on other countries, thereby producing welfare losses.

We explore how the distortions in the model (i.e. the failure to commit monetary policy and strategic debt policy) may be alleviated through appropriate institutional adjustments. In the special case of identical countries, imposing a state-contingent inflation target on the common central bank leads the economies to become second best. In particular, the inflation target needs to be made contingent on the average union debt level. More debt accumulation leads to higher expected future inflation and therefore requires a tightening of the inflation target. With fiscal policy decentralized to heterogeneous countries, however, the optimal state-contingent inflation target needs to be complemented by country-specific debt targets to establish the second best. In this way, inflation targets address the lack of commitment in monetary policy, while debt targets eliminate the strategic interaction among

heterogeneous governments with different views about the common monetary policy.

We emphasize that the debt targets that are part of the optimal institutional set-up should be country-specific, unlike the restrictions that the Maastricht Treaty and the Stability and Growth Pact impose on the EMU participants. In particular, our debt targets are differentiated according to the differences in initial debt and countries' specific time profiles in product and labour market distortions. In addition, the debt targets have been adjusted for unexpected asymmetric shocks.

The analysis assumes that the inflation and debt targets are credible. Inflation targets may be enforced by giving the central banker financial incentives to meet the target or by making reappointment of the central banker dependent upon achievement of the target. Also, reputational considerations may induce the central banker to stick to the inflation target.

Debt targets may be enforced through peer pressure and fines (such as conforming to the Stability and Growth Pact), although many people are sceptical whether this will work in practice. Further research should therefore pay attention to the question of whether and how debt targets (or other budgetary restrictions) can be enforced. Such an analysis would model the incentives of individual governments to enforce the compliance of other countries to the targets. It would also require a dynamic framework that takes into account reputational considerations. Indeed, the incentives to enforce adherence to the targets are likely to depend on how a failure to enforce the targets affects the future behaviour of governments.

1. Introduction

Although the Maastricht Treaty has laid the institutional foundations for European Monetary Union (EMU), how these institutions can best be operated in practice remains to be seen in the coming years. For example, the European Central Bank (ECB) has announced a two-tier monetary policy strategy based on a reference value for money growth and an indicator that is based on a number of other measures, such as output gaps, inflation expectations, etcetera (see European Central Bank, 1999). Over time the ECB may well shift to implicit targeting of inflation. Indeed, a number of economists has argued (e.g. see Svensson, 1998) that also the Bundesbank has pursued such a strategy. Furthermore, how the Excessive Deficit Procedure and the Stability and Growth Pact (see Beetsma and Uhlig, 1999) will work in practice is not yet clear.

This paper deals with the interaction between inflation targets and constraints on decentralized fiscal policy in a monetary union. To do so, we extend our earlier work on the interaction between a common monetary policy and decentralized fiscal policies in a monetary union. In particular, in Beetsma and Bovenberg (1999) we showed that monetary union raises debt accumulation, because in a monetary union countries only partly internalize the effects of their debt policies on future monetary policy. This additional debt accumulation is actually welfare enhancing (if the governments share societal preferences). We showed that, in the absence of shocks, making the central bank sufficiently conservative (in the sense of Rogoff, 1985, that is by imposing on the central bank a loss function that attaches a sufficiently high weight to price stability) can lead the economy to the second-best equilibrium. However, this is no longer the case in the presence of common shocks, as the economies are confronted with a trade off between credibility and flexibility.

While Beetsma and Bovenberg (1999) emphasized the effects of lack of commitment in monetary policy, this paper introduces another complication in the form of strategic interactions between decentralized fiscal policymakers who have different views on the stance of the common monetary policy.¹ These different views originate in differences among the economies in the monetary union. In particular, we allow for systematic differences in labour and product market distortions, public spending requirements and initial public debt levels. We also allow for idiosyncratic stochastic shocks hitting the countries. In combination with the decentralization of fiscal policy these differences lead to conflicts about the preferred future stance of the common monetary policy. In particular, countries that suffer from severe distortions in labor and commodity markets, feature

¹Our earlier model incorporated another potential distortion: the possibility that governments discount the future at a higher rate than their societies do. We ignore this distortion throughout the current paper.

higher public spending or initial debt levels or are hit by worse shocks prefer a laxer future stance of monetary policy. These conflicts about monetary policy induce individual governments to employ their debt policy strategically, so as to induce the union's central bank to move monetary policy into the direction they prefer. This strategic behavior imposes negative externalities on other countries, thereby producing welfare losses.

In contrast to Beetsma and Bovenberg (1999), we do not address the distortions in the model by making the common central bank sufficiently conservative. Instead, we focus on state-contingent inflation targets which, in contrast to a conservative central bank, can lead the economy to the second-best equilibrium if countries are identical. Hence, as stressed by Svensson (1997) in a model without fiscal policy and debt accumulation, inflation targets eliminate the standard credibility-flexibility trade-off. If fiscal policy is decentralized to heterogeneous countries, however, the optimal state-contingent inflation targets need to be complemented by (country-specific) debt targets to establish the second best. In this way, inflation targets address the lack of commitment in monetary policy, while debt targets eliminate strategic interaction among heterogeneous governments with different views about the common monetary policy stance.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 discusses the second-best equilibrium in which not only monetary but also fiscal policy is centralized and in which monetary policy is conducted under commitment. This is the second-best optimum that can be attained under monetary union, assuming that the supranational authorities attach an equal weight to the preferences of each of the participating countries. Section 4 derives the equilibrium for the case of a common, discretionary monetary policy with decentralized fiscal policies. Section 5 explores institutional arrangements (i.e. inflation targets and public debt targets) that may alleviate the welfare losses arising from the lack of monetary policy commitment and the wasteful strategic interaction among the decentralized governments. Finally, Section 6 concludes the main body of this paper. The derivations are contained in the appendix.

2. The model

A monetary union, which is small relative to the rest of the world, is formed by n countries.² A common central bank (CCB) sets monetary policy for the entire union, while fiscal policy is determined at a decentralized, national level by the n governments. There are two periods.

²Monetary union is taken as given. Hence, we do not explore the incentives of countries to join a monetary union.

Workers are represented by trade unions who aim for some target real wage rate (e.g. see Alesina and Tabellini, 1987, and Jensen, 1994). They set nominal wages so as to minimize the expected squared deviation of the realized real wage rate from this target. Monetary policy (i.e., the inflation rate) is selected after nominal wages have been fixed. In each country, firms face a standard production function with decreasing returns to scale in labour. Output in period t is taxed at a rate τ_{it} . Therefore, output in country i in periods 1 and 2, respectively, is given by³

$$x_{i1} = \omega (\mu_{1i} \mu_{1i}^e \tau_{i1}) \mu_{1i}^{-1} \mu_{1i}^{-2} z_i; \quad (2.1)$$

$$x_{i2} = \omega (\mu_{2i} \mu_{2i}^e \tau_{i2}) \mu_{2i}^{-1} \mu_{2i}^{-2} z_i; \quad (2.2)$$

where μ_{1i} represents a common union-wide shock, while z_i stands for an idiosyncratic shock that solely hits country i . μ_{tj}^e denotes the inflation rate for period t expected at the start of period t (that is, before period t shocks have materialized, but after period $t-1$; $t-2$; ... shocks have hit). We assume that $E[z_i] = 0$; $E[\mu_{1i}^2] = 0$; $E[\mu_{2i}^2] = 0$; $E[\mu_{1i} \mu_{2i}] = 0$; and that $\sigma^2 = \frac{1}{n} \sum_{i=1}^n z_i^2 = 0$.⁴ The variances of μ_{1i} and z_i are given by $\sigma_{\mu_1}^2$ and σ_z^2 , respectively. We abstract from shocks in the second period, because they would not affect debt accumulation.

Each country features a social welfare function which is shared by the government of that country. Hence, governments are benevolent. In particular, the loss function of government i is defined over inflation, output and public spending:

$$V_{S;i} = \frac{1}{2} \sum_{t=1}^h -\tau_{it} \mu_{it}^2 + (x_{it} - \bar{x}_{it})^2 + \omega_g (g_{it} - \bar{g}_{it})^2; \quad 0 < \tau_{it} < 1; \quad \omega_{\mu}; \omega_g > 0; \quad (2.3)$$

Welfare losses increase in the deviations of inflation, (log) output and government spending (g_{it} is government spending as a share of output in the absence of distortions) from their targets (or first-best levels or "bliss points"). For convenience, the target level for inflation corresponds to price stability. The target level for output is denoted by $\bar{x}_{it} > 0$. Two distortions reduce output below this optimal level. First, the output tax τ_{it} drives a wedge between the social and private benefits of additional output. Second, market power enables unions to drive the real wage above its level in the absence of distortions. Hence, even in the absence of taxes, output is below the first-best output level $\bar{x}_{it} > 0$. The first-best

³Details on the derivations of these output equations can be found in Beetsma and Bovenberg (1999).

⁴Without this assumption, the mean σ^2 of the z_i 's would play the same role as μ_{1i} does. In the outcomes given below, μ_{1i} would then be replaced by $\mu_{1i} + \sigma^2$. For convenience, we assume that $\sigma^2 = 0$.

level of government spending, g_{it} , can be interpreted as the optimal share of non-distortionary output to be spent on public goods if (non-distortionary) lump-sum taxes would be available (see Debelle and Fischer, 1994). The target levels for output and government spending can differ across countries. Parameters α_y and α_g correspond to the weights of the price stability and government spending objectives, respectively, relative to the weight of the output objective. Finally, β denotes society's subjective discount factor.

Government i 's budget constraint can be approximated by (e.g., see Appendix A in Beetsma and Bovenberg, 1999):

$$g_{it} + (1 + \frac{1}{2}) d_{i;t-1} = \zeta_{it} + \hat{A} \frac{1}{4}_t + d_{it}; \quad (2.4)$$

where $d_{i;t-1}$ represents the amount of public debt carried over from the previous period into period t , while d_{it} stands for the amount of debt outstanding at the end of period t . All public debt is real, matures after one period, and is sold on the world capital market against a real rate of interest of $\frac{1}{2}$. This interest rate is exogenous because the countries making up the monetary union are small relative to the rest of the world.⁵ ζ_{it} and \hat{A} (a constant) stand for, respectively, distortionary tax revenue and real holdings of base money as shares of non-distortionary output. All countries share equally in the seigniorage revenues of the CCB, so that the seigniorage revenues accruing to country i amount to $\hat{A} \frac{1}{4}_t$.

We combine (2.4) with the expression for output, (2.1) or (2.2), to eliminate ζ_{it} . The resulting equation can be rewritten to yield the government financing requirement of period t :

$$\begin{aligned} \text{GFR}_{it} &= \mathcal{K}_{it} + (1 + \frac{1}{2}) d_{i;t-1} - d_{it} + \pm_t (1 + 2_i) \frac{1}{4}_t \\ &= [(\mathcal{X}_{it} - \mathcal{X}_{it}^e) + \hat{A} \frac{1}{4}_t + (g_{it} - g_{it}^e) + (\frac{1}{4}_t - \frac{1}{4}_t^e)]; \end{aligned} \quad (2.5)$$

where \pm_t is an indicator function, such that $\pm_1 = 1$ and $\pm_2 = 0$, and where

$$\mathcal{K}_{it} \equiv g_{it} + \mathcal{X}_{it}^e.$$

The government financing requirement, GFR_{it} , consists of three components. The first component, \mathcal{K}_{it} , amounts to the government spending target, g_{it} , and an output subsidy aimed at offsetting the implicit output tax due to labor- or product-market distortions, \mathcal{X}_{it}^e . The second component involves net debt-servicing costs,

⁵In the following, we will occasionally explore what happens when the number of union participants becomes infinitely large (i.e. $n \rightarrow \infty$) in order to strengthen the intuition behind our results. In these exercises the real interest rate remains beyond the control of union-level policymakers.

$(1 + \frac{1}{2}) d_{i,t-1} - d_{it}$. The deterministic component (in period 1 only) is the stochastic shock (scaled by σ), $(1 + z_i) \epsilon_t$. The last right-hand side of (2.5) represents the sources of finance: the shortfall (scaled by σ) of output from its target (henceforth referred to as the output gap), $(x_{it} - x_{it}^e)$, seigniorage revenues, \hat{M}_t , the shortfall of government spending from its target (henceforth referred to as the spending gap), $(g_{it} - g_{it}^e)$, and the inflation surprise, $(\pi_t - \pi_t^e)$.

All public debt is paid off at the end of the second period ($d_{i2} = 0; i = 1; \dots; n$). Under this assumption, while taking the discounted (to period one) sums of the left- and right-hand sides of (2.5) ($t = 1; 2$), we obtain the intertemporal government financing requirement:

$$\begin{aligned} \text{IGFR}_i &= F_i + (1 + z_i) \epsilon_t \\ &= \sum_{t=1}^2 (1 + \frac{1}{2})^{t-1} [(x_{it} - x_{it}^e) + \hat{M}_t + (g_{it} - g_{it}^e) + (\pi_t - \pi_t^e)]; \end{aligned} \quad (2.6)$$

where $F_i = K_{i1} + (1 + \frac{1}{2}) d_{i0} + K_{i2} = (1 + \frac{1}{2})$ stands for the deterministic component of the intertemporal government financing requirement.

Monetary policy is delegated to a common central banker (CCB), who has direct control over the union's inflation rate. One could assume that the CCB has certain intrinsic preferences regarding the policy outcomes. Alternatively, and this is the interpretation we prefer, one could assume that the CCB is assigned a loss function by means of an appropriate contractual agreement. More specifically, this agreement shapes the CCB's incentives in such a way (by appropriately specifying its salary and other benefits – for example, possible reappointment – conditional on its performance) that it chooses to maximize the following loss function:

$$V_{\text{CCB}} = \frac{1}{2} \sum_{t=1}^2 \beta^{t-1} \left(\frac{1}{\sigma} (\pi_t - \pi_t^e)^2 + \frac{1}{n} \sum_{i=1}^n (x_{it} - x_{it}^e)^2 + \frac{1}{g} (g_{it} - g_{it}^e)^2 \right); \quad (2.7)$$

where π_t^e is the inflation target in period t , which may be different from the socially-optimal inflation rate, which was set at zero.

If $\pi_1^e = \pi_2^e = 0$, the CCB's objective function corresponds to an equally-weighted average of the individual societies' objective functions. We assume that π_2^e is a linear function of $d_{i1}; i = 1; \dots; n$. This linearity assumption suffices for our purposes: we will see later on that the optimal second-period inflation target is indeed a linear function of $d_{i1}; i = 1; \dots; n$. The optimal first-period inflation target will be a function of d_{i0} , which is exogenous.

3. The second-best equilibrium

As a benchmark for the remainder of the analysis, we discuss the equilibrium resulting from centralized fiscal and monetary policies under commitment. Monetary policy is set by the CCB. Fiscal policy is conducted by a centralized fiscal authority, which minimizes:

$$V_U \sim \frac{1}{n} \sum_{i=1}^n V_{S;i} \quad (3.1)$$

where the $V_{S;i}$ are given by (2.3), $i = 1; \dots; n$. Equation (3.1) assumes that countries have equal bargaining power as regards to the fiscal policy decisions taken at the union level. Government spending is residually determined, so that the CCB, when it selects monetary policy, internalizes the government budget constraints. The resulting equilibrium is Pareto optimal. In the sequel, we refer to this equilibrium as the second-best equilibrium. In the absence of first-best policies (such as the use of lump-sum taxation and the elimination of product- and labor-market distortions), it is the equilibrium with the smallest possible welfare loss (3.1), given monetary union. The derivation of the second-best equilibrium is contained in Appendix A.

3.1. Inflation, the output gap and the public spending gap

Table 1 contains the outcomes for inflation, the output gap,⁶ $x_{it} - x_{it}^*$, and the spending gap, $g_{it} - g_{it}^*$. We write each of these outcomes as the sum of two deterministic and two stochastic components. F_i^{Φ} is the deviation of country i 's deterministic component of its intertemporal government financing requirement from the cross-country average, defined by F . Formally, $F = \frac{1}{n} \sum_{j=1}^n F_j$ and $F_i^{\Phi} = F_i - F$. The factor between square brackets in each of the entries of Table 1 makes clear how, within a given period, the government financing requirement is distributed over the financing sources (seigniorage, the output gap, the spending gap and an inflation surprise). Indeed, for each period these factors add up to unity, both across the deterministic and across the stochastic components. For example, for the first period one has:

⁶Throughout, we present the outcome for the output gap instead of the outcome for the tax rate. The reason is that, in contrast to the latter, the former directly enters the welfare loss functions.

$$\begin{aligned}
& \left[(x_{i1} \text{ i } x_{i1}) \frac{1}{3} \right] + \hat{A} \frac{1}{4} + (g_{i1} \text{ i } g_{i1}) + (\frac{1}{4} \text{ i } \frac{1}{4}^e) \\
= & \frac{1 - \alpha(1 + \frac{1}{2})}{1 + \alpha(1 + \frac{1}{2})} \bar{F} + \bar{F}_i^{\Phi} + \frac{1 - \alpha(1 + \frac{1}{2})(P^{\pi} = P)}{1 + \alpha(1 + \frac{1}{2})(P^{\pi} = P)} \frac{1}{\sigma} + \frac{z_i}{\sigma} \\
= & \kappa_{i1} + (1 + \frac{1}{2}) d_{i0} \text{ i } d_{i1}^S + (1 + z_i) = 0; \tag{3.2}
\end{aligned}$$

where d_{i1}^S is the second-best debt level. The last equality can be checked by substituting (3.4)-(3.7) into (3.3) (all given below) and substituting the resulting expression into the last line of (3.2). For each of the outcomes, the terms that follow the factor in square brackets regulate the intertemporal allocation of the intertemporal government financing requirement.

The coefficients of the common stochastic shock $\frac{1}{\sigma}$ (in the fourth column of Table 1, σ_2) differ in two ways from the coefficients of the common deterministic component of the intertemporal government financing requirement \bar{F} (in the second column of Table 1, σ_0). The first difference is with respect to the first-period, intratemporal, allocation of the government financing requirement over the financing sources. The deterministic components of the government financing requirement are anticipated and thus correctly incorporated in expected inflation. The common shock, in contrast, is unanticipated and, hence, not taken into account when inflation expectations are formed. The predetermination of the inflation expectation is exploited by the central policymakers so as to finance part of this common shock through an inflation surprise. Indeed, whereas the coefficient of $\frac{1}{4} \text{ i } \frac{1}{4}^e$ is zero in the second column in Table 1, this coefficient is positive in the fourth column, indicating that part of the common shock is financed through an inflation surprise in the first period. With surprise inflation absorbing part of the common shock, the output gap and the spending gap have to absorb a smaller share of this shock.

In the second period, the allocation over the financing sources for the stochastic component $\frac{1}{\sigma}$ is the same as for the deterministic component \bar{F} . The reason is that the first-period shock $\frac{1}{\sigma}$ has materialized before second-period inflation expectations are formed. The effect of $\frac{1}{\sigma}$ on the second-period outcomes will thus be perfectly anticipated. Indeed, the share of $\frac{1}{\sigma}$ that is transmitted into the second period through debt policy becomes part of the deterministic component of the second-period government financing requirement (when viewed from the start of the second period).

The second way in which the coefficient of the stochastic shock $\frac{1}{\sigma}$ differs from the coefficient of \bar{F} , involves the intertemporal allocation of the government financing requirement. In particular, the share of $\frac{1}{\sigma}$ absorbed in the first period (relative to the second period) is larger than that of \bar{F} ($(1 - \alpha)(P^{\pi} = P) c_1 > (1 - \alpha) c_0$ and $c_1 < c_0$, where c_0 and c_1 are defined in Table 1). The reason is again that first-

period inflation expectations are predetermined when the stochastic shock hits. This enables the policymakers to absorb a relatively large share of the stochastic shock in the first period through an inflation surprise.

The responses of the output and government spending gaps to F_i^Φ and $\frac{z_i}{\sigma}$ differ from the responses to F and $\frac{z}{\sigma}$. Since inflation is attuned to cross-country averages, it cannot respond to country-specific circumstances as captured by F_i^Φ and $\frac{z_i}{\sigma}$. Accordingly, taxes (the output gap) and the government spending gap have to fully absorb these country-specific components of the government financing requirements.

3.2. Public debt policy

The solution for debt accumulation in the second-best equilibrium can be written as:

$$d_{i1}^S = d_1^{e;S} + d_{i1}^{\Phi;e;S} + d_1^{d;S} + d_{i1}^{\pm;S}; \quad (3.3)$$

where

$$d_1^{e;S} = \frac{h}{1 + \beta^{-n} (1 + \frac{1}{2})} \frac{\bar{K}_1 + (1 + \frac{1}{2}) d_0 \bar{K}_2 + (1 + \beta^{-n}) \bar{K}_2}{1 + \beta^{-n} (1 + \frac{1}{2})}; \quad (3.4)$$

$$d_{i1}^{\Phi;e;S} = \frac{h}{1 + \beta^{-n} (1 + \frac{1}{2})} \frac{\bar{K}_{i1}^\Phi + (1 + \frac{1}{2}) d_{i0}^\Phi \bar{K}_{i2}^\Phi + (1 + \beta^{-n}) \bar{K}_{i2}^\Phi}{1 + \beta^{-n} (1 + \frac{1}{2})}; \quad n > 1; \quad (3.5)$$

= 0; n = 1;

$$d_1^{d;S} = \frac{1}{1 + \beta^{-n} (1 + \frac{1}{2})} \frac{1}{\sigma}; \quad (3.6)$$

$$d_{i1}^{\pm;S} = \frac{1}{1 + \beta^{-n} (1 + \frac{1}{2})} \frac{z_i}{\sigma}; \quad n > 1; \quad (3.7)$$

= 0; n = 1;

where the superscript "S" stands for "second-best equilibrium", the superscript "e" denotes the expectation of a variable, an upperbar above a variable indicates its cross-country average (except for variables carrying a tilde, like \bar{K}_1 , where the cross-country average is indicated by dropping the country-index), a superscript " Φ " denotes an idiosyncratic deviation of a deterministic variable from its cross-country average (for example, $\bar{K}_{i1}^\Phi = \bar{K}_{i1} - \bar{K}_1$), a superscript "d" denotes

the response to a common shock, a superscript “±” indicates the response to an idiosyncratic shock, and where

$$\begin{aligned}
 P^{\pm} &= (1 + \frac{1}{2}) ; & (3.8) \\
 P &= \hat{A}^2 = \frac{\sigma^2}{\sigma_g^2} + 1 = \sigma^2 + 1 = \sigma_g ; \\
 P^{\pm} &= (\hat{A} + 1)^2 = \frac{\sigma^2}{\sigma_g^2} + 1 = \sigma^2 + 1 = \sigma_g ;
 \end{aligned}$$

Hence, optimal debt accumulation (3.3) is the sum of two deterministic components and two stochastic components. The component $d_1^{e;S}$ optimally distributes over time the absorption of the cross-country averages of the deterministic components of the government financing requirements. Therefore, it is common across countries. The country-specific components $d_{i1}^{e;S}$ intertemporally distribute the idiosyncratic deterministic components of the government financing requirements. The common (across countries) component $d_1^{d;S}$ represents the optimal debt response to the common shock ϵ_1 , while $d_{i1}^{d;S}$ stands for the optimal debt response to the country-specific shock, ϵ_i .

The debt response to the common shock is less active than the response to the idiosyncratic shock (since $P^{\pm} = P > 1$). The common inflation rate can exploit the predetermination of inflation expectations only in responding to the common shock, because the common inflation rate can not be attuned to idiosyncratic shocks. Hence, the share of the common shock that can be absorbed in the first period can be larger than the corresponding share of the idiosyncratic shock. Public debt thus needs to respond less vigorously to the common shock.

4. Discretionary monetary policy with decentralized fiscal policy

This section introduces two distortions compared with the second-best equilibrium explored in the previous section. First, the CCB is no longer able to commit to monetary policy announcements. Second, fiscal policy is decentralized to individual governments, which may result in wasteful strategic interaction among heterogeneous governments.

From now on, the timing of events in each period is as follows. At the start of the period, the institutional parameters are set. That is, an inflation target is imposed on the CCB for the coming period and, if applicable, the debt targets on the individual governments are set. The inflation target may be conditioned on the state of the world. In particular, the inflation target may depend on the average debt level in the union.⁷ Furthermore, the debt target, which represents

⁷The optimal inflation target can either be optimally reset at the start of each period, or

the amount of public debt that a government has to carry over into the next period, may be shock-contingent.⁸ After the institutional parameters have been set, inflation expectations are determined (through the nominal wage-setting process). Third, the shock(s) materialize. Fourth, taking inflation expectations as given, the CCB selects the common inflation rate and the fiscal authorities simultaneously select taxes and, in the absence of a debt target, public debt. Each of the players takes the other players' policies at this stage as given. Finally, public spending levels are residually determined. As a result, the CCB internalizes the effect of its policies on the government budget constraints.

This section explores the outcomes under pure discretion, i.e. in the absence of both inflation targets (i.e., $\pi_1^a = \pi_2^a = 0$) and debt targets. The complete derivation of the equilibrium is contained in Appendix B. The suboptimality of the resulting equilibrium compared to the second best motivates the exploration of inflation and debt targets in Section 5.

4.1. Inflation, the output gap and the public spending gap

Table 2 contains the solutions for the inflation rate, the output gap and the spending gap. The main difference compared to the outcomes under the second-best equilibrium (see Table 1) is that, for a given amount of debt d_{i1} to be carried over into the second period, expected first-period inflation (and, hence, seigniorage if $\hat{A} > 0$) will be higher (compare the term between the square parentheses in the second column and the second row of Table 2 with the corresponding term in Table 1 and observe that $[\hat{A}(\hat{A} + 1)^{-\frac{1}{\mu}}] = S > (\hat{A}^2)^{-\frac{1}{\mu}} = P$, where $S > \hat{A}(\hat{A} + 1)^{-\frac{1}{\mu}} + 1 = \sigma^2 + 1 = \frac{1}{g}$). The source of the higher expected inflation rate under pure discretion is the inability to commit to a stringent monetary policy, which yields the familiar inflation bias (Barro and Gordon, 1983). The outcomes for inflation, the output gap and the spending gap deviate from the outcomes under the second-best equilibrium also because debt accumulation under pure discretion differs from debt accumulation under the second best. These differences are discussed below.

4.2. Public debt policy

Government i 's debt can, analogous to (3.3), be written as:

$$d_{i1}^D = d_{i1}^{e;D} + d_{i1}^{\phi;e;D} + d_{i1}^{d;D} + d_{i1}^{z;D}; \quad (4.1)$$

be determined according to a state-contingent rule selected at the beginning of the first period. These two alternative interpretations yield equivalent results.

⁸Debt at the end of the second period is restricted to be zero. Hence, the second period features a debt target of zero.

where the superscript "D" is used to indicate the solution of the purely discretionary equilibrium with decentralized ...scal policies and where

$$d_1^{e;D} = \frac{h \kappa_1 + (1 + \frac{1}{2}) d_0 \kappa_2 + [1 - \alpha (S^a = S)] \kappa_2}{1 + \alpha (1 + \frac{1}{2}) (S^a = S)}; \quad (4.2)$$

$$d_{i1}^{c;e;D} = \frac{h \kappa_{i1}^c + (1 + \frac{1}{2}) d_{i0}^c \kappa_{i2}^c + [1 - \alpha (Q = S)] \kappa_{i2}^c}{1 + \alpha (1 + \frac{1}{2}) (Q = S)}; \text{ if } n > 1; \quad (4.3)$$

$$= 0; \text{ if } n = 1;$$

$$d_1^{d;D} = \frac{1}{1 + \alpha (1 + \frac{1}{2}) (S^a = S) (P^a = S)}; \quad (4.4)$$

$$d_{i1}^{s;D} = \frac{1}{1 + \alpha (1 + \frac{1}{2}) (Q = S)}; \text{ if } n > 1; \quad (4.5)$$

$$= 0; \text{ if } n = 1;$$

and where

$$S \sim \hat{A} (\hat{A} + 1)^{\frac{1}{4}} + 1 = \alpha^2 + 1 = \alpha_g; \quad (4.6)$$

$$S^a \sim \hat{A} (\hat{A} + 1)^{\frac{1}{4}} + (\hat{A} + 1) = (n^{\frac{1}{4}}) + 1 = \alpha^2 + 1 = \alpha_g;$$

$$Q \sim [(n - 1) \hat{A} (\hat{A} + 1)^{\frac{1}{4}}] + 1 = \alpha^2 + 1 = \alpha_g;$$

4.2.1. Response to the common deterministic components of the government ...nancing requirements

Positive analysis:

This subsection explores the solution for expected average debt $d_1^{e;D}$ in (4.2). Whereas current inflation expectations are predetermined at the moment that debt is selected, future inflation expectations still need to be determined. A reduction in debt reduces the future government ...nancing requirement and, thus, the tax rate in the future. This, in turn, weakens the CCB's incentive to raise future inflation in order to protect employment. Hence, by restraining debt accumulation, governments help to reduce future inflation expectations, which are endogenous from a ...rst-period perspective. The reduction in future inflation expectations implies a lower inflation bias in the future. In other words, asset accumulation is an indirect way to enhance the commitment of a central bank to low future inflation.

Expected average debt $d_1^{e:D}$ increases in the size of the union (because $\partial d_1^{e:D} / \partial [r^m (S^m=S)] < 0$ and because $S^m=S$ is decreasing in n – see Beetsma and Bovenberg, 1999): in a larger union, each individual government perceives a smaller effect of a unilateral reduction in public debt on the common future inflation rate. Hence, the incentive to restrain debt becomes weaker.⁹ Indeed, in a monetary union, the credibility of the common monetary policy has the features of a public good.

Normative analysis:

Expected average debt accumulation is suboptimally low (because $S^m=S > 1$, $r^m (S^m=S) > r^m$ and, hence, $d_1^{e:D} < d_1^{e:S}$ – see also Beetsma and Bovenberg, 1999). The source of this underaccumulation of debt is the lack of commitment in monetary policy, which gives rise to an inflation bias. In order to strengthen the credibility of future monetary policy and thus reduce the inflation bias, governments try to exploit the predetermined nature of first-period inflation expectations by absorbing a relatively large part of the intertemporal government financing requirement in the first period. In equilibrium, these attempts to exploit the fixed nature of inflation expectations are ineffective, however, because the private sector anticipates the incentive facing the governments to exploit first-period inflation expectations and thus sets first-period wages (inflation expectations) higher than under commitment. Hence, the first-period equilibrium inflation rate, output gap and spending gap will be higher than under commitment. If the union becomes arbitrarily large (i.e. $n \rightarrow \infty$), individual governments are no longer able to affect the credibility of the common future monetary policy. This source of underaccumulation of debt thus disappears. Hence, as $n \rightarrow \infty$, $d_1^{e:D} = d_1^{e:S}$.

4.2.2. Response to country-specific deterministic components of the government financing requirements

Positive analysis:

This subsection investigates the response of public debt to the deviations of the deterministic components of the government financing requirements from the corresponding cross-country averages, $d_1^{c:e}$. In equilibrium, these country-specific deviations of the financing requirements are fully absorbed by first- and second-period deviations of output and public spending from their targets. The reason is that monetary policy and, hence, inflation is attuned to the average conditions

⁹An analogous mechanism features in Cukierman and Lippi (1999), who explore how the (nominal) wage demands of trade unions change as a result of a switch from national monetary policymaking to a monetary union. In a monetary union, trade unions internalize the inflationary consequences of higher wage demands to a lesser extent. Hence, the incentive to restrain wages is weakened.

in the union and thus does not respond to the country-specific components of the government financing requirements.

Inspection of (4.3) and the definition of Q reveals that, if $\hat{A} > 0$, an increase in n weakens the (positive) response of debt to the first-period component $K_{i1}^c + (1 + \frac{1}{2})d_{i0}^c > 0$, while it strengthens the negative response to $K_{i2}^c > 0$ (i.e., the response becomes more negative). To explain the intuition behind these effects, we first consider the case with $K_{i1}^c + (1 + \frac{1}{2})d_{i0}^c > 0$ and $K_{i2}^c = 0$. In that case, expected debt accumulation in country i exceeds average expected debt accumulation in the union, i.e. $d_{i1}^{c,e;D} > 0$. Hence, government i has a relatively large need for seigniorage revenues in the second period so that its preferred second-period inflation rate is relatively high (if $\hat{A} > 0$). This discrepancy between government i 's preferred second-period inflation rate and the preferred second-period inflation rate of the average union member, as well as the fact that government i in the first period acts as a Stackelberg leader against the CCB in the second period, induces government i to strategically further raise its debt.¹⁰ The resulting higher second-period tax rate forces the CCB to raise the second-period inflation rate (of course, in equilibrium, the inflation rate is unaffected by country-specific factors, but this is neglected by the individual, optimizing government). However, in a larger union the effect of such an increase in debt on the common inflation rate will be diluted and, hence, the incentive to use debt strategically is weaker.

Now, suppose that $K_{i2}^c > 0$ and $K_{i1}^c + (1 + \frac{1}{2})d_{i0}^c = 0$. Debt accumulation by country i will be below average, while the need for seigniorage revenues in the second period will be relatively large. Hence, government i 's preferred second-period inflation rate will be relatively high (if $\hat{A} > 0$). Therefore, it raises debt strategically (compared with the case in which seigniorage is absent) to force the CCB to bring the future inflation rate more in line with its own preferred future inflation rate. Again, in a larger union the effect of unilateral changes in debt accumulation on the common inflation rate is weaker and, hence, the incentive to raise debt strategically is weaker. Hence, in a larger union, $d_{i1}^{c,e;D}$ will be lower, ceteris paribus.¹¹

Normative analysis:

¹⁰First-period debt and first-period inflation are simultaneously chosen, while the monetary and fiscal policymakers take each other's decisions as given. Therefore, when selecting debt, governments ignore the effect of first-period debt on first-period seigniorage.

¹¹If $\hat{A} = 0$, preferences concerning the second-period inflation rate are the same across governments and, hence, governments will not strategically use debt. Indeed, if $\hat{A} = 0$, $d_{i1}^{c,e;D}$ is unaffected by n . Within the context of our simple model, therefore, the presence of seigniorage revenues is crucial for countries to engage in strategic debt accumulation. However, governments may differ in their preferences about future monetary policy for other reasons as well (for example, a different timing of business cycles). In that case, governments may also strategically use debt policy to affect the future stance of monetary policy.

Countries with relatively severe product and labor market distortions, or high initial debt levels (i.e., countries with $F_i^C > 0$), feature a relatively high country-specific expected debt component. In fact, these countries overaccumulate debt (at least as far as the debt component $d_{i1}^{C;e;D}$ is concerned, $d_{i1}^{C;e;D} > d_{i1}^{C;e;S}$).¹² The opposite holds true for countries with a relatively low intertemporal government financing requirement (i.e. $F_i^C < 0$). Governments featuring a high intertemporal government financing requirement (i.e. $F_i^C > 0$) overaccumulate debt in order to encourage the CCB to raise inflation in the second period, thereby bringing second-period inflation more in line with these governments' preferred inflation rate. These governments fail to internalize the negative externalities of this behavior on the governments with relatively small distortions or low initial debt. Similarly, governments with low intertemporal government financing requirements (i.e. $F_i^C < 0$) do not internalize the negative externalities on other governments associated with the underaccumulation of debt. Hence, although in equilibrium average expected debt and inflation are unaffected, this failure to internalize the negative externalities of suboptimal debt accumulation leads to wasteful strategic behavior by causing the country-specific deterministic components of public debt to deviate from their values in the second-best equilibrium. A larger union weakens the incentive for strategic behavior:

$$\frac{\partial d_{i1}^{C;e;D}}{\partial n} = \frac{\mu}{\partial d_{i1}^{C;e;D}} \frac{\partial \pi}{\partial n} \frac{\partial d_{i1}^{C;e;D}}{\partial \pi} = i \frac{-\pi(1+\frac{1}{2})}{[1+\pi(1+\frac{1}{2})(Q=S)]^2} F_i^C \frac{\partial (Q=S)}{\partial n} ; \text{ if } n \geq 2; \quad (4.7)$$

which is negative (positive) if $F_i^C > (<)0$. Furthermore, as $n \rightarrow 1$, $d_{i1}^{C;e;D}$ converges to $d_{i1}^{C;e;S}$. As n rises, the difference between $d_{i1}^{C;e;D}$ and $d_{i1}^{C;e;S}$ thus becomes smaller so that the welfare loss that originates in strategic behavior declines.

4.2.3. Response to the common shock.

Positive analysis:

This subsection turns to the response of debt policy to the common shock ϵ^1 . As is the case for the second-best equilibrium, the share of the common stochastic shock ϵ^1 that is absorbed in the first period is larger than the share of the common deterministic component of the intertemporal government financing requirement F that is absorbed in the first period (see Table 2 and note that $P^\pi > S$).

Monetary unionification (i.e., an increase from $n = 1$ to $n > 1$) intensifies the response of public debt to unanticipated supply shocks (see (4.4) and note that

¹²Since $-\pi(Q=S) < -\pi$ and $\partial d_{i1}^{C;e;D} / \partial [-\pi(Q=S)] < (>)0$ if $F_i^C > (<)0$ (see also (4.7), below), $d_{i1}^{C;e;D} > (<)d_{i1}^{C;e;S}$ if $F_i^C > (<)0$.

π^e ($S^e=S$) is declining in n). Hence, in a monetary union, governments engage in more active debt stabilization policies to union-wide shocks than under national policy making.

The reason for more active debt stabilization is as follows. Fiscal authorities choose to absorb a relatively large part of an adverse first-period supply shock immediately in order to exploit the predetermination of first-period inflation expectations. As a result, second-period inflation expectations will to a lesser extent be affected by the common shock and, hence, inflation variability will be smaller in the second period. In a monetary union, however, this effect is smaller than under national monetary policymaking because each individual union member perceives a relatively small effect of its actions on the future variability of the union-wide inflation rate. A monetary union thus yields more variability of public debt and future inflation. Accordingly, even if shocks are shared by the countries, monetary union results in more active debt stabilization.

Normative analysis:

Is the additional variability of debt produced by monetary union excessive? Beetsma and Bovenberg (1999) show that, given that the CCB is not able to commit, the socially-optimal response of debt to common shocks amounts to:

$$d_1^{d:opt} = \frac{1}{1 + \pi^e (1 + \frac{1}{2}) (P^e=S)^2} \frac{1}{\sigma}; \quad (4.8)$$

This response is actually attained with national monetary policymaking (i.e., $n = 1$). As explained above, monetary union ($n > 1$) leads to a more active response of debt to common shocks (i.e. for $\pi^e > 0$, $d_1^{d:D} > d_1^{d:opt}$). The larger response of debt to uniform shocks is welfare reducing. Intuitively, reducing future inflation variability is a public good in a monetary union. Hence, individual countries freeride on each other when taking measures to reduce inflation variability.

The coefficient of $\frac{1}{\sigma}$ in (4.8) is smaller than the corresponding coefficient in (3.6) in the second-best equilibrium (since $S^2 < P^e P$). Hence, the response of debt to common shocks, as prescribed by (4.8), is too "conservative" when compared to the second best (i.e. for a bad shock ($\pi^e > 0$), $d_1^{d:opt} < d_1^{d:S}$). By absorbing more of π^e in the first period, this shock has less of an effect on future inflation expectations and, hence, the future inflation bias due to a lack of commitment in monetary policy.

Debt policy given by $d_1^{d:opt}$ thus deviates from $d_1^{d:S}$ as a result of the trade-off between future monetary policy credibility and a suboptimal distribution of welfare losses over time. This trade-off is a variant of the well-known "credibility-flexibility" trade-off. In this particular case, flexibility refers to activeness of the response of debt, rather than inflation, to shocks; to enhance the credibility of

future monetary policy, the government reduces its flexibility in employing public debt to absorb shocks.

4.2.4. Response to the idiosyncratic shock.

Positive analysis:

Like the country-specific deterministic components of the intertemporal government financing requirement (see Subsection 4.2.2), the idiosyncratic shock z_i is exclusively dealt with at the national level through fiscal policy. A bad idiosyncratic shock (i.e., $z_i > 0$) raises the idiosyncratic debt component, $d_{i1}^{z;D}$.¹³ Inspection of (4.5) and the definition of Q makes clear that, if $\hat{A} > 0$, an increase in n weakens the response of $d_{i1}^{z;D}$ to z_i (i.e., the coefficient of $z_i=0$ in (4.5) becomes smaller). In other words, and in contrast to the response of debt to a common shock, debt responds less actively to idiosyncratic shocks as the union becomes larger. Hence, contrary to common wisdom, in a larger union a country will use less debt stabilization in response to country-specific shocks.

The explanation is the same as the earlier explanation for the dependency of $d_{i1}^{z;D}$ on the number of union participants. If country i is hit by a bad shock $z_i > 0$, it issues more debt than the average government in the union. Hence, if $\hat{A} > 0$, government i has a relatively large need for seigniorage revenues in the second period and, hence, its preferred inflation rate in that period is relatively high. Government i thus strategically raises debt further to bring future inflation more in line with its preferred inflation rate. In a larger union, the perceived influence of government i 's policies on the common monetary policy is reduced and, hence, the incentive to use debt strategically is weakened. If $\hat{A} = 0$, all governments share the same preferences concerning the common second-period inflation rate.¹⁴ Hence, governments have no reason to use debt strategically. Indeed, in that case, $d_{i1}^{z;D}$ does not depend on the number of countries.

¹³ $d_{i1}^{z;D}$ is zero in the case of national monetary policymaking (i.e., $n = 1$). In that case, $z_i = 0$ (as we have assumed that the cross-country average of the idiosyncratic shocks is zero). This constraint has explicitly been used in the derivation of the equilibrium. If we had assumed that $z_i \neq 0$, then $\frac{1}{\sigma}$ in (4.4) would have been replaced with $\frac{1+z_i}{\sigma}$, but $d_{i1}^{z;D}$ would have remained at zero.

¹⁴Although country i may have been hit by a relatively bad shock $z_i > 0$, suggesting a larger need for inflation as a stabilizing tool, its preferred second-period inflation rate does not differ from the other governments' preferred inflation rate. The reason is that second-period inflation expectations adjust for the part of the shock that is transmitted into the second period. Hence, in the second period inflation no longer has a role in stabilizing the effects of z_i on the economy.

Normative analysis:

A comparison of $d_{i1}^{\pm;D}$ and $d_{i1}^{\pm;S}$ reveals that the response of debt to idiosyncratic shocks is more vigorous than under the second-best equilibrium. A larger union renders the debt response less active. The associated reduction in wasteful strategic interaction boosts welfare. If the union becomes infinitely large (i.e., $n \rightarrow 1$), $d_{i1}^{\pm;D}$ converges to $d_{i1}^{\pm;S}$ and the response of debt to idiosyncratic shocks becomes optimal. In that case, all wasteful strategic interactions are eliminated.

4.2.5. Summary of the effects of a larger union

If the number of countries goes to infinity and all strategic interactions among the governments disappear, the debt components $d_1^{e;D}$, $d_{i1}^{\phi;e;D}$ and $d_{i1}^{\pm;D}$ all converge to the same response coefficients $1 = (1 + \alpha^{-1}(1 + \frac{1}{2}))$. This response is in fact optimal (compare (4.2) with (3.4), (4.3) with (3.5) and (4.5) with (3.7) for $n \rightarrow 1$). The response $d_1^{d;D}$ to the common shock, in contrast, is optimal and equal to (4.8) if $n = 1$. It becomes excessive if $n > 1$, and the more so the larger the union becomes. Hence, only the response of debt to the common shock will be suboptimal for $n \rightarrow 1$. Actually, the welfare loss associated with this response is largest in an infinitely large union. Even though for $n \rightarrow 1$ the debt response to idiosyncratic shocks is more active than the response to common shocks, the former is optimal, while the latter is excessive.

5. Optimal institutional arrangements

This section investigates institutional adjustments that can help the monetary union to reach better equilibria. In particular, we allow for inflation targets. The second-period inflation target may depend on first-period debt levels and, thereby, indirectly, on first-period shocks. In addition, we allow for debt targets. These debt targets should be hit exactly. Hence, they act as debt ceilings when they prevent overaccumulation of debt, while they act as debt floors when they prevent underaccumulation of debt. The debt targets may depend on the shocks, i.e. one can write $d_{i1}^T = d_{i1}^T(1; z_i)$, where the superscript "T" stands for "target". Inflation targets can be viewed as a contractual way to deal with the commitment problem in monetary policy. In the same way, debt targets are a contractual way to address externalities.

Inflation targets can be enforced by giving the central banker financial incentives to meet the target or by making reappointment of the central banker dependent upon meeting the target.¹⁵ Reputational considerations may also help to

¹⁵This is the case for New Zealand – see Walsh (1995) for a detailed account of this arrangement.

enforce inflation targets. The announcement of an inflation target provides market participants with a benchmark against which the central banker can be held accountable. Failure to meet the target indicates a lack of willingness or ability on the side of the central banker to stick to the announcements. This information will be taken into account when future inflation expectations are formed.

Debt targets may be enforced through peer pressure (the loss of political prestige if the target is missed) and fines. Indeed, under the so-called Stability and Growth Pact (SGP), EMU participants who persistently violate the ceilings on public deficits will be fined (for more details on the SGP, see Artis and Winkler, 1998, and Eichengreen and Wyplosz, 1998).¹⁶ In the sequel, we assume that both the inflation and the debt targets can be enforced.

This section explores how institutional rearrangements (inflation targets and debt targets) can induce optimal responses of debt to the various components of the intertemporal government financing requirement. In particular, it will investigate whether these arrangements ensure that debt accumulation mimics debt accumulation in the second-best equilibrium.

5.1. Identical economies ($K_{i1} = K_1$, $K_{i2} = K_2$, $d_{i0} = d_0$ and $z_i = 0$, δ_i)

This subsection assumes that the union participants are completely identical. Hence, the deterministic components of the government financing requirements are equal across the countries ($K_{i1} = K_1$, $K_{i2} = K_2$ and $d_{i0} = d_0$, δ_i), while idiosyncratic shocks are absent ($z_i = 0$, δ_i). As a result, all governments adopt identical policies. The solution for d_{i1} consists only of the response to the deterministic components of the average intertemporal government financing requirement and the response to the common shock. Potential strategic interactions arising from a disagreement about the common monetary policy are absent. Hence, optimal institutional design needs to address only the lack of commitment of monetary policy. Appendix C proves the following proposition:

Proposition 1. Suppose that $K_{i1} = K_1$, $K_{i2} = K_2$, $d_{i0} = d_0$ and $z_i = 0$, δ_i . In that case, the following combination of state-contingent inflation targets

$$\pi_1^s(h^e) = \frac{h}{P} \frac{1-\theta}{\theta} K_1 + (1 + \frac{1}{2}) d_0 \frac{h}{P} ; \quad (5.1)$$

$$\pi_2^s(h) = \frac{h}{P} \frac{1-\theta}{\theta} K_2 + (1 + \frac{1}{2}) h ; \quad (5.2)$$

with h equal to the cross-country average realized debt level d_1 , ensures that the decentralized, discretionary equilibrium coincides with the second-best equilibrium.

¹⁶Under exceptional circumstances (in particular, a large fall in GDP) the sanctions envisaged by the Pact may be waived. This aspect of the Pact to some extent resembles the contingent nature of the debt targets explored in this section.

Proposition 1 reveals that, in contrast to making the central banker conservative à la Rogo α (1985) – see Beetsma and Bovenberg (1999) –, the state-contingent inflation target succeeds in establishing the second best (see also Svensson, 1997, and Beetsma and Jensen, 1999). A conservative central bank addresses the commitment problem (the inflation bias), but at the same time distorts the stabilization of shocks. A state-contingent inflation target, in contrast, addresses the inflation bias without distorting stabilization. Thus, the contractual solution to the commitment problem (à la Svensson, 1997) dominates the solution of delegation (à la Rogo α , 1985). The contract, however, needs to be quite rich. In particular, the inflation target needs to be state-contingent, because the size of the (second-period) inflation bias depends on the amounts of debt issued by the union participants. A state-independent inflation target would not be able to establish the second best.

By addressing the commitment problem through a state-contingent inflation target, one also eliminates the intertemporal distortions that originate in the lack of monetary policy credibility. In particular, by setting the second-period inflation target at the proposed level (5.2), the second-period inflation bias is eliminated. Accordingly, without a second-period inflation bias, governments in the first period no longer perceive the need to underaccumulate (for the purpose of enhancing the credibility of monetary policy in the second period) debt in response to the deterministic components of the government financing requirement. The absence of the second-period inflation bias also takes away the need to absorb an excessively large share of the common shock in the first period and thus ensures an optimal debt response to ϵ . Finally, by setting the first-period inflation target conform (5.1), the first-period inflation bias is eliminated.

5.2. Differences among countries

This subsection allows for cross-country differences both in the deterministic components of the government financing requirements (i.e., K_{i1} ; K_{i2} and d_{i0}) and in the stochastic shocks (i.e., the ϵ_i 's are no longer assumed to be zero). As explained in Section 4, such differences among countries produce conflicts about the preferred future stance of the common monetary policy. The conflicts result in wasteful strategic interactions between decentralized fiscal policymakers. Debt targets eliminate these conflicts of interest and the associated costly strategic interactions.

The following proposition is proven in Appendix D:

Proposition 2. With deterministic differences in the government financing requirements and with idiosyncratic shocks, the combination of inflation and debt targets that minimizes V_U is obtained by setting the first- and second-period inflation targets at, respectively, (5.1) and (5.2), with h equal to the cross-country average realized debt level d_1 , and the (specific) debt target d_{i1}^T on country i ($i = 1; \dots; n$) at $d_{i1}^T = d_1^{e;S} + d_{i1}^{c;e;S} + d_1^{d;S} + d_{i1}^{z;S}$. The resulting equilibrium coincides with the second-best equilibrium.

By imposing debt targets attuned to each country's specific situation, the union can thus eliminate the externalities associated with the strategic behavior of the individual governments. Hence, with the proposed inflation and debt targets in place, the responses of public debt (and the other policy instruments) mimic their counterparts under the second-best equilibrium. In particular, compared with the purely discretionary equilibrium, debt targets restrict debt to be less active in response to the country-specific components of the government financing requirements (F_i^c and $\frac{z_i}{\sigma}$). They thus operate as a ceiling on the associated debt responses if $F_i^c > 0$ or $\frac{z_i}{\sigma} > 0$, and as a floor if $F_i^c < 0$ or $\frac{z_i}{\sigma} < 0$. Only if $\hat{A} = 0$ or $n = 1$, are debt targets redundant and are optimal inflation targets sufficient for the discretionary equilibrium to coincide with the second-best equilibrium. In that case, no debt targets are needed, because the effect of a unilateral change in debt on seigniorage revenues becomes negligible and any strategic effects disappear.

6. Conclusion

This paper investigated the interaction between fiscal policy and monetary policy in a monetary union. Our analysis has allowed for two imperfections. One is the lack of monetary policy commitment. The other involves spillovers among decentralized fiscal policymakers. We explored how inflation targets and debt targets can alleviate the welfare losses arising from these imperfections.

With identical economies, imposing the optimal state-contingent inflation target on the common central bank is sufficient to establish the second-best equilibrium. If countries are heterogeneous, however, the inflation target needs to be complemented by debt targets. These debt targets eliminate the strategic, welfare-reducing interactions among governments arising from differences among the union participants on the preferred stance of monetary policy. The inflation and debt targets can be viewed as a contractual solution to the lack of commitment in monetary policy and the spillovers among the decentralized fiscal policymakers.

The analysis can be extended into a variety of directions. One extension would be to allow for a longer modelling horizon and to explore the optimal, dynamic paths for the inflation and debt targets. In particular, it would be interesting to

explore whether debt targets converge over time for countries with different initial debt levels. Another extension is to investigate whether and how the debt targets can be enforced. Indeed, the enforcement of the Stability and Growth Pact is subject to some doubt. Such an analysis would require a dynamic framework that accounts for reputational considerations. The countries with the smallest product- and labor-market distortions and the countries with the lowest initial debt level can be expected to most favor strict enforcement of the targets. In a multiperiod context, these countries will take into account the effects on the future behavior of governments (in terms of debt policies) of a failure to enforce the targets.

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Appendix

Notation:

We will use the following conventions: superscript e denotes the subjective expectation of a variable. When no confusion is possible, it is also used to denote the mathematical (model-induced) expectation. The deviation of some variable y_{it} from its expected value is denoted by superscript “ d ”: $y_{it}^d = y_{it} - y_{it}^e$. An upperbar denotes the cross-country average of a variable: $\bar{y}_t = \frac{1}{n} \sum_{i=1}^n y_{it}$. The exception is a variable with a tilde, whose cross-country average is denoted by the omission of the country index: $\bar{y}_t = \frac{1}{n} \sum_{i=1}^n \tilde{y}_{it}$. Note that $\bar{y}_t^e = \frac{1}{n} \sum_{i=1}^n y_{it}^e$ and that $\bar{y}_t^d = \bar{y}_t - \bar{y}_t^e$. Superscript “ Φ ” denotes the deviation of a variable from its cross-country average: $y_{it}^\Phi = y_{it} - \bar{y}_t$. Note that $y_{it}^{\Phi:e} = y_{it}^e - \bar{y}_t^e$. Finally, superscript “ \pm ” is used to denote the deviation of a country-specific variable from the sum of its expected value and the cross-country average difference between this variable and its expected value: $y_{it}^\pm = y_{it} - y_{it}^e + \bar{y}_t^d = y_{it} - \bar{y}_t^e - y_{it}^{\Phi:e} + \bar{y}_t^d$. Hence, using our notation, we can decompose a variable y_{it} into $y_{it} = \bar{y}_t^e + y_{it}^{\Phi:e} + \bar{y}_t^d + y_{it}^\pm$, i.e. the sum of the cross-country average expectation, the difference between the expectation of y_{it} and the average expectation, the cross-country average prediction error, and the difference between y_{it} and its expectation plus the cross-country average prediction error.

Assumption:

As in the main text, the sum of the idiosyncratic shocks is assumed to be zero: $\sum_{i=1}^n \varepsilon_i = 0$.

1. Derivation of the second-best equilibrium

Both monetary and fiscal policymaking are centralized. Moreover, monetary policy is conducted under commitment. The supranational centralized fiscal authority (CFA) minimizes an equally-weighted average of the participating countries’ social objectives. In solving for the equilibrium, we work backwards, starting with the second period.

1.1. Period 2

First, we solve for the second-period outcomes, given the first-period debt choices. The CCB’s Lagrangian is:

$$2\mathcal{L}_2^{\text{CCB}} = \frac{\alpha}{2} \bar{y}_2^2 +$$

$$\frac{1}{n} \sum_{i=1}^n \left[\frac{1}{2} (y_{i2} - y_{i2}^e - z_{i2}) - x_{i2} \right]^2 + \lambda_g \left[\frac{1}{2} (1 + \frac{1}{2}) d_{i1} + z_{i2} + \hat{A} y_{i2} - g_{i2} \right]^2 + 2\mu_2 \left[\frac{1}{2} (y_{i2} - y_{i2}^e) - g_{i2} \right]$$

where μ_2 is the Lagrange multiplier associated with the rational expectations constraint in period 2. The CCB's first-order conditions with respect to y_{i2} and y_{i2}^e can be written as:

$$\lambda_g y_{i2} + \frac{1}{n} \sum_{i=1}^n \left[\frac{1}{2} (y_{i2} - y_{i2}^e - z_{i2}) - x_{i2} \right] + \hat{A} \lambda_g (g_{i2} - g_{i2}) g_{i2} + \mu_2 = 0;$$

$$E \left[\frac{1}{n} \sum_{i=1}^n \left[\frac{1}{2} (y_{i2} - y_{i2}^e - z_{i2}) - x_{i2} \right] + \mu_2 \right] = 0;$$

We combine these two equations to:

$$\lambda_g y_{i2} + \frac{1}{n} \sum_{i=1}^n \left[\frac{1}{2} (y_{i2} - y_{i2}^e - z_{i2}) - x_{i2} \right] + \hat{A} \lambda_g (g_{i2} - g_{i2}) g_{i2} = 0; \quad (1.1)$$

$$E \left[\frac{1}{n} \sum_{i=1}^n \left[\frac{1}{2} (y_{i2} - y_{i2}^e - z_{i2}) - x_{i2} \right] + \mu_2 \right] = 0;$$

The loss function to be minimized by the CFA in the second period is:

$$\frac{1}{2n} \sum_{i=1}^n \left[\frac{1}{2} (y_{i2} - y_{i2}^e - z_{i2}) - x_{i2} \right]^2 + \lambda_g \left[\frac{1}{2} (1 + \frac{1}{2}) d_{i1} + z_{i2} + \hat{A} y_{i2} - g_{i2} \right]^2 :$$

The first-order conditions with respect to the z_{i2} can be written as:

$$\left[\frac{1}{2} (y_{i2} - y_{i2}^e - z_{i2}) - x_{i2} \right] = \lambda_g \left[\frac{1}{2} (1 + \frac{1}{2}) d_{i1} + z_{i2} + \hat{A} y_{i2} - g_{i2} \right]; \quad (1.2)$$

Imposing rationality of expectations (and noting that, in equilibrium, realizations coincide with expectations), we can write (1.1) and (1.2) as:

$$\lambda_g y_{i2} = \hat{A} \lambda_g (g_{i2} - g_{i2}); \quad (1.3)$$

$$\left[\frac{1}{2} (y_{i2} - y_{i2}^e - z_{i2}) - x_{i2} \right] = \lambda_g \left[\frac{1}{2} (1 + \frac{1}{2}) d_{i1} + z_{i2} + \hat{A} y_{i2} - g_{i2} \right]; \quad (1.4)$$

Using the second-period government budget constraints, one can rewrite (1.3) as:

$$\frac{1}{g} + \frac{\hat{A}^2}{\hat{A}^{\frac{2}{4}}} \frac{1}{4} = \frac{\hat{A}}{\hat{A}^{\frac{2}{4}}} K_2 + (1 + \frac{1}{2}) d_1 \quad ; \quad (1.5)$$

Take cross-country averages of (1.4) and combine this with (1.5) to eliminate $\frac{1}{4} + \frac{x_2}{g}$. The resulting equation can be rewritten to yield the solution for inflation, given average union debt, d_1 :

$$\frac{1}{4} = \frac{\hat{A}^{\frac{2}{4}}}{P} K_2 + (1 + \frac{1}{2}) d_1 \quad ; \quad (1.6)$$

where

$$P = \hat{A}^{\frac{2}{4}} + 1 = \frac{1}{g} + 1 \quad ; \quad (1.7)$$

as defined in the main text. Combine this with (1.4) to eliminate $\frac{1}{4}$ and rewrite the resulting equation to yield the solution for $\frac{1}{4} + \frac{x_2}{g}$:

$$\frac{1}{4} + \frac{x_2}{g} = \frac{\hat{A}}{1 = \frac{1}{g} + 1 = \frac{1}{g}} K_{i2} + (1 + \frac{1}{2}) d_{i1} \quad ; \quad \frac{\hat{A}^{\frac{2}{4}}}{P} K_2 + (1 + \frac{1}{2}) d_1 \quad ; \quad 8i: \quad (1.8)$$

Combining this with (2.2) and using that $\frac{1}{4} = \frac{1}{4}^e$, we obtain:

$$x_{i2} - x_{i2}^e = \frac{\hat{A}}{1 = \frac{1}{g} + 1 = \frac{1}{g}} K_{i2} + (1 + \frac{1}{2}) d_{i1} \quad ; \quad \frac{\hat{A}^{\frac{2}{4}}}{P} K_2 + (1 + \frac{1}{2}) d_1 \quad ; \quad 8i: \quad (1.9)$$

Take expectations of (1.2), use that in period 2 realizations match expectations and combine the result with (1.8) to obtain the solution for $\frac{1}{4} - \frac{1}{4}^e$:

$$\frac{1}{4} - \frac{1}{4}^e = \frac{\hat{A}}{1 = \frac{1}{g} + 1 = \frac{1}{g}} K_{i2} + (1 + \frac{1}{2}) d_{i1} \quad ; \quad \frac{\hat{A}^{\frac{2}{4}}}{P} K_2 + (1 + \frac{1}{2}) d_1 \quad ; \quad 8i: \quad (1.10)$$

The second-period loss of both the CCB and the CFA is given by:

$$L_2 = \frac{1}{2} \frac{\hat{A}^{\frac{2}{4}}}{P^2} K_2 + (1 + \frac{1}{2}) d_1 \quad ; \quad + \frac{1}{2n} \frac{1}{1 = \frac{1}{g} + 1 = \frac{1}{g}} \sum_{j=1}^n K_{j2} + (1 + \frac{1}{2}) d_{j1} \quad ; \quad \frac{\hat{A}^{\frac{2}{4}}}{P} K_2 + (1 + \frac{1}{2}) d_1 \quad ; \quad 10_2:$$

1.2. Period 1

We now move back to solve for the first-period outcomes. The Lagrangian of the CCB in period 1 is:

$$2\mathcal{L}_1^{\text{CCB}} = \sum_{i=1}^n \frac{1}{n} \left(\frac{1}{2} (\pi_{1i} - \pi_{1i}^e - \zeta_{i1}) - \pi_{1i} - \pi_{1i}^2 - \pi_{1i} x_{i1} \right)^2 + \sum_{i=1}^n \frac{1}{n} \left(\frac{1}{2} d_{i0} + \zeta_{i1} + \hat{A} \pi_{1i} + d_{i1} - g_{i1} \right)^2 + 2^{-1} L_2 + 2\mu_1 \sum_{i=1}^n \frac{1}{n} (\pi_{1i} - \pi_{1i}^e) g_{i1};$$

where μ_1 is the Lagrange multiplier associated with the rational expectations (of inflation) constraint in period 1. Note that L_2 is not affected by π_{1i} . Therefore, the CCB's first-order conditions with respect to π_{1i} and π_{1i}^e can be written as:

$$\sum_{i=1}^n \frac{1}{n} (\pi_{1i} - \pi_{1i}^e - \zeta_{i1}) - \pi_{1i} - \pi_{1i}^2 - \pi_{1i} x_{i1} + \hat{A} (g_{i1} - g_{i1}) g_{i1} + \mu_1 = 0;$$

$$\sum_{i=1}^n \frac{1}{n} (\pi_{1i} - \pi_{1i}^e - \zeta_{i1}) - \pi_{1i} - \pi_{1i}^2 - \pi_{1i} x_{i1} + \mu_1 = 0;$$

which can be combined to give:

$$\sum_{i=1}^n \frac{1}{n} (\pi_{1i} - \pi_{1i}^e - \zeta_{i1}) - \pi_{1i} - \pi_{1i}^2 - \pi_{1i} x_{i1} + \hat{A} (g_{i1} - g_{i1}) g_{i1} + \sum_{i=1}^n \frac{1}{n} (\pi_{1i} - \pi_{1i}^e - \zeta_{i1}) - \pi_{1i} - \pi_{1i}^2 - \pi_{1i} x_{i1} = 0; \quad (1.11)$$

The loss function to be minimized by the CFA in the first period is:

$$\sum_{i=1}^n \frac{1}{n} \left(\frac{1}{2} (\pi_{1i} - \pi_{1i}^e - \zeta_{i1}) - \pi_{1i} - \pi_{1i}^2 - \pi_{1i} x_{i1} \right)^2 + 2^{-1} L_2;$$

The first-order conditions with respect to ζ_{i1} and d_{i1} ($i = 1; \dots; n$) can be written as:

$$(\pi_{1i} - \pi_{1i}^e - \zeta_{i1}) - \pi_{1i} - \pi_{1i}^2 - \pi_{1i} x_{i1} + (g_{i1} - g_{i1}) g_{i1} = 0; \quad (1.12)$$

$$\frac{1}{n} (g_{i1} - g_{i1}) g_{i1} = -\frac{\partial L_2}{\partial d_{i1}}; \quad (1.13)$$

where

$$\begin{aligned}
L_{2,t} &= \frac{h}{h} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P^2} K_2 + (1 + \frac{1}{2}) d_1 \frac{1}{n} (1 + \frac{1}{2}) + \\
&\frac{1}{n} \frac{1}{1 = \sigma^2 + 1 = \hat{\theta}_g} \frac{1}{h} \frac{1}{P} \frac{1}{nh} K_{i2} + (1 + \frac{1}{2}) d_{i1} \frac{1}{i} \frac{h}{h} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P} K_2 + (1 + \frac{1}{2}) d_1 \frac{1}{\alpha} \\
&(1 + \frac{1}{2}) \frac{1}{i} \frac{1}{P} \frac{1}{nh} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P} K_{j2} + (1 + \frac{1}{2}) d_{j1} \frac{1}{i} \frac{h}{h} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P} K_2 + (1 + \frac{1}{2}) d_1 \frac{1}{\alpha} \\
&\frac{1}{n} \frac{1}{1 = \sigma^2 + 1 = \hat{\theta}_g} \frac{1}{h} \frac{1}{P} \frac{1}{nh} K_{j2} + (1 + \frac{1}{2}) d_{j1} \frac{1}{i} \frac{h}{h} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P} K_2 + (1 + \frac{1}{2}) d_1 \frac{1}{\alpha} \\
&= \frac{h}{h} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P^2} K_2 + (1 + \frac{1}{2}) d_1 \frac{1}{n} (1 + \frac{1}{2}) + \\
&\frac{1}{n} \frac{1}{1 = \sigma^2 + 1 = \hat{\theta}_g} \frac{1}{h} \frac{1}{P} \frac{1}{nh} K_{i2} + (1 + \frac{1}{2}) d_{i1} \frac{1}{i} \frac{h}{h} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P} K_2 + (1 + \frac{1}{2}) d_1 \frac{1}{\alpha} (1 + \frac{1}{2}) \frac{1}{i} \\
&\frac{1}{n} \frac{1}{1 = \sigma^2 + 1 = \hat{\theta}_g} \frac{1}{h} \frac{1}{P} \frac{1}{nh} K_{j2} + (1 + \frac{1}{2}) d_{j1} \frac{1}{i} \frac{h}{h} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P} K_2 + (1 + \frac{1}{2}) d_1 \frac{1}{\alpha} \frac{1}{n} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P} \frac{1}{\alpha} \\
&(1 + \frac{1}{2}) \\
&= \frac{1}{n} \frac{1}{1 = \sigma^2 + 1 = \hat{\theta}_g} \frac{1}{h} \frac{1}{P} \frac{1}{nh} K_{i2} + (1 + \frac{1}{2}) d_{i1} \frac{1}{i} \frac{h}{h} \frac{\hat{A}^2 = \hat{\theta}_{1/2}}{P} K_2 + (1 + \frac{1}{2}) d_1 \frac{1}{\alpha} (1 + \frac{1}{2}) :
\end{aligned}$$

Finally, we can rewrite the ...rst-period government budget constraint as:

$$K_{i1} + (1 + \frac{1}{2}) d_{i0} = (\dot{c}_{i1} + x_{i1} = \circ) + \hat{A} \frac{1}{4} + (g_{i1} i g_{i1}) + d_{i1}; 8i; \quad (1.14)$$

The system of ...rst-order conditions to be solved is thus given by (1.11), (1.12), (1.13) and (1.14). Take some country-speci...c policy variable y_{i1} ($y_{i1} = \frac{1}{4} i \dot{c}_{i1}; g_{i1}$ or d_{i1}). We will solve for y_{i1} by solving for each of the components of the decomposition of y_{i1} into $y_{i1} = \hat{y}_1^e + y_{i1}^{\Phi:e} + \hat{y}_1^d + y_{i1}^D$, where \hat{y}_1^e will be the response to the cross-country average of the deterministic components of the government ...nancing requirements, $y_{i1}^{\Phi:e}$ will be the response to the country-speci...c deterministic components of the government ...nancing requirements, \hat{y}_1^d will be response to the common shock and y_{i1}^D will be the response to the idiosyncratic shock. We thus compute the solutions to these variables in four steps.

1.2.1. Steps 1 and 2

Take expectations across (1.11), (1.12), (1.13) and (1.14) to yield:

$$\hat{\theta}_{1/2}^e = \hat{A} \hat{\theta}_g (g_{i1} i g_{i1}^e); \quad (1.15)$$

$$\sigma^2 (\dot{c}_{i1}^e + x_{i1} = \circ) = \hat{\theta}_g (g_{i1} i g_{i1}^e); 8i; \quad (1.16)$$

$$\frac{1}{n} \otimes_g (\mathfrak{g}_{i1} \text{ i } \mathfrak{g}_{i1}^e) = \text{ }^{\circ} (\text{L}_2 = \text{d}_{i1})^e ; 8i; \quad (1.17)$$

$$\mathbb{K}_{i1} + (1 + \frac{1}{2}) d_{i0} = (\mathcal{J}_{i1}^e + \mathbf{x}_{i1} = \text{ }^{\circ}) + \hat{A} \mathbb{V}_1^e + (\mathfrak{g}_{i1} \text{ i } \mathfrak{g}_{i1}^e) + d_{i1}^e ; 8i; \quad (1.18)$$

Step 1: Computation of expectations of cross-country averages. Take cross-country averages across (1.15)-(1.18) to yield:

$$\otimes_g \mathbb{V}_1^e = \hat{A} \otimes_g (\mathfrak{g}_1 \text{ i } \mathfrak{g}_1^e); \quad (1.19)$$

$$\text{ }^{\circ 2} (\mathcal{J}_1^e + \mathbf{x}_1 = \text{ }^{\circ}) = \otimes_g (\mathfrak{g}_1 \text{ i } \mathfrak{g}_1^e); \quad (1.20)$$

$$\frac{1}{n} \otimes_g (\mathfrak{g}_1 \text{ i } \mathfrak{g}_1^e) = \frac{1}{n} \text{ }^{\circ} (1 + \frac{1}{2}) \frac{\mathbf{h} \text{ i } \mathbf{h}}{\mathbb{P}} \mathbb{K}_2 + (1 + \frac{1}{2}) d_1^e \text{ i}; \quad (1.21)$$

$$\mathbb{K}_1 + (1 + \frac{1}{2}) d_0 = (\mathcal{J}_1^e + \mathbf{x}_1 = \text{ }^{\circ}) + \hat{A} \mathbb{V}_1^e + (\mathfrak{g}_1 \text{ i } \mathfrak{g}_1^e) + d_1^e; \quad (1.22)$$

Using (1.19), (1.20) and (1.22), one can solve for the expected cross-country averages of the outcomes, given d_1^e :

$$\mathbb{V}_1^e = \frac{\mathbf{h} \text{ }^{\circ} \mathbf{h}}{\mathbb{P}} \mathbb{K}_1 + (1 + \frac{1}{2}) d_0 \text{ i } d_1^e \text{ i}; \quad (1.23)$$

$$\mathcal{J}_1^e + \mathbf{x}_1 = \text{ }^{\circ} = \frac{\mathbf{h} \text{ }^{\circ 2} \mathbf{h}}{\mathbb{P}} \mathbb{K}_1 + (1 + \frac{1}{2}) d_0 \text{ i } d_1^e \text{ i};$$

$$\mathfrak{g}_1 \text{ i } \mathfrak{g}_1^e = \frac{\mathbf{h} \text{ }^{\circ} \mathbf{h}}{\mathbb{P}} \mathbb{K}_1 + (1 + \frac{1}{2}) d_0 \text{ i } d_1^e \text{ i};$$

Combining this last equation with (1.21) to eliminate $\mathfrak{g}_1 \text{ i } \mathfrak{g}_1^e$, we can solve for the equilibrium value of d_1^e as:

$$d_1^{e;S} = \frac{\mathbf{h} \text{ }^{\circ} \mathbf{h} \mathbb{K}_1 + (1 + \frac{1}{2}) d_0 \text{ i } \mathbb{K}_2 + (1 \text{ i } \text{ }^{\circ} \mathbf{h}) \mathbb{K}_2}{1 + \text{ }^{\circ} (1 + \frac{1}{2})}; \quad (1.24)$$

where

$$\text{ }^{\circ} = \text{ }^{\circ} (1 + \frac{1}{2}); \quad (1.25)$$

and where (here and in the sequel) a superscript "S" stands for "second best".

Substitute (1.24) back into the expressions for \mathbb{V}_1^e , $\mathcal{J}_1^e + \mathbf{x}_1 = \text{ }^{\circ}$ and $\mathfrak{g}_1 \text{ i } \mathfrak{g}_1^e$ to give:

$$\frac{1}{4}_1^e = \frac{\hat{A}=\textcircled{g}}{P} \frac{\mathbf{h}}{1+\alpha} \frac{\mathbf{i}}{1+\frac{1}{2}} \mathbf{F}; \quad (1.26)$$

$$\zeta_1^e + \mathbf{x}_1^o = \frac{\mathbf{h}}{P} \frac{1=\textcircled{2}}{1+\alpha} \frac{\mathbf{i}}{1+\frac{1}{2}} \mathbf{F}; \quad (1.27)$$

$$\vartheta_1 \mathbf{i} \hat{g}_1^e = \frac{\mathbf{h}}{P} \frac{1=\textcircled{g}}{1+\alpha} \frac{\mathbf{i}}{1+\frac{1}{2}} \mathbf{F}; \quad (1.28)$$

$$\mathbf{x}_1 \mathbf{i} \mathbf{x}_1^e = \frac{\mathbf{h}}{P} \frac{1=\textcircled{o}}{1+\alpha} \frac{\mathbf{i}}{1+\frac{1}{2}} \mathbf{F}; \quad (1.29)$$

where the last expression follows upon combining (1.27) and (2.1).

Step 2: Computation of country-specific expected deviations from cross-country expected averages. Subtract (1.19)-(1.22) from (1.15)-(1.18) to give the system:

$$\textcircled{2} \zeta_{i1}^{\text{C};e} + \mathbf{x}_{i1}^{\text{C};o} = \textcircled{g} \vartheta_{i1}^{\text{C}} \mathbf{i} \hat{g}_{i1}^{\text{C};e}; \quad (1.30)$$

$$\textcircled{g} \vartheta_{i1}^{\text{C}} \mathbf{i} \hat{g}_{i1}^{\text{C};e} = \frac{\mathbf{h}}{1=\textcircled{2}+1=\textcircled{g}} \mathbf{K}_{i2}^{\text{C}} + (1 + \frac{1}{2}) d_{i1}^{\text{C};e}; \quad (1.31)$$

$$\mathbf{K}_{i1}^{\text{C}} + (1 + \frac{1}{2}) d_{i0}^{\text{C}} = \zeta_{i1}^{\text{C};e} + \mathbf{x}_{i1}^{\text{C};o} + \vartheta_{i1}^{\text{C}} \mathbf{i} \hat{g}_{i1}^{\text{C};e} + d_{i1}^{\text{C};e}; \quad (1.32)$$

where we note that the first equation has dropped out. Hence, from this system we obtain:

$$\zeta_{i1}^{\text{C};e} + \mathbf{x}_{i1}^{\text{C};o} = \frac{\mathbf{h}}{1=\textcircled{2}+1=\textcircled{g}} \mathbf{K}_{i1}^{\text{C}} + (1 + \frac{1}{2}) d_{i0}^{\text{C}} \mathbf{i} d_{i1}^{\text{C};e}; \quad (1.33)$$

$$\vartheta_{i1}^{\text{C}} \mathbf{i} \hat{g}_{i1}^{\text{C};e} = \frac{\mathbf{h}}{1=\textcircled{2}+1=\textcircled{g}} \mathbf{K}_{i1}^{\text{C}} + (1 + \frac{1}{2}) d_{i0}^{\text{C}} \mathbf{i} d_{i1}^{\text{C};e}; \quad (1.34)$$

If we combine the last equation with (1.31), we can solve for $d_{i1}^{\text{C};e}$ as:

$$d_{i1}^{\text{C};e} = \frac{\mathbf{h}}{1 + \alpha} \frac{\mathbf{K}_{i1}^{\text{C}} + (1 + \frac{1}{2}) d_{i0}^{\text{C}} \mathbf{i} \mathbf{K}_{i2}^{\text{C}} + (1 + \alpha) \mathbf{K}_{i2}^{\text{C}}}{1 + \alpha (1 + \frac{1}{2})}; \quad (1.35)$$

1.2.2. Steps 3 and 4

Subtract the system (1.15)-(1.18) from the system (1.11), (1.12), (1.13) and (1.14). This yields:

$$\textcircled{g} \frac{1}{4}_1^d + \textcircled{2} \frac{1}{4}_1^d \mathbf{i} \zeta_1^d \mathbf{i} \frac{1}{\alpha} + \hat{A} \textcircled{g} \hat{g}_1^d = 0; \quad (1.36)$$

$$\mathbf{i} \textcircled{2} \frac{1}{4}_1^d \mathbf{i} \zeta_{i1}^d \mathbf{i} \frac{1+\alpha}{\alpha} + \textcircled{g} \vartheta_{i1}^d = 0; \quad (1.37)$$

$$i^{\otimes} g_{i1}^d = -\alpha (1 + \frac{1}{2}) \frac{h}{1=\sigma^2+1=\otimes_g} i n d_{i1}^d i \frac{h}{P} i^{\otimes} d_1^d ; 8i; \quad (1.36)$$

$$0 = \zeta_{i1}^d + \hat{A} \frac{1}{4} i g_{i1}^d + d_{i1}^d ; 8i; \quad (1.37)$$

Step 3: Responses to common shocks. Take cross-country averages across (1.34)-(1.37) to give:

$$\otimes \frac{1}{4} \frac{1}{4}^d + \sigma^2 \frac{1}{4}^d i \zeta_1^d i \frac{1}{\sigma} + \hat{A} \otimes g_1^d = 0; \quad (1.38)$$

$$i \sigma^2 \frac{1}{4}^d i \zeta_1^d i \frac{1}{\sigma} + \otimes g_1^d = 0; \quad (1.39)$$

$$i^{\otimes} g_1^d = -\alpha (1 + \frac{1}{2}) \frac{h}{P} i d_1^d; \quad (1.40)$$

$$0 = \zeta_1^d + \hat{A} \frac{1}{4} i g_1^d + d_1^d; \quad (1.41)$$

The solution for $\frac{1}{4}^d$, ζ_1^d , g_1^d and x_1^d , given d_1^d , is:

$$\frac{1}{4}^d = \frac{h}{P^{\alpha}} \frac{(\hat{A}+1)=\otimes}{\sigma} i^{\otimes} i d_1^d ;$$

$$x_1^d = i \frac{h}{P^{\alpha}} i^{\otimes} i d_1^d ;$$

$$g_1^d = i \frac{h}{P^{\alpha}} i^{\otimes} i d_1^d ;$$

where

$$P^{\alpha} = (\hat{A} + 1)^2 =\otimes \frac{1}{4} + 1=\sigma^2 + 1=\otimes_g; \quad (1.42)$$

as defined in the main text. Combine the expression for g_1^d with (1.41) to give the solution of d_1^d as:

$$d_1^{d:S} = \frac{1}{1 + -\alpha (1 + \frac{1}{2}) (P^{\alpha}=P)} \frac{1}{\sigma}; \quad (1.43)$$

Step 4: Computation of responses to idiosyncratic shocks. Subtract the system (1.38)-(1.41) from (1.34)-(1.37) to give:

$$\sigma^2 \zeta_{i1}^{\ddagger} + \frac{2}{\sigma} + \otimes g_{i1}^{\ddagger} = 0; \quad (1.44)$$

$$i^{\otimes} g_{i1}^{\pm} = -^{\alpha} (1 + \frac{1}{2}) \frac{h}{1=\circ 2+1=\otimes g} i^{\#} d_{i1}^{\pm}; \quad (1.45)$$

$$0 = \zeta_{i1}^{\pm} i^{\#} g_{i1}^{\pm} + d_{i1}^{\pm}; \quad (1.46)$$

Using (1.44) and (1.46) we can solve $\zeta_{i1}^{\pm} + \frac{2_i}{\circ}$ and g_{i1}^{\pm} for given d_{i1}^{\pm} :

$$\zeta_{i1}^{\pm} + \frac{2_i}{\circ} = \frac{h}{1=\circ 2+1=\otimes g} i^{\#} \frac{2_i}{\circ} i^{\#} d_{i1}^{\pm};$$

$$g_{i1}^{\pm} = i^{\#} \frac{h}{1=\circ 2+1=\otimes g} i^{\#} \frac{2_i}{\circ} i^{\#} d_{i1}^{\pm};$$

Combine this with (1.45) to yield:

$$d_{i1}^{\pm S} = \frac{1}{1 + -^{\alpha} (1 + \frac{1}{2})} \frac{2_i}{\circ}; \quad (1.47)$$

2. Derivation of the decentralized equilibrium

We now solve the model for the case of a centralized, discretionary monetary policy and a decentralized fiscal policy. We allow for the possibility of a constant (possibly zero) inflation target or a state-contingent inflation target which is a linear function of the individual countries' debt choices. The case of pure discretion is obtained when the inflation target is restricted to zero in both periods. The model is solved through backwards induction.

2.1. Period 2

We compute the second-period policy outcomes, conditional on first-period debt choices. Substitute (2.2) and

$$g_{i2} = i^{\#} (1 + \frac{1}{2}) d_{i1} + \zeta_{i2} + \hat{A} \frac{1}{2};$$

into (2.3). Hence, in period 2 the CCB minimizes over $\frac{1}{2}$:

$$\frac{1}{2} : \frac{1}{n} \sum_{i=1}^n \left(\frac{1}{2} i^{\#} \frac{1}{2} i^{\#} \zeta_{i2} \right)^2 + \frac{1}{n} \sum_{i=1}^n \left(i^{\#} (1 + \frac{1}{2}) d_{i1} + \zeta_{i2} + \hat{A} \frac{1}{2} i^{\#} g_{i2} \right)^2;$$

where $\frac{1}{2}$ is a constant (possibly zero) or a linear function of the individual countries' debt choices. The CCB's first-order condition is:

$$\mathbb{R}_{\frac{1}{4}} (\mathcal{Y}_2 \text{ i } \mathcal{Y}_2^{\text{n}}) + \frac{1}{\text{n}} \sum_{\text{i}=1}^{\text{n}} [\text{o} (\text{o} (\mathcal{Y}_2 \text{ i } \mathcal{Y}_2^{\text{e}} \text{ i } \mathcal{L}_{\text{i}2}) \text{ i } \mathbf{x}_{\text{i}2}) + \hat{\mathbf{A}}^{\text{R}}_{\text{g}} (\mathfrak{g}_{\text{i}2} \text{ i } \mathfrak{g}_{\text{i}2})] = 0; \quad (2.1)$$

The ...scal authority of country i minimizes over $\mathcal{L}_{\text{i}2}$:

$$\frac{1}{2} \sum_{\text{i}=1}^{\text{n}} \mathbb{R}_{\frac{1}{4}} \mathcal{Y}_2^2 + [\text{o} (\mathcal{Y}_2 \text{ i } \mathcal{Y}_2^{\text{e}} \text{ i } \mathcal{L}_{\text{i}2}) \text{ i } \mathbf{x}_{\text{i}2}]^2 + \mathbb{R}_{\text{g}} [\text{i} (1 + \frac{1}{2}) \text{d}_{\text{i}1} + \mathcal{L}_{\text{i}2} + \hat{\mathbf{A}} \mathcal{Y}_{\text{i}2} \text{ i } \mathfrak{g}_{\text{i}2}]^2 \text{o} :$$

The ...rst-order condition is:

$$\text{i} \text{o} [\text{o} (\mathcal{Y}_2 \text{ i } \mathcal{Y}_2^{\text{e}} \text{ i } \mathcal{L}_{\text{i}2}) \text{ i } \mathbf{x}_{\text{i}2}] + \mathbb{R}_{\text{g}} (\mathfrak{g}_{\text{i}2} \text{ i } \mathfrak{g}_{\text{i}2}) = 0; \text{8i}; \quad (2.2)$$

Furthermore, we can write the second-period government budget constraint as:

$$\mathbb{K}_{\text{i}2} + (1 + \frac{1}{2}) \text{d}_{\text{i}1} = (\mathcal{L}_{\text{i}2} + \mathbf{x}_{\text{i}2}=\text{o}) + \hat{\mathbf{A}} \mathcal{Y}_2 + (\mathfrak{g}_{\text{i}2} \text{ i } \mathfrak{g}_{\text{i}2}); \quad (2.3)$$

Take (2.1), (2.2) and (2.3) together and impose that $\mathcal{Y}_2^{\text{e}} = \mathcal{Y}_2$. The system to be solved is then given by:

$$\mathbb{R}_{\frac{1}{4}} (\mathcal{Y}_2 \text{ i } \mathcal{Y}_2^{\text{n}}) = \text{o}^2 (\mathcal{L}_2 + \mathbf{x}_2=\text{o}) + \hat{\mathbf{A}}^{\text{R}}_{\text{g}} (\mathfrak{g}_2 \text{ i } \mathfrak{g}_2); \quad (2.4)$$

$$\text{o}^2 (\mathcal{L}_{\text{i}2} + \mathbf{x}_{\text{i}2}=\text{o}) = \mathbb{R}_{\text{g}} (\mathfrak{g}_{\text{i}2} \text{ i } \mathfrak{g}_{\text{i}2}); \quad (2.5)$$

$$\mathbb{K}_{\text{i}2} + (1 + \frac{1}{2}) \text{d}_{\text{i}1} = (\mathcal{L}_{\text{i}2} + \mathbf{x}_{\text{i}2}=\text{o}) + \hat{\mathbf{A}} \mathcal{Y}_2 + (\mathfrak{g}_{\text{i}2} \text{ i } \mathfrak{g}_{\text{i}2}); \quad (2.6)$$

Take the cross-country average of (2.5) and combine the resulting equation with (2.4) to eliminate $(\mathfrak{g}_2 \text{ i } \mathfrak{g}_2)$ from (2.4). We have, after rewriting the result:

$$\mathcal{Y}_2 = \mathcal{Y}_2^{\text{n}} + \frac{\text{h} (\hat{\mathbf{A}}+1)\text{o}^2 \text{i}}{\mathbb{R}_{\frac{1}{4}}} (\mathcal{L}_2 + \mathbf{x}_2=\text{o}); \quad (2.7)$$

Take the cross-country average of (2.6) and combine this with (2.7) to eliminate \mathcal{Y}_2 from (2.6). Combine the result with the cross-country average of (2.5) to eliminate $\mathfrak{g}_2 \text{ i } \mathfrak{g}_2$. We end up with:

$$\mathcal{L}_2 + \mathbf{x}_2=\text{o} = \frac{\text{h} \text{1}=\text{o}^2 \text{i} \text{h}}{\text{S}} \mathbb{K}_2 + (1 + \frac{1}{2}) \text{d}_1 \text{ i } \hat{\mathbf{A}} \mathcal{Y}_2^{\text{n}} \text{ i}; \quad (2.8)$$

where

$$\text{S} = \hat{\mathbf{A}} (\hat{\mathbf{A}} + 1) = \mathbb{R}_{\frac{1}{4}} + 1=\text{o}^2 + 1=\mathbb{R}_{\text{g}} \text{S}; \quad (2.9)$$

as in the main text. Combine (2.8) with (2.7) to give:

$$\begin{aligned}
\frac{1}{4}_2 &= \frac{1}{4}_2^g + \frac{h^{(\hat{A}+1)=\frac{\circledast}{4}}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \hat{A} \frac{1}{4}_2^i \\
&= \frac{h^{1=\circledast} + 1=\frac{\circledast}{g}}{S} \frac{1}{4}_2^g + \frac{h^{(\hat{A}+1)=\frac{\circledast}{4}}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i : \quad (2.10)
\end{aligned}$$

Furthermore, we can combine (2.5) and (2.6) to yield

$$\mathbf{K}_{i2} + (1 + \frac{1}{2}) d_{i1} = 1 + \frac{\circledast}{g} (\zeta_{i2} + x_{i2}=\circ) + \hat{A} \frac{1}{4}_2 :$$

Combine this with (2.10) to eliminate $\frac{1}{4}_2$. Rewrite the result to yield:

$$\begin{aligned}
& \frac{\zeta_{i2} + x_{i2}=\circ}{h} \frac{1=\circledast}{1=\circledast + 1=\frac{\circledast}{g}} \mathbf{K}_{i2} + (1 + \frac{1}{2}) d_{i1}^i \frac{h}{S} \frac{\hat{A}(\hat{A}+1)=\frac{\circledast}{4}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \frac{\hat{A}(1=\circledast + 1=\frac{\circledast}{g})^{\frac{3}{4}}}{S} \frac{1}{4}_2^{\frac{3}{4}} \\
&= \frac{h}{3} \frac{1=\circledast}{1=\circledast + 1=\frac{\circledast}{g}} \mathbf{K}_{i2}^{\circledast} + (1 + \frac{1}{2}) d_{i1}^{\circledast} + \frac{h^{1=\circledast}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \hat{A} \frac{1}{4}_2^i \\
&= \zeta_{i2}^{\circledast} + x_{i2}=\circ + \frac{h^{1=\circledast}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \hat{A} \frac{1}{4}_2^i : \quad (2.11)
\end{aligned}$$

Combine this with (2.5) to find that:

$$\begin{aligned}
& \frac{g_{i2}^i}{h} \frac{1=\frac{\circledast}{g}}{1=\circledast + 1=\frac{\circledast}{g}} \mathbf{K}_{i2} + (1 + \frac{1}{2}) d_{i1}^i \frac{h}{S} \frac{\hat{A}(\hat{A}+1)=\frac{\circledast}{4}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \frac{\hat{A}(1=\circledast + 1=\frac{\circledast}{g})^{\frac{3}{4}}}{S} \frac{1}{4}_2^{\frac{3}{4}} \\
&= \frac{h}{3} \frac{1=\frac{\circledast}{g}}{1=\circledast + 1=\frac{\circledast}{g}} \mathbf{K}_{i2}^{\circledast} + (1 + \frac{1}{2}) d_{i1}^{\circledast} + \frac{h^{1=\frac{\circledast}{g}}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \hat{A} \frac{1}{4}_2^i \\
&= g_{i2}^{\circledast} + \frac{h^{1=\frac{\circledast}{g}}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \hat{A} \frac{1}{4}_2^i : \quad (2.12)
\end{aligned}$$

Finally,

$$\begin{aligned}
& \frac{x_{i2}^i}{h} \frac{1=\circ}{1=\circledast + 1=\frac{\circledast}{g}} \mathbf{K}_{i2} + (1 + \frac{1}{2}) d_{i1}^i \frac{h}{S} \frac{\hat{A}(\hat{A}+1)=\frac{\circledast}{4}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \frac{\hat{A}(1=\circledast + 1=\frac{\circledast}{g})^{\frac{3}{4}}}{S} \frac{1}{4}_2^{\frac{3}{4}} \\
&= \frac{h}{3} \frac{1=\circ}{1=\circledast + 1=\frac{\circledast}{g}} \mathbf{K}_{i2}^{\circledast} + (1 + \frac{1}{2}) d_{i1}^{\circledast} + \frac{h^{1=\circ}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \hat{A} \frac{1}{4}_2^i \\
&= x_{i2}^{\circledast} + \frac{h^{1=\circ}}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^i \hat{A} \frac{1}{4}_2^i : \quad (2.13)
\end{aligned}$$

Using (2.10), (2.12) and (2.13), the CCB's and government i's second-period losses are, respectively:

$$L_2^{CCB} = \frac{1}{2} \frac{h(\hat{A}+1)^2 = \theta_{1/4}}{S^2} K_2 + (1 + \frac{1}{2}) d_{1i} \hat{A} \frac{1}{4}_2^{\pi} + \frac{1}{2n} \frac{h}{1 = \theta^2 + 1 = \theta_g} \sum_{j=1}^n K_{j2} + (1 + \frac{1}{2}) d_{j1i} \frac{h(\hat{A}+1) = \theta_{1/4}}{S} K_2 + (1 + \frac{1}{2}) d_{1i} \frac{h}{\hat{A}(1 = \theta^2 + 1 = \theta_g)} \frac{1}{4}_2^{\pi} ;$$

$$L_{i2}^G = \frac{1}{2} \frac{nh}{1 = \theta^2 + 1 = \theta_g} \frac{1}{4}_2^{\pi} + \frac{h(\hat{A}+1) = \theta_{1/4}}{S} K_2 + (1 + \frac{1}{2}) d_{1i} \frac{1}{4}_2^{\pi} + \frac{1}{2} \frac{h}{1 = \theta^2 + 1 = \theta_g} \sum_{j=1}^n K_{j2} + (1 + \frac{1}{2}) d_{j1i} \frac{h(\hat{A}+1) = \theta_{1/4}}{S} K_2 + (1 + \frac{1}{2}) d_{1i} \frac{h}{\hat{A}(1 = \theta^2 + 1 = \theta_g)} \frac{1}{4}_2^{\pi} ; \quad (2.14)$$

2.2. Period 1

The CCB minimizes over $\frac{1}{4}_1$:

$$\left(\frac{1}{2} \theta_{1/4} (\frac{1}{4}_1 i \frac{1}{4}_1^{\pi})^2 + \frac{1}{n} \sum_{i=1}^n [\theta (\frac{1}{4}_1 i \frac{1}{4}_1^e i \zeta_{i1}) i^1 i^2 i \mathbf{x}_{i1}]^2 + \hat{A} \theta_g (g_{i1} i \mathbf{g}_{i1})^2 + -L_2^{CCB} \right) ;$$

Hence, the first-order condition is:

$$\theta_{1/4} (\frac{1}{4}_1 i \frac{1}{4}_1^{\pi}) + \frac{1}{n} \sum_{i=1}^n [\theta (\frac{1}{4}_1 i \frac{1}{4}_1^e i \zeta_{i1}) i^1 i^2 i \mathbf{x}_{i1}] + \hat{A} \theta_g (g_{i1} i \mathbf{g}_{i1})] = 0; \quad (2.15)$$

The government of country i minimizes over ζ_{i1} and d_{i1} :

$$\left(\theta_{1/4} \frac{1}{4}_1^2 + [\theta (\frac{1}{4}_1 i \frac{1}{4}_1^e i \zeta_{i1}) i^1 i^2 i \mathbf{x}_{i1}]^2 + \theta_g [i (1 + \frac{1}{2}) d_0 + \zeta_{i1} + \hat{A} \frac{1}{4}_1 + d_{i1} i \mathbf{g}_{i1}]^2 + -L_{i2}^G \right) ;$$

The first-order conditions are:

$$i \theta [\theta (\frac{1}{4}_1 i \frac{1}{4}_1^e i \zeta_{i1}) i^1 i^2 i \mathbf{x}_{i1}] + \theta_g (g_{i1} i \mathbf{g}_{i1}) = 0; \quad (2.16)$$

$$\theta_g (g_{i1} i \mathbf{g}_{i1}) = - \frac{\partial}{\partial d_{i1}} L_{i2}^G ; \quad (2.17)$$

where

$$\begin{aligned} \mathbb{E}_t L_{i2}^G = \mathbb{E}_t d_{i1} &= \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{nh}{S} \frac{1=\sigma^2+1=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} \frac{1}{2} \frac{\mathbb{E}_t}{\mathbb{E}_t} + \frac{h}{S} \frac{(\hat{A}+1)=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} \frac{h}{\mathbb{E}_t} \mathbb{K}_2 + (1 + \frac{1}{2}) d_1^i \mathbb{E}_t \\ &= \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{1}{n} (1 + \frac{1}{2}) \frac{h}{\mathbb{E}_t} \frac{(\hat{A}+1)=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} + \frac{h}{S} \frac{1=\sigma^2+1=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{1}{2} \frac{\mathbb{E}_t}{\mathbb{E}_t} + \\ &= \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{1}{1=\sigma^2+1=\mathbb{E}_t} \frac{\mathbb{E}_t}{\mathbb{E}_t} \\ &= \frac{1}{2} \frac{h}{\mathbb{E}_t} \mathbb{K}_{i2} + (1 + \frac{1}{2}) d_{i1}^i \frac{h}{\mathbb{E}_t} \frac{(\hat{A}+1)=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} \frac{h}{\mathbb{E}_t} \mathbb{K}_2 + (1 + \frac{1}{2}) d_1^i \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{\hat{A}(1=\sigma^2+1=\mathbb{E}_t)}{S} \frac{1}{2} \frac{\mathbb{E}_t}{\mathbb{E}_t} \\ &= \frac{1}{2} (1 + \frac{1}{2}) \frac{h}{\mathbb{E}_t} \frac{1}{n} \frac{\hat{A}(\hat{A}+1)=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} \frac{\hat{A}(1=\sigma^2+1=\mathbb{E}_t)}{S} \frac{1}{2} \frac{\mathbb{E}_t}{\mathbb{E}_t} : \end{aligned}$$

Finally, we can rewrite the ...rst-period government budget constraint as:

$$\mathbb{K}_{i1} + (1 + \frac{1}{2}) d_{i0} = (\zeta_{i1} + \mathbf{x}_{i1}=\mathbb{E}_t) + \hat{A} \mathbb{V}_1 + (\mathbb{g}_{i1} - \mathbb{g}_1) + d_{i1}: \quad (2.18)$$

The system of ...rst-order conditions to be solved is thus given by (2.15), (2.16), (2.17) and (2.18). Take some country-specific policy variable y_{i1} ($y_{i1} = \mathbb{V}_{i1}; \zeta_{i1}; \mathbb{g}_{i1}$ or d_{i1}). We will solve for y_{i1} by solving for each of the components of the decomposition of y_{i1} into $y_{i1} = \mathbb{y}_1^e + \mathbb{y}_{i1}^{\mathbb{C};e} + \mathbb{y}_1^d + \mathbb{y}_{i1}^{\pm}$, where \mathbb{y}_1^e will be the response to the cross-country average of the deterministic components of the government ...nancing requirements, $\mathbb{y}_{i1}^{\mathbb{C};e}$ will be the response to the country-specific deterministic components of the government ...nancing requirements, \mathbb{y}_1^d will be response to the common shock and \mathbb{y}_{i1}^{\pm} will be the response to the idiosyncratic shock. Note that $\mathbb{V}_{i1}^{\mathbb{C};e} = \mathbb{V}_{i1}^{\pm} = 0$. We thus compute the solutions of the variables in four steps.

2.2.1. Steps 1 and 2

Take expectations across (2.15), (2.16), (2.17) and (2.18) to yield:

$$\mathbb{E}_t (\mathbb{V}_1^e - \mathbb{V}_1^{\mathbb{C};e}) = \sigma^2 (\mathbb{z}_1^e + \mathbf{x}_1=\mathbb{E}_t) + \hat{A} \mathbb{E}_t (\mathbb{g}_1 - \mathbb{g}_1^e); \quad (2.19)$$

$$\sigma^2 (\zeta_{i1}^e + \mathbf{x}_{i1}=\mathbb{E}_t) = \mathbb{E}_t (\mathbb{g}_{i1} - \mathbb{g}_{i1}^e); \quad (2.20)$$

$$\begin{aligned} &= \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{nh}{S} \frac{1=\sigma^2+1=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} \frac{1}{2} \frac{\mathbb{E}_t}{\mathbb{E}_t} + \frac{h}{S} \frac{(\hat{A}+1)=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} \frac{h}{\mathbb{E}_t} \mathbb{K}_2 + (1 + \frac{1}{2}) d_1^i \mathbb{E}_t \\ &= \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{1}{n} (1 + \frac{1}{2}) \frac{h}{\mathbb{E}_t} \frac{(\hat{A}+1)=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} + \frac{h}{S} \frac{1=\sigma^2+1=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{1}{2} \frac{\mathbb{E}_t}{\mathbb{E}_t} + \\ &= \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{1}{1=\sigma^2+1=\mathbb{E}_t} \frac{\mathbb{E}_t}{\mathbb{E}_t} \\ &= \frac{1}{2} \frac{h}{\mathbb{E}_t} \mathbb{K}_{i2} + (1 + \frac{1}{2}) d_{i1}^e \frac{h}{\mathbb{E}_t} \frac{(\hat{A}+1)=\mathbb{E}_t}{S} \frac{i}{\mathbb{E}_t} \frac{h}{\mathbb{E}_t} \mathbb{K}_2 + (1 + \frac{1}{2}) d_1^e \frac{\mathbb{E}_t}{\mathbb{E}_t} \frac{\hat{A}(1=\sigma^2+1=\mathbb{E}_t)}{S} \frac{1}{2} \frac{\mathbb{E}_t}{\mathbb{E}_t} \end{aligned}$$

$$\frac{1}{2} \left(1 + \frac{1}{2}\right) \frac{h}{n} \frac{\hat{A}(\hat{A}+1)^{\otimes_{\frac{1}{4}}}}{S} \mathbf{i} \frac{\hat{A}(1=\circ^2+1=\otimes_g)^{\circ} \frac{\otimes_{\frac{1}{4}}}{d_{i1}}}{S} \mathbf{i} \quad (2.21)$$

$$\mathbf{K}_{i1} + (1 + \frac{1}{2}) d_{i0} = (\mathbf{j}_{i1}^e + \mathbf{x}_{i1}=\circ) + \hat{A}\frac{1}{4}_1^e + (\mathbf{g}_{i1} \mathbf{i} \mathbf{g}_{i1}^e) + d_{i1}^e \quad (2.22)$$

Here, $\frac{1}{4}_2^{\pi;e}$ is the expectation about the second-period inflation target, formed before ...rst-period shocks have occurred. Furthermore, we have used that $\frac{\otimes_{\frac{1}{4}}}{d_{i1}}$ is constant (possibly zero), because $\frac{1}{4}_2^{\pi}$ is a constant or a linear function of the individual countries' debt choices.

Step 1: Computation of expectations of cross-country averages. Take cross-country averages across (2.19)-(2.22) to yield the following system (again making use of the constancy of $\frac{\otimes_{\frac{1}{4}}}{d_{i1}}$):

$$\otimes_{\frac{1}{4}} (\mathbf{1}_1^e \mathbf{i} \mathbf{1}_1^{\pi}) = \circ^2 (\mathbf{j}_1^e + \mathbf{x}_1=\circ) + \hat{A}^{\otimes_g} (\mathbf{g}_1 \mathbf{i} \mathbf{g}_1^e) \quad (2.23)$$

$$\circ^2 (\mathbf{j}_1^e + \mathbf{x}_1=\circ) = \otimes_g (\mathbf{g}_1 \mathbf{i} \mathbf{g}_1^e) \quad (2.24)$$

$$\begin{aligned} & \otimes_g (\mathbf{g}_1 \mathbf{i} \mathbf{g}_1^e) \\ = & -\otimes_{\frac{1}{4}} \frac{h}{n} \frac{1=\circ^2+1=\otimes_g}{S} \mathbf{i} \frac{1}{4}_2^{\pi;e} + \frac{h}{S} \frac{(\hat{A}+1)^{\otimes_{\frac{1}{4}}}}{S} \mathbf{i} h \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^e \mathbf{i} \circ \\ & \frac{1}{h} (1 + \frac{1}{2}) \frac{h}{n} \frac{(\hat{A}+1)^{\otimes_{\frac{1}{4}}}}{S} \mathbf{i} + \frac{h}{S} \frac{1=\circ^2+1=\otimes_g}{S} \mathbf{i} \frac{\otimes_{\frac{1}{4}}}{d_{i1}} \mathbf{o} + \\ & - \frac{1}{1=\circ^2+1=\otimes_g} \mathbf{i} \\ & \frac{1}{2} h \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^e \mathbf{i} \frac{h}{S} \frac{\hat{A}(\hat{A}+1)^{\otimes_{\frac{1}{4}}}}{S} \mathbf{i} h \mathbf{K}_2 + (1 + \frac{1}{2}) d_1^e \mathbf{i} \frac{\hat{A}(1=\circ^2+1=\otimes_g)^{\circ} \frac{1}{4}_2^{\pi;e}}{S} \mathbf{i} \\ & \frac{1}{2} (1 + \frac{1}{2}) \frac{h}{n} \frac{\hat{A}(\hat{A}+1)^{\otimes_{\frac{1}{4}}}}{S} \mathbf{i} \frac{\hat{A}(1=\circ^2+1=\otimes_g)^{\circ} \frac{\otimes_{\frac{1}{4}}}{d_{i1}}}{S} \mathbf{i} \quad (2.25) \end{aligned}$$

$$\mathbf{K}_1 + (1 + \frac{1}{2}) d_0 = (\mathbf{j}_1^e + \mathbf{x}_1=\circ) + \hat{A}\frac{1}{4}_1^e + (\mathbf{g}_1 \mathbf{i} \mathbf{g}_1^e) + d_1^e \quad (2.26)$$

Combine (2.23) and (2.24) to eliminate $\otimes_g (\mathbf{g}_1 \mathbf{i} \mathbf{g}_1^e)$ and obtain:

$$\frac{1}{4}_1^e \mathbf{i} \frac{1}{4}_1^{\pi} = \frac{h}{\otimes_{\frac{1}{4}}} \frac{(\hat{A}+1)^{\circ^2}}{S} \mathbf{i} (\mathbf{j}_1^e + \mathbf{x}_1=\circ) :$$

Combine this equation and (2.24) with (2.26) to eliminate both $\mathbf{g}_1 \mathbf{i} \mathbf{g}_1^e$ and $\frac{1}{4}_1^e$, to obtain after rewriting:

$$(\mathbf{j}_1^e + \mathbf{x}_1=\circ) = \frac{h}{S} \frac{1=\circ^2}{S} \mathbf{i} h \mathbf{K}_1 + (1 + \frac{1}{2}) d_0 \mathbf{i} d_1^e \mathbf{i} \hat{A}\frac{1}{4}_1^{\pi} \mathbf{i} \quad (2.27)$$

Hence,

$$\frac{1}{4}_1^e = \frac{1}{4}_1^a + \frac{h^{(\hat{A}+1)=\textcircled{g}}}{S} \mathbf{i} \mathbf{h} \mathbf{K}_1 + (1 + \frac{1}{2}) d_{0i}^j d_{1i}^e \hat{A} \frac{1}{4}_1^a; \quad (2.28)$$

$$g_{1i}^e = \frac{h^{1=\textcircled{g}}}{S} \mathbf{i} \mathbf{h} \mathbf{K}_1 + (1 + \frac{1}{2}) d_{0i}^j d_{1i}^e \hat{A} \frac{1}{4}_1^a; \quad (2.29)$$

Hence, combining this last equation with (2.25) to eliminate g_{1i}^e and rewriting yields:

$$\begin{aligned} & \mathbf{K}_1 + (1 + \frac{1}{2}) d_{0i}^j d_{1i}^e \hat{A} \frac{1}{4}_1^a \\ = & - \frac{1}{n} \frac{1}{\textcircled{2}} + \frac{1}{\textcircled{g}} \mathbf{i} \frac{(n-1)\hat{A}S}{S} \frac{1}{4}_2^{a;e} + \frac{S^a \cdot h}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_{1i}^e \mathbf{i} = \\ & - \frac{\textcircled{g}(1=\textcircled{2}+1=\textcircled{g})}{S} \frac{nh}{\textcircled{2}} + \frac{1}{\textcircled{g}} \mathbf{i} \frac{1}{4}_2^{a;e} + \frac{h^{(\hat{A}+1)=\textcircled{g}}}{S} \mathbf{i} \mathbf{h} \mathbf{K}_2 + (1 + \frac{1}{2}) d_{1i}^e \mathbf{i} \frac{\textcircled{2}}{\textcircled{d}_{11}} \frac{1}{4}_2^a \\ & - \frac{\hat{A}(1=\textcircled{2}+1=\textcircled{g})}{S} \frac{h}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_{1i}^e \hat{A} \frac{1}{4}_2^{a;e} \frac{\textcircled{2}}{\textcircled{d}_{11}} \end{aligned} \quad (2.30)$$

where

$$S^a = \hat{A}(\hat{A} + 1) = \textcircled{g} + (\hat{A} + 1) = (n \textcircled{g}) + 1 = \textcircled{2} + 1 = \textcircled{g}; \quad (2.31)$$

as defined in the main text. Hence,

$$\begin{aligned} d_{1i}^e = & \frac{[\mathbf{K}_1 + (1 + \frac{1}{2}) d_{0i}^j \mathbf{K}_2] + [1 - \frac{1}{n} (S^a - S)] \mathbf{K}_2}{1 + \frac{1}{n} (1 + \frac{1}{2}) (S^a - S)} \mathbf{i} \\ & \frac{1}{1 + \frac{1}{n} (1 + \frac{1}{2}) (S^a - S)} \hat{A} \frac{1}{4}_1^a + \frac{1}{1 + \frac{1}{n} (1 + \frac{1}{2}) (S^a - S)} \hat{A} \frac{1}{4}_2^{a;e} \mathbf{i} \\ & \frac{1}{n} \frac{(\hat{A}+1)(P-S)}{1 + \frac{1}{n} (1 + \frac{1}{2}) (S^a - S)} \frac{1}{4}_2^{a;e} \mathbf{i} \\ & - \frac{h^{(\hat{A}+1)=\textcircled{g}}}{1 + \frac{1}{n} (1 + \frac{1}{2}) (S^a - S)} \frac{\textcircled{g}(1=\textcircled{2}+1=\textcircled{g})}{S} \frac{nh}{\textcircled{2}} \\ & + \frac{h^{(\hat{A}+1)=\textcircled{g}}}{1 + \frac{1}{n} (1 + \frac{1}{2}) (S^a - S)} \mathbf{i} \frac{1}{\textcircled{g}} \frac{1}{4}_2^{a;e} + \frac{h^{(\hat{A}+1)=\textcircled{g}}}{S} \mathbf{i} \mathbf{h} \mathbf{K}_2 + (1 + \frac{1}{2}) d_{1i}^e \mathbf{i} \frac{\textcircled{2}}{\textcircled{d}_{11}} \frac{1}{4}_2^a \\ & + \frac{h^{(\hat{A}+1)=\textcircled{g}}}{1 + \frac{1}{n} (1 + \frac{1}{2}) (S^a - S)} \mathbf{i} \frac{\hat{A}(1=\textcircled{2}+1=\textcircled{g})}{S} \mathbf{K}_2 + (1 + \frac{1}{2}) d_{1i}^e \hat{A} \frac{1}{4}_2^{a;e} \frac{\textcircled{2}}{\textcircled{d}_{11}} \end{aligned} \quad (2.32)$$

One obtains $d_{1i}^{e;D}$ by imposing a zero inflation target in both periods. Hence, the terms involving $\frac{1}{4}_1^a$, $\frac{1}{4}_2^{a;e}$ and $\frac{\textcircled{2}}{\textcircled{d}_{11}}$ in (2.32) all drop out.

Step 2: Computation of country-specific expected deviations from cross-country expected averages. Subtract (2.24)-(2.26) from (2.20)-(2.22). This gives the following system:

$$\textcircled{2} \dot{z}_{i1}^{c;e} + x_{i1}^{c;o} = \textcircled{g} g_{i1}^c \mathbf{i} g_{i1}^{c;e}; \quad (2.33)$$

$$\begin{aligned} \mathbb{g}_{i1}^{\mathbb{C}} i \mathbb{g}_{i1}^{\mathbb{C};e} &= - \frac{\hbar}{1=\circ^2+1=\mathbb{g}} \frac{i \hbar}{S} \mathbb{K}_{i2}^{\mathbb{C}} + (1 + \frac{1}{2}) d_{i1}^{\mathbb{C};e} i \\ (1 + \frac{1}{2}) \frac{\hbar}{1} i \frac{1}{n} \frac{\hat{A}(\hat{A}+1)=\mathbb{g}}{S} i & \frac{\hat{A}(1=\circ^2+1=\mathbb{g})}{S} \frac{\mathbb{g}_{i2}^{\mathbb{C}}}{\mathbb{d}_{i1}} \end{aligned} \quad (2.34)$$

$$\mathbb{K}_{i1}^{\mathbb{C}} + (1 + \frac{1}{2}) d_{i0}^{\mathbb{C}} = \mathbb{L}_{i1}^{\mathbb{C};e} + \mathbb{X}_{i1}^{\mathbb{C};\circ} + \mathbb{g}_{i1}^{\mathbb{C}} i \mathbb{g}_{i1}^{\mathbb{C};e} + d_{i1}^{\mathbb{C};e} \quad (2.35)$$

Equations (2.33) and (2.35) can be combined to yield outcomes conditional on $d_{i1}^{\mathbb{C};e}$,

$$\mathbb{L}_{i1}^{\mathbb{C};e} + \mathbb{X}_{i1}^{\mathbb{C};\circ} = \frac{\hbar}{1=\circ^2+1=\mathbb{g}} \frac{i \hbar}{S} \mathbb{K}_{i1}^{\mathbb{C}} + (1 + \frac{1}{2}) d_{i0}^{\mathbb{C}} i d_{i1}^{\mathbb{C};e} i \quad (2.36)$$

$$\mathbb{g}_{i1}^{\mathbb{C}} i \mathbb{g}_{i1}^{\mathbb{C};e} = \frac{\hbar}{1=\circ^2+1=\mathbb{g}} \frac{i \hbar}{S} \mathbb{K}_{i1}^{\mathbb{C}} + (1 + \frac{1}{2}) d_{i0}^{\mathbb{C}} i d_{i1}^{\mathbb{C};e} i \quad (2.37)$$

Combining the latter equation with (2.34) gives:

$$\begin{aligned} \mathbb{K}_{i1}^{\mathbb{C}} + (1 + \frac{1}{2}) d_{i0}^{\mathbb{C}} i d_{i1}^{\mathbb{C};e} &= - \frac{\hbar}{1=\circ^2+1=\mathbb{g}} \frac{i \hbar}{S} \mathbb{K}_{i2}^{\mathbb{C}} + (1 + \frac{1}{2}) d_{i1}^{\mathbb{C};e} i \\ (1 + \frac{1}{2}) \frac{\hbar}{1} i \frac{1}{n} \frac{\hat{A}(\hat{A}+1)=\mathbb{g}}{S} i & \frac{\hat{A}(1=\circ^2+1=\mathbb{g})}{S} \frac{\mathbb{g}_{i2}^{\mathbb{C}}}{\mathbb{d}_{i1}} \end{aligned} \quad (2.38)$$

Setting $\frac{\mathbb{g}_{i2}^{\mathbb{C}}}{\mathbb{d}_{i1}} = 0$, one can easily solve for $d_{i1}^{\mathbb{C};e;D}$ given in the main text.

2.2.2. Steps 3 and 4

Subtract equations (2.19)-(2.22) from equations (2.15)-(2.18), respectively, to give the following system in terms of deviations of realizations of variables from their expectations:

$$\mathbb{g}_{i1}^{\mathbb{C}} \mathbb{g}_{i1}^{\mathbb{C};e} + \mathbb{L}_{i1}^{\mathbb{C};e} i \mathbb{L}_{i1}^{\mathbb{C};e} i \frac{1}{\circ} + \hat{A} \mathbb{g}_{i1}^{\mathbb{C}} = 0; \quad (2.39)$$

$$i \circ^2 \mathbb{g}_{i1}^{\mathbb{C}} i \mathbb{L}_{i1}^{\mathbb{C};e} i \frac{1+\circ^2}{\circ} + \mathbb{g}_{i1}^{\mathbb{C}} = 0; \quad (2.40)$$

$$\begin{aligned} i \mathbb{g}_{i1}^{\mathbb{C}} \mathbb{g}_{i1}^{\mathbb{C};e} &= - \frac{\hbar}{1=\circ^2+1=\mathbb{g}} \frac{i \hbar}{S} \mathbb{K}_{i2}^{\mathbb{C}} + (1 + \frac{1}{2}) d_{i1}^{\mathbb{C};e} i \\ \frac{1}{n} \frac{\hbar}{1} (1 + \frac{1}{2}) \frac{\hat{A}(\hat{A}+1)=\mathbb{g}}{S} i & + \frac{\hbar}{1=\circ^2+1=\mathbb{g}} \frac{i \hbar}{S} \mathbb{K}_{i1}^{\mathbb{C}} + (1 + \frac{1}{2}) d_{i0}^{\mathbb{C}} i d_{i1}^{\mathbb{C};e} i \\ - \frac{1}{1=\circ^2+1=\mathbb{g}} & \\ \frac{1}{2} (1 + \frac{1}{2}) d_{i1}^{\mathbb{C};e} i & \frac{\hat{A}(\hat{A}+1)=\mathbb{g}}{S} i (1 + \frac{1}{2}) d_{i1}^{\mathbb{C};e} i \frac{\hat{A}(1=\circ^2+1=\mathbb{g})}{S} \frac{\mathbb{g}_{i2}^{\mathbb{C}}}{\mathbb{d}_{i1}} \\ \frac{1}{2} (1 + \frac{1}{2}) \frac{\hbar}{1} i \frac{1}{n} \frac{\hat{A}(\hat{A}+1)=\mathbb{g}}{S} i & \frac{\hat{A}(1=\circ^2+1=\mathbb{g})}{S} \frac{\mathbb{g}_{i2}^{\mathbb{C}}}{\mathbb{d}_{i1}} \end{aligned} \quad (2.41)$$

$$0 = \zeta_{i1}^d + \hat{A} \mathcal{V}_1^d \mathbf{i} \mathbf{g}_{i1}^d + \mathbf{d}_{i1}^d; \quad (2.42)$$

where $\mathcal{V}_2^{\pi;d} = \mathcal{V}_2^{\pi} \mathbf{i} \mathcal{V}_2^{\pi;e}$.

Step 3: Responses to common shocks. In step 3 we solve for $\mathcal{V}_1^d, \zeta_1^d = \zeta_1 \mathbf{i} \zeta_1^e$, $\mathbf{g}_1^d = \mathbf{g}_1 \mathbf{i} \mathbf{g}_1^e$ and $\mathbf{d}_1^d = \mathbf{d}_1 \mathbf{i} \mathbf{d}_1^e$, which are the policy responses to the common shock ϵ_1 . We take the cross-country averages of the system we just obtained and use the assumption that the cross-country average of the ζ_1 's equals zero. This yields:

$$\mathbb{E} \left[\mathcal{V}_1^d + \mathbb{E} \left[\mathcal{V}_1^d \right] \right] = \mathbb{E} \left[\zeta_1^d + \mathbb{E} \left[\zeta_1^d \right] \right] \mathbf{i} \hat{A} \mathbb{E} \left[\mathbf{g}_1^d \right]; \quad (2.43)$$

$$\mathbf{i} \mathbb{E} \left[\mathcal{V}_1^d \right] \mathbf{i} \zeta_1^d + \mathbb{E} \left[\zeta_1^d \right] \mathbf{i} \mathbb{E} \left[\mathbf{g}_1^d \right] = 0; \quad (2.44)$$

$$\begin{aligned} \mathbf{i} \mathbb{E} \left[\mathbf{g}_1^d \right] &= - \mathbb{E} \left[\mathcal{V}_1^d \right] \frac{\mathbf{h}}{\mathbf{S}} \frac{1-\mathbb{E}[\zeta_1^d] + 1-\mathbb{E}[\mathbf{g}_1^d]}{\mathbf{h}} \mathcal{V}_2^{\pi;d} \mathbf{i} + \frac{\mathbf{h} (\hat{A}+1) \mathbb{E}[\zeta_1^d]}{\mathbf{S}} (1 + \mathbb{E}[\zeta_1^d]) \mathbf{d}_1^d \mathbf{o} \mathbf{x} \\ &\quad - \frac{1}{\mathbf{S}} (1 + \mathbb{E}[\zeta_1^d]) \frac{(\hat{A}+1) \mathbb{E}[\zeta_1^d]}{\mathbf{S}} \mathbf{i} + \frac{\mathbf{h} (1-\mathbb{E}[\zeta_1^d] + 1-\mathbb{E}[\mathbf{g}_1^d])}{\mathbf{S}} \frac{\mathbb{E}[\zeta_1^d]}{\mathbb{E}[\mathbf{d}_1^d]} \mathbf{o} \mathbf{x} \\ &\quad - \frac{1}{\mathbf{S}} (1 + \mathbb{E}[\zeta_1^d]) \mathbf{d}_1^d \mathbf{i} \hat{A} \mathcal{V}_2^{\pi;d} \mathbf{i} \mathbf{x} \\ &\quad \frac{1}{2} \frac{\mathbf{h}}{(1 + \mathbb{E}[\zeta_1^d])} \mathbf{i} \frac{1}{\mathbf{n}} \frac{\hat{A} (\hat{A}+1) \mathbb{E}[\zeta_1^d]}{\mathbf{S}} \mathbf{i} \mathbf{x} \frac{\hat{A} (1-\mathbb{E}[\zeta_1^d] + 1-\mathbb{E}[\mathbf{g}_1^d])}{\mathbf{S}} \frac{\mathbb{E}[\zeta_1^d]}{\mathbb{E}[\mathbf{d}_1^d]} \mathbf{x}; \end{aligned} \quad (2.45)$$

$$0 = \zeta_1^d + \hat{A} \mathcal{V}_1^d \mathbf{i} \mathbf{g}_1^d + \mathbf{d}_1^d. \quad (2.46)$$

One can solve (2.43), (2.44) and (2.46) to obtain the solutions for the variables for given \mathbf{d}_1^d :

$$\begin{aligned} \mathcal{V}_1^d &= \frac{\mathbf{h} (\hat{A}+1) \mathbb{E}[\zeta_1^d]}{\mathbf{P}^{\pi}} \mathbf{i} \frac{1}{\mathbb{E}[\zeta_1^d]} \mathbf{i} \mathbf{d}_1^d; \zeta_1^d + \frac{1}{\mathbb{E}[\zeta_1^d]} = \frac{\mathbf{h} (\hat{A}+1) \mathbb{E}[\zeta_1^d] + 1-\mathbb{E}[\mathbf{g}_1^d]}{\mathbf{P}^{\pi}} \mathbf{i} \frac{1}{\mathbb{E}[\zeta_1^d]} \mathbf{i} \mathbf{d}_1^d; \\ \mathbf{x}_1^d &= \mathbf{i} \frac{\mathbb{E}[\zeta_1^d]}{(\hat{A}+1) \mathbb{E}[\zeta_1^d]} \mathcal{V}_1^d; \mathbf{g}_1^d = \mathbf{i} \frac{\mathbb{E}[\zeta_1^d]}{(\hat{A}+1) \mathbb{E}[\zeta_1^d]} \mathcal{V}_1^d; \end{aligned} \quad (2.47)$$

Finally, we can solve for \mathbf{d}_1^d by combining (2.45) and (2.47). We can solve for \mathbf{d}_1^d if we set $\mathcal{V}_2^{\pi} = 0$ (hence, $\mathcal{V}_2^{\pi;d} = \frac{\mathbb{E}[\zeta_1^d]}{\mathbb{E}[\mathbf{d}_1^d]} = 0$).

Step 4: Computation of responses to idiosyncratic shocks. These responses are defined as $\zeta_{i1}^{\ddagger} = \zeta_{i1}^d \mathbf{i} \zeta_1^{\ddagger}$, $\mathbf{g}_{i1}^{\ddagger} = \mathbf{g}_{i1}^d \mathbf{i} \mathbf{g}_1^{\ddagger}$ and $\mathbf{d}_{i1}^{\ddagger} = \mathbf{d}_{i1}^d \mathbf{i} \mathbf{d}_1^{\ddagger}$. The relevant system that needs to be solved is obtained by subtracting (2.43)-(2.46) from (2.39)-(2.42), respectively. This yields (note that the first equation drops out):

$$\mathbf{i} \mathbb{E} \left[\zeta_{i1}^{\ddagger} \right] \mathbf{i} \zeta_1^{\ddagger} \mathbf{i} \mathbf{d}_1^{\ddagger} + \mathbb{E} \left[\zeta_{i1}^{\ddagger} \right] \mathbf{i} \mathbb{E} \left[\mathbf{g}_{i1}^{\ddagger} \right] = 0; \quad (2.48)$$

$$\frac{\partial}{\partial g} \log g_{i1}^{\pm} = -\alpha \frac{h}{1-\alpha^2+1-\alpha g} \frac{i}{2} \frac{h}{S} \frac{i}{i} \frac{\hat{A}(1-\alpha^2+1-\alpha g)^{\alpha}}{S} \frac{\partial}{\partial d_{i1}} \frac{3}{4} d_{i1}^{\pm}; \quad (2.49)$$

$$0 = \zeta_{i1}^{\pm} \frac{i}{i} g_{i1}^{\pm} + d_{i1}^{\pm}; \quad (2.50)$$

where

$$Q = [(n \frac{i}{i} - 1) = n] [\hat{A} (\hat{A} + 1) = \alpha \frac{h}{S}] + 1-\alpha^2 + 1-\alpha g; \quad (2.51)$$

as defined in the main text. Combine (2.48) and (2.50) to eliminate ζ_{i1}^{\pm} and solve for g_{i1}^{\pm} to yield:

$$g_{i1}^{\pm} = \frac{h}{1-\alpha^2+1-\alpha g} \frac{i}{i} \frac{h}{S} \frac{i}{i} \frac{2}{\alpha} \frac{h}{i}; \quad (2.52)$$

By substituting the right-hand side of (2.52) into (2.49), we can solve for d_{i1}^{\pm} . In addition, if we set $\frac{h}{2} = 0$ (hence, $\frac{\partial}{\partial d_{i1}} = 0$), we obtain the solution for $d_{i1}^{\pm D}$.

3. Proof of Proposition 1

Let $K_{i1} = K_1$, $K_{i2} = K_2$, $d_{i0} = d_0$ and $z_i = 0$, $8i$. Hence, $d_{i1} = d_1$. We pursue the following strategy in proving Proposition 1. First, we derive the value $\frac{h}{2}(d_1)$ for $\frac{h}{2}$, which can be written as $\frac{h}{2}(d_1) = \frac{h}{2}{}^{e}(d_1) + \frac{h}{2}{}^{d}(d_1)$, that replicates the second-period policy outcomes under the second best, for given d_1 .¹⁷ Then, we derive the value $\frac{h}{1}(d_1)$ for $\frac{h}{1}$ that replicates the first-period policy outcomes under the second best, for given d_1 . We plug $\frac{h}{2}{}^{e}(d_1)$ and $\frac{h}{1}(d_1)$ for $\frac{h}{2}{}^{e}$ and $\frac{h}{1}$, respectively, into (2.30), which is then solved for d_1^e . Finally, we plug $\frac{h}{2}{}^{d}(d_1)$ into (2.45) to solve for d_1^d . The resulting solutions for d_1^e and d_1^d turn out to coincide with the corresponding outcomes in the second-best equilibrium.

For given d_1 , the optimal inflation target $\frac{h}{2}(d_1)$ is the one that yields the second-best outcome for $\frac{h}{2}$, i.e. $\frac{h}{2} = \frac{\hat{A}-\alpha \frac{h}{S}}{P} K_2 + (1 + \frac{1}{2}) d_1$. Hence, we solve for $\frac{h}{2}(d_1)$ from the following equation:

$$\frac{h}{P} \frac{\hat{A}-\alpha \frac{h}{S}}{i} K_2 + (1 + \frac{1}{2}) d_1 = \frac{h}{S} \frac{1-\alpha^2+1-\alpha g}{i} \frac{h}{2} + \frac{h}{S} \frac{(\hat{A}+1)-\alpha \frac{h}{S}}{i} K_2 + (1 + \frac{1}{2}) d_1;$$

which is obtained by equating the right-hand side of the first equation in (1.6) with the right-hand side in (2.10). The solution is:

$$\frac{h}{2}(d_1) = \frac{h}{P} \frac{1-\alpha \frac{h}{S}}{i} K_2 + (1 + \frac{1}{2}) d_1; \quad (3.1)$$

¹⁷These are the outcomes that would have been obtained from the computations in Appendix B had we constrained debt accumulation in country i to d_1 .

It is easy to check that for this value of the inflation target, the other second-period variables attain their second-best values, given d_1^e . Further, note that,

$$\frac{\partial \mathcal{W}_2^{\pi}}{\partial d_1} = i \frac{1}{n} \frac{h}{P} \frac{1-\theta}{P} (1 + \frac{1}{2}) : \quad (3.2)$$

For given d_1^e , the optimal inflation target $\mathcal{W}_1^{\pi}(d_1^e)$ is the one that yields $\mathcal{W}_1^e = \frac{h}{P} \frac{A-\theta}{P} K_1 + (1 + \frac{1}{2}) d_0 i d_1^e$. Hence, we solve $\mathcal{W}_1^{\pi}(d_1^e)$ from the following equation:

$$\frac{h}{P} \frac{A-\theta}{P} K_1 + (1 + \frac{1}{2}) d_0 i d_1^e = \frac{h}{S} \frac{1-\theta^2+1-\theta}{S} \mathcal{W}_1^{\pi} + \frac{h}{S} \frac{(A+1)-\theta}{S} K_1 + (1 + \frac{1}{2}) d_0 i d_1^e ;$$

which is derived upon equating the right-hand sides of (1.23) and (2.28). The solution is:

$$\mathcal{W}_1^{\pi}(d_1^e) = i \frac{h}{P} \frac{1-\theta}{P} K_1 + (1 + \frac{1}{2}) d_0 i d_1^e : \quad (3.3)$$

It is easy to check that for this value of the inflation target, the other first-period variables attain their second-best equilibrium values, given d_1^e .

Having noticed that for these inflation targets the intratemporal allocation of the government financing requirements is optimal (i.e. according to the second-best equilibrium), for given debt policy, we now need to check that for these inflation targets debt policy coincides with debt policy in the second-best equilibrium. To this end, substitute \mathcal{W}_1^{π} , given by (3.3), $\frac{\partial \mathcal{W}_2^{\pi}}{\partial d_1}$, given by (3.2), and

$$\mathcal{W}_2^{\pi;e} = i \frac{h}{P} \frac{1-\theta}{P} K_2 + (1 + \frac{1}{2}) d_1^e i ;$$

into (2.30). It is easy to see that the final two terms in (2.30) cancel out against each other. It is then straightforward to solve the resulting equation to yield $d_1^e = d_1^{e;S}$.

As the final step, substitute

$$\mathcal{W}_2^{\pi;d} = i \frac{h}{P} \frac{1-\theta}{P} (1 + \frac{1}{2}) d_1^d ;$$

(obtained by subtracting the expectation of (3.1) from (3.1)) into (2.45). Let's work out the various terms after this substitution. For the left-hand side we have:

$$i \frac{\partial \mathcal{G}_1^d}{\partial d_1} = \frac{1}{P^{\pi}} \frac{1}{\sigma} i d_1^d :$$

For the right-hand side we have:

$$\begin{aligned} - \frac{h}{P} \frac{1-\theta^2+1-\theta}{S} \mathcal{W}_2^{\pi;d} + \frac{h}{S} \frac{(A+1)-\theta}{S} (1 + \frac{1}{2}) d_1^d &= - \frac{h}{P} \frac{A-\theta}{PS} i d_1^d ; \\ \frac{1}{n} (1 + \frac{1}{2}) \frac{h}{S} \frac{(A+1)-\theta}{S} i + \frac{h}{S} \frac{1-\theta^2+1-\theta}{S} \frac{\partial \mathcal{W}_2^{\pi}}{\partial d_1} &= \frac{1}{n} (1 + \frac{1}{2}) \frac{h}{PS} \frac{A-\theta}{PS} i ; \end{aligned}$$

$$-\frac{1}{S} \frac{h}{(1 + \frac{1}{2})} d_1^d \quad \hat{A} \frac{1}{2} d_1^d = -\frac{1}{P} d_1^d;$$

$$(1 + \frac{1}{2}) \frac{h}{1} \quad \frac{1}{n} \frac{\hat{A}(\hat{A}+1)=\frac{1}{2}}{S} \quad \frac{\hat{A}(1=\sigma^2+1=\frac{1}{2})}{S} \frac{1}{2} = (1 + \frac{1}{2}) \frac{R}{P};$$

where

$$R = [(n - 1) = n] \hat{A}^2 = \frac{1}{2} + 1 = \sigma^2 + 1 = \frac{1}{2}; \quad (3.4)$$

Hence, we obtain the following equation to be solved for d_1^d :

$$\frac{1}{P} \frac{1}{2} d_1^d = -\frac{1}{2} (1 + \frac{1}{2}) \frac{1}{n} \frac{\hat{A}^2 = \frac{1}{2}}{P^2} d_1^d + -\frac{1}{P} d_1^d (1 + \frac{1}{2}) \frac{R}{P};$$

The right-hand side of this equation can be written as $-\frac{1}{2} (1 + \frac{1}{2}) d_1^d = P$. Using this, we can solve the equation to give $d_1^d = d_1^{d:S}$. This completes the proof.

4. Proof of Proposition 2

First, we derive the equilibrium when a debt target d_{i1}^T is imposed on country i , $i = 1; \dots; n$. This debt target is the exact amount of debt that country i has to carry over into the second period. After having derived the equilibrium, we prove Proposition 2.

4.1. Derivation of the equilibrium with debt targets d_{i1}^T , $i = 1; \dots; n$.

In deriving the equilibrium, we closely follow the derivations of the decentralized equilibrium in Appendix B.

4.1.1. Period 2

We replace d_{i1} with d_{i1}^T in the derivation of the second-period outcomes in Appendix B and, consistent with the notation we used so far, we use d_1^T to denote the cross-country average of the individual countries' debt targets. The second-period policy outcomes can then be written as:

$$\frac{1}{2} = \frac{1}{2} + \frac{h}{S} \frac{(\hat{A}+1)=\frac{1}{2}}{S} \quad K_2 + (1 + \frac{1}{2}) d_1^T \quad \hat{A} \frac{1}{2}; \quad (4.1)$$

$$\frac{g_{i2}}{g_{i2}} = \frac{h}{1=\sigma^2+1=\frac{1}{2}} \frac{1}{S} \quad K_{i2} + (1 + \frac{1}{2}) d_{i1}^T \quad \frac{h}{S} \frac{\hat{A}(\hat{A}+1)=\frac{1}{2}}{S} \quad K_2 + (1 + \frac{1}{2}) d_1^T \quad \frac{1}{2}; \quad (4.2)$$

$$\begin{aligned} \mathbf{x}_{i2} &= \frac{1-\alpha}{1-\alpha^2 + \alpha^2 \beta} \mathbf{h} \\ \mathbf{K}_{i2} &+ (1 + \frac{1}{2}) d_{i1}^T \mathbf{h} \frac{\hat{A}(\hat{A}+1)^{\alpha}}{S} \mathbf{h} \\ \mathbf{K}_2 &+ (1 + \frac{1}{2}) d_1^T \mathbf{h} \end{aligned} \quad (4.3)$$

analogous to (2.10), (2.12) and (2.13), respectively.

4.1.2. Period 1

Because ...rst-period debt is no longer a choice variable, the relevant ...rst-order conditions for government i are:

$$\beta^{\frac{1}{\alpha}} (\mathcal{V}_1^e + \mathcal{V}_1^a) + \frac{1}{n} \sum_{i=1}^n [\beta^{\frac{1}{\alpha}} (\mathcal{V}_1^e + \mathcal{V}_1^e + \mathcal{L}_{i1}) + \beta^{\frac{1}{\alpha}} (\mathbf{x}_{i1}) + \hat{A}^{\frac{1}{\alpha}} (g_{i1} + \mathfrak{g}_{i1})] = 0; \quad (4.4)$$

$$\beta^{\frac{1}{\alpha}} [\beta^{\frac{1}{\alpha}} (\mathcal{V}_1^e + \mathcal{V}_1^e + \mathcal{L}_{i1}) + \beta^{\frac{1}{\alpha}} (\mathbf{x}_{i1}) + \hat{A}^{\frac{1}{\alpha}} (g_{i1} + \mathfrak{g}_{i1})] = 0; \quad (4.5)$$

$$\mathbf{K}_{i1} + (1 + \frac{1}{2}) d_{i0} = (\mathcal{L}_{i1} + \mathbf{x}_{i1}^e) + \hat{A} \mathcal{V}_1^e + (g_{i1} + \mathfrak{g}_{i1}) + d_{i1}^T; \quad (4.6)$$

analogous to (2.15), (2.16) and (2.18), respectively.

We solve now for the ...rst-period outcomes. As before, we solve for these in four steps. Take expectations across the ...rst-order conditions (4.4), (4.5) and (4.6), to yield:

$$\beta^{\frac{1}{\alpha}} (\mathcal{V}_1^e + \mathcal{V}_1^a) = \beta^{\frac{1}{\alpha}} (\mathcal{L}_1^e + \mathbf{x}_1^e) + \hat{A}^{\frac{1}{\alpha}} (g_1 + \mathfrak{g}_1^e); \quad (4.7)$$

$$\beta^{\frac{1}{\alpha}} (\mathcal{L}_{i1}^e + \mathbf{x}_{i1}^e) = \hat{A}^{\frac{1}{\alpha}} (g_{i1} + \mathfrak{g}_{i1}^e); \quad (4.8)$$

$$\mathbf{K}_{i1} + (1 + \frac{1}{2}) d_{i0} = (\mathcal{L}_{i1}^e + \mathbf{x}_{i1}^e) + \hat{A} \mathcal{V}_1^e + (g_{i1} + \mathfrak{g}_{i1}^e) + d_{i1}^{T,e}; \quad (4.9)$$

Step 1: Solution in terms of cross-country averages. Take cross-country averages across the previous three equations. This yields:

$$\beta^{\frac{1}{\alpha}} (\mathcal{V}_1^e + \mathcal{V}_1^a) = \beta^{\frac{1}{\alpha}} (\mathcal{L}_1^e + \mathbf{x}_1^e) + \hat{A}^{\frac{1}{\alpha}} (g_1 + \mathfrak{g}_1^e); \quad (4.10)$$

$$\beta^{\frac{1}{\alpha}} (\mathcal{L}_1^e + \mathbf{x}_1^e) = \hat{A}^{\frac{1}{\alpha}} (g_1 + \mathfrak{g}_1^e); \quad (4.11)$$

$$\mathbf{K}_1 + (1 + \frac{1}{2}) \mathbf{d}_0 = (\mathbf{z}_1^e + \mathbf{x}_1^e) + \hat{\mathbf{A}} \mathbf{y}_1^e + (\mathbf{g}_1 \text{ i } \mathbf{g}_1^e) + \mathbf{d}_1^{T:e}; \quad (4.12)$$

Following the steps in Appendix B, we solve for the outcomes as a function of $\mathbf{d}_1^{T:e}$:

$$\mathbf{x}_1 \text{ i } \mathbf{x}_1^e = \frac{\mathbf{h}_{1=\circ} \text{ i } \mathbf{h}}{\mathbf{S}} \mathbf{K}_1 + (1 + \frac{1}{2}) \mathbf{d}_0 \text{ i } \mathbf{d}_1^{T:e} \text{ i } \hat{\mathbf{A}} \mathbf{y}_1^e; \quad (4.13)$$

$$\mathbf{y}_1^e = \mathbf{y}_1^a + \frac{\mathbf{h}_{(\hat{\mathbf{A}}+1)=\circ} \text{ i } \mathbf{h}}{\mathbf{S}} \mathbf{K}_1 + (1 + \frac{1}{2}) \mathbf{d}_0 \text{ i } \mathbf{d}_1^{T:e} \text{ i } \hat{\mathbf{A}} \mathbf{y}_1^a; \quad (4.14)$$

$$\mathbf{g}_1 \text{ i } \mathbf{g}_1^e = \frac{\mathbf{h}_{1=\circ} \text{ i } \mathbf{h}}{\mathbf{S}} \mathbf{K}_1 + (1 + \frac{1}{2}) \mathbf{d}_0 \text{ i } \mathbf{d}_1^{T:e} \text{ i } \hat{\mathbf{A}} \mathbf{y}_1^a; \quad (4.15)$$

Step 2: Country-specific expected deviations from cross-country expected averages. Subtract (4.11) and (4.12) from (4.8) and (4.9), respectively, which gives the following pair of equations:

$$\circ^2 \mathbf{z}_{i1}^{\Phi:e} + \mathbf{x}_{i1}^{\Phi:e} = \circ_g \mathbf{g}_{i1}^{\Phi} \text{ i } \mathbf{g}_{i1}^{\Phi:e};$$

$$\mathbf{K}_{i1}^{\Phi} + (1 + \frac{1}{2}) \mathbf{d}_{i0}^{\Phi} = \mathbf{z}_{i1}^{\Phi:e} + \mathbf{x}_{i1}^{\Phi:e} + \mathbf{g}_{i1}^{\Phi} \text{ i } \mathbf{g}_{i1}^{\Phi:e} + \mathbf{d}_{i1}^{T;\Phi:e};$$

These are solved to give the outcomes conditional on $\mathbf{d}_{i1}^{T;\Phi:e}$;

$$\begin{aligned} \mathbf{x}_{i1}^{\Phi} \text{ i } \mathbf{x}_{i1}^{\Phi:e} &= \frac{\mathbf{h}_{1=\circ} \text{ i } \mathbf{h}}{1=\circ^2+1=\circ_g} \mathbf{K}_{i1}^{\Phi} + (1 + \frac{1}{2}) \mathbf{d}_{i0}^{\Phi} \text{ i } \mathbf{d}_{i1}^{T;\Phi:e}; \\ \mathbf{g}_{i1}^{\Phi} \text{ i } \mathbf{g}_{i1}^{\Phi:e} &= \frac{1=\circ_g}{1=\circ^2+1=\circ_g} \mathbf{K}_{i1}^{\Phi} + (1 + \frac{1}{2}) \mathbf{d}_{i0}^{\Phi} \text{ i } \mathbf{d}_{i1}^{T;\Phi:e}; \end{aligned}$$

Step 3: Responses to common shocks. Subtract (4.7)-(4.9) from (4.4)-(4.6), respectively, to give the system:

$$\circ_g \mathbf{y}_1^d + \circ^2 \mathbf{y}_1^d \text{ i } \mathbf{z}_1^d \text{ i } \frac{1}{\circ} + \hat{\mathbf{A}} \circ_g \mathbf{g}_1^d = 0; \quad (4.16)$$

$$\text{i } \circ^2 \mathbf{y}_1^d \text{ i } \mathbf{z}_{i1}^d \text{ i } \frac{1+\mathbf{z}_1}{\circ} + \circ_g \mathbf{g}_{i1}^d = 0; \quad (4.17)$$

$$0 = \mathbf{z}_{i1}^d + \hat{\mathbf{A}} \mathbf{y}_1^d \text{ i } \mathbf{g}_{i1}^d + \mathbf{d}_{i1}^{T;d}; \quad (4.18)$$

Next, take cross-country averages of this system, to yield:

$$\circ_g \mathbf{y}_1^d + \circ^2 \mathbf{y}_1^d \text{ i } \mathbf{z}_1^d \text{ i } \frac{1}{\circ} + \hat{\mathbf{A}} \circ_g \mathbf{g}_1^d = 0; \quad (4.19)$$

$$i^{\circ 2} \frac{1}{P} \mathcal{W}_1^d + \mathcal{L}_1^d + \frac{1}{\sigma} \mathcal{X}_1^d + \mathbb{R}_g \mathcal{G}_1^d = 0; \quad (4.20)$$

$$0 = \mathcal{L}_1^d + \bar{A} \mathcal{W}_1^d + \mathcal{G}_1^d + d_1^{T;d}; \quad (4.21)$$

The solution of this system is:

$$\mathcal{W}_1^d = \frac{h}{P} \frac{(\bar{A}+1)^{\circ \frac{1}{\sigma}}}{P} \mathcal{X}_1^d + \frac{1}{\sigma} \mathcal{L}_1^{T;d}; \quad \mathcal{X}_1^d = i \frac{h}{(\bar{A}+1)^{\circ}} \mathcal{W}_1^d; \quad \mathcal{G}_1^d = i \frac{h}{(\bar{A}+1)^{\circ g}} \mathcal{W}_1^d; \quad (4.22)$$

Step 4: Computation of responses to idiosyncratic shocks. The relevant system to be solved is obtained by subtracting (4.20) and (4.21) from (4.17) and (4.18), respectively, to give:

$$i^{\circ} \frac{h}{P} \mathcal{L}_{i1}^{\pm} + \mathcal{L}_{i1}^{\pm} + \mathbb{R}_g \mathcal{G}_{i1}^{\pm} = 0;$$

$$0 = \mathcal{L}_{i1}^{\pm} + \mathcal{G}_{i1}^{\pm} + d_{i1}^{T;\pm};$$

The solution of this system is:

$$\mathcal{X}_{i1}^{\pm} = \frac{h}{1-\sigma^2+1-\mathbb{R}_g} \mathcal{L}_{i1}^{T;\pm} + \frac{2}{\sigma} d_{i1}^{T;\pm}; \quad (4.23)$$

$$\mathcal{G}_{i1}^{\pm} = \frac{h}{1-\sigma^2+1-\mathbb{R}_g} \mathcal{L}_{i1}^{T;\pm} + \frac{2}{\sigma} d_{i1}^{T;\pm}; \quad (4.24)$$

4.2. Proof of Proposition 2

We show that by setting the individual countries' debt targets at $d_{i1}^T = d_1^{e;S} + d_{i1}^{c;e;S} + d_1^{d;S} + d_{i1}^{\pm;S}$ ($i = 1; \dots; n$), and the inflation targets at

$$\mathcal{W}_1^{\pi} = i \frac{h}{P} \mathbb{K}_1 + (1 + \frac{1}{2}) d_0 + \mathcal{L}_1^{T;\pi}; \quad (4.25)$$

$$\mathcal{W}_2^{\pi} = i \frac{h}{P} \mathbb{K}_2 + (1 + \frac{1}{2}) d_1^{T;\pi}; \quad (4.26)$$

as proposed in Proposition 2, the second-best equilibrium is attained. Hence, this combination of inflation and debt targets mimimizes V_U .

Substitute expression (4.26) into (4.1) in order to eliminate \mathcal{W}_2^{π} . The resulting expression can be simplified to:

$$\mathcal{W}_2 = \frac{h}{P} \mathbb{K}_2 + (1 + \frac{1}{2}) d_1^{T;\pi};$$

Because we set $d_1^T = d_1^{e;S} + d_1^{d;S}$ in Proposition 2, we substitute the solutions for $d_1^{e;S}$ and $d_1^{d;S}$ obtained in Section 3 into this expression for y_2 , to yield:

$$y_2 = \frac{h_{A=0} \cdot i \cdot h}{P} \frac{1+\frac{1}{2}}{1+\frac{1}{2} \cdot P} F + \frac{h_{A=0} \cdot i \cdot h}{P} \frac{1+\frac{1}{2}}{1+\frac{1}{2} \cdot (P^2=P)} \frac{1}{\sigma} ;$$

which is the expression for y_2 given in Table 1.

We can proceed in a similar fashion to show that under the proposed combination of targets $g_{i2} \mid g_{i2}$, $x_{i2} \mid x_{i2}$, y_1 , $g_{i1} \mid g_{i1}$ and $x_{i1} \mid x_{i1}$ all coincide with their second-best counterparts. This completes the proof.

Table 1. Second-best policy outcomes

variable	$\frac{h}{P} \frac{1-\alpha}{P} C_0$	$\frac{h}{P} \frac{1-\alpha}{P} C_0$	$\frac{h}{P} \frac{1-\alpha}{P} C_1$	$\frac{h}{P} \frac{1-\alpha}{P} C_0$
$\hat{A}_{1/4}$	$\frac{\hat{A}^2 = \frac{1}{4}}{P} \frac{1-\alpha}{P} C_0$	0	$\frac{\hat{A}(\hat{A}+1) = \frac{1}{4}}{P} \frac{1-\alpha}{P} C_1$	0
$\frac{x_{11}}{x_{10}}$	$\frac{1-\alpha}{P} \frac{1-\alpha}{P} C_0$	$\frac{1-\alpha}{1-\alpha^2+1-\alpha g} \frac{1-\alpha}{P} C_0$	$\frac{1-\alpha}{P} \frac{1-\alpha}{P} C_1$	$\frac{1-\alpha}{1-\alpha^2+1-\alpha g} \frac{1-\alpha}{P} C_0$
g_{11}	$\frac{1-\alpha}{P} \frac{1-\alpha}{P} C_0$	$\frac{1-\alpha}{1-\alpha^2+1-\alpha g} \frac{1-\alpha}{P} C_0$	$\frac{1-\alpha}{P} \frac{1-\alpha}{P} C_1$	$\frac{1-\alpha}{1-\alpha^2+1-\alpha g} \frac{1-\alpha}{P} C_0$
$\frac{1}{4}_1$	0	0	$\frac{(\hat{A}+1) = \frac{1}{4}}{P} \frac{1-\alpha}{P} C_1$	0
$\hat{A}_{2/4}$	$\frac{\hat{A}^2 = \frac{1}{4}}{P} \frac{1-\alpha}{P} C_0$	0	$\frac{\hat{A}^2 = \frac{1}{4}}{P} \frac{1-\alpha}{P} C_1$	0
$\frac{x_{12}}{x_{10}}$	$\frac{1-\alpha}{P} \frac{1-\alpha}{P} C_0$	$\frac{1-\alpha}{1-\alpha^2+1-\alpha g} \frac{1-\alpha}{P} C_0$	$\frac{1-\alpha}{P} \frac{1-\alpha}{P} C_1$	$\frac{1-\alpha}{1-\alpha^2+1-\alpha g} \frac{1-\alpha}{P} C_0$
g_{12}	$\frac{1-\alpha}{P} \frac{1-\alpha}{P} C_0$	$\frac{1-\alpha}{1-\alpha^2+1-\alpha g} \frac{1-\alpha}{P} C_0$	$\frac{1-\alpha}{P} \frac{1-\alpha}{P} C_1$	$\frac{1-\alpha}{1-\alpha^2+1-\alpha g} \frac{1-\alpha}{P} C_0$
$\frac{1}{4}_2$	0	0	0	0

Note: for each variable z_{it} the outcome can be written in the format $z_{it} = \alpha_0 F + \alpha_1 F_i + \alpha_2 \frac{1}{\sigma} + \alpha_3 \frac{1}{\sigma}$. Further, $C_0 = \frac{1+\frac{1}{2}}{1-\alpha(1+\frac{1}{2})}$, $P = (1+\frac{1}{2})$, $C_1 = \frac{1+\frac{1}{2}}{1-\alpha(1+\frac{1}{2})(P-\alpha)}$, $\hat{A}^2 = \frac{1}{4} + 1-\alpha^2 + 1-\alpha g$ and $P = (\hat{A}+1)^2 = \frac{1}{4} + 1-\alpha^2 + 1-\alpha g$.

Table 2. Purely discretionary policy outcomes

variable	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{S} i_0^3 \cdot C_0^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_1^3 \cdot C_1^D$	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{P^{\otimes} S^{\otimes}} i_2^3 \cdot C_2^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_3^3 \cdot C_1^D$
$\bar{A}_{1/1}$	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{S} i_0^3 \cdot C_0^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_1^3 \cdot C_1^D$	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{P^{\otimes} S^{\otimes}} i_2^3 \cdot C_2^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_3^3 \cdot C_1^D$
$\frac{x_{11} \circ x_{11}}$	$h \frac{1=^{\otimes_2}}{S} i_0^3 \cdot C_0^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_1^3 \cdot C_1^D$	$h \frac{1=^{\otimes_2}}{P^{\otimes} S^{\otimes}} i_2^3 \cdot C_2^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_3^3 \cdot C_1^D$
$g_{11} i g_{11}$	$h \frac{1=^{\otimes_g}}{S} i_0^3 \cdot C_0^D$	$h \frac{1=^{\otimes_g}}{1=^{\otimes_2}+1=^{\otimes_g}} i_1^3 \cdot C_1^D$	$h \frac{1=^{\otimes_g}}{P^{\otimes} S^{\otimes}} i_2^3 \cdot C_2^D$	$h \frac{1=^{\otimes_g}}{1=^{\otimes_2}+1=^{\otimes_g}} i_3^3 \cdot C_1^D$
$\frac{1}{4}_1 i \frac{1}{4}_1^{\circ}$	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{S} i_0^3 \cdot C_0^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_1^3 \cdot C_1^D$	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{P^{\otimes} S^{\otimes}} i_2^3 \cdot C_2^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_3^3 \cdot C_1^D$
$\bar{A}_{1/2}$	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{S} i_0^3 \cdot C_0^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_1^3 \cdot C_1^D$	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{S} i_2^3 \cdot C_2^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_3^3 \cdot C_1^D$
$\frac{x_{12} \circ x_{12}}$	$h \frac{1=^{\otimes_2}}{S} i_0^3 \cdot C_0^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_1^3 \cdot C_1^D$	$h \frac{1=^{\otimes_2}}{S} i_2^3 \cdot C_2^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_3^3 \cdot C_1^D$
$g_{12} i g_{12}$	$h \frac{1=^{\otimes_g}}{S} i_0^3 \cdot C_0^D$	$h \frac{1=^{\otimes_g}}{1=^{\otimes_2}+1=^{\otimes_g}} i_1^3 \cdot C_1^D$	$h \frac{1=^{\otimes_g}}{S} i_2^3 \cdot C_2^D$	$h \frac{1=^{\otimes_g}}{1=^{\otimes_2}+1=^{\otimes_g}} i_3^3 \cdot C_1^D$
$\frac{1}{4}_2 i \frac{1}{4}_2^{\circ}$	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{S} i_0^3 \cdot C_0^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_1^3 \cdot C_1^D$	$h \frac{A(\bar{A}+1)^{\otimes_{1/4}}}{S} i_2^3 \cdot C_2^D$	$h \frac{1=^{\otimes_2}}{1=^{\otimes_2}+1=^{\otimes_g}} i_3^3 \cdot C_1^D$

Note: for each variable z_{it} the outcome can be written in the format

$$h z_{it} = {}^{\circ}_0 F + {}^{\circ}_1 F_i^C + {}^{\circ}_2 F_i^C + {}^{\circ}_3 F_i^C \cdot i$$

Further $h^{-\otimes} = -(1 + \frac{1}{2})$, i

$$C_0^D \cdot \frac{1+\frac{1}{2}}{1+^{-\otimes}(1+\frac{1}{2})(S^{\otimes}=S)}, C_1^D \cdot \frac{1+\frac{1}{2}}{1+^{-\otimes}(1+\frac{1}{2})(Q=S)}, C_2^D \cdot \frac{1+\frac{1}{2}}{1+^{-\otimes}(1+\frac{1}{2})(S^{\otimes}=S)(P^{\otimes}=S)}$$

$$P^{\otimes} \cdot (\bar{A} + 1)^2 =^{\otimes_{1/4}} + 1=^{\otimes_2} + 1=^{\otimes_g}, Q \cdot [(n_j - 1)=n] [\bar{A} (\bar{A} + 1) =^{\otimes_{1/4}}] + 1=^{\otimes_2} + 1=^{\otimes_g},$$

$$S \cdot \bar{A} (\bar{A} + 1) =^{\otimes_{1/4}} + 1=^{\otimes_2} + 1=^{\otimes_g} \text{ and}$$

$$S^{\otimes} \cdot \bar{A} (\bar{A} + 1) =^{\otimes_{1/4}} + (\bar{A} + 1) = (n^{\otimes_{1/4}}) + 1=^{\otimes_2} + 1=^{\otimes_g}.$$