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Tommaso M Valletti

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Tommaso M Valletti, London School of Economics and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: http://www.cepr.org

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ABSTRACT

Location Choice and Price Discrimination in a Duopoly*

This paper analyzes the problem of price discrimination in a market where consumers have heterogeneous preferences over both a horizontal parameter (brand) and a vertical one (quality). A model with two firms competing over locations and non-linear contracts is analyzed. Discriminatory contracts are first characterized at each location. It is then shown that locations have a big impact on the firms' discriminatory ability and that equilibrium locations are non-monotonic in consumer types, however, firms never locate too far away from the first and third quartiles.

JEL Classification: D43, L13
Keywords: location, price discrimination, horizontal and vertical differentiation

Tommaso M Valletti
Department of Economics
London School of Economics
Houghton Street
London WC2A 2AE
UK
Tel: (44 171) 955 7551
Fax: (44 171) 831 1840
Email: t.valletti@lse.ac.uk

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NON-TECHNICAL SUMMARY

The rationale for price discrimination stems from the fact that consumers may have heterogeneous preferences over product characteristics, hence firms can try to design different solutions for different customers. Discriminatory practices are common in oligopolistic industries, but the economic analysis of this setting is not entirely understood. The purpose of this paper is twofold: first, its aim is to characterize the competition between two rival firms that offer alternative price contracts designed to discriminate between different groups of consumers; secondly it studies how competition over locations affects the discriminatory ability of firms.

The presence of more than one firm gives the customer the right to buy products from different suppliers. The notion of competition plays a central role in understanding how contracts change with respect to a discriminatory monopolist since contracts in a competitive environment should not give room to rivals. Firms typically differ in the relative appeal to customers, so it is interesting to study the properties of equilibrium contracts with different intensity of competition.

I consider a model of two firms located at some points of a line segment along which consumers are located. Consumers have heterogeneous preferences both over a horizontal parameter (brand) and a vertical one (quality). It is assumed that firms observe the location parameter while vertical preferences are private information. The difference in types gives a rationale for non-linear contracts, while the horizontal dimension is used at first to control for the intensity of price competition.

In the monopoly case, the only option left to the consumer, other than buying a product, is inaction: now the customer has the right to buy from another supplier. In the context of this model, oligopolistic interaction can be reduced to reformulating the individual rationality constraints since the outside option is endogenised by the presence of a rival firm.

For given firms' locations, I characterize discriminatory contracts that change according to preferences over brand and quality. In particular, I discuss how there are three different discriminatory mechanisms at work that define three corresponding regions according to consumers' tastes.

In a first region (region A), I find 'monopoly-type' discrimination with 'standard' constraints binding. The same quality distortions result as in a monopoly situation, thus competition has no impact on efficiency but simply redistributes surplus. Low types are offered better deals because they would otherwise change supplier. High types are not directly affected by the presence of their

alternative good, but they indirectly benefit through the effects on low types in order to maintain the compatibility between contracts.

In an intermediate region (region B), a firm has to take into account that the rival good also becomes attractive for high-type consumers. Compared to region A, higher discounts are necessary to retain them and the force at work is now the presence of a potential rival good rather than the compatibility between the contracts alone. In order to have low types self-select the contract designed for them, a firm starts reducing the distortion of quality, so we have ‘intermediate’ discrimination. Competition not only redistributes surplus as in region A, but also yields efficiency gains. The quality distortion is reduced for those consumers which are less sensitive to brand differences and such consumers pay higher prices for higher quality.

In a final region (region C), quality reaches efficient levels for all types. Firms are not concerned any more by the compatibility of contracts. Discrimination is ‘competitive’ in the sense that it is driven only by cost differences.

As one moves from region A to region C, a firm sees its monopoly power being reduced and its production choice is disciplined towards efficiency. When low-type customers are not too sensitive to the rival option (region A), the incumbent still offers them inefficiently low quality as a screening device that allows him to discriminate among heterogeneous customers. When low-type consumers have weaker preferences for a firm’s brand (regions B and C), the rival good has a bigger impact because it is valued more by all consumers along the horizontal dimension and competition drives to allocative efficiency.

By providing a closed-form solution to contracts, I can proceed one step further and address another question that represents the second theme of this paper. I am able to endogenize firms’ locations, thus contributing to an extensive literature on spatial competition in Hotelling-type models where firms first choose locations and then price schedules.

When a firm chooses its location taking the rival’s location as given, it has to find a trade-off among several effects. When a firm gets closer to the rival, it causes:

- an increase in market share;
- a change in total profits that can be obtained at a generic location;
- a change in the discriminatory ability of the firm.

The last point is the novel aspect of this paper in the context of the literature on spatial discrimination. Other things being constant, we can expect that the firm will try to extend region A as much as possible in order to exercise ‘monopoly-type’ discrimination over quality. Indeed, the interval of validity of each region depends on an exogenous parameter of vertical heterogeneity but also on the horizontal location of each firm. In particular, I show how the symmetric location equilibrium is non-monotonic in the difference between types. When types are relatively similar, firms tend to locate closer to each other as the difference between types increases and ‘monopoly-type’ discriminatory contracts prevail in the last stage of the game. When the difference is high enough, the reverse is true and firms locate further apart as the difference between types increases. Finally, when the difference is very high, efficient discriminatory contracts emerge everywhere and firms also choose socially optimal locations.

My results also contribute to shed additional light on the old question as to whether firms tend to agglomerate or differentiate in Hotelling-type models. I obtain that when firms compete over locations and non-linear contracts, they tend to agglomerate more than a monopoly firm. Their locations depend in a non-monotonic way on customer heterogeneity, however they never agglomerate ‘too much’ (in particular their distance is at most 0.5 and at least 0.458), thus reinforcing the validity of a ‘Principle of almost intermediate differentiation’, in the sense that firms always locate ‘around’ the first and third quartiles.

1. Introduction

The purpose of this paper is twofold: firstly, its aim is to characterise competition between two rival firms that offer alternative price contracts designed to discriminate between different groups of consumers; secondly it studies how competition over locations affects the discriminatory ability of firms.

As far as price discrimination is concerned, much is known about the analysis of such contracts under monopoly.[1] In practice, however, discriminatory practices are common in oligopolistic industries, but the economic analysis of this setting is not entirely well understood. The rationale for discrimination stems from the fact that consumers have heterogeneous preferences over product characteristics. If a monopolist producer knew exactly the preferences of his customers (the customer 'type'), he would offer the most preferred variety to each type, and then charge a price not exceeding the surplus created: the problem of the firm is relatively simple, facing only one kind of constraints, usually called 'participation' or 'individual rationality' (IR) constraints. This would be a case of first-degree price discrimination: allocations are efficient and the firm appropriates the entire surplus. However, perfect discrimination is very unlikely in practice either for legal reasons or because the firm does not observe each type, but is simply aware of its overall distribution. In a context of imperfect information, a producer faces additional constraints. Different contracts, in fact, have to be freely chosen by each consumer: this is what is usually called a 'self-selection' or 'incentive compatibility' (IC) constraint. It is intuitive that the firm will be more 'cautious' with those consumers with a high willingness-to-pay. One should expect to find efficient allocations for high types, because any other variety would cause a sharp decline in the surplus they enjoy, and this would have to be compensated by a big decline in their price. On the other hand, distortions on low types can be introduced by the producer in order to make sure that high types will never decide to select a bundle different to the one designed for them.

The argument that I have just sketched in an informal way has received a great deal of attention in the literature, following a seminal paper of Mussa and Rosen (1978) that has initiated a family of principal-agent problems illustrating the equivalence between price discrimination using quantity discounts (second-degree discrimination) and monopoly pricing of products of differing quality. They show that a monopolist offers a quality range that is broader than that required for efficiency (cf. also Maskin and Riley, 1983, for a general treatment). This is because by exaggerating quality differences, the firm can effectively screen different customers and discriminate between them, and it is in this respect that non-linear pricing is a particular kind of product differentiation. Efficiency, however, is achieved 'at the top', among those customers with the highest willingness-to-pay.

In the simplest case one could think of, with just two types, one of two situations can happen. If the differences between types are very big, then any attempt to make low types buy the product would have a chain effect on contracts offered to high types. The firm is better off by reducing the size of its market by dismissing completely low types, concentrating only on high types over which it can exercise full monopoly power. In the more interesting case with type differences that are not too marked, the firm is willing to serve both categories of users. Under a wide range of circumstances, it can be shown that only two 'standard' constraints (out of four) are binding: IR for low types and IC for high types. Prices extract all the surplus from low types, while some 'informational rent' is left to high types.

Now in the oligopolistic setting, matters are more complex. The presence of more than one firm gives the customer the right to buy products from different suppliers. If firms can offer perfect substitutes, then we can expect Bertrand-type outcomes. Prices will be brought in line

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[1] See Phlips (1983), Varian (1989) and Wilson (1993).

with costs, and customers will buy their preferred quality, paying just production costs. The efficiency properties in standard models of perfect competition are well known. However, if firms offer imperfect substitutes (think of horizontal brand preferences), then the analysis is less clear. First of all, it is unlikely that firms will decide to dismiss completely low types even when there are huge variations in the intensity of consumer preferences. A market can be left unserved by a firm only if it does not leave the possibility of profitable entry by a rival. If a market can be potentially covered by two firms, then all consumers will be served, no matter what the difference between types is.

The notion of competition plays a central role in understanding how contracts change with respect to a discriminatory monopolist since contracts in a competitive environment should not give room to rivals. Firms typically differ in the relative appeal to customers, so it is interesting to study the properties of equilibrium contracts with different intensity of competition. One implication is that the presence of a rival will have a positive impact on consumer surplus. It is not obvious, however, whether the mechanism at work is simply a transfer between buyers and sellers, or whether allocations are affected as well.

It is helpful, to fix ideas, to begin with the simple extreme of a rival that does not represent a substantial threat. The discriminatory contracts offered by an incumbent will roughly follow the same line of reasoning typical of an uncontested monopolist: the only difference is that some rent has to be left also to low types (the minimal amount such that the rival will never be able to offer a good to them in a profitable way). Thus, one could think of a monopolist 'adjusted' problem, with the two standard binding constraints: IC for high types and 'adjusted' IR for low types. The discriminatory mechanism is the same one as under pure monopoly, thus competition has no impact on efficiency but simply redistributes surplus.

Turning to the more general case of firms offering goods that are not too imperfect substitutes, then the picture changes. Now high types can find appealing not only the low-type bundle, but also the rival good. The forces at work are both the presence of a potential rival good and the compatibility between the contracts offered to different consumers. The incumbent firm loses some of its screening ability, which means that the quality distortion cannot be as big as under monopoly. Competition then should yield efficiency gains.

I have already said that the screening potential is completely eliminated when firms offer perfectly substitutable goods. Does this imply that efficiency is reached only in that case, or is 'sufficient' substitutability enough? Among the results derived below, I will show that the latter case is true. Efficiency is typically reached, as it is intuitive, in a region characterised by brand preferences that are not too strong, but also when brand preferences are strong *and* differences between types are substantial. This is because the willingness-to-pay for any good increases with the intensity of preferences. A very high type could still enjoy quite a lot of surplus even from an outside good that is quite distant from his ideal brand.

I consider a model of two firms located at some points of a line segment along which consumers are located. Consumers have heterogeneous preferences both over a horizontal parameter (brand) and a vertical one (quality). It is assumed that firms observe the location parameter while vertical preferences are private information. The difference in types gives a rationale for non-linear contracts, while the horizontal dimension is used at first to control for the intensity of price competition. It should be noted that previous work has been done on the symmetric case of unobservable horizontal parameters and observable vertical ones (Spulber, 1989; Hamilton and Thisse, 1997), while the case dealt by this paper has not been studied before, with the exception of Stole (1995).[2]

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[2] The mechanisms that Stole identifies are very similar to those that emerge in this paper and they have been derived independently. In section 4 of his paper, Stole (1995) deals with the more general case of 'vertical uncertainty' that closely parallels the basic model presented here. While he considers a continuum of types over the vertical dimension, I have discrete types. This has an impact on the regions of validity of 'non standard' binding constraints. In Stole's paper all the consumers' IR constraints are binding only when consumers are located in the midpoint between firms, while in this paper IR constraints for all types result to be binding *everywhere* when vertical preferences are very heterogeneous, and in any case they bind at some locations other than the midpoint. In Valletti (1996), I also discuss price dispersion, i.e. the observed range of prices for class of customers, and present

For given firms' locations, I characterise discriminatory contracts that change according to preferences over brand and quality. In particular, I discuss how there are three different discriminatory mechanisms at work that define three corresponding regions according to consumers' tastes. By providing a closed-form solution to contracts, I can proceed one step further and address another question that represents the second theme of this paper. I am able to endogenise firms' locations, thus contributing to an extensive literature on spatial competition in Hotelling-type models where firms first choose locations and then price schedules. While Lederer and Hurter (1986) consider the case of perfect spatial price discrimination with identical consumers with inelastic demand, Hamilton and Thisse (1992), study perfect spatial price discrimination and quantity-dependent price discrimination when customers have downward-sloping demands. However, Hamilton and Thisse assume perfect observability also on customers' types, hence they study first-degree discrimination without adverse selection in the pricing game. On the other hand, in this paper I tackle the more interesting case of second-degree price discrimination (still with perfect spatial price discrimination)[3] and I introduce a novel aspect in the spatial analysis: the location choice of a firm affects, among other things, also the firm's discriminatory ability in the last stage of the game. In particular, I show how the symmetric location equilibrium is non-monotonic in the difference between types. When types are relatively similar, firms tend to locate closer to each other as the difference between types increases and "monopoly-type" discriminatory contracts prevail in the last stage of the game. When the difference is high enough, the reverse is true and firms locate further apart as the

an extension of the model with capacity constraints together with an application to the UK mobile telecommunications market. See also Martimort (1992) and Stole (1991) on the more general problem of multiprincipal incentive theory. Earlier works on third-degree price discrimination under imperfect competition include Borenstein (1985) and Holmes (1987).

[3] Unobservability on both horizontal and vertical parameters is an interesting but also very challenging task. Spulber (1981) represents one of the first attempts to obtain solutions for non-linear mill pricing and he compares it with local price discrimination. However, he is able to obtain solutions only for the monopoly case. In general, multi-dimensional screening models can easily become very difficult to solve. See Armstrong and Rochet (1999) for the monopoly case where screening is done over two dimensions, each with a binary distribution; see also Rochet and Stole (1998) for a duopoly setting that does not address the location problem.

difference between types increases. Finally, when the difference is very high, efficient discriminatory contracts emerge everywhere and firms also choose socially optimal locations.

The basic set up of the model is presented in section 2. The solutions to the first-best and to the monopoly case are derived in section 3. As is common in location models, in the duopoly I restrict the analysis to subgame perfect Nash equilibria with locations chosen first and then contracts. Solving backwards, sections 4 and 5 discuss the more complex case of discriminatory contracts in a duopoly, with particular reference to the effects of competition on consumer participation. Section 6 studies the location game and section 7 concludes.

2. The Model

Consumers with heterogeneous tastes buy a single unit of a certain product. They differ in the ideal brand, and this is modelled as in traditional spatial models of horizontal product differentiation. The brand space is represented by a line of unit length along which consumers are distributed. Each consumer is identified with her own location *d*. The loss of surplus to the consumer when she buys a product which does not coincide with her ideal is dependent on the distance between the product bought and the consumer's ideal, that is the distance between the seller's and consumer's locations.

Products are also assumed to be differentiated in terms of a vertical attribute u referred to as quality. At each location there is an equal measure of two types of consumers, a high type and a low type with the former valuing a given u more than the latter. This difference is taken into account by a parameter $\theta \in \{\underline{\theta}, \overline{\theta}\}, \overline{\theta} \geq \underline{\theta} > 0.$ Both types are uniformly distributed along the line, with mass 1 each.

When a consumer of type θ buys at a price p a product of quality u produced by a firm i located at d_i, which is then at a distance $|d - d_i|$ from her, she enjoys a net surplus:[4]

$$V = U(\theta, u; d) - p = \theta(1 - |d - d_i|)u - p.$$

Turning to the production technology, a unit of a good of quality u can be supplied at a quadratic cost:

$$C(u) = u^2/2.$$

This set up could also be used to address the problem of second-degree price discrimination. Under monopoly conditions, in fact, the quantity-pricing problem is isomorphic to the quality-pricing problem of a monopolist (Varian, 1989). For instance, with a transformation $x = u^2/2$, preferences can be rewritten as $U(\theta, d, x) = \theta(1 - |d - d_i|)(2x)^{1/2}$, where x can be interpreted as quantity. The cost function becomes $C(x) = x$, which means that each unit is produced at a constant marginal cost equal to unity. However, I prefer to stick to the original formulation of a 'quality' problem. This is because with multiple suppliers the choice to buy only from one firm should rise endogenously in the 'quantity' problem, which could impose additional constraints.

In the remaining part of the paper I will assume that the location parameter d can be perfectly observed, while this is not the case for the preferences over the vertical attribute. However, the producer knows the distribution of θ, hence he can practice price discrimination

[4] More precisely, $V(\cdot)$ is the conditional indirect utility function of type θ and its formulation follows a tradition that goes back to Mussa and Rosen (1978). Peitz (1995) shows that it is possible to construct an associated direct

at a given location. The role of the horizontal parameter is two-fold. Firstly, when I obtain discriminatory contracts offered by competing firms over quality at each location, the horizontal parameter regulates the intensity of price competition. Its perfect observability allows me to consider the effects of ’pure’ price discrimination when the producer has varying degrees of market power along the line. Secondly, the closed-form solutions obtained in the last stage of the game are manageable so that location choices can be endogenised.

3. Benchmarks: Efficiency and Monopoly

3.1 Efficiency

Assume that a single firm is located at a location d_i. The efficient quality allocation is the one that maximises social surplus at every point along that line:

$$\max_{\underline{u},\overline{u}} \underline{U}(\underline{u};d)+\overline{U}(\overline{u};d)-C(\underline{u})-C(\overline{u})=\underline{\theta}(1-|d-d_i|)\underline{u}+\overline{\theta}(1-|d-d_i|)\overline{u}-\frac{\underline{u}^2}{2}-\frac{\overline{u}^2}{2}$$

where the underlined variables refer to low types and overlined ones to high types. The efficient allocation and the corresponding social surplus are:

$$\begin{cases}\underline{u}^e(d)=\underline{\theta}(1-|d-d_i|)\\ \overline{u}^e(d)=\overline{\theta}(1-|d-d_i|)\end{cases}\quad\begin{cases}\underline{U}-C(\underline{u})=\underline{u}^e(d)^2/2\\ \overline{U}-C(\overline{u})=\overline{u}^e(d)^2/2\end{cases}$$

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utility function, despite continuity problems, so that the behaviour of discrete choice and unit demand adopted here can be derived from utility maximisation.

Since goods of higher qualities are more expensive to produce, it turns out to be efficient to allocate them only to those consumers whose valuation of quality is sufficiently high, i.e. consumers with strong preferences over brand or over the vertical attribute. This explains why the efficient *u* increases with type and decreases with the distance between the firm and a customer.

The optimal location of plant(s) is also easily determined: plant(s) should be set in order to minimise total losses depending on the distance between plant(s) and customer. Since social surplus at a given location is proportional to $(1-|d-d_i|)^2$, every plant should be set at the mid-point of the market it serves.[5] Hence, if only one plant is available, it should be set at d_1 = 1/2. If two plants are available, they should be placed at the quartiles 1/4 and 3/4.

3.2 Monopoly

Consider now the case of a monopolist operating a plant at d_i. In general, the strategy space of the monopolist would consist of a family of schedules, one for each value of *d*, labelled $p(u; d)$. Since there are only two types at each location, I can confine the attention to the special case of two contracts being offered at each *d*. In addition, the monopolist can also perfectly discriminate over distance (there cannot be arbitrage between consumers at different locations), so that each pair of contracts can be treated separately since there are no linkages between sales at different points. As a consequence, I can drop the dependence of contracts on *d* for simplicity

[5] A function $F(d_i) = \int_a^b K(1-|x-d_i|)^2 \mathrm{d}x$, where *K* is a constant and $a \bullet d_i \bullet b$, attains a maximum when $x = (a + b)/2$. This result would also be true if one adopted a different model to address discrimination over quantity *x* rather than quality. As it was mentioned before, this would exactly apply to the utility function $U(\theta, d, x) = \theta(1 - |d - d_i|)(2x)^{1/2}$ when the producer has unit marginal costs. Another example is the common case of linear demand with transportation costs: in particular the correspondence would be exact with demand $p = \theta - x$ and transportation costs $\theta|d - d_i|$ per unit carried to consumer θ.

of notation. At every location, the monopolist offers pairs of contracts $(\underline{p},\underline{u}),(\overline{p},\overline{u})$ designed for the two types that have to self-select them. His aim is to maximise total profits at *d*, subject to participation (IR) and incentive compatibility (IC) constraints for both types:

$$\max_{(\underline{p},\underline{u})(\overline{p},\overline{u})} \Pi = \underline{\Pi} + \overline{\Pi} = \underline{p} - \frac{\underline{u}^2}{2} + \overline{p} - \frac{\overline{u}^2}{2}$$

subject to

$$\begin{aligned}
&\underline{\mathrm{IR}} : \underline{\theta}(1-|d-d_i|)\underline{u} - \underline{p} \geq 0 \\
&\overline{\mathrm{IR}} : \overline{\theta}(1-|d-d_i|)\overline{u} - \overline{p} \geq 0 \\
&\underline{\mathrm{IC}} : \underline{\theta}(1-|d-d_i|)\underline{u} - \underline{p} \geq \underline{\theta}(1-|d-d_i|)\overline{u} - \overline{p} \\
&\overline{\mathrm{IC}} : \overline{\theta}(1-|d-d_i|)\overline{u} - \overline{p} \geq \overline{\theta}(1-|d-d_i|)\underline{u} - \underline{p}
\end{aligned}$$

As it is standard in this kind of problem with adverse selection, the only binding constraints are IR for the low type and IC for the high type. The algebraic solution is:

$$(1)\qquad \begin{aligned}
&\begin{cases} \underline{u}^m(d) = (1-|d-d_i|)(2\underline{\theta} - \overline{\theta}) < \underline{u}^e(d) \\ \overline{u}^m(d) = (1-|d-d_i|)\overline{\theta} = \overline{u}^e(d) \end{cases} \\
&\begin{cases} \underline{p}^m(d) = (1-|d-d_i|)^2(2\underline{\theta} - \overline{\theta})\underline{\theta} \\ \overline{p}^m(d) = (1-|d-d_i|)^2(2\overline{\theta}^2 - 3\overline{\theta}\underline{\theta} + 2\underline{\theta}^2) > \underline{p}^m(d) \end{cases} \\
&\begin{cases} \underline{\Pi}^m(d) = (1-|d-d_i|)^2(2\underline{\theta} - \overline{\theta})\overline{\theta}/2 \\ \overline{\Pi}^m(d) = (1-|d-d_i|)^2(3/2\,\overline{\theta}^2 - 3\overline{\theta}\underline{\theta} + 2\underline{\theta}^2) > \underline{\Pi}^m(d) \end{cases} \\
&\begin{cases} \underline{V}^m(d) = 0 \\ \overline{V}^m(d) = (1-|d-d_i|)^2(\overline{\theta} - \underline{\theta})(2\underline{\theta} - \overline{\theta}) \end{cases}
\end{aligned}$$

The solution displays ’efficiency at the top’, while the quality offered to the low type is distorted away from the efficient one. The monopolist widens the quality spectrum in order to effectively discriminate among the consumers he faces. Prices extract the entire surplus from low types, while some informational rent is left to high types. Finally, the market is completely served if $(2\underline{\theta} - \overline{\theta}) \geq 0$. It will be convenient to refer to the ratio between the two type parameters, then the previous condition can be rewritten as $1 \leq k = \overline{\theta} / \underline{\theta} \leq 2$. This restriction on k says that all consumers are economically interesting for a monopolist.[6]

While distortions arise in the allocation of qualities due to asymmetric information problems, on the contrary the location of plant(s) is always efficient. In fact total profits at a given location are proportional to $(1 - | d - d_i |)^2$, hence the location problem for a monopolist is not different from the location problem for a social planner (see the remark at footnote 5) and every plant should be set at the mid-point of the market it serve: if the monopolist owns only 1 plant, this is set at 1/2; if he owns two plants, one is set at 1/4 and serves customers between the origin of the line and 1/2, while the second plant is set at 3/ 4 and it supplies customers between 1/2 and 1.

4. Effects of Competition on Consumers' Participation Constraints

The aim of this and the following sections is to analyse the effects on quality, prices and locations induced by the presence of a second, independent firm. Firms first simultaneously choose locations, and then, given the pair of locations, they choose simultaneously discriminatory contracts that depend on location. I will refer with 1 (respectively 2) to the variables related to the left firm located at d_1 closest to the origin of the line (respectively the right firm at d_2 from the end of the line). I will drop supercripts when ambiguities do not arise.

[6] When $k > 2$, the monopolist would prefer not to supply low types at any location. High types would consume the efficient quality and would have their surplus completely extracted by the price. I will show that competitive duopolists are always forced to provide goods to both types.

Both firms have the same technology of production and this symmetry, combined with the spatial model of preferences, immediately suggests that in equilibrium the market will be split in two parts. Intuitively, consumers in $[0, (1 + d_1 - d_2)/2]$ will buy from firm 1 and consumers in $((1 + d_1 - d_2)/2, 1]$ from firm 2 because each firm enjoys an advantage over the rival firm for those customers who are relatively closer. Suppose, on the contrary, that a customer at $d < (1 + d_1 - d_2)/2$ is served by firm 2 in equilibrium. A minimum rationality requirement implies that firm 2 is making non-negative profits on that customer. By mimicking the same price-quality schedule, firm 1 can attract the customer while securing the same level of profits as firm 2. Therefore such customer cannot be served by the more distant firm.

It is clear that firms cannot behave as simple monopolists in their own markets, hence contracts offered by a firm in its market must differ from those described in section 3. In particular, contracts must take into account that no profitable entry into firm 1's market by firm 2 should occur. Absence of entry does not mean absence of competition, and in fact potential entry imposes a real constraint on firm 1's actions. Firm 1 has to make offers to its customers that cannot be matched by the rival. Using a standard undercutting argument, the best price-quality schedule offered by a firm to its rival's customers must follow a zero-profit condition. The best that firm 2 can offer to 1's customers is to give them the technology at its cost, hence quality u at cost $u^2/2$. Consumer θ would choose $u^2 = \text{argmax } \theta[1 - |1 - d - d_2|]u - u^2/2 = \theta(d + d_2)$, enjoying a net surplus $V^2 = (d + d_2)^2\theta^2/2$. The existence of equilibrium contracts under more general conditions has been provided by Stole (1995) and his arguments are immediately translated into our model:

Proposition 1. For given locations, there exists an equilibrium in which customers in $[0, (1 + d_1 - d_2)/2]$ will buy from firm 1 and consumers in $((1 + d_1 - d_2)/2, 1]$ from firm 2. All customers are served and the reservation value of each consumer in $(0, (1 + d_1 - d_2)/2]$ is greater than zero and increases in d: $V^1 \bullet (d + d_2)^2\theta^2/2$.

The possibility of entry gives the customer the right to buy the other firm's product under particular conditions and the outside option for each captive consumer is endogenised by the presence of a potential rival firm. In the monopoly case the only option left to the consumer is inaction, now the customer has the right of buying a more distant product. In the context of this model, oligopolistic interaction can be reduced to reformulating the individual rationality constraints. It is important to mention that the net surplus V is higher for higher types not only because they value any given quality more than lower types, but also because they would choose a higher quality variant of the distant firm's product. Once the participation constraints have been modified, then the optimal solution for each firm, within its individual market, can be thought of as a monopolist's problem as before.[7]

5. Discriminatory Contracts in the Duopoly Case

The previous section has developed the idea that each incumbent firm has to readjust its optimal policy when it faces a rival. In particular, no schedule offered by the rival should be able to steal customers profitably. The main implication is that both participation constraints are affected by the presence of a competing firm and they become type-dependent. Firm 1 has to find the optimal price-quality schedules in its market $[0, (1 + d_1 - d_2)/2]$ as a solution to the following maximisation problem at each location (superscripts referring to firm 1 are omitted for simplicity of notation):

[7] See Jullien (1997) on the problem of optimal contracts when the agent's reservation utility is type-dependent.

$$\max_{(\underline{p},\underline{u})(\overline{p},\overline{u})} \Pi = \underline{\Pi} + \overline{\Pi} = \underline{p} - \frac{\underline{u}^2}{2} + \overline{p} - \frac{\overline{u}^2}{2}$$

subject to

$$\underline{\text{IR}}:\ \underline{\theta}(1-|d-d_1|)\underline{u} - \underline{p} \geq (d+d_2)^2 \frac{\underline{\theta}^2}{2}$$

$$\overline{\text{IR}}:\ \overline{\theta}(1-|d-d_1|)\overline{u} - \overline{p} \geq (d+d_2)^2 \frac{\overline{\theta}^2}{2}$$

$$\underline{\text{IC}}:\ \underline{\theta}(1-|d-d_1|)\underline{u} - \underline{p} \geq \underline{\theta}(1-|d-d_1|)\overline{u} - \overline{p}$$

$$\overline{\text{IC}}:\ \overline{\theta}(1-|d-d_1|)\overline{u} - \overline{p} \geq \overline{\theta}(1-|d-d_1|)\underline{u} - \underline{p}$$

In principle there could be many combinations of the four constraints in firm 1's maximisation problem but only three of them are plausible (see the Appendix, that also contains all the details). Each one of these three combinations corresponds to a particular mechanism, which gives birth to an optimal solution in a certain region. I am also going to show how such regions of validity depend on the two parameters *k* and *d*.

A solution can be found in the first region, labelled A, which runs from the origin to a certain location $d^*(k)$ to the right of d_1. The solution takes into account only $\underline{\text{IR}}$ and $\overline{\text{IC}}$ binding:

$$(2)\qquad \begin{cases} \underline{u}(d;\text{A}) = \underline{u}^m(d) \\ \overline{u}(d;\text{A}) = \overline{u}^m(d) \end{cases}$$
$$\begin{cases} \underline{\Pi}(d;\text{A}) = (1-|d-d_1|)^2(2\underline{\theta}-\overline{\theta})\overline{\theta}/2 - (d+d_2)^2\underline{\theta}^2/2 \\ \overline{\Pi}(d;\text{A}) = (1-|d-d_1|)^2(3/2\,\overline{\theta}^2 - 3\overline{\theta}\underline{\theta} + 2\underline{\theta}^2) - (d+d_2)^2\underline{\theta}^2/2 \end{cases}$$

The basic discriminatory mechanism at work is the same as the one discussed in section 3.2, hence in region A we have "monopoly-type" discrimination with 'standard' constraints binding. The same quality distortions result, thus competition has no impact on efficiency but simply redistributes surplus. The effects of the alternative good on surplus are identical in absolute terms for both types of consumers but they are caused by two different reasons. Low types are offered better deals because they would otherwise change supplier. High types are not directly affected by the presence of their alternative good, but they indirectly benefit through the effects on low types in order to maintain the compatibility between contracts ($\overline{\text{IC}}$ binds). It also results relatively easy for the firm to separate the two classes of customers. This is true so long as the rival good does not become interesting to high types too, which is more likely for high values of $\overline{\theta}$. As a result, the extension of region A narrows as *k* increases.

Proposition 2. When *d* is in region A = [0, max{0, *d**(*k*)}], "monopoly-type" discriminatory contracts are given by eq. (2). Quality is as in the monopoly case. The width of the interval decreases with *k*.

When consumers located at *d**(*k*) are reached, the $\overline{\text{IR}}$ constraint starts binding and the solution changes. We enter a new region, labelled B, that runs from *d**(*k*) to $\hat{d}(k)$. Three constraints are binding simultaneously and the solution results as follows:

$$\begin{cases} \underline{u}^e(d) > \underline{u}(d;\text{B}) > \underline{u}^m(d) \\ \overline{u}(d;\text{B}) = \overline{u}^m(d) \end{cases}$$

(3)
$$\begin{cases} \underline{\Pi}(d;\text{B}) = \dfrac{(d+d_2)^2}{2}[\underline{\theta}\overline{\theta} - \dfrac{(d+d_2)^2}{(1-|d-d_1|)^2}\dfrac{(\underline{\theta}+\overline{\theta})^2}{4}] \\ \overline{\Pi}(d;\text{B}) = [(1-|d-d_1|)^2 - (d+d_2)^2]\overline{\theta}^2/2 \end{cases}$$

In region B, firm 1 has to take into account that the rival good has also become attractive for high-type consumers. Compared to region A, higher discounts are necessary to retain them and the force at work is now the presence of a potential rival good rather than the compatibility between the contracts alone. In region B the firm is particularly constrained, so it first optimises on the most profitable type, and then adjusts the other contract accordingly. In order to have low types self-select the contract designed for them, firm 1 starts reducing the distortion of quality, so we have "intermediate" discrimination. Competition not only redistributes surplus as in region A, but also yields efficiency gains. The quality distortion is reduced for those consumers which are less sensitive to brand differences and they pay higher prices for higher quality.

Proposition 3. When d is in region B = [max{0, $d^*(k)$}, max{0, $\hat{d}(k)$}], "intermediate" discriminatory contracts are given by eq. (3). High-type customers are offered the same quality as in the monopoly case while distortions in the quality offered to low-type customers are reduced. The width of the interval increases with k when k • k_A, then it decreases.

When the last consumer in region B is supplied, the nature of the problem changes again since the $\overline{\text{IC}}$ constraint is not binding anymore. The solution in the region close to the centre of the market, labelled C, is the last one relevant to this model and it is characterised by:

$$(4)\qquad \begin{cases} \underline{u}(d;\mathrm{C}) = \underline{u}^e(d) \\ \overline{u}(d;\mathrm{C}) = \overline{u}^e(d) \end{cases} \qquad \begin{cases} \underline{\Pi}(d;\mathrm{C}) = [(1-|d-d_1|)^2 - (d+d_2)^2]\underline{\theta}^2/2 \\ \overline{\Pi}(d;\mathrm{C}) = [(1-|d-d_1|)^2 - (d+d_2)^2]\overline{\theta}^2/2 \end{cases}$$

If consumer θ were to buy from the rival, she would choose $u^2 = \theta(d + d_2)$, enjoying a net surplus $V^2 = (d + d_2)^2\theta^2/2$. The difference in the quality potentially available for the two types increases approaching the centre of the market. In practice, once the outside good is taken into account, such a difference separates the two problems and in region C firm 1 does not have to worry about the compatibility between contracts but only about both participation constraints. We saw before that in region B the distortion in the quality offered to low types is gradually reduced: at a certain location, which coincides with $\hat{d}$, $\underline{u}$ reaches the efficient level. After that location, the firm is not concerned by the compatibility of contracts, and it is not necessary to overshoot the efficient quality offered to low types. In the entire region C the efficient allocation is reached for all consumers and we have "competitive" discrimination. Since firm 1 still enjoys an advantage deriving from brand preferences, the price charged to customers allows for positive profits. Zero profits result only for the marginal consumers at $d = (1 + d_1 - d_2)/2$. These are the customers who are exactly indifferent between the two brands, therefore the intensity of competition is maximal and drives away all profits. Finally, the extension of region C depends only on its left bound, since its right bound is fixed at $(1 + d_1 - d_2)/2$. The left bound $\hat{d}(k)$ decreases with k, therefore region C widens as k increases.

Proposition 4. When d is in region C = $[\max\{0, \hat{d}(k)\}, (1 + d_1 - d_2)/2]$, "competitive" discriminatory contracts are given by eq. (4). Both types of customers are offered the efficient quality. The width of the interval first increases with k as long as $k \bullet k_B$, then it remains constant.

The extension of each of the three regions that I have identified depends on k, the ratio between the taste parameters. Figure A1 in the Appendix draws a phase diagram that gives regions A, B and C as functions of d and k. It is useful to recapitulate the three different discriminatory mechanisms at work in each region and their dependence on location and vertical tastes. In region A, the rival good is so distant that it imposes a weak constraint on firm

1's policy. No quality adjustments are required compared to the monopoly case, rather price reductions are sufficient to outperform the rival firm. In terms of consumer tastes, region A is characterised by either strong brand preferences for firm 1's variety, and/or small differences in the evaluation of the vertical attribute. In practice, firm 1 has to offer better deals to low types, which induces price reductions also to high types in order to preserve self-selection. As *d* and *k* increase, the outside option becomes more valuable for high types either because it is closer to their ideal, or because they value quality more. In region B, the incumbent is constrained by the potential outside choices of both classes of consumers that still have to be induced to choose their designed contract. The self-selection problem disappears in region C, which is valid close to the centre of the line and for high enough values of *k*. This is because the rival goods cause a significant difference in the reservation utility for the two types. When the incumbent ensures its customers the same level of surplus as the outside option, there is not the risk that high types should try to report a type different from their own.

As one moves from region A to region C, firm 1 sees its monopoly power being reduced and its production choice is disciplined towards efficiency. When low-type customers are not too sensitive to the rival option (region A), the incumbent still offers them inefficiently low quality as a screening device that allows him to discriminate among heterogeneous customers. When low-type consumers have weaker preferences for firm 1's brand (regions B and C), the rival good has a bigger impact because it is valued more by all consumers along the horizontal dimension and competition drives to allocative efficiency. At first sight, region B shows a paradoxical result: prices for low types increase with *d*, and they may be higher than the monopoly price at the same location. This is consistent with the fact that firm 1 is also increasing the quality offered, therefore customers are willing to pay a premium price for a level of quality which is closer to the efficient one. The effects of competition are more 'evident' on prices rather than quality in region A, while the reverse is true in region B.

I conclude this section with a brief remark when k • k_A. It is easy to realise that there are only two solutions in two different intervals. Solutions (3) and (4) are still valid respectively in region B = [0, max{0, $\hat{d}(k)$}] and C. Discriminatory contracts are induced by competitive entry. A monopolist would make offers only to high types when $k > 2$, while the presence of a rival firm obliges producers to serve their entire markets. In this sense, when taste parameters over quality are sufficiently different, quality discrimination is observed *only* in a competitive environment.

6. The Equilibrium Locations

In the previous section, I have described optimal contracts for firm 1's customers, for given locations of the two competing firms. Now I turn to study the location equilibrium. Firm 1 has to maximise w.r.t. d_1 the following profit expression:

$$(5) \qquad \pi_1(d_1, d_2) = \begin{cases} \displaystyle\int_0^{d^*(k)} [\underline{\Pi}(x;\mathrm{A}) + \overline{\Pi}(x;\mathrm{A})]\mathrm{d}x + \int_{d^*(k)}^{\hat{d}(k)} [\underline{\Pi}(x;\mathrm{B}) + \overline{\Pi}(x;\mathrm{B})]\mathrm{d}x \\ \displaystyle\quad + \int_{\hat{d}(k)}^{(1+d_1-d_2)/2} [\underline{\Pi}(x;\mathrm{C}) + \overline{\Pi}(x;\mathrm{C})]\mathrm{d}x \quad \text{if } k \le k_\mathrm{A} \\ \displaystyle\int_0^{\hat{d}(k)} [\underline{\Pi}(x;\mathrm{B}) + \overline{\Pi}(x;\mathrm{B})]\mathrm{d}x + \int_{\hat{d}(k)}^{(1+d_1-d_2)/2} [\underline{\Pi}(x;\mathrm{C}) + \overline{\Pi}(x;\mathrm{C})]\mathrm{d}x \quad \text{if } k_\mathrm{A} < k < k_\mathrm{B} \\ \displaystyle\int_0^{(1+d_1-d_2)/2} [\underline{\Pi}(x;\mathrm{C}) + \overline{\Pi}(x;\mathrm{C})]\mathrm{d}x \quad \text{if } k \ge k_\mathrm{B} \end{cases}$$

When firm 1 chooses d_1 taking d_2 as given, it has to find a trade-off among several effects. An increase in d_1 would cause:

• an increase in market share;

• a change in total profits that can be obtained at a generic location *d*;

• a change in the discriminatory ability of the firm.

The last point is the novel aspect of this paper in the context of the literature on spatial discrimination. In the previous section, I have shown how discriminatory contracts differ in the three different regions labelled A, B and C. Total profit at a given location is greater the less constrained the firm is, hence it decreases going from region A to region C:

$$[\underline{\Pi}(d;\mathrm{A})+\overline{\Pi}(d;\mathrm{A})]\geq[\underline{\Pi}(d;\mathrm{B})+\overline{\Pi}(d;\mathrm{B})]\geq[\underline{\Pi}(d;\mathrm{C})+\overline{\Pi}(d;\mathrm{C})]$$

where the equality signs hold only at the boundary of two adjacent regions. Other things being constant, we can expect that the firm will try to extend region A as much as possible in order to exercise "monopoly-type" discrimination over quality. Indeed, the interval of validity of each region depends on the exogenous parameter *k* (vertical heterogeneity) but also on the horizontal location of each firm. In particular, in the Appendix I obtain the following:

(6) $$\frac{\partial d^*(k)}{\partial d_1}\begin{cases}>0 \text{ if } d^*(k)\geq d_1\\ <0 \text{ if } d^*(k)<d_1\end{cases}\quad \frac{\partial \hat{d}(k)}{\partial d_1}\begin{cases}>0 \text{ if } \hat{d}(k)\geq d_1\\ <0 \text{ if } \hat{d}(k)<d_1\end{cases}$$

For a given *k*, the first binding boundary for region A (d^*) is always to the right of d_1 when *k* is low enough, hence an increase in firm 1's location always extends region A. If we expand fictitiously region A all over the entire market share of firm 1, and comparing eq. (1) with eq. (2), one could write:

(7) $$\pi_1(d_1,d_2)=\int_0^{\bar{d}}[\underline{\Pi}(x;\mathrm{A})+\overline{\Pi}(x;\mathrm{A})]\mathrm{d}x=\int_0^{\bar{d}}[\underline{\Pi}^m(x)+\overline{\Pi}^m(x)-(x+d_2)^2\underline{\theta}^2]\mathrm{d}x$$

where $\bar{d}$ = (1 + d_1 - d_2)/2. Using Leibnitz' rule, the maximisation w.r.t. d_1 of the previous expression would differ from the monopolist's problem only for one term:

(8) $$\frac{\partial\pi_1(d_1,d_2)}{\partial d_1}=\frac{\partial\pi^m(d_1)}{\partial d_1}+[\underline{\Pi}^m(\bar{d})+\overline{\Pi}^m(\bar{d})-(\bar{d}+d_2)^2\underline{\theta}^2]\frac{\partial\bar{d}}{\partial d_1}=$$
$$[(2-\bar{x})(\bar{x}-2d_1)(\bar{\theta}+2\underline{\theta})+\frac{(\bar{\theta}-\underline{\theta})}{8}(1+d_1+d_2)^2](\bar{\theta}-\underline{\theta})$$

From the previous expression, it can be seen that the optimal location *exactly* coincides with the monopolist's choice (d_1 = 1/4) when k = 1: in such a case, in fact, regions B and C are empty, so that eq. (7) represents firm 1's total profit. As k increases, the positive additional term in the FOC given by eq. (8) implies that the optimal location should lie *to the right* of 1/4: to obtain the exact value one would obviously have to include also the contributions from regions B and C, however they are of negligible extensions for low k.

As k increases, region A becomes narrower, thus the "monopoly-type" discriminatory ability is substituted with "intermediate" discrimination in region B and "competitive" discrimination in region C. Concentrating only on the contribution to total profits obtained in region C, we can write:

$$\pi_1(d_1,d_2)=\int_{\hat{d}(k)}^{\bar{d}}[\underline{\Pi}(x;\mathrm{C})+\overline{\Pi}(x;\mathrm{C})]\mathrm{d}x$$

(9) $$\frac{\partial\pi_1(d_1,d_2)}{\partial d_1}=-[\underline{\Pi}(\hat{d};\mathrm{C})+\overline{\Pi}(\hat{d};C)]\frac{\partial\hat{d}}{\partial d_1}+\frac{(\bar{\theta}+\underline{\theta})^2}{8}(1-3d_1-d_2+2\hat{d})(3-d_1+d_2+2\hat{d})$$

From the previous FOC, it can be seen that when the left boundary of region C is at the origin ($\hat{d} = 0$), then the first term in eq. (9) is zero, and the symmetric equilibrium would be identical to the monopolist's choice ($d_1 = 1/4$). In the Appendix I show that this is indeed the case when vertical heterogeneity is big enough (k • 17). If one lowers k slightly below 17, then the total profit for firm 1 would have to include also the profit extracted in region B. However, at first, the latter contribution is negligible since region B would be very narrow; hence from eq. (9) it can be seen that the optimal location should lie *to the right* of 1/4 (the first term in eq. (9) is positive since $\hat{d}$ is to the left of d_1 hence its derivative w.r.t. d_1 is negative - see eq. (6)). When k is lowered even further, region B becomes more and more relevant and I have to rely on numerical solutions to solve for the symmetric equilibrium. Results are reported in the Appendix and they are summarised in figure 1 and in the following:

Proposition 5. There exists a symmetric two-stage Nash equilibrium. Equilibrium locations are non-monotonic in k. When $k = 1$, locations are socially optimal ($d_i = 1/4$) and there is no discrimination over quality (types are identical). When k is low enough ($1 < k$ • 1.7) firms tend to locate inside the second and third quartiles: d_i increases with k and there is a prevalence of (inefficient) "monopoly-type" discriminatory contracts over quality. Firms locate closest when k • 1.7 (d_i • .271). As k increases even further ($1.7 < k < 17$) firms still locate inside the second and third quartiles but d_i decreases with k: "intermediate" and "competitive" discriminatory contracts become more relevant while "monopoly-type" contracts disappear for $k > 1.8$. When k is big enough (k • 17), the unique equilibrium involves firms locating efficiently at the quartiles and offering only efficient "competitive" discriminatory contracts.

These results can be contrasted with previous works that have dealt with spatial price discrimination in a variety of pricing strategies used by firms. Lederer and Hurter (1986) find that competing firms would locate efficiently at the quartiles when consumer demands are perfectly inelastic. In their model there is no scope for discrimination over quantity, and their result is also obtained here where $k = 1$ so that there is no difference between types. Hamilton and Thisse (1992) also find an efficiency result when consumers have downward-sloping demands at each location and firms can *perfectly* price discriminate. They interpret their result in a ’negative’ way, arguing that the level of information and contractual complexity required to sustain the efficient allocations is very high. In fact, they show elsewhere (Hamilton *et al*., 1989) that if firms *do not* price discriminate at all at a given location, they would locate closer to the centre according to the magnitude of transportation costs.

In this paper, I have considered the intermediate case where, at each location, I require neither an implausibly high level of information to sustain first-degree price discrimination, nor a constraint on firms’ pricing flexibility. My model is one of second-degree price discrimination, where firms compete by varying their location as well as the shape of their non-linear price schedules. Incidentally, it should be noticed that the results of Hamilton and Thisse (1992) are also re-obtained here in a different setting when k • 17 (efficiency on both locations and quality allocations); however their efficiency result in the location game derives from perfect price discrimination while here it stems from "competitive" discrimination, hence the implications in terms of distribution of surplus among firms and customers are different. I should also remark that while efficiency over locations is attained if k is either very small or very high, in the former case "monopoly-type" contracts over quality would prevail with the associated distortions over quality as a screening device, while in the latter case efficiency would extend also over quality allocations.[8]

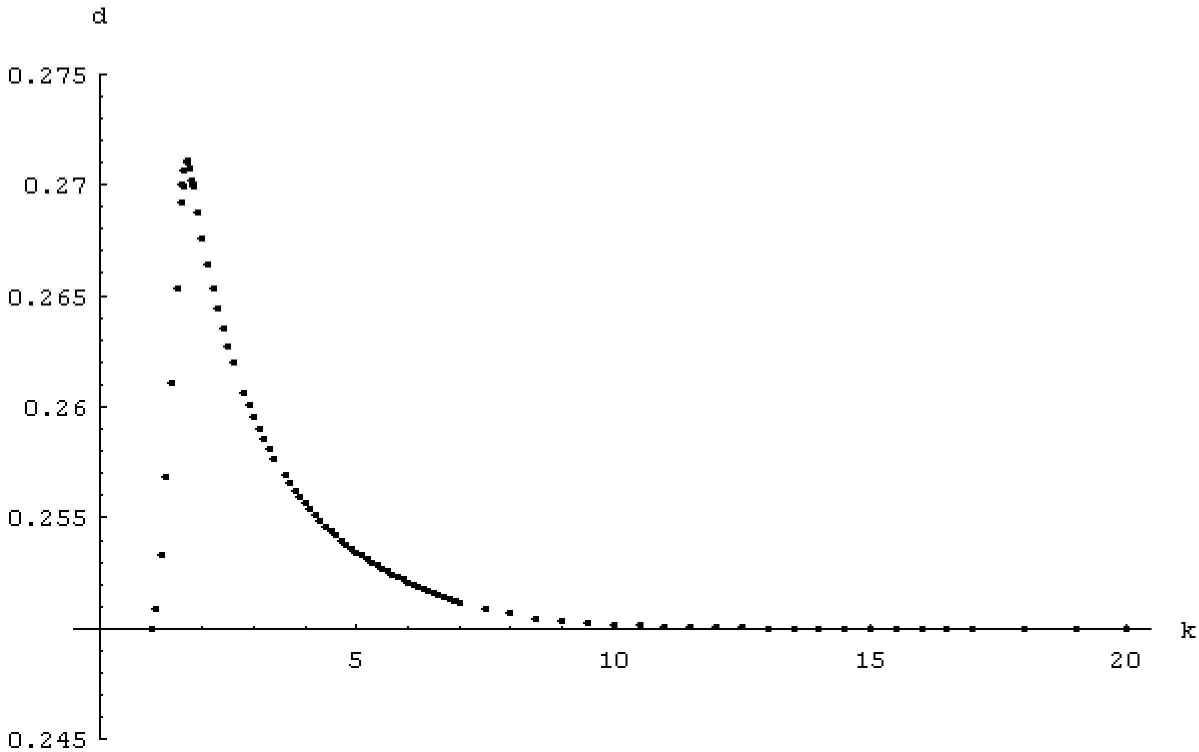

Figure 1 - Location equilibria

To conclude, it is also of some interest to observe that, when firms compete over locations, they are caught in a sort of prisoner's dilemma over the firms' discriminatory ability. Taking the point of view of the left firm, for intermediate values of *k* it has an incentive to locate to the right of 1/4; however this is also true for its rival: an increase in d_2 always cause a downward shift of the boundaries of validity of each region (see the Appendix), hence narrowing the more profitable regions (A and B) and causing the emergence of the

8 My results could also contribute to shed additional light on the old question as to whether firms tend to agglomerate or differentiate in Hotelling-type models. I have obtained that when firms compete over locations and non-linear contracts, they tend to agglomerate more than a monopoly firm. Their locations depend in a non-monotonic way on customer heterogeneity, however they never agglomerate 'too much' (in particular their distance is at most 1/2 and at least .458), thus reinforcing the validity of a "Principle of almost intermediate differentiation"

"competitive" region C for rather small values of k in equilibrium. If firms collude over locations, but still compete over discriminatory contracts, they would locate their plants much further away both for the standard reason of reducing the intensity of competition around the centre of the market and for the additional reason of extending the more profitable regions A and B as much as possible in order to benefit from their screening ability that is otherwise eliminated in region C.[9]

7. Concluding Remarks

This paper has analysed price discrimination and location choice in a duopoly game. Optimal contracts have been characterised and it has been shown that contracts change according to taste parameters over brand and quality.

When firms, at a given location, offer discriminatory contracts, they behave as if the rival firm was offering the best possible deal to its own customers: consumers' participation constraints become type-dependent and the solution of the contract game can be summarised as follows:

• When vertical preferences are not too different, there are three different discriminatory mechanisms at work that define three corresponding regions according to consumers' tastes. As brand preferences become weaker and/or differences between customers are more marked, quality distortions are reduced gradually until they are eliminated.

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recently proposed by Hinloopen and van Marrewijk (1999) in a Hotelling model with linear transport costs, when customers have finite reservation prices and the market is fully covered.

[9] I have computed the semi-collusive symmetric equilibrium (collusion over location in the first stage but competition over contracts in the last stage): the equilibrium locations are also non-monotonic in k, starting at 1/8 when $k = 1$, then first increasing with k and then decreasing for higher values. "Monopoly-type" contracts would be observed until $k = 1.95$, while "intermediate" discriminatory contracts would still be practised for $k < 97$.

• There is an entire range of vertical taste parameters which support discriminatory contracts *only* in a competitive environment.

Once location choices are endogenised, each firm takes into account the effect that its choice has on its discriminatory ability. The symmetric equilibrium in locations is not monotonic in vertical taste parameters. As vertical heterogeneity increases, firms first get closer and then further apart. However, product differentiation is 'intermediate' since firms always locate 'around' the first and third quartiles. Finally, when vertical taste parameters are sufficiently different, efficiency results both on locations and quality allocations.

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Appendix

Proof of Propositions 2-4

I discuss here the solution to firm 1's problem under duopoly. I assume first that $\underline{\text{IR}}$, $\overline{\text{IR}}$ and $\overline{\text{IC}}$ all bind: the firm has in principle four choice variables, but the three binding and independent constraints allow only for one degree of freedom. Contracts are:

$$\text{(A1)} \qquad \begin{cases} \underline{u} = \dfrac{(d+d_2)^2}{1-|d-d_1|}\dfrac{\underline{\theta}+\overline{\theta}}{2} \\ \underline{p} = (d+d_2)^2\underline{\theta}\overline{\theta}/2 \end{cases} \quad \begin{cases} \overline{u} = (1-|d-d_1|)\overline{\theta} \\ \overline{p} = \overline{\theta}^2[(1-|d-d_1|)^2 - (d+d_2)^2/2] \end{cases}$$

To show that I have found an equilibrium, I still have to check the $\underline{\text{IC}}$ constraint. The latter is not binding when, after manipulations, the following is true:

$$(1-|d-d_1|)^2\overline{\theta} - (d+d_2)^2\frac{\overline{\theta}+\underline{\theta}}{2} > 0$$

which is true for all $0 \bullet d \bullet (1 + d_1 - d_2)/2$ and $d_1 + d_2 \bullet 1$.

Finally, the Lagrangian multipliers of the three binding constraints are:

$$\lambda(\underline{\text{IR}}) = \frac{(1-|d-d_1|)^2 k - (d+d_2)^2(k+1)/2}{(1-|d-d_1|)^2(k-1)}$$

$$\lambda(\overline{\text{IR}}) = \frac{-(1-|d-d_1|)^2(2-k) + (d+d_2)^2(k+1)/2}{(1-|d-d_1|)^2(k-1)}$$

$$\lambda(\overline{\text{IC}}) = \frac{(1-|d-d_1|)^2 - (d+d_2)^2(k+1)/2}{(1-|d-d_1|)^2(k-1)}$$

The last two expressions are monotonic in d and have a single root in $(0, (1 + d_1 - d_2)/2)$. Let $d^*(k)$ and $\hat{d}(k)$ respectively be the locations that make $\lambda(\overline{\text{IR}}) = 0$ and $\lambda(\overline{\text{IC}}) = 0$ (notice that $d^*(1) = \hat{d}(1) = (1 + d_1 - d_2)/2$):

(A2) $$d^*(k) = \begin{cases} \dfrac{k(2d_1 - d_2 + 2) - 4 - 4d_1 - d_2 + (1 + d_1 + d_2)\sqrt{2(k+1)(2-k)}}{3(k-1)} & \text{if } d^*(k) \geq d_1 \\ \dfrac{k(2d_1 - d_2 - 2) + 4 - 4d_1 - d_2 + (1 + d_1 + d_2)\sqrt{2(k+1)(2-k)}}{3(k-1)} & \text{if } d^*(k) < d_1 \end{cases}$$

(A3) $$\hat{d}(k) = \begin{cases} \dfrac{(1+d_1)[\sqrt{2(k+1)} - 2] - d_2[k + 1 - \sqrt{2(k+1)}]}{k-1} & \text{if } \hat{d}(k) \geq d_1 \\ \dfrac{(1-d_1)[\sqrt{2(k+1)} + 2] - d_2[k + 1 + \sqrt{2(k+1)}]}{k-1} & \text{if } \hat{d}(k) < d_1 \end{cases}$$

• $\lambda(\underline{\text{IR}})$ is always non-negative for 0 • d • $(1 + d_1 - d_2)/2$ and $d_1 + d_2$ • 1.

• For small values of d, $\lambda(\overline{\text{IR}})$ is negative. The corresponding constraint is therefore not binding and the maximisation problem *sub* $\underline{\text{IR}}$ and $\overline{\text{IC}}$ gives the following solution (superscripts *m* refer to the monopoly solution, described by eq. (1) in the text):

(A4) $$\begin{cases} \underline{u}(d;\text{A}) = \underline{u}^m(d) \\ \overline{u}(d;\text{A}) = \overline{u}^m(d) \end{cases} \begin{cases} \underline{p}(d;\text{A}) = \underline{p}^m(d) - (d + d_2)^2 \underline{\theta}^2/2 \\ \overline{p}(d;\text{A}) = \overline{p}^m(d) - (d + d_2)^2 \underline{\theta}^2/2 \end{cases} \begin{cases} \underline{V}(d;\text{A}) = \underline{V}^2(d) \\ \overline{V}(d;\text{A}) = \overline{V}^m(d) + \underline{V}^2(d) \end{cases}$$

$$\begin{cases} \underline{\Pi}(d;\text{A}) = (1 - |d - d_1|)^2 (2\underline{\theta} - \overline{\theta})\overline{\theta}/2 - (d + d_2)^2 \underline{\theta}^2/2 \\ \overline{\Pi}(d;\text{A}) = (1 - |d - d_1|)^2 (3/2\,\overline{\theta}^2 - 3\overline{\theta}\underline{\theta} + 2\underline{\theta}^2) - (d + d_2)^2 \underline{\theta}^2/2 \end{cases}$$

The interval of validity is denoted as region A = [0, max{0, $d^*(k)$}]. It can also be shown that the region is non-empty when the difference between vertical preferences is relatively small:

$$k \leq k_{\text{A}} = (4 - 8d_1 + 4d_1^2 - d_2^2)/(2 - 4d_1 + 2d_1^2 + d_2^2) \leq 2$$

• For intermediate values of d, the three multipliers are positive. The contracts given by eq. (A1) are optimal, and the full solution is reported here:

$$
\text{(A5)}\quad
\begin{cases}
\underline{u}(d;\mathrm{B}) = \dfrac{(d+d_2)^2}{1-|d-d_1|}\dfrac{\underline{\theta}+\overline{\theta}}{2} > \underline{u}^m(d) \\
\overline{u}(d;\mathrm{B}) = \overline{u}^m(d)
\end{cases}
\begin{cases}
\underline{p}(d;\mathrm{B}) = (d+d_2)^2\underline{\theta}\overline{\theta}/2 \\
\overline{p}(d;\mathrm{B}) = \overline{\theta}^2[(1-|d-d_1|)^2-(d+d_2)^2/2]
\end{cases}
$$

$$
\begin{cases}
\underline{\Pi}(d;\mathrm{B}) = \dfrac{(d+d_2)^2}{2}[\underline{\theta}\overline{\theta} - \dfrac{(d+d_2)^2}{(1-|d-d_1|)^2}\dfrac{(\underline{\theta}+\overline{\theta})^2}{4}] \\
\overline{\Pi}(d;\mathrm{B}) = [(1-|d-d_1|)^2-(d+d_2)^2]\overline{\theta}^2/2
\end{cases}
\begin{cases}
\underline{V}(d;\mathrm{B}) = \underline{V}^2(d) \\
\overline{V}(d;\mathrm{B}) = \overline{V}^2(d)
\end{cases}
$$

The interval of validity is denoted as region B = [max {0, $d^*(k)$}, max{0, $\hat{d}(k)$}]. In region B the difference between preferences over vertical attributes cannot exceed a limiting value:

$$
k \le k_{\mathrm{B}} = (2-4d_1+2d_1^{\,2}-d_2^{\,2})/d_2^{\,2} > k_{\mathrm{A}}
$$

• For high values of d and k, $\lambda(\overline{\mathrm{IC}})$ is negative. The corresponding constraint is therefore not binding and the maximisation problem *sub* $\underline{\mathrm{IR}}$ and $\overline{\mathrm{IR}}$ gives the following solution:

$$
\text{(A6)}\quad
\begin{cases}
\underline{u}(d;\mathrm{C}) = \underline{u}^e(d) \\
\overline{u}(d;\mathrm{C}) = \overline{u}^e(d)
\end{cases}
\begin{cases}
\underline{p}(d;\mathrm{C}) = [(1-|d-d_1|)^2-(d+d_2)^2/2]\underline{\theta}^2 \\
\overline{p}(d;\mathrm{C}) = [(1-|d-d_1|)^2-(d+d_2)^2/2]\overline{\theta}^2
\end{cases}
$$

$$
\begin{cases}
\underline{\Pi}(d;\mathrm{C}) = [(1-|d-d_1|)^2-(d+d_2)^2]\underline{\theta}^2/2 \\
\overline{\Pi}(d;\mathrm{C}) = [(1-|d-d_1|)^2-(d+d_2)^2]\overline{\theta}^2/2
\end{cases}
\begin{cases}
\underline{V}(d;\mathrm{C}) = \underline{V}^2(d) \\
\overline{V}(d;\mathrm{C}) = \overline{V}^2(d)
\end{cases}
$$

The interval of validity is denoted as region C = [max{0, $\hat{d}(k)$}, (1 + d_1 - d_2)/2].
By differentiating the relevant conditions, I can discuss how each boundary changes with k:

$$
\frac{\partial d^*(k)}{\partial k} = -\frac{(1-|d^*-d_1|)^2+(d^*+d_2)^2/2}{(d^*+d_2)(k+1)\mp 2(1-|d^*-d_1|)(2-k)} < 0
$$

$$
\frac{\partial \hat{d}(k)}{\partial k} = -\frac{(\hat{d}+d_2)^2}{(\hat{d}+d_2)^2(k+1)\mp 2(1-|\hat{d}-d_1|)} < 0
$$

After simple but tedious manipulations it is also possible to show that $\partial[\hat{d}(k)-d^*(k)]/\partial k > 0$ as long as k • k_{A}. Figure A1 draws the three different regions and their boundaries (thick lines).

It is worthy discussing the value taken by positive Lagrangian multipliers in the three regions. As one would expect, in region A, $\lambda(\underline{\text{IR}}) = 2$ and $\lambda(\overline{\text{IC}}) = 1$, as in the monopoly case. If low types are given 1 unit less of surplus, the increase of the firm's profit function is double because of the chain effect from contracts compatibility. On the other hand, the relaxation of self-selection has a 1:1 effect on profits. Region B shows an intermediate case with 1 • $\lambda(\underline{\text{IR}})$ • 2, 0 • $\lambda(\overline{\text{IR}})$ • 1, 1 • $\lambda(\overline{\text{IC}})$ • 0. The multipliers of the constraints in region C are $\lambda(\underline{\text{IR}}) = \lambda(\overline{\text{IR}}) = 1$: the two contracts are completely independent and the relaxation of one constraint does not have any indirect effect on the profit function.

Proof of Proposition 5 (sketch)

From equations (A2) and (A3) it is possible to derive the effect on the boundaries of each region due to a change in the location of firm 1 and firm 2 (a change in the location can change the region itself, say from A to B or from B to C, hence these comparisons are valid only when the region does not change at a given d). The arrows in the phase diagram of figure A1 give a qualitative indication of the change of the boundaries of regions A, B and C caused by an increase of the location of firm 1, initially at d_1. Since $d^*(k)$ is defined only when k • k_A • 2:

$$\frac{\partial d^*(k)}{\partial d_1} = \begin{cases} \dfrac{\sqrt{2(k+1)(2-k)} - (2-k)}{3(k-1)} > 0 \text{ if } d^*(k) \geq d_1 \\ \dfrac{\sqrt{2(k+1)(2-k)} + (2-k)}{-3(k-1)} < 0 \text{ if } d^*(k) < d_1 \end{cases}$$

$$\frac{\partial \hat{d}(k)}{\partial d_1} = \begin{cases} \dfrac{\sqrt{2(k+1)} - 2}{k-1} > 0 \text{ if } \hat{d}(k) \geq d_1 \\ \dfrac{\sqrt{2(k+1)} + 2}{-(k-1)} < 0 \text{ if } \hat{d}(k) < d_1 \end{cases}$$

$$\frac{\partial d^*(k)}{\partial d_2} = \begin{cases} \dfrac{k+1-\sqrt{2(k+1)(2-k)}}{-3(k-1)} < 0 \text{ if } d^*(k) \geq d_1 \\ \dfrac{k+1+\sqrt{2(k+1)(2-k)}}{-3(k-1)} < 0 \text{ if } d^*(k) < d_1 \end{cases}$$

$$\frac{\partial \hat{d}(k)}{\partial d_2} = \begin{cases} \dfrac{k+1-\sqrt{2(k+1)}}{-(k-1)} < 0 \text{ if } \hat{d}(k) \geq d_1 \\ \dfrac{k+1+\sqrt{2(k+1)}}{-(k-1)} < 0 \text{ if } \hat{d}(k) < d_1 \end{cases}$$

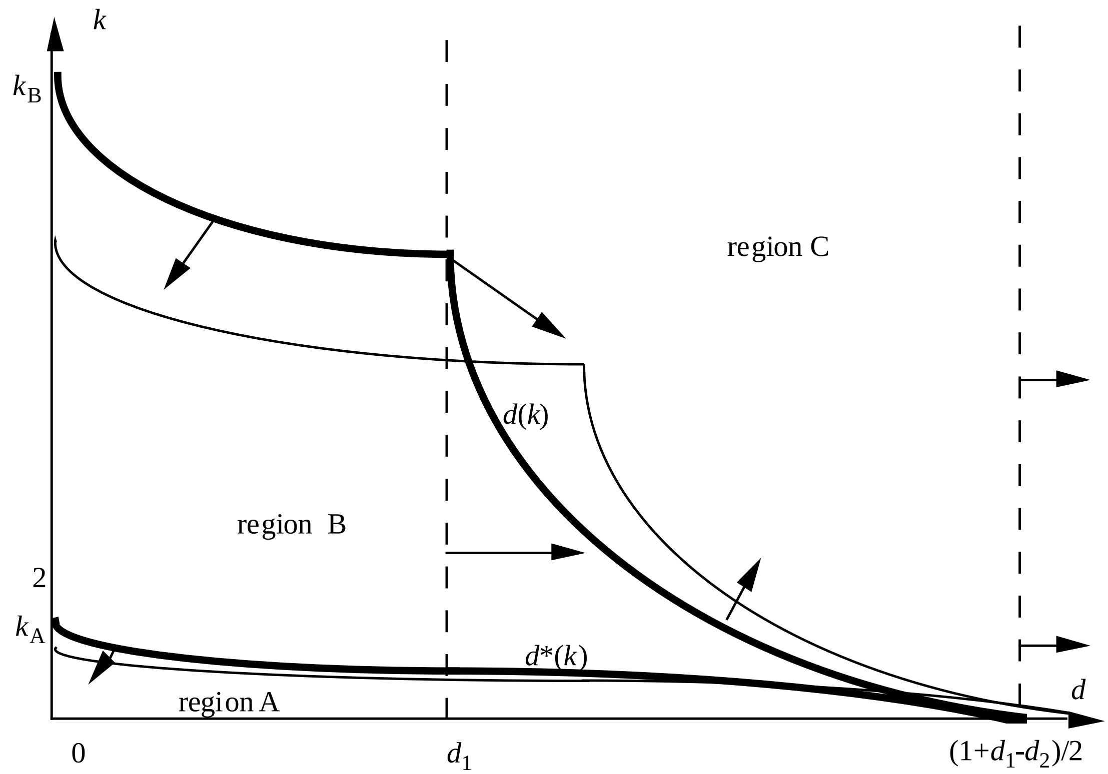

Figure A1 - Discriminatory contracts: regions of validity and effects caused by a change of location

The expression for the profits for firm 1 is given by eq. (5) in the main text where each integrand takes the values defined by equations (A4), (A5) and (A6) and hence it takes different values according to whether a customer is located to the left or to the right of firm 1; d^* and $\hat{d}$ are defined respectively by eq. (A2) and (A3).

The expression for $\bullet_1(\cdot)$ can be computed but it is not reported here because it becomes rather cumbersome, mainly due to the effect deriving from $\underline{\Pi}(\mathrm{B})$. On the other hand, when k is high enough, only region C is valid everywhere and one immediately gets: $\max_{d_1} \pi_1(d_1, d_2) \Rightarrow$ $d_1 = (1-d_2)/3$, which is valid when $k \geq (8+8d_2-7d_2^{\,2})/9d_2^{\,2}$. Since the profit function in that region is concave everywhere, there is a unique symmetric solution $d_1(k) = d_2(k) = 1/4$ when k • 17.

When k takes lower values, it is not possible to obtain a closed-form solution for the location problem. I have however found numerical values to the maximisation of $\bullet_1(\cdot)$ following Leibnitz' rule and imposing symmetry at equilibrium. Numerical solutions have been found starting with k = 1 and augmenting k by intervals of 0.1 and increasing the number of intermediate points whenever the solution showed sudden changes. Along the previous grid I have also calculated the value taken by the second-order condition, which always resulted negative at the symmetric equilibrium. Finally, for selected values of d_2 I have also plotted $\bullet_1(\cdot)$ as a function of d_1, allowing the possibility of leapfrogging, always finding interior solutions, and I have the conjecture that the symmetric equilibrium is also unique.

The following table reports the numerical solutions and the corresponding regions of validity (the plot of $d_1(k)$ corresponds to figure 1 in the main text).

K	d_1	Region A	Region B	Region C
1	.25	[0, .5]	-	-
1.2	.2533	[0, .4401]	[.4401, .4820]	[.4820, .5]
1.6	.2692	[0, .2797]	[.2797, .4496]	[.4496, .5]
1.62	.2699	[0, .2672]	[.2672, .4481]	[.4481, .5]
1.7	.2710	[0, .1374]	[.1374, .4423]	[.4423, .5]
1.8	.2702	[0, .0091]	[.0091, .4354]	[.4354, .5]
2	.2675	-	[0, .4225]	[.4225, .5]
3	.2595	-	[0, .3697]	[.3797, .5]
5	.2535	-	[0, .2981]	[.2981, .5]
6.92	.2513	-	[0., .2513]	[.2513, .5]
8	.2507	-	[0, .1941]	[.1941, .5]
10	.2502	-	[0, .1211]	[.1211, .5]
$\geq$ 17	.25	-	-	[0, .5]