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ABSTRACT

From Physical to Human Capital Accumulation: Inequality in the Process of Development*

This paper presents a novel approach for the dynamic implications of income inequality on the process of development. The proposed theory provides an intertemporal reconciliation between conflicting viewpoints about the effect of inequality on economic growth. It argues that the replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth has changed the qualitative impact of inequality on the process of development. In early stages of industrialization as physical capital accumulation is a prime source of economic growth, inequality enhances the process of development by channelling resources towards individuals whose marginal propensity to save is higher. In later stages of development, however, as the return to human capital increases due to capital-skill complementarity, human capital becomes the prime engine of growth and equality, given credit constraints, stimulating investment in human capital and economic growth. As wages increase, however, credit constraints become less binding and the overall effect of inequality becomes insignificant.

JEL Classification: O11, O15, O40
Keywords: income distributions, growth, credit constraints, human capital

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NON-TECHNICAL SUMMARY

This Paper presents a unified approach for the dynamic implications of income inequality on the process of development. It argues that the replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth has changed the qualitative impact of inequality on the process of development. In early stages of industrialization, as physical capital accumulation is a prime source of economic growth, inequality enhances the process of development by channelling resources towards individuals whose marginal propensity to save is higher. In later stages of development, however, as the return to human capital increases due to capital-skill complementarity, human capital becomes the prime engine of growth and equality, in the presence of credit constraints, stimulating investment in human capital and promoting economic growth. As wages increase, however, credit constraints become less binding, the adverse effect of inequality on human capital accumulation and growth subsides, and the effect of inequality on the growth process becomes insignificant.

The central insight of this approach stems from the recognition that human capital accumulation and physical capital accumulation are fundamentally asymmetric. In contrast to physical capital, human capital is inherently embodied in humans and its aggregate stock would therefore be larger if its accumulation would be widely spread among individuals in society. This asymmetry between human and physical capital accumulation suggests therefore that equality is conducive to human capital accumulation as long as credit constraints are largely binding, whereas provided that the marginal propensity to save increases with income, inequality is conducive for physical capital accumulation. Inequality therefore stimulates economic growth in stages of development in which physical capital accumulation is the prime engine of growth, whereas equality enhances economic growth in stages of development in which human capital accumulation is the dominating engine of economic growth and credit constraints are still largely binding.

Existing theories regarding the effect of income distribution on the process of development can be classified into two categories distinguished by their conflicting predictions. The Classical approach suggests that inequality stimulates capital accumulation and thus promotes economic growth, whereas strands of the recent capital market imperfection approach argue in contrast that for sufficiently wealthy economies equality stimulates investment in human capital and hence enhances economic growth.

The proposed theory provides an intertemporal reconciliation between these conflicting viewpoints about the effect of inequality on economic growth. It suggests that the classical viewpoint, regarding the positive effect of inequality on the process of development, reflect the state of the world in early stages of industrialization when physical capital accumulation was the prime engine of economic growth. In contrast, the credit market imperfection approach regarding the positive effect of equality on economic growth reflects later stages of development when human capital accumulation becomes a prime engine of economic growth, and credit constraints are largely binding.

The Classical approach was originated by Adam Smith (1776) and was further interpreted and developed by Keynes (1920), Lewis (1954), and Kaldor (1957). According to this approach, saving rates are an increasing function of wealth, and inequality therefore channels resources towards individuals whose marginal propensity to save is higher, increasing aggregate savings and capital accumulation and enhancing the process of development.

The Modern paradigm has been dominated by two complementary approaches. The capital market imperfection approach has argued that, in the presence of credit markets imperfection, equality in sufficiently wealthy economies stimulates investment in human capital or in (individual specific) projects, and enhances economic growth. The political economy approach has argued that equality diminishes the tendency for socio-political instability, or distortionary redistribution, and hence it stimulates investment and economic growth.

This Paper, in contrast, develops a unified growth model in which the process of development is marked by an endogenous transition from the domination of physical capital as a prime engine of economic growth to a gradual increase in the importance of human capital accumulation for the growth process. It argues that the replacement of physical capital accumulation by human capital accumulation as the prime engine of growth has changed the qualitative impact of inequality on the process of development. Following the Classical as well as the Credit Market Imperfection approaches, inequality is conducive to physical capital accumulation whereas equality is conducive to human capital accumulation when credit constraints are binding. Inequality, therefore, has a positive effect on economic growth in early stages of development and a negative effect in later stages of development, prior to a significant reduction in the effectiveness of the credit constraints.

The model is based upon three fundamental elements that are well supported by empirical evidence. First, the process of development is characterized by complementarity between capital and skills. Second, the marginal propensity to save and to bequeath increases with wealth. Third, credit market imperfections result in under-investment in human capital.

In every period, inequality has two opposing effects on the process of development. Inequality has a positive effect on capital accumulation and a negative effect on human capital accumulation as long as credit constraints are sufficiently binding. In early stages of industrialization physical capital is scarce, the rate of return to human capital is lower than the rate of return to physical capital and the process of development is fuelled by capital accumulation. The positive effect of inequality on aggregate saving dominates therefore the negative effect on investment in human capital and, since the marginal propensity to save is an increasing function of the individual's wealth, inequality increases aggregate savings and capital accumulation and enhances the process of development. In later stages of development, however, due to capital-skill complementarity, the accumulation of physical capital raises the rate of return to human capital sufficiently so as to induce human capital accumulation and physical capital as well as human capital accumulation fuel the process of development. Since human capital is embodied in individuals and individual's investment in human capital is subjected to diminishing marginal returns, the aggregate return to investment in human capital is maximized if the marginal returns are equalized across individuals. Given credit constraints, equality therefore has a positive effect on the aggregate level of human capital and economic growth. Moreover, as wages increase, the differences in the marginal propensities to save across individuals narrow, and the negative effect of equality on aggregate saving declines. In later stages of development therefore, as long as credit constraints are sufficiently binding, the positive effect of inequality on aggregate saving is dominated by the negative effect on investment in human capital and equality stimulates economic growth.

The empirical implications of the proposed model are consistent with existing evidence regarding the relationship between inequality and the return to education and economic development. Consistently with Kuznets (1955) the proposed theory argues that income inequality widens in early phases of economic growth and narrows in later stages of development. In mature stages of development, however, inequality may widen once again due to skilled or ability-biased technological change induced by human capital accumulation. Moreover, consistently with empirical evidence equality promotes economic growth, via investment in human capital.

1 Introduction

This paper presents a novel approach for the dynamic implications of income inequality on the process of development. It argues that the replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth has changed the qualitative impact of inequality on the process of development. In early stages of industrialization as physical capital accumulation is a prime source of economic growth, inequality enhances the process of development by channeling resources towards individuals whose marginal propensity to save is higher. In later stages of development, however, as the return to human capital increases due to capital-skill complementarity, human capital becomes the prime engine of growth and equality, in the presence of credit constraints, stimulates investment in human capital and promotes economic growth. As wages increase, however, credit constraints become less binding, the adverse effect of inequality on human capital accumulation and growth subsides, and the effect of inequality on the growth process becomes insignificant.

The central insight of this approach stems from the recognition that human capital accumulation and physical capital accumulation are fundamentally asymmetric. In contrast to physical capital, human capital is inherently embodied in humans and its aggregate stock would therefore be larger if its accumulation would be widely spread among individuals in society. This asymmetry between human and physical capital accumulation suggests therefore that equality is conducive for human capital accumulation as long as credit constraints are largely binding, whereas provided that the marginal propensity to save increases with income, inequality is conducive for physical capital accumulation. Inequality therefore stimulates economic growth in stages of development in which physical capital accumulation is the prime engine of growth, whereas equality enhances economic growth in stages of development in which human capital accumulation is the dominating engine of economic growth and credit constraints are still largely binding.[1]

Existing theories regarding the effect of income distribution on the process of development can be classified into two categories distinguished by their conflicting predictions. The Classical approach suggests that inequality stimulates capital accumulation and thus promotes economic growth, whereas strands of the recent capital market imperfection approach argue in contrast that for sufficiently wealthy economies equality stimulates investment in human capital and hence enhances economic growth.

The proposed theory provides an intertemporal reconciliation between these conflicting viewpoints about the effect of inequality on economic growth.[2] It suggests that the classical viewpoint, regarding the positive effect of inequality on the process of development, reflects the state of the world in early stages of industrialization when physical capital accumulation was the prime engine of economic growth. In contrast, the credit market imperfection approach regarding the positive effect of equality on economic growth reflects later stages of development when human capital accumulation becomes a prime engine of economic growth, and credit constraints are largely binding.

The Classical approach was originated by Adam Smith (1776) and was further interpreted and developed by Keynes (1920), Lewis (1954), Kaldor (1957), and Bourguignon (1981). According to this approach, saving rates are an increasing function of wealth, and inequality therefore channels resources towards individuals whose marginal propensity to save is higher, increasing aggregate savings and capital accumulation and enhancing the process of development.

The Modern paradigm has been dominated by two complementary approaches. The capital market imperfection approach (Galor and Zeira (1993) and Banerjee and Newman (1993)) has argued that, in the presence of credit markets imperfection, equality

[1]Credit on investment in human capital is constrained since embodied human capital is viewed as poor collateral by lenders.

[2]Fishman and Shimhon (1998) analyze the effect of income distribution on economic growth in a model that combines the classical approach and the capital market imperfection approach. They argue that Galor and Zeira (1993)'s hypothesis, that equality contributes to long-run growth, holds in monopolistically competitive economy only if individuals differ in their saving rates.

in sufficiently wealthy economies stimulates investment in human capital or in (individual specific) projects, and enhances economic growth.[3] The political economy approach (Alesina and Rodrik (1994), Persson and Tabelini (1994) and Alesina and Perotti (1995)) has argued that equality diminishes the tendency for socio-political instability, or distortionary redistribution, and hence it stimulates investment and economic growth.[4]

This paper, in contrast, develops a unified growth model in which the process of development is marked by an endogenous transition from the domination of physical capital as a prime engine of economic growth to a gradual increase in the importance of human capital accumulation for the growth process. It argues that the replacement of physical capital accumulation by human capital accumulation as the prime engine of growth has changed the qualitative impact of inequality on the process of development. Following the Classical as well as the Credit Market Imperfection approaches, inequality is conducive for physical capital accumulation whereas equality is conducive for human capital accumulation when credit constraints are binding. Inequality, therefore, has a positive effect on economic growth in early stages of development and a negative effect in later stages of development, prior to a significant reduction in the effectiveness of the credit constraints.

The model is based upon three fundamental elements that are well supported by empirical evidence. First, the process of development is characterized by complementarity between capital and skills as documented empirically by Goldin and Katz (1998). Second, the marginal propensity to save and to bequeath increases with wealth.[5] Third, credit markets imperfections results in under-investment in human capital.[6]

In every period, inequality has two opposing effects on the process of development.

[3]See Benabou (1996), Durlauf (1996), Fernandez and Rogerson (1996), and Aghion and Bolton (1997) as well.

[4]See Bertola (1993) and Benabou (1996) as well.

[5]See for example Dynan, Skinner and Zeldes (1996), for evidence that saving rates increase with wealth and Tomes (1981) for evidence that the marginal propensity to bequeath increases with wealth.

[6]See Flug et. al. (1998) for evidence regarding the adverse effect of credit markets imperfection in the presence of inequality on human capital investment.

Inequality has a positive effect on capital accumulation and a negative effect on human capital accumulation as long as credit constraints are sufficiently binding. In early stages of industrialization physical capital is scarce, the rate of return to human capital is lower than the rate of return to physical capital and the process of development is fueled by capital accumulation. The positive effect of inequality on aggregate saving dominates therefore the negative effect on investment in human capital and, since the marginal propensity to save is an increasing function of the individual's wealth, inequality increases aggregate savings and capital accumulation and enhances the process of development. In later stages of development, however, due to capital-skill complementary, the accumulation of physical capital raises the rate of return to human capital sufficiently so as to induce human capital accumulation and physical capital as well as human capital accumulation fuel the process of development. Since human capital is embodied in individuals and individual's investment in human capital is subjected to diminishing marginal returns, the aggregate return to investment in human capital is maximized if the marginal returns are equalized across individuals. Given credit constraints, equality has therefore a positive effect on the aggregate level of human capital and economic growth. Moreover, as wages increase, the differences in the marginal propensities to save across individuals narrow, and the negative effect of equality on aggregate saving declines. In later stages of development therefore, as long as credit constraints are sufficiently binding, the positive effect of inequality on aggregate saving is dominated by the negative effect on investment in human capital and equality stimulates economic growth.

The empirical implications of the proposed model are consistent with existing evidence regarding the relationship between inequality, the return to education and economic development. Consistently with Kuznets (1955)[7] the proposed theory argues that

[7]Kuznets' inverted U hypothesis has been debated in the last few decades. It was confirmed by a number of cross-section empirical studies, most recently by Barro (1999), and it was refuted by others. Time series analysis indicates that the hypothesis is consistent with the experience of most of the developed world. See Brenner et. al. (1991) for a most recent sequence of confirming studies covering the British, Swedish, Belgian, German, Australian, Austrian and the American experience.

income inequality widens in early phases of economic growth and narrows in later stages of development.[8] In mature stages of development, however, inequality may widen once again due to skilled or ability-biased technological change induced by human capital accumulation.[9] Moreover, consistently with Perotti (1996), equality promotes economic growth, via investment in human capital.[10]

2 The Basic Structure of the Model

Consider an overlapping-generations economy in a process of development. In every period the economy produces a single homogeneous good that can be used for consumption and investment. The good is produced using physical capital and human capital. Output per-capita grows over time due to the accumulation of these factors of production. The stock of physical capital in every period is the output produced in the preceding period net of consumption and human capital investment, whereas the level of human capital in every period is the outcome of individuals' education decisions in the preceding period, subject to borrowing constraints.

[8]Further, consistent with the pattern observed by Durlauf and Johnson (1995), the return to human capital increases in early stages of development and decreases temporarily as credit constraints become less binding. As documented by Goldin and Katz (1998, 1999) despite the presence of capital skill complementarity, the skill premium declined in the United States over the period 1890-1950 due to the high school movement and the associated reduction in the effectiveness of credit constraints.

[9]This line of research was explored theoretically by Galor and Tsiddon (1997), Acemoglu (1998), Galor and Moav (1998), and Iyigun and Owen (1999), and is supported empirically by Autor, Katz and Krueger (1998).

[10]Recent studies of Barro (1999) and Forbes (1998) in contrast have argued on the basis of a panel study that inequality stimulate economic growth in wealthy economies. However, it appears that these conflicting evidence stem from the fact that Perotti examines the medium run effect of inequality, whereas Barro and Forbes focus on the short run effect of inequality on economic growth. Further, for poor economies Barro (1999) finds a negative effect of inequality on growth, whereas consistently with Perotti (1996), Forbes (1998) finds a positive effect .

2.1 Production of Final Output

Production occurs within a period according to a neoclassical, constant-returns-to-scale, production technology. The output produced at time t, Y_t, is

$$Y_t = F(K_t, H_t) \equiv H_t f(k_t) = AH_t k_t^{\alpha}; \quad k_t \equiv K_t/H_t; \quad \alpha \in (0,1), \tag{1}$$

where K_t and H_t are the quantities of physical capital and human capital (measured in efficiency units) employed in production at time t, and A is the level of technology.[11] The production function, $f(k_t)$, is therefore strictly monotonic increasing, strictly concave satisfying the neoclassical boundary conditions that assure the existence of an interior solution to the producers' profit-maximization problem.

Producers operate in a perfectly competitive environment. Given the wage rate per efficiency unit of labor, w_t, and the rate of return to capital, r_t , producers in period t choose the level of employment of capital, K_t, and the efficiency units of labor, H_t, so as to maximize profits. That is, $\{K_t, H_t\} = \arg\max\,[H_t f(k_t) - w_t H_t - r_t K_t]$. The producers' inverse demand for factors of production is therefore

$$\begin{aligned} r_t &= f'(k_t) = \alpha A k_t^{\alpha-1} \equiv r(k_t); \\ w_t &= f(k_t) - f'(k_t)k_t = (1-\alpha)Ak_t^{\alpha} \equiv w(k_t). \end{aligned} \tag{2}$$

2.2 Individuals

In every period a generation which consists of a continuum of individuals of measure 1 is born. Each individual has a single parent and a single child.[12] Individuals, within as well as across generations, are identical in their preferences and innate abilities. They may differ, however, in their family wealth and thus, due to borrowing constraints, in their investment in human capital.

[11] For simplicity, the basic model abstracts from technological change. As discussed in the Concluding Remarks, the introduction of endogenous technological change does not affect the qualitative results.

[12] As discussed in the Concluding Remarks, a more realistic family structure, based upon endogenous marriages and fertility decisions, would enrich the micro-foundations but would not affect the qualitative results.

Individuals live for two periods. In the first period of their lives individuals devote their entire time for the acquisition of human capital. The acquired level of human capital increases if their time investment is supplemented with capital investment in education.[13] In the second period of their lives (adulthood), individuals supply their efficiency units of labor and allocate the resulting wage income, along with their inheritance, between consumption and transfers to their children. The resources devoted to transfers are allocated between an immediate finance of their offspring's expenditure on education and saving for the future wealth of their offspring.

2.2.1 Wealth and Preferences

In the second period life, an individual i born in period t (a member i of generation t) supplies the acquired efficiency units of labor, h^i_{t+1}, at the competitive market wage, w_{t+1}. In addition, the individual receives an inheritance of x^i_{t+1}. The individual's second period wealth, I^i_{t+1}, is therefore

$$I^i_{t+1} = w_{t+1}h^i_{t+1} + x^i_{t+1}. \tag{3}$$

The individual allocates this wealth between consumption, c^i_{t+1}, and transfers to the offspring, b^i_{t+1}. That is,

$$c^i_{t+1} + b^i_{t+1} \leq I^i_{t+1}. \tag{4}$$

The transfer of a member i of generation t, b^i_{t+1}, is allocated between an immediate finance of their offspring's expenditure on education, e^i_{t+1}, and saving, s^i_{t+1}, for the future wealth of their offspring.[14] That is, the saving of a member i of generation t, s^i_{t+1}, is

$$s^i_{t+1} = b^i_{t+1} - e^i_{t+1}. \tag{5}$$

[13] The qualitative results would not be affected if the time investment in education (foregone earnings) is the prime factor in the production of human capital, as long as physical capital would be needed in order to finance consumption over the education period. Both formulations assure that in the presence of capital markets imperfections investment in human capital depends upon family wealth.

[14] Parents finance the education of their offspring directly, subtracting the cost from the total intended bequest. This formulation of the saving function is consistent with the view that bequest as a saving motive is perhaps more important than life cycle considerations (e.g., Deaton (1992)).

The inheritance of a member i of generation t, x_{t+1}^i, is therefore the return on the parental saving, s_t^i.

$$x_{t+1}^i = s_t^i R_{t+1} = (b_t^i - e_t^i) R_{t+1} \tag{6}$$

where $R_{t+1} \equiv 1 + r_{t+1} - \delta \equiv R(k_{t+1})$. For simplicity the rate of capital depreciation $\delta = 1$.[15]

Preferences of a member i of generation t are defined over consumption during adulthood,[16] c_{t+1}^i, and the value in period $t+1$ of total transfer to their offspring, b_{t+1}^i (i.e., the sum of the immediate finance of the offspring's investment in human capital, e_{t+1}^i, and the saving for the offspring's future wealth, s_{t+1}^i). They are represented by a log-linear utility function that as will become apparent captures the spirit of Kaldorian-Keynesian saving behavior (i.e., the saving rate is an increasing function of wealth),[17]

$$u_t^i = (1-\beta) \log c_{t+1}^i + \beta \log(\overline{\theta} + b_{t+1}^i), \tag{7}$$

where $\beta \in (0,1)$ and $\overline{\theta} > 0$.[18]

2.2.2 The Formation of Human Capital

In the first period of their lives individuals devote their entire time for the acquisition of human capital. The acquired level of human capital increases if their time investment is supplemented with capital investment in education. However, even in the absence of real expenditure individuals acquire one efficiency unit of labor - basic skills. The number of efficiency units of labor of a member i of generation t in period $t+1$, h_{t+1}^i, is a strictly

[15]This is clearly just a simplifying assumption. $\delta \in [0,1]$ would not alter any of the qualitative results.

[16]The consumption of the child may be viewed as part of the consumption of the parent.

[17]Unlike Kaldor (1957) who assumes that the capitalists and workers differ in their saving behavior, the current formulation suggests that individuals are ex-ante identical in their intertemporal preferences although due to differences in income their marginal propensity to save may differ. Moav (1998) shows that persistent inequality may exist in Galor and Zeira (1993) if this type of a "Keynesian saving function" replaces the assumption of non-convexities in the production of human capital.

[18]This form of altruistic bequest motive (i.e., the "joy of giving") is the common form in the recent literature on income distribution and growth. It is supported empirically by Altonji, Hayashi and Kotlikoff (1997) and Wilhelm (1996).

increasing, strictly concave function of the individual's real expenditure on education in period t, e_t^i.[19]

$$h_{t+1}^i = h(e_t^i), \tag{8}$$

where $h(0) = 1$, $\lim_{e_t^i \to 0^+} h'(e_t^i) = \gamma < \infty$, and $\lim_{e_t^i \to \infty} h'(e_t^i) = 0$.[20]

Given that the indirect utility function is a strictly increasing function of the individual's second period wealth, individual i of generation t chooses the real expenditure on education, e_t^i, so as to maximize the second period wealth, I_{t+1}^i. In the absence of borrowing constraints, the optimal real expenditure on education in every period t, e_t^i, is given by

$$e_t^i = \arg\max[w_{t+1}h(e_t^i) + (b_t^i - e_t^i)R_{t+1}].$$

Hence, as follows from the properties of $h(e_t^i)$, the optimal *unconstrained* real expenditure on education in every period t, e_t, is unique and identical across members of generation t.

If $R_{t+1} > w_{t+1}\gamma$ then $e_t = 0$, otherwise e_t is given by

$$w_{t+1}h'(e_t) = R_{t+1}. \tag{9}$$

Moreover, since $w_{t+1} = w(k_{t+1})$ and $R_{t+1} = R(k_{t+1})$, it follows that $e_t = e(k_{t+1})$.

Given the properties of $f(k_t)$, there exists a unique capital-labor ratio $\widetilde{k}$, below which individuals do not invest in human capital (i.e., do not acquire non-basic skills).

[19] A more realistic formulation would link the cost of education to (teacher's) wages, which may vary in the process of development. For instance, $h_{t+1}^i = h(e_t^i/w_t)$ implies that the cost of education is a function of the number of efficiency units of teachers that are used in the education of individual i. As will become apparent from (9) and (10), under both formulation the optimal capital expenditure on education, e_t^i, is an increasing function of the capital-labor ratio in the economy, and the qualitative results are therefore identical under both formulations.

[20] The assumption $\lim_{e_t^i \to 0^+} h'(e_t^i) = \gamma < \infty$ assures that under some market conditions (non-basic) investment in human capital is not optimal. This assumption assures that in the early stage of development the sole engine of growth is physical capital accumulation. It permits, therefore, a sharp presentation of the results regarding the positive role of inequality in this early stage..

That is, $R(\widetilde{k}) = w(\widetilde{k})\gamma$, where $\lim_{e_t^i \to 0^+} h'(e_t^i) = \gamma$. As follows from (2), $\widetilde{k} = \alpha/(1-\alpha)\gamma \equiv \widetilde{k}(\gamma) > 0$ where $\widetilde{k}'(\gamma) < 0$. Since $R'(k_{t+1}) < 0$, $w'(k_{t+1}) > 0$, and $h''(e_t) < 0$, it follows that

$$e_t = e(k_{t+1}) \begin{cases} = 0 \quad if \quad k_{t+1} \leq \widetilde{k} \\ \\ > 0 \quad if \quad k_{t+1} > \widetilde{k}, \end{cases} \tag{10}$$

where $e'(k_{t+1}) > 0$ if $k_{t+1} > \widetilde{k}$. Hence, if the capital-labor ratio in the next period is expected to be below $\widetilde{k}$ individuals do not acquire non-basic skills.

Suppose that individuals can not borrow in order to finance the education expenditure of their offspring.[21] It follows that the expenditure on education of a member i of generation t, e_t^i is limited by the aggregate transfer, b_t^i, that the individual receives. As follows from (9) and the strict concavity of $h(e_t)$, $e_t^i = b_t^i$ if $b_t^i \leq e_t$, whereas $e_t^i = e_t$ if $b_t^i > e_t$. That is,

$$e_t^i = \min[e(k_{t+1}), b_t^i]. \tag{11}$$

where e_t^i is a non-decreasing function of k_{t+1} and b_t^i.

2.2.3 Optimal Consumption and Transfers

A member i of generation t chooses the level of second period consumption, c_{t+1}^i, and a non-negative aggregate level of transfers to the offspring, b_{t+1}^i, so as to maximize the utility function subject to the second period budget constraint (4).[22]

Hence the optimal transfer of a member i of generation t is:

$$b_{t+1}^i = b(I_{t+1}^i) \equiv \begin{cases} \beta(I_{t+1}^i - \theta) & if \quad I_{t+1}^i \geq \theta; \\ \\ 0 & if \quad I_{t+1}^i \leq \theta, \end{cases} \tag{12}$$

where $\theta \equiv \overline{\theta}(1-\beta)/\beta$. As follows from (12), the transfer rate $b_{t+1}^i / \ I_{t+1}^i$ is increasing in I_{t+1}^i. Moreover, as follows from (5) and (10) the saving of a member i of generation $t-1$, s_t^i, is

[21] Alternative specifications of capital markets imperfections e.g., finite differences between the interest rates for borrowers and lenders, would not affect the qualitative results.

[22] It should be noted that the transfer, b_{t+1}^i, is necessarily non-negative due to the assumption that the offspring has no income in the first period of life.

$$s_t^i = \begin{cases} b_t^i & if \quad k_{t+1} \leq \widetilde{k}; \\ \\ b_t^i - e_t^i & if \quad k_{t+1} > \widetilde{k}. \end{cases} \tag{13}$$

Hence, since b_{t+1}^i / I_{t+1}^i is increasing in I_{t+1}^i, it follows from (11) that s_{t+1}^i / I_{t+1}^i is increasing in I_{t+1}^i as well. The transfer function and the implied saving function capture the properties of the Kaldorian-Keynesian saving hypothesis.

2.3 Aggregate Physical and Human Capital

Suppose that in period 0 the economy consists of two groups of adult individuals - Capitalists and Workers. They are identical in their preferences and differ only in their initial capital ownership. The Capitalists, denoted by R (Rich), are a fraction λ of all adult individuals in society, who equally own the entire *initial* physical capital stock . The Workers, denoted by P (Poor), are a fraction $1-\lambda$ of all adult individuals in society, who have no ownership over the *initial* physical capital stock.[23] Since individuals are ex-ante homogenous *within* a group, the uniqueness of the solution to their optimization problem assures that their offspring are homogenous as well. Hence, in every period a fraction λ of all adults are homogenous descendents of the Capitalist, denoted by members of group R, and a fraction $1-\lambda$ are homogenous descendents of Workers, denoted by members of group P.

The optimization of groups P and R of generations $t-1$ and t in period t, determines the levels of physical capital, K_{t+1}, and human capital, H_{t+1}, in period $t+1$,

$$K_{t+1} = \int_0^1 s_t^i di = \lambda s_t^R + (1-\lambda) s_t^P = \lambda(b_t^R - e_t^R) + (1-\lambda)(b_t^P - e_t^P), \tag{14}$$

where $K_0 > 0$.

[23] As will become apparent this class distinction will Dissipate over time. In particular, the descendents of the working class will ultimately own some physical capital.

$$H_{t+1} = \int_0^1 h_{t+1}^i di = \lambda h(e_t^R) + (1-\lambda)h(e_t^P), \tag{15}$$

where in period 0 there is no (non-basic) human capital, i.e., $h_0^i = 1$ for all $i = R, P$ and thus $H_0 = 1$.[24]

Hence, (11) implies that,

$$\begin{aligned} H_{t+1} &= H(b_t^R, b_t^P, k_{t+1}); \\ K_{t+1} &= K(b_t^R, b_t^p, k_{t+1}). \end{aligned} \tag{16}$$

where (10),(11) and $e'(k_{t+1}) \geq 0$, imply that $\partial H_{t+1}/\partial k_{t+1} \geq 0$, $\partial K_{t+1}/\partial k_{t+1} \leq 0$, $H_{t+1} = H(b_t^R, b_t^P, 0) = 1$, and $K_{t+1} = K(b_t^R, b_t^P, 0) > 0$ for $b_t^R > 0$.

The capital-labor ratio in period $t+1$ is therefore,

$$k_{t+1} = \frac{K(b_t^R, b_t^P, k_{t+1})}{H(b_t^R, b_t^P, k_{t+1})}, \tag{17}$$

where the initial level of the capital labor ratio, k_0, is assumed to be

$$k_0 \in (0, \widetilde{k}). \tag{A1}$$

As follows from (10), this assumption is consistent with the assumption that the initial level of human capital is $H_0 = 1$.

Hence, it follows from (17) and the properties of the functions in (16) that there exists a continuous single valued function $\kappa(b_t^R, b_t^P)$ such that the capital-labor ratio in period $t+1$ is fully determined by the level of transfer of groups R and P in period t.

$$k_{t+1} = \kappa(b_t^R, b_t^P), \tag{18}$$

where $\kappa(0,0) = 0$ (since in the absence of transfers and hence savings the capital stock in the subsequent period is zero).

[24] Note that as long as $k_{t+1} \leq \widetilde{k}$, there is no expenditure on education in the economy as a whole. Hence, $H_{t+1} = 1$ and $k_{t+1} = K_{t+1}$.

2.4 The Evolution of Transfers Within Dynasties

The evolution of transfers within each group $i = R, P$, as follows from (12), is

$$b_{t+1}^i = \max\{\beta[w_{t+1}h(e_t^i) + (b_t^i - e_t^i)R_{t+1} - \theta], 0\}; \qquad i = R, P. \tag{19}$$

Hence, it follows from (11) that

$$\begin{aligned} b_{t+1}^i &= \max\left\{ \begin{cases} \beta[w(k_{t+1})h(b_t^i) - \theta] & if \quad b_t^i \le e(k_{t+1}) \\ \beta[w(k_{t+1})h\left(e(k_{t+1})\right) + \left(b_t^i - e(k_{t+1})\right)R(k_{t+1}) - \theta] & if \quad b_t^i > e(k_{t+1}) \end{cases}, 0 \right\} \\ &\equiv \phi(b_t^i, k_{t+1}). \end{aligned} \tag{20}$$

Let $\widehat{k}$ be the critical level of the capital-labor ratio below which individuals who do not receive transfers from their parents (i.e., $b_t^i = 0$ and therefore $h(b_t^i) = 1$) do not transfer income to their offspring. That is, $w(\widehat{k}) = \theta$. As follows from (2), $\widehat{k} = [\theta/(1-\alpha)A]^{1/\alpha} \equiv \widehat{k}(\theta)$, where if $k_{t+1} \le \widehat{k}$ then $w(k_{t+1}) \le \theta$, whereas if $k_{t+1} > \widehat{k}$ then $w(k_{t+1}) > \theta$. Hence,

$$b_{t+1}^i = \phi(0, k_{t+1}) \begin{cases} = 0 & if \quad k_{t+1} \le \widehat{k}; \\ \\ > 0 & if \quad k_{t+1} > \widehat{k}. \end{cases} \tag{21}$$

In order to reduce the number of feasible scenarios for the evolution of the economy, suppose that once wages increase sufficiently such that members of group P transfer resources to their offspring, i.e., $k_{t+1} > \widehat{k}$, investment in human capital is profitable, i.e., $k_{t+1} > \widetilde{k}$. That is,[25]

$$\widetilde{k} \le \widehat{k}. \tag{A2}$$

Let $\widetilde{t}+1$ be the first period in which the capital labor ratio exceeds $\widetilde{k}$ (i.e., $k_{\widetilde{t}+1} > \widetilde{k}$). That is, since $k_0 < \widetilde{k}$, it follows that $k_{t+1} \le \widetilde{k}$ for all $0 \le t < \widetilde{t}$. Let $\widehat{t}+1$ be the first

[25]Clearly, since $\widehat{k} = \widehat{k}(\theta)$, where $\widehat{k}'(\theta) > 0$, it follows that for any given γ, there exists θ sufficiently large such that $\widetilde{k}(\gamma) \le \widehat{k}(\theta)$.

period in which the capital labor ratio exceeds $\widehat{k}$. That is, $k_{t+1} \leq \widehat{k}$ for all $0 \leq t < \widehat{t}$. It follows from Assumption A2 that $\widetilde{t} \leq \widehat{t}$.

Since $k_{t+1} = \kappa(b_t^R, b_t^P)$, the evolution of transfers within *each* of the two groups is fully determined by the evolution of transfers within *both* types of dynasties. Namely,

$$b_{t+1}^i = \phi(b_t^i, k_{t+1}) = \phi(b_t^i, \kappa(b_t^R, b_t^P)) \equiv \psi^i(b_t^R, b_t^P); \qquad i = R, P, \tag{22}$$

where the initial transfers of the Capitalists and the Workers are

$$\begin{aligned} b_0^R &= \max[\beta[w(k_0) + k_0 R(k_0)/\lambda - \theta]\ , 0]; \\ b_0^P &= \max[\beta[w(k_0) - \theta]\ , 0], \end{aligned} \tag{23}$$

since the level of human capital of every adult i in period 0 is $h_0^i = 1$ and the entire stock of capital in period 0 is distributed equally among the Capitalists . Hence, the initial transfers are uniquely determined by the initial levels and distribution of physical and human capital.

Lemma 1 $b_t^R \geq b_t^P$ *for all t.*

Proof. As follows from (19+1) b_{t+1}^i is increasing in b_t^i. Hence, since (23) implies that $b_0^R \geq b_0^P$ it follows that $b_t^R \geq b_t^P$ for all t. □

3 The Process of Development

This section analyzes the endogenous evolution of the economy from early to mature stages of development. Since $k_{t+1} = \kappa(b_t^R, b_t^P)$, it follows from (22), that the dynamical system is uniquely determined by the sequence $\{b_t^P, b_t^R\}_{t=0}^{\infty}$ such that

$$\begin{aligned} b_{t+1}^P &= \psi^P(b_t^R, b_t^P); \\ b_{t+1}^R &= \psi^R(b_t^R, b_t^P), \end{aligned} \tag{24}$$

where b_0^P and b_0^R are given by (23).

As will become apparent, if additional plausible restrictions are imposed on the basic model, the economy endogenously evolves through two fundamental regimes:

- Regime I: In this early stage of development the rate of return to human capital is lower than the rate of return to physical capital and the process of development is fueled by capital accumulation.

- Regime II: In these mature stages of development, the rate of return to human capital increases sufficiently so as to induce human capital accumulation, and the process of development is fueled by human capital as well as physical capital accumulation.

3.1 Regime I: Physical Capital Accumulation

Regime I is defined as the time interval $0 \leq t < \widetilde{t}$. In this early stage of development the capital-labor ratio in period $t+1$, k_{t+1}, which determines the return to investment in human capital in period t, is lower than $\widetilde{k}$. The rate of return to human capital is therefore lower than the rate of return to physical capital, and the process of development is fueled by capital accumulation. As follows from (10) the level of real expenditure on education in Regime I is therefore zero and members of both groups acquire only basic skills. That is, $h(e(k_{t+1})) = 1$.

Lemma 2 *Under Assumptions A1 and A2,*

$$b_t^P = 0 \quad \textit{for} \quad 0 \leq t \leq \widehat{t}$$

Proof. As follows from the definition of $\widehat{k}$, if $k_t \leq \widehat{k}$ then $w(k_t) \leq \theta$. Hence, since $k_0 < \widehat{k}$ it follows from (23) that $b_0^P = \max[\beta[w(k_0) - \theta]\ , 0] = 0$. Furthermore, for $1 \leq t \leq \widehat{t}$, as long as $b_{t-1}^P = 0$ the descendents of members of group P do not invest in human capital in period $t-1$, $h_t^P = 1$, and therefore $b_t^P = \max[\beta[w(k_t) - \theta]\ ,0] = 0$. $\square$

As follows from (14)-(18), and Lemma 2, since $e_t^R = e_t^P = b_t^P = 0$ in the time interval $0 \leq t < \widetilde{t}$, the capital-labor ratio $k_{t+1} = \kappa(b_t^R, 0) = \lambda b_t^R$ for $0 \leq t < \widetilde{t}$ (i.e., for $k_{t+1} \in (0, \widetilde{k})$). Alternatively,

$$k_{t+1} = \kappa(b_t^R, 0) = \lambda b_t^R \quad \text{for} \quad b_t^R \in [0, \widetilde{b}], \tag{25}$$

where $\widetilde{b} \equiv \widetilde{k}/\lambda = \alpha / [(1-\alpha)\gamma\lambda]$.[26]

The Dynamics of Transfers

A. Unconditional Dynamics

As follows from (24) and Lemma 2, the evolution of the economy for $b_t^R \in [0, \widetilde{b}]$ is given by

$$\begin{aligned} b_{t+1}^R &= \psi^R(b_t^R, 0) = \max[\beta[w(\lambda b_t^R) + b_t^R R(\lambda b_t^R) - \theta], 0]; \\ b_{t+1}^P &= \psi^P(b_t^R, 0) = \max[\beta[w(\lambda b_t^R) - \theta], 0] = 0, \end{aligned} \tag{26}$$

where $b_0^P = 0$ and b_0^R is given by (23).

In order to assure that the economy would ultimately take off from Regime I to Regime II, and from Stage I to Stage II (within Regime II), it is assumed that the technology is sufficiently productive. That is,[27]

$$A \geq \underline{A} \equiv A(\alpha, \gamma, \lambda, \beta, \theta). \tag{A3}$$

[26] Note that one can assure that the economy remains in Regime I for several periods. For instance, since $k_0 \in (0, \widetilde{k}(\gamma))$ there exist a sufficiently large θ and a sufficiently small γ such that the economy is in Regime I in period 0. As follows from Lemma 2, b_0^R is decreasing in θ and is independent of γ. Furthermore, $\widetilde{k}$ is decreasing in γ and $\widehat{k}$ is increasing in θ. Hence, since $k_1 = \lambda b_0^R$ if $\lambda b_0^R \leq \widetilde{k}$ there exist a sufficiently small level of γ and a sufficiently large level θ such that $k_1 \leq \widetilde{k}$ and the economy is in Regime I in period 0.

[27] The precise value of $\underline{A}$ is a cumbersome expression of these five parameters.

Lemma 3 *Under Assumptions A2 and A3, there exists* $\underline{b} \in (0, \widetilde{b})$ *such that the properties of* $\psi^R(b_t^R, 0)$ *in the interval* $b_t^R \in [0, \widetilde{b}]$ *are*

$$\begin{array}{ll} \psi^R(b_t^R, 0) = 0 & for \quad b_t^R \leq \underline{b} \\ \partial\psi^R(b_t^R, 0)/\partial b_t^R > 0 & for \quad \underline{b} < b_t^R \leq \widetilde{b} \\ \partial^2\psi^R(b_t^R, 0)/\partial[b_t^R]^2 < 0 & for \quad \underline{b} < b_t^R \leq \widetilde{b} \\ \psi^R(b_t^R, 0) > b_t^R & for \quad b_t^R = \widetilde{b} \end{array}$$

Proof. Follows from (2) and (26), noting that $\underline{b} = [\theta/A\lambda^\alpha(1-\alpha+\alpha/\lambda)]^{1/\alpha}$ decreases in A and $\widetilde{b} = \alpha/[(1-\alpha)\lambda\gamma]$ is independent of A. □

Corollary 1 *Under Assumptions A2 and A3, the dynamical system* $\psi^R(b_t^R, 0)$ *has two steady-state equilibria in the interval* $b_t^R \in [0, \widetilde{b}]$; *A locally stable steady-state,* $\bar{b} = 0$, *and an unstable steady-state,* $\bar{b}^u \in (\underline{b}, \widetilde{b})$.

Figure 1 depicts the properties of $\psi^R(b_t^R, 0)$ over the interval $b_t^R \in (0, \widetilde{b}]$. If $b_t^R < \bar{b}^u$ then the transfers within each dynasty of type R contract over time and the system converges to the steady-state equilibrium $\bar{b} = 0$. If $b_t^R > \bar{b}^u$ then the transfers within each dynasty of type R expand over the entire interval $(\bar{b}^u, \widetilde{b}]$, crossing into Regime II. To assure that the process of development starts in Regime I and ultimately reaches Regime II, it is assumed that[28]

$$b_0^R \in (\bar{b}^u, \widetilde{b}). \tag{A4}$$

B. Conditional Dynamics

[28] As follows from (23), there exists a feasible set of parameters A, $\alpha, \beta, k_0, \theta$, and λ that satisfy Assumptions A1-A3 such that $b_0^R \in (\bar{b}^u, \widetilde{b})$. In particular, given the initial level of capital, if the number of Capitalist in the initial period is sufficiently small $b_0^R > \bar{b}^u$.

In order to visualize the evolution of the threshold for the departure of members of group P from the zero transfer state, the dynamics of transfers within dynasties is depicted in Figure 2(a) for a given k. This conditional dynamical system is given by (19). For a given $k \in (0, \widetilde{k}]$,

$$b_{t+1}^i = \phi(b_t^i; k) = \max\{\beta[w(k) + b_t^i R(k) - \theta], 0\}. \tag{27}$$

Hence, there exist a critical level $b(k)$ such that

$$\begin{aligned} &\phi(b_t^i; k) = 0 && for \quad 0 \leq b_t^i \leq b(k); \\ &\partial\phi(b_t^i; k)/\partial b_t^i = \beta R(k) > 1 && for \quad b_t^i > b(k). \end{aligned} \tag{28}$$

Note that under Assumption A3 $\beta R(k) > 1$. Otherwise $\psi^R(b^R, 0) < b^R$ for $b^R \in (0, \widetilde{b}]$, in contradiction to Lemma 3.

As depicted in Figure 2(a), in Regime I, members of group P are trapped in a zero transfer temporary steady-state equilibrium, whereas the level of transfers of members of group R increases from generation to generation. As the transfers of members of group R increase the capital-labor ratio increases and the threshold level of transfer, $b(k)$, that enables dynasties of type P to escape the attraction of the no-transfer temporary steady-state equilibrium, eventually declines.

Redistribution and the Dynamics of Output Per Worker

The evolution of output per worker, Y_t, in Regime I, follows from (1),(2),(25) and (26). Provided that Assumption A4 is satisfied,

$$Y_{t+1} = A\left[\beta\left\{\lambda[(1-\alpha)Y_t - \theta] + \alpha Y_t\right\}\right]^\alpha \equiv Y(Y_t), \tag{29}$$

where $Y'(Y_t) > 0$.

In order to examine the effect of inequality on economic growth, suppose that income in period t is distributed differently between group R and group P.[29] That is,

[29] Although one can view the change as a non-distortionary transfer from group R to group P, we advocate a different interpretation. That is, a comparison between two hypothetical paths starting from

the income of members of group i, $\check{I}_t^i$, is

$$\begin{aligned} \check{I}_t^R &= I_t^R - \varepsilon_t \equiv I^R(I_t^R, \varepsilon_t); \\ \check{I}_t^P &= I_t^P + \lambda\varepsilon_t/(1-\lambda) \equiv I^P(I_t^P, \varepsilon_t), \end{aligned} \tag{30}$$

where ε_t is sufficiently small in absolute value such that: (i) the economy does not depart from its current stage of development, and (ii) the net income of members of group P remains below that of member of group R. The transfer of member i of generation t to their offspring is therefore

$$b_t^i = \max\{\beta[I^i(I_t^i, \varepsilon_t) - \theta], 0\} \equiv b^i(I_t^i, \varepsilon_t) \quad i = P, R. \tag{31}$$

Proposition 1 *(The effect of inequality on economic growth in Regime I). Suppose that income would have been distributed differently in Regime I. Under Assumptions A2-A4, in every period in which income is redistributed less equally (between groups) the growth rate of output per worker increases and output per worker increases in all subsequent periods.*

Proof. As long as the economy is in Regime I, $I^P(I_t^P, \varepsilon_t) < \theta$, and $\beta[I^R(I_t^R, \varepsilon_t) - \theta] \in (\overline{b}^u, \widetilde{b})$. Hence, it follows from (31) that $\partial b_t^P/\partial\varepsilon_t = 0$ and $\partial b_t^R/\partial\varepsilon_t < 0$. Hence $Y_{t+1} = A[\lambda b_t^R]^\alpha = A\{\lambda\beta[I^R(I_t^R, \varepsilon_t) - \theta]\}^\alpha$ declines in ε_t and the growth rate of Y_t increases if income is redistributed less equally (i.e., $\varepsilon_t < 0$). Moreover, as follows from (29), Y_{t+2} increases in Y_{t+1} and output increases in all the subsequent periods of Regime I. $\square$

Inequality enhances the process development in Regime I since a transfer of wealth from members of group R to members of group P would increase aggregate consumption, decrease aggregate intergenerational transfers, and thus would slow capital accumulation and the process of development.

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different initial conditions in a given stage of development.

Remark 1 *If income is redistributed less equally within groups (i.e., if additional income groups are created), then redistribution would not affect output per-worker as long as the marginal propensity to save remains equal among all sub-groups of each of the original groups (i.e., β for group R and 0 for group P). Otherwise, since saving is a convex function of wealth, inequality would promote economic growth.*

3.2 Regime II: Human Capital Accumulation

In these mature stages of development, the rate of return to human capital increases sufficiently so as to induce human capital accumulation, and the process of development is fueled by human capital as well as physical capital accumulation. In stages I and II members of group P are credit constrained and their marginal rate of return to investment in human capital is higher than that on physical capital, whereas those marginal rates of returns are equal for members of group R who are not credit constrained. In stage III all individuals are not credit constrained and the marginal rate of return to investment in human capital is equal to the marginal rate of return on investment in physical capital.

3.2.1 Stage I: Selective Human Capital Accumulation

Stage I of Regime II is defined as the time interval $\widetilde{t} \leq t \leq \widehat{t}$. In this time interval $k_{t+1} \in (\widetilde{k}, \widehat{k})$ and the marginal rate of return on investment in human capital is higher than the rate of return on investment in physical capital for individuals who are credit constrained (members of group P), whereas those rates of returns are equal for members of group R.[30]

As follows from (10) and Lemma 2, $e_t^R > 0$ and $e_t^P = 0$. Hence, given (17), it

[30] In all stages of development members of group R are not credit constrained.. That is, $e_t < b_t^R$, and the level of investment in human capital, e_t, permits therefore a strictly positive investment in physical capital, $b_t^R - e_t$, by the members of group R. If $e_t \geq b_t^R$ and hence, as follows from Lemma 1, $e_t > b_t^P$ there would be no investment in physical capital, the return to investment in human capital would be zero and $e_t = 0 < b_t^R$. A Contradiction.

follows that k_{t+1} in the interval $k_{t+1} \in (\widetilde{k}, \widehat{k})$ is given by

$$k_{t+1} = \frac{\lambda(b_t^R - e(k_{t+1}))}{1 - \lambda + \lambda h(e(k_{t+1}))}. \qquad (32)$$

Since $e'(k_{t+1}) > 0$, it follows that $k_{t+1} = \kappa(b_t^R, 0)$ where $\partial\kappa(b_t^R, 0)/\partial b_t^R > 0$. Hence, there exist a unique value $\widehat{b}$ of the level of b_t^R such that $k_{t+1} = \widehat{k}$. That is, $\kappa(\widehat{b}, 0) = \widehat{k}$.

The Dynamics of Transfers

A. Unconditional dynamics

As follows from (22) and (24) the evolution of the economy for $b_t^R \in [\widetilde{b}, \widehat{b}]$ is given by[31]

$$\begin{aligned} b_{t+1}^R &= \psi^R(b_t^R; 0) = \beta[w(k_{t+1})h(e(k_{t+1})) + (b_t^R - e(k_{t+1}))R(k_{t+1}) - \theta]; \qquad (33) \\ b_{t+1}^P &= \psi^P(b_t^R; 0) = 0. \end{aligned}$$

In order the assure that the process of development does not come to a halt in this pre-mature stage of development (i.e., in order to assure that there is no steady-state equilibrium in stage I of Regime II) it is sufficient that $\beta[w(\lambda\widehat{b}) + \widehat{b}R(\lambda\widehat{b}) - \theta] > \widehat{b}$ - a condition that is satisfied under Assumption A3.[32] This condition assures that if the equation of motion in Regime I would remain in place in Stage I of Regime II, then there is no steady-state in Stage I. As will be established below this condition is sufficient to assure that given the actual equation of motion in Stage I of Regime II, the system has no steady-state in this Stage.

Lemma 4 *Under Assumptions A2 and A3, the properties of* $\psi^R(b_t^R, 0)$ *in the interval* $b_t^R \in [\widetilde{b}, \widehat{b}]$ *are*

[31] $b_{t+1}^R > 0$ in this interval since as established in Lemma 3 $b_{\widetilde{t}}^R > 0$, and as follows from Lemma 4 $\partial\psi^R(b_t^R, 0)/\partial b_t^R > 0$.

[32] For any given $b > \widetilde{b}$, (where $\widetilde{b}$ is independent of A) since $\beta[w(\lambda b) + bR(\lambda b) - \theta]$ is strictly increasing in A, there exists a sufficiently large A such that $\beta[w(\lambda b) + bR(\lambda b) - \theta] > b$. Note that $\widehat{b}$ decreases with A, however a sufficiently large θ assures that $\widehat{k} > \widetilde{k}$.

$$\partial \psi^R(b_t^R, 0)/\partial b_t^R > 0$$

$$\psi^R(b_t^R, 0) > b_t^R$$

Proof. $\partial \psi^R(b_t^R, 0)/\partial b_t^R > 0$ as follows from the properties of (2). Moreover, Lemma 3 and the condition $\beta[w(\lambda\widehat{b}) + \widehat{b}R(\lambda\widehat{b}) - \theta] > \widehat{b}$, imply that in the absence of investment in human capital $\beta[w(\lambda b_t^R) + b_t^R R(\lambda b_t^R) - \theta] > b_t^R$ for $b_t^R \in [\widetilde{b}, \widehat{b}]$. Since $\partial \psi^R(b_t^R, 0)/\partial e_t^R > 0$ for $b_t^R \in (\widetilde{b}, \widehat{b}]$,and $e_t^R \in [0, e_t]$, it follows therefore that $\psi^R(b_t^R, 0) \geq \beta[w(\lambda b_t^R) + b_t^R R(\lambda b_t^R) - \theta] > b_t^R$ for $b_t^R \in [\widetilde{b}, \widehat{b}]$. □

Corollary 2 *The dynamical system $\psi^R(b_t^R, 0)$ has no steady-state equilibria in the interval $b_t^R \in [\widetilde{b}, \widehat{b}]$.*

Figure 1 depicts the properties of $\psi^R(b_t^R, 0)$ over the interval $b_t^R \in [\widetilde{b}, \widehat{b}]$. The transfers within each dynasty of type R expand over the entire interval crossing into Stage II.

B. Conditional dynamics

In order to visualize the evolution of the threshold for the departure of dynasties of type P from the zero transfer state, the dynamics of transfers within dynasties is depicted in Figure 2(b) for a given k. This conditional dynamical system is given by (19+1). For a given $k \in (\widetilde{k}, \widehat{k}]$

$$\begin{aligned} b_{t+1}^i &= \max\left\{ \begin{array}{ll} \beta[w(k)h(b_t^i) - \theta] & if \quad b_t^i \leq e(k) \\ \beta[w(k)h\left(e(k)\right) + \left(b_t^i - e(k)\right) R(k) - \theta] & if \quad b_t^i > e(k) \end{array}, 0 \right\} \\ &\equiv \phi(b_t^i, k). \end{aligned} \tag{34}$$

Hence, there exist a critical level $b(k)$ such that for a given $k \in (\widetilde{k}, \widehat{k})$[33]

[33]Note that the condition $\beta[w(\lambda\widehat{b}) + \widehat{b}R(\lambda\widehat{b}) - \theta] > \widehat{b}$ that follows from Assumption A3 and assures that there is no steady-state in Stage I of Regime II, implies that $\beta R(\widehat{k}) \geq 1$.

$$\begin{aligned}
&\phi(b_t^i;k)=0 && for\quad 0\le b_t^i\le b(k);\\
&\partial\phi(b_t^i;k)/\partial b_t^i>\beta R(k)>0 && for\quad b(k)<b_t^i<e(k);\\
&\partial^2\phi(b_t^i;k)/\partial b_t^{i2}<0 && for\quad b(k)<b_t^i<e(k);\\
&\partial\phi(b_t^i;k)/\partial b_t^i=\beta R(k)>1 && for\quad b_t^i\ge e(k).
\end{aligned}\tag{35}$$

Note that $\phi(b_t^i,k)>b_t^i$ for all $b^i>\widetilde{b}$.

As depicted in Figure 2(b), in Stage I of Regime II, members of group P are still trapped in a zero transfer temporary steady-state equilibrium, whereas the level of transfers of members of group R increases from generation to generation. As the transfer of members of group R increases the capital-labor ratio increases and the threshold level of transfer, $b(k)$, that enables members of group P to escape the attraction of the no-transfer temporary steady-state equilibrium, eventually declines.

Redistribution and the Dynamics of Output Per Worker

The evolution of output per worker, Y_t, in Stage I of Regime II , follows from (1),(2),(32) and (33).

$$Y_{t+1}=Y^I(Y_t),\tag{36}$$

where $Y^{I\prime}(Y_t)>0$.

Proposition 2 *(The effect of inequality on economic growth in Stage I of Regime II.) Suppose that income would have been distributed differently in Stage I of Regime II. Under Assumptions A2-A4, in every period in which income is redistributed less equally (between groups) the growth rate of output per worker increases and output per worker increases in all subsequent periods.*

Proof. As long as redistribution is sufficiently small in absolute value such that the economy remains in Stage I of Regime II (i.e., $I^P(I_t^P,\varepsilon_t)<\theta$, and $\beta[I^R(I_t^R,\varepsilon_t)-\theta]\in$

$(\widetilde{b}, \widehat{b}))$ it follows from (31) that $\partial b_t^P / \partial \varepsilon_t = 0$ and $\partial b_t^R / \partial \varepsilon_t < 0$. Hence, as follows from (30) and (32), $k_{t+1} = \kappa(b_t^R, 0) = \kappa(\beta[I^R(I_t^R, \varepsilon_t) - \theta], 0)$ decreases in ε_t. Moreover, (8) and (10) imply that $h(e(k_{t+1}))$ and hence the stock of human capital in period $t+1$, $H_{t+1} = 1 - \lambda + \lambda h(e(k_{t+1}))$, declines in ε_t. Hence Y_{t+1} declines in ε_t as well. Therefore the growth rate of Y_t increases if income is redistributed less equally (i.e., $\varepsilon_t < 0$). Moreover as follows from the properties of (36), Y_{t+2} increases in Y_{t+1} and output increases in all the subsequent periods. □

Inequality enhances the process development in Stage I of Regime II since a transfer of wealth from members of group R to members of group P would increase aggregate consumption, decrease aggregate intergenerational transfers, and thus would slow physical and human capital accumulation and the process of development.

Remark 2 *If income is redistributed less equally within groups (i.e., if additional income groups are created), then redistribution would not affect output per-worker as long as the marginal propensity to save remains equal among all sub-groups of each of the original groups (i.e., β for group R and 0 for group P). Moreover, as long as the redistribution among members of group R does not cause the credit constraints to bind for any member of the group, redistribution has no effect on investment in human capital. Otherwise, inequality has an ambiguous effect on output per worker. Inequality enhances investment of members of sub-groups of P, and has an ambiguous effect on the aggregate investment of members of sub-groups of R (i.e., investment in human capital declines, whereas investment in physical capital increases).*

3.2.2 Stage II: Universal Human Capital Investment

Stage II of Regime II is defined as the time interval $\widehat{t} < t < t^*$, where t^* is the time period in which the credit constraints are no longer binding for members of group P, i.e., $b_{t^*}^P \geq e_{t^*}$. In this time interval, the marginal rate of return on investment in human capital is higher than the marginal rate of return on investment in physical capital for members of group P, whereas these rates of return are equal for members of group R. As established before once $t > \widehat{t}$ the economy exits Stage I of Regime II and enters Stage II of Regime II. In the initial period $k_{\widehat{t}+1} > \widehat{k}$ and therefore $b^P_{\widehat{t}+1} > 0$ and consequently as established below, the sequence $\{b_t^R, b_t^P\}$ increases monotonically over the time interval $\widehat{t} < t < t^*$.

As follows from (10), (11), and (17), in Stage II $e_t^P = b_t^P < e_t$ and $e_t^R = e_t$ and therefore

$$k_{t+1} = \frac{\lambda(b_t^R - e(k_{t+1}))}{(1-\lambda)h(b_t^P) + \lambda h(e(k_{t+1}))}. \tag{37a}$$

Since $e'(k_{t+1}) > 0$, it follows that $k_{t+1} = \kappa(b_t^R, b_t^P)$ where $\partial\kappa(b_t^R, b_t^P)/\partial b_t^R > 0$ and $\partial\kappa(b_t^R, b_t^P)/\partial b_t^P < 0$.

The Dynamics of Transfers

A. Unconditional dynamics

As long as $b_t^P < e_t$ the evolution of the economy as follows from (19) and (24) is given by

$$\begin{aligned} b_{t+1}^R &= \psi^R(b_t^R, b_t^P) = \beta[w(k_{t+1})h(e(k_{t+1})) + (b_t^R - e(k_{t+1}))R(k_{t+1}) - \theta]; \\ b_{t+1}^P &= \psi^P(b_t^R, b_t^P) = \max\{\beta[w(k_{t+1})h(b_t^P) - \theta], 0\}, \end{aligned} \tag{38}$$

where $k_{t+1} = \kappa(b_t^R, b_t^P)$.

Lemma 5 *Under Assumption A2-A4, $\partial\psi^i(b_t^R, b_t^P)/\partial b_t^j > 0$ for all $i, j = P, R$ in the time interval $\widehat{t} < t < t^*$.*

Proof. Follows from (1),(9), (37a) and (38), noting that (i) $h'(b_t^P) > \alpha/(1-\alpha)k_{t+1}$, and (ii) an increase in b_t^P increases output per worker, and hence aggregate wage income, and decreases e_t. □

Lemma 6 *Under Assumptions A2-A4, $b_t^p > 0$ in the time interval $\widehat{t} < t < t^*$.*

Proof. Given Lemma 4 and the definition of $\widehat{t}$, $b^R_{\widehat{t}+1} > b^R_{\widehat{t}} > 0$ and $b^P_{\widehat{t}+1} > b^P_{\widehat{t}} = 0$. Hence it follows from (38) and the positivity of $\partial\psi^i(b^R_t, b^P_t)/\partial b^j_t$ for all $i, j = P, R$, that $b^p_t > 0$ in the time interval $\widehat{t} < t < t^*$. □

Lemma 7 *Under A2-A4, there exists no steady-state equilibrium in Stage II of Regime II.*

Proof. A steady-state equilibrium is a triplet (k, b^P, b^R) such that $b^R = \phi(b^R, k)$, $b^P = \phi(b^P, k)$, and $k = \kappa(b^R, b^P)$. If there exist a non-trivial steady state in Stage II of Regime II then Lemma 1 and 6 implies that $(k, b^P, b^R) >> 0$. As follows from (28),(35) and (40), for any k there exits at most one $b^i = \phi(b^i, k) > 0$. Hence, since ϕ is independent of $i = P, R$, if there exist a non-trivial steady-state then $b^P = b^R > 0$ and therefore $b^P_t > e_t$, and the steady-state is not in stage II of Regime II. □

Corollary 3 *Under A2-A4, (b^R_t, b^p_t) increases monotonically in Stage II of Regime II.*

Proof. Given Lemma 4 and the definition of $\widehat{t}$, $b^R_{\widehat{t}+1} > b^R_{\widehat{t}} > 0$ and $b^P_{\widehat{t}+1} > b^P_{\widehat{t}} = 0$. Hence since as follows from Lemma 5-7 $\partial\psi^i(b^R_t, b^P_t)/\partial b^j_t > 0$ for all $i, j = P, R$, and there exists no steady-state equilibrium in Stage II, (b^R_t, b^p_t) increases monotonically in Stage II of Regime II. □

It follows from Corollary 3 that the economy exits Stage II of Regime II and enters in period t^* Stage III of Regime II.

B. Conditional dynamics

The evolution of transfers within dynasties is depicted in Figure 2(c) for a given $k > \widehat{k}$.[34] This conditional dynamical system is given by (19+1). For a given $k > \widehat{k}$,

$$\begin{aligned} b^i_{t+1} &= \left\{ \begin{array}{ll} \beta[w(k)h(b^i_t) - \theta] & if \quad b^i_t \le e(k) \\ \beta[w(k)h\left(e(k)\right) + \left(b^i_t - e(k)\right) R(k) - \theta] & if \quad b^i_t > e(k) \end{array} \right\} \\ &\equiv \phi(b^i_t, k). \end{aligned} \qquad (39)$$

[34]Note that k_t in stage II of Regime II may decline below $\widehat{k}$. In this case, conditional dynamics are described by (35). However, b^P_t is non-decreasing in stage II of Regime II, that is, b^P_t is above the threshold level $b = \phi(b, k)$ of (35).

Hence, for a given $k > \widehat{k}$,

$$\begin{aligned}
&\phi(b_t^i; k) > 0 && for \quad b_t^i \geq 0 \\
&\partial\phi(b_t^i; k)/\partial b_t^i > \beta R(k) > 0 && for \quad 0 < b_t^i < e(k); \\
&\partial^2\phi(b_t^i; k)/\partial b_t^{i2} < 0 && for \quad 0 < b_t^i < e(k); \\
&\partial\phi(b_t^i; k)/\partial b_t^i = \beta R(k) && for \quad b_t^i \geq e(k).
\end{aligned} \tag{40}$$

Note that for $k > \widehat{k}$ it follows that $\phi(b_t^i, k) > b_t^i$ for at least a strictly positive range $b_t^i \in [0, b]$, where $b > \widehat{b}$.

As depicted in Figure 2(c), in Stage II of Regime II, members of group P depart from the zero transfer temporary equilibrium. The level of transfers of members of group P increases from generation to generation. Eventually members of group P are not credit constrained, i.e., $b_t^P \geq e_t$ and the economy endogenously enters into stage III of Regime II.

Redistribution and the Dynamics of Output Per Worker

Since in stage II and III of Regime II the income of each individual is greater than θ, it follows from (12) that the marginal propensity to transfer is equal to β among all individuals. The aggregate transfers of members of generation t, $\lambda b_t^R + (1-\lambda)b_t^P$, is therefore simply a fraction β of $Y_t - \theta > 0$. That is,

$$\lambda b_t^R + (1-\lambda)b_t^P = \beta(Y_t - \theta). \tag{41}$$

The evolution of output per worker, Y_t, in Stage II of Regime II, as follows from (1),(14),(15), noting that $e_t^R = e_t$ and $e_t^P = b_t^P$, is therefore

$$Y_{t+1} = AK_{t+1}^{\alpha} H_{t+1}^{1-\alpha} = A[\beta(Y_t - \theta) - \lambda e_t - (1-\lambda)b_t^P]^{\alpha}[\lambda h(e_t) + (1-\lambda)h(b_t^P)]^{1-\alpha}. \tag{42}$$

Since $e_t = \arg\max\,[w_{t+1}h(e_t) - R_{t+1}e_t] = \arg\max\, Y_{t+1}$ (and since therefore $\partial Y_{t+1}/\partial e_t = 0$), it follows that

$$Y_{t+1} \equiv Y(Y_t, b_t^P), \tag{43}$$

where $\partial Y(Y_t, b_t^P)/\partial Y_t > 0$ and $\partial Y(Y_t, b_t^P)/\partial b_t^P > 0$, noting that as follows from (2) and (9), $h'(b_t^P) > h'(e_t) = \alpha/[(1-\alpha)k_{t+1}]$.

Lemma 8 *Under A2-A4, Y_t increases monotonically over Stage II.*

Proof. Follows from (41) and Corollary 3. □

Proposition 3 *(The effect of inequality on economic growth in Stage II of Regime II.) Suppose that income would have been distributed differently in Stage II of Regime II. Under Assumptions A2-A4, in every period in which income is redistributed more equally between groups the growth rate of output per worker increases and output per worker increases in all subsequent periods.*

Proof. As long as redistribution is sufficiently small in absolute value such that the economy remains in Stage II of Regime II (i.e., $I^P(I_t^P, \varepsilon_t) > \theta$ and $\beta[I^P(I_t^P, \varepsilon_t) - \theta] < e_t$) it follows from (31) that $\partial b_t^P/\partial \varepsilon_t > 0$ and $\partial b_t^R/\partial \varepsilon_t < 0$. Hence, as follows from the properties of the function in (43)

$$\frac{\partial Y_{t+1}}{\partial \varepsilon_t} = \frac{\partial Y(Y_t, b_t^P)}{\partial b_t^P}\frac{\partial b_t^P}{\partial \varepsilon_t} > 0, \tag{44}$$

and therefore

$$\frac{\partial Y_{t+2}}{\partial \varepsilon_t} = \frac{\partial Y_{t+2}}{\partial b_{t+1}^P}\frac{\partial b_{t+1}^P}{\partial b_t^P}\frac{\partial b_t^P}{\partial \varepsilon_t} + \frac{\partial Y_{t+2}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial \varepsilon_t} > 0. \tag{45}$$

Hence, $\partial Y_{t+j}/\partial \varepsilon_t > 0$ for $j = 1, 2, 3, 4, ...,$ and the Proposition follows. □

Inequality negatively effects the process development in Stage II of Regime II. A transfer of wealth from members of group R to members of group P would not affect aggregate consumption, and aggregate intergenerational transfers, but due to liquidity constraints would allow for a more efficient allocation of aggregate investment between physical and human capital.

Remark 3 *If income is redistributed less equally within groups then redistribution would not affect the aggregate level of intergenerational transfers as long as the marginal propensity to transfer, β, is equal among all member of the economy . However, redistribution of income among members of group P implies a less efficient allocation of human capital due to the liquidity constraints and the concavity of $h(e_t^P)$. Redistribution among members of group R, as long as all the members of sub-groups of R remain unaffected by credit constraint, will not affect output. If however redistribution makes some members of sub-groups of R credit constrained, less equal redistribution will decrease the growth rate of output per worker and the level of output per worker in all subsequent periods.*

3.2.3 Stage III - Unconstrained Investment in Human Capital

Stage III of Regime II is defined as $t \geq t^*$ where credit constraints are no longer binding (i.e.,. $b_t^R \geq b_t^P \geq e_t$). In this time interval the marginal rate of return on investment in human capital is equal to the marginal rate of return on investment in physical capital for all individuals.

As follows from (11), in stage III of Regime II $e_t^P = e_t^R = e_t$. Hence, given (17) and (41) it follows that k_{t+1} is given by

$$k_{t+1} = \frac{\beta[Y_t - \theta] - e(k_{t+1})}{h(e(k_{t+1}))}. \qquad (46)$$

Since $e'(k_{t+1}) > 0$, it follows that $k_{t+1} = k(Y_t)$ where $k'(Y_t) > 0$ and $\lim_{Y_t \to \infty} k_{t+1} = \infty$.

The Dynamics of Transfers and Output Per Worker

As follows from (22) and (24) the evolution of the economy in stage III of Regime II is given by

$$b_{t+1}^R = \psi^R(b_t^R, b_t^P) = \beta[w(k_{t+1})h(e(k_{t+1})) + (b_t^R - e(k_{t+1}))R(k_{t+1}) - \theta];$$
$$b_{t+1}^P = \psi^P(b_t^R, b_t^P) = \beta[w(k_{t+1})h(e(k_{t+1})) + (b_t^P - e(k_{t+1}))R(k_{t+1}) - \theta]. \tag{47}$$

The evolution of output per worker, Y_t, in Stage III of Regime II, is independent of the distribution of intergenerational transfers. As follows from (1) and (41)

$$Y_{t+1} = A[\beta(Y_t - \theta) - e_t]^\alpha [h(e_t)]^{1-\alpha}. \tag{48}$$

Since $e_t = \arg\max Y_{t+1}$, it follows that $\partial Y_{t+1}/\partial e_t = 0$ and therefore

$$Y_{t+1} = Y^{III}(Y_t), \tag{49}$$

where $Y^{III\,\prime}(Y_t) = \beta\alpha A k_t^{\alpha-1} > 0$, $Y^{III\,\prime\prime}(Y_t) < 0$ and $\lim_{Y_t\to\infty} Y^{III\,\prime}(Y_t) = 0$ since $\lim_{Y_t\to\infty} k_{t+1} = \infty$.

Lemma 9 *Under A2-A4, Y_t increases monotonically in Stage III of Regime II and converges to $\overline{Y} > 0$.*

Proof. As follows from the properties of the functions in (43), (48) and (49), $Y_{t+1} = Y^{III}(Y_t) = \max Y(Y_t, b_t^P)$. Hence, it follows from Lemma 8 that once the system enters Stage III $Y_{t+1} > Y_t$. Moreover, since $Y^{III}(Y_t)$ is strictly concave and since $\lim_{Y_t\to\infty} Y^{III\,\prime}(Y_t) = 0$, output increases monotonically converging to a unique, *locally stable,* steady-state equilibrium, $\overline{Y} > 0$. □

Proposition 4 *Under A2-A4, the economy converges to a steady-state equilibrium where $b^P = b^R > 0$.*

Proof. As follows from the properties of (46) and Lemma 9 the economy converges to a unique steady-state level of the capital labor ratio, $\overline{k} = k(\overline{Y})$. As follows from (28),(35) and (40), given $\overline{k}$ it follows that $b^i = \overline{b}^i$ where $\overline{b}^i = \phi(\overline{b}^i, \overline{k})$, otherwise (since $\partial\phi(b^i, \overline{k})/\partial b^i \geq 0$) either [$b^i$ decreases (increases) for all i and thus k decreases (increases)] or [b^R increases indefinitely and b^P decreases to zero, and thus k increases] in contradiction to the stationarity of $\overline{k}$. Hence, $\overline{b}^R = \phi(\overline{b}^R, \overline{k})$, $\overline{b}^P = \phi(\overline{b}^P, \overline{k})$, and $\overline{k} = \kappa(\overline{b}^R, \overline{b}^P)$. As follows from Lemma 3 and 4 there is no non-trivial steady-state equilibrium under which $b^P = 0$. Hence the steady-state equilibrium is $(b^R, b^P) >> 0$, where $b^P = b^R$ since ϕ is independent of $i = P, R$. □

Redistribution and the Dynamics of Output Per Worker

Proposition 5 *(The effect of inequality on economic growth in Stage III of Regime II.) Suppose that income would have been distributed differently in Stage III of Regime II. Redistribution has no effect on output and growth.*

Proof. Follows from the fact that Y_{t+1} in (49) is independent of the distribution of output per worker in period t between the two groups. □

4 Inequality and Development

Theorem 1 *Under Assumption A1-A4 (a) In early stages of development when the process of development is driven by capital accumulation, inequality raises the rate of growth of output per worker.*
(b) In mature stages of development when the process of development is driven by human capital accumulation, investment in human capital is common, and credit constraints are binding, equality raises the growth rate of output per worker.

Proof. The Theorem is a corollary of Propositions 1-3 and Remarks 1-3. □

In the early stage inequality is conducive for economic development. In this early stage of development the rate of return to human capital is lower than the rate of return to physical capital and the process of development is fueled by capital accumulation. Since capital accumulation is the prime engine of growth and since the marginal propensity to save is an increasing function of the individual's wealth, inequality increases aggregate savings and capital accumulation and enhances the process of development.

In these mature stages of development, the rate of return to human capital increases sufficiently so as to induce human capital accumulation, and the process of development is fueled by human capital as well as physical capital accumulation.
Since human capital is embodied in individuals and each individual's investment is subjected to diminishing marginal returns, the aggregate return to investment in human capital is maximized if the marginal returns are equalized across individuals. Given credit constraints, since individuals are homogenous (and hence ability is independent of wealth), equality has a positive effect on the aggregate level of human capital and economic growth. In this later stages of development, inequality has therefore two opposing effects on the process of development. Inequality has a positive effect on capital accumulation and a negative effect on human capital accumulation. As the process of development proceeds, the relative importance of physical capital declines and that of human capital rises. Hence, in mature stages of development, the negative effect of inequality becomes the dominating factor and equality therefore stimulates the process of development.

5 Concluding Remarks

This paper presents a novel approach for the dynamic implications of income inequality on the process of development. The proposed theory provides an intertemporal reconciliation for conflicting viewpoints about the effect of inequality on economic growth. The paper argues that the replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth has changed the qualitative impact of inequality on the process of development. In early stages of industrialization as physical capital accumulation is a prime source of economic growth, inequality enhances the process of development by channeling resources towards the owners of capital whose marginal propensity to save is higher.[35] In later stages of development, however, as the return to human capital increases due to capital-skill complementarity, human capital becomes the prime engine of growth. Since human capital is inherently embodied in humans and its accumulation is larger if it is shared by a larger segment of society, equality, in the presence of credit constraints, stimulates investment in human capital and promotes economic growth. As credit constraints are gradually diminished, the adverse effect of inequality on human capital accumulation subsides, and the effect of inequality on economic growth becomes insignificant.[36]

The theory suggests that in the currently less developed economies, equality is largely beneficial for economic growth.[37] In these economies, the presence of international capital inflow diminishes the role of inequality in stimulating physical capital accumulation. Moreover, the adoption of skilled-biased technologies, increases the return to human capital and thus, given credit constraints, strengthens the positive effect of equality on human capital accumulation and economic growth.

The incorporation of endogenous fertility decisions into the basic model will greatly enrich the understanding of the reasons for the changing role of inequality in the process of development, without affecting the qualitative results. If individuals gain utility from the quantity and the wealth of their children, then as long as the income of poor families is insufficient to provide bequest for their children, poor individuals will choose high fertility rates that will negatively affect the wages and thus the income of their offspring. However, once wages have increased sufficiently due to capital accumulation and the

[35] In earlier stages of development, however, inequality that results in a larger share of income to the aristocrats may be harmful for the process of development, provided that their marginal propensity to consume is indeed high.

[36] If heterogeneity in ability would be incorporated into the analysis, inequality at this mature stages of development may rise the incentives for investment and hence stimulates economic growth. See Galor and Tsiddon (1997), Hassler and Rodriguez-Mora (1998), and Maoz and Moav (1999). This hypothesis regarding the positive role of inequality in mature stages of development is consistent with the finding of the recent study of Barro (1999) who argues that inequality stimulates growth in sufficiently wealthy economies.

[37] This hypothesis is consistent with the finding of Barro (1999) who argues that equality stimulates growth in poor countries.

poor can afford bequeathing, there is an incentive to reduce the number of children, increasing the share of bequest to each child. The transition from physical to human capital accumulation in the process of development would be therefore accelerated.

The introduction of endogenous technological progress that is fueled by human capital accumulation would not affect the qualitative results. If human capital accumulation is conducive for economic growth, the optimal evolution of the economy would require the fastest capital accumulation in early stages of development so as to raise the incentive to invest in human capital. Inequality in early stages of development would therefore stimulate the process of development.

Finally, it is interesting to note that the effect of inequality on economic growth is qualitatively similar to the effect of assortative marriages on economic growth. In early stages of development since inequality is beneficial for growth, assortative marriages (i.e., sorting of couples by income) raise inequality and promote growth. However, in later stages of development in which equality contributes to economic growth, mixed marriages promote growth.

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Figure 1. The dynamical system in Regime I and Stage I of Regime II

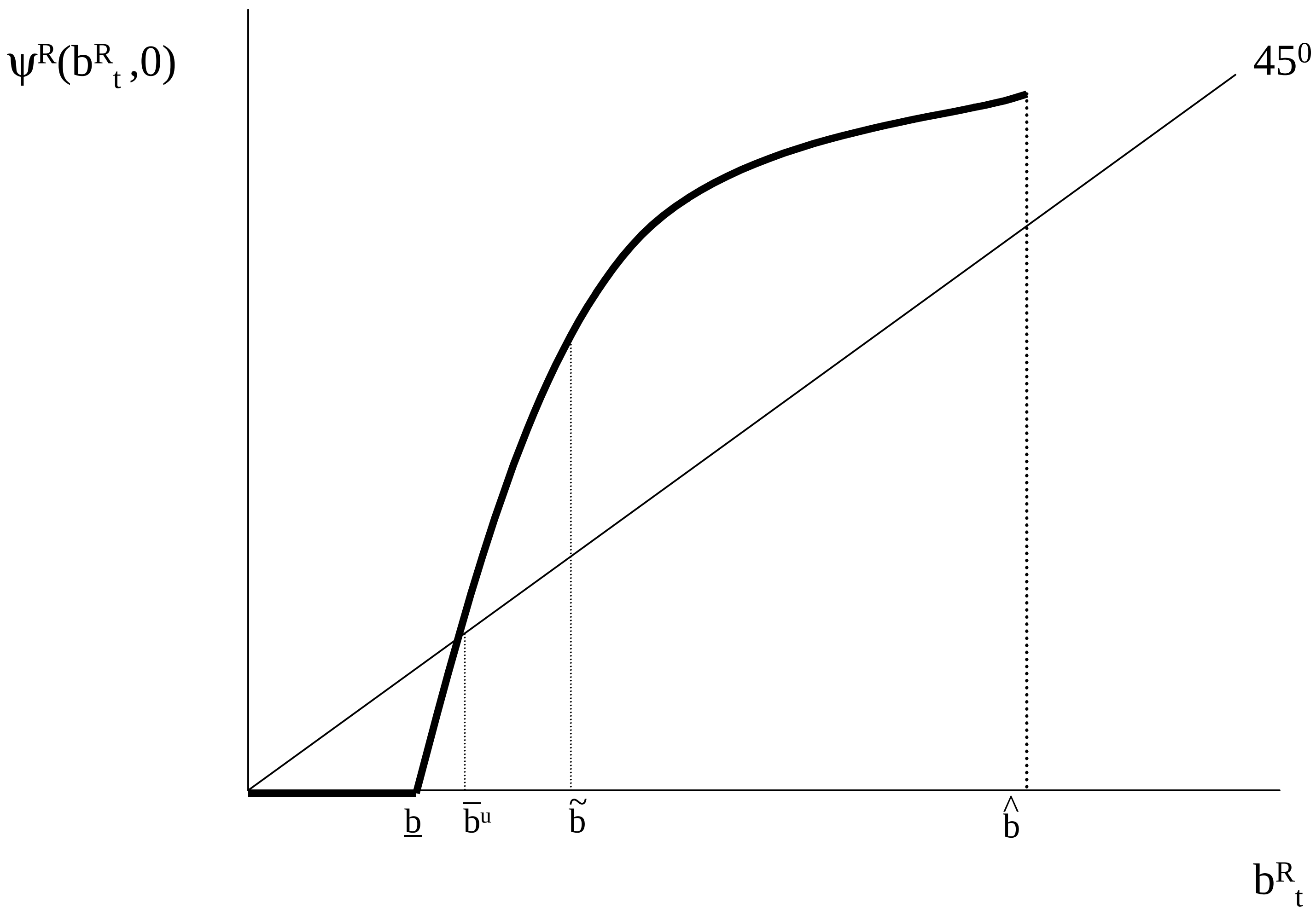

Figure 2(a). The conditional dynamical system in Regime I

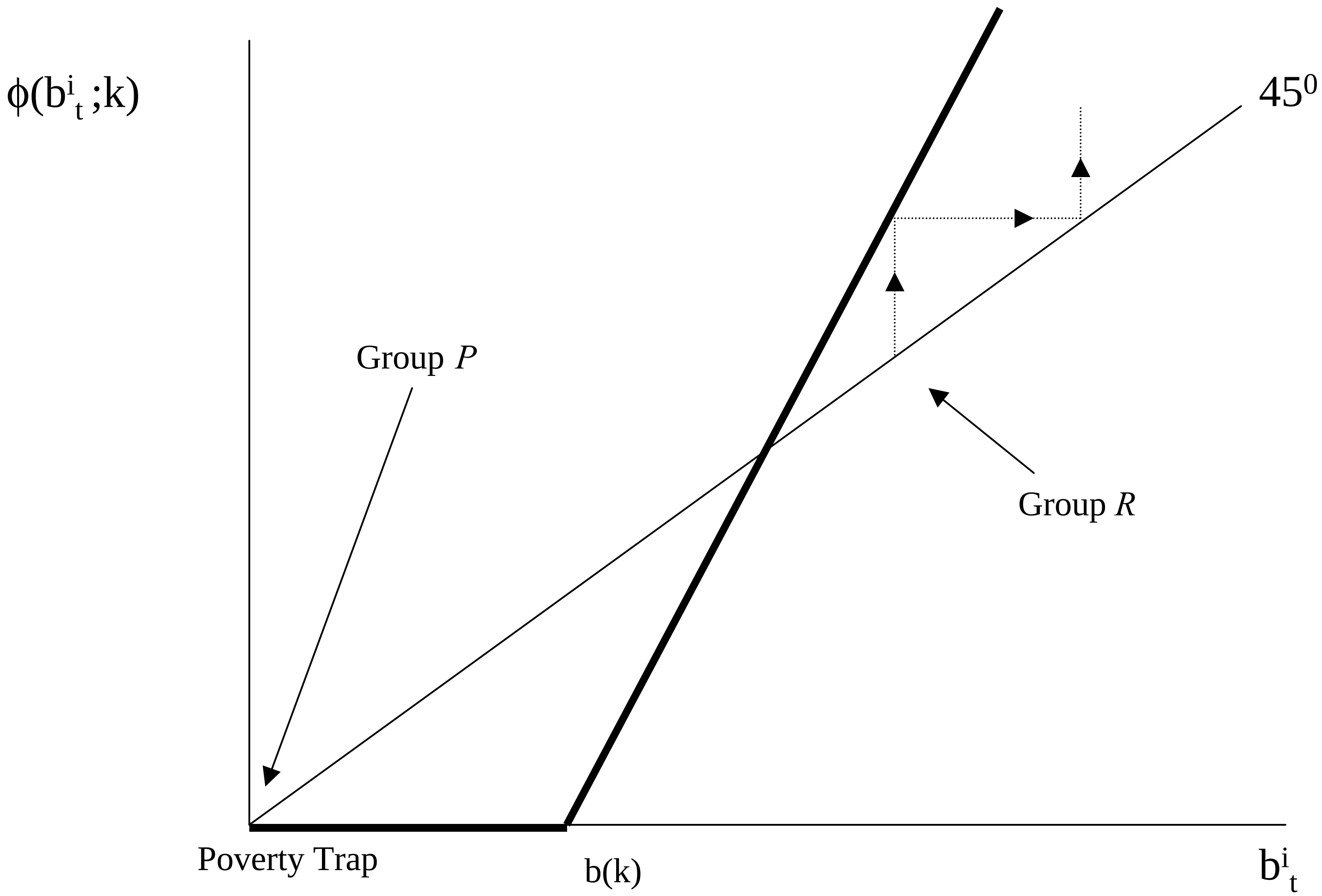

Figure 2(b). The conditional dynamical system in Stage I of Regime II

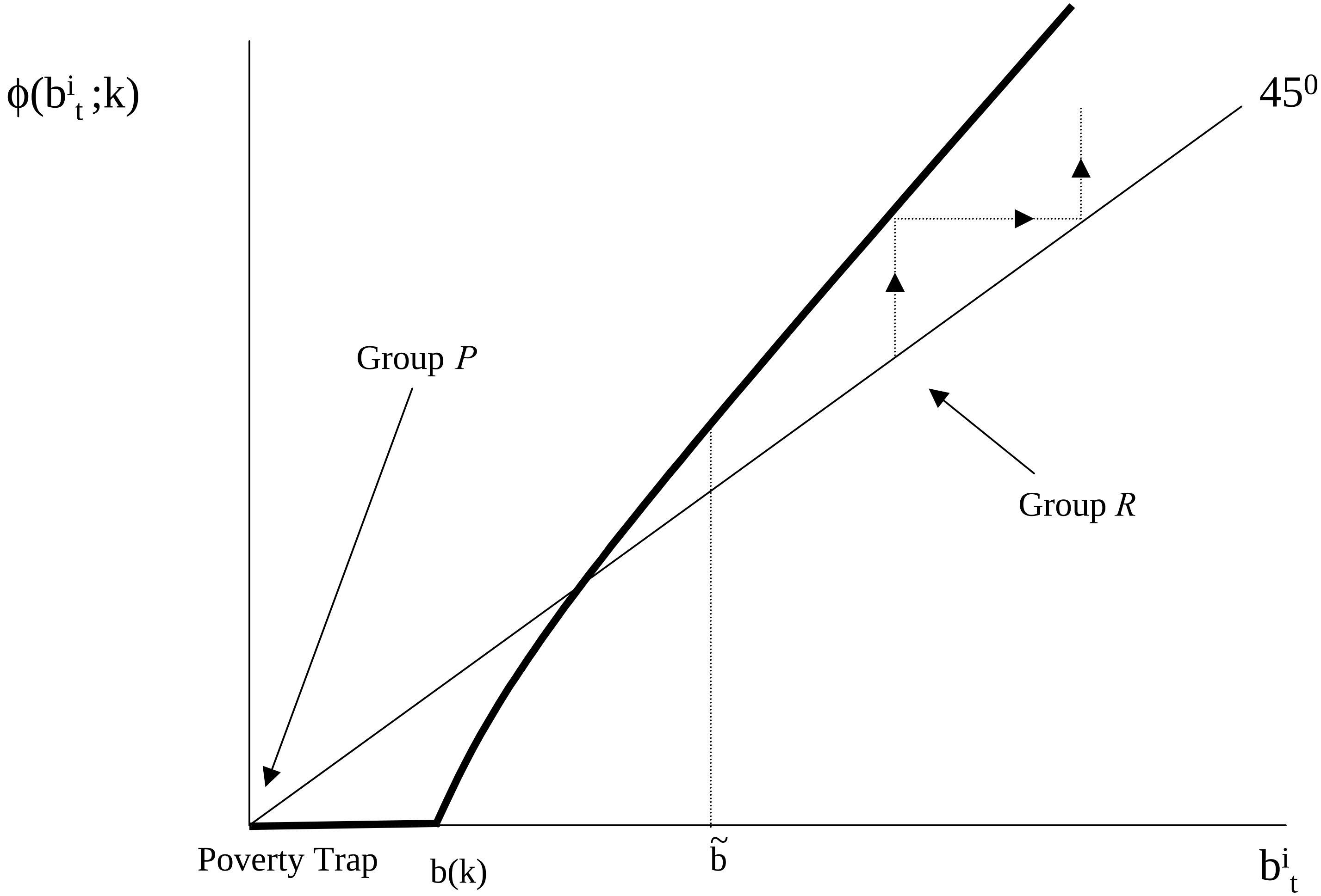

Figure 2(c). The conditional dynamical system in Stage II and III of Regime II