No. 2306

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INTERNATIONAL MACROECONOMICS



Centre for Economic Policy Research

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Discussion Paper No. 2306 December 1999

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ABSTRACT

Revisiting the Case for a Populist Central Banker*

It has been argued that the inflationary bias of discretionary monetary policy can be eliminated, and welfare maximized, by the appointment of a central banker who does not care at all about inflation (a 'populist central banker'). We show that this result hinges crucially on the assumption that wage bargaining occurs in terms of the real wage. When the strategic variable chosen by the unions is the nominal wage, the above result is true only in the special case of a single, all-encompassing union. In the more general case of multiple unions, however, inflation increases linearly with their number and a populist central bank may turn out to be bad for welfare. The Paper also shows that whether unions bargain their wages in nominal or in real terms influences the number of channels through which monetary policy can have systematic effects on real variables.

JEL Classification: E50, J50

Keywords: monetary policy games, central bank conservatism, wage-setting,

non-atomism

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Submitted 14 October 1999

^{*} The views are personal and do not involve the responsibility of the institutions with which the author is affiliated.

NON-TECHNICAL SUMMARY

Most central banks are concerned with inflation and in many countries this concern has been emphasized and made more explicit in recent years. The theoretical justification for this behaviour usually rests on Rogoff's (1985) idea that, in the presence of credibility problems, the government may be better off by delegating monetary policy to an agent who is known for being 'conservative' (i.e. more concerned with inflation than the government). In most models, such a delegation reduces the equilibrium inflation rate without having systematic effects on real variables and is therefore welfare-improving.

The above 'conventional wisdom' is challenged in a recent paper by Guzzo and Velasco (1999) who propose a model in which, due to an assumption of wage-setters' aversion to inflation, the appointment of a 'populist' central banker, i.e. one who does not care at all about inflation, leads to first best (inflation and employment) outcomes. This happens because a 'populist' banker presents the unions with an awesome inflation threat, namely that if employment is below the optimal level, an infinitely high inflation rate will be chosen by the central bank. Faced with such a (credible) threat, inflation-averse unions moderate their wage requests in order to avoid the catastrophe of a hyperinflation.

This Paper shows that the result of Guzzo and Velasco (GV) hinges crucially on the assumption that wage bargaining occurs in terms of *real* wages. When unions bargain *nominal* wages, as it is done almost everywhere, the welfare effects of the populist central banker may change radically. It is shown that under nominal wage bargaining and a populist central banker, equilibrium inflation is linearly increasing in the number of unions. A similar result is obtained by Cukierman and Lippi (1999), who also assume that bargaining occurs in terms of nominal wages. Provided the government cares sufficiently about inflation, this result restores the 'conventional wisdom' about the welfare effects of a conservative central bank.

It may appear at first sight that, since unions have rational inflation expectations, modeling their choices in terms of the *real* wage rather than the *nominal* wage would lead to identical outcomes. But this is not quite correct if unions are non-atomistic. A simple intuition for understanding the difference between nominal wage bargaining (NWB) and real wage bargaining (RWB) is the following. Under NWB each union selects its own nominal wage taking the *nominal* wages of other unions as given. Since a large union understands the inflationary impact of its actions, a unit increase of that union's real wages (caused by a nominal wage increase larger than the expected inflation increase) is perceived by that union to reduce the real wages of all other unions (since inflation increases and the other unions' nominal wages are

assumed to be constant). Instead, under RWB, a unit increase in a union's real wage does not affect the other unions' real wages by assumption. Thus, the true difference between RWB and NWB concerns each union's perception of the relation between its real wage and the other unions' real wages. Under RWB there is no link between those variables, while such a link exists under NWB. This interrelation, created by NWB, leads to substantial changes in the equilibrium outcomes. For instance, under NWB each union has the perception that it can raise its individual real wage without causing hyperinflation even with a 'populist' central banker, since the higher employment that is generated by the fall of the other unions' real wages compensates the lower employment of that union's workers (thus preventing the central bank from producing hyper-inflation).

We reformulate the GV problem in terms of nominal wage bargaining and demonstrate the difference between NWB and RWB. As anticipated, it will appear that inflation and hence welfare, are not necessarily optimal under the populist central banker. Moreover, it turns out that whether unions bargain their wages in nominal or in real terms also influences the overall number of channels through which monetary policy can have effects on real variables. Under nominal wage bargaining there is an additional channel through which monetary institutions can affect real variables, even when unions do not care about inflation. Both results are similar to those obtained by Cukierman and Lippi (1999). This shows that, despite the different macroeconomic models, the assumption about nominal, as opposed to real, wage bargaining is key to different results of those models. understanding the Finally, the appropriateness of modeling discretionary monetary policy under RWB is discussed: this is in fact problematic since RWB implies that policy-makers cannot create surprise inflation and hence that there is no inflationary bias.

Overall, the results suggest that, with the exception of the special case in which there is a single all-encompassing union (setting nominal wages for all of the workers) the optimality of a populist central banker is not robust. In particular, if society (i.e. workers) is sufficiently interested in inflation, a 'conservative' central bank may indeed be welfare-improving. This casts serious doubts on the normative implication, which is implicit in Guzzo and Velasco's result, that central banks should not be concerned with price stability.

1. Introduction

Some recent contributions show that the macroeconomic effects of monetary institutions may depend on the structure of the labor market. Among the main variables that characterize the latter are the number of unions that bargain wages in an independent manner and the unions' degree of inflation aversion. Cukierman and Lippi (1999) and Guzzo and Velasco (1999) provide models were the macroeconomic effects of central bank independence on inflation and employment depend on both those labor market features.¹

But the models of Cukierman and Lippi and Guzzo and Velasco (CL and GV henceforth) produce rather different results. Perhaps the most striking difference is that in GV inflation and employment are at their socially optimal level when the central banker attaches a zero weight to inflation (what they label a "populist central banker"), while this is not generally true in CL. Given the overriding importance that many central banks attribute to the goal of price stability, the robustness of such a proposition seems relevant.

This paper shows that several of the different results between the two papers, among which the proposition concerning the optimality of a "populist" central banker, are not caused by the different underlying models of the economy which are used in those papers. Rather, they are caused by the different assumptions that CL and GV make concerning the wage bargaining process. CL explicitly assume that the strategic variable chosen by each union in the bargaining process is the nominal wage, i.e. that each union chooses its nominal wage taking the nominal wages of the other unions as given. GV *claim* that unions select the

¹A survey of the previous literature which has focussed on labor market structure and monetary policy is provided by Cukierman and Lippi (1999).

²CL label the equivalent concept as the "ultra liberal central banker". Here the more parsimonious terminology of GV is used.

nominal wage, but they implicitly solve the unions' problem by making each union choose its *real* wage taking the *real* wages of other unions as given. It will be shown that, if unions bargain wages in nominal terms, the GV model produces exactly the same result obtained by CL: under a populist central banker welfare is maximized *only* if there is a single union. As the number of unions increases, inflation rises linearly with it and welfare is not necessarily maximized.

It may appear at first blush that, since unions have rational inflation expectations, modelling their choices in terms of the real wage rather than the nominal wage amounts to the same thing. But this is not quite correct if unions are nonatomistic. A simple intuition for understanding the crucial difference between nominal wage bargaining (NWB) and real wage bargaining (RWB) is the following. Under NWB each union selects its own nominal wage taking the nominal wages of other unions as given. Since a large union understands the inflationary impact of its actions, a unit increase of that union's real wages (caused by a nominal wage increase larger than the expected inflation increase) is perceived by that union to reduce the real wages of all other unions (since inflation increases and the other unions' nominal wages are assumed to be constant).³ Instead, under RWB, a unit increase in a union's real wage does not affect the other unions' real wages by assumption. Thus, the true difference between RWB and NWB concerns each union's perception of the relation between its real wage and the other unions' real wages. Under RWB there is no link between those variables, while such a link exists under NWB. This interrelation, created by NWB, leads to substantial changes in the equilibrium outcomes.

We reformulate the GV problem in terms of nominal wage bargaining and demonstrate the difference between NWB and RWB. As anticipated, it will appear that inflation, and hence welfare, are not necessarily optimal under the populist central banker. Moreover, it turns out that whether unions bargain their wages

³If unions were atomistic each of them would perceive that the impact of its individual wages on inflation is nil. That is why the assumption of non-atomism is essential to our argument.

in nominal or in real terms also influences the overall number of channels through which monetary policy can have effects on real variables. Under nominal wage bargaining there is an additional channel through which monetary institutions can affect real variables, even when unions do not care about inflation. Both results are similar to those obtained by Cukierman and Lippi (1999). This shows that, despite the different macroeconomic models, the assumption about nominal as opposed to real wage bargaining is key to understand the different results of those models. Finally, the appropriateness of modeling discretionary monetary policy under RWB is discussed: this is in fact problematic since RWB implies that policy makers cannot create surprise inflation and hence that there is no inflationary bias.

This paper is organized as follows. The next section briefly collects the essential equations of the GV model, making a few simple transformations which are necessary to model wage-setting in nominal terms. The third section shows the essential difference between NWB and RWB, and proves that GV actually model wage setting assuming RWB. The fourth section derives the unions' equilibrium strategies under NWB and presents the equilibrium outcomes. A comparison between outcomes under NWB and those of GV appears in section 5. A final section draws conclusions.

2. The Essential Elements of the Model

The economy is populated by a profit-maximizing representative firm, producing the single consumption good, and a continuum of symmetric workers (indexed by i and arranged in the unit interval) who supply labor, receive dividends from the firm, and consume. Workers are organized in $n \geq 1$ unions⁴ (indexed by j), each of which has a set of members of measure n^{-1} on whose behalf it sets wages.

 $^{^4}$ GV assume $n \geq 2$. However, as will become clear later, only when n = 1 the "populist" central banker is unconditionally optimal. Therefore $n \geq 1$ is assumed here in order to consider the special case of n = 1.

There is also a government, which sets the rate of inflation after nominal wages. Workers who earn wage income and firms' profits in the form of dividends. Worker i's utility (subscript i) is given by (see equation 2.8 in GV)

$$U_i = \log C_i - \frac{\gamma}{2} (\log L_i)^2 - \frac{\beta_p}{2} \pi^2, \beta_p \ge 0, \quad \gamma > \alpha > 0$$
 (2.1)

where γ and β_p are preference parameters and C_i , L_i and π are, respectively, consumption, labor-supplied by individual i and the inflation rate.⁵

Each union j is assumed to represent the workers that lie contiguously in the interval $(j - n^{-1}, j)$. The representative union maximizes the utility of its members (see equation 2.10 in GV)

$$V_{j} = n \int_{i-n^{-1}}^{j} U_{i} di.$$
 (2.2)

Two reduced form equations are needed to study the unions' problem. The first is the expression for the demand of labor type i, obtained from the profit-maximizing behavior of firms and the equilibrium relation between wages and output, yielding (see equation B.2 in GV)

$$L_i = \alpha^{\frac{1}{1-\alpha}} \left(\frac{W_i}{W}\right)^{-\sigma} W^{-\frac{1}{1-\alpha}} \qquad 0 \le \alpha \le 1; \sigma > 1.$$
 (2.3)

where W_i and W are, respectively, the real wage for worker i and the aggregate real wage. The second equation is the representative worker's budget constraint

$$C_i = W_i L_i + D_i = \alpha^{\frac{1}{1-\alpha}} \left(\frac{W_i}{W}\right)^{1-\sigma} W^{-\frac{\alpha}{1-\alpha}} + D_i$$
 (2.4)

It is hypothesized that unions, no matter how large, take D_i as given when setting wages.

The real wage is defined as (see equations 2.11 and 2.4 of GV)

$$W_i \equiv \frac{1+\omega_i}{1+\pi} \tag{2.5}$$

⁵The assumption $\gamma > \alpha$ ensures that the utility function is concave in leisure.

and
$$W \equiv \left(\int_0^1 W_i^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$$
 (2.6)

where ω_i is the percent increases in the *nominal* wage of the union to which worker i belongs (the previous period real wage of worker i is normalized to unity). It is assumed that ω_i (for all $i \in j$) is the control variable of union j. Identical results are obtained if the unions' choice variable is the nominal wage *level*.⁶ For the purpose of this paper, it is convenient to express the aggregate real wage in terms of the aggregate *nominal* wage growth (ω): substituting equation (2.5) into (2.6) yields

$$W = \frac{1+\omega}{1+\pi}, \quad \text{where } \omega \equiv \left[\int_0^1 (1+\omega_i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} - 1 \tag{2.7}$$

Let ω_j be the nominal wage growth of the workers of union j. This growth rate is identical across all workers of union j (i.e. $\omega_i = \omega_j$; all $i \in j$) since the union targets the same utility level for each of them.⁷ Equation (2.7) implies that, in a symmetric equilibrium, $\frac{d\omega}{d\omega_j} = \frac{1}{n}$.⁸ Hence, union j perceives that the growth of the nominal wages of its members increases aggregate nominal wage growth by a factor of 1/n, which is directly related to the union's size.

The objective function of the government is

$$J = \int_0^1 \left[\log C_i - \frac{\gamma}{2} (\log L_i)^2 \right] di - \frac{\beta_g}{2} \pi^2, \qquad \beta_g \ge 0.$$
 (2.8)

Note that the government objectives differ from the individual unions' objectives in that the government accounts for all workers in the economy, that β_g may differ from β_p , and that the government does not take D_i as given when choos-

⁶Since the previous period nominal wage does not affect equilibrium outcomes (see section 4.2) and nominal wage changes are costless for the unions.

⁷This is true since union members' preferences, the way their labor enters into the firm's technology, and the weight the union places on their welfare, are all symmetric.

⁸The partial derivative of ω with respect to ω_j is $\frac{d\omega}{d\omega_j} = \frac{(1+\omega)^{\sigma}}{1-\sigma} \int_{i\in j} (1-\sigma)(1+\omega_i)^{-\sigma} di = \frac{1}{n} (\frac{1+\omega}{1+\omega_j})^{\sigma}$ where the last equality holds since the wages of union j's workers are identical. In a symmetric equilibrium, where the wages of all unions are equal, then $\omega = \omega_j$ and $\frac{d\omega}{d\omega_j} = \frac{1}{n}$.

ing monetary policy. Replacing the expression for the dividends into (2.4), the budget constraint (of the individual worker) faced by the government becomes 10

$$C_i = \left[\alpha^{\frac{1}{1-\alpha}} \left(\frac{W_i}{W} \right)^{1-\sigma} + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} \right] W^{-\frac{\alpha}{1-\alpha}}.$$
 (2.9)

2.1. The Monetary Policy Reaction Function to Nominal Wages

A two-stage game is considered. In the second stage the central bank observes the nominal wage contracts, previously signed by the unions, and sets the inflation rate. The monetary authorities' problem amounts to maximizing (2.8) with respect to π subject to (2.5), (2.7) and (2.9). The first order condition implies the reaction function (see equation 3.1 of GV)

$$\pi = \frac{\alpha - \gamma \int_0^1 \log L_i di}{(1 - \alpha) \beta_g}.$$
 (2.10)

Since the strategic variable chosen by the unions is the nominal wage growth ω_j , we express the reaction function (2.10) in terms of the central bank reaction to nominal wages by using equation (2.3) into (2.10), which yields¹¹

$$\pi = \frac{\alpha (1 - \alpha) - \gamma \log \alpha + \gamma \left[(1 - \alpha) \sigma \int_0^1 (\omega_i - \omega) di + \omega \right]}{(1 - \alpha)^2 \beta_g + \gamma}.$$
 (2.11)

Key to our results is that a non-atomistic union perceives that the growth of its nominal wages raises inflation, in a way which is determined by (2.11). The perceived impact effect of ω_j on the rate of inflation, evaluated along the reaction

⁹As mentioned, these assumptions are exact replicas of the GV assumptions. Lippi (1999) studies how the outcomes of this game vary under alternative assumptions about union's behavior (e.g. internalization of dividends or externalization of general equilibrium effects).

¹⁰In equilibrium, dividends are given by $D = (1 - \alpha)Y$. Using the expression for equilibrium output yields $D_i = (1 - \alpha)\alpha^{\frac{1}{1-\alpha}}W^{-\frac{\alpha}{1-\alpha}}$ which is used in the text.

¹¹Equations (2.5) and (2.7) are used to write the labor demand (eq. 2.3) in terms of nominal wages and inflation. As done in GV, the approximations $\log W_i \cong \omega_i - \pi$ and $\log W \cong \omega - \pi$ are used throughout.

function (2.11) while taking the nominal wages of other unions (label those ω_{-j}) as given, is

$$\frac{d\pi}{d\omega_j}\Big|_{\omega_{-j}} = \frac{\gamma}{n\left[(1-\alpha)^2\beta_g + \gamma\right]} \equiv s(\beta_g, n) \in (0, 1)$$
(2.12)

which we label s.¹² It appears that the impact effect depends on the central bank inflation aversion β_g and on the size of the union. Atomistic unions $(n \to \infty)$ perceive their impact on inflation (s) is zero. A non-atomistic union, however, perceives that an increase in its nominal wages increases the inflation rate (s > 0), and that this increase is smaller if the central bank is more inflation averse (higher β_g).

3. The difference between NWB and RWB

In this section we show that despite GV claim (on p.1324) that "the union sets the rate of increase of the nominal wages of its members. [..] In doing so, it takes the nominal wages set by other unions as given", they actually solve the unions' problem by making each union choose the real wage variable W_i , taking the real wages of other unions as given.¹³

Intuitively, the crucial difference between NWB and RWB is the following: under NWB, each union j chooses its nominal wage (ω_j) , taking the nominal wages of other unions (which we label ω_{-j}) as given and knowing that a unit increase in its nominal wage increases inflation (by s units). Hence, a unit increase

¹²Equation (2.12) gives the impact effect of ω_j on inflation evaluated at a symmetric equilibrium, where all wages are identical. This implies that in the derivative of (2.11) with respect to ω_j the term $\frac{d}{d\omega_j} \left[\int_0^1 (\omega_i - \omega) di \right]$ is equal to zero. The reason to measure the impact of ω_j on inflation at a symmetric equilibrium is that we want to analyze each union's incentive to deviate from the Nash equilibrium of the wage setting game, that will be shown to be symmetric.

¹³The proof of this statement is given below, where it is shown that the elasticity of labor demand they use (in the first order condition of the representative union's problem, equation C.2 on p.1341 of their paper) is correct only under the assumption of RWB.

in the real wages of union j, W_j , (caused by an increase in ω_j equal to $\frac{1}{1-s}$)¹⁴, for given nominal wages of the other unions (ω_{-j}) , reduces the real value of those wages (i.e. W_{-j} is reduced by the increase in ω_j). Instead, under RWB, a unit increase in the real wage of union j is assumed not to have an impact on the real wages of the other unions. Therefore the impact of a unit increase in a union's real wage on the aggregate real wage is smaller under NWB than under RWB since under the former the real wages of the other unions fall. This leads to different perceived elasticity of labor demand under NWB and RWB, and hence to different equilibrium outcomes.

The above arguments can be formalized by studying the impact of a unit increase in the real wage of union j on the aggregate real wage under the two scenarios. Under RWB the impact effect of W_j on W is calculated taking the real wages of other unions, W_{-j} , as given $(\frac{dW}{dW_j}\Big|_{W_{-j}})$, while under NWB the impact effect is calculated taking the nominal wages of other unions, ω_{-j} , as given $(\frac{dW}{dW_j}\Big|_{\omega_{-j}})$. Using the definition (2.6) to calculate those impacts at a symmetric equilibrium (where $W_i = W$ for all i's) reveals that under RWB (see Appendix A)

$$\frac{dW}{dW_j}\Big|_{W_{-j}} = \frac{\partial W}{\partial W_j} + \frac{\partial W}{\partial W_{-j}} \cdot \frac{\partial W_{-j}}{\partial W_j}\Big|_{W_{-j}} = \frac{1}{n}$$
(3.1)

since the term $\frac{\partial W_{-j}}{\partial W_j}\Big|_{W_{-j}}$ is zero by assumption under RWB. Instead, under NWB (see Appendix A)

$$\frac{dW}{dW_i}\Big|_{\omega_{-j}} = \frac{\partial W}{\partial W_i} + \frac{\partial W}{\partial W_{-j}} \cdot \frac{\partial W_{-j}}{\partial W_i}\Big|_{\omega_{-j}} = \frac{1}{n} - \frac{(n-1)s}{n(1-s)} > 0.$$
 (3.2)

It appears from (3.2) that the impact effect of a unit increase in W_j on W under NWB is given by two terms: the first term (1/n) is the direct impact of the wages of union j on the aggregate wage, which is proportional to the size of union

¹⁴The (semi)elasticity of the nominal wage growth with respect to the real wage level is given by $\frac{d\omega_j}{d\log W_j} = \frac{1}{1-s}$ (where the approximation $\log W_j \cong \omega_j - \pi$ is used). This gives the increase in the nominal wage that is associated to a unit increase of the real wage.

j. The second term is the effect that an increase in W_j exerts on W because it reduces the real wages of the other unions. Since a unit increase in the nominal wages of union j increases inflation by s, then a unit increase in the real wages of union j increases inflation by $\frac{s}{1-s}$ units. Hence the other unions' real wages fall by the same amount (see appendix A). The reduction of the aggregate real wage due to this effect is given by the fall of the other unions' wages $\left(-\frac{s}{1-s}\right)$ times their weight in the aggregate real wage $\left(\frac{n-1}{n}\right)$.

Equations (3.1) and (3.2) show that, provided $1 < n < \infty$, the impact of a unit increase in the real wage of union j on the aggregate real wage is *smaller* under NWB than under RWB. This occurs because the increase in the real wage of union j causes an increase in inflation that reduces the real wages of the other unions under NWB.

Those different impacts of union j wages on the aggregate wage, under RWB and under NWB, carry over to differences in the elasticity of labor demand with respect to real wages as perceived by each union. Under RWB the elasticity at a symmetric equilibrium is (see Appendix B)

$$\psi \equiv -\frac{d \log L_j}{d \log W_j} \Big|_{W_{-j}} = \sigma - \frac{\sigma(1-\alpha) - 1}{(1-\alpha)n}$$
(3.3)

This is expression 2.13 in GV paper, which shows that they implicitly assume RWB. Instead, in a symmetric equilibrium under NWB (see Appendix B)

$$\tilde{\psi} \equiv -\frac{d \log L_j}{d \log W_j} \Big|_{\omega_{-j}} = \left[\psi + \frac{\sigma(1-\alpha) - 1}{(1-\alpha)} \cdot \frac{(n-1)s}{n(1-s)} \right]$$
(3.4)

which we label $\tilde{\psi}$. 15

The comparison of the elasticity of labor demand under RWB and NWB shows that whether the elasticity under NWB is larger or smaller than under RWB (in absolute value) depends on the sign of the expression $\sigma(1-\alpha)-1$. This has a simple interpretation. It appears from equation (2.3) that the final impact of the real

In terms of the basic model parameters the elasticity under NWB is equal to $\tilde{\psi} = \frac{1}{(1-\alpha)} + \left(\sigma - \frac{1}{(1-\alpha)}\right) \frac{(1-\alpha)^2 \beta_g + \gamma}{\frac{n}{n-1}(1-\alpha)^2 \beta_g + \gamma}$.

wage of union j, W_j , on the labor demand, depends on its impact on the relative wage term $(\frac{W_j}{W})$ and on the aggregate real wage W^{16} Simple algebra shows that under NWB the first effect is larger (see equations 3.1 and 3.2) while the second effect is smaller than under RWB.¹⁷ Hence, the elasticity under NWB is larger than under RWB when the impact of W_i on the relative wage term dominates the impact on the aggregate wage term. This occurs when $\sigma(1-\alpha) > 1$.

Notice that the impacts of W_j on $\frac{W_j}{W}$ and W also depend on s under NWB. In particular, a higher inflationary perception (higher s) decreases the impact of W_i on W (see equation 3.2) and increases the impact of W_j on $\frac{W_j}{W}$ (see footnote 17). Hence the elasticity of labor demand is increasing in s when the relative wage effect dominates the aggregate wage effect (i.e. when $\sigma(1-\alpha) > 1$), while it is decreasing otherwise.

Finally, note that in the special case in which there is a single union in the economy (n = 1), NWB and RWB give rise to identical results. This is obvious since in this case the single union encompasses the whole labor force and hence there are no "other" unions whose wages can be reduced by an increase of that union's wages. Moreover, equivalence also occurs when unions are atomistic $(n \rightarrow$ ∞) as in that case each union perceives its impact on inflation is zero (s=0), which implies that the perceived impact on the real wages of other unions is zero even under NWB.

¹⁶Note that one can write the elasticity as $-\frac{d \log L_j}{d \log W_j} |.= \frac{1}{(1-\alpha)} \cdot \frac{d \log W}{d \log W_j} |.+\sigma \cdot \frac{d \log \frac{W_j}{W}}{d \log W_j} |.$ ¹⁷This can be seen immediately by noting that $\frac{d \log W_j}{d \log W_j} = 1 - \frac{d \log W}{d \log W_j}$, which shows that a larger impact effect of W_j on the aggregate real wage implies a smaller impact on the relative wage.

4. Equilibrium under NWB

4.1. Wage Setting under NWB

The problem faced by the typical union j under NWB is to maximize (2.2) with respect to ω_j , subject to (2.3), (2.4) and taking ω_{-j} as given. The first order condition of this problem implies (see Appendix C)

$$-\beta_p \pi \frac{s}{1-s} - \alpha \left[\tilde{\psi} - 1 \right] + \gamma \tilde{\psi} \log L = 0 \tag{4.1}$$

which indicates that an increase in the wages of union j has two opposing effects on the utility of workers: on the one hand, it decreases utility since it increases inflation and reduces consumption (the two negative terms in (4.1)).¹⁸ On the other hand, it increases utility since it raises leisure. Equation (4.1) shows that union j trades off those marginal benefits and costs, when choosing ω_j , according to its preferences about inflation, consumption and leisure (as shown by the preference parameters β_p and γ).

4.2. Equilibrium Outcomes under NWB

Equilibrium outcomes under NWB are obtained combining the monetary policy reaction function at a symmetric equilibrium (equation 2.10 with $L_i = L$) and the unions' first order condition (4.1). This yields

$$\log L = \left(\frac{\alpha}{\gamma}\right)\tilde{\phi} \in (0,1) \tag{4.2}$$

where
$$0 \le \tilde{\phi} \equiv 1 - \frac{(1-\alpha)(1-s)\beta_g}{\beta_p s + (1-\alpha)(1-s)\beta_q \tilde{\psi}} \le 1.$$
 (4.3)

A comparison of $\tilde{\phi}$ with the corresponding equilibrium variable in GV, ϕ , reveals that the two solutions coincide only in the special cases when n=1 or

¹⁸A wage increase reduces consumption as it reduces the resources available for consumption (W_iL_i) . This occurs because the fall in labor demand dominates the increase in the real wage (as indicated by the fact that $\tilde{\psi} > 1$).

 $n \to \infty$ (see appendix D). This should not be surprising since, as noticed, in these two cases the elasticity of labor demand under NWB and RWB is the same. If $1 < n < \infty$, the comparison depends on whether the elasticity under NWB $(\tilde{\psi})$ is larger than the one under RWB and also on the union's preferences about inflation (β_p) . If the union is indifferent to inflation (i.e. if $\beta_p = 0$), then employment is larger under NWB if $\tilde{\psi} > \psi$, smaller otherwise.¹⁹ If $\beta_p > 0$, however, the comparison also has to take into account that the impact on inflation of a given increase in a union's real wages is always smaller under NWB than under RWB (because the reduction in the other unions' wages leads the CB to react by less). This effect, taken in isolation, induces less moderation on the part of unions under NWB (hence lower employment). The final effect on employment depends on the combination of the two effects.

Since inflation is also related to the level of employment in this model, equilibrium inflation under NWB is, in general, different from inflation under RWB. Equation (4.2) and (2.10) imply²⁰

$$\pi = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{1-\tilde{\phi}}{\beta_a}\right) \tag{4.4}$$

which is the equilibrium rate of inflation obtained under NWB and discretionary monetary policy.

5. A Comparison with Guzzo and Velasco's results

In this section we show how the results obtained under NWB differ from those obtained by Guzzo and Velasco (1999) under RWB. First, the "populist central banker" result is reconsidered. Second, it is shown that under NWB the degree of central bank conservatism can affect real outcomes even when unions do not care about inflation, as in Cukierman and Lippi (1999) and Lippi (1999). Third, the

 $^{^{19}}$ Note, from appendix D, that when $\beta_p=0$ then $\phi=1-\frac{1}{\tilde{\psi}}$ and $\tilde{\phi}=1-\frac{1}{\tilde{\psi}}.$

²⁰Analytical results for output can be immediately obtained noting that, in a symmetric equilibrium, $\log Y = \alpha \log L$.

effects of labor market decentralization are studied. Fourth, the appropriateness of assuming RWB in a model of time-inconsistency is discussed.

5.1. Revisiting the Case for a Populist Central Banker

When $\beta_g = 0$, the impact on inflation as perceived by each trade union is (from equation 2.12) equal to $s = \frac{1}{n}$. The equilibrium level for employment is derived from equation (4.2), yielding

$$\log L = \frac{\alpha}{\gamma} \tag{5.1}$$

Equation (5.1) shows that, as in GV, under a populist CB employment is at its optimal level. The intuitive reason is that, under a populist central bank, if employment is below the level desired by the CB an infinite inflation would be produced. Since unions are averse to inflation ($\beta_p > 0$) they therefore set the real wage at a level which avoids such a catastrophe.

The equilibrium level for inflation is derived from equation (4.4), yielding

$$\pi = \frac{\alpha(n-1)}{\beta_p}. (5.2)$$

This result is in stark contrast with the one of GV, where the populist central bank produces zero inflation at all n's. Under NWB, this occurs only if there is a single union (n = 1). There is an intuitive reason for why this happens. When n = 1, the single union does not perceive the possibility to increase its real wage above the "optimal" level (i.e. the level consistent with the optimal employment in 5.1) because a unit increase in ω_j when n = 1 is matched by a unit increase in inflation (s = 1). Thus the union has no incentive to increase its nominal wage since that would raise inflation with no beneficial effects in terms of leisure (i.e. real wage).

If there is more than one union in the economy, however, each union perceives that a unit increase in its nominal wages pushes its own real wages above the optimal level since inflation rises by less than one for one (s < 1). The reason

why inflation does not jump to infinity after *one* union's wage increase above the optimal wage level, even in the presence of a populist central banker, is that the inflation caused by this wage increase reduces the other unions' real wages to the point where average employment is at the level desired by the CB (thus avoiding the catastrophic event of hyper-inflation).²¹ Thus, when n > 1, each individual union balances the beneficial effects of nominal wages (i.e. higher real wages and hence leisure) with its costs (i.e. higher inflation). In a symmetric equilibrium *all* unions increase their nominal wages by identical amounts, which are transformed *fully* in inflation by the populist CB.

Note from equation (5.2) that inflation is higher the larger the number of unions in the economy. This occurs because the smaller each union is, the smaller is the perceived impact on inflation (naturally, as each union accounts for a smaller portion of the aggregate nominal wage). This makes the marginal cost of inflation decreasing in the number of unions, hence the equilibrium nominal wage growth chosen by each union, and therefore equilibrium inflation, increases with n.²²

At this point, it should be clear that the GV result on the optimality of the populist central banker is unconditionally valid (i.e. valid for any $\beta_p > 0$) only in the special case of n = 1, where both inflation and unemployment lie at their socially optimal levels. As n increases, the level of inflation increases linearly with it. In the limit, as n goes towards infinity, so does inflation while unemployment remains at its optimal level. Therefore, when n > 1, the optimal level of CB

$$-\beta_p \pi \frac{n}{n-1} - \alpha \left[\sigma - 1\right] + \gamma \sigma \log L = 0$$

which reveals that inflation costs are decreasing in n.

²¹Notice the difference with the case of the populist CB under RWB under which, each union perceives that an increase in its own wages above the full-employment level lowers *aggregate* employment (since the real wages of the other unions are unchanged) and hence leads to a hyper-inflation.

²²The first order condition of the representative union (4.1), when $\beta_g = 0$ (note that 3.4 implies $\tilde{\psi} = \sigma$ when $\beta_g = 0$), is

inflation aversion (i.e. the optimal β_g) depends on the preferences of the workers with respect to inflation and unemployment (β_p) . For instance, in a decentralized labor market (high n) inflation will be high under the populist central banker, which makes it an improbable social optimum (i.e. given n, it is always possible to find a sufficiently large β_p for which this type of banker is not optimal).

5.2. Two Channels for Strategic Non-Neutralities

Another difference between NWB and RWB concerns the number of channels through which monetary policy institutions, as characterized by the degree of central bank inflation aversion, can affect real variables. The first channel, which also appears in GV and CL, occurs because if unions are inflation averse ($\beta_p > 0$) their real wage choices will depend on how much inflation will be created, which in turn depends on the degree of CB inflation aversion. Hence, a CB who is only mildly inflation averse tends to moderate trade unions' real wage requests. This channel operates under both nominal and real wage bargaining.

Even if unions are not inflation averse ($\beta_p = 0$), however, there is a second channel through which the inflation aversion of the central bank affects real variables. This is similar to the channel identified by CL and is present only under NWB.²³ A simple way to highlight the second non-neutrality effect is to rewrite the expression for employment in the special case when ($\beta_p = 0$). From (4.2) this gives

$$\log L \Big|_{\beta_{p}=0} = \left(\frac{\alpha}{\gamma}\right) \left[1 - \frac{1}{\tilde{\psi}}\right]. \tag{5.3}$$

The sign of the partial derivative of (5.3) with respect to β_g is determined by the sign of the expression

$$\frac{dL}{d\beta_q} \approx \frac{ds}{d\beta_q} \left[\frac{\sigma(1-\alpha) - 1}{(1-\alpha)} \left(1 - \frac{1}{n} \right) \right]$$
 (5.4)

²³This explains why in CL, who model NWB, there are two channels for non-neutralities while in GV there is only one.

which shows that only in the extreme cases of n = 1 or $n \to \infty$ the derivative is zero. In all other cases, an increase in the degree of inflation aversion of the central bank is going to decrease employment if $\sigma(1-\alpha) > 1$, to increase it otherwise.

The reason why β_g influences employment is that it affects the perceived impact on inflation, s, and, through it, the elasticity of labor demand (see section 3). We showed that (the absolute value of) the elasticity of labor demand is increasing in s if the relative wage effect dominates the aggregate wage effect (which occurs if $\sigma(1-\alpha) > 1$). A decrease in s (for instance due to a higher β_g) makes each union perceive that the impact of its wage on the relative wage is smaller, while the impact on the aggregate wage is larger. Then, if $\sigma(1-\alpha) > 1$, the relative wage effect dominates the aggregate wage effect on the labor demand and the elasticity of labor demand falls.

It appears from the union's first order condition that a lower labor demand elasticity leads each union to aim for a higher real wage, producing a lower employment level. The opposite effect (i.e. higher employment) occurs if $\sigma(1-\alpha) < 1$ since in that case the increase in CB inflation aversion raises the elasticity of labor demand, leading the unions to demand lower real wages.

An important consequence of the existence of the second non-neutrality is that it shows that GV "result 1" (reported on p. 1328), where it is claimed that the traditional Barro and Gordon (1983) neutrality results can be obtained as special cases from their model when $\beta_p = 0$, is not true under NWB (apart from the special case of n = 1). The traditional results are obtained when unions are atomistic, whether or not they care about inflation.

5.3. The Effects of Labor Market Decentralization

The number of unions that independently bargain wages in the economy can be used to investigate the effects of the degree of decentralization on economic performance. Some algebra shows that the sign of the partial derivative of the equilibrium rate of employment (4.2) with respect to n is given by the sign of the

expression

$$\frac{dL}{dn} \approx \sigma(1 - \alpha) - 1 - \frac{\beta_p \gamma}{\beta_q^2 (1 - \alpha)}.$$
 (5.5)

which shows that employment is either increasing or decreasing in the number of unions depending on whether the elasticity of labor demand with respect to the relative wage term (σ) is sufficiently high.

When unions do not care about inflation ($\beta_p = 0$), the effect of n on employment occurs because n influences the elasticity of labor demand. In particular, a higher n increases the elasticity (and hence employment) if the impact of own wage increases on the relative wage term dominates the impact on the aggregate wage term, which occurs if $\sigma(1-\alpha) > 1$.

Moreover, when unions are inflation averse $(\beta_p > 0)$ a change in n also decreases each union's perception of how inflationary its actions are. This gives the union an incentive to push for higher wages. This effect alone would tend to make employment decreasing in n. The combination of the effects of n on the labor demand elasticity and on the inflationary perceptions of unions determines the sign of the final impact.

5.4. The Inconsistency between RWB and Credibility Games

In the previous subsections we have shown that under NWB the equilibrium levels of inflation and employment are different from what described by GV under their implicit assumption of RWB, in particular it was shown that the populist central banker maximizes welfare only in the very special case of a single union.

The mechanism that determines the final impact of n on L is analogous to the one that was discussed for the impact of β_g on L. As n increases, the impact of W_j on the relative wage term $\frac{W_j}{W}$ increases while the impact on the aggregate wage W decreases (this can be derived from equation 3.2). As before, which of those effects dominates depends on whether the elasticity of the labor demand to the relative wage term (σ) dominates the elasticity of the labor demand with respect to the aggregate real wage term $(\frac{1}{1-\alpha})$.

But there is another, perhaps more fundamental, objection to the results of GV. This concerns the fact that if RWB is taken seriously, then the central bank would have no incentives to surprise private agents since it would be clear that employment cannot be increased by a monetary expansion. In other words, under RWB (for instance in a setting where wages are fully indexed), the government does not perceive that it can affect employment, since it does not take nominal wages as given. Hence, if the government realizes that real wages cannot be altered because of, say, indexation, then it faces no short run tradeoff between inflation and employment and hence inflation would be zero. In fact, this is why time-inconsistency models emphasize that unions bargain nominal wages which cannot be renegotiated ex-post (see Persson and Tabellini (1999, section 2.3) and Walsh (1998, chapter 8)).

6. Concluding Remarks

Most central banks are concerned with inflation and in many countries this concern has been emphasized and made more explicit in the recent years (see Cukierman, 1998). The theoretical justification for this usually rests on Rogoff's (1985) idea that, in the presence of credibility problems, the government may be better off by delegating monetary policy to an agent who is known for being "conservative" (i.e. more concerned with inflation than the government). In most models, such a delegation reduces the equilibrium inflation rate without having systematic effects on real variables, and is therefore welfare improving.

Recently, Guzzo and Velasco (1999) have proposed a model in which, due to an assumption of wage setters' (unions) aversion to inflation, the appointment of a "populist" central banker, i.e. one who does not care at all about inflation, leads to a first best outcome. This happens because a "populist" banker produces an infinite inflation if employment differs from the optimal level. Hence, inflation-averse unions moderate their wage requests in order to avoid hyper-inflation.

This paper has shown that the result of Guzzo and Velasco hinges crucially

on the assumption that wage bargaining occurs in terms of the real wage rather than in terms of nominal wages. When unions bargain nominal (as opposed to real) wages, an assumption that is also appealing in terms of realism, the welfare effects of the populist central banker may change radically. The simple intuition for this is that under nominal wage bargaining each union perceives that an increase of its own nominal wages causes some inflation which in turn reduces the real wages of the other unions (since their nominal level is given to the union). Hence, even with a populist central banker, each union has the perception that it can raise its individual real wage without causing hyper-inflation, since the higher employment that is generated by the fall of the other unions' real wages compensates the lower employment of that union's workers (thus preventing the central bank from producing hyper-inflation). We showed that under nominal wage bargaining and a populist central banker, equilibrium inflation is linearly increasing in the number of unions. A similar results is obtained by Cukierman and Lippi (1999), who also assume that bargaining occurs in terms of nominal wages. These results imply that, with the exception of the special case in which there is a single all-encompassing union that sets nominal wages for all of the workers, the optimality of a populist central banker is not robust. In particular, if society (i.e. workers) is sufficiently interested in inflation, it is possible to show that a "conservative" central bank may indeed be welfare improving. Overall, this casts serious doubts on the normative implication, which is implicit in Guzzo and Velasco's result, that central banks should not be concerned with price stability.

A. Appendix: The Impact of W_i on W

The partial derivative of W with respect to W_j taking W_{-j} as given, from equation (2.6), is

$$\frac{dW}{dW_j}\Big|_{W_{-j}} = \frac{W^{\sigma}}{1-\sigma} \int_{i \in j} (1-\sigma) W_i^{-\sigma} di = \frac{1}{n} \left(\frac{W}{W_j}\right)^{\sigma}$$

since the wages of union j workers are identical we can integrate across them, obtaining the last equality. Equation (3.1) holds in a symmetric equilibrium (where $W = W_i$).

Under NWB instead (using the real wage definition 2.5)

$$\frac{dW}{dW_j}\Big|_{\omega_{-j}} = \frac{W^{\sigma}}{1-\sigma} \left[\int_{i \in j} (1-\sigma)W_i^{-\sigma} di + \int_{i \in -j} (1-\sigma)W_i^{-\sigma} \frac{d\left(\frac{1+\omega_i}{1+\pi}\right)}{dW_j}\Big|_{\omega_{-j}} di \right]$$

since the wage is the same for the workers of union j (label this W_j), and within the group of the workers belonging to "other unions" (i.e. all W_i for which $i \in -j$, label the W_{-j}), we can integrate across each of these groups obtaining

$$\frac{dW}{dW_j}\Big|_{\omega_{-j}} = W^{\sigma} \left[\frac{1}{n} W_j^{-\sigma} + \frac{n-1}{n} W_{-j}^{-\sigma} \frac{d\left(\frac{1+\omega_{-j}}{1+\pi}\right)}{dW_j} \Big|_{\omega_{-j}} \right]. \tag{A.1}$$

Using (2.12), rewrite the derivative

$$\frac{d\left(\frac{1+\omega_{-j}}{1+\pi}\right)}{dW_{j}}\Big|_{\omega_{-j}} = \frac{W_{-j}}{W_{-j}}\frac{\partial W_{-j}}{\partial \omega_{j}}\Big|_{\omega_{-j}} \frac{W_{j}}{W_{j}}\frac{\partial \omega_{j}}{\partial W_{j}} =
= \frac{W_{-j}}{W_{j}}\frac{\partial \log W_{-j}}{\partial \omega_{j}}\Big|_{\omega_{-j}} \cdot \frac{\partial \omega_{j}}{\partial \log W_{j}} =
= -\frac{W_{-j}}{W_{i}}\frac{s}{1-s}$$

where the last line is obtained using the approximation $\log W_i = \omega_i - \pi$. Plugged into (A.1) this yields

$$\left. \frac{dW}{dW_j} \right|_{\omega_{-j}} = \left(\frac{W}{W_{-j}} \right)^{\sigma} \left[\frac{1}{n} \left(\frac{W_j}{W_{-j}} \right)^{-\sigma} + \frac{n-1}{n} \left(-\frac{W_{-j}}{W_j} \frac{s}{1-s} \right) \right]$$

which gives equation (3.2) in the text at a symmetric equilibrium (where $W = W_j = W_{-j}$).

B. Appendix: Derivation of the labor demand elasticity

From equation (2.3) we get

$$\log L_i = \frac{1}{1 - \alpha} \log \alpha - \sigma \log W_i + (\sigma - \frac{1}{1 - \alpha}) \log W.$$
 (B.1)

Define the elasticity of labor demand with respect to the real wage (in absolute value) as $-\frac{d \log L_i}{d \log W_i}$, straightforward algebra reveals that under NWB

$$\tilde{\psi} \equiv -\frac{d \log L_i}{d \log W_i} \Big|_{\omega_{-j}} = \sigma - (\sigma - \frac{1}{1 - \alpha}) \frac{d \log W}{d \log W_i} \Big|_{\omega_{-j}} =
= \sigma - (\sigma - \frac{1}{1 - \alpha}) \frac{W_i}{W} \frac{dW}{dW_i} \Big|_{\omega_{-j}}$$
(B.2)

At a symmetric equilibrium (where $W = W_i$) using expression (3.2) into (B.2) yields equation (3.4) in the text. An analogous reasoning for the case of RWB (where the expression (3.1) is used in the place of (3.2)) yields equation (3.3) in the text.

C. Appendix: Derivation of the unions' first order condition under NWB

The typical union j solves the problem

$$\max_{\omega_j} n \int_{i \in j} \left[\log C_i - \frac{\gamma}{2} \left(\log L_i \right)^2 - \frac{\beta_p}{2} \pi^2 \right] di$$
 (C.1)

with respect to ω_j subject to $C_i = W_i L_i + D_i$, $\frac{d\pi}{d\omega_j}\Big|_{\omega_{-j}} = s$ (equation 2.12) and taking ω_{-j} as given. The partial derivative of C.1 with respect to ω_j (i.e. ω_i for $i \in j$) yields

$$n \int_{i \in j} \left[\frac{1}{C_i} \frac{dC_i}{d\omega_i} \Big|_{\omega_{-j}} - \gamma \log L_i \frac{d \log L_i}{d\omega_i} \Big|_{\omega_{-j}} - \beta_p \pi s \right] di = 0$$

Since nominal wages of union j members are all identical (as implied by the union's preferences), we can integrate across them, obtaining

$$\frac{1}{C_j} \frac{dC_j}{d\omega_j} \Big|_{\omega_{-j}} - \gamma \log L_j \frac{d \log L_j}{d\omega_j} \Big|_{\omega_{-j}} - \beta_p \pi s = 0$$

Simple algebraic manipulations yield $\frac{1}{C_j} \frac{dC_j}{d\omega_j} \Big|_{\omega_{-j}} = \frac{W_j L_j}{C_j} \left[\frac{d \log W_j}{d\omega_j} + \frac{d \log L_j}{d\omega_j} \Big|_{\omega_{-j}} \right]$. Making use of the fact that in equilibrium $\frac{W_j L_j}{C_j} = \alpha$ (i.e. the labor share in consumption), and of the approximation $\log W_j \cong \omega_j - \pi$, the first order condition can be rewritten as

$$\alpha \left[1 - s + \frac{d \log L_j}{d\omega_j} \Big|_{\omega_{-j}} \right] - \gamma \log L_j \frac{d \log L_j}{d\omega_j} \Big|_{\omega_{-j}} - \beta_p \pi s = 0$$
 (C.2)

Noting that $\tilde{\psi} \equiv -\frac{d \log L_j}{d \log W_j}\Big|_{\omega_{-j}} = -\frac{d \log L_j}{d \omega_j}\Big|_{\omega_{-j}} \frac{d \omega_j}{d \log W_j} = -\frac{d \log L_j}{d \omega_j}\Big|_{\omega_{-j}} \frac{1}{1-s}$ (the approximation $\log W_j \cong \omega_j - \pi$ is used in the last passage), equation (4.1) in the text is obtained dividing expression (C.2) by 1-s (this can be done harmlessly since s<1 for all parameters configurations except when: $\beta_g=0$ and n=1; the first order condition for this special case (in which s=1) can be derived from footnote 22).

D. Appendix: Comparison of equilibrium employment under NWB and RWB

In order to compare $\tilde{\phi}$ with ϕ (expression 3.4 in the GV paper) we express s in terms of the elasticity $\tilde{\psi}$. To do this, take the partial derivative of 2.10 with respect to ω_j taking ω_{-j} as given. This yields

$$s \equiv \frac{d\pi}{d\omega_{j}}\Big|_{\omega_{-j}} = \frac{-\gamma}{(1-\alpha)\beta_{g}} \left[\frac{1}{n} \frac{d\log L_{j}}{d\log W_{j}} \Big|_{\omega_{-j}} \frac{d\log W_{j}}{d\omega_{j}} + \frac{n-1}{n} \frac{d\log L_{-j}}{d\omega_{j}} \Big|_{\omega_{-j}} \right] =$$

$$= \frac{-\gamma}{(1-\alpha)\beta_{g}n} \left[-\tilde{\psi}(1-s) + (n-1) \left(\sigma s + (\sigma - \frac{1}{1-\alpha})(\frac{1}{n} - s) \right) \right] =$$

$$= \frac{\gamma}{(1-\alpha)\beta_{g}n} \left[\tilde{\psi}(1-s) - (n-1) \left(-\tilde{\psi}(1-s) + \sigma \right) \right] =$$

$$= \frac{\gamma}{(1-\alpha)\beta_{g}n} \left[n\tilde{\psi}(1-s) - (n-1)\sigma \right]$$

which implies that s is equal to

$$s = \frac{\gamma \left[n\tilde{\psi} - (n-1)\sigma \right]}{n \left[(1-\alpha)\beta_g + \gamma \tilde{\psi} \right]} \text{ and therefore } \frac{s}{1-s} = \frac{\gamma \left[n\tilde{\psi} - (n-1)\sigma \right]}{n(1-\alpha)\beta_g + (n-1)\gamma\sigma} .$$

Replacing the expression for $\frac{s}{1-s}$ into (4.3) yields

$$\tilde{\phi} \equiv 1 - \frac{n \left(1 - \alpha\right)^2 \beta_g^2 + (n - 1) \left(1 - \alpha\right) \sigma \beta_g \gamma}{\tilde{\psi} \beta_p \gamma n + \tilde{\psi} n \left(1 - \alpha\right)^2 \beta_g^2 + (n - 1) \left[(1 - \alpha) \sigma \beta_g \gamma \tilde{\psi} - \beta_p \gamma \sigma \right]}$$

which can be compared with the corresponding GV expression (see their equation 3.4)

$$\phi \equiv 1 - \frac{n (1 - \alpha)^2 \beta_g^2}{\psi \beta_p \gamma + \psi n (1 - \alpha)^2 \beta_g^2}.$$

Recall that if n=1 or $n\to\infty$ then value of the elasticity is the same $(\psi=\tilde{\psi};$ see equation 3.4). Therefore, it immediately appears from the above expressions that under either n=1 or $n\to\infty$, $\tilde{\phi}$ and ϕ coincide.

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