No. 2284

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INDUSTRIAL ORGANIZATION

Centre for Economic Policy Research

ISSN 0265-8003

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Discussion Paper No. 2284
November 1999

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CEPR Discussion Paper No. 2284

November 1999

ABSTRACT

Licensing the Market for Technology*

In technology-based industries, incumbent firms often license their technology to other firms that will potentially compete with them. Such a strategy is difficult to explain within traditional models of licensing. This Paper extends the literature on licensing by relaxing the assumption of a monopolist technology holder. We develop a model with many technological trajectories for the production of a differentiated good. We find that competition in the market for technology induces licensing of innovations and that the number of licenses can be inefficiently large. A strong testable implication of our theory is that the number of licenses per patent holder decreases with the degree of product differentiation.

JEL Classification: D23, D43, L13
Keywords: licensing, market structure, oligopoly theory

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*This Paper was written while Andrea Fosfuri was visiting the Heinz School of Public Policy and Management, Carnegie Mellon University. The hospitality of the centre is gratefully acknowledged. We are indebted to Alfonso Gambardella for ongoing suggestions and discussions. We thank Stephan Arping, Eric Brousseau, Bruno Cassiman, Robert Merges, Massimo Motta, Thomas Ronde and participants in seminars at University of Stuttgart, University of Urbino and Fondazione Eni Enrico Mattei (Milan) for helpful comments. All errors, of course, remain our own.

Submitted 3 September 1999

NON-TECHNICAL SUMMARY

The importance of licensing as a means of profiting from innovation is increasing in a number of technology-intensive industries. A recent study by Arora et al. (1999) reports that the size of the market for technology licensing is about 25 billion dollars a year in North America and is of the order of 6.6 and 8.3 billion dollars a year in Europe and Japan. In some sectors, for instance chemicals and semiconductors, it is not uncommon to observe large established companies consciously adopting a strategy of licensing some of their technology for generating revenues. The noteworthy feature is that often these firms license their technology to other firms that will potentially compete with them.

This constitutes something of a challenge to traditional wisdom, which holds that an innovator can best profit from innovations by commercializing them itself (e.g. Teece, 1988). In this view, licensing is undesirable because the innovator has to share some of the rents with the licensee, because there are considerable transaction costs in writing contracts upon technological knowledge and especially because licensing increases competition, hence dissipating rents.

Traditional explanations for licensing build on the idea that firms license if they are less efficient at exploiting the invention than the potential licensees, or they attempt to establish their technology as a de-facto standard, for instance when network externalities are important. Although these explanations have some validity, they cannot explain the kind of licensing behaviour we are witnessing. In this Paper we propose a different explanation. Specifically, we argue that the interaction between a market for technology, where firms sell their technology through licensing and a product market, where firms sell their output, helps to explain licensing decisions by firms. The key to our argument is that competition in the product market creates a strategic incentive to license. Indeed, licensing imposes a negative pecuniary externality upon other incumbents in the product market, which is not taken into account by the licensor. As a result, if there are two or more incumbent firms that have proprietary technologies that are substitutes for each other, both firms may find it privately profitable to license, although their joint profits would be higher in the absence of any licensing.

To better understand the intuition behind the results derived in the Paper, we shall focus our attention on two main effects that licensing generates on the profits of the licensor. The first, the *revenue effect*, is given by the rents earned by the licensee that will accrue to the patent holder in the form of licensing payments. The second, the *rent dissipation effect*, is given by the erosion of profits due to another firm competing in the downstream market. If there is only one incumbent in the product market the *rent dissipation effect* dominates the *revenue effect* whenever industry profits are maximized by a monopoly (as is typically the case). Instead, when another incumbent exists, the losses due to increased competition are shared with the other incumbent in the product market so that the licensor does not fully internalize the *rent dissipation effect*. If the *revenue effect* is larger than the *rent dissipation effect*, then firms compete not only to supply the products but also to supply their technologies.

The *revenue effect* depends on transaction costs and the relative bargaining power of the licensor and licensee. Our results confirm that lower transaction costs and greater bargaining power of the licensor lead to more licensing by increasing the *revenue effect*. The degree of product differentiation across technologies has an important influence on the magnitude of the two effects. If the goods are differentiated the licensee will be a stronger competitor with the technology holder in the product market than with the other producers. This enhances the *rent dissipation effect* (which is now internalized to a greater extent by the licensor) and reduces the profitability of the licensing strategy. Thus we find that licensing will be more widespread, the lower the degree of product differentiation. Moreover, one would expect that the *rent dissipation effect* would depend on the production and commercial capabilities of the licensor: Large, well-established producers have less to gain from licensing and more to lose from competition. Our results confirm that, all else held equal, research laboratories license more. Interestingly enough, the model also throws up a less straightforward result, namely that the presence of independent laboratories may induce a producer-innovator to license more as well.

We also derive conditions for a policy that stimulates licensing to be welfare-improving. Licensing is typically welfare-improving when we have few patent holders. Instead, when competition in the market for technology is high, we show that one can actually have too much licensing compared to the socially efficient level of licensing.

Finally, we show that incumbent firms would benefit from restricting their licensing activity. This suggests that technology holders might have incentives to collude in order to reduce or stop licensing and hence increase profits. Surprisingly, this also implies that factors such as stronger patent protection, which stimulate licensing by lowering transaction costs, might hurt rather than benefit technology holders.

1. Introduction

The importance of licensing as a means for generating revenues from innovations is increasing in a number of technology-intensive industries. In 1996 U.S. corporations received $66 billion in royalty income from unaffiliated entities (Degnan, 1998). A recent study by Arora et al. (1999) reports nearly 8000 transactions worldwide in technology during the period 85-97. Transactions were found to be steadily increasing over time and concentrated in few sectors notably chemicals, software, electrical and non-electrical machinery, and engineering and professional services.[1]

There are some well-known examples of large companies consciously adopting a strategy of licensing for generating revenues. For instance, in chemicals, Union Carbide and Montecatini have actively licensed their polyethylene and polypropylene technology. Currently, a number of firms including Dow Chemicals, Exxon, Union Carbide, Nova Chemicals and Phillips Petroleum are actively licensing their metallocene catalyst technology for producing plastics (Arora and Fosfuri, 1998). In semiconductors, industry observers expect IBM to generate $750 million in 1998 from licensing. Texas Instruments is reported to have earned royalties of over $1.8 billion between 1986 to 1993, a figure comparable to Texas Instruments' cumulative net income during this period (Grindley and Teece, 1997). Other firms licensing semi-conductor technology include AT&T, and SGS-Thompson.[2]

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[1] Anand and Khanna (1997) in their study of strategic alliances find that licensing is common in other sectors such as biotechnology and computers as well, accounting for about 20 to 33% of all alliances, depending on the sector. They also find that licensing has increased in frequency between 1990-1993, the time period that they study. A recent survey, by a consulting firm, of 133 companies in the US, Japan and Western Europe in automotive, engineering, bio-pharmaceuticals and electronics found that 75% of firms license in technology and nearly 66% license out their technology. Furthermore, expenditure on licensing in technology are about 12% of total R&D budget (IPR Market Benchmark Study, 1998).

[2] Note also that network externalities are not important in the examples, so that this licensing activity is not driven by a desire to establish the technology as a de-facto standard.

This constitutes something of a challenge to traditional wisdom, which holds that an innovator can best profit from innovations by commercializing them itself (e.g., Teece 1988). In this view, licensing is undesirable because the innovator has to share some of the rents with the licensee, because there are considerable transaction costs in writing contracts upon technological knowledge and especially because licensing increases competition, and hence, dissipates rents. However, as Merges (1998) and Arora (1995) have argued, stronger patent protection can enhance the efficiency of licensing transactions. Levin et al. (1987) find that patent protection was generally inadequate in many industries, with some important exceptions such as chemicals. Interestingly enough, licensing is widespread in chemicals, and chemicals is a good exemplar of the theory developed in this paper.[3]

In this paper we develop a theoretical framework where the interaction between a market for technology, where firms sell their technology through licensing, and a product market, where firms sell their output, helps explaining licensing decisions by firms.[4] The key to our argument is that competition in the product market creates a strategic incentive to license. Thus, if there is more than one technology holder, they will compete to license their technology, even though this reduces their joint profits. We use this framework to explore how licensing decisions are affected by factors such as the nature of demand, transaction costs and patent protection.

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[3] Elsewhere, we document the widespread incidence of licensing in the chemical industry (Arora, 1997, and Arora and Fosfuri, 1998). In the chemical industry, the question of whether or not licensing is sensible has been a matter of considerable debate. For instance, Union Carbide has been criticized for its liberal licensing of its polypropylene/polyethylene Unipol process, with the claim that licensing reduced profitability, both for the industry, and also for Union Carbide itself (Spitz, 1988).

[4] A distinction between 'product market' and 'market for technology' (and 'market for innovation') is explicitly made in the Antitrust Guidelines for the Licensing of Intellectual Property (U.S. Department of Justice, 1995).

As we show in the next section, our key departure from the literature on licensing is the relaxing of the assumption of a monopolist innovator. We formalize this intuition in Section 3, where we characterize the conditions under which the equilibrium involves licensing and the effects of the nature of demand, of transaction costs and of the bargaining power of the licensor. Section 4 examines the socially efficient level of licensing and compares it to the equilibrium level of licensing. The licensing behavior of small versus large producers is analyzed in Section 5. The next section shows that, with multiple licensors, increasing the efficiency of licensing contracts can diminish profitability and hence the incentives for R&D. Section 7 extends our analysis to capture the interaction between market for technology and market for innovation. It shows that, under fairly general conditions, allowing free entry into the market for innovation leaves earlier results qualitatively unchanged. Section 8 brings together our main findings and concludes the paper.

2. Incentives to licensing with one and many technology holders

Why do firms license their technology? The typical answer is that they license if they are less efficient (or unable) at exploiting the invention than potential licensees or they attempt to establish their technology as a de-facto standard, for instance when network externalities are important. Both of these motivations are well known and accordingly we ignore them here. Instead, we focus on the role of licensing in rapidly expanding the use of technology. Typically, there are significant firm level adjustment costs and other constraints that restrict how rapidly an innovator can expand output. Thus, a technology holder can turn to licensing as a way of exploiting the technology more aggressively. In our model, these constraints are endogenous as the output of technology holders is constrained by the type of commitment problem that is well known in Cournot competition.[5] In turn, this implies that we allow non-exclusive licensing contracts, a common practice in technology licensing. (See for example, Anand and Khanna, 1997).

We analyze the case in which there are at least two technology holders in the market.[6] Although the introduction of multiple technology holders might appear to be a minor modification with respect to the acquired literature on licensing, it turns out to have important effects on the predictions of the model and on the set of results one can derive. Specifically, licensing imposes a negative pecuniary externality upon other incumbents in the product market, which is not taken into account by the licensor.[7] As a result, if there are two or more incumbent firms that have proprietary technologies that are substitutes for each other, both firms may find it privately profitable to license, although their joint profits may well be higher in the absence of any licensing.

Our results are driven by two main effects that licensing generates on the profits of the licensor. The first, the *revenue effect*, is given by the rents earned by the licensee which will accrue to the patent holder in the form of licensing payments. The second, the *rent dissipation effect*, is given by the erosion of profits due to another firm competing in the product market.

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[5] A firm can more credibly commit itself to an expansion in production when it transfers the output decision to a separate entity (the licensee) rather than when it keeps such decision within the firm's boundaries.

[6] For instance, Lenzig and Courtaulds, both developed technology for producing lyocell, the first man made fiber in 30 years (Financial Times, January 8, 1998). The chemical industry is another rich source of such examples. For instance, as shown in Arora (1997) Union Carbide, Himont and Mobil compete with each other in selling polypropylene licenses; BP and Du Pont compete in polyethylene process technology; UOP, Mobil-BP and Phillips Petroleum in methyl tert butyl ethers (MTBE).

[7] Critical to the argument is competition in the product market. In our model, only firms with access to the technology can produce. In principle, however, one can think of fringe firms that compete in the product market but do not have proprietary technology to license. These fringe firms can, by reducing the *rent dissipation effect*, induce licensing. We briefly discuss this case in Section 3.1 but our main focus in this paper is on competition between technology holders.

If there is only one incumbent in the product market the *rent dissipation effect* dominates the *revenue effect* whenever industry profits are maximized by a monopoly (as is typically the case).[8] Instead, when another incumbent exists the losses due to increased competition are shared with the other incumbent in the product market so that the licensor does not fully internalize the *rent dissipation effect*. If the *revenue effect* is larger than the *rent dissipation effect*, then firms compete not only to supply the products but also to supply their technologies.

The *revenue effect* depends on transaction costs and the relative bargaining power of the licensor and licensee. Our results confirm that lower transaction costs and greater bargaining power of the licensor lead to more licensing. The degree of product differentiation across technologies has an important influence on the magnitude of the two effects. If the goods are differentiated the licensee will be a stronger competitor of the technology holder in the product market than of the other producers. This enhances the rent *dissipation effect* (which is now internalized to a greater extent by the licensor) and reduces the profitability of the licensing strategy. Thus we find that licensing will be more widespread, the lower the degree of product differentiation. Moreover, one would expect that the *rent dissipation effect* would depend on the production and commercial capabilities of the licensor: Large, well-established producers have less to gain from licensing and more to lose from competition. Our results confirm that, all else held equal, research labs license more. Interestingly enough, the model also throws up a less straightforward result, namely that the presence of independent labs may induce a producer-innovator to license more as well.

[8] In general, the *revenue effect* may prevail whenever there is more than one incumbent. If $\pi(N)$ represents the profit of the typical firm when there are N firms in the industry, then an incumbent's marginal payoff from licensing (assuming the licensor captures all the surplus) is $2\pi(N + 1) - \pi(N)$. For $N = 1$, it is typically the case

We also derive conditions for a policy aimed to stimulate licensing to be welfare improving. Licensing is typically welfare improving when we have few patent holders. Instead, when competition in the market for technology is high, we show that, at the equilibrium, one can actually have too much licensing compared to the socially efficient level of licensing. This result links our paper to the literature on excessive entry in oligopolistic markets (see for instance, Suzumura and Kiyono, 1987).

Finally, we show that incumbent firms would benefit from restricting their licensing activity. This suggests that technology holders might have incentives to collude in order to reduce or stop licensing and hence increase profits. Interestingly enough, this also implies that factors which stimulate licensing – lower transaction costs, greater bargaining power of the licensor – might hurt rather than benefit technology holders.

The importance of strategic effects in vertically linked markets has been discussed in many other multistage models of oligopoly. For instance, Besanko and Perry (1993) analyze how the externalities to other brands from a franchiser's investments in its retailers provides incentives for exclusive dealing in a differentiated oligopoly. In our model, a technology holder's actions have only pecuniary externalities for others. Fershtman and Judd (1987) analyze how the nature of product market competition distorts incentives that owners provide to managers.[9] Similar effects are possible in licensing. For instance, output based royalties can be used to affect the "toughness" of product market competition. In Cournot competition, output based royalty contracts are inferior to pure lump sum contracts. Indeed, the insight from Brander and Spencer (1983) is that a licensor may even wish to subsidize its licensees.

that the marginal payoff is negative. However, notice that although the latter is a useful benchmark, it is not crucial for the comparative statics we work out in the paper.

The key difference in our paper is that our basic results on why firms license do not depend on whether product market competition is Cournot or Bertrand – on whether the strategies in the product market are complements or substitutes (Bulow et al., 1985). Indeed, although we analyze Cournot competition in differentiated products in the text, in Appendix I we find that licensing would take place even with Bertrand competition and multinomial demand, and many of the results are common to both Cournot and Bertrand specification.

We apply the insights from the literature on vertically linked markets to explain why and how much licensing takes place. By contrast, much of the literature on licensing has focused on the optimal licensing behavior of the monopolist inventor once it has developed and patented a new technology or production process (see Gallini and Wright, 1990; Kamien and Tauman, 1986). Katz and Shapiro (1986) discuss the optimal number of licensees for a single technology holder who does not compete in the product market. Instead, we analyze how the number of licenses sold is affected by competition from other technology holders, by the strength of patent protection and nature of demand. When the innovator is also active in the product market then either only minor innovations are licensed (Gallini, 1984; Katz and Shapiro, 1985; Rockett, 1990a) or licensing is used strategically to enhance demand (Shepard, 1987) or to choose competitors after the patent expires (Rockett, 1990b) or to deter entry (Gallini, 1984). Some of the literature also focuses on how licensing might encourage or refrain firms from investing in R&D (Katz and Shapiro, 1985). We formally model the R&D stage of the game in Section 7, and we shall comment on this issue both in Section 6 and Section 7.

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[9] Absent the uncertainty in demand or costs assumed by Fershtman and Judd (1987), the standard Cournot outcome is also an equilibrium. By contrast, licensing is privately profitable even if other technology holders do not license, as long as there is some competition in the product market.

3. The model

Consider a sector where *N* firms have independently developed and patented proprietary technologies for the production of a good. Such a good can either be perfectly homogeneous across technologies or differentiated. When the good is differentiated, each variety is assumed to be an equally imperfect substitute for all the others.

Besides the *N* patent holders we assume that there exist many potential entrants who do not have innovative capabilities but can produce if they receive the rights to use the technology from one of the incumbents. We also assume that startups are costless to the licensees and that their opportunity cost is zero.[10] Incumbents can therefore both produce themselves (by using their installed production facilities) and license their technology to potential entrants. A licensee produces the same variety of the good as the original licensor.

Let $k_i - 1$ be the number of licenses sold out by firm $i = 1, 2, \ldots, N$. Hence, the total number of firms that have the technology and can produce the (differentiated) good is equal to $\sum_{i=1}^{N} k_i$. Qualitative results would not change if one assumes the existence of a fringe of firms that can get access to the technology without obtaining a license from one of the technology holders (for instance, through imitation). For analytical tractability we shall consider k_i and N to be continuous variables.

Technology transfer from the licensor to the licensee involves a fixed cost, $F \geq 0$, which captures the transaction costs of licensing. These are the deadweight losses arising from the costs of writing contracts, as well as the cost of transferring know-how, which has

[10] This assumption could be removed at the cost of messier algebra. Either positive startup costs or positive opportunity cost would simply put an upper bound on the number of licenses per patent holder. Our analysis would hold unchanged except for this additional constraint. If the constraint binds, it determines the number of licenses at equilibrium and the comparative statics would be driven by what happens to the (gross) profits of each licensee in the second stage of the game.

been found to be an important component of the costs of technology transfer (Teece, 1977). We also assume that $\sigma \in [0,1]$ is the share – collected up front – accruing to the licensor of total profits earned by its licensee through the use of the technology. In particular, we do not allow for contracts with per-unit output royalties.[11] Whereas F captures the inefficiencies in arm's length contracting, σ accounts for the inability of the licensor to capture all rents. We shall interpret σ as the bargaining power of the innovator in the licensing negotiation.[12] Both F and σ depend on the actual form of the licensing contract and the strength of patent protection. To keep the analysis manageable, we treat both as parameters.

We analyze the following three-stage game. First, potential innovators have to invest in R&D in order to develop and patent their proprietary technology ("competition in the market for innovation"). Second, each patent holder decides how many licenses to sell out to potential entrants ("competition in the market for technology") and third, all firms that have got the technology supply the (differentiated) good ("competition in the product market"). Notice that these three stages correspond to the distinction between 'goods market', 'technology market' and 'innovation market' introduced by the Antitrust Guidelines for the

[11] For instance, the licensor might choose a combination of royalties and lump sum payments to maximize profits. The design of the optimal license contract, for both exclusive and non-exclusive contracts has been studied by Katz and Shapiro (1986) and Gallini and Wright (1990) among others. Output based royalties are typically a response to asymmetric information or moral hazard, or to induce licensees to reduce output. We ignore information problems here. Restricting licensee output is sub-optimal for the licensor, because the same outcome can be achieved more efficiently by reducing the number of licensees and saving on transaction costs. Therefore, a lump sum payment contract is the optimal contract in our model. We express this fixed sum as a fraction, σ, of the profits.

[12] Given our assumption of a large number of potential licensees, σ would be set equal to 1 if it were endogenous to the model. We prefer to keep it as a free parameter to better match the empirical finding that on average licensors capture only a share of the total rents generated in a licensing contract (e.g., Caves et al., 1983). There are several other reasons for the licensor being unable to extract the full rents generated by its technology, including asymmetric information and the ability of the licensee to invent around the patent once licensing negotiations have begun, due to weak intellectual property rights (Gallini, 1992).

Licensing of Intellectual Property (1995). For much of the paper we focus on the last two stages of the game taking *N* as exogenous. However, in Section 7 we extend our analysis to the whole game showing that all results remain qualitatively unchanged. We proceed by backward induction.

<u>Competition in the product market</u>

We assume Cournot competition in the product market. Inverse demand function for each variety *i* has the following linear schedule[13]:

$$p_i = 1 - \sum_{k_i} x_i - \mu \sum_{N \setminus i} \sum_{k_j} x_j \qquad (1)$$

for any $i = 1, 2, \ldots, N$, where p_i denotes the price, the first summation is across quantities supplied by firms producing *i*, and the second summation is across all quantities supplied by firms endowed with technology different from *i* ($N \setminus i$ stands for all varieties but *i*).

Here, a key parameter is μ, which captures for the degree of product differentiation across varieties. We assume that $\mu \in [0,1]$, with varieties being homogeneous for $\mu = 1$ and completely differentiated (independent) for $\mu = 0$. To keep things as simple as possible, we assume that all technologies allow production at zero marginal cost. Also, notice that we are implicitly imposing that the good is perfectly homogeneous within the group (i.e. all firms using the same technology) and equally differentiated across all groups.

Denote by $\pi^i(k_i, k_{-i}, \mu, N)$ the profits accruing to each firm endowed with technology *i* in the last stage of the game, where k_{-i} stands for the vector $\{k_1, k_2, \ldots, k_{i-1}, k_{i+1}, \ldots k_N\}$ of firms

[13] Notice that this demand structure could be derived from the maximization of a quadratic utility function of the form $U(X_1, \ldots, X_N) = \sum_i X_i - 0.5 \sum_i X_i^2 - \mu \sum_i \sum_{j \neq i} X_i X_j$, where $X_i = \sum_{k_i} x_i$ (e.g., Singh and Vives, 1984, and Sutton, 1998).

endowed with technologies different from *i*. In Appendix A we show that

$$\pi^i(k_i, k_{-i}, \mu, N) = A^{-2}B^{-2} \text{ where } A = [1 + (1-\mu)k_i] \text{ and } B = \left[1 + \sum_N \frac{\mu k_j}{1 + (1-\mu)k_j}\right].$$

Furthermore, π^i is decreasing and convex in k_i. Notice that for $\mu = 0$ each firm's profit only depends on the number of firms producing that given variety. Instead, for $\mu = 1$, π^i depends on the total number of firms active in the market $\sum_{i=1}^{N} k_i$.

<u>Competition in the market for technology</u>

Given the results of quantity competition in the last stage of the game one can express each patent holder's profit as a function of the number of firms producing each variety of the good, k_i and k_{-i}. That is:

$$V^i(k_i, k_{-i}, \sigma, \mu, N, F) = [1 + \sigma(k_i - 1)]\pi^i(k_i, k_{-i}, \mu, N) - (k_i - 1)F. \tag{2}$$

Each technology holder *i* chooses k_i in order to maximize its total profit given by the expression above. The first order condition is therefore:

$$V_k^i = \sigma\pi^i + [1 + \sigma(k_i - 1)]\pi_k^i - F \le 0 \quad \text{for } k_i - 1 \ge 0. \tag{3}$$

In Appendix B we show that the second order condition is satisfied at any interior equilibrium. The first order condition (3) shows the two effects discussed in the introduction.[14] The first one, $\sigma\pi^i$, is positive and corresponds to the *revenue effect*, which is the increase in profits from additional licensing revenues due to an additional licensee. The second, $[1 + \sigma(k_i - 1)]\pi_k^i$, is negative and corresponds to the *rent dissipation effect* due to an increase in the final stage competition (through licensing the patent holder creates more

[14] A third effect is due to the additional transaction costs for each new license agreement.

competition which lowers profits). The magnitude of these two effects and the value of *F* determine whether firms license at equilibrium, and if so, how many licenses are sold.

Before studying in detail the licensing equilibrium with multiple technology holders, we focus on a simpler case in which a single technology holder faces a fringe of downstream competitors with no access to the licensing strategy. This case nicely illustrates how the presence of other incumbents in the product market affects licensing decisions in the market for technology, independent of the competitive pressure from other technology holders.

3.1 The case of a single technology holder with competitors in the product market

We analyze the case where only one firm, firm *i*, can license its technology whereas the remaining $N - 1$ firms do not have proprietary technology to license, but are able to produce and compete in the product market. For simplicity and without loss of generality, assume that μ tends to 1. The first order condition for firm *i* can be written as

$$V_k^i = -\frac{\sigma}{(k_i + N)^3}\left[k_i - 1 - (N+1) + \frac{2}{\sigma}\right] - F \; . \tag{4}$$

It is easy to see that this expression is always negative for any $k_i \geq 1$ if $N = 1$. However, if $N \geq 2$, this expression can become positive for small values of *F* and σ close to 1, evaluated at $k_i = 1$. This result illustrates the key forces behind the licensing decision of the technology holder. When other firms are also able to supply the product ($N > 1$), by licensing firm *i* expands its market share at the expenses of all other producers. (More precisely, what expands is the market share of producers using technology *i*.) Indeed, with a homogenous good ($\mu = 1$), by licensing firm *i* increases its market share from $\frac{1}{N}$ to $\frac{k_i}{N + k_i - 1}$ while all other producers observe a reduction in their respective market shares from $\frac{1}{N}$ to

$\frac{1}{N + k_i - 1}$.[15] However, the presence of additional competitors in the product market results in lower prices. For some range of parameter values the benefits of an expanded market share outweigh the losses of increased competition and hence firm *i* chooses to license its technology. Instead, when the innovator is also a monopolist in the product market ($N = 1$), the licensing strategy would not expand firm *i*'s market share, but it does increase competition in the product market. Thus a monopolist patent holder would never license.

Finally, notice that firm *i*'s profits are at least as large with the possibility of licensing as without it and that factors which make licensing more appealing (a reduction in transaction costs or a larger σ) always increase licensor's profits. As we shall see later this is not the case when there are other technology holders that can react by licensing their technology as well.

3.2 The case of multiple symmetric technology holders

We now solve the game where N incumbents compete in the market for technology. We derive the symmetric equilibrium of the second stage of the game and explore how the optimal number of licenses is affected by the parameters of our model. For completeness, in Appendix C we prove that a symmetric Nash equilibrium generically exists. Further, both the non-licensing equilibrium (NLE) and the symmetric licensing equilibrium (SLE) can coexist. Finally, we prove that at most one stable symmetric licensing equilibrium can exist.[16] Notice that, for the range of parameter values supporting multiple equilibria, no firm would license out its technology if the competitors would not do it, but when all the other firms are licensing the best response is to license as well. This highlights the importance of competition in the

[15] In the case of product differentiation ($\mu < 1$), these two effects on market shares are smaller.

[16] In Appendix D we also provide necessary conditions for the existence of the stable SLE and for the existence of multiple equilibria.

market for technology in shaping the licensing decision of the innovator. We proceed now by analyzing the symmetric stable equilibrium.

Proposition 1: *In a symmetric stable equilibrium, k is increasing in* σ *and decreasing in F.*

Proof. The key to the proof is that in a symmetric stable equilibrium, the direction of change of *k* depends only on the sign of the cross-partial of the payoff function. One can directly verify that $V_{k\sigma} \geq 0$ and $V_{kF} \leq 0$, thus giving us the required results. ■

Note that Proposition 1 is completely general and does not rely on the assumption of Cournot competition in the downstream market. Moreover, it applies both to the NLE and stable SLE. It implies that any factor that increases the bargaining power of licensors, or decreases the transaction costs involved in licensing will increase licensing. The interpretation with respect to *F* is more interesting from a policy perspective.[17] Arora (1995), and Merges (1998), have argued that stronger patent protection reduces the transaction costs of technology licensing. The results reported by Anand and Khanna (1997) provide empirical support for this proposition. Based on a sample of 1612 licensing agreements, they find that sectors where patents are strong are also those with a higher incidence of licensing activity, while sectors with weak patents tend to have joint ventures and other such bundled

[17] Notice that *F* should be read relative to the size of the market. Assume that we have a mass ϕ of identical consumers with a quadratic utility function generating the inverse demand function in (1). (See Singh and Vives (1984) for a similar specification.) Therefore, ϕ stands for market size. Each patent holder's profit would be $V^i(k_i, k_{-i}, \sigma, \mu, N, F, \phi) = \phi[1 + \sigma(k_i - 1)]\pi^i(k_i, k_{-i}, \mu, N) - (k_i - 1)F$ and the first order condition $V_k^i = \sigma\pi^i + [1 + \sigma(k_i - 1)]\pi_k^i - F/\phi \leq 0$ for $k_i - 1 \geq 0$. Therefore, an increase (decrease) in *F* can be reinterpreted as a decrease (increase) in market size.

arrangements for transferring technology. We discuss the effect of patent protection on licensing further in Section 7.

We now examine how the number of competitors, N, and the degree of product differentiation in the market, μ, affect licensing.

Proposition 2: *For F = 0, in a stable SLE, the number of licenses by each technology holder is (weakly) increasing in N.*

Proof. At the stable SLE, the direction of change of k depends only on the sign of the cross-partial of the payoff function. By differentiating V_k with respect to N, one obtains:

$$V_{kN} = \sigma\pi_N + [1+\sigma(k_i - 1)]\pi_{kN} . \tag{5}$$

Using $V_k = 0$ we can rewrite (5) as:

$$V_{kN} = \sigma\left(\pi_N - \pi\frac{\pi_{kN}}{\pi_k}\right) + F\frac{\pi_{kN}}{\pi_k} \tag{6}$$

One can show that: $\pi_N = -2\mu k A^{-3}B^{-3}$ and $\pi_{kN} = 2\mu k A^{-5}B^{-4}[2(1-\mu)AB + 3\mu]$. It follows that

$\pi_N - \pi\frac{\pi_{kN}}{\pi_k} = \frac{\mu^2 k A^{-3}B^{-3}}{(1-\mu)AB+\mu}$, which is always non-negative. Hence, $V_{kN} \geq 0$ at $F = 0$. ■

From expression (6) one can see that an increase in the number of incumbents involves two forces working in opposite directions. More competition reduces the magnitude of the *revenue effect*, reducing the payoff from licensing. However, more competition also reduces the size of the *profit dissipation effect* and hence reduces the opportunity cost of licensing. At $F = 0$ the second force prevails. Under positive transaction costs, the two forces have different magnitudes. In particular, for large values of F an increase in N could decrease the equilibrium number of licenses per firm. We can partially characterize the situation as follows:

Result 1: *At $N = 2$, k is increasing in N. Further, we conjecture that there exists an $N^* > 2$, such that, in a stable SLE, k is increasing in N for any $N < N^*$, and decreasing in N for any $N > N^*$, and N^* is decreasing in μ, F and σ.*

Proof. In Appendix E we show analytically that k is increasing in N for $N = 2$. Our conjecture is supported by extensive numerical simulations briefly described in Appendix E. ■

This result implies that even with positive transaction costs, increases in competition in the market for technology initially increase licensing. Only later, as the number of technology holders increases, further increases in competition decrease licensing by reducing the profitability of the product market. Note that this result refers to the number of licenses per technology holder, k, and not to the total number of licenses, $N \times (k - 1)$.

We now state and prove one of the most robust results of the paper, which can also be empirically tested most easily, namely that the extent of licensing decreases with the degree of product differentiation. The intuition is quite straightforward. When the good is highly differentiated, each firm has a well-defined market niche. Any entrant licensed by the technology holder will be a close competitor to the technology holder itself, and the increased competition will be internalized to a greater extent. Instead, when the good is homogeneous, the negative effect due to increased competition is spread across all incumbents, while only the licensor shares in the profits of the new entrants. As with the comparative static with respect to N, there are two forces at work. On the one hand, less differentiation implies a smaller *revenue effect*, but on the other hand, it also reduces the *profit dissipation effect*. It turns out that the second force always prevails. This result is robust and holds also for Bertrand competition with multinomial logit demand, as we show in Appendix I.

Proposition 3: *At any stable SLE that involves at least one license per technology holder, k is increasing in* μ.

Proof. We prove by contradiction. First, notice that in a stable SLE, the direction of change of *k* depends only on the sign of the cross-partial of the payoff function. By differentiating V_k with respect to μ, one obtains:

$$V_{k\mu} = \sigma\pi_{\mu} + [1+\sigma(k_i - 1)]\pi_{k\mu} \tag{7}$$

where $\pi_{\mu} < 0$ and $\pi_{k\mu} > 0$ (see Appendix G.1). Then, suppose that for some values of σ, $V_{k\mu} < 0$. This implies that $V_{k\mu\sigma} = \pi_{\mu} + k_i\pi_{k\mu} < 0$. However, if at $\sigma = 1$, $V_{k\mu}$ is positive, then $V_{k\mu\sigma} < 0$ implies that $V_{k\mu}$ cannot be negative for any σ, resulting in a contradiction. Indeed, one can show that for all admissible parameter values, $k \geq 2$ implies $V_{k\mu} > 0$.[18] Hence, $V_{k\mu}$ must be positive at any $k \geq 2$ for all admissible values of σ. ■

Patterns of technology licensing in the chemical industry provide empirical support for Proposition 3. Elsewhere (see Arora and Fosfuri, 1998) we find that the per-firm number of licenses decreases with the degree of product differentiation in the chemical industry. Homogeneous sectors like air separation, pulp and paper, and petrochemicals are marked by

[18] Although licensing implies that *k* should be no less than two, in Appendix G we show that the result reported in Proposition 3 holds at any $k > 1$ when either $F = 0$, or $N < 15$, or if product differentiation is low enough.

extensive licensing, while we observe only limited licensing by producers in differentiated product groups like pharmaceuticals and organic chemicals.

4. Welfare

Does a policy that stimulates licensing always increase welfare? The answer is not always in the affirmative and it turns out that under certain circumstances we might have too much licensing with respect to the socially efficient level.

In computing the socially efficient level of licensing we assume that the market structure is given (i.e. *N* is exogenous). Therefore, the social planner can only choose the symmetric number of licenses per-firm, *k*.

Social welfare is the sum of consumer surplus and net industry profits, i.e.

$$W = N\left\{\underbrace{\tfrac{1}{2}\left(k^2x^2\left[1+\mu(N-1)\right]\right)}_{\text{consumer surplus}} + \underbrace{kx^2-(k-1)F}_{\text{net industry profits}}\right\} \tag{8}$$

where $x=\left[1+(1-\mu)k+\mu Nk\right]^{-1}$. Maximizing this expression with respect to *k*, one obtains that the socially efficient number of licenses per-firm is $k^S=\frac{1}{1+(N-1)\mu}\left(\frac{1}{\sqrt[3]{F}}-1\right)$

Notice that k^s is decreasing in *N* and μ and tends to infinity as *F* goes to zero. Furthermore, k^s does not depend on σ, which is helpful in proving our main result with respect to welfare.

Proposition 4: *There exists an* $N^o\geq 2$ *such that for any* $N < N^o$, $k^s > k^*$ *and for any* $N \geq N^o$, $k^s \leq k^*$. *Further,* N^o *is decreasing in* σ *and* μ.

Proof. Take the first order condition evaluated at the symmetric $k_i = k_j = k^S$, i.e. $V_{k_i}\big|_{k_i=k_j=k^S}$. Then, solve for the value of *N*, such that $V_{k_i}\big|_{k_i=k_j=k^S}=0$. One can show that

$$N^{o}(\sigma,\mu,F)=\frac{\left(\frac{1-\sqrt[3]{F}}{u}\right)\left[2\sigma\mu+(1-\mu)\left(1+\frac{\sigma}{\sqrt[3]{F}}\right)\right]}{\sigma-2(1-\mu)(1-\sigma)-\sqrt[3]{F}\,[1+2\mu(1-\sigma)]}+\frac{\mu-1}{\mu}$$. It is easy to see that for any N $< N^o$, $V_{k_i}\big|_{k_i=k_j=k^S}<0$ which implies that $k^* < k^S$. We shall prove that N^o is decreasing in μ.

First, notice that $\frac{dN^o}{d\mu}=\frac{N}{D}$, where $D=\sqrt[3]{F}\mu^2\left[2+\sqrt[3]{F}-3\sigma-2\left(1-\sqrt[3]{F}\right)(1-\sigma)\mu\right]^2>0$ and

$N=-\left[\sigma+\sqrt[3]{F}(3-4\sigma)\right]\left[-2-\sqrt[3]{F}+3\sigma+2\left(1-\sqrt[3]{F}\right)(1-\sigma)\mu\left(2-\mu\left(1-\sqrt[3]{F}\right)\right)\right]^2>0$. One can then show that, for all admissible values of our parameters, $N^o > 0$ implies that $D < 0$. Finally,

$\frac{dN^o}{d\sigma}=\frac{-2(1-\sqrt[3]{F})(1-\mu+\mu\sqrt[3]{F})(1-\mu+\sqrt[3]{F}(2+\mu))}{Du}$, which shows that N^o is decreasing in σ.

Hence N^o reaches a lower bound at $\sigma = 1$. Evaluating N^o at $\sigma = 1$, one obtains

$\underline{N}=N^o\big|_{\sigma=1}=2+\frac{1-\mu}{\mu\sqrt[3]{F}}\geq 2$. ■

The reason for possibly excessive licensing in equilibrium is the deadweight loss of the transaction costs associated with licensing. As N becomes larger these deadweight losses grow. Indeed, the following corollary helps to highlight this aspect of Proposition 4:

Corollary 1: *At $F = 0$, private licensing is always insufficient, i.e. $k^* < k^s$.*

Proof. Notice that N^o tends to infinity as F tends to zero, which implies that for any $N <$ ∞, $k^* < k^s$. ■

Finally, we consider possible policy interventions aimed to increase the share of profits accruing to the licensor, σ. For instance, policies that lower the cost of patent enforcement, or that raise the penalties for patent infringement will increase the share of profits that the licensor can extract. Corollary 2 below shows that a policy aimed raising σ is likely to improve welfare when there are few technology holders, transaction costs are small and products are differentiated. Note also that this provides only a sufficient condition for such policies to be welfare improving, and such policies may be desirable even otherwise.

Corollary 2 : *Any policy that increases* σ *is welfare improving if* $N \leq \underline{N} = 2 + \frac{1-\mu}{\mu \sqrt[3]{F}}$.

Proof. Since N^o is decreasing in σ, $\underline{N} < N^o$, where $\underline{N} = N^o|_{\sigma=1}$. Hence, by Proposition 4 private licensing is insufficient. By Proposition 2, k^* is non-decreasing in σ, therefore an increase in σ enhances welfare. ■

5. Small firms and research labs

In this section we analyze how the presence of firms with limited production capability (i.e. small firms and research labs) affects licensing behavior. To this end, consider a slightly more general profit function for the innovator:

$$V^i = [\lambda + \sigma(k_i - 1)]\pi^i - (k_i - 1)F \tag{9}$$

where $\lambda \geq 0$ captures for the size of the installed production facilities. Large values of λ are associated with big corporations, while $\lambda = 0$ corresponds to the case of research labs. As one might expect, small firms and research labs tend to license more than big corporations (see also Gans and Stern, 1997). The intuition behind this result is fairly simple: having little or no production capability means that the extent of the *profit dissipation effect* is smaller and, hence, licensing is a more appealing strategy. This is formalized below:

Result 2: k_i^* *is decreasing in* λ.

Proof. Consider the first order condition for firm *i*: $V_k^i = [\lambda + \sigma(k_i - 1)]\pi_k^i + \sigma\pi^i - F$.

Take any pair $(\underline{\lambda}, \overline{\lambda})$ such that $\underline{\lambda} < \overline{\lambda}$. Since $\pi_k^i < 0$, it is easy to see that

$V_k^i = [\overline{\lambda} + \sigma(k_i(\underline{\lambda}) - 1)]\pi_k^i + \sigma\pi^i - F < 0$, which implies that $k_i(\overline{\lambda}) < k_i(\underline{\lambda})$. ■

Perhaps less obvious is how the presence of research labs influences the licensing behavior of firms with installed production facilities. Consider the case of a duopoly where instead of having two producers (i.e. two firms with installed production facilities), one of them is replaced by a research lab. Then, we have the following result:

Proposition 5: *In a duopoly setting, substituting a producer with a research lab increases the number of licenses sold by the remaining producer*.

Proof. Consider the cross-partial of the profit function in (9) with respect to k_i and k_j, i.e. $V_{k_i k_j} = \sigma[(k_i - 1)\pi_{k_i k_j} + \pi_{k_j}] + \lambda\pi_{k_i k_j}$. If $V_{k_i k_j} \geq 0$, the result follows directly because for a technology holder, replacing a rival technology holding producer with a research lab implies that licensing by the rival will be greater. Hence, suppose that for some value of σ, $V_{k_i k_j} \leq 0$. Since $\lambda\pi_{k_i k_j} > 0$, this implies that $[(k_i - 1)\pi_{k_i k_j} + \pi_{k_j}] < 0$ and therefore that $V_{k_i k_j \sigma} < 0$. Hence, $V_{k_i k_j}(k)|_{\sigma=1} \leq V_{k_i k_j}(k)|_{\sigma<1} \leq 0$. Finally, one can show that there are no parameter values such that at $\sigma = 1$ and $k_i = k_j$, $V_{k_i k_j}(k) \leq 0$ at any $k > 1$, implying that $V_{k_i k_j} \geq 0$ for all admissable values of σ. ■

What this proposition illustrates is an inducement effect: The presence of a research lab stimulates the licensing activity of the big firm at a level that it would not have reached otherwise.[19] Elsewhere (see Arora and Fosfuri, 1998) we provide empirical support for this inducement effect. Using data from the chemical industry, we find both that firms without production facilities tend to license more and that in sectors where such firms operate more intensively, large chemical producers themselves tend to license more.

6. Incentives for R&D

In Section 4, we have established sufficient conditions for an increase in the per-firm number of licenses to be socially desirable. Note well that our definition of welfare takes the number of technology holders as exogenous. But as well understood in the literature on innovations, an increase in the rate of diffusion also implies a smaller incentive to develop the innovation in the first place. This is precisely what occurs in our model where a larger *k* means lower per-firm profits.

Proposition 6: *With ex-ante symmetric licensors, the possibility of licensing reduces profits per innovator.*

Proof. The licensor's profits are:

$V(k^*) = [1+\sigma(k^*-1)][1+(1-\mu)k^* + \mu N k^*]^{-2} - (k^*-1)F$. Taking the derivative with respect to k^* it is easy to show that it is always negative. ■

[19] Does Proposition 5 fully generalize? In Appendix H we show that at *F = 0*, $V_{k_i k_j} > 0$ which implies that in a model with no transaction costs the presence of research labs always increases the number of licenses sold by all other producers. For positive values of *F*, Proposition 5 holds unchanged provided that *N* is small enough.

Proposition 6 also suggests that technology holders might have incentives to collude in order to reduce or stop licensing and hence increase profits. An example of such a practice is provided by the history of the chemical sector. Before WWII, cartels were widespread. The major technology leaders, which were typically European firms, adopted a strict control over their licensing policies in order to keep market shares, deter entry and sustain prices above competitive levels (see Arora, 1997). Such collusion in the market for technology is explicitly analyzed in the Antitrust Guidelines for the Licensing of Intellectual Property (example 2).

Moreover, Proposition 6 underscores the ambiguous effects on profits of technology holders of factors that increase the efficiency of licensing transactions or enhance the licensor's share. Using the envelope theorem one can show that

$V_\sigma^i = (k-1)\pi^i + \sum_{j \neq i} \frac{\partial V^i}{\partial k_j} \frac{dk_j}{d\sigma}$, where the second term is negative and

$V_F^i = -(k-1) + \sum_{j \neq i} \frac{\partial V^i}{\partial k_j} \frac{dk_j}{dF}$, where the second term is positive. The effect of σ (or F) on firms' profits is ambiguous : on the one hand, a larger σ (or a smaller F) increases licensor's profits; on the other hand, it also stimulates (by Proposition 1) the licensing activity of all the other competitors and hence reduces profits. In particular, it is readily apparent that at $k^* = 1$, a reduction in the inefficiency of the licensing contracts would actually hurt technology holders.[20] This is stated formally in the following result:

Result 3: *At parameter values such that $k^* = 1$, an increase in σ or a decrease in F reduces profits of each technology holder.*

[20] Interesting enough, an increase in the size of market that – as we show in footnote 17 – works as a decrease in *F*, might also hurt the licensors.

Notice that the above relationship between licensor profits and the parameters σ and F holds for a larger parameter space than the one given in Result 3. Indeed, through numerical simulations (available from the authors upon request) we have been able to construct several examples for which an increase in σ or a decrease in F reduces profits of technology holders. Thus, insofar as stronger patent protection encourages licensing (through higher σ or lower F), it may even lower industry R&D or reduce the number of firms investing in R&D!

7. The 'market for innovation': making *N* endogenous

So far we have assumed that the number of incumbents, N, is exogenously given. However, it is likely that in order to develop and patent a proprietary technology, firms have to incur substantial costs (for instance, expenditures in R&D). A forward-looking firm would not commit resources unless future profits are sufficiently large to recover initial investments. Thus, factors affecting the market for technology are also likely to affect the number of potential licensors, which in turn will also affect licensing behavior.

This section provides sufficient conditions such that the results presented so far are qualitatively unchanged in a three-stage game where initially firms decide whether to invest or not in R&D and if they do then they play the game analyzed in Section 3. Specifically, we show that even after modeling the R&D decision, k is increasing in σ, decreasing in F and increasing in μ. A sufficient condition for the results to hold is that $\frac{dk}{dN} \geq 0$ (see Proposition 2 and Result 1 above, which provide sufficient conditions).

We assume that there are $\overline{N}$ firms (with $\overline{N}$ sufficiently large) that have the ability to invest in R&D and develop new technology. The cost of R&D required to develop a new technology is *G*, which is fixed. For simplicity, we assume away uncertainty and assume that if the firm does invest *G*, it will develop a proprietary technology. A potential technology holder that does not invest *G* enjoys an outside opportunity that is normalized to zero.[21] We confine our analysis to the case in which *G* is small enough to support at least two firms. For larger values of *G* either only one firm or no firm will invest in R&D. The following result is quite intuitive, which we state for completeness.

Result 4: *A larger R&D fixed cost reduces the number of incumbents, N, and the per-firm number of licenses, k.*

Proof: See Appendix F.

The following two propositions, proved Appendix F, generalize the results we derived in Section 3 with exogenous *N*.

Proposition 1': *The per-firm number of licenses, k, is increasing in* σ *and decreasing in F.*

Proposition 3': *The more homogenous are the products, the smaller is the number of incumbents, N, and the larger is the per-firm number of licenses, k.*

[21] Notice that a firm that does not become a technology holder could obtain a license in the second stage of the game from one of the other innovators, and thereby possibly earn positive profits, at least in the last stage game. However, given the large number of potential licensees, the probability of becoming a licensee is small. Further, any rents earned in the product market are likely to get dissipated in rent seeking activities (that we do not model) by potential licensees. In any event, our results simply require that firms seeking to develop technology behave as if their outside options are independent of the parameters of the model. Normalizing the value of the outside option to zero is a convenient simplification.

Proposition 3' has a strong empirical implication. We should expect that differentiated markets have many technology holders but little licensing activity. Conversely, homogenous markets should be characterized by a smaller number of more active licensors. Notice that we cannot unambiguously predict the effect of changes in σ and F on N since, as explained in Section 6, the effect of changes in these two parameters on per-firm profits is a priori ambiguous. A further qualification concerns the effects due to a change in the patent regime. In Section 3 we argued that stronger patents can increase licensing by reducing F. Stronger patents could also strengthen the bargaining power of the licensor by reducing the ability to invent around patents (Gallini, 1992), implying an increase in σ and a greater propensity to license. However, broader patent scope or stronger novelty requirements may also increase G and hence reduce the number of firms that develop proprietary technology.

8. Conclusion

There is increasing evidence that firms in some sectors are looking to profit from their intellectual property not just by embodying it in their own output but also by licensing their intellectual property to others, including potential competitors. Such behavior is difficult to understand in the context of models with only a monopolist technology holder, who by definition faces no competition in the product market. By relaxing the widespread assumption of a monopolist patent holder, our paper shows that licensing might be the result of firms' strategic behavior. Indeed, the presence of competition drastically changes the incentives for an incumbent to license its technology to potential entrants. In particular, when there are multiple technology holders, not only do they compete in the product market, they also compete in the market for technology. Thus, our paper provides a framework for analyzing the nature and properties of markets for technology.

Within this framework, we showed that increases in the efficiency of licensing contracts and reductions in transaction costs increase the propensity to license. Although licensing profits increase, stronger product market competition may reduce overall profits of the innovators. This implies that stronger patents may be a mixed blessing for firms in technology intensive industries. Although stronger patents raise barriers against imitation by rivals, they may ultimately result in increased product market competition by facilitating licensing.

Since licensing partially substitutes for production, firms lacking adequate downstream commercialization (production and marketing) capabilities are naturally more aggressive licensors. Interestingly enough, our results indicate that their presence induces more aggressive licensing by their larger rivals with commercialization capabilities as well. Therefore, managers in technology based firms must guard against a common tendency to treat in-house technology like the "family jewels", particularly when smaller, technology focused firms also develop substitute technologies. Instead, they must be prepared to become effective technology licensors and compete in the market for technology.

The challenges that markets for technology pose for managers are strongest in sectors with relatively homogenous products. Unless the established incumbents also have strong brand identities or other ways of differentiating themselves from others, even proprietary process technologies will not be effective entry barriers if the transaction costs of licensing are low enough. Our model thus provides another important insight – increasing product differentiation not only softens price competition in the product market, it also reduces the propensity to license in the technology market.

Finally, from an economic policy perspective, the existence of a market for technology implies technology diffusion and increased entry, which improves the static efficiency of the market. However, by inducing entry in the product market, a market for technology may reduce the incentives to undertake R&D and hence restrict entry in the market for innovation. Moreover, if licensing involves transaction costs, our paper suggests that the presence of competitors in the market for technology might induce firms to an inefficiently high level of licensing. In general, policies aimed to stimulate licensing are likely to be welfare improving when there are few technology holders and products are differentiated.

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Appendix

A. Deriving the producer's profit function

Take any firm *i* (either patent holder or licensees) producing variety *i*. By maximizing firm *i*'s profits with respect to its own quantity we obtain the following first order condition:

$$1-\sum_{k_i} x_i - \mu \sum_{N\setminus i} \sum_{k_j} x_j - x_i = 0 . \tag{A.1}$$

First, impose symmetry across firms using the same technology. Then, by adding and subtracting $\mu \sum_{k_i} x_i$ we obtain:

$$1-k_i x_i - \mu \sum_{N} k_j x_j - x_i + \mu k_i x_i = 0 \tag{A.2}$$

from which

$$x_i = \frac{1-\mu \sum_N k_j x_j}{1+(1-\mu)k_i} . \tag{A.3}$$

Now, multiply both sides by μk_i and sum up across all possible varieties to obtain:

$$\sum_N \mu k_j x_j = \left[1-\sum_N \mu k_j x_j\right] \sum_N \frac{\mu k_j}{1+(1-\mu)k_j} \tag{A.4}$$

which after some manipulation can be rewritten as

$$\sum_N \mu k_j x_j = \frac{1}{1+\frac{1}{\sum_N \frac{\mu k_j}{1+(1-\mu)k_j}}} . \tag{A.5}$$

Then, substituting (A.5) in (A.3) and simplifying we obtain the equilibrium quantity by each firm (either patent holder or licensee) producing variety *i* as a function of the numbers of firms active in the production of all varieties ($k_1, k_2, \ldots, k_N$):

$$x_i = \left[1+(1-\mu)k_i\right]^{-1} \left[1+\sum_N \frac{\mu k_j}{1+(1-\mu)k_j}\right]^{-1} . \tag{A.6}$$

Replacing expression (A.6) in (1) we can compute the equilibrium price for each variety, and then profits as reported in the text. Furthermore, one can show that

$\pi_k^i = -2A^{-4}B^{-3}\left[AB(1-\mu)+\mu\right] < 0$ and $\pi_{kk}^i = 6A^{-6}B^{-4}\left[AB(1-\mu)+\mu\right]^2 > 0$.

B. Second order condition is satisfied at any stable SLE

We want to show that $V_{kk}^{i} < 0$. The second order condition is

$V_{kk}^{i} = 2\sigma\pi_{k}^{i} + [1+\sigma(k_i - 1)]\pi_{kk}^{i}$. Using the first order condition, one can write

$V_{kk}^{i} = 2\sigma\pi_{k}^{i} + \frac{F-\sigma\pi^{i}}{\pi_{k}^{i}}\pi_{kk}^{i} = 2\sigma\pi_{k}^{i} + \frac{F-\sigma\pi^{i}}{\pi_{k}^{i}}\left(\pi_{k}^{i}\right)^{2}\frac{3}{2\pi^{i}}$. Simplifying further we obtain

$V_{kk}^{i} = \pi_{k}^{i}\left[\frac{\sigma}{2} + \frac{3F}{2\pi^{i}}\right] < 0$. ■

C. Characterizing existence, stability and uniqueness of equilibrium.

Since we focus here on symmetric equilibria alone, we can simplify the analysis of existence and stability by defining $\Phi(k,\sigma,\mu,N) \equiv V_{k_i}^{\ i}\left(k_i = k, k_{-i} = k, \sigma, \mu, N\right)$, i.e. as the marginal revenue from licensing evaluated at a symmetric licensing level, k. Note that the set of symmetric equilibria is given by $\Phi(k) = F$. Notice further that stability requires that $\Phi_k(k) < 0$. Recall that $\Phi(k)$ is the marginal revenue from licensing evaluated at the symmetric licensing configuration. We can rewrite $\Phi(k) = g(k) f(k)$, where $g(k)$ is quadratic in k, with $g_{kk} < 0$, and $f(k) = \frac{[1+k(1+u(N-1))]^{-3}}{1+(1-\mu)k} > 0$.

Consider any value of $\mu < 1$. (At $\mu = 1$ and $F = 0$, a symmetric Nash equilibrium does not exist). Let k_1 and k_2 be the two roots of $g(k) = 0$, with $k_2 > k_1$. Notice that since $g(k) < 0$ at any $k > k_2$ then at any $k > k_2$, $\Phi(k)$ must be negative ($f(k)$ is always positive). Now, let $\Phi(1) - F < 0$. Hence, a symmetric Nash equilibrium exists where no firms license. On the contrary, let $\Phi(1) - F > 0$. Since $\Phi(k) < 0$ for any $k > k_2$ then it must be that $k_2 > 1$. Since $\Phi(k)$ is continuous for $k > 0$, then at least one stable SLE must exist. Figure 1 shows a SLE.

To show uniqueness of the stable SLE we take some intermediate steps. First, notice that $\Phi_k = 0$ is cubic in k. Hence, $\Phi(k)$ has three inflection points. Second, we show that one inflection point is on the right of k_2. To see this, note that $\Phi(k)$ tends to zero as k approaches infinity. Moreover, $\Phi(k) < 0$ at any $k > k_2$, which implies that there must exist a $k_3 > k_2$ such that $\Phi_k(k_3) = 0$. Third, we show that $\Phi(k)$ has only one inflection point for $\Phi(k) > 0$. (See Figure 2a and 2b.) Suppose that $\Phi_k(1) > 0$, then since $\Phi_k(k_2) < 0$, we only have an odd number of inflection points in the interval [1, k_2]. Since the maximum number of inflection points for $k < k_2$ is two, then there must be only one inflection point in the interval [1, k_2]. By a similar argument one can exclude multiplicity of inflection points in the interval [1, k_2] for $\Phi(k)$, when $\Phi_k(1) < 0$ and $\Phi(1) < 0$. Finally, consider the case where $\Phi_k(1) < 0$ and $\Phi(1) > 0$. One can show that, under these two conditions, $\Phi_k(k) < 0$ for any $k < k_2$. Indeed, $\Phi_k(1) < 0$ and $\Phi(1) > 0$ imply that $g_k(1) < 0$ which implies that $g_k < 0$ at all $k > 1$. But $g_k < 0$ implies $\Phi_k(k) < 0$ for any k belonging to the interval [1, k_2]. Only one inflection point in the interval [1, k_2] means that $\Phi(k) = F$ can at most have two roots. It is then easy to see that only the larger root is stable.

Finally, a necessary and sufficient condition for having multiple equilibria is that the smallest root of $\Phi(k)= F$ is greater than 1. (See Figure 3.) We show necessary conditions for the existence of multiple equilibria in Appendix D.

D. Necessary conditions for the existence of the stable SLE and for the existence of multiple equilibria

We first derive the set of conditions which insure the existence of a stable SLE at $F = 0$ (for all values of $\mu < 1$). Such conditions are only necessary at $F > 0$ (while they are also sufficient at $F = 0$).

Let k_1 and k_2 be the two roots of $\Phi(k) = 0$, with $k_2 > k_1$. For the existence of a stable SLE we want k_2 be a real number greater than 1 (greater than two if we are only interested in integer values of k). It turns out this is the case when the following two conditions are satisfied: $\beta^2 - 4\alpha\gamma \geq 0$ and $\alpha + \beta + \gamma < 0$, where $\beta = \sigma(1-\mu) + [1+\mu(N-1)][2(1-\sigma)(1-\mu) - \sigma]$, $\gamma = 2 - 3\sigma$ and $\alpha = \sigma(1-\mu)[1+\mu(N-1)]$. It is easy to check that both conditions are satisfied for large values of μ, σ and N.

To derive necessary conditions for the existence of multiple equilibria (sufficient at $F = 0$) one has to force k_1 to be greater than 1. This is the case when the following conditions are simultaneously satisfied: $\beta^2 - 4\alpha\gamma \geq 0$, $2\alpha + \beta < 0$ and $\alpha + \beta + \gamma > 0$. It is easy to check that all conditions hold for small values of σ, intermediate values of N and large values of μ (for instance, $\sigma = 0.25$, $N = 5$, $\mu = 0.98$).

E. Proof of Result 1

First, notice that at *N = 1*, $V_{kN} > 0$. Second, we prove that $V_{kN} > 0$ at *N = 2*. Indeed, $V_{kN}\big|_{N=2,\sigma=1} = 2\mu k\tilde{C}^{-3}\left[-\tilde{A}\tilde{C} + 2\tilde{C}k(1-\mu) + 3\mu k\right]$ where $\tilde{A} = [1+(1-\mu)k]$ and $\tilde{C} = [1+(1+\mu)k]$. Notice also that if V_{kN} is not negative at $\sigma = 1$, then it is not negative at any σ. Simplifying further we obtain that $sign\{V_{kN}\big|_{N=2,\sigma=1}\} = sign\{k^2 - 1 + \mu k - \mu^2 k^2\}$, which is always positive. Hence, $V_{kN}\big|_{N=2} > 0$.

To show that $V_{kN} > 0$ for *N* < *N** and $V_{kN} < 0$ for *N* > *N** we use simulations. (Mathematica files are available from the authors upon request.) In the figures 4a, 4b and 4c we plot the equilibrium value of *k* as a function of *N*, holding constant all other parameters (σ, μ, *F*), but one. We obtain *k*(N)* by solving $\Phi(k, \overline{\sigma}, \overline{\mu}, N) = \overline{F}$. In each figure the value of *N** corresponds of the inflection point of *k*(N)*. Notice that the curves joining the inflection points show the response of *N** to σ, μ, and *F* respectively in figures 4a-c. ■

F. Proofs of propositions from Section 7

Define the symmetric per firm profits as:

$$V^S = [1+\sigma(k-1)][1+(1-\mu)k+\mu Nk]^{-2} - (k-1)F \qquad \text{(F.1)}$$

where $k \equiv k(N, \sigma, \mu, F)$ solves equation (3) after having imposed symmetry across all k_i.

F.1. Proof of Result 4

The key of the proof is to show that per-firm profits are decreasing in *N*. Indeed, one can show that $V_N^S = \pi_N[1+\sigma(k-1)] + \xi(.)\frac{dk}{dN} < 0$, where $\xi(.) = \sigma\pi + [1+\sigma(k-1)]\pi_k - F < 0$ and $\pi_N < 0$. Hence, as *G* increases, *N* decreases and so does *k* ($\frac{dk}{dN} > 0$). ■

F.2. Proof of Proposition 1’

Taking the total differential of (F.1) with respect to σ and *N*, one obtains:

$$\frac{dN}{d\sigma} = \frac{\pi(k-1) + \xi(.)\dfrac{\partial k}{\partial \sigma}}{\pi_N[1+\sigma(k-1)] + \xi(.)\dfrac{\partial k}{\partial N}}$$. Then, the net effect of a higher σ on k is given by:

$\frac{dk}{d\sigma} = \left.\frac{\partial k}{\partial \sigma}\right|_N + \frac{\partial k}{\partial N}\frac{dN}{d\sigma}$. Substituting and simplifying one obtains that

$\frac{dk}{d\sigma} = \frac{\left.\frac{\partial k}{\partial \sigma}\right|_N \pi_N[1+\sigma(k-1)] - \frac{\partial k}{\partial N}(k-1)\pi}{D} > 0$. The proof follows along the same line for $\frac{dk}{dF}$ and it is omitted to save space. ■

F.3. Proof of Proposition 3'

Taking the total differential of (F.1) with respect to μ and N, one obtains:

$\frac{dN}{d\mu} = \frac{\pi_\mu[1+\sigma(k-1)] + \xi(.)\frac{\partial k}{\partial \mu}}{\pi_N[1+\sigma(k-1)] + \xi(.)\frac{\partial k}{\partial N}} < 0$, where $\xi(.) < 0$, $\pi_\mu < 0$ and $\pi_N < 0$. Then, the net effect of a higher μ on k is given by: $\frac{dk}{d\mu} = \left.\frac{\partial k}{\partial \mu}\right|_N + \frac{\partial k}{\partial N}\frac{dN}{d\mu}$. Substituting and simplifying one obtains that $\frac{dk}{d\mu} = \frac{[1+\sigma(k-1)]\left(\left.\frac{\partial k}{\partial \mu}\right|_N \pi_N - \frac{\partial k}{\partial N}\pi_\mu\right)}{D}$, where $D = \pi_N[1+\sigma(k-1)] + \xi(.)\frac{\partial k}{\partial N} < 0$.

Hence, $sign\left(\frac{dk}{d\mu}\right) = sign\left(1 - \frac{\frac{\partial k}{\partial N}\pi_\mu}{\left.\frac{\partial k}{\partial \mu}\right|_N \pi_N}\right)$, which after some substitutions is equal to the sign of $1 - \frac{V_{kN}\pi_\mu}{V_{k\mu}\pi_N}$. Finally, we show that $\frac{V_{kN}\pi_\mu}{V_{k\mu}\pi_N} < 1$, so that $\frac{dk}{d\mu} > 0$. Notice that $\frac{V_{kN}\pi_\mu}{V_{k\mu}\pi_N} < 1 \Leftrightarrow V_{kN}\pi_\mu > V_{k\mu}\pi_N \Leftrightarrow \pi_{k\mu}\pi_N < \pi_{kN}\pi_\mu$. Some additional algebra shows that $\pi_{k\mu}\pi_N - \pi_{kN}\pi_\mu = -4k^2(N-1)\mu^2 A^{-8} B^{-6} < 0$. ■

G. Additional results related to Proposition 3

G.1 $V_{k\mu} > 0$ at any *k > 1* and *F = 0.*

By differentiating V_k with respect to μ, one obtains: $V_{k\mu} = \sigma\pi_{\mu} + [1+\sigma(k_i - 1)]\pi_{k\mu}$. Using $V_k = 0$ we can rewrite: $V_{k\mu} = \sigma\left(\pi_{\mu} - \pi\frac{\pi_{k\mu}}{\pi_k}\right)$. With some additional algebra one can show that $\pi_{\mu} = -2A^{-3}B^{-3}k(N-1)$ and $\pi_{k\mu} > 0$. Simplifying further one obtains that

$$V_{k\mu} = \sigma\left(\pi_{\mu} - \pi\frac{\pi_{k\mu}}{\pi_k}\right) = \frac{\sigma A^{-3}B^{-3}uk(N-1)(B+1)}{AB(1-\mu)+\mu} > 0. \ ■$$

G.2 $V_{k\mu} > 0$ at any *k > 1* and μ *= 1.*

One can show that $V_{k\mu}\Big|_{\mu=1} = \frac{2k(N-1)[(4+kN)(1-\sigma)+\sigma(4k-1+(k-1)kN)]}{(1+kN)^4} > 0$. ■

G.3 $V_{k\mu} > 0$ at any *k > 1* and $N <$ *15.*

Notice that $V_{k\mu\sigma} < 0$. Take σ *= 1*. If we prove that $V_{k\mu}$ cannot be negative at σ *= 1*, then it must be the case that it cannot be negative for any σ. Solve $V_{k\mu}\Big|_{\sigma=1} = 0$ with respect to *N*. Only the largest root, *N^(k,* μ*)* is of interest to us. $V_{k\mu}$ is negative at any *N > N^*. Such a root is increasing in *k*. Take *k = 1*. Then, one can show that the lower bound of *N^* is at $\mu = 2\left(2-\sqrt{3}\right)$ and it is equal to 14.92. ■

H. $V_{k_i k_j} > 0$ at *F = 0.*

Write $V_{k_i k_j} = \sigma\pi_{k_j} + [1+\sigma(k_i - 1)]\pi_{k_i k_j}$. Using the first order condition one obtains

$V_{k_i k_j} = \sigma\left(\frac{\pi_{k_j}\pi_{k_i} - \pi\pi_{k_i k_j}}{\pi_{k_i}}\right)$, where $\pi_{k_j} = \frac{-2A^{-2}B^{-3}}{[1+(1-\mu)k_j]^2}$ and

$\pi_{k_i k_j} = \frac{2A^{-4}B^{-4}\mu}{[1+(1-\mu)k_j]^2}[2AB(1-\mu)+3\mu]$. Some additional algebra shows that

$$V_{k_i k_j} = -\frac{2\sigma A^{-6}B^{-6}\mu}{\pi_{k_i}[1+(1-\mu)k_j]^2} > 0. \ ■$$

I. BERTRAND COMPETITION WITH MULTINOMIAL LOGIT DEMANDS

In this section, we move from quantity competition to a more realist framework where firms set prices. Notation and assumptions are as in Section 3. Since competition is based on prices, we need to differentiate firms also when they use the same technology. Therefore, we denote by ρ_2 the degree of differentiation within the 'group' (where 'group' stands for all firms using the same technology). Instead, the parameter ρ_1 captures for the degree of product differentiation across groups as discussed above. Notice that with this notation a larger ρ means more differentiation. We assume that $\rho_1 > \rho_2$, which amounts to saying that firms using the same technology are closer to each other than to firms using different technologies. For simplicity, assume that total demand is normalized to one and marginal production costs are equal to zero for all firms.

We use the following notation. S_i is the share of total demand corresponding to group *i*, $S_{i|m}$ is firm *m*'s share of group *i*'s demand. Assuming a logistic distribution for demand, one can show that (see Anderson et al., 1992)

$$S_i = \frac{\left[\sum_{h=1}^{k_i} \exp\left(\frac{-p_{ih}}{\rho_2}\right)\right]^{\rho_2/\rho_1}}{\sum_{j=1}^{N}\left[\sum_{h=1}^{k_j} \exp\left(\frac{-p_{jh}}{\rho_2}\right)\right]^{\rho_2/\rho_1}} \qquad \text{and} \qquad S_{i|m} = \frac{\exp\left[\frac{-p_{im}}{\rho_2}\right]}{\sum_{h=1}^{k_i}\exp\left[\frac{-p_{ih}}{\rho_2}\right]}.$$

Hence, market share of firm *m* belonging to group *i* is simply $S_{im} = S_{i|m} \times S_i$ and profits are $\pi_{im} = p_{im} \times S_{im}$.

We can then solve for the first stage of the game. First order condition[22] for firm *m* is:

$$\frac{\partial \pi_{im}}{\partial p_{im}} = S_{im} + p_{im}\left(S_i \frac{\partial S_{m|i}}{\partial p_{im}} + S_{m|i}\frac{\partial S_i}{\partial p_{im}}\right). \tag{I.1}$$

Letting (I.1) be equal to zero and simplifying, we get

$$p_{im} = \frac{\rho_1\rho_2}{\rho_1\left(1 - S_{m|i}\right) + \rho_2\left(S_{m|i} - S_i\right)}. \tag{I.2}$$

[22] We assume that both the second order condition for a local maximum and the stability condition for the equilibrium are satisfied. The second order condition is always satisfied for the simple case where all firms are equally differentiated both within and across groups. In general,the stability condition is satisfied for *N* small.

By imposing symmetry within the group (i.e. $p_{im} = p_i$ and $S_{m|i} = \frac{1}{k_i}$) one obtains

$$p_i = \frac{\rho_2 k_i}{(k_i - 1) + \theta(1 - S_i)} \tag{I.3}$$

where $\theta = \frac{\rho_2}{\rho_1} < 1$. By substituting (I.3) in the profit expression we have

$$\pi_{im} = \frac{p_i S_i}{k_i} = \frac{\rho_2 S_i}{(k_i - 1) + \theta(1 - S_i)}, \quad \forall m. \tag{I.4}$$

We now derive a series of results for the case in which $p_j = \overline{p}, k_j = \overline{k}, S_j = \overline{S}, j \neq i$.

Result I: *In a symmetric equilibrium in prices, an increase in the number of licenses within group i, increases group i's price more than other groups' prices.*

Proof. Define X = $\frac{p_i - \overline{p}}{\rho_1}$. Then using (I.3) and the adding up condition,

$S_i + (N-1)\overline{S} = 1$, one can write $\frac{\partial X}{\partial k_i}\left(1 + \frac{k_i}{p_i}\frac{\theta^2}{D^2}\right) = -\frac{\theta}{D^2}[(1-\theta) + S_i\theta(1 - S_i\theta)] < 0$, where

$D = (k_i - 1) + \theta(1 - S_i)$. ■

Result II: *An increase in the number of licenses within group i increases group i's market share less than proportionately.*

Proof. $\frac{\partial S_i}{\partial k_i}\frac{k_i}{S_i} = \left(\frac{\theta}{k_i} - \frac{\partial X}{\partial k_i}\right)(N-1)S_i k_i$, which is positive by Result I above. Recall that

we are evaluating all expressions at a symmetric outcome, so that $S_i = \frac{1}{N}\forall i$. This implies

that $\frac{\partial S_i}{\partial k_i}\frac{k_i}{S_i} = \left(\theta - k_i\frac{\partial X}{\partial k_i}\right)\frac{(N-1)}{N}$. Using Result I, one can show by induction that

$-k_i\frac{\partial X}{\partial k_i} < \frac{N}{(N-1)} - \theta$, so that $\frac{\partial S_i}{\partial k_i}\frac{k_i}{S_i} < 1$. Notice that this implies $-k_i\frac{\partial X}{\partial k_i} < \frac{N}{(N-1)}$. ■

Result III: $\dfrac{\partial \pi_i}{\partial k_i} < 0$ *in a neighborhood of the symmetric equilibrium.*

Proof. $\dfrac{\partial \pi_i}{\partial k_i} = \dfrac{\rho_2}{D^2}\left[\dfrac{\partial S_i}{\partial k_i}\dfrac{k_i}{S_i} - 1 - (1-\theta)\right]$ and it is negative by Result II. This is a common result in almost all models of oligopolistic competition for which an increase in the number of firms reduces the per-firm profit. ■

Notice that when θ increases the degree of product differentiation within the group and across the groups become similar. For ρ_2 held constant, this means that the differentiation across groups falls, groups compete more closely and profits decrease. On the other hand, when groups are less differentiated, a decrease in profits due to an increase in competition, i.e. a larger k_i, is shared across all groups.

Result IV: *Holding ρ_2 constant,* $\dfrac{\partial \pi_i}{\partial \theta} < 0$ *and* $\dfrac{\partial^2 \pi_i}{\partial k_i \partial \theta} > 0$.

Proof. In a symmetric equilibrium, each group's share is unchanged. Therefore, $\dfrac{\partial \pi}{\partial \theta} = -\dfrac{\pi(1-S)}{D} < 0$. Then, $\dfrac{\partial^2 \pi}{\partial \theta \partial k} = -\dfrac{\pi_k(1-S)}{D} + \dfrac{\pi}{D}\left(\dfrac{\partial S}{\partial k}\right) + \dfrac{\pi(1-S)}{D^2}\left(1 - \theta\dfrac{\partial S}{\partial k}\right)$. Note that the first two terms are positive. Since $\dfrac{\partial S_i}{\partial k_i} < \dfrac{S}{k_i} = \dfrac{1}{Nk_i} < 1$, the third term is positive as well, giving us the result. ■

Result V: *At any symmetric licensing equilibrium,* $V^i_{k\theta} > 0$.

Proof. $V^i_{k\theta} = \dfrac{1-S}{D}\left[\sigma\pi(k-1) - F\right] + \left[1 + \sigma(k-1)\right]\dfrac{\pi}{D}\dfrac{\partial S}{\partial k}\left(1 - \dfrac{\theta}{D}\right)$. Note that $\theta > D$. Also, for all $k \geq 1$, $\sigma\pi(k-1) - F > 0$ at $V^i_k = 0$. Thus, both the terms positive, giving us the result. ■

Proposition I: *In a symmetric equilibrium, the per-group number of firms, k, is decreasing in the degree of product differentiation across groups, holding constant the degree of within group differentiation.*

Proposition I follows because the sign of the effect of an increase in θ depends only on the sign of $V_{k\theta}$. This proposition is analogous to the proposition derived for the differentiated Cournot model in the text.

Result VI: $V^i_{k\rho_2} > 0$.

Proof. $V_{k\rho_2} = \frac{V_{k\theta}}{\rho_1} + \frac{1}{\rho_1}\left(1+\sigma(k-1)\pi_k + \sigma\pi_k\right) = \frac{V_{k\theta}}{\rho_1} + \frac{F}{\rho_2} > 0$. ■

Proposition II: *For any given level of product differentiation across group, an increase of the degree of product differentiation within the group increases the per-group number of firm, k.*

An increment in the degree of product differentiation within the group makes the market more profitable. Each licensee competes less fiercely with other firms belonging to the same group, thus increasing the attractiveness of licensing. Notice that, unlike the Cournot model, it might well be that a 'unique' patent holder would like to license out its technology to competing firms. Indeed, when ρ_2 is large enough industry profits might be maximized by an oligopoly (even if $N = 1$). However, the presence of competing technologies still has an effect on firms' licensing behavior, which is to increase the number of licenses, k. Although we did not formally prove this result, one can argue that the case of $N = 1$ corresponds to a situation where firms are completely differentiated, that by proposition II means less licensing at equilibrium. Hence, when we move to $N > 1$ we should expect more licensing at equilibrium.

Reference:

Anderson, S., Palma, A., and Thisse, J., 1992, *Discrete Choice Theory of Product Differentiation*, Cambridge, Mass., MIT Press.

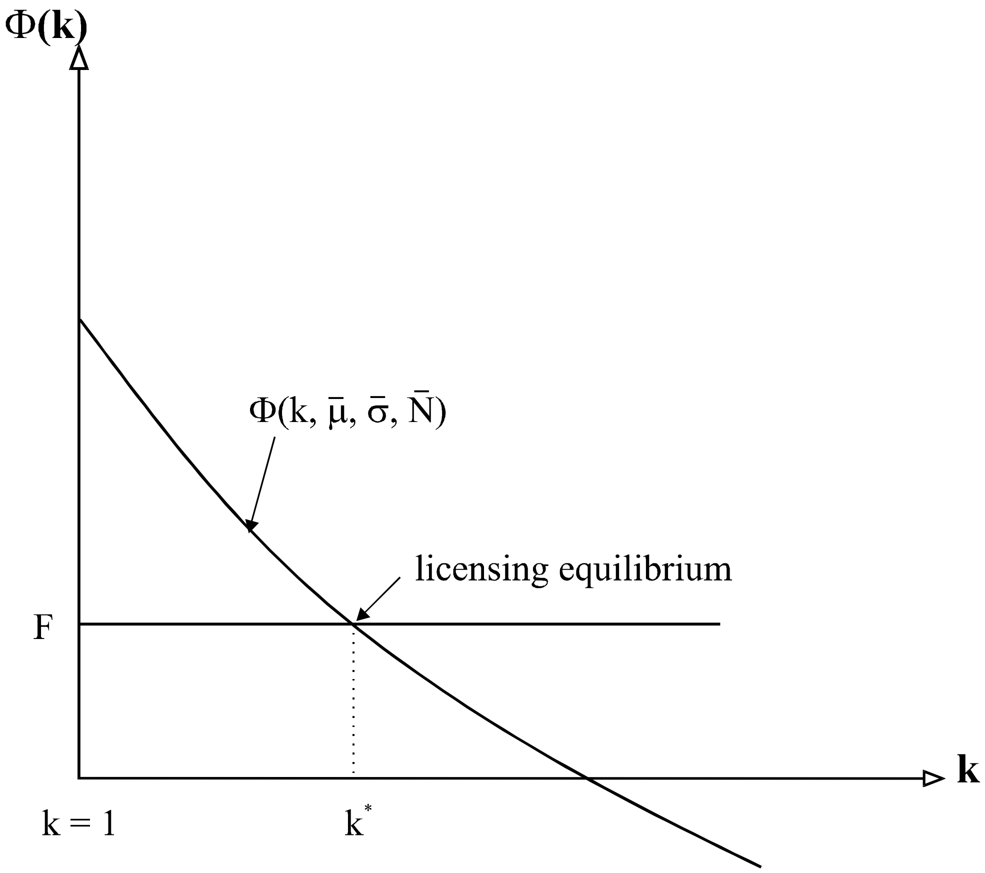

Figure 1: A unique symmetric stable licensing equilibrium

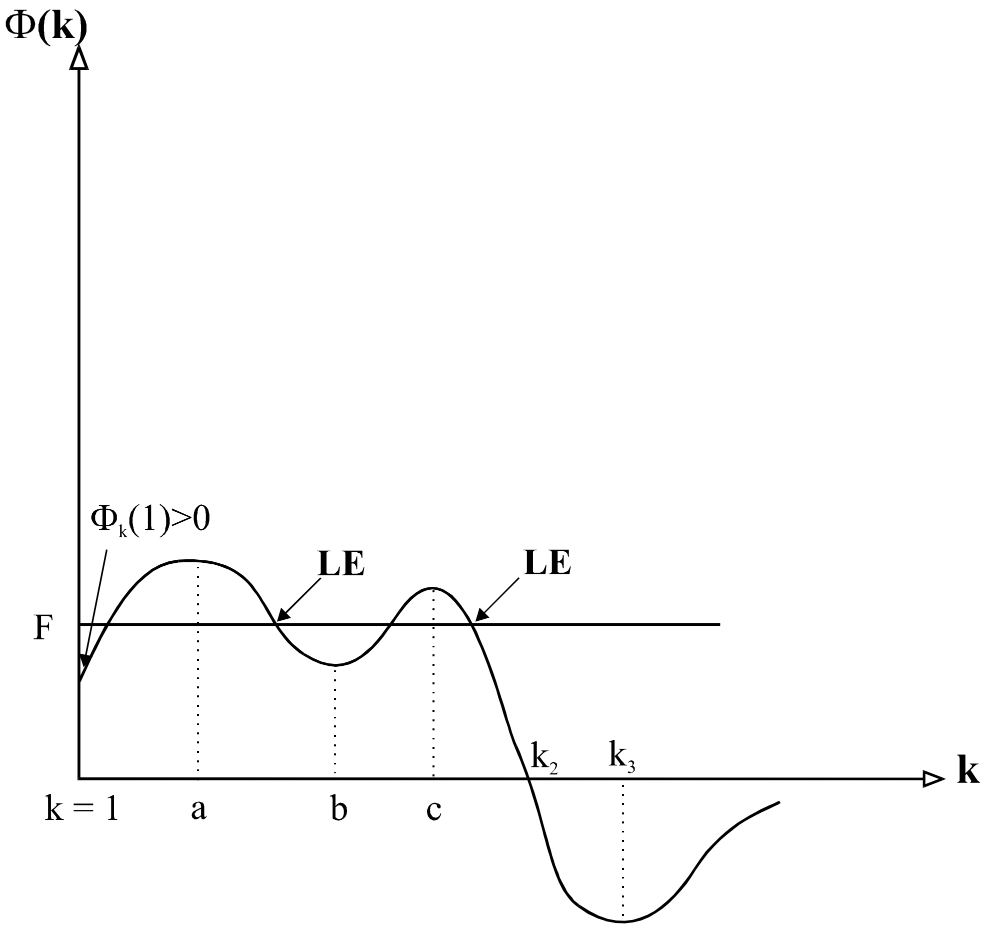

Figure 2a: Not possible, *Φ(k)* has at most 3 inflection points

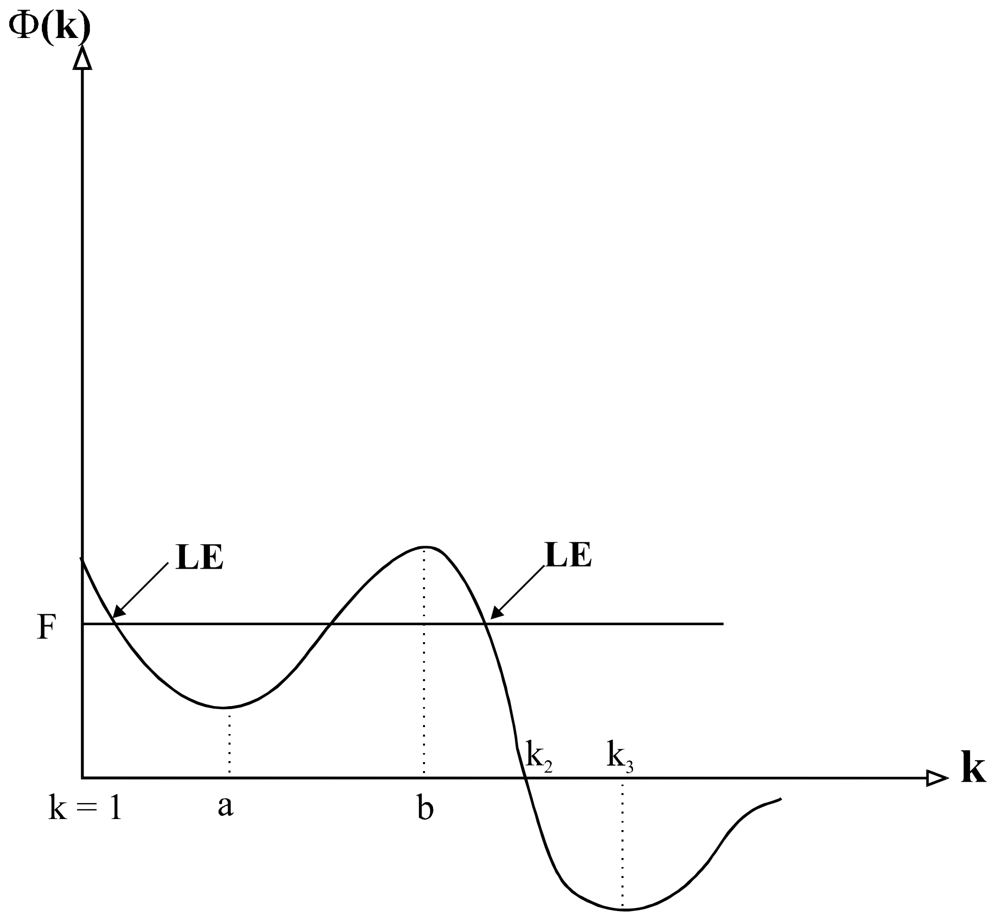

Figure 2b: Not possible, $\Phi_k(1) < 0$ and $\Phi(1) - F > 0$
implies $\Phi_k < 0$ for any $k < k_2$

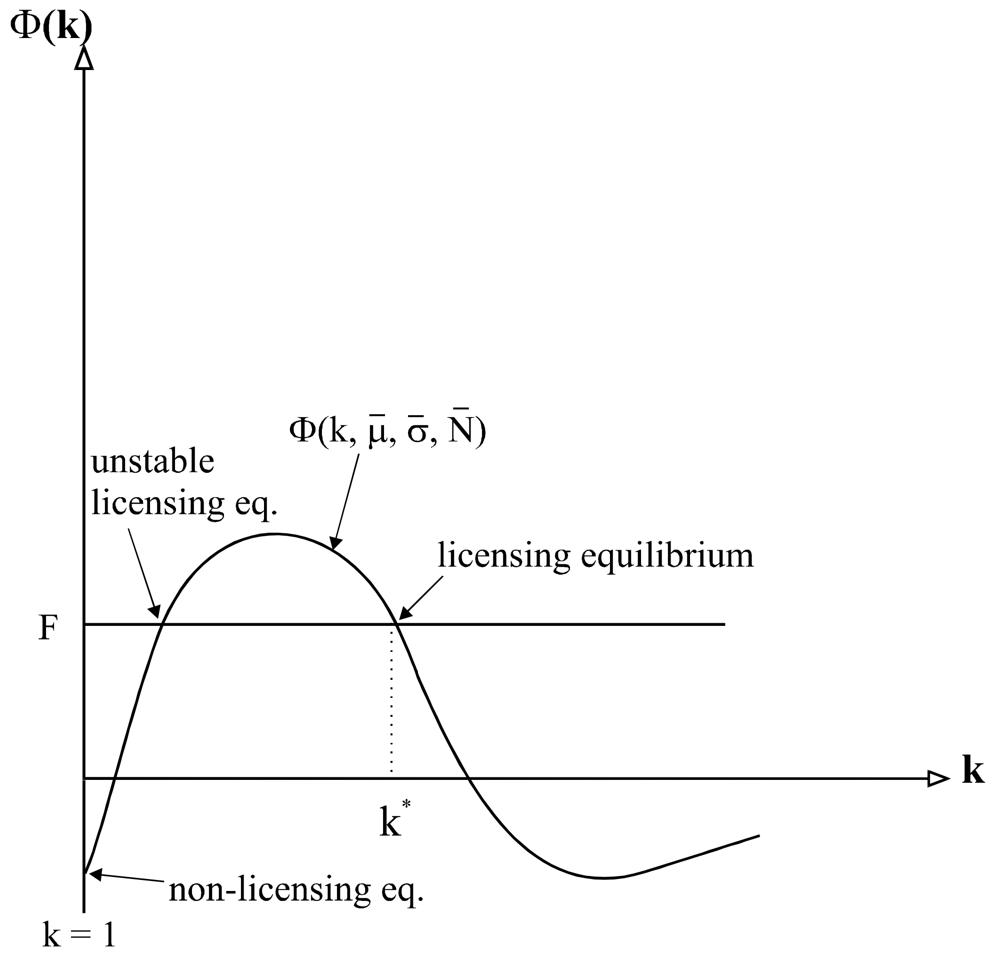

Figure 3: Multiple equilibria

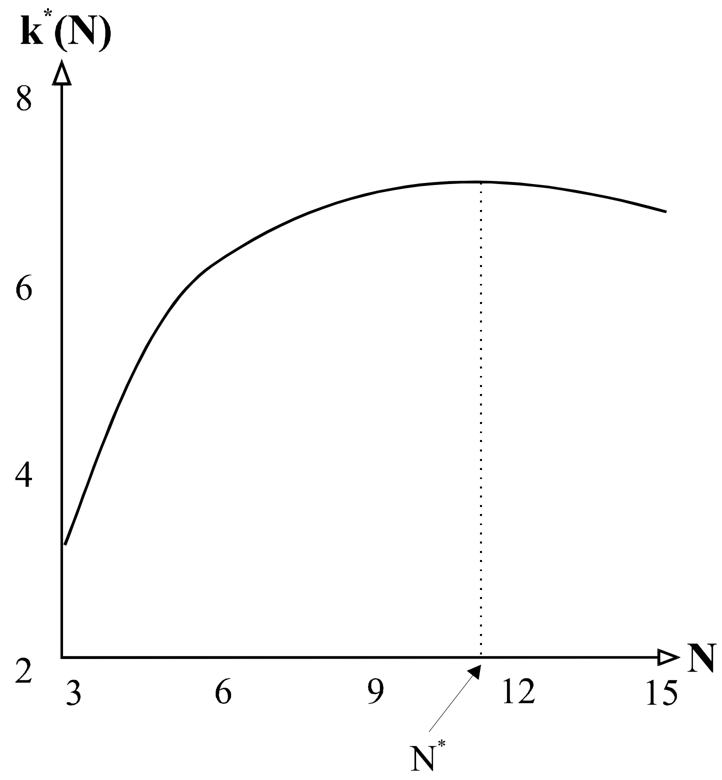

Figure 4: $V_{kN} > 0$ if $N < 11$, $V_{kN} < 0$ if $N > 11$
($F = 0.00001$, $\mu = 0.9$, $\sigma = 0.8$)

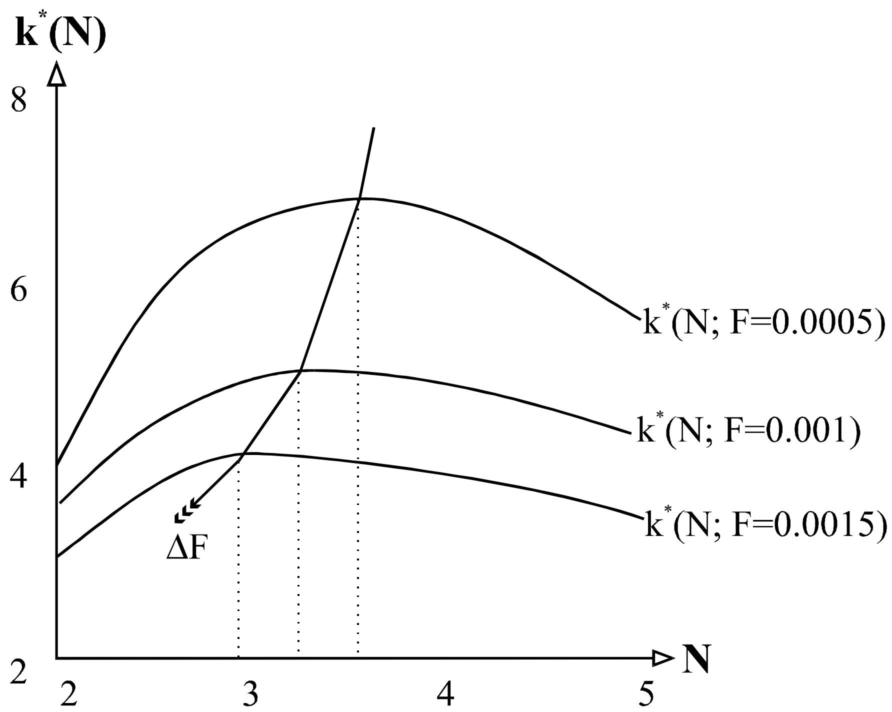

Figure 4a: N^* is decreasing in F ($\sigma = 1$, $\mu = 0.98$)

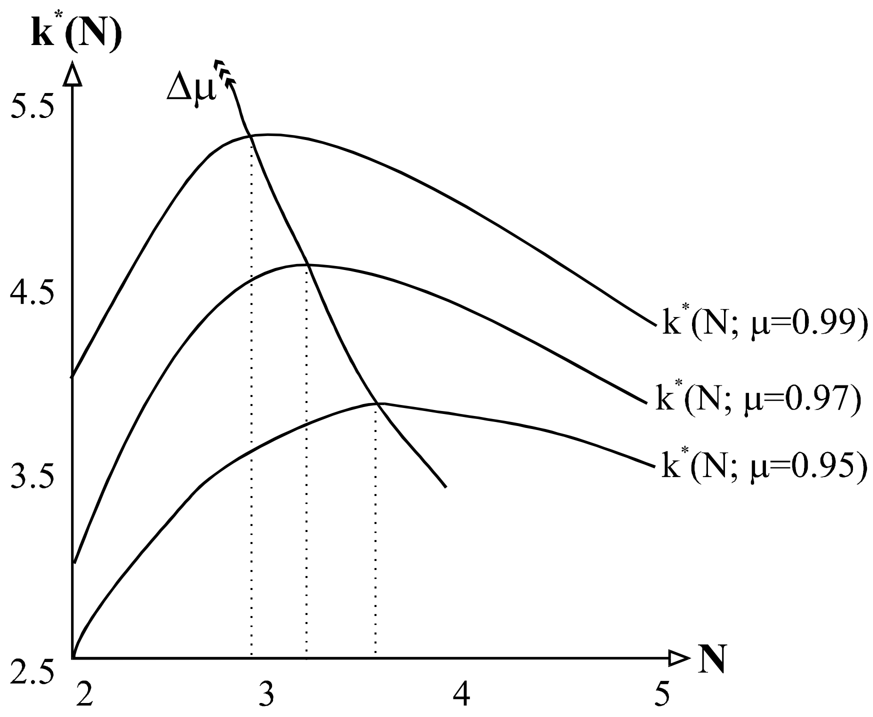

Figure 4b: N^* is decreasing in μ (σ = 1, F = 0.001)

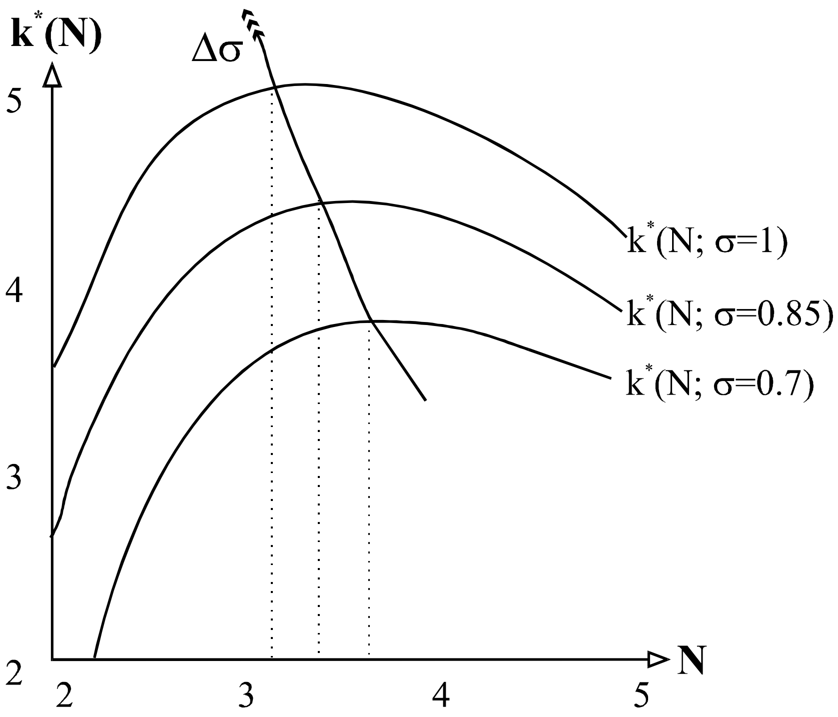

Figure 4c: N^* is decreasing in σ (μ = 0.98, F = 0.001)