No. 2269

COMPETITIVE INSURANCE MARKETS UNDER ADVERSE SELECTION AND CAPACITY CONSTRAINTS

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INDUSTRIAL ORGANIZATION

Centre for Economic Policy Research

ISSN 0265-8003

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Discussion Paper No. 2269
October 1999

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CEPR Discussion Paper No. 2269

October 1999

ABSTRACT

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Ever since the seminal work by Rothschild and Stiglitz (1976) on competitive insurance markets under adverse selection the equilibrium-non-existence problem has been one of the major puzzles in insurance economics. We extend the original analysis by considering firms that face capacity constraints due to limited capital. Two scenarios are considered: if the demand at any insurer exceeds the capacity then either consumers are rationed, or they are served, but the insurer faces a larger risk of bankruptcy. We show under mild assumptions that a pure strategy equilibrium exists, where every consumer buys his appropriate Rothschild-Stiglitz contract.

JEL Classification: C72, C78, D82, G22
Keywords: insurance markets, asymmetric information, competitive equilibrium, capacity constraints

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*We thank Klaus Schmidt for helpful comments.

Submitted 3 August 1999

NON-TECHNICAL SUMMARY

Adverse selection on insurance markets has been the subject of research in economic theory for a long time. This refers to the notion that the insured have more information on their risk type than the insurer might have. Take as an example genetic tests, where so far it is unclear whether insurance companies are allowed to demand results from previously undertaken tests, or even undertake tests themselves. If testing and the test results are private information, consumers are potentially much better informed about their illness and death risk than the insurer. They might then use this information when buying insurance.

To evaluate policy proposals for such a market with asymmetric information, a consistent model is required. However, ever since the work by Rothschild and Stiglitz (1976), researchers have been aware that the standard way of modelling such a market leads to non-existence of equilibrium in those cases, where there are just a few high risks in the market. This situation can be considered very relevant for the health insurance market. In the simplest set-up, insurer offer contracts and the potential customers choose the best contract available. The only possible equilibrium in pure strategies is a separating one, where the risk types obtain different contracts. Both contracts are priced fairly given the risk of the type, but the lower risks are underinsured. However, sometimes a single pooling contract might attract all customers and destabilise the so-called Rothschild-Stiglitz separating equilibrium.

There are several attempts in the literature on obtaining a well-defined model. In some models firms are allowed to withdraw contracts after the insured have made their choice (Wilson (1977), Hellwig (1987)). Another avenue of research is to let firms decide whether they want to divulge information about their customers with other firms (Jaynes (1979), Hellwig (1988)). Further models allow the firms to renegotiate with their customers once the first contract is signed (Asheim and Nilssen, (1997)). Although in all those models equilibria in pure strategies exist, it is probably fair to say that an overall agreed upon model has not been established. So it does not come as a surprise that most researchers still use the original Rothschild-Stiglitz model (or the extension by Wilson (1977), Miyazaki (1977), Spence (1978)) to discuss policy proposals.

In this paper we modify the original set-up by Rothschild and Stiglitz by introducing capacity constraints. With this rather innocuous modification, equilibrium in pure strategies can be established. Interestingly, these contracts are the same as the original ones proposed by Rothschild and Stiglitz: high risks obtain full insurance at their fair premium, while low risks are underinsured at their fair premium.

Capacity constraints on the firm’s side can arise for different reasons. We consider two of these in the paper, both of which are connected to the insurer’s limited capital:

1. Solvency regulation: for a given size of capital, only a finite number of risks can be added to the portfolio of the insurer, as otherwise, depending on how the solvency requirement is specified, the ratio of premium income to capital, or the ratio of risk exposure to capital, exceeds a given size.

2. Bankruptcy probability: even if no regulation is installed, the increase in bankruptcy probability due to the acquisition of further contracts will make each policy less attractive to the potentially insured, thus in effect restricting the firm into arbitrarily selling many contracts.

If firms cannot arbitrarily serve many customers, the dynamics that led to the non-existence of the original paper no longer work. If all firms offer the Rothschild-Stiglitz contracts, and a single firm intends to deviate, it cannot be sure that it will attract the desired mix of risk types it requires to make the deviation profitable. Such a firm attracts more customers than it can serve, so rationing will occur. Parties wishing to take out high risk insurance will gain much more from the deviating offer than will those taking out low risk insurance, whereas in previous contracts there was little differentiation between the two levels. Therefore, even if the deviation is intended to attract low risk parties, only high-risk parties will arrive at this firm. This in turn makes deviation unattractive, and thus stabilises the Rothschild-Stiglitz contracts as equilibrium outcomes.

This model adds confidence to the use of the Rothschild-Stiglitz outcome as a prediction for markets where asymmetric information is present. This applies not only to the insurance markets, but also to other economic circumstances, such as the credit market, where the creditor better knows the profitability of a project than the bank does.

Competitive Insurance Markets under Adverse Selection and Capacity Constraints

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July 1999

Abstract

Ever since the seminal work by Rothschild and Stiglitz (1976) on competitive insurance markets under adverse selection the equilibrium-non-existence problem has been one of the major puzzles in insurance economics.

We extend the original analysis by considering firms which face capacity constraints due to limited capital. Two scenarios are considered, if the demand at any insurer exceeds the capacity: Either consumers are rationed, or they are served, but the insurer faces a larger risk of bankruptcy. We show under mild assumptions that a pure strategy equilibrium exists, where every consumer buys his appropriate Rothschild-Stiglitz contract.

JEL classification: C72, C78, D82, G22

Keywords: Insurance Markets, Asymmetric Information, Competitive Equilibrium, Capacity Constraints

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Acknowledgements: We thank Klaus Schmidt for helpful comments.

1 Introduction

Ever since the seminal work by Rothschild and Stiglitz (1976) on competitive insurance markets under adverse selection the equilibrium-non-existence problem has been one of the major puzzles in insurance economics. The origin of this problem lies in the fact that only zero profit making separating contracts can constitute an equilibrium in the sense of Rothschild and Stiglitz, while in some cases a single pooling contract or a pair of cross-subsidizing contracts may be preferred by everyone and will therefore upset the Rothschild-Stiglitz equilibrium contracts.

There are many approaches to this problem in the literature. One way out of it is to allow firms to have mixed strategies (Dasgupta and Maskin, 1986), however the economic interpretation of this modification is not clear. Another possibility is to propose different equilibrium concepts (Wilson, 1977; Miyazaki, 1977; Spence, 1978; Riley, 1979), which however lack a game-theoretic foundation. There exist a few attempts of introducing some form of dynamics explicitly in a non-cooperative model (Jaynes, 1978; Hellwig, 1987, 1988; Asheim and Nilssen, 1997).[1] These models have contributed to our understanding of insurance markets to a large extent, but it is probably fair to say that an overall agreed upon model has not yet been derived.

In this paper we want to add one aspect to the discussion of the non-existence problem which so far has not received any attention in the insurance literature, and which lies at the heart of the non-existence problem: If a deviating firm offers a new set of contracts, who chooses these contracts? So far it was always assumed that any new contract offer can potentially serve the whole market, so the pooling contract which destabilized the Rothschild-Stiglitz equilibrium is taken by everyone, and the pair of cross-subsidizing contracts attracts all high and low risks.

[1] Recently, an explicitly dynamic evolutionary model of the insurance market has been proposed (Ania et al., 1998). If firms copy profit making contracts and experiment with their own contracts locally, the unique long run outcome is that all firms offer the Rothschild-Stiglitz contracts.

Here we assume instead that firms face capacity constraints. In that case it is no longer guaranteed that a new offer may attract a fair selection of the market. Indeed, the distribution of risk types applying for a (deviating) contract at a given firm is now determined endogenously. Capacity constraints on the side of the firms may arise for different reasons. We consider two, which are both consequences of limited capital available to any single insurer:

First, solvency regulation: For a given size of capital, only a finite number of risks can be added to the portfolio of the insurer, as otherwise, depending on how the solvency requirement is specified, the ratio of premium income to capital or the ratio of risk exposure to capital exceeds a given size. Second, bankruptcy probability: Even if no regulation is installed, the increase in bankruptcy probability due to the acquisition of further contracts will make each policy less attractive to the potential insured, thus in effect restricting the firm to sell arbitrary many contracts (see also Rees et al., 1999).[2] We consider both forms of capacity constraints in turn.[3]

Under capacity constraints, our main result is that the Rothschild-Stiglitz (RS) contracts are stable, even if they are not equilibrium contracts of the original game. The intuition is most easily grasped for the case of fixed insurance capacity. For an illustration, consider pooling contracts which were used to destabilize the RS contracts in the original paper. If the new contract is supposed to also attract low-risk types and if the proposer intends to realize a strictly positive profit, the coverage of the low-risk type must increase compared to the RS contract. Observe now that the high-risk type's incentive compatibility constraint is binding under the RS allocation and that he bene-

[2] In a recent article, the ECONOMIST (16th January 1999, 'The Insurance Bust') argues that there is too much capacity, i.e., capital, in the insurance market which lead to falling premiums. The author recommends insurers to pay back capital to the shareholders.

[3] Another argument why a firm might not serve the whole market could be the mere size of the firm, the number of employees, the size of the computer system, etc., which makes it difficult to process more than a given number of policies.

fits strictly more from an increase in the coverage (due to the single-crossing property). Hence, the high-risk type's utility will increase strictly more under the deviating contract. As a consequence, high risks are prepared to endure a more severe rationing in case a firm's capacity constraint becomes binding. This intuitive property can now be applied to make any deviating offer, even with a pair of contracts, unprofitable as it simply will not assure the firm the desired mix of types.

The rest of this paper is organized as follows. Section 2 introduces the two variants of the model. The scenario with limited capacity is solved in detail in Section 3. The argument is extended to the scenario with a possibility of bankruptcy in Section 4. Other equilibria are discussed in Section 5, before we conclude in Section 6 by relating our approach to recent technical contributions on contractual markets with adverse selection.

2 The Model

The insurance market is populated by $F = \{1, ..., F\}$ risk-neutral firms, each with a fixed capacity of $k_f > 0$. In Scenario A the capacity of an individual form k_f represents a physical constraint on the number of contracts which can be obtained from this firm. In Scenario B the firm is physically capable of signing more than k_f contracts. However, if the number of contracts exceeds this threshold, the risk of bankruptcy become non-negligible. On the demand side there are $N = \{1, ..., N\}$ customers. We assume that $k_f < N$ holds for all $f \in F$ and that $\sum_{f \in F'} k_f \geq N$ holds for all sets $F' = F/\{f\}$ with $f \in F$. Hence, no single firm can serve the whole market, while all but one firm together are sufficient to serve all customers.[4] The customers face a risk of loosing a sum

[4]This assumption is a simplification of the capacity problem due to limited capital. Given any amount of capital, k_f will in general depend on the form of the contracts offered and the types of the insured buying these contracts. However, we conjecture that making k_f an endogenous variable will not change the result. The important assumption we require is that there are enough firms to serve the least-cost separating contracts without incurring capacity constraints, while any single firm will run

S. An individual may have either a high risk probability of π_H or a low risk probability $\pi_L < \pi_H$. The respective risk type of customer n is denoted by $t_n \in T = \{L, H\}$. All individuals have the same von Neuman-Morgenstern utility function $U(w)$. Below we will invoke a further assumption on the severeness of the capacity constraint to support an equilibrium.

The game is modelled as follows:

Stage 0: The risk type of each individual is chosen by nature. Each person has the chance of γ_H $(1-\gamma_H)$ to be a high (low) risk type. This draw is taken independent across individuals, so that overall the expected number of high risks is $\gamma_H N$.

Stage 1: Firm f, $f = 1, ..., F$, sets a menu of contracts $\{\omega_1^f, \omega_2^f, ..., \omega_k^f\}$ where each ω_l^f specifies a premium P_l^f and a net indemnity payment I_l^f.

Stage 2: Each customer either chooses a firm f and a contract ω_l^f or decides not to visit any firm.

If the number of customers choosing firm f, which we denote by n_f, does not exceed k_f, then each customer obtains his desired contract. The expected utility if the chosen contract ω specifies the premium P and the net indemnity I is abbreviated by

$$U_t^E(\omega) = (1-\pi_t)U(w-P) + \pi_t U(w-S+I),$$

where the risk type t is either H or L. If n_f is larger than k_f, the firm runs into capacity constraints. As discussed above, we consider two scenarios:

Scenario A:

Here capacity constraints, which arise e.g. as a consequence of regulation, imply that firms are not able to serve more than k_f customers. If $n_f > k_f$ customers queue for a contract at firm f, the firm applies a rationing scheme. We assume that rationing occurs randomly over all applicants. Hence, each individual is rationed with probability $\rho_f = 1 - \min\{1, k_f/n_f\}$. If some customers do not obtain a contract, Stage 3 follows.

into severe capacity problems if it tries to serve a significant fraction of the market.

Stage 3: The customer can either choose to remain uninsured or he can visit another firm f' which still has free capacity available and pick a contract $\omega_l^{f'}$ from the menu of contracts of firm f'. The search for a new firm is costly. We measure these costs in utility units and assume that approaching another firm results in costs $u > 0$, which are independent of the risk type.[5] If the customer has chosen a firm f' where again demand exceeds supply, and he did not obtain a contract, Stage 3 is repeated.

Scenario B:

In this scenario all customers are served. However, due to the limited capital available, firms face the risk of going bankrupt. This bankruptcy risk depends on the number of customers, and is denoted by a function $\alpha(\rho_f)$ which depends positively on ρ_f with $\alpha(0) = 0$. For a given risk of bankruptcy α and a contract ω with premium P and net indemnity I, we abbreviate the expected utility by

$$
\begin{aligned}
U_t^R(\omega, \alpha) &= (1-\pi_t)U(w-P) + \pi_t(1-\alpha)U(w-S+I) + \pi_t \alpha U(w-S-P) \\
&= U_t^E(\omega) - \alpha\pi_t \left[U(w-S+I) - U(w-S-P)\right],
\end{aligned}
$$

where the risk type t is either H or L.

3 Rothschild-Stiglitz Contracts as Equilibrium Contracts

In this section we consider in detail Scenario A where firms have (strictly) limited capacity. The extension to Scenario B is given in the next section. Our main result is that we can support Rothschild-Stiglitz contracts under some reasonable assumptions on the costs u and the structure of the economy.

[5]Note that we assume that the first visit is free. Our results continue to hold if the costs of a first visit do not exceed the difference between the utility derived by the low-risk type under his Rothschild-Stiglitz contract and his utility without insurance.

Before showing this, let us recall the pair of RS contracts, which are the least-cost separating contracts. They are uniquely derived by the following conditions.

The RS contract ω_H^{RS} for the high-risk type specifies full coverage with $I_H^{RS} = S - P_H^{RS}$, while the premium is determined by the zero-profit condition for the risk-neutral insurer as $P_H^{RS} = \pi_H S$. We denote the realized (expected) utility by $U_H^{RS} = U_H^E(\omega_H^{RS})$.

The RS contract ω_L^{RS} for the low-risk type is chosen to maximize $U_L^E(\omega)$ subject to the firms' participation constraint $P \geq I\pi_L/(1-\pi_L)$ and the high-risk type's incentive compatibility constraint $U_H^{RS} \geq U_H^E(\omega)$. It can be shown that both constraints become binding, while the contract provides less than full coverage with $I_L^{RS} < S - P_L^{RS}$. We denote the utility by $U_L^{RS} = U_L^E(\omega_L^{RS})$. Denote $U_t^0 = U_t^E(0,0)$ for the expected utility without insurance coverage. It is clear that U_L^{RS} exceeds the expected utility from staying uninsured U_L^0.

Throughout this paper we focus on (subgame-perfect) equilibria where firms play pure strategies in Stage 1. As the number of firms is finite, the market will always clear after a finite numbers of repetitions of Stage 3. Hence, once contracts are in place, we face a finite (continuation) game, which therefore has always an equilibrium in (possibly mixed) strategies.

As indicated in the introduction, the novel feature of our approach to the insurance market is that the distribution of types applying for a possibly deviating offer will be determined endogenously. Customers evaluate a new proposal in comparison to what they can get at other firms and decide whether it is worthwhile to go for a better contract even at the risk of being rationed, which is associated with strictly positive costs. More precisely, we intend to abandon the particular specification in the Rothschild-Stiglitz environment that a deviator can either serve the whole market or can always assure himself a fair selection of risks. For our result to hold we require that the capacity problem is sufficiently severe. This contains three elements:

First, the search costs u should not be negligible. Suppose otherwise: For search costs of zero there are no direct costs of being rationed by a firm which makes a deviating offer. Thus again, everyone might try to obtain such a contract which in turn makes the distribution of risks equal to the distribution in society.

Second, search costs u should not be too high. Otherwise, consumers only have one possibility to search around. If they do not receive a contract at their first firm, they prefer to stay uninsured. As high risks suffer more from being uninsured, by offering a contract which is going to be rationed firms might deter high risks from choosing this contract.

Third, the capacity of a single firm must be sufficiently low compared to the economy. If not, any firm would by offering a deviating contract attract maybe not all of the population, but nearly all. This would make the risk distribution more and more favorable.

We next provide a formalization of these assumptions. (A.1) assures that rationed players prefer to newly approach an insurer to sign their respective RS contract instead of staying uninsured.

Assumption (A.1)

$$U_t^{RS} - u > U_t^0 \quad \text{for} \quad t \in \{L, H\} \tag{1}$$

We next derive a combined requirement which puts a lower boundary on the search costs u, while assuring that capacity is sufficiently dispersed. Let $k^M = \max_{f \in F} k_f$ be the maximum capacity of a single firm. Recall next that at Stage 0 the type of an individual is determined randomly. Therefore, the true distribution of types in the population is unknown to all market participants.[6] Suppose that individual n expects that all high-risk individuals choose to visit a particular firm f with the maximum capacity k^M, while all low-risk individuals pick different firms. This allows us to calculate an expected rationing probability for individual n if he chooses firm f as well. We denote this probability by ρ^M. [7] Let U_H^P be the expected utility of a high risk under

[6] By the law of large numbers this uncertainty vanishes as the number of individuals increases.

[7] Formally, suppose that there are N_H high risks visiting firm f in addition to individuum i. This

a full insurance contract at the fair pooling premium, i.e. the utility from a contract ω satisfying $P\left[\gamma_H(1-\pi_H)+(1-\gamma_H)(1-\pi_L)\right]=I\left[\gamma_H\pi_H+(1-\gamma_H)\pi_L\right]$ and $I=S-P$.

Assumption (A.2).

$$(1-\rho^M)U_H^P+\rho^M\left(U_H^{RS}-u\right)<U_H^{RS}. \tag{2}$$

Assumption (A.2) implies that if individuum n is a high risk, he is better off buying his RS contract than queueing at a firm together with all other high risks for the full insurance contract at the pooling premium, and in case he is unlucky in the draw, buying the RS contract in the next round. That is the expected rationing at one single firm is sufficiently severe if all high risks are expected to turn up.

Note that (A.2) holds for a given level of u if the number of customers N is sufficiently large and if the capacity is sufficiently dispersed among firms. To see this, consider a sequence of economies where the expected fraction of high-risk types $0<\gamma_H<1$ is kept fixed together with the maximum capacity k^M of a single insurer. If the size of the economy N increases, ρ^M will converge to 1, so that (2) is satisfied for any positive costs $u>0$.

Under (A.1)-(A.2) we can now prove our main result.

Proposition 1. *Under Scenario A and if Assumptions (A.1)-(A.2) hold, there exists an equilibrium where any costumer $n\in N$ realizes his respective RS contract at stage 2.*

Proof:

We claim that one possible equilibrium strategy of firms is to offer the two RS contracts each. Given that these contracts are offered, the customers face a coordination problem. We solve the problem by ordering individuals as follows. Customer n turns to firm f where the index f satisfies $\sum_{f'=1}^{f}k_{f'}\geq n$ and $\sum_{f'=1}^{f-1}k_{f'}<n$. He demands the

gives rise to the rationing probability $\rho^M(N_H)=1-min\left\{1,k^M/(N_H+1)\right\}$. The probability that there are exactly m high risks in the population is equal to $Pr(N_H=m)=\begin{pmatrix}N-1\\ m\end{pmatrix}\gamma_H^m(1-\gamma_H)^{N-1-m}$, such that $\rho^M=\sum_{m=0}^{N-1}Pr(N_H=m)\rho^M(N_H)$.

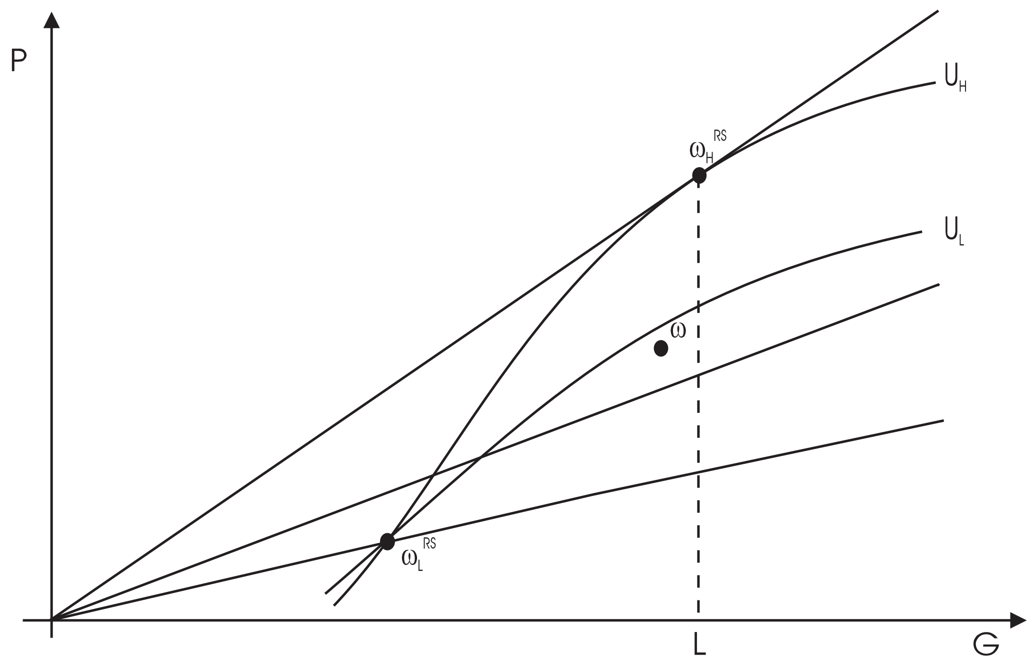

Figure 1: Rothschild-Stiglitz contracts.

high- or low-risk contract, depending on his type t_n. First note, that given the contract offers, no customer has an incentive to deviate, as all are served with the best possible contract on offer and no rationing occurs. Moreover, all firms make zero profit with these contracts. To support the resulting allocation as an equilibrium, it thus remains to specify strategies if a single firm deviates to a different menu of contracts in order to show that there are no profitable deviations.[8]

Recall that in the standard discussion by Rothschild and Stiglitz, a pooling profit-making contract might be just such a profitable deviation. This is shown in Figure 1.

On the two axes are the premium and the gross indemnity $G = I + P$. Full insurance is obtained at the vertical line which is given by $G = S$. The three straight lines denote the zero-profit lines, if only high risks (the top line), only low risks (the bottom line),

[8]We do not specify the whole equilibrium strategy of the customers, which depends on his type realization at Stage 0 and on all possible contract offers by the firms. Instead, we only discuss that part of the strategies which conditions on the relevant contract offers.

or a mixture of the two risks buy such a contract. Recall that the least-cost separating contracts are denoted by ω_L^{RS} and ω_H^{RS}. As drawn, a pooling contract like ω would be preferred by everyone and would be strictly profitable if it can assure a fair selection. It thus makes the least-cost separating contracts not an equilibrium of the original game.

Coming back to our model, we must at this stage consider any possible deviation by some firm $\bar{f}$ consisting of a menu $\{\bar{\omega}_1^{\bar{f}}, \bar{\omega}_2^{\bar{f}}, ..., \bar{\omega}_k^{\bar{f}}\}$ of contracts. Note that rationing, if it occurs, is the same for all customers of a single firm, independent of which contract they have chosen. Therefore we can, without loss of generality, reduce the menu to two contracts, one for each type, which are denoted by $\bar{\omega}_H$ and $\bar{\omega}_L$.[9] Moreover, these contracts must be incentive compatible. To support the asserted equilibrium, we must find for any deviating pair $(\bar{\omega}_H, \bar{\omega}_L)$ offered by a single firm a continuation equilibrium which renders this offer unprofitable.

Recall that an individual customer can only observe the realization of his own type and that each realization represents an independent random draw. As a consequence, the length and the constitution of the queue forming at an individual firm represents a random variable. This gives rise to an expected rationing probability, which we denote by ρ. If there is rationing at $\bar{f}$ in Stage 2, we specify that individuals who are not allocated a contract and who choose to visit another firm in Stage 3 will again perfectly resolve the coordination problem in Stage 3 and will thus realize their respective utilities $U_t^{RS} - u$ with one period delay. (Note that there is enough capacity as the remaining $F - 1$ firms offer the RS menu and as $\sum_{f \in F/\{\bar{f}\}} k_f \geq N$.) The expected utility of a type t who visits the deviating firm is therefore equal to

$$U_t^D(\bar{\omega}_t, \rho) = (1 - \rho)U_t^E(\bar{\omega}_t) + \rho(U_t^{RS} - u). \tag{3}$$

It is obvious that we can restrict attention to offers which are supposed to attract (some) low-risk types as otherwise there would be no scope to increase profits above

[9] With a slight abuse of notation this also covers the case where only a single contract is offered. In this case set $\bar{\omega}_L = \bar{\omega}_H$.

zero. This already implies $U_L^E(\bar{\omega}_L) \geq U_L^{RS}$. If expected profits are positive, this can only hold if this offer involves more coverage $\bar{I}_L > I_L^{RS}$ and also a higher premium $\bar{P}_L > P_L^{RS}$ than the RS contract for the low-risk type. We claim that for any menu satisfying these conditions for the low-risk contract there exists a (continuation) equilibrium where only high-risk types turn up at the deviator.

First, note that the high risks strictly prefer the contract designed for the low risks to their RS contract, i.e. $U_H^E(\bar{\omega}_L) > U_H^{RS}$. To see this, recall that the RS contracts were chosen such that $U_H^E(\omega_L^{RS}) = U_H^{RS}$. As the deviating menu satisfies $U_L^E(\bar{\omega}_L) \geq U_L^{RS}$ and $\bar{I}_L > I_L^{RS}$, the asserted strict inequality is immediate from the single-crossing property, which was illustrated in Figure 1. As a consequence, it holds that $U_H^E(\bar{\omega}_H) > U_H^{RS}$, while $U_H^D(\bar{\omega}_H, \rho)$ is strictly decreasing in the expected rationing probability ρ.

We show now that we can specify a continuation equilibrium where for a value $0 < \phi < 1$ all individuals $n \in \{1, .., N\}$ visit $\bar{f}$ in Stage 2 with probability ϕ if and only if they are of the high-risk type. With probability $1 - \phi$ a high-risk type turns to a firm where he obtains his RS contract with probability one. Low-risk types choose a firm other than $\bar{f}$ with probability one in order to buy their RS contract. (As any firm, including $\bar{f}$, is dispensable to serve the whole market, customers can indeed perfectly resolve their coordination problem when choosing a firm other than $\bar{f}$.) For a given ϕ we can calculate the expected rationing probability $\rho(\phi)$ for an individual who contemplates choosing firm $\bar{f}$.[10] Note that $\rho(\phi)$ is strictly increasing in ϕ.

We show first that there exists a unique $0 < \phi < 1$ satisfying

$$U_H^D(\bar{\omega}_H, \rho(\phi)) = U_H^{RS}. \tag{4}$$

That is, there exists a probability ϕ smaller than one at which high risks are indeed indifferent between joining the queue and choosing the RS contract. Recall that $U_H^D(\bar{\omega}_H, \rho)$ is strictly decreasing and continuous in ρ, while $\rho(\phi)$ is strictly increasing and contin-

[10]Formally, $\rho(\phi) = 1 - \sum_{m=0}^{N-1} \begin{pmatrix} N-1 \\ m \end{pmatrix} (\phi\gamma_H)^m (1 - \phi\gamma_H)^{N-1-m} min\left\{1, k_{\bar{f}}/(m+1)\right\}$.

uous in ϕ. Moreover, it holds that $U_H^D(\bar{\omega}_H, \rho(0)) > U_H^{RS}$. It remains to prove that $U_H^D(\bar{\omega}_H, \rho(1)) \leq U_H^{RS}$. The argument is by contradiction. Suppose there exists a profitable deviation such that $U_H^D(\bar{\omega}_H, \rho(1)) \geq U_H^{RS}$, i.e., where every high-risk individual could turn up and (in expectancy) realize not less than by buying instead the RS contract. Note next that $\rho(1) = \rho^M$, which was defined before invoking (A.2). To ensure that the deviating firm $\bar{f}$ still realizes nonnegative payoffs if it is visited by all available high-risk types, $U_H^E(\bar{\omega}_H)$ must be bounded from above by U_H^P, as defined before (A.2). With these preliminary remarks, it follows immediately that the claim $U_H^D(\bar{\omega}_H, \rho(1)) \geq U_H^{RS}$ contradicts (A.2).

It now remains to show that, given the uniquely chosen $0 < \phi < 1$ satisfying (4), it does not pay any low-risk type to visit $\bar{f}$. Recall first that by (A.1) and our specification of continuation strategies at Stage 3, an individual implements his RS contract at Stage 3 if he was rationed when visiting $\bar{f}$ at Stage 2. It therefore remains to show that $U_L^D(\bar{\omega}_L, \rho(\phi)) \leq U_L^{RS}$. This again holds if $U_H^E(\bar{\omega}_H) - U_H^{RS}$ is not below $U_L^E(\bar{\omega}_L) - U_L^{RS}$, which after substitution of $U_H^E(\omega_L^{RS}) = U_H^{RS}$ and $U_H^E(\bar{\omega}_H) \geq U_H^E(\bar{\omega}_L)$ holds if

$$U_L^E(\bar{\omega}_L) - U_L^E(\omega_L^{RS}) \leq U_H^E(\bar{\omega}_L) - U_H^E(\omega_L^{RS}).$$

But this must be satisfied even strictly due to the single-crossing property, $\bar{I}_L > I_L^{RS}$, and $\bar{P}_L > P_L^{RS}$.[11]

Q.E.D.

Proposition 1 has a simple intuition which comes out most clearly if we suppose that a deviating offer intends to attract a mixed set of types. To realize a profit with low-risk types, the offer must specify a higher coverage than the RS contract designated for these types. By the single-crossing property, high-risk types gain more under the new offer

[11] Note that $U_L^E(\bar{\omega}_L) - U_L^E(\omega_L^{RS}) \leq U_H^E(\bar{\omega}_L) - U_H^E(\omega_L^{RS})$ is equivalent to

$$(\pi_H - \pi_L)[U(w - \bar{P}_L) - U(w - P_L^{RS})] + (\pi_L - \pi_H)[U(w - S + \bar{I}_L) - U(w - S + I_L^{RS})] \leq 0.$$

than low-risk types. They are thus prepared to accept a higher (expected) rationing probability than low-risk types who would not apply for the deviating offer at this level of congestion.

4 Insurer Facing Bankruptcy Risk

The argument used for the proof of Proposition 1 carries over to Scenario B. Here rather than being rejected by an insurer if she reaches the capacity constraint, consumers face a risk of not being served in case of damage due to the possibility of bankruptcy. For a full analysis of the game we would have to specify an insurer's payoff function under the risk of bankruptcy, while it would also be necessary to endogenize the bankruptcy function $\alpha(\cdot)$. Some of the analytical complications involved with this formulation have been addressed in Rees et al. (1999). In this section we restrict ourselves to indicate how the main argument of this paper can be extended if the risk of bankruptcy instead of the possibility of being rationed is used to endogenize the distribution of types for a deviating offer.

Take a particular firm f offering a single deviating contract $\omega = (P, I)$. For any expected bankruptcy probability α with $0 \leq \alpha < 1$ which will occur at this firm along the continuation equilibrium, we can write:

$$U_t^R(\omega, \alpha) = U_t^E(\tilde{\omega}(\alpha))$$

where $\tilde{\omega}(\alpha) = (P, \tilde{I}(\alpha))$ is such that

$$(1 - \alpha)U(w - S + I) + \alpha U(w - S - P) = U(w - S + \tilde{I}(\alpha)).$$

Both types are indifferent between obtaining contract ω at a firm which has bankruptcy probability α or obtaining $\tilde{\omega}(\alpha)$ at a safe insurer. Due to risk aversion it follows that

$$(1 - \alpha)(I) + \alpha(-P) > \tilde{I}(\alpha).$$

Assume that only low risks turn up at the deviating firm. Then it's expected profit per person will be equal to $P - \pi_l(1-\alpha)(I+P)$ which is smaller than $P - \pi_l(\tilde{I}(\alpha) + P)$. The last term is the expected profit of an insurer who never goes bankrupt offering $\tilde{\omega}(\alpha)$ to low risks only. As argued in the proof of Proposition 1, we can restrict attention to contracts ω which satisfy $U_t^E(\omega) \geq U_t^{RS}$ for both types. If at the same time the contract offer should assure a positive payoff for the firm, it follows that $P > P_l^{RS}$ and $\tilde{I}(\alpha) > I_l^{RS}$, which implies $I > I_l^{RS}$.

Recall next that the risk of bankruptcy was strictly increasing in the number of sold contracts whenever these exceed the capacity k_f. For a given contract ω and a type t we determine a critical bankruptcy risk $\alpha_t(\omega)$ satisfying

$$\alpha_t(\omega) = \frac{U_t^E(\omega) - U_t^{RS}}{\pi_t \left[U(w - S + I) - U(w - S - P)\right]}.$$

Hence, if the bankruptcy risk at f offering ω is equal to $\alpha_t(\omega)$, type t is just indifferent between implementing ω at this level of bankruptcy risk and implementing his respective RS contract without any risk of bankruptcy. We show next that $\alpha_H(\omega) > \alpha_L(\omega)$. To see that this holds, note that the inequality $\frac{U_H^E(\omega) - U_H^{RS}}{\pi_H} > \frac{U_L^E(\omega) - U_L^{RS}}{\pi_L}$ transforms from $U_H^{RS} = U_H^E(\omega_L^{RS})$ to

$$\frac{U(w-P) - U(w - P_L^{RS})}{\pi_H} > \frac{U(w-P) - U(w - P_L^{RS})}{\pi_L}.$$

This holds as $U(w-P) - U(w - P_L^{RS}) < 0$ follows from $P \geq P_L^{RS}$. Hence, for any possibly profitable deviation, high risks are prepared to accept a higher risk of bankruptcy than low risks. In other words, high risks are more likely than low risks to visit the deviating firm even if the (expected) demand already exceeds the capacity threshold at which there would be no risk of bankruptcy. This property was all that was used to prove Proposition 1.

5 Discussion of the Equilibrium

We now return to Scenario A. So far we have constructed a specific continuation equilibrium for each possible deviation to support the RS allocation as an equilibrium. However, this does not need to be the only equilibrium outcome of the game.

In the proof of Proposition 1 it was required that customers can perfectly coordinate their choices along the equilibrium path. In contrast, if buyers were to play mixed strategies, they would suffer from a coordination problem. For a one-shot set-up without adverse selection Peters (1984) shows that the possibility of coordination failure implied by mixed strategies (both on and off the equilibrium path) leads to multiple equilibria where -in his case- sellers can realize strictly positive expected profits even if they strictly outnumber buyers. In our model, as nature randomly chooses a buyer's type, coordination failure may occur even if all buyers play pure (type-dependent) strategies.

We can, however, obtain uniqueness of the RS allocation under quite reasonable restrictions on strategies. Indeed, uniqueness of the RS allocation follows if the following three requirements are imposed: First, insurers have free capacity in equilibrium. Second, there is at least one idle firm, which from a conceptual level is equivalent to assuming that free entry is possible. Third, no consumer is made worse off if an additional contract is offered.

To formalize these requirements, let us first define by $\Omega = \{\omega_j^f, f \in \{1, ..., F\}, j \in \{1, ..., k\}\}$ the set of all contract offers by the different firms. Now we can define the following property:

Definition: *We call the set of equilibrium strategies consistent, if the following holds: Consider any two sets of contracts Ω_1 and Ω_2, where the allocation given by the continuation equilibrium induced by Ω_1 is a feasible outcome if the set Ω_2 is offered. Then in the continuation equilibrium induced by Ω_2 no one will be made worse off than in Ω_1.*

Thus consistency implies that if, for example, a firm which was idle in Ω_1 with probability 1, offers different contracts leading to Ω_2, then no buyer should be made worse off by the additional choice they now have.

Proposition 2. *In any equilibrium where each firm f has no more than $k_f - 1$ customers with probability 1, and where at least one firm stays idle with probability 1, and where the strategies of the consumers are consistent, customers realize their respective RS contracts.*

Proof: As the proof works quite similar to the original proof in Rothschild and Stiglitz, and the analysis on Bertrand competition under capacity constraints, we only show the next claim rigorously and leave the rest of the proof to the reader:

Claim: In equilibrium, no firm can offer a contract with which it makes a strictly positive profit.

Proof: Assume otherwise. Then there exist firms which sell contracts to some risk type or a distribution of risks with which they make a profit. It is easy to see that if the contract makes a profit with the high risks, the idle firm could by offering a slightly better contract attract some individuals and make a profit.[12] Therefore assume that profit is made with the low risks. Call one such profit making contract ω_L. Due to the single crossing property, there exists a contract ω'_L in the vicinity of ω_L which the low (high) risks do (do not) prefer to ω_L, and which still makes a profit if only low risks buy this contract. Due to the assumption that there is no rationing in equilibrium even if one more person queues at any active firm, the high risks obtain an expected utility which is larger or equal to that if they were to buy contract ω_L. If the idle firm now offers contract ω'_L, no high risk type will turn up at this firm in the continuation equilibrium, due to the assumption of consistency. However, due to subgame perfection it cannot be that with probability 1 no low risk will choose this firm. Therefore by offering ω'_L the idle firm would make a strictly positive expected profit, which violates the equilibrium assumption.

[12] This part of the proof works even without invoking the consistency and the no-rationing requirements.

Having shown that no contract can make a profit, the next steps in the proof work similar to the analysis of Rothschild and Stiglitz. First, no contract can make a loss in equilibrium as otherwise the firm would make a loss overall. Therefore the contract for each type must lie on the zero profit line of this respective type. One can show that the incentive constraint binds from the high risks to the low risks, therefore the high risks obtain their full insurance contract, while the low risks obtain the according incentive compatible contract on their zero profit line, which together constitute the RS contracts.[13]

Q.E.D.

The requirements leading to the uniqueness asserted in Proposition 2 do not seem to be too restrictive for the insurance market. However, we would like to stress that other equilibria are conceivable, in particular some, where rationing occurs.

6 Conclusion

We showed how an existence result in pure strategies can be obtained for an insurance market if there is limited capacity to write contracts, which moreover is sufficiently dispersed among the competing firms. A family of (least-cost separating) Rothschild-Stiglitz contracts cannot be destabilized by a supposedly pooling deviation as the congestion resulting from applying high-risk types, who will always have more to gain, will make low-risk types strictly prefer to take up their prescribed equilibrium contract at one of the other firms.

This paper is not the first to use rationing and congestion as an equilibrating device in

[13] The equilibrium strategies we used in Proposition 1 satisfy all requirements of Proposition 2 apart from the first one: All firms have no more than $k_f - 1$ customers with probability 1. However, it is easy to generalize our proof to include this property, if we modify the assumption made on the overall capacity of the market to: $\sum_{f \in F'} k_f \geq N + F$ holds for all sets $F' = F / \{f\}$ with $f \in F$. Thus an equilibrium exsits which satisfies the conditions of Proposition 2.

markets with adverse selection. However, the literature has so far restricted attention to atomistic markets with typically a continuum of buyers and sellers who may trade at most once. The Walrasian approach of Gale (1992, 1996) takes a one-shot perspective where individuals who are rationed at a first stage will subsequently realize some exogenous reservation value. Moreover, he does not solve a fully developed game as in our paper where contracts are offered by one side and subsequently accepted by the other side of the market. Instead, he considers an abstract market for contracts where each separate contract is traded in a different submarket. To reduce the multiplicity of equilibria he introduces refinements which restrict players' beliefs at unpopulated submarkets. As contracts are offered by the uninformed side in our game and as we consider a sequential time structure, these issues do not arise in our setting.

Congestion has also been applied as an equilibrating device in Inderst (1998) and in Inderst and Müller (1999a/b). These papers study a search-market environment where (expected) delay to trade is used as a separating device in product and labor markets. In contrast to the present paper, this work uses again an axiomatic equilibrium concept which was pioneered by Moen (1997) under complete information.

As remarked above, the possibility of coordination failure among customers creates space for equilibria where other contracts than those specified in the RS allocation will be implemented. In particular, firms may be able to realize positive expected payoffs even though each individual seller is negligible. This issue has been discussed under complete information in Peters (1984). In Inderst (1999) two ways are discussed how prices will converge to the unique competitive price in a setting with complete information where any seller is dispensable. First, increasing the number of buyers (while keeping the structure of the economy fixed) facilitates coordination as it allows buyers to predict more accurately the congestion prevailing at an individual seller. Secondly, coordination failure becomes less serious if the costs of visiting another seller decrease. Further work is required to see whether these arguments can be exploited to put boundaries on the set of equilibrium allocations of the insurance model analyzed in this paper.[14]

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[14] In Inderst and Wambach (1999) we study a labor market with adverse selection where a finite number of firms with limited vacancies compete for a continuum of workers. Assuming that rationed workers will subsequently stay unemployed, we can indeed show that any equilibrium must support the RS allocation.

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