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**RE-EMPLOYMENT PROBABILITIES
AND RETURNS TO MATCHING**

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LABOUR ECONOMICS

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ABSTRACT

Re-Employment Probabilities and Returns to Matching*

The assumption of constant returns in the matching function, embodied in most bilateral search models, is crucial to ensure the uniqueness of the unemployment rate along a steady state growth path. This Paper explores whether this is an acceptable assumption by estimating individual re-employment probabilities on a sample of entrants into unemployment. This is done by applying hazard models to survey data on both completed and uncompleted unemployment durations. The hypothesis of constant returns to matching is not rejected, on the basis of the evidence that the job-finding hazard only depends on local labour market tightness and it is independent of its size.

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NON-TECHNICAL SUMMARY

Search models of the labour market hinge on the existence of a hiring or matching function that describes the technology of the job formation process by relating hirings to unemployment and vacancies. The equilibrium properties of such models depend crucially on the characteristics of the matching technology. In particular, the assumption of constant returns to scale in the matching function ensures a constant unemployment rate along a balanced-growth path. Multiple (rankable) equilibria arise instead when the matching function exhibits increasing returns. Multiplicity raises obvious policy questions. In particular, with multiple equilibria, even temporary policies may pull the economy out of an inefficient unemployment level. Also, multiplicity has the potential to explain why economies may get stuck at high levels of unemployment, even though the initial adverse shock was only temporary, like the oil price rises of the 1970s. Finally, multiplicity is closely related to hysteresis of unemployment in the sense that, with hysteresis, equilibrium unemployment is always close to the actual rate.

Given the implications of increasing versus constant returns on theoretical grounds, it is insightful to explore what the data can reveal about this issue. The aim of this Paper is to test the empirical relevance of the constant returns hypothesis, by estimating individual hazard functions on a sample of unemployment entrants.

For this purpose, I use a well known link between aggregate matching conditions and individual job-finding hazards. The hazard rate denotes the probability of a transition out of unemployment within some small time interval, conditional on the worker still being unemployed when the interval started. If the underlying matching technology displays constant returns to scale, the hazard rate for an unemployed worker (and, conversely, unemployment duration) should only depend on the degree of labour market tightness, measured by the vacancy/unemployment ratio and not on the absolute size of the pool of job-seekers. If instead matching displays increasing returns, the hazard rate should depend positively on the size of the market, once labour market tightness is controlled for. This is precisely the hypothesis that is tested on individual duration data.

Compared to aggregate matching functions, hazard function specifications have the main advantage of being rather flexible. They allow for a wide spectrum of functional forms for duration distributions and control for a number of individual characteristics whose importance is only implicit in an aggregate matching function. More specifically, they have the potential of explaining different stages of the search process, being the combination of two probabilities: the probability of receiving a job offer and the probability of

accepting the offer. The first of these depends on the set of characteristics that describe a worker's productivity (such as age, education, experience, etc.) and on labour demand conditions. This latter effect is basically the only one captured by aggregate matching functions. The second probability depends on a worker's reservation wage and therefore on the expected distribution of wages, family needs, the cost of search, unemployment income and, once more, labour demand conditions. Furthermore, hazard functions can introduce duration dependence of exit rates from unemployment, which is generally controlled for in aggregate estimates by conditioning job formation on single *ad hoc* regressors such as the incidence of long-term unemployment.

The data used in this Paper comes from the Survey of Incomes In and Out of Work, which examines labour market transitions of a sample of British workers who registered as unemployed in Spring 1987. In order to avoid a geographical aggregation bias, exit rates from unemployment are conditioned on local labour market variables, measured within travel-to-work areas, which are the closest approximation to self-contained labour markets. The constant returns hypothesis is tested by checking whether re-employment probabilities only depend on local labour market tightness, or whether they are also enhanced by the absolute number of traders.

The econometric analysis is led in the context of two alternative continuous-time duration models: a fully parametric hazard model, with Weibull duration dependence and Gamma-distributed unobserved heterogeneity and a more general Cox proportional hazard model, that does not impose a specific functional form for the baseline hazard. The two specifications delivered very consistent estimates.

The results obtained broadly confirm previous findings on the determinants of re-employment probabilities for men and women and are generally consistent with the predictions of a job search framework. The probability of receiving a job offer should be related to the individual's educational attainment and to the state of local labour demand, which in fact positively affect re-employment probabilities for men. No effect of local labour demand is instead detected in re-employment probabilities for women, although this may be at least in part a consequence of how local labour market conditions are measured. The probability of accepting a job offer should in turn depend on the replacement ratio and on the family composition of the individual, which in fact have the expected effect on re-employment probabilities of both men and women. Concerning the shape of the baseline hazard, clear evidence is found of negative duration dependence in hazard rates for both males and females, when re-employment probabilities are conditioned on the whole evolution of local labour demand during the unemployment spell. This result is also robust to the introduction of Gamma-distributed unobserved heterogeneity.

In no specification does the absolute coefficient on (the log of) local unemployment differ from that on the (log of) number of vacancies, implying that the absolute size of the searching pool does not affect matching rates. This therefore allows us not to reject the constant returns hypothesis in the matching technology between unemployment and vacancies. For methodological purposes, this finding in turn implies that the results of several aggregate studies were not too seriously biased by aggregation problems. In a broader perspective, the results of this Paper suggest that thicker and more active markets do not necessarily lead to easier trading, at least as far as the number of matches are concerned.

1 Introduction

Search models of the labor market hinge on the existence of a hiring or matching function that describes the technology of the job formation process by relating hirings to unemployment and vacancies. The equilibrium properties of such models crucially depend on the characteristics of the matching technology. In particular, the assumption of constant returns to scale in the matching function ensures a constant unemployment rate along a balanced-growth path, as shown in Pissarides (1990), Aghion and Howitt (1994) and Mortensen and Pissarides (1998). The same property has also been incorporated in real business cycle models with labor market search (Merz 1995, Andolfatto 1996, and Yashiv 1998), leading to a unique level of equilibrium unemployment. Multiple (rankable) equilibria arise instead when the matching function exhibits increasing returns, as in Diamond (1982, 1984).

Multiplicity raises obvious policy questions. In particular, with multiple equilibria, even temporary policies may pull the economy out of an inefficient unemployment level. Also, multiplicity has the potential to explain why economies may get stuck at high levels of unemployment, even though the initial adverse shock was only temporary, like the oil price rises of the 1970s. Finally, multiplicity is closely related to hysteresis of unemployment in the sense that, with hysteresis, equilibrium unemployment is always close to the actual rate.

Given the implications of increasing versus constant returns on theoretical grounds, it is insightful to explore what the data can tell on this issue. The aim of this paper is to test the empirical relevance of the constant returns hypothesis, by estimating individual hazard functions on a sample of unemployment entrants.

For this purpose, I use a well known link between aggregate matching conditions and individual job-finding hazards. The hazard rate denotes the probability of a transition out of unemployment within some small time interval, conditional on the worker being still unemployed when the interval started. If the underlying matching technology displays constant returns to scale, the hazard rate for an unemployed worker (and - conversely - unemployment duration) should only depend on the degree of labor market tightness, measured by the vacancy÷unemployment ratio, and not on the absolute size of the pool of job-seekers. If instead matching displays increasing returns, the hazard rate should depend positively on the size of the market, once labor market tightness is controlled for. This is precisely the hypothesis that will be tested on individual duration data.

Compared to aggregate matching functions, hazard function specifications have the main advantage of being rather flexible. They allow for a

wide spectrum of functional forms for duration distributions, and control for a number of individual characteristics whose importance is only implicit in an aggregate matching function. More specifically, they have the potential of explaining different stages of the search process, being the combination of two probabilities: the probability of receiving a job offer and the probability of accepting the offer. The first of these depends on the set of characteristics that describe a worker's productivity (such as age, education, experience, etc.) and on labor demand conditions. This latter effect is basically the only one captured by aggregate matching functions. The second probability depends on a worker's reservation wage, and therefore on the expected distribution of wages, family needs, the cost of search, unemployment income and, once more, labor demand conditions. Furthermore, hazard functions can introduce duration dependence of exit rates from unemployment, which is generally controlled for in aggregate estimates by conditioning job formation on single ad hoc regressors such as the incidence of long term unemployment.

Despite the importance of micro duration analysis for the understanding of aggregate matching performance, macro and micro approaches have so far mainly been used in the empirical search literature for answering different questions. On the one hand, aggregate matching function studies have mainly addressed the issue of aggregate search effectiveness and of the returns to scale in the matching technology, in the tradition of Pissarides (1986) and Blanchard and Diamond (1989). On the other hand, hazard functions were mainly used to investigate the determinants of exits from unemployment, without concern for the structure of the underlying matching technology. Devine and Kiefer (1991) review a number of hazard function studies, and among them only Nickell (1979) and Atkinson *et al.* (1984) include labor market tightness as a determinant of the exit rate from unemployment, but they do not test for constant returns to scale in the matching technology by controlling for labor market size.

An exception to this general approach is the work by Lindeboom *et al.* (1994), who exploit the link between aggregate matching function and hazard rate specifications for evaluating the relative effectiveness of alternative search channels. The empirical analysis of this paper however differs from theirs mainly on the grounds of the specification of the hazard. In particular, this is the first attempt to use individual data to estimate the returns to scale in the matching technology.

The data used in this paper comes from the Survey of Incomes In and Out of Work, which examines labor market transitions of a sample of British workers who registered as unemployed in Spring 1987. In order to avoid a geographical aggregation bias, exit rates from unemployment will be conditioned on local labor market variables, measured within travel-to-work areas, which

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are the closest approximation to self-contained labor markets. The constant returns hypothesis is tested by checking whether re-employment probabilities only depend on local labor market tightness, or whether they are also enhanced by the absolute number of traders.

The organization of the paper is as follows. Section 2 describes how returns to scale are generated in a matching environment, and overviews the main empirical findings on the issue. Section 4 gives details on the data set used. Section 3 specifies the alternative econometric models to be estimated: a fully parametric hazard model with Weibull duration dependence, and a semi-parametric Cox proportional hazard model. Section 5 provides the estimation results and Section 6 concludes.

2   Returns to matching: Theory and evidence

In a matching environment, the returns to search for each trader depend crucially on what other traders do. Finding a job is harder when many people are searching and few new openings are being posted. At the same time, filling an extra vacancy is more costly when the economy is booming and fewer workers are searching. Such search externalities determine the relationship between matching efficiency and market size, as measured by the number of agents that involve in search. In other words, they determine the homogeneity degree of the matching technology.[1]

Widely used models of bilateral search, such as Pissarides (1990), consider two different kinds of externalities that each searcher generates. Suppose that each agent of one type can only trade with agents of the other type. An agent's decision to involve in search produces a positive (*thin market*) externality, by enhancing the probability of finding a trading partner among agents of the opposite type, and therefore decreasing the cost of search to the other side of the market. At the same time she generates a negative (*congestion*) externality on agents of her same type, by increasing the number of competitors for potential trading partners, and therefore increasing the cost of search on her same side of the market. It can be argued that the net effect of positive and negative externalities from trade leaves the matching efficiency of a marketplace independent of the number of traders, so that constant returns in the matching technology can be used a plausible starting point for search models. In particular, if this is the case for the labor market matching technology, there exists a unique level of unemployment and

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[1]Clearly, such externalities also imply that optimal search decisions at the individual level may not maximize social output, opening the question of equilibrium inefficiency (see Pissarides 1986, 1990 Chapter 7).

vacancies where unemployment inflows and outflows are equal.

Diamond (1982) however argues that, if greater search effort on one side of the market not only decreases the cost of search on the other side but also leads the other side to increase its own search effort, then matching may display increasing returns, potentially leading to a multiplicity of unemployment equilibria.

More recent studies by Coles (1994) and Coles and Smith (1996, 1998) take into consideration possible alternatives to a random, space-independent technology, and argue that it is theoretically plausible that the matching technology exhibit increasing returns. Coles and Smith (1996) infer increasing returns from a replication argument, in which replicated marketplaces can interact with one another. However, it is not clear why interactions between spatially distinct markets should not generate both positive and negative trading externalities, whose net result in terms of matching efficiency is ambiguous. Coles (1994) and Coles and Smith (1998) consider instead a non random matching process, in which the stock of unmatched traders on one side of the market can only match with the flow of traders on the other side. This derives from a plausible sampling assumption in which two agents will never trade after they meet once and decide not to trade. In this case, although a conventional matching function defined over stocks of traders may display constant or even decreasing returns to scale, the correct matching function in stocks and flows may exhibit increasing returns.

Determining whether there are constant or increasing returns in the matching function is ultimately an empirical matter. Empirical studies on the matching function cannot reject in most cases the constant returns hypothesis or find, in few cases, evidence of weakly increasing returns to scale. However, possible misspecification problems (such as those arising from temporal or geographical aggregation), inducing a downward bias in the resulting estimates, would still leave the question quite open for further research.

The benchmark study of Blanchard and Diamond (1989) finds evidence of constant or weekly increasing returns estimating a Cobb-Douglas matching function for the US aggregate economy. They find more clear-cut evidence in favor of increasing returns when they restrict the estimation to the US manufacturing sector and use a set of instruments for unemployment and vacancies. On the whole, they tend to dismiss the result of strongly increasing returns to scale.

Their analysis was updated and modified in several ways by later work. As far as aggregation biases are concerned, Burdett *et al.* (1994) show that temporal aggregation problems are nearly irrelevant in the work or Blanchard and Diamond, given the sufficiently high (monthly) frequency of the time series they use and the relatively low frequency of cycles in the conditioning

variables. According to Coles and Smith (1996), geographical aggregation should also play no role in practice, despite the claimed validity of the increasing returns hypothesis on theoretical grounds. They estimate in fact a cross section of matching functions for England and Wales, and cannot reject the constant returns hypothesis in matching, even within perfectly integrated labor markets such as travel-to-work areas. Similar results are obtained by Bennet and Pinto (1994), who estimate separate matching functions for local districts in Britain, and by Burda and Profit (1996) and Burgess and Profit (1998), who analyze the effects of regional migration and commuting on local matching conditions in the Czech Republic and Britain, respectively.

Increasing returns are obtained instead by Anderson and Burgess (1995). They estimate a matching function using panel data on state-industry level matches in four US states, using therefore a lower level of aggregation than Blanchard and Diamond. They also estimate separate matching functions, using hires from employment and hires from non-employment in turn as dependent variables. Constant returns cannot be rejected in the first case, while they are strongly rejected in the second.

The studies mentioned estimate Cobb-Douglas matching functions in unemployment and vacancies. On one hand, this specification is globally well-behaved, in the sense that it adequately embodies the property that no jobs can be created when one of the inputs of the function is zero. On the other hand, it is rather restrictive, and the number of studies who attempt alternative specifications is surprisingly low. An exception to this modeling is Warren (1996), who explicitly addresses the issue of increasing versus constant returns to scale using a flexible (translog) specification of the matching technology. This is supposed to give the least biased estimate of the degree of returns to scale of a known technology (compared with the generalized-Leontief and extended generalized Cobb-Douglas forms[2]). The constant returns hypothesis is rejected in favor of increasing returns.

On the whole, those who estimate log linear matching functions on highly aggregated data tend to find evidence in favor of constant returns. Using a lower level of aggregation and/or estimating a more flexible functional form generally induces to reject the constant returns hypothesis in favor of increasing returns.

An alternative way of assessing the relationship between market size and matching efficiency - pursued in this paper - consists in estimating re-employment probabilities using a hazard function approach. The next section illustrates the data set used for this purpose.

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[2]See Guilkey *et al.* (1983).

3   The data

The data used comes from the Survey of Incomes In and Out of Work. The Survey collects individual information on a British representative sample of men and women who started a spell of unemployment, and registered at any of the 88 Unemployment Benefit Offices (UBO) selected, in the four weeks starting March 16, 1987.

By focusing on unemployment entrants, the use of these data does not involve a stock sample bias, and allows the adoption of semi-parametric methods such as the Cox proportional hazard model that do not condition on the elapsed duration at the first interview date.

Information was collected from two personal interviews. The first interview was carried out shortly after unemployment began - between April and July 1987 - and a total of 3003 interviews was completed with the selected respondents. The second interview was held about nine months later, in January 1988, on respondents who had been interviewed in 1987 and had consented to a second interview. A total of 2146 interviews was completed at this second stage.[3]

The first interview focused on individuals' personal details and their employment history during the 12 months proceeding the interview, including employment and unemployment income, type of job(s) held and job search activities while unemployed. The follow-up interview covered individuals' employment history since their first interview.

Given the competing-risk framework described, the duration of unemployment - treated in continuous time - is measured as the number of days between the date the worker signed at the UBO and the date she re-entered employment, provided she did not leave the unemployment register before that. In the case that the worker left the register before finding a job, the unemployment spell is censored and is measured as the duration of registered unemployment. Similarly, in the case that by the time of the second interview (or the first interview for those who only had one interview) she is not back yet into employment and she has not left the unemployment register, the unemployment spell is censored and is measured as the number of days between the day of signing at the UBO and the interview.

As said above, unemployment duration or, conversely, re-employment probabilities, depend on the probability of receiving a job offer and the probability of accepting the offer. The first of these depends on local labor demand,

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[3]There is clearly some attrition in the data collected, with 28% of the observations being lost by the time of the second interview. Although we use available information also on those who only had one interview, we nevertheless need to assume that attrition is random.

human capital variables such as education and the past (un)employment history, and personal characteristics such as sex, age, race and health status. The second probability is clearly influenced by everything that determines the reservation wage, and therefore by the opportunity cost of being employed, measured by the replacement ratio, the family composition of the unemployed, and, again, local labor demand.

As far as the characterization of local labor markets is concerned, for confidentiality reasons the Survey does not attach explicit geographic identifiers to interviewees. The only geographical information that can be used is the code of the UBO at which the worker was registered. The first two digits of the UBO code denote the region where the UBO is located. Therefore the mapping between British regions and UBOs is non controversial.

However, in order to characterize more precisely local labor market conditions, it is advisable to switch to a narrower definition of a local market such as the travel-to-work-area. TTWAs are approximations to self-contained labor markets, i.e. areas in which people live and work, or look for jobs. According to the most recent definition, TTWAs meet the following criteria: a minimum of working population of 3,500; 75% of those living in the area work there; 75% of those working in the area live there.

The mapping between TTWAs and UBOs is more problematic. Using information from the NOMIS database it is possible to associate a name with each UBO code. The mapping is then constructed using the TTWA classification provided by NOMIS in order to obtain the closest match between TTWAs and UBOs (or Jobcentres). UBOs which had the same name as a TTWA (e.g. Leeds) were easily located within the corresponding TTWA. This allowed to locate 49 of the 88 UBOs selected in the Survey. Further progress is made using some implicit geographical information contained in the Survey. Attached to each worker is in fact the unemployment rate of the TTWA in which her UBO is situated. This permitted to locate 26 more UBOs making cross-section comparisons between the unemployment rates attached. Finally, 9 further UBOs were located using unambiguous associations between the name of the UBO and that of the TTWA (e.g. Stockport-Manchester). 4 remaining UBOs could not be located precisely and the corresponding 220 observations were dropped. Once the mapping is done, the unemployment and vacancy data for the 61 resulting TTWAs are obtained from NOMIS. They refer to the unemployment claimant count and the number of vacancies advertised at Jobcentres.

Further 838 observations were dropped because of missing data on the replacement ratio, leading to a final sample of 1239 men and 706 women. Tables 1 and 2 report the relevant descriptive statistics of individuals included in the sample and of the local labor markets where they live, respectively.

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Table 1: Sample characteristics of the unemployment inflow

| | Males | | Females | |
Variables	Mean	St.dev.	Mean	St.dev.
% exit in employment	54.2		46.5	
% exit in non-employment	7.7		15.7	
% stay unemployed	38.1		37.8	
uncensored duration	11.9	10.7	11.0	10.6
censored duration	27.4	17.5	23.5	17.4
age	37.7	11.5	36.8	11.0
% not white	7.1		6.7	
% with health problems	34.4		36.7	
% with high education	43.8		42.5	
% married	79.3		68.6	
No. of dep. children	1.2	1.4	0.9	0.9
% with children<6yrs in household	30.8		25.9	
% home owners	51.5		59.3	
% lost full-time job	93.3		67.7	
% union members	36.1		23.5	
past unemployment	1.2	3.9	1.9	5.6
replacement ratio	0.51	0.80	0.54	0.49
search methods used (%):				
media advertisement	0.61		0.60	
Jobcentres	0.50		0.50	
personal contact	0.41		0.29	
contacts with employers	0.22		0.09	
private agencies	0.04		0.03	
other	0.13		0.03	
No. of cases	1239		706	

Notes. *High education*: includes all those who attended school or vocational training courses until the age of 18, plus those with higher education. *Past unemployment:* denotes the number of weeks spent unemployed during the previous year. The *replacement ratio* is computed as the ratio between the total weekly benefits received by the worker (general + supplementary + housing benefits) and the weekly take-home pay in the last job before registering at the UBO. A *search method* is used when it is employed at least once a week. Source: SIIOW.

Table 2: Local labor markets in Britain

Variables	Mean	St.dev.
V/U - April 1987	0.085	0.057
V/U - July 1987	0.103	0.070
V/U - October 1987	0.121	0.084
V/U - January 1988	0.100	0.081
geographical size (acres)	71,669	80,566
population - April 1987	427,567	805,517

Notes. No. of observations: 61. Source: NOMIS.

Search variables described in Table 1 are not included in the estimation of hazard functions, because the use of various search channels has proved to be largely endogenous, as argued by Gregg and Wadsworth (1996) and Thomas (1997). For example, the use of media advertisement tends to be preferred at low durations, and, if search is unsuccessful, people tend to switch to Jobcentres.

I use instead information on search behavior as a first screening of the constant returns hypothesis versus increasing returns. If, as argued by Diamond (1982), increasing returns stem from the positive externality of favorable labor market conditions onto search effort, one would expect unemployed search effort to be higher in those TTWAs characterized by higher V/U ratios. In order to test this hypothesis I simply regress the proportion of workers using each search channel in each TTWA on (the log of) local labor market tightness, assuming implicitly that vacancies advertised at Jobcentres are a proxy for local labor demand. The results of this exercise are reported in Table 3. The fit of all equations is very poor, especially for women. The use of most search channels is negatively influenced by local labor market tightness, except for the residual category - including contacts with trade unions, search for self-employed jobs and other contacts. If anything, it seems that search is used as a substitute rather than a complement to favorable labor market conditions, therefore rejecting the hypothesis that increasing returns to scale may result from this kind of externality.

Table 3: Search channels and labor market tightness

	Media	Jobcentres	Friends	Employers	Private agencies	Other
Males						
$\ln(V/U)$	−0.087 (0.022)	−0.084 (0.025)	−0.054 (0.022)	−0.037 (0.022)	0.008 (0.009)	0.038 (0.014)
constant	0.382 (0.060)	0.276 (0.068)	0.253 (0.060)	0.117 (0.061)	0.062 (0.024)	0.239 (0.038)
R^2	0.210	0.163	0.092	0.045	0.015	0.116
Females						
$\ln(V/U)$	−0.017 (0.030)	−0.008 (0.033)	−0.005 (0.020)	0.004 (0.016)	0.001 (0.011)	0.017 (0.012)
constant	0.538 (0.082)	0.476 (0.091)	0.272 (0.055)	0.091 (0.045)	0.033 (0.030)	0.090 (0.033)
R^2	0.006	0.001	0.001	0.001	0.001	0.033

Notes. The dependent variable is the proportion of individuals using each search method in each TTWA. Estimation method: OLS. Standard errors in brackets. No. of observations: 61. Source: SIIOW and NOMIS.

4 The model

In order to study the determinants of the exit from unemployment, I apply hazard models to data on the duration of unemployment spells.[4]

The probability distribution of durations can be specified by the cumulative distribution function $F(t) = \Pr(T < t)$, that gives the probability that a continuous random variable T denoting duration is less than some value t. The corresponding density function is $f(t) = dF(t)/dt$. The joint probability distribution of a sample of n observations t_i can be represented by the log-likelihood function

$$\ln L = \sum_{i=1}^{n} \ln f(t_i). \tag{1}$$

Some of the n observations in my sample are right censored, and hence represent uncompleted spells. The likelihood contribution of each censored observation is the survivor function $S(t) = 1 - F(t)$, denoting the probability that the duration is longer than t. Let us introduce the censoring indicator c_i, such that $c_i = 1$ if the ith observation is uncensored, and $c_i = 0$ otherwise. The likelihood function is given by

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[4]Econometric applications of hazard models are extensively described in Kiefer (1988) and Lancaster (1979, 1990).

$$\ln L = \sum_{i=1}^{n} c_i \ln f(t_i) + \sum_{i=1}^{n} (1 - c_i) \ln S(t_i). \tag{2}$$

It is convenient to express (2) in terms of the hazard rate $\lambda(t)$, which denotes the probability of completing duration in the short interval of length dt after t, conditional on duration being still uncompleted at time t. The hazard rate is given by $\lambda(t) = f(t)/S(t) = -d\ln S(t)/dt$.

Making this substitution, equation (2) becomes

$$\ln L = \sum_{i=1}^{n} c_i \ln \lambda(t_i) + \sum_{i=1}^{n} \ln S(t_i). \tag{3}$$

with $S(t) = \exp\left(-\int_0^t \lambda(s)ds\right)$.

Besides duration t, a set of explanatory variables can affect the hazard function $\lambda(t)$. Below I consider the general case in which at least some of the regressors are time-varying, i.e. they assume more than one value during individuals' unemployment spells. In particular, this serves to condition re-employment probabilities on the whole evolution of local labor market variables during job search.

We consider a proportional hazard model

$$\lambda(t, x(t)) = \phi_1(t)\phi_2(x(t)), \tag{4}$$

with the survivor function being given by

$$S(t, x(t)) = \exp\left\{-\int_0^t \phi_1(s)\phi_2(x(s))ds\right\}. \tag{5}$$

The baseline hazard $\phi_1(.)$ is a functional form for the dependence of λ on duration. The second component $\phi_2(.)$ describes the way in which λ shifts, at given duration t, between individuals endowed with different x's.

In order to assess the impact of duration on unemployment exit rates, the baseline hazard can be represented by an explicit function of duration, e.g. the Weibull

$$\phi_1(t) = \alpha t^{\alpha - 1}, \tag{6}$$

where $\alpha \gtreqless 1$ denotes positive, zero or negative duration dependence, respectively.

The term $\phi_2(.)$ is conveniently specified as

$$\phi_2(x(t)) = \exp(x(t)'\beta), \tag{7}$$

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in order to ensure a non-negative hazard without constraining the parameter space for β.

The model outlined specifies the determinants of a single risk: that of leaving the unemployment register. Unemployment duration can terminate with job finding or alternative states. Given that I am interested in the first type of transition, I need to consider a competing risk model, that distinguishes exit into employment from exit into alternative states.

Suppose that there are J alternative states: then the contribution of the ith individual with destination k to the log-likelihood is

$$
\begin{aligned}
\ln L_i &= c_{ik} \ln \lambda_k(t_i) + \sum_{j=1}^{J} \ln S_j(t_i) \\
&= c_{ik} \ln \lambda_k(t_i) + \ln S_k(t_i) + \sum_{j \neq k} \ln S_j(t_i). \quad (8)
\end{aligned}
$$

The full log-likelihood is $\ln L = \sum_i \ln L_i = \sum_j \ln L_j$, with

$$
\ln L_j = \sum_{i=1}^{n} c_{ij} \ln \lambda_j(t_i) + \sum_{i=1}^{n} \ln S_j(t_i). \quad (9)
$$

Equation (9) shows that the parameters of a given cause-specific hazard can be estimated by treating durations finishing for other reasons as censored at time of exit (see Narendranathan and Stewart 1993). We therefore treat all durations that end in non-employment as censored at the time the worker left the unemployment register. Having done this, the proportional hazard specification (4) used for the single-risk model can be applied to the job-finding hazard.

In order to exploit the link between aggregate matching conditions and re-employment probabilities, explanatory variables $x(t)$ to be included into the job-finding hazard are determined by a simple labor market matching model. For this purpose I consider the standard matching function in unemployment and vacancies (see Pissarides 1990), augmented with a search-effectiveness parameter:

$$
M_t = m\left(\overline{e}U_t, V_t\right). \quad (10)
$$

This relates the amount of job creation M_t to efficiency units of unemployment $\overline{e}U_t$ and the number of vacant jobs V_t. $\overline{e}$ therefore represents the average search effectiveness of the unemployed faced by employers. Ignoring for the moment duration dependence, the re-employment probability for an unemployed worker, $\lambda\left(x_i(t)\right),$ is given by

$$\lambda\left(x_i(t)\right) = e_i \, \frac{M_t}{\overline{e}U_t}, \tag{11}$$

where e_i denotes individual search effectiveness. Using a Cobb-Douglas specification for the function $m\left(.\right)$, with elasticities a and b respectively, equation (11) becomes

$$\lambda\left(x_i(t)\right) = \exp\left(\ln e_i - (1-a)\ln\overline{e} - (1-a)\ln U_t + b\ln V_t\right), \tag{12}$$

so that $x_i(t)'\beta = \ln e_i - (1-a)\ln\overline{e} - (1-a)\ln U_t + b\ln V_t$.

Personal characteristics are used as proxies for individual search effectiveness e_i. Average search effectiveness $\overline{e}$ is captured by the constant term in $x_i(t)$. Finally, U_t and V_t are measured in the local labor market where the ith individual lives and supposedly looks for a job.

If the matching function (10) displays constant returns to scale, $a+b=1$, so that the hazard rate (12) only depends on the labor market tightness $\theta_t = V_t/U_t$. If instead matching displays increasing returns, one expects a lower absolute coefficient on $\ln U_t$ than on $\ln V_t$.

The effect of possibly omitted regressors in the exit from unemployment is controlled for by conditioning the hazard rate on an individual's unobserved characteristics, summarized into the variable v. The hazard rate and the survivor function are therefore rewritten as $\lambda(v,t,x(t)) = v\phi_1(t)\phi_2(x(t))$ and $S(v,t,x(t)) = \exp\left\{-v\int_0^t \phi_1(s)\phi_2(x(s))ds\right\}$ respectively. Following Lancaster (1979), I assume that v is distributed as a Gamma variate of unit mean and variance σ^2, taking the form

$$f(v) \propto v^{\sigma^{-2}-1}\exp\left(-\sigma^{-2}v\right). \tag{13}$$

Equation (13) assumes that v is independent of t and $x(t)$. The resulting proportional hazard specification $\lambda(v,t,x(t))$ therefore identifies the three sources of variation among individual hazard rates: the duration of search (t), the observable differences among individuals ($x(t)$) and the unobservable ones (v). However, in a competing risk framework, allowing for a random disturbance term in each of the cause-specific hazard requires an additional assumption, that imposes the independence of these disturbance terms across the cause-specific hazards.[5]

The hazard function and the survivor function, conditional on included regressors only, are computed as $\int_0^\infty \lambda(v,t,x(t))f(v)dv$ and $\int_0^\infty S(v,t,x(t))f(v)dv$,

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[5] The alternative approach would be to assume perfect correlation (as opposed to zero correlation) between the cause-specific disturbance terms. See Narendranathan and Stewart (1993) for a discussion of advantages and disadvantages of the two methods.

which give[6]

$$\lambda(t, x(t)) = \frac{\phi_1(t)\phi_2(x(t))}{1 + \sigma^2 \int_0^t \phi_1(s)\phi_2(x(s))ds} \tag{14}$$

$$S(t, x(t)) = \left\{ 1 + \sigma^2 \int_0^t \phi_1(s)\phi_2(x(s))ds \right\}^{-\sigma^{-2}}. \tag{15}$$

The discussion so far concerned a fully parametric specification of the hazard. However, for identifying the impact of explanatory variables $x(t)$ on the hazard rate λ, there is no need to impose an explicit functional form for the baseline hazard $\phi_1(.)$, in which case estimation is semi-parametric, as in the Cox (1972) proportional hazard model. This model exploits the ranking of observed durations: $t_1 < t_2 < ... < t_i < ... < t_n$. The conditional probability that some observation i could have completed a spell at duration t_i, given that all those observations with longer duration could have completed a spell at the same duration, is $\lambda(t_i, x_i(t))/\sum_{j=i}^n \lambda(t_j, x_j(t))$, which reduces to $\phi_2(x_i(t))/\sum_{j=i}^n \phi_2(x_j(t))$ for the proportional hazard model (4). The resulting partial log likelihood is therefore

$$L = \sum_{i=1}^n \left\{ \ln \phi_2(x_i(t)) - \ln \left(\sum_{j=i}^n \phi_2(x_j(t)) \right) \right\}. \tag{16}$$

Having described the likelihood functions that are the objective of the present analysis (more details are reported in the Appendix), I turn to the description of estimation results.

5 Empirical results

I move next to estimate hazard models described in the previous section. In doing this, I let local labor market variables embodied in $x(t)$ to vary monthly, because this is the highest frequency available for unemployment and vacancy data. Re-employment probabilities are therefore conditioned on the series of monthly U_t and V_t during the individual's unemployment spell.

On the one hand, using time-varying regressors allows to capture the effect of seasonality and/or other fluctuations in activity. On the other hand, U_t is not a fully predetermined regressor, in the sense that the evolution of the unemployment stock reflects the intensity of the outflow rate from the

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[6]See Lancaster (1979).

unemployment pool. In particular, this mechanism implies that the unemployment stock is depleted by the unemployment outflow, potentially generating a downward bias in the resulting elasticity of the job-finding hazard with respect to the unemployment stock (see Burdett *et al.* 1994, and Berman 1997). For this reason, the likelihood functions are also estimated using time-invariant U and V. In this case the values used for U and V are those recorded in April 1987, when most workers in the sample started their unemployment spell.

One further local labor market variable that is included in the hazard is the geographical size of the TTWA where the worker lives. This should reveal whether an increase in the geographical density of searchers would improve the efficiency of search (see Hall 1989). An additional way to control for density effects – pursued below in Table 6 – consists in deflating U and V by local population, in order to represent unemployment and vacancy density respectively.

The model is estimated separately for both men and women, given that not only re-employment probabilities differ across genders, but they also tend to respond differently to some of the controls used (see also Lynch 1989). In particular, when controlling for family composition of workers, males re-employment probabilities are conditioned on the total number of dependent children, while female ones are conditioned on the presence of children under the age of six in the household.

Table 4 provides the estimation results using time-invariant regressors.[7]

Column I reports the estimates of re-employment probabilities for men, not controlling for unobserved heterogeneity. The results look fairly consistent with the predictions of a simple search model and with previous empirical findings (see also the results collected in Devine and Kiefer 1991). Personal characteristics that lower the re-employment probabilities of men include age, the replacement ratio, the time spent as unemployed during the year proceeding the survey, belonging to ethnic minorities, suffering from health problems and having been union member during the last job held[8]. Higher education increases instead the probability of finding a job, and so does home ownership and being married, while the total number of dependent children does not.

Contrary to expectations, having lost a full-time job in the past does not

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[7]All estimates reported are obtained using a *quasi-Newton* method (the Broyden, Fletcher, Goldfarb, Shannon method), with the covariance matrix computed as the inverse of the Hessian. Alternative methods used provided equivalent results.

[8]This last effect may proxy low relocation opportunities for workers who are displaced from heavily unionized industries, such as mining or manufacturing. No industry indicators were included directly because they were missing for 15% of the observations.

Table 4: Maximum likelihood estimates of re-employment probabilities with time-invariant regressors

Baseline hazard	Weibull		Weibull		Non parametric	
	Males I	Males II	Females III	Females IV	Males V	Females IV
constant	2.271 (0.778)	5.894 (1.956)	−1.162 (1.109)	0.675 (6.328)	—	—
ln(age)	−1.066 (0.196)	−1.693 (0.284)	0.069 (0.171)	0.629 (0.727)	−1.004 (0.154)	0.086 (0.200)
not white	−0.267 (0.168)	−0.343 (0.277)	−0.034 (0.269)	−0.299 (0.527)	−0.235 (0.165)	−0.039 (0.250)
health problems	−0.173 (0.086)	−0.316 (0.149)	−0.086 (0.124)	−0.319 (0.253)	−0.168 (0.086)	−0.086 (0.121)
high education	0.532 (0.081)	0.628 (0.138)	0.366 (0.118)	0.845 (0.253)	0.479 (0.081)	0.343 (0.115)
married	0.511 (0.217)	0.884 (0.238)	−0.114 (0.146)	0.007 (0.109)	0.479 (0.133)	−0.101 (0.138)
children	0.037 (0.074)	0.096 (0.058)	−1.233 (0.181)	−2.236 (0.356)	0.037 (0.034)	−1.154 (0.182)
home owner	0.145 (0.089)	0.240 (.149)	0.165 (0.138)	−0.196 (0.267)	0.148 (0.089)	0.158 (0.137)
had FT job	0.061 (0.253)	0.045 (0.098)	−0.218 (0.129)	−0.387 (0.255)	0.050 (0.168)	−0.184 (0.125)
union member	−0.265 (0.094)	−0.442 (0.150)	−0.411 (0.153)	−0.899 (0.306)	−0.254 (0.089)	−0.387 (0.148)
past unemp.	−0.028 (0.013)	−0.034 (0.019)	−0.007 (0.012)	−0.027 (0.023)	−0.026 (0.012)	−0.009 (0.011)
ln(repl. ratio)	−0.392 (.043)	−0.689 (0.099)	−0.109 (0.059)	−0.351 (0.133)	−0.366 (0.042)	−0.102 (0.057)
$\ln(U)$	−0.205 (0.072)	−0.356 (0.124)	0.063 (0.167)	0.123 (0.533)	−0.190 (0.065)	0.044 (0.094)
$\ln(V)$	0.171 (0.082)	0.294 (0.136)	−0.069 (0.160)	−0.076 (0.534)	0.158 (0.070)	−0.052 (0.098)
ln(area)	−0.011 (0.057)	−0.074 (0.143)	−0.146 (0.111)	−0.513 (0.289)	−0.015 (0.068)	−0.140 (0.100)
α	0.910 (0.029)	1.406 (0.102)	0.868 (0.039)	1.914 (0.201)	—	—
σ^2	—	1.292 (0.133)	—	1.655 (0.249)	—	—
χ^2	1.28	1.85	0.03	0.33	0.98	0.27
mean log-likelihood	-2.374	-2.351	-2.066	-2.027	- 3.488	-2.722
No. of cases	1239	1239	706	706	1239	706

Notes. Asymptotic standard errors in brackets. The χ^2 statistic is the result of a Wald test of H_0: coef[$ln(U)$]+coef[$ln(V)$] = 0. Critical value $\chi^2(1, 0.05) = 3.84$. Source: SIIOW and NOMIS.

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enhance significantly the probability of finding a new job. This is possibly explained by the negative correlation (for both men and women) between the replacement ratio and the full-time control. Workers that lost a full-time job have lower replacement ratios, and it is difficult to distinguish the two effects on re-employment probabilities. Estimation was also performed dropping the replacement ratio, delivering a positive and highly significant effect on the full-time status.

Local labor market variables have the expected impact on the hazard. Moreover, the coefficient on $\ln U$ is not significantly different from the one on $\ln V$, as shown by the χ^2 statistic reported at the bottom of the Table, providing evidence in favor of constant returns to scale in the matching function. The geographical size of the local labor market has instead no significant impact on individual hazards, implying a non-significant density effect in matching. Concerning duration dependence, the estimated Weibull coefficient α is significantly lower than 1, implying that the hazard is slightly declining with duration.

Turning to re-employment probabilities for women in column III, they seem to be affected positively by educational qualifications, and negatively by the presence of young children in the household, union membership, the replacement ratio, and past full-time status. Similarly as for men, the full-time variable had instead a positive significant impact when the replacement ratio was dropped.

Unemployment and vacancy variables have an opposite sign to what one would have expected, although neither coefficient is significantly different from zero. This can be at least partly explained considering that the controls used - the number of registered unemployed and the number of vacancies advertised at Jobcentres - typically reflect males' rather than females' labor market variables. The design of the British unemployment insurance system is in fact such that out-of-work women are less likely to be registered unemployed (see Gregg 1994), so that the related figures are much closer to the male rather than the female unemployment rate. Furthermore, the information given by the SIIOW shows that the proportion of unemployed women who find a job through a Jobcentre is lower than that for unemployed men (although the use of Jobcentres across genders is very similar), so that vacancies advertised there may only affect weakly the probability of a woman going back into work. Interestingly enough, there seems to be a moderate density effect in female re-employment probabilities, given that coefficient on the geographical size of the local labor market is significantly lower than zero at the 10% significance level. Similarly as for men, unemployment duration negatively affects female re-employment rates.

However, before concluding that there is negative duration dependence in

the transition probabilities from unemployment to employment, we should consider the possibility that the estimates obtained in column I are biased due to the omission of unobserved variables. As Lancaster (1979) recognizes, the estimate for α is in fact at least in part an index of the misspecification of the model, measuring the extent of unobserved heterogeneity within the sample. With the present sample, this is found in column II and IV, where the control for gamma-distributed unobserved heterogeneity delivers a value of α well above 1 for both males and females. If anything, the presence of unobserved heterogeneity seems more relevant in the female sample, as shown by higher values of α and σ^2 for women than for men. After controlling for higher α, the effect of most covariates on unemployment durations from columns II and IV is closely comparable to that found in columns I and III. The coefficients on marital status and on home ownership status switch sign in the regression for females, when unobserved heterogeneity is controlled for. However in no case are they significantly different from zero.

Even so, it cannot be concluded at this stage that re-employment probabilities are genuinely increasing with duration. The sample is in fact constructed in such a way that it is not possible to distinguish between genuine duration dependence and calendar time dependence, given that all individuals have started an unemployment spell within the same four weeks. As it is shown by the results that follow, this is a serious problem in the estimates provided, given that the British economy experienced some recovery during 1987 (see also the θ ratios reported in Table 1). This may have improved re-employment prospects for all those who have been jobless long enough to benefit from the recovery, thus introducing some spurious positive duration dependence in hazard functions.

It can be argued that the dependence of re-employment probabilities on the state of the labor market is a combination of two factors: a purely aggregate factor, represented by business cycle and seasonal fluctuations that affect equally all workers in the sample, irrespective of the area where they live; and local deviations from these aggregate trends, represented by the time pattern of local labor market characteristics. While in the sample used the first component cannot be distinguished from the genuine duration dependence, an attempt to control for local labor market trends can be made by conditioning re-employment rates on the time pattern of local labor demand during the whole unemployment spell.

The estimation results using time-varying regressors are reported in Table 5.

The sign and the significance of most explanatory variables in columns I-IV have hardly changed, for both males and females, with respect to the case in which all regressors are time-invariant. In particular, local labor

Table 5: Maximum likelihood estimates of re-employment probabilities with time-varying regressors

Baseline hazard	Weibull		Weibull		Non parametric	
	Males I	Males II	Females III	Females IV	Males V	Females VI
constant	2.006 (0.935)	2.005 (0.935)	-1.542 (1.263)	-1.542 (1.268)	—	—
ln(age)	-0.980 (0.158)	-0.978 (0.158)	0.098 (0.200)	0.100 (0.208)	$-.931$ (0.153)	0.115 (0.201)
not white	-0.236 (0.159)	-0.237 (0.160)	-0.027 (0.267)	-0.021 (0.268)	-0.209 (0.165)	-0.045 (0.250)
health problems	-0.168 (0.088)	-0.169 (0.088)	-0.075 (0.121)	-0.075 (0.124)	-0.161 (0.086)	-0.091 (0.121)
high education	0.482 (0.082)	0.482 (0.082)	0.344 (0.116)	0.345 (0.129)	0.451 (0.081)	0.310 (0.115)
married	0.471 (0.141)	0.471 (0.141)	-0.117 (0.137)	-0.118 (0.139)	0.446 (0.133)	-0.117 (0.138)
children	0.033 (0.036)	0.033 (0.036)	-1.150 (0.177)	-1.149 (0.212)	0.033 (0.034)	-1.086 (0.182)
home owner	0.128 (0.090)	0.128 (0.090)	0.146 (0.136)	0.146 (0.137)	0.122 (0.089)	0.130 (0.137)
had FT job	0.027 (0.172)	0.025 (0.172)	-0.214 (0.126)	-0.213 (0.128)	0.017 (0.168)	-0.178 (0.124)
union member	-0.245 (0.091)	-0.245 (0.091)	-0.404 (0.147)	-0.404 (0.161)	-0.232 (0.089)	-0.378 (0.148)
past unemp.	-0.027 (0.011)	-0.027 (0.011)	-0.007 (0.012)	-0.007 (0.012)	-0.025 (0.012)	-0.008 (0.011)
ln(repl. ratio)	-0.361 (0.041)	-0.361 (0.041)	-0.090 (0.051)	-0.091 (0.055)	-0.346 (0.043)	-0.084 (0.058)
$\ln(U_t)$	-0.224 (0.075)	-0.224 (0.075)	0.107 (0.098)	0.107 (0.099)	-0.206 (0.071)	0.051 (0.098)
$\ln(V_t)$	0.206 (0.082)	0.206 (0.082)	-0.119 (0.103)	-0.119 (0.105)	0.182 (0.077)	-0.061 (0.103)
ln(area)	-0.022 (0.070)	-0.022 (0.070)	-0.139 (0.105)	-0.139 (0.108)	-0.015 (0.067)	-0.137 (0.099)
α	0.863 (0.039)	0.863 (0.039)	0.849 (0.057)	0.851 (0.109)	—	—
σ^2	—	0.005 (1.112)	—	0.100 (1.759)	—	—
χ^2	0.31	0.31	0.06	0.06	0.51	0.05
mean log-likelihood	-2.437	-2.437	-2.121	-2.121	-3.533	-2.765
No. of cases	1239	1239	706	706	1239	706

Notes. Asymptotic standard errors in brackets. The χ^2 statistic is the result of a Wald test of H_0: coef$[ln(U_t)]$+coef$[ln(V_t)] = 0$. Critical value $\chi^2(1, 0.05) = 3.84$. Source: SIIOW and NOMIS.

market variables have the expected sign on the re-employment probabilities for men. Coefficients on $\ln U_t$ and $\ln V_t$ are closely comparable to those on time-invariant regressors $\ln U$ and $\ln V$ (see Table 4) and confirm the presence of constant returns to scale in matching. Similarly as in Table 4, local labor market conditions do not affect significantly the re-employment probabilities of women.

What changes significantly from Table 4 is the relative importance of state dependence versus unobserved heterogeneity. When re-employment probabilities are conditioned on the whole evolution of the state of local labor markets over time, there is evidence of negative duration dependence of hazard rates, as shown in columns I-IV of Table 5. The inclusion of time-varying regressors captures in fact the rise in re-employment probabilities due to the improving prospects of the British economy through the second half of 1987. Interestingly, there is no residual unobserved heterogeneity to be accounted for in individual hazard functions, as shown by the non significant value of σ^2 for both males and females.

Concerning the robustness of the constant returns result, it may be argued that the fully parametric approach adopted, where the functional form for duration dependence is specified as a Weibull distribution, has imposed some unnecessary restrictions on the shape of re-employment probabilities. In order to obtain some more general results, a Cox (1972) proportional hazard model is also estimated. This model is semiparametric in the sense that it does not specify any functional form for duration dependence, and therefore does not predict whether the hazard is upward or downward sloping with duration. The results obtained are reported in the last two columns of Tables 4 and 5, using time-invariant and time-varying regressors in turn.

Column V and VI of Table 4 contain two vectors of estimated coefficients that are virtually unchanged for both men and women from those obtained using a fully parametric model with Weibull duration dependence. The Weibull baseline hazard therefore seems to be a reasonable characterization of the duration distribution of unemployment spells. For both genders the effect of local labor market variables replicates pretty closely the results of columns I and II. Very similar considerations hold for estimates that use time-varying regressors, reported in columns V and IV of Table 5: the impact of local labor market variables does not lead to reject the assumption of constant returns in matching for males, while is not significantly different from zero for females.

The last experiment performed consists in repeating all the estimates by rescaling the number of local unemployed and local vacancies by the population of each TTWA. Clearly, there is considerable variation in TTWA sizes, ranging from 1,389 unemployed and 23 vacancies in Galashiels to nearly

350,000 unemployed and 30,000 vacancies in London (data recorded in April 1987). So it may be plausible that, when time-varying regressors are used, it is the cross-sectional rather than the time series variation of U_t and V_t that is mostly driving the results, delivering strikingly close coefficients on $\ln U_t$ and $\ln V_t$. In order to check this I report in Table 6 a new set of results where the ratios $(U/Pop)_t$ and $(V/Pop)_t$ are used as regressors.[9]

While the effect of most regressors is pretty much unchanged from the results reported in Table 5, nevertheless the coefficients on local unemployment and vacancies are not so close in absolute value as they were when using just the level of U_t and V_t. However, in the re-employment probabilities of men, we cannot reject the hypothesis that they are not significantly different from each other at the conventional confidence levels.

6 Conclusions

The assumption of constant returns in the matching function is a property embodied in most bilateral search models, ensuring the uniqueness of the unemployment rate along a steady state growth path. This paper has investigated whether this is a plausible assumption, by estimating re-employment probabilities on a British sample of entrants into unemployment.

The analysis was led in the context of two alternative continuos-time duration models: a fully parametric hazard model, with Weibull duration dependence and Gamma-distributed unobserved heterogeneity, and a more general Cox proportional hazard model, that does not impose a specific functional form for the baseline hazard. The two specifications delivered very consistent estimates.

The results obtained broadly confirm previous findings on the determinants of re-employment probabilities for men and women (see Devine and Kiefer 1991 for a survey), and are generally consistent with the predictions of a job search framework. The probability of receiving a job offer should be related to the individual's educational attainment and to the state of local labor demand, which in fact affect positively re-employment probabilities for men. No effect of local labor demand is instead detected in re-employment probabilities for women, although this may be at least in part a consequence of how local labor market conditions are measured. The probability of accepting a job offer should in turn depend on the replacement ratio and on the

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[9]An alternative way to control for density effects in matching consists in excluding the London area from the sample, being the TTWA with highest density and size. All the results remained virtually unchanged when London observations were dropped.

Table 6: Maximum likelihood estimates of re-employment probabilities with time-varying regressors - Relative labor market covariates

Baseline hazard	Weibull	Weibull	Weibull	Weibull	Non parametric	Non parametric
	Males	Males	Females	Females	Males	Females
	I	II	III	IV	V	VI
constant	2.429 (1.216)	2.353 (1.218)	−0.179 (1.659)	−0.191 (1.666)	—	—
ln(age)	−0.972 (0.158)	−0.967 (0.158)	0.104 (0.200)	0.105 (0.206)	−0.928 (0.153)	0.121 (0.201)
not white	−0.245 (0.159)	−0.246 (0.159)	−0.049 (0.264)	−0.049 (0.265)	−0.233 (0.163)	−0.063 (0.248)
health problems	−0.169 (0.088)	−0.169 (0.088)	−0.072 (0.122)	−0.072 (0.124)	−0.162 (0.086)	−0.089 (0.121)
high education	0.484 (0.082)	0.484 (0.082)	0.347 (0.116)	0.348 (0.128)	0.455 (0.081)	0.312 (0.114)
married	0.466 (0.141)	0.466 (0.141)	−0.107 (0.135)	−0.108 (0.136)	0.445 (0.133)	−0.107 (0.134)
children	0.032 (0.036)	0.033 (0.036)	−1.155 (0.176)	−1.156 (0.211)	0.034 (0.034)	−1.090 (0.181)
home owner	0.128 (0.090)	0.128 (0.090)	0.147 (0.137)	0.147 (0.137)	0.119 (0.089)	0.130 (0.136)
had FT job	0.026 (0.173)	0.021 (0.173)	−0.222 (0.126)	−0.222 (0.128)	0.015 (0.168)	−0.185 (0.124)
union member	−0.248 (0.091)	−0.249 (0.091)	−0.414 (0.148)	−0.414 (0.161)	−0.227 (0.089)	−0.387 (0.148)
past unemp.	−0.027 (0.011)	−0.027 (0.011)	−0.007 (0.012)	−0.007 (0.012)	−0.025 (0.012)	−0.008 (0.011)
ln(repl. ratio)	−0.357 (0.041)	−0.356 (0.041)	−0.088 (0.051)	−0.088 (0.054)	−0.343 (0.042)	−0.083 (0.057)
$\ln(U/Pop)_t$	−0.172 (0.132)	−0.175 (0.132)	0.186 (0.180)	0.177 (0.183)	−0.239 (0.127)	0.197 (0.174)
$\ln(V/Pop)_t$	0.255 (0.103)	0.251 (0.104)	0.007 (0.141)	0.006 (0.141)	0.178 (0.106)	0.045 (0.139)
ln(area)	−0.045 (0.061)	−0.042 (0.061)	−0.165 (0.092)	−0.165 (0.096)	−0.034 (0.061)	−0.159 (0.088)
α	0.862 (0.039)	0.862 (0.039)	0.850 (0.057)	0.851 (0.109)	—	—
σ^2	—	0.005 (1.022)	—	0.058 (2.968)	—	—
χ^2	0.22	0.18	1.43	1.40	0.11	1.00
mean log-likelihood	-2.437	-2.437	-2.120	-2.121	-3.533	-2.764
No. of cases	1239	1239	706	706	1239	706

Notes. Asymptotic standard errors in brackets. The χ^2 statistic is the result of a Wald test of $H_0 : \text{coef}[ln(U_t)] + \text{coef}[ln(V_t)] = 0$. Critical value $\chi^2(1, 0.05) = 3.84$. Source: SIIOW and NOMIS.

family composition of the individual, which in fact have the expected effect on re-employment probabilities of both men and women.

Concerning the shape of the baseline hazard, clear evidence is found of negative duration dependence in hazard rates for both males and females, when re-employment probabilities are conditioned on the whole evolution of local labor demand during the unemployment spell. This result is also robust to the introduction of Gamma-distributed unobserved heterogeneity.

In no specification does the absolute coefficient on local unemployment differ from that on the number of vacancies, implying that the absolute size of the searching pool does not affect matching rates. This therefore allows not to reject the constant returns hypothesis in the matching technology between unemployment and vacancies. For methodological purposes, this finding in turn implies that the results of several aggregate studies *à la* Blanchard and Diamond (1989) were not too seriously biased by aggregation problems. In a broader perspective, the results of this paper suggest that thicker and more active markets do not necessarily lead to easier trading, at least as far as the number of matches are concerned. One possible avenue of future research consists in assessing whether the matching process may instead display increasing returns as far as the *quality* - as opposed to the *number* - of matches are concerned. This idea would rest on the premise that thick markets provide better matching opportunities to highly specialized labor, and therefore tend to enhance average productivity and wages.

A   Some likelihood functions

According to (12), the $x(t)$ vector includes some variables that are time-invariant, represented by y, and some that are time-varying, represented by $z(t)$, so that

$$\phi_2(x(t)) = \exp\left(y'\gamma + z(t)'\delta\right).$$

Therefore

$$\ln S(t, x(t)) = -\exp(y'\gamma) \int_0^t \alpha s^{\alpha-1} \exp(z(s)'\delta) ds. \tag{17}$$

Suppose now that variables in $z(s)$ assume a finite number of values between time 0 and time t, say 2 for simplicity, such that $z(s) = z_1$ for $0 < s < u$, and $z(s) = z_2$ for $u < s < t$, implying

$$\ln S(t, x(t)) = -\exp(y'\gamma) \left[\exp(z_1'\delta_1)u^\alpha + \exp(z_2'\delta_2)\left(t^\alpha - u^\alpha\right)\right]. \tag{18}$$

Equation (3) can hence be rewritten as

$$
\begin{aligned}
L &= \sum_{i=1}^n c_i \left\{\ln\alpha + (\alpha - 1)\ln t_i + x_i(t)'\beta\right\} \\
&\quad - \sum_{i=1}^n \exp(y_i'\gamma)\left[\exp(z_{i1}'\delta_1)u_i^\alpha + \exp(z_{i2}'\delta_2)\left(t_i^\alpha - u_i^\alpha\right)\right]. \tag{19}
\end{aligned}
$$

Finally, the log-likelihood function with unobserved heterogeneity takes the form

$$
\begin{aligned}
L &= \sum_{i=1}^n c_i \left\{\ln\alpha + (\alpha - 1)\ln t_i + x_i(t)'\beta\right\} \tag{20} \\
&\quad - \sum_{i=1}^n c_i \ln\left\{1 + \sigma^2 \exp(y_i'\gamma)\left[\exp(z_{i1}'\delta_1)u_i^\alpha + \exp(z_{i2}'\delta_2)\left(t_i^\alpha - u_i^\alpha\right)\right]\right\} \\
&\quad - \sigma^{-2}\sum_{i=1}^n \ln\left\{1 + \sigma^2 \exp(y_i'\gamma)\left[\exp(z_{i1}'\delta_1)u_i^\alpha + \exp(z_{i2}'\delta_2)\left(t_i^\alpha - u_i^\alpha\right)\right]\right\},
\end{aligned}
$$

which tends to the log-likelihood in (19) as $\sigma^2 \to 0$.

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