No. 2234

COMPLEMENTARITY, GROWTH AND TRADE

Seppo Honkapohja and Arja Turunen-Red

INTERNATIONAL MACROECONOMICS AND INTERNATIONAL TRADE

Centre for Economic Policy Research

ISSN 0265-8003

COMPLEMENTARITY, GROWTH AND TRADE

Seppo Honkapohja, Academy of Finland and University of Helsinki
Arja Turunen-Red, University of New Orleans

Discussion Paper No.2234
September 1999

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: http://www.cepr.org

CEPR Discussion Paper No. 2234

September 1999

ABSTRACT

Complementarity, Growth and Trade*

We consider an endogenous growth model that includes international trade in capital goods. The model yields several distinct balanced growth solutions that can be classified using stability under adaptive learning. Some of the equilibria can involve growth rates much higher (or lower) than others. The impacts of international trade on the equilibria include *local* (differential) effects and global *bifurcation* (global changes). If a favourable bifurcation occurs, equilibria associated with low growth disappear. This phenomenon suggests a possible explanation for observations in which active international trade by some countries seems to have been associated with periods of exceptionally high growth. We show that equivalent bifurcation effects can be induced in autarky using domestic industry subsidies. However, such subsidization can be very costly.

JEL Classification: F12, F15, 041
Keywords: innovation, international trade, technology policy, multiple equilbria

Seppo Honkapohja
Department of Economics
FIN-00014 University of Helsinki
FINLAND
Tel: (358 9) 1918876
Fax: (358 9) 1918877
Email: seppo.honkapohja@helsinki.fi

Arja Turunen-Red
Department of Economics and Finance
University of New Orleans
2000 Lakeshore Drive
New Orleans, LA 70148
USA
Tel: (1 504) 280 6912
Fax: (1 504) 280 6397
Email: ahtef@mindspring.com

*Financial support from the Academy of Finland and the Yrjo Jahnsson Foundation is gratefully acknowledged.

Submitted 2 September 1999

NON-TECHNICAL SUMMARY

The notion that growth speeds up when an economy is opened to international trade has recently been studied in context, using models of endogenous growth. These have demonstrated a positive relationship between international integration and growth. Free flow of ideas and trade in new inventions can enhance growth as basic economic intuition would suggest.

The literature has mostly relied on models involving single steady state solutions. These solutions are shifted by exogenous events such as, for example, changes in trade policy, and the consequences can be studied as exercises in comparative statistics. The shifts induced by opening a country for trade in such models may be called *local effects* as attention is focused on a given equilibrium. If, however, a model exhibits multiple equilibria for a given configuration of fundamental parameters, then a discrete (non-differential) change can alter the structure of the set of equilibria. The number of steady state solutions may change and consequently an exogenous event may induce large sudden changes, which we call *bifurcation effects*, in growth and other economic variables. The possibility of bifurcation effects on growth arising from international exchange is the focus of this paper.

We examine an open economy extension of the recent endogenous growth model of Evans, Honkapohja and Romer (1998). This model has fully-fledged microfoundations including technological innovations which appear in the form of new capital goods that are produced by profit-seeking entrepreneurs. In the open economy version we develop we assume that (i) all capital goods are complementary to each other and (ii) the opportunity cost of aggregate capital in terms of final consumption increases as production of capital goods and inventions expands. Complementarity among intermediate capital goods and the two-sector production structure imply that multiple balanced growth equilibria can exist in a model of this type, given reasonable values of the model parameters.

In order to classify the various feasible steady state equilibria that may arise we invoke a disequilibrium stability criterion, known as *stability under adaptive learning*. Under this criterion, stable equilibria will be approached via an expectational adjustment process in which agents learn about the equilibrium values of the model variables by observing the outcome of the economic process at any point in time. As a stable equilibrium is approached, expectational errors diminish directing the economy toward a steady state solution.

The model generates a multiplicity of steady state equilibria which are stable under adaptive learning. This feature allows us to show that trade in capital goods between two countries can lead to both local and bifurcation effects. Local effects yield the expected prediction (‘trade implies a higher rate of growth’), assuming that the opportunity cost of aggregate capital does not rise too rapidly. The new and potentially significant possibility concerns the bifurcation effects. We show that trade in capital goods is likely to alter the structure of the set of steady state equilibria. Balanced growth equilibria with low rates of growth tend to disappear, whereas steady state solutions with high rates of growth are maintained and shifted to higher rates of growth. International trade may therefore not only enhance a country's growth performance but may propel it to an entirely new regime of higher growth. This may provide an explanation for empirical observations in which active international trade appears to have been associated with periods of exceptionally high growth.

We also show that the local and bifurcation effects of international trade in capital goods should be similar to the effects of technology subsidies within a closed growing economy. In our symmetric trade model free trade in intermediate capital goods is equivalent in its growth effects to a suitably chosen subsidy (in the corresponding autarkic economy) on purchases of capital goods by a country's competitive producers. This equivalence allows us to numerically simulate the effects of international trade using the simpler autarky framework for one nation. The equivalence also implies that autarkic countries may in principle replicate the growth performance of freely trading but otherwise identical nations by subsidizing the purchases of capital goods. However, such subsidizing can be very costly and we argue that the cost of the subsidy increases with the degree of technological complementarity in the economy. International trade therefore appears to provide a superior vehicle for enhanced growth for most nations.

1 Introduction

The notion that growth speeds up when an economy is opened to international trade has received a significant amount of attention in recent literature. The new models of endogenous growth that have demonstrated a positive relationship between international integration and growth include, e.g., Rivera-Batiz and Romer (1991) and Grossman and Helpman (1990). These models have shown that free flow of ideas and trade in new inventions can enhance growth as basic economic intuition would suggest.[1]

The literature on endogenous growth and international trade has mostly relied on models involving single steady state solutions; these solutions are shifted by exogenous events such as, for example, changes in trade policy, and the consequences can be studied as exercises in comparative statics. The shifts induced by opening a country for trade in such models may be called *local effects* as attention is focused on a given equilibrium.[2] If, however, a model exhibits multiple equilibria for a given configuration of fundamental parameters, then a discrete (non-differential) change can alter the structure of the set of equilibria.[3] The number of steady state solutions may change and consequently an exogenous event may induce large sudden changes, which we call *bifurcation effects*, in growth and other economic variables. The possibility of bifurcation effects on growth arising from international exchange is the focus of this paper.

We examine an open economy extension of the recent endogenous growth model of Evans, Honkapohja and Romer (1998) (subsequently referred to as EHR). This model has fully fledged microfoundations including technological innovations which appear in the form of new capital goods that are produced by profit-seeking entrepreneurs. In contrast to other literature, the closed economy model by EHR and the open economy version we develop assume that (i) all capital goods are complementary to each other and (ii) the opportunity cost of aggregate capital in terms of final consumption increases as production of capital goods and inventions expands.[4] As shown by EHR,

[1]Recent literature has studied various specific aspects of trade, technology and growth. For example, see Feenstra (1996), Dinopoulus and Segerstrom (1999) and Kim (1999) which may be consulted for further references.

[2]This terminology seems acceptable as long as attention is directed at a given solution, even if the comparative statics exercise may involve a discrete (non-differential) change.

[3]Barring exceptional cases, differential changes in model parameters do not alter the structure of the equilibrium set.

[4]Assumption (ii) implies that the relative price of capital to consumption may vary

complementarity among intermediate capital goods and the two-sector production structure imply that multiple balanced growth equilibria can exist in a model of this type, given reasonable values of the model parameters.

In order to classify the various feasible steady state equilibria that may arise, we follow EHR and invoke a disequilibrium stability criterion, known as *stability under adaptive learning.*[5] Under this criterion, stable equilibria will be approached via an expectational adjustment process in which agents learn about the equilibrium values of the model variables by observing the outcome of the economic process at any point in time. As a stable equilibrium is approached, expectational errors diminish directing the economy toward a steady state solution. In contrast, ever increasing forecast errors by agents near unstable equilibria would cause the economy to drift toward the stable solutions.

Our open economy model generates a multiplicity of steady state equilibria which are stable under adaptive learning. This feature allows us to show that trade in capital goods between two countries can lead to both local and bifurcation effects. Local effects yield the expected prediction ("trade implies a higher rate of growth"), assuming that the opportunity cost of aggregate capital does not rise too rapidly. The new and potentially significant possibility, however, concerns the bifurcation effects. We show that trade in capital goods is likely to alter the structure of the set of steady state equilibria. More precisely, balanced growth equilibria with low rates of growth tend to disappear, whereas steady state solutions with high rates of growth are maintained and shifted to even higher rates of growth. International trade may therefore not only enhance a country's growth performance but may propel it to an entirely new regime of higher growth. This observation may be of interest in the context of economic development and it may provide some explanation for empirical observations in which active international trade appears to have been associated with periods of exceptionally high growth.

As trade in capital goods expands the variety of capital goods accessible to a country's production sector, it is perhaps not surprising that the local and bifurcation effects of international trade in capital goods should be similar to the effects of technology subsidies within a closed growing economy. We show that, in our symmetric trade model, free trade in intermediate

across steady states; this does not mean that the relative price of capital increases over time.

[5] An extensive survey of stability under learning is provided in Evans and Honkapohja (1999).

capital goods is equivalent in its growth effects to a suitably chosen subsidy (in the corresponding autarkic economy) on purchases of capital goods by a country's competitive producers.[6] This equivalence allows us to numerically simulate of the effects of international trade using the simpler autarky framework for one nation. The equivalence also implies that countries in autarky may in principle replicate the growth performance of freely trading but otherwise identical nations by subsidizing the purchases of capital goods. However, such subsidizing can be very costly and we argue that the cost of the subsidy increases with the degree of technological complementarity in the economy. International trade therefore appears to provide a superior vehicle for enhanced growth for most nations.[7]

The possibility of sudden changes in an economy due to switches between multiple equilibria has been raised in other contexts. Krugman (1991) and Matsuyama (1991) have discussed the possibility of amplified changes in systems with multiple equilibria in economic geography.[8] In the context of economic development Baland and Francois (1996) have investigated the role of a country's industrial structure in determining whether or not a drastic escape from an underdevelopment trap can be accomplished. Young (1993) constructed a model of invention in which threshold effects arising from small policy changes can alter the economy's opportunity set and growth rate. Recently, Baldwin, Martin and Ottaviano (1998) have combined features from frameworks of economic geography and endogenous growth to study cases of growth and income divergence using a model with multiple equilibria.[9] In macroeconomics, the possibility of multiple equilibria leading to coordination failures has been widely discussed (references are given, e.g., in Section III of EHR). Matsuyama (1995) has noted that monopolistic competition often provides a central underpinning for these phenomena in different contexts.

The organization of the paper is as follows. Section 2 introduces the

[6]In this paper, we only consider subsidies financed by lump sum taxation of the consumer sector. If lump sum taxes are not available, the effects of subsidies are most likely less favorable.

[7]Our model incorporates the assumption that domestically produced and imported innovations are interchangeable in the production of final goods. If this were not the case, then domestic policies would not be able to replicate the growth impact of trade.

[8]The February 1999 Special Issue of *European Economic Review* on International Geography and Trade has several further papers with models of multiple equilibria.

[9]Recently, Mountford (1998) has considered the dynamics in an overlapping generations version of the standard Heckscher-Ohlin model. This model also allows for a multiplicity of stable steady state equilibria under perfect foresight dynamics.

model and characterizes the balanced growth paths for two symmetric nations. A criterion for stability under adaptive learning is developed in order to classify the equilibria. Sections 3 discusses the local and bifurcation effects arising from international exchange of capital goods and Section 4 compares the growth impact of international trade to that of domestic subsidy policy. Section 5 concludes.

2 The Model

In this section, we expand the closed economy model of EHR to include two countries which trade in intermediate capital goods. In order to parameterize the degree of openness of the two economies we assume that all imports are subject to a tariff (which is common for the two countries). The tariff is introduced merely as a convenient means of determining the local impact of trade in the model, and we are not going to focus on trade policy questions.[10] As the economies of the two countries are assumed symmetric, we specify the model by discussing the home country only.[11]

2.1 Basic Assumptions

A competitive production sector in the home country produces a single aggregate final product which is either consumed or invested as raw capital. The home consumer maximizes the discounted utility function

$$U = \sum_{i=0}^{\infty} \frac{\beta^{t+i} C_{t+i}^{1-\sigma}}{1-\sigma}, \tag{1}$$

where C_t denotes total consumption in period t, the parameter β represents the consumer's subjective time preference, and σ^{-1} equals the constant intertemporal elasticity of substitution. (We assume that $\sigma < 1$.)

Final goods are produced according to the technological relationship

$$Y_t = L^{1-\alpha}\left(\int_0^{A_t} x_t(i)^{\gamma}\, di + \int_0^{A_t^*} x_t(i^*)^{\gamma}\, di^*\right)^{\phi}. \tag{2}$$

[10] We use tariffs rather than transport costs as a device to model the degree of openness. The reason is that here trade involves durable capital goods.

[11] The closed economy aspects of our model, including the (common) expectations formation and learning rules of the two identical agents, are similar to those of EHR; we refer the reader to that paper for further details.

In (2), L indicates the fixed amount of labor supplied by the aggregate consumer and $x_t(i)$ (resp. $x_t(i^*)$) denotes the quantities of the A_t (resp. A_t^*) domestic (resp. imported) intermediate capital goods, indexed by i (resp. i^*), which have been invented and are available for use at the time t. The parameter ϕ indicates the degree of technological substitutability or complementarity among the capital inputs; if $\phi > 1$, which we assume, all domestic and imported capital varieties are complements in production.

The complementarity assumption implies that the marginal productivity of each type of machine increases as more of other varieties of capital goods are being produced. This sort of interaction is meant to roughly capture the interconnections of, e.g., computers, printers, fax machines, communications networks etc. We impose the restriction $\alpha = \gamma\phi$ on the technological parameters in order to preserve linear homogeneity of the production process with respect to labor and intermediate inputs.

The demand for the intermediate capital goods by the home country's competitive production sector is determined by the inverse demand functions

$$R_t(i) = L^{1-\alpha}(\int_0^{A_t} x_t(i)^\gamma \, di + \int_0^{A_t^*} x_t(i^*)^\gamma \, di^*)^{\phi-1} \phi\gamma x_t(i)^{\gamma-1}, \quad (3)$$

$$R_t(i^*) = L^{1-\alpha}(\int_0^{A_t} x_t(i)^\gamma \, di + \int_0^{A_t^*} x_t(i^*)^\gamma \, di^*)^{\phi-1} \phi\gamma x_t(i^*)^{\gamma-1}, \quad (4)$$

where $R_t(i)$ (resp. $R_t(i^*)$) is the rental price of domestic (resp. imported) capital varieties.[12]

All intermediate capital goods are supplied by monopolistic competitors. Since the two countries are identical, only capital goods are traded internationally. Introducing an *ad valorem* tariff, we postulate that a fraction c of revenue on exported items arrives to the country of origin (so that the fraction $1 - c$ denotes the import tariff of each country).[13] The profit from producing each of the capital goods which have been developed in the home country then equals

$$\pi_t^i = R_t(i)x_t(i) + cR_t^*(i)x_t^*(i) - (r_t + d_t)p_z^t(x_t(i) + x_t^*(i)), \quad (5)$$

[12] We measure all prices in terms of the final consumption good.

[13] When c equals unity, trade in capital goods is free and costless but when c decreases, such trade becomes progressively more restricted. The value $c = 0$ represents autarky. We assume that all tariff revenue in each country is distributed to the consumer sector as lump sum income.

where $cR_t^*(i)x_t^*(i)$ denotes the after-tariff export revenue on capital variety i. Expression (5) assumes that all capital goods are produced by converting aggregate (raw) capital Z_t into usable machinery in one-to-one ratio. Given that production is realized in the end of each time period (as we assume), the capital goods producers must pay the cost $r_t p_z^t(x_t(i) + x_t^*(i))$ for $(x_t(i) + x_t^*(i))$ units of output, given the price of raw capital, p_z^t, and the interest rate, r_t. The cost of production in (5) also includes a depreciation term, $d_t p_z^t(x_t(i) + x_t^*(i))$, on all intermediate capital varieties.[14]

Substituting (3) and the foreign market equivalent of (4) into (5) and maximizing with respect to $x_t(i)$ and $x_t^*(i)$ we obtain the domestic producers' mark-up rules

$$R_t(i) = \frac{(r_t + d_t)p_z^t}{\gamma}, \; R_t^*(i) = \frac{R_t(i)}{c}. \tag{6}$$

According to (6), all capital goods are equally priced if trade is free and costless (i.e., $c = 1$). However, if the tariff is positive (i.e., $c < 1$), by (6), the rental charged for exported capital goods will exceed the price charged in the domestic market.[15] Still, equations (6) imply that the output quantities $x_t(i)$ and $x_t^*(i)$ do not depend on the index of capital variety, i; this greatly simplifies analysis.

By symmetry we can assume that $A_t = A_t^*$.[16] Given that the capital goods developed in the home and foreign country correspond to different technological innovations, the intra-industry trade in capital varieties between the two countries will be balanced. Using (3) and (4) and noting that A_t and A_t^* are equal, we obtain expressions for machinery output both for the domestic market and for export:

$$x_t = (1 + c^{\frac{\alpha}{\phi-\alpha}})^{\xi} A_t^{\xi} L \left(\frac{R_t}{\alpha}\right)^{\frac{1}{\alpha-1}}; \; x_t^* = c^{\frac{\phi}{\phi-\alpha}} x_t; \; \xi \equiv \frac{\phi - 1}{1 - \alpha}. \tag{7}$$

By (7), an increase in the tariff $(1 - c)$ reduces all capital exports.

[14] Unlike much of the existing literature on endogenous growth, our model includes a depreciation term. The specification in (5) implies that the exporter pays for the depreciation of exported capital goods. This is consistent with the services of machinery being rented out and exported. We assume that new designs (innovations) do not depreciate.

[15] In autarky (i.e., when $c = 0$), we drop the equation for the capital export markup, R^*, and set $x^* = 0$ which then maximizes (5).

[16] This assumption will hold at symmetric balanced growth equilibria; it implies that the number of capital varieties expands at the same rate in both countries.

Following EHR, we assume that each new invention, indexed by i, costs i^{ξ} ($\xi > 0$) units of raw capital to develop. This specification implies that later designs of capital goods are more expensive; however, they are also more valuable, given the complementarity of capital goods.[17] In equilibrium, the discounted stream of monopoly rents arising from the invention of the latest new variety A_t of intermediate capital (obtained using (5) and (7)) must equal the fixed cost of the invention, i.e., we require that

$$p_t^z A_t^{\xi} = \sum_{s=0}^{\infty}(1+r_{t+s})^{-(s+1)}\left[k(c)\Omega A_{t+s}^{\xi}((r_{t+s}+d_{t+s})p_{t+s}^z)^{\frac{\alpha}{\alpha-1}}\right], \qquad (8)$$

where $\Omega \equiv \phi^{\frac{1}{1-\alpha}}(1-\gamma)\gamma^{\frac{1+\alpha}{1-\alpha}}L$ (>0) and the impact of the tariff parameter c on the profitability of the monopolistically competitive industry is represented by the term

$$k(c) = (1+c^{\frac{\phi}{\phi-\alpha}})(1+c^{\frac{\alpha}{\phi-\alpha}})^{\xi}. \qquad (9)$$

The function $k(c)$ is positive and increasing in c, so that *ceteris paribus* a reduction in the tariff $(1-c)$ raises the profits of the intermediate capital producers (i.e., the square-bracketed expression in (8)).[18]

The market value of aggregate (raw) capital in each country is determined using the competitive production sector's trade-off between investment and final consumption. Following EHR, we postulate a standard production possibility frontier between consumption and investment, given by

$$C_t = Y_t - Z_t\chi(\frac{Z_{t+1}-Z_t+D_t}{Z_t}),\ D_t \equiv d_t((1+c^{\frac{\phi}{\phi-\alpha}})x_tA_t), \qquad (10)$$

where χ is a convex cost function, D_t denotes the quantity of depreciated capital, and the total stock of gross investment (raw capital) equals

$$Z_t = \int_0^{A_t}(x_t(i)+x_t^*(i))di + \int_0^{A_t} i^{\xi}di = (1+c^{\frac{\phi}{\phi-\alpha}})x_tA_t + \frac{A_t^{1+\xi}}{1+\xi}. \qquad (11)$$

Using (10), we obtain the price of aggregate capital in terms of consumption:

$$p^z = -\frac{dC_t}{dZ_{t+1}} = \chi'(\frac{Z_{t+1}-Z_t+D_t}{Z_t}). \qquad (12)$$

[17] The specification of the invention cost guarantees that the model will exhibit balanced growth equilibria with constant growth rates.

[18] Parameter Ω can generally be interpreted as reflecting total factor productivity of capital.

If the cost function χ is linear ($\chi'' = 0$), the price of capital is a constant, determined by (12). This corresponds to the case where the production technologies for all three industries in the economy (the competitive sector, the R&D sector and the machinery producers) are the same. If, however, the cost function is strictly convex ($\chi'' > 0$), then technologies between the perfectly competitive and monopolistically competitive producers differ, whereby the opportunity cost of capital increases as the economy's investment in R&D and/or intermediate capital production increases (see EHR for a detailed discussion of the different cases).

2.2 Balanced Growth

In a symmetric perfect foresight equilibrium with balanced growth and constant depreciation ($d_t \equiv d$), the interest rate and the growth rate of capital designs, $g_A \equiv A_{t+1}/A_t$, are constant and equal in both countries. Since the growth rate of aggregate capital, g_z, must equal $g_A^{1+\xi}$, we obtain, using (8), that

$$g_z = \left[1 + r - k(c)\Omega(p_z)^{\frac{1}{\alpha-1}}(r+d)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{\phi-\alpha}{\phi-1}}. \tag{13}$$

By (12), the growth rate of aggregate capital must also satisfy the equation

$$p^z = \chi'(g_z - 1 + d\eta_t), \tag{14}$$

where the fraction of (raw) capital invested in machines is defined as

$$\eta_t \equiv (1 + c^{\frac{\phi}{\phi-\alpha}})x_t A_t / Z_t$$

and equals

$$\eta_t = 1 - \frac{1}{(1+\xi)\mu_t}, \; \mu_t \equiv \left\{\frac{1}{1+\xi} + k(c)L\left[\frac{R_t}{\alpha}\right]^{\frac{1}{\alpha-1}}\right\}. \tag{15}$$

In the subsequent sections, changes in η_t arising from differential and discrete perturbations of the trade parameter c will play an important role in determining the growth consequences of international trade.

Conditions (13) and (14) define the combinations of the interest rate, r, and the capital growth rate, g_z, which are consistent with the production structure of the model. On a symmetric balanced growth path the Euler equation for consumer's optimization implies that aggregate consumption will grow at the rate

$$g_{cons} \equiv \frac{C_{t+1}}{C_t} = [\beta(1+r)]^{1/\sigma} . \tag{16}$$

Combining (16) with (13), (14) and (15) and observing that $g = g_{cons} = g_z$ in a steady state equilibrium, we obtain, for a given level of the parameter c, a complete description of feasible steady state equilibria in each country.

Given a fixed tariff rate $(1-c)$ and applying the solution for the relative price of (raw) capital in (14), equations (13) and (16) can be used to illustrate the determination of steady state equilibria in (g, r)-space.[19]

FIGURE 1 HERE

The upward sloping curve CC in Figure 1 depicts the combinations (g, r) that satisfy the consumer side equilibrium condition (16), whereas the curve TT graphs the combinations of the interest rate and the growth rate of capital for which the production equilibrium conditions (13) and (14) hold.

The slope of TT is determined by the curvature of the production possibility frontier between consumption and investment (χ''), the degree of capital depreciation (d) and the level of technological complementarity between capital goods (ϕ).[20] Given our basic assumption that capital goods are complementary to each other (i.e., $\phi > 1$), the curve TT is upward sloping for all g if the economy exhibits a single aggregate productive sector. Further, TT is locally upward sloping whenever the relative price of (raw) capital is constant (i.e., whenever $\chi'' = 0$).[21] However, TT can easily possess downward sloping segments if the relative price of capital is allowed vary (i.e., $\chi'' > 0$). In this case the profitability of the monopolistically competitive sector may be sufficiently sensitive to changes in the price of capital or, given $d > 0$, the cost of replacing capital equipment may become sufficiently large so as to require a reduction in the interest rate as the economy grows faster.

In Figure 1 steady state equilibria appear as intersections of the curves CC and TT. Since CC is upward sloping while TT can have both upward and

[19] Figure 1 is our open economy version of Figures 2 and 3 of EHR.

[20] Expressions for the slopes of the curves CC and TT are given in the Appendix.

[21] In economic terms, the positive slope arises from the observation that, due to complementarity, current capital goods become more valuable when the economy grows faster. The equilibrium interest rate must therefore rise in order to maintain the intertemporal zero profit condition of the monopolistically competitive industry.

downward sloping segments, multiple intersections can occur; a case of four intersections, denoted by $E1$ to $E4$, is illustrated in Figure 1. Each of the four intersections of CC and TT in Figure 1 corresponds to a distinct balanced growth path and the equilibrium rates of growth associated with these steady states vary quite significantly. Depending on the model parameters and subsequently the relative shapes of the CC and TT curves, the model can yield equilibria involving growth rates much higher (lower) than others.

2.3 The Stability Condition

This model, like many other models of endogenous growth, possesses no traverse dynamics. This means that, starting from any initial condition, the economy will immediately evolve along a balanced growth path.[22] Therefore, the multiple steady states that are obtained as solutions to equations (13)-(16) cannot be classified using stability under perfect foresight. Nevertheless some of the possible equilibria can be discarded as being *unstable under adaptive learning*. This type of instability involves, as a part of the model, a notion of expectation formation and learning undertaken by economic agents.

We apply the adaptive learning rule of EHR which centers around discrepancies between the actual interest rate and the rate agents expected to obtain.[23] As agents adjust their expectations toward the realized rate of interest the economy will not find its way to any of the unstable equilibria. Rather, stable equilibria will be approached through an expectational adjustment process when the economy starts from nearby values of g and r. In the following the terms 'stability' and 'stable solution' will refer to the concept of stability under adaptive learning.

The following lemma formally states a condition which is sufficient for stability of equilibria under adaptive learning in our model.

Lemma 1 *A steady state solution to equations (13)-(16), denoted by (g^*, r^*), is stable under adaptive learning if*

$$\mathcal{B} \equiv \frac{dr_z(g^*)}{dg} - \frac{dr_{cons}(g^*)}{dg} < 0, \tag{17}$$

[22] As in EHR, the present model has only one state variable, Z, the initial state of which does not determine the solutions (g, r) for equations (13)-(16). Therefore, given any initial value of Z, the economy can select any one of the possible growth paths characterized by the intersections of the curves CC and TT.

[23] For the precise formulation of this adaptive learning rule, see EHR (Section II.F).

where $r_{cons}(g)$ and $r_z(g)$ denote the inverse solutions to equations (13) and (16), given (14) and (15). A steady state is unstable if $\mathcal{B} > 0$.

In (g, r)-space (Figure 1), condition (17) can be interpreted using the relative slopes of the CC and TT curves. Specifically, at a balanced growth solution, the technology curve TT must cut the preference curve CC, i.e. the inverse function of (16), from above.[24] In Figure 1, two equilibria - one associated with a low rate of growth, $E1$, and one with high growth, $E3$ - satisfy this condition. There are also two equilibria ($E2$ and $E4$) in the figure which are unstable. As the CC curve is monotonically increasing whereas the slope of TT may vary it is evident that, barring exceptional cases, any two stable equilibria must be separated by one that is unstable and vice versa.

According to condition (17) the positive slope of the CC curve also implies that a stable equilibrium may involve either a negative or a positive slope for TT. Figure 1 has been drawn to show a positive slope for TT at the low growth stable equilibrium $E1$, whereas the slope of TT is negative at the stable high growth steady state $E3$. By (17), an unstable steady state must necessarily involve a positive slope for TT which is steeper than that of CC. This is the case at equilibria $E2$ and $E4$ in Figure 1.

The stability condition (17) can be written in full using the definitions of the functions $r_z(g)$ and $r_{cons}(g)$. This involves fairly complex expressions which can be interpreted as the impact of changes in the interest rate on the production and consumption sectors of the economy while also taking into account possible changes in the relative price of (raw) capital. The complete condition, including the definitions of the various derivatives, is given in the Appendix.

3 Local and Bifurcation Effects of International Trade

Having characterized the stable (symmetric) balanced growth paths of the model, we study the impact of an expansion in international trade on these solutions. We first consider a differential change in the trade parameter c; this allows us to determine the local impact of trade on equilibria. Secondly,

[24] The stability condition utilizing slopes of curves was formulated in EHR. Since the TT curve can have both upward and downward sloping segments, it is convenient to describe the stability condition in terms of the inverse functions.

we determine how the set of feasible steady states responds as openness toward trade, as described by c, is altered. The latter exercise illustrates the bifurcation effects of international trade.

3.1 Local Effects

A small perturbation in the trade parameter c affects the solutions to equations (13), (14) and (15). Geometrically, in Figure 1, such a change yields a shift in the technology curve TT, while the CC curve remains unchanged.[25]

For clarity, it is convenient to distinguish between what might be called the *direct* and *indirect* effects of a change in the parameter c. The direct effect of an increase in c, which, according to (5), corresponds to an increase in the proportion of export revenue flowing to the exporting countries, is to increase the supply of foreign capital goods in each country; this expansion in the number of capital varieties tends to increase the equilibrium growth rate. However, higher growth also tends to raise the price of (raw) capital and the cost of capital replacement, and this indirect effect dampens the growth impact of expanded trade.

3.1.1 The Direct Impact of Expanded Trade

The direct effect of a perturbation in the trade parameter c is isolated by assuming that the production possibility frontier between consumption and (raw) capital is either globally or locally linear (i.e., $\chi'' = 0$). In this case differentiation of the model (13)-(16) at a steady state solution (g^*, r^*) yields

$$\frac{dg}{dc}\mid_{g=g^*} = \mathcal{A}\frac{dr}{dc}\mid_{r=r^*} = -\frac{\mathcal{C}\mathcal{F}}{\mathcal{B}\left[\mathcal{C}(1-\mathcal{E})\right]}, \tag{18}$$

where $\mathcal{B}$ comes from the stability condition in Lemma 1 and the terms $\mathcal{A}$, $\mathcal{C}$, $\mathcal{F}$ and $(1-\mathcal{E})$ are positive.[26] The sign of the denominator $\mathcal{B}$ is determined by the stability properties of the equilibrium in question. We thus obtain

Proposition 2 *Let the trade parameter c be given and let a corresponding equilibrium solution to (13)-(16) be stable (resp. unstable) under adaptive*

[25] The CC curve is independent of the parameter c in the present model because we have assumed that tariff revenue is returned to the consumer sector as lump sum income.

[26] The terms $\mathcal{A}$, $\mathcal{C}$, $\mathcal{F}$ and $\mathcal{E}$ are defined in the detailed discussion of the stability condition in the Appendix.

learning. Then, assuming that the opportunity cost of (raw) capital is constant (i.e., $\chi'' = 0$*) around the initial equilibrium, an increase in the degree of openness to international trade in capital goods (i.e., a differential change* $dc > 0$*) will raise (resp. reduce) the economy's growth rate,* g*.*

Proposition 2 can be illustrated using Figure 1. Expanded trade in capital goods will shift the curve TT upwards along its positively sloped segments[27] (i.e., corresponding to any g_z, increased access to complementary capital imports mandates an increase in the interest rate so as to maintain zero profitability of the monopolistically competitive producers), whereas the curve CC will not change. Consequently, the unstable equilibria $E2$ and $E4$ located on the upward sloping portions of the TT curve will now exhibit lower growth but the stable low growth equilibrium $E1$ is shifted up toward higher growth.

This stable response illustrates the intuitive conclusion of Proposition 2: "a more open (symmetric) world adaptively learns to grow faster" - at least, if one only allows for the positive effect of the increased access to imported capital goods. Proposition 2 cannot be applied to infer the effect of increased capital goods trade on the stable steady state equilibrium $E3$ since the TT curve is locally downward-sloping (i.e. $\chi'' > 0$) nearby. Further analysis of this case is required.

3.1.2 The Indirect Impact of Expanded Trade:

The conclusions of Proposition 2 accord well with our basic intuition regarding growth and trade. Yet, the full consequences of trade in capital goods are not always as straightforward. The indirect effect of trade on the opportunity cost of capital must also be taken into account, and such a worsening of the economy's cost conditions can even lead to locally lower growth. In Figure 1 this paradoxical case corresponds to a downward shift in the technology curve TT along its downward sloping segments (where the relative price of capital increases as the economy grows faster).

Adding the indirect effects to the growth derivative (18) so as to take into account the adjustment in the price of (raw) capital, we obtain

$$\frac{dg}{dc}\,|_{g=g^*} = -\frac{(\mathcal{CF} + \frac{\mathcal{CD}}{\mathcal{S}}\chi'' d\frac{d\eta_t}{dc})}{\mathcal{B}\left[\mathcal{C}(1-\mathcal{E}) - \frac{\mathcal{CD}}{\mathcal{S}}\chi'' d\frac{d\eta_t}{dr}\right]}, \tag{19}$$

[27] As indicated in Section 2.2, the slope of TT is positive if $\chi'' = 0$.

where the terms $\mathcal{C}$, $\mathcal{F}$, $\mathcal{S}$ and $\frac{d\eta_t}{dc}$ are positive, $\mathcal{D}$ and $\frac{d\eta_t}{dr}$ are negative, and the sign of $\mathcal{B}$ follows from the stability condition. The terms involving χ'' in (19) spell out the growth impact of the reduction in the profitability of the monopolistically competitive industry due to an increase in the price of capital and the possible subsequent increase in the cost of depreciation replacement.[28] It is evident from (19) that these adverse effects may exceed the positive direct impact of expanded trade even at stable equilibria.

Proposition 3 *Let the trade parameter c be given and the corresponding steady state solution to (13)-(16) be stable (resp. unstable) under adaptive learning. Then, an expansion in international trade in capital goods ($dc > 0$) will raise (resp. reduce) the growth rate, g, if and only if*

$$\left[\mathcal{C}(1-\mathcal{E}) - \frac{\mathcal{C}\mathcal{D}}{\mathcal{S}}\chi'' d\frac{d\eta_t}{dr}\right]^{-1}\left(\mathcal{C}\mathcal{F} + \frac{\mathcal{C}\mathcal{D}}{\mathcal{S}}\chi'' d\frac{d\eta_t}{dc}\right) > 0 \ (< 0). \qquad (20)$$

According to Proposition 3 the local growth impact of international trade in capital goods is determined by the relative size of its direct and indirect effects. If the direct effect (indicated by the terms $\mathcal{C}(1-\mathcal{E}) > 0$ and $\mathcal{C}\mathcal{F} > 0$ in (20)) exceeds the reduction in profits arising from the higher depreciation cost of capital goods (represented by the terms $-\frac{\mathcal{C}\mathcal{D}}{\mathcal{S}}\chi'' d\frac{d\eta_t}{dr} < 0$ and $\frac{\mathcal{C}\mathcal{D}}{\mathcal{S}}\chi'' d\frac{d\eta_t}{dc} < 0$), then growth accelerates at stable solutions. This happens in particular if the opportunity cost of capital is nearly constant or if the depreciation parameter is small. In Figure 1, this "normal" case corresponds to an upward shift of the stable equilibrium $E3$. However, when the economy's transformation frontier between consumption and total capital is significantly curved (i.e., $\chi'' > 0$ is large) and the depreciation rate of physical capital is high, the technology curve TT may shift locally down around equilibria such as $E3$ in Figure 1 resulting in slower growth.

Condition (20) makes it clear that the possible negative impact on growth arises from the depreciation cost of capital and not from the higher cost of developing or manufacturing new capital goods. Accordingly, without depreciation condition (20) and Proposition 3 yield

[28] Since η_t represents the fraction of (raw) capital invested in machinery, the term $\chi'' d\frac{d\eta_t}{dc}$ equals the increase in the replacement cost of capital given that the price of capital will change as a result of a change in c. The term $\chi'' d\frac{d\eta_t}{dr}$ takes into account the change in the replacement cost arising from the corresponding change in the interest rate.

Corollary 4 *Assume that capital goods do not depreciate, i.e., $d = 0$. Then, an increase in the degree of openness to international trade in capital goods raises (resp. reduces) the economy's growth rate at stable (resp. unstable) steady state equilibria.*

Despite the above corollary, a conclusion arising from Proposition 3 is that increased openness to trade can sometimes lead to slower economic growth. Such a case would probably be rarely observed, but it can happen if the production possibility frontier between consumption and investment is very curved (i.e., χ'' is large). In such economies, the price of aggregate (raw) capital may be sufficiently sensitive to changes in capital investment so as to nullify the local positive growth impulse from trade. Starting from an equilibrium with low growth, such economies could experience a local worsening in their performance.

3.1.3 Welfare Consequences

The growth effects of international trade do not directly translate into changes in welfare. Still, under balanced growth, the aggregate consumer's lifetime utility, defined in (1), equals

$$V = \frac{C_0^{1-\sigma}}{1-\sigma}\left(\frac{1}{1-\beta g^{1-\sigma}}\right). \tag{21}$$

By (21) an unanticipated change in the trade parameter c can affect welfare by altering the growth rate g (and the interest rate r) and by changing the level of initial consumption, C_0. In the "normal " case a reduction in the tariff $(1-c)$ speeds up growth and this yields a positive impact on welfare in (21). The corresponding decrease in current consumption, C_0, however, must reduce the consumer's lifetime utility. The total effect is theoretically ambiguous, though in most cases the growth effect is likely to dominate. This happens in all numerical illustrations reported below.

3.2 Bifurcation Effects

We now specify the model further and postulate that the transformation frontier between consumption and aggregate capital takes a continuous, nearly piecewise-linear form with three linear segments, connected by two regions of sharp curvature. (This form was used by EHR (1998) in the closed economy context. See the Appendix for some details.) The linear segments of the transformation curve yield upward sloping segments on the economy's technology curve TT (analogously to Figure 1). On these linear segments the price of capital, p_z, is locally constant. In contrast, the opportunity cost of capital increases at a rapid rate on the curved parts of the production possibility frontier near the junctions of the linear segments. Corresponding to these segments of the transformation frontier we obtain regions of the TT curve which are downward-sloping.

For a given level of the trade parameter c the specification of the production possibility frontier allows us to numerically graph the equivalent of Figure 1 once a set of the model parameters has been chosen. Figure 2a below is obtained using the following parameter values.[29]

α	Ω	ϕ	d	β	σ	L
0.39	0.0225	1.30	0.09	0.95	0.22	0.3249

TABLE 1: Parameter values for numerical evaluation.

The dashed technology curve TT in Figure 2a represents the locus of solutions to equations (13)-(14) in autarky (i.e., when $c = 0$). The curve exhibits upward and downward sloping segments as discussed above. Given the upward-sloping preference curve CC, we obtain two steady state solutions which are stable under adaptive learning (one with a high and one with a low rate of growth).

3.2.1 A Bifurcation Example

In Figure 2a, the bifurcation effect of international trade corresponds to a shift of the TT curve which is sufficiently large to alter the set of equilibria. An example of such an effect is obtained by comparing the set of feasible steady state equilibria in autarky (i.e. when $c = 0$) to those obtained under free and costless international exchange of capital goods (i.e. when $c = 1$). In Figure 2b, the technology curve corresponding to the parameter value $c = 1$ is graphed as the dashed TT curve.

FIGURES 2 HERE

[29]Complete *Mathematica* programs that have been used to generate the data in this paper are available from the first author upon request.

Evidently, in the case depicted in Figures 2, the introduction of international trade in capital varieties has led to a *favorable bifurcation*: the set of stable autarky equilibria which includes both low and high growth states has been replaced by a single, stable, high growth equilibrium under free trade.

Figure 2b suggests that countries that have been persistently stuck on a low growth path in autarky can transform themselves to high growth achievers when access to foreign capital goods is introduced. During the period of adjustment the countries' economies adaptively learn their way to the high growth state and, once there, a presumably much higher level of welfare is permanently obtained. Another possibility is that the countries may have experienced random growth cycles (i.e., a sequence of low and high growth equilibria) in autarky.[30] Due to the bifurcation effect arising from international exchange, such growth cycles would entirely disappear. Such "stabilization via bifurcation" should provide additional welfare gains as resources are redirected to productive uses from purely hedging or speculation type activities.

3.2.2 The Existence of Bifurcation Effects

Figures 2 give only one example of a change in the set of equilibria. Since such changes have potentially large welfare consequences the existence of bifurcations is of general interest. We have seen that changes in the countries' degree of openness, as reflected in the parameter c, shift the technology curve TT while the preference curve CC is independent of c. The direction of the shifts in TT (an expansion of trade, i.e. an increase in c, usually shifts TT upwards in (g, r) space[31]) suggests that, under fairly general conditions, increasing the scope for international trade can lead to favorable bifurcation effects.

Consider, for example, the situation shown in Figure 1 where two stable equilibria are separated by an unstable steady state. If we now allow the trade parameter c to increase, the curve TT will shift up and eventually, when some critical value c^* is attained, one of the segments of the TT curve near its local minima will lie above the CC curve, except for one point touching CC. (Note that normally a segment around the other stable steady state will

[30] The existence of such cycles in a closed economy version of the model was demonstrated by EHR.

[31] As noted in Proposition 3, the TT curve can shift down along its downward sloping segments in exceptional cases. We ignore this possibility in the ensuing discussion.

still lie below CC when $c = c^*$.) If the parameter c is increased further, both a stable and an unstable steady state will disappear. Should this happen first for the low growth stable steady state, a *favorable growth bifurcation* has taken place. The economy will then move to a state of permanent high growth.

The preceding numerical example in Figures 2 is an illustration of a favorable growth bifurcation obtained when the frontier $\chi(.)$ between consumption and aggregate capital is almost piecewise linear and specified by particular parameter values. More generally for economies characterized by piecewise linear opportunity frontiers, we know that if the rounding off of corners in the $\chi(.)$ function takes place in a short interval for its argument, the segments of the TT curve will slope downward very steeply. The existence of a favorable bifurcation can then be described in terms of the corners in the TT curve. We can identify these corners by finding the lowest values of the variables g and r which are consistent with equation (13) and the (fixed) values of the capital price p_z that pertain to the upward sloping segments of TT.

Specifically, given c, we may first set $p_z = p_z(high)$, where $p_z(high)$ is the highest (fixed) relative price of capital along the linear segments of the frontier $\chi(.)$, and then choose the lowest values of g and r, denoted by $g_H(c)$ and $r_H(c)$, which solve (13) for $p_z = p_z(high)$. This identifies the equivalent of the upper local minimum of the TT curve in Figure 1. Likewise, we may determine $g_M(c)$ and $r_M(c)$ which are the lowest values for g and r consistent with (13) when we set $p_z = p_z(med)$, where $p_z(med)$ is the price of capital on the middle linear segment of $\chi(.)$. The following result gives a subsequent condition for presence of a favorable bifurcation.

Proposition 5 *Assume that the opportunity frontier $\chi(.)$ is piecewise linear with three linear segments (and rounded corners). Suppose that when $c = 0$ the model exhibits two steady state equilibria which are stable under adaptive learning and that for $c = 1$ only a unique stable equilibrium exists. Then, there exists a critical level of the trade parameter, c^*, such that for any value of c lower than the critical level (i.e. $c < c^*$), the model exhibits two stable steady states, whereas for all values of c larger than the critical level (i.e. $c > c^*$), only one stable equilibrium exists. If $g_M(c^*) = [\beta(1 + r_M(c^*)]^{1/\sigma}$ and $g_H(c^*) < [\beta(1 + r_H(c^*)]^{1/\sigma}$ the bifurcation at c^* is favorable.*

The above condition characterizing the presence of a favorable growth bifurcation guarantees that it is the low growth steady state and the nearby unstable equilibrium that disappear when the trade parameter c exceeds the critical value c^*. This ensures that the economy can only find its way to a high growth state once the expansion of trade is sufficiently large.[32,33]

In terms of Figure 1, a favorable bifurcation is likely to occur if the TT curve is more sensitive to changes in the trade parameter c (and therefore shifts up more as c increases) for relatively low values of r and g and less sensitive when the interest rate and growth rate are high. Under these circumstances, an expansion in countries' trade opportunities can more easily shift the lower minimum of the TT curve above the curve CC, whereas the high minimum of TT is likely to remain below CC. This sort of asymmetry in the shift reactions of the TT curve seems intuitively quite plausible as the equilibria at low values of r and g are likely to involve relatively small numbers of capital goods in production. In such situations, expanding access to complementary capital imports is likely to cause a larger increase in profitability of the few existing types of machinery, thus necessitating a larger increase in the equilibrium interest rate (and thus a larger shift up in the TT curve) for any level of g. On the other hand, when the interest rate and growth rate are already high and many capital goods have already been developed, additional access to more varieties of machinery is likely to have a smaller positive impact on the profitability of any particular existing variety and the resulting shift in TT is therefore relatively small.

Proposition 5 shows that international exchange of capital goods can produce a significant accelerating effect on growth. If the shift in the countries' trade orientation is sufficiently large so that the trade parameter c exceeds the critical level, the low and middle growth equilibria associated with restricted trade regimes can entirely disappear, leaving only a path of very high growth as the feasible stable equilibrium. In this case (illustrated by the above example) the countries learn to realize a much higher level of welfare than one would expect merely on the basis of the positive local growth impact of trade. Even in cases where the local growth effect of trade may be negative (i.e., for small increases in c, growth decelerates due to an increasing price of (raw) capital), there still exist values of c sufficiently large so as to

[32] When $g_M(c^*) = [\beta(1 + r_M(c^*)]^{1/\sigma}$, the lower corner (local minimum) of TT lies on the curve CC and, since $g_H(c^*) < [\beta(1+r_H(c^*)]^{1/\sigma}$, the upper corner must lie below CC. Therefore, when $c > c^*$, the only stable equilibrium must involve high growth.

[33] Clearly, if the frontier $\chi(.)$ exhibits more than three linear segments, analogous arguments can be made to determine conditions which are sufficient to guarantee the existence of various sets of stable equilibria (associated with different levels of growth).

guarantee the bifurcation "growth-jump". Thus, the local and bifurcation effects of international trade do *not* necessarily work in the same direction and it "pays to be bold" (i.e. the shift in the trade parameter c should exceed the critical threshold).

Consideration of the learning dynamics yields further implications. Suppose that initially the trade restricting tariff is relatively high and the two countries are in a low growth steady state. Then a reduction in (symmetric) tariffs to levels lower than $(1 - c^*)$ will take, via learning dynamics, the two countries to the unique high growth equilibrium. In reverse, however, if the initial situation involves low tariffs and high growth, a movement toward more restricted trade (i.e., lowering of c), for tariff levels higher than $(1 - c^*)$ will *not* necessarily take the countries to a low growth state. Learning dynamics will keep the countries near the high growth steady state, provided such a state continues to exist after the increase in tariffs. The possibility of an *unfavorable bifurcation* emerges only if the increase in tariffs leads to a situation where only the low growth stable steady state exists. With learning dynamics included the model can therefore exhibit *hysteresis effects*.[34] This possibility seems to strengthen the growth-based argument for trade liberalization as, having experienced high growth under liberal regimes, countries may be less susceptible to negative growth consequences of perhaps temporary protectionist regimes.

Proposition 5 is crucially dependent on the assumption that multiple stable steady states exist for the initial parameter configuration:

Corollary 6 *Bifurcational changes in the set of equilibria will not exist if the economy has a unique stable steady state.*

The possibility of a unique stable equilibrium arises, for example, (i) if the economy has a single aggregate production sector (i.e., $\chi'' = 0$), or (ii) if the economy possesses a two-sector production structure but capital varieties are not complementary in production.[35] More generally, the shape of the technology curve TT and in particular the steepness of its (middle) positive segment in relation to the curve CC determine the feasibility of multiple (stable) equilibria. The slope of the middle upward segment of TT

[34] A detailed discussion of such effects in the context of a simple overlapping generations model with increasing returns can be found in Evans and Honkapohja (1993).

[35] See EHR for more details on these cases. We remark that in case (i) further conditions on the curvature of the TT curve need to be imposed.

is affected by the supply side parameters, including the productivity term Ω and the complementarity parameter ϕ; for low values of ϕ and Ω the middle segment of the TT curve may not be sufficiently steep for the high growth equilibria of Figure 2a to exist. In such a case, the only feasible balanced growth path may involve relatively low growth. Figures 3 illustrate.

FIGURES 3 HERE

Figures 3 are obtained using the parameter values of Table 1, except that we have replaced the technological complementarity parameter by the low value $\phi = 1.01$ while also reducing the total factor productivity, so that $\Omega = 0.01$. In Figure 3a, which is drawn for autarky, the unique steady state now involves low growth. From Figure 3b illustrating the free trade outcome we see that the two countries will still gain from the opportunity to exchange capital goods but the gain will only manifest itself as a small local growth effect around the initial (low) equilibrium. In this economy access to foreign varieties of capital goods fails to induce, through capital complementarity, sufficient new investment for a stable high growth equilibrium to appear. If it is the case that advancing technology spurs more capital complementarity, then Figures 3 suggest that countries with the lowest levels of technological sophistication and low total factor productivity are the least likely to experience bifurcational growth spurts due to trade.

4 Domestic Subsidies and Growth

The relative roles of international trade and domestic policy in spurring growth and development have generated much discussion. Accordingly, we can ask whether the local and bifurcation effects of trade can be replicated using solely domestic policy within the present model.

As trade in capital goods expands the variety of intermediate inputs accessible to a country's production sector, one expects that the local and bifurcation effects of trade resemble the corresponding effects of a suitably chosen technology subsidy. We consider an *ad valorem* subsidy for purchases of all capital varieties in each country.[36] The subsidy finances the fraction $(1 - s)$ (where $0 < s < 1$) of the purchase price of all capital varieties. I.e.

[36] For simplicity we assume that the subsidy is financed by a lump sum tax on the consumption sector.

given the subsidy, the consumer price of each unit of capital services (paid by the producers of final consumption goods) equals $sR_t(i)$, whereas the corresponding producer price received by the monopolistically competitive firms remains at $R_t(i)$.

Following the procedure outlined in Section 2, we obtain the equations which characterize the closed economy steady state equilibria:[37]

$$g_z = \left[1 + r - s^{\frac{1}{\alpha-1}}\Omega(p_z)^{\frac{1}{\alpha-1}}(r+d)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{\phi-\alpha}{\phi-1}} \tag{22}$$

$$p^z = \chi'(g_z - 1 + d\eta_t),\ \eta_t \equiv x_t A_t / Z_t, \tag{23}$$

$$\eta_t = 1 - \frac{1}{(1+\xi)\mu_t},\ \mu_t \equiv \left\{\frac{1}{1+\xi} + L\left[\frac{sR_t}{\alpha}\right]^{\frac{1}{\alpha-1}}\right\}. \tag{24}$$

A comparison of these conditions and equations (13) and (15) reveals that the two sets of equilibrium conditions are equivalent if we choose the capital subsidy according to the equation $s^{\frac{1}{\alpha-1}} = k(c)$.[38] Denoting this subsidy solution by s_c, we obtain that there exists a *trade-equivalent rate of capital subsidy*, $1 - s_c$, in the closed economy yielding the set of equilibria for (g, r) which is identical to that obtained under international exchange of capital goods when the trade openness parameter takes a given value c. The following proposition states the solution for s_c.

Proposition 7 *Consider the model (13)-(16) and let Q_c be the set of equilibrium solutions corresponding to the trade parameter c $(0 < c \leq 1)$. The equilibrium solutions Q_c can also be attained by choosing in the closed economy model (22)- (24) and (16), the capital subsidy $1 - s_c$, where*

$$s_c \equiv \frac{(1 + c^{\frac{\phi}{\phi-\alpha}})^{1-\phi}}{(1 + c^{\frac{\alpha}{\phi-\alpha}})^{1-\alpha}}\ (<1). \tag{25}$$

Proposition 7 indicates that in our symmetric two country model trade in intermediate capital goods is equivalent in its growth effects to a suitably chosen domestic subsidy on purchases of capital goods. This equivalence implies that in the absence of international trade the countries may replicate

[37] We replace the left hand side of equation (3) by $sR_t(i)$ and set $x^*(i) = x(i^*) = 0$ in (5) and solve for the equivalents of equations (13)-(14).

[38] Note that $c = 0$ in autarky in (14).

the growth performance of a freely trading but otherwise identical world by subsidizing the purchase of all capital varieties. However, we now show that the cost of such subsidizing can quickly exceed plausible limits.

FIGURE 4 HERE

Figure 4 graphs the trade equivalent subsidy, $1 - s_c$, as a function of the trade parameter, c, and the complementarity parameter, ϕ, keeping other parameter values fixed at levels indicated in Table 1. Along the c-axis the trade equivalent degree of capital subsidization monotonically increases with openness to trade and reaches its maximum at $c = 1$, i.e. when the exchange of capital goods is free and costless. From (25) it can be determined that for $c = 1$ the subsidy parameter s_c equals $2^{\alpha-\phi}$. Given the parameter values of Table 1, we obtain $1 - s_c = .4678$, i.e. in order to replicate the growth performance of a free freely trading world the two symmetric countries would have to finance 46.78% of the purchase price of all intermediate goods using public funds. The trade equivalent subsidy also increases as the degree of technological complementarity increases. For example, if $\phi = 2$ the free trade equivalent subsidy equals $1 - s_c = .6748$.

Even excluding the inevitable additional economic costs arising from more realistic distortionary financing of the capital subsidy (which we have not considered in our model), these figures suggest that free exchange of capital goods provides the cheaper alternative path to faster economic growth.[39]

5 Conclusions

We have studied the effects of international trade in capital goods using a symmetric two country model with endogenous growth and incorporating technological complementarity and an increasing opportunity cost of aggregate (raw) capital. Since the model gives rise to a multiplicity of equilibria which are stable under adaptive learning, we have been able to study the impact of international trade on the set of stable steady state solutions. We have shown that the equilibria associated with relatively low growth may suddenly disappear when trade in capital goods is sufficiently liberalized (a

[39]Naturally, a closed economy must also pay the R&D development costs associated with new capital goods, whereas trade provides each country direct access to innovations developed elsewhere.

favorable bifurcation effect). Such bifurcation effects arising from trade can have a much larger impact on national growth rates and welfare than the local perturbations around single steady states which have been discussed in the literature thus far.

Regarding the local effect of trade on equilibria, we have shown that the intuitive conclusion of increasing openness spurring growth must be qualified when changes in the price of aggregate capital are taken into account. When the production possibility frontier between final consumption and capital has curvature (i.e. the relative price of capital with respect to consumption can vary), increasing international trade will generally increase the price of capital and this effect will reduce the growth impact of trade as the cost of capital replacement increases. If these additional costs are sufficiently large, we cannot rule out the unintuitive outcome that a more open world grows more slowly.

In the present model both small local changes in growth and large, rapid growth spurts can be caused by changes in the countries' openness toward international exchange. Our examples suggest that a high degree of technological complementarity and a relatively low rate of capital depreciation make favorable growth bifurcations more likely. The opposite is the case if capital goods are less complementary to each other and the rate of depreciation is high.

Finally, we have briefly considered the question of whether or not domestic policy combined with restricted international trade can provide an alternative route to high growth. Our results indicate that, even if equivalent outcomes can be attained through both suitable domestic capital subsidies and expanding trade in capital goods, the cost of domestic policy intervention quickly exceeds plausible limits even when we exclude the distortionary effects of any realistic financing of such subsidization. This provides additional support for outward-orientation as an effective international growth strategy.

A Appendices

A.1 The Slopes of the CC and TT Curves

The total differential of the model (13)-(16) at a steady state solution (r^*, g^*), where $g^* = g_c(r^*) = g_z(r^*)$, is the following:

$$dg = \mathcal{C}(1-\mathcal{E})dr - \mathcal{C}\mathcal{D}dp_z - \mathcal{C}\mathcal{F}dc, \tag{26}$$

$$\mathcal{S}dp_z = \chi'' dg_z + \chi'' d\frac{d\eta_t}{dr}dr + \chi'' d\frac{d\eta_t}{dc}dc, \tag{27}$$

$$dg = \mathcal{A}dr. \tag{28}$$

The multipliers in (26)-(28) are defined by

$$\mathcal{A} = \frac{1}{\sigma}\beta^{\frac{1}{\sigma}}(1+r^*)^{\frac{1-\sigma}{\sigma}} \ (>0), \tag{29}$$

$$\mathcal{C} = \frac{\phi-\alpha}{\phi-1}(1+r^* - k(c)\Omega p_z^{\frac{1}{\alpha-1}}(r^*+d)^{\frac{-\alpha}{1-\alpha}})^{\frac{1-\alpha}{\phi-1}} \ (>0), \tag{30}$$

$$\mathcal{E} = \frac{k(c)\alpha\Omega}{\alpha-1}(r^*+d)^{\frac{1}{\alpha-1}}p_z^{\frac{1}{\alpha-1}} \ (<0), \tag{31}$$

$$\mathcal{D} \equiv \frac{\Omega}{\alpha-1}k(c)(r^*+d)^{\frac{\alpha}{\alpha-1}}p_z^{\frac{2-\alpha}{\alpha-1}}(<0), \tag{32}$$

$$\mathcal{S} \equiv 1 - \chi'' d\frac{d\eta_t}{dp} \ (>0), \tag{33}$$

$$\mathcal{F} \equiv \Omega(r+d)^{\frac{-\alpha}{1-\alpha}}p_z^{\frac{1}{\alpha-1}}\frac{dk(c)}{dc} \ (>0), \tag{34}$$

$$\frac{d\eta_t}{dr} = \frac{1}{(1+\xi)(\mu_t)^2}\frac{L(1+c^{\frac{\alpha}{\phi-\alpha}})^{\xi}}{\alpha(\alpha-1)}\left(\frac{R}{\alpha}\right)^{\frac{2-\alpha}{\alpha-1}}\frac{p_z}{\gamma} \ (<0), \tag{35}$$

$$\frac{d\eta_t}{dp} = \frac{1}{(1+\xi)(\mu_t)^2}\frac{L(1+c^{\frac{\alpha}{\phi-\alpha}})^{\xi}}{\alpha(\alpha-1)}\left(\frac{R}{\alpha}\right)^{\frac{2-\alpha}{\alpha-1}}\frac{(r^*+d)}{\gamma} \ (<0), \tag{36}$$

$$\frac{d\eta_t}{dc} = \frac{1}{(1+\xi)(\mu_t)^2}L\left(\frac{R}{\alpha}\right)^{\frac{1}{\alpha-1}}m(c) \ (>0), \tag{37}$$

where $m(c) = \left[(1+c^{\frac{\phi}{\phi-\alpha}})(1+c^{\frac{\alpha}{\phi-\alpha}})^{\xi-1}\xi\left(\frac{\alpha}{\phi-\alpha}\right)c^{\frac{-\phi}{\phi-\alpha}} + (1+c^{\frac{\alpha}{\phi-\alpha}})^{\xi}\left(\frac{\phi}{\phi-\alpha}\right)\right]$.

The slope of the curve CC equals

$$dr_{cons}(g)/dg = \mathcal{A}^{-1} \ (>0) \tag{38}$$

as indicated by (28). Substituting (27) into (26), we obtain that

$$\frac{dr_z(g)}{dg} = (1+\frac{\mathcal{C}\mathcal{D}}{\mathcal{S}}\chi'')\left[\mathcal{C}(1-\mathcal{E}) - \frac{\mathcal{C}\mathcal{D}}{\mathcal{S}}\chi'' d\frac{d\eta_t}{dr}\right]^{-1} \equiv \mathcal{L}\mathcal{M}^{-1}. \tag{39}$$

which is the slope of the curve TT at a steady state equilibrium. Clearly, assuming that $\phi > 1$, the slope of TT is positive if the production possibility frontier between consumption and (raw) capital is linear ($\chi'' = 0$). When $\chi'' > 0$, however, the slope of TT may become negative. If there is no depreciation ($d = 0$), the slope is equal to $(1 + (\mathcal{CD}/\mathcal{S})\chi'')\,[\mathcal{C}(1-\mathcal{E})]^{-1}$; this expression is negative if $\mathcal{L} = 1 + (\mathcal{CD}/\mathcal{S})\chi'' < 0$. If $d > 0$, the second expression $\mathcal{M}^{-1}$ in (39) may also be negative or positive. Thus in general the expression $\frac{dr_z(g)}{dg} = \mathcal{L}\mathcal{M}^{-1}$ can have either sign.

A.2 Stability under Learning

Let (g^*, r^*) be a solution to (13)-(16). Using the solutions for the slopes of the curves CC and TT in (38) and (39) we obtain

$$\left(\frac{dr_{cons}(g^*)}{dg} - \frac{dr_z(g^*)}{dg}\right) = \mathcal{A}^{-1} - \mathcal{L}\mathcal{M}^{-1} > 0$$

or

$$\mathcal{L}\mathcal{M}^{-1} < \mathcal{A}^{-1} \tag{40}$$

as the necessary and sufficient condition for a given steady state to be stable under adaptive learning (cf. EHR). The right-hand side of (40) is positive, while the left-hand side can have either sign.

If the production possibility frontier between consumption and (raw) capital is linear ($\chi'' = 0$), we have $\mathcal{L} = 1$, $\mathcal{M} = \mathcal{C}(1-\mathcal{E})$ and so the stability condition (40) simplifies to $\mathcal{A} < \mathcal{C}(1-\mathcal{E})$.

It is possible to interpret the stability condition in different ways, since the mappings describing the CC and TT curves are locally invertible. Using the inverse mappings, this condition requires that a differential perturbation in the interest rate alters the aggregate consumer's desired growth rate of consumption by less than the growth rate of (raw) capital will change (while consistent with zero profitability of industry). The full stability condition (40) takes into account the subsequent change in the price of (raw) capital and demands that the combined effect of a perturbation in the interest rate and the price of capital on the growth rate of capital exceeds the impact of the interest rate on the growth rate of consumption.

A.3 Proof of Proposition 2

Equating (28) and (26) and substituting in (27), we obtain

$$\frac{dr}{dc} = -\frac{1}{\mathcal{AL}-\mathcal{M}}(\mathcal{CF}+\frac{\mathcal{CD}}{\mathcal{S}}\chi'' d\frac{d\eta_t}{dc}) \tag{41}$$

and

$$\frac{dg}{dc} = -\frac{\mathcal{A}}{\mathcal{AL}-\mathcal{M}}(\mathcal{CF}+\frac{\mathcal{CD}}{\mathcal{S}}\chi'' d\frac{d\eta_t}{dc}). \tag{42}$$

Assuming that $\chi''=0$, the stability condition (40) implies that the denominator in (41) is negative and the numerator is positive. Therefore, we obtain $\frac{dg}{dc}, \frac{dr}{dc} > 0$ at stable equilibria, whereas $\frac{dg}{dc}, \frac{dr}{dc} < 0$ when the equilibrium is unstable under adaptive learning. This yields Proposition 2.

A.4 Proof of Proposition 3:

The result follows using equation (41) and the proof of Proposition 2.

A.5 The Piecewise Linear Transformation Frontier:

We adopt the numerical example from EHR for the (nearly) piecewise linear consumption-investment frontier. The price of capital is set at $p_z = 0.5$ (linear segment 1); for intermediate values of Z_t, we choose $p_z = 1.2941$ (linear segment 2) and, for high levels of aggregate investment, we set $p_z = 3$ (linear segment 3). Figure A.1 illustrates this frontier in the (C_t, Z_{t+1})-space. The smoothly curved segments of the frontier were constructed by fitting quadratic arcs to remove the sharp kinks. For details of this last construction, see EHR (Appendix).

FIGURE A.1 HERE

References

[1] Baland J.-M. and P. Francois (1996), Innovation, Monopolies and the Poverty Trap, *Journal of Development Economics*, vol.49, 151-178.

[2] Baldwin, R., Martin P. and G.I.P. Ottaviano (1998), Global Income Divergence, Trade and Industrialization: The Geography of Growth Take-Offs, CEPR Working Paper No. 1803, February.

[3] Dinopoulos, E. and P. Segerstrom (1999), A Schumpeterian Model of Protection and Relative Wages, *American Economic Review*, vol.89, 450-472.

[4] *European Economic Review* (1999), Special Issue on International Geography and Trade, vol.43, No.2.

[5] Evans G.W. and S. Honkapohja (1993),Adaptive Forecasts, Hysteresis and Endogenous Fluctuations, *Federal Reserve Bank of San Francisco Economic Review*, nr.1, 3-13.

[6] Evans G.W. and S. Honkapohja (1998), Learning Dynamics, forthcoming in *Handbook of Macroeconomics, Vol.I,* ed. by J.B. Taylor and M. Woodford, Elsevier: Amsterdam.

[7] Evans G.W., S. Honkapohja and P. Romer (1998), Growth Cycles, *American Economic Review*, vol.88, 495-515.

[8] Feenstra, R. C. (1996), Trade and Uneven Growth, *Journal of Development Economics*, vol.49, 229-256.

[9] Grossman G.M. and E. Helpman (1990), Comparative Advantage and Long-Run Growth, *American Economic Review*, vol.80, 796-815.

[10] Kim T.-H. (1999), International Trade, Technology Transfer, Growth and Welfare in A Schumpeterian Model of Endogenous Growth, *Review of International Economics*, vol.7, 37-49.

[11] Krugman, P. (1991), History versus Expectations, *Quarterly Journal of Economics*, vol.106, 651-667.

[12] Matsuyama, K. (1991), Increasing Returns, Industrialization, and Indeterminacy of Equilibrium, *Quarterly Journal of Economics*, vol.106, 617-650.

[13] Matsuyama K. (1995), Complementarities and Cumulative Processes in Models of Monopolistic Competition, *Journal of Economic Literature*, vol.35, 701-729.

[14] Mountford, A. (1998), Trade, Convergence and Overtaking, *Journal of International Economics*, vol. 46, 167-182.

[15] Rivera-Batiz L.A. and P. Romer (1991), Economic Integration and Endogenous Growth, *Quarterly Journal of Economics*, vol.106, 531-555.

[16] Young, A. (1993), Substitution and Complementarity in Endogenous Innovation, *Quarterly Journal of Economics* 108, 775-808.

Figure 1: steady states

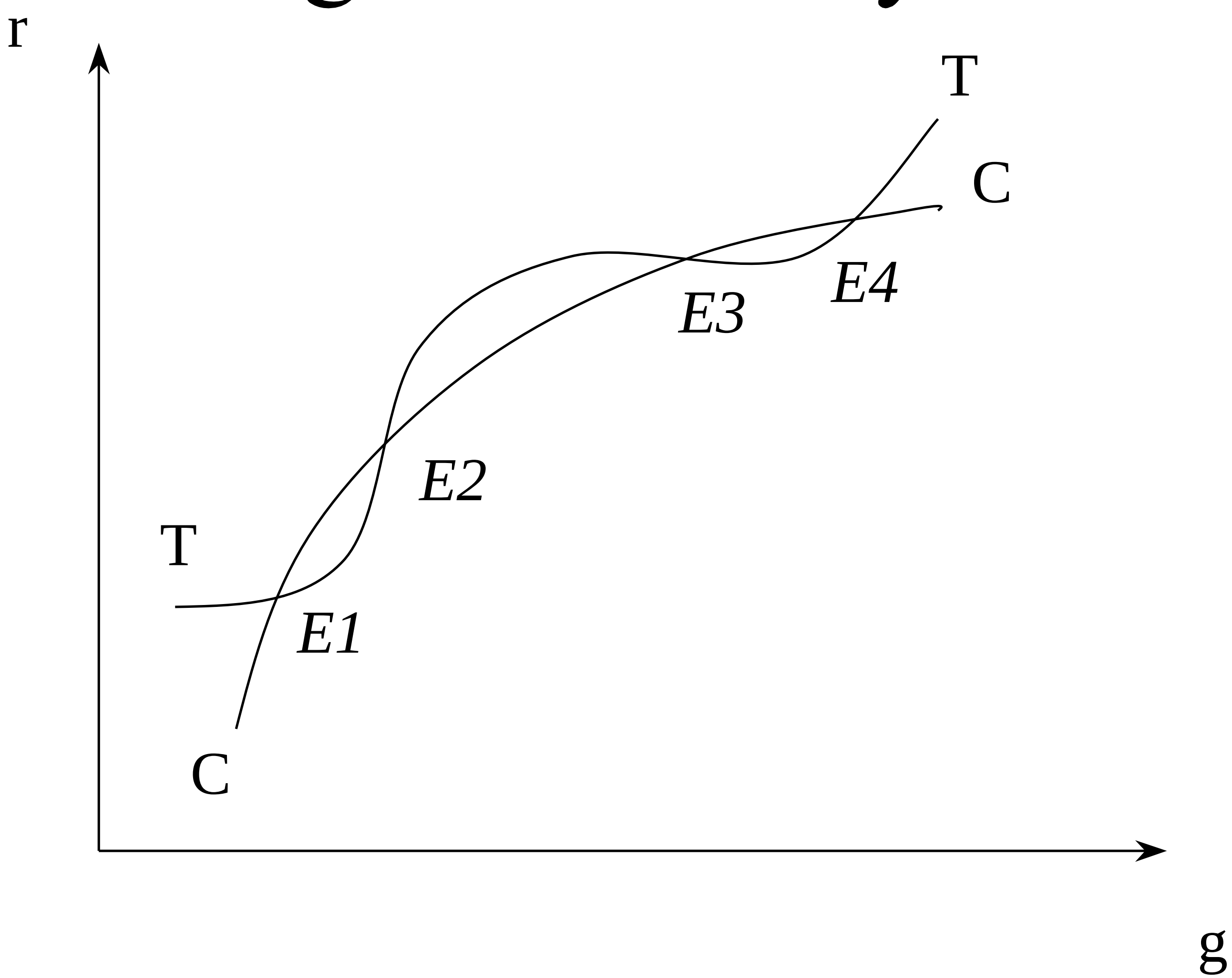


Figure A.1

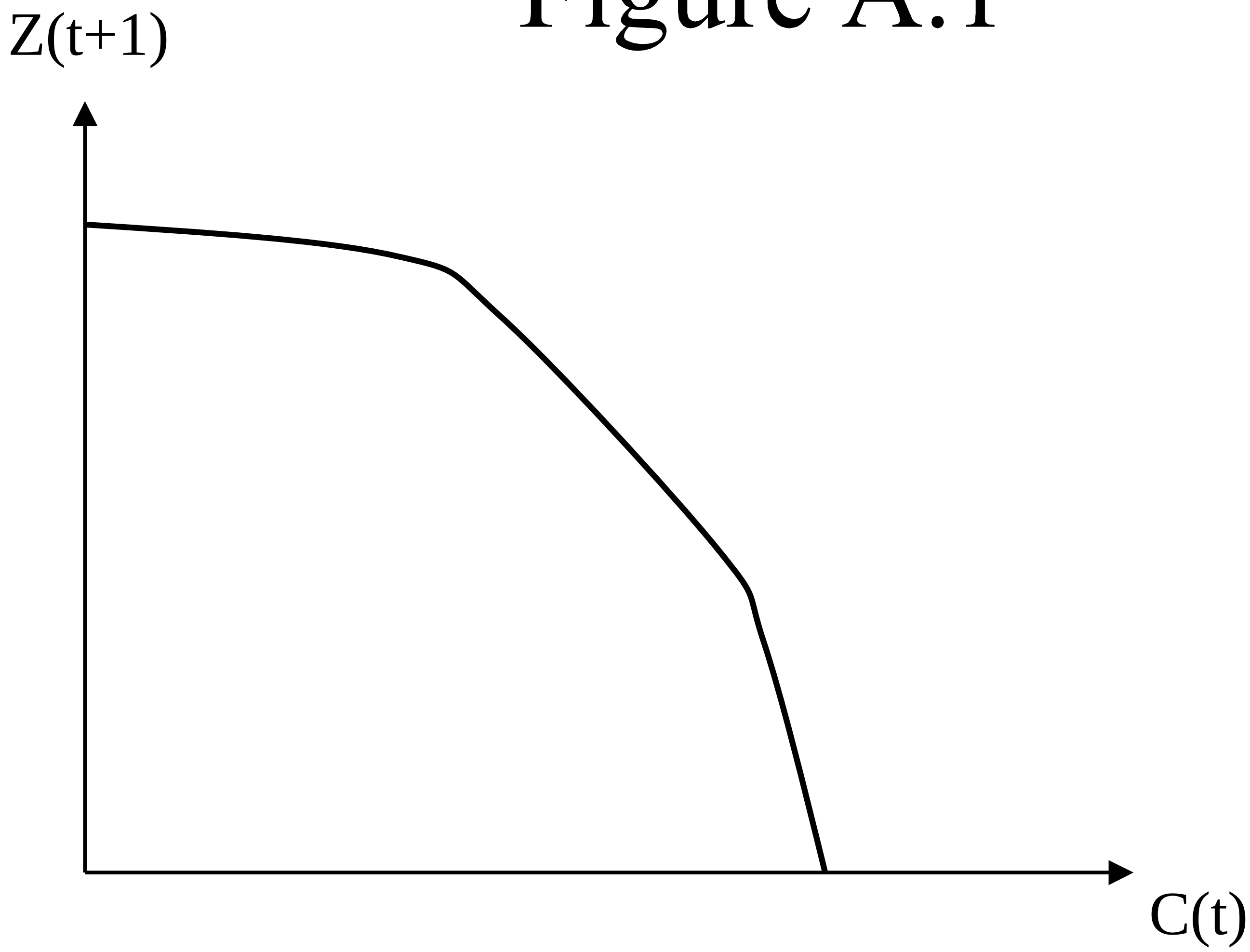


FIGURE 2a (autarky)

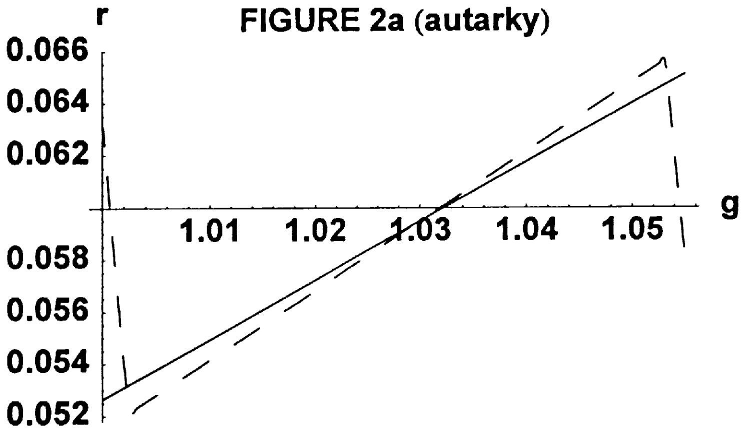


FIGURE 2b (free trade)

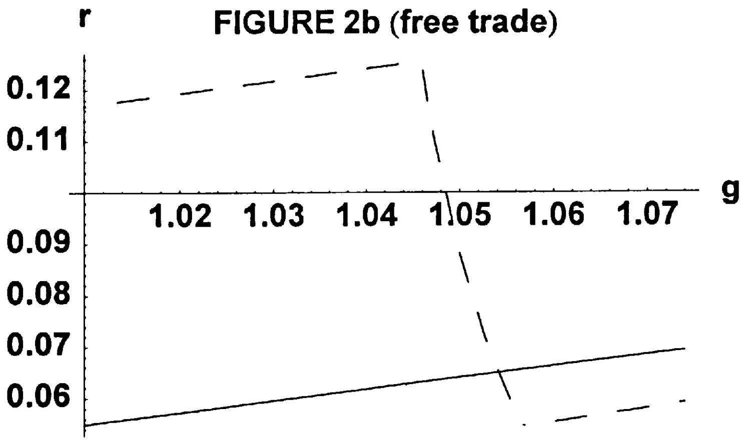


FIGURE 3a (autarky)

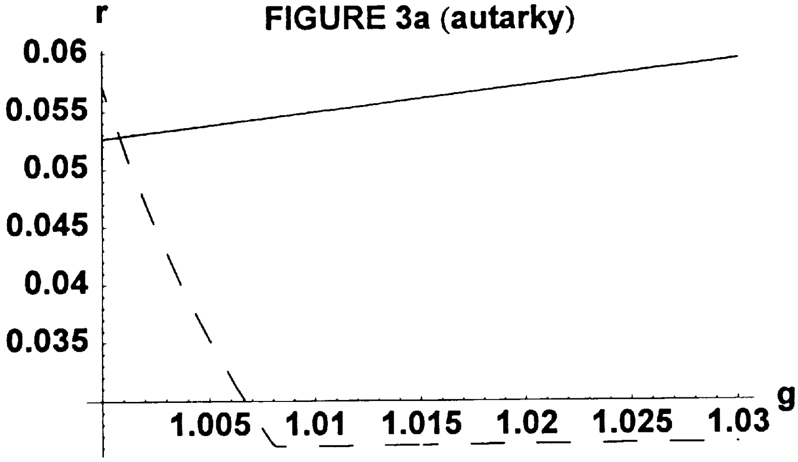


FIGURE 3b (free trade)

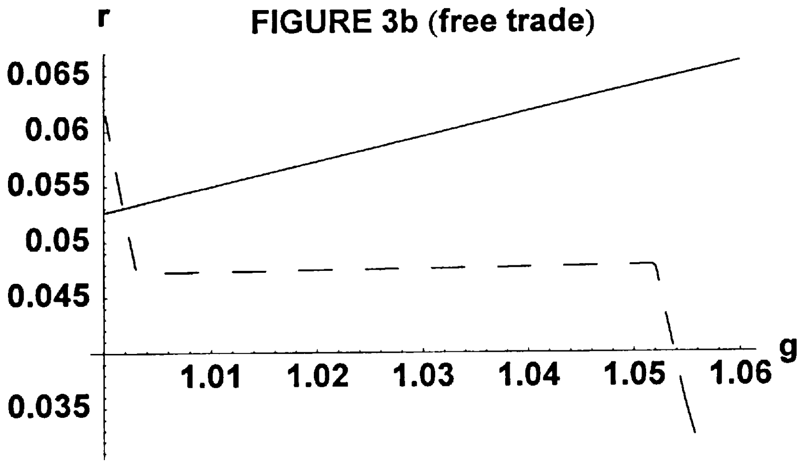
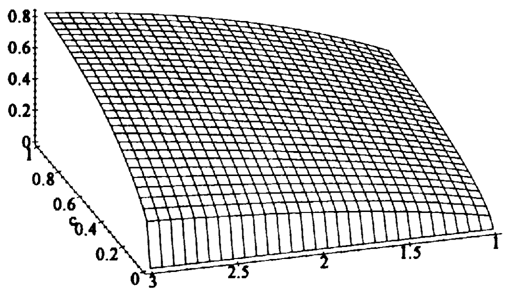


Figure 4: The trade equivalent subsidy as a function of ϕ and c.



The subsidy $(1 - s_c)$ as a function of ϕ and c.