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ABSTRACT

Entry Mistakes, Entrepreneurial Boldness and Optimism*

We analyse the investment decision of a population of time-inconsistent entrepreneurs who overweight current payoffs relative to future returns. We show that, in order to avoid inefficient procrastination, agents may find it optimal to keep optimistic about their chances of success and 'blindly invest'. This explains entrepreneurial boldness and entry mistakes (or an excessive level of investment in the economy) without assuming the existence of 'intrinsically optimistic' managers who feel bound to be rational. We also prove that: (i) there is a negative correlation between the risk-free rate and the proportion of bold entrepreneurs in the economy, (ii) realist and bold agents can coexist and achieve the same payoff and (iii) entrepreneurs with highest ability are most likely to keep optimistic prospects and make entry mistakes.

JEL Classification: A12, D81, D92, G39
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NON-TECHNICAL SUMMARY

There is extensive evidence that the rate of failure of new businesses during their first few years is very high. Three alternative explanations for this high turnover rate have been suggested. The first one says that entries are profitable, but only for a short period (hit-and-run strategies). According to the second one, business success is a low probability, high return event; investing is therefore profitable on average, but it is natural to observe few 'winners'. Last, firms could mistakenly enter too often. The reason commonly argued for excessive entry is simple: intrinsic optimists, i.e. individuals who tend to overestimate their capacity to succeed in business, self-select into being entrepreneurs. Overall, there is a consensus that the first two explanations invoke standard rational arguments while the third one is based on the existence of individuals who feel bound to be rational and the debate is centred on the plausibility of each story. Given the importance of this question to understanding the performance of markets, it is surprising to notice that scholars have not challenged the *a priori* not so straightforward link between excessive failure rates and overconfidence. This work offers an explanation of what are labelled as 'entry mistakes' that do not require the assumption of intrinsic optimism. More precisely, we show that rational entrepreneurs may decide to keep optimistic prospects about their chances of success and invest boldly. This attitude leads (on average) to an excessive level of investment and therefore to high failure rates.

The Paper considers a population of cash-constrained agents, each of them willing to borrow capital from banks in order to undertake a risky investment. We depart from standard models by assuming that agents have dynamically inconsistent preferences so that short-term events are discounted at a higher rate than long-term ones. The special characteristics relevant to our model are twofold. First, investment requires a current cost (in terms of effort to find a potentially valuable project, or foregone fixed outside salary) and yields a delayed benefit (profit if the project is successful). Second, individuals do not know the probability of success of their selected project, although they can learn it at no cost.

Combining the taste for immediate gratification with this temporal gap between costs and benefits of entrepreneurial activities, we first show that rational individuals make excessively optimistic predictions about the value of their project and choose to make uninformed investments even when information acquisition is free. Information can usually help to avoid the undertaking of bad projects, but it can also lead to a state of inefficient procrastination: the individual finds the project valuable and, at the same time, he is not willing to undertake it when the date at which the cost has to be exerted comes.

Overall, ignorance is beneficial when it helps to avoid procrastination, i.e. it must induce an extra desire to invest. As a result, our economy can only be composed of ‘realist’ agents (who invest according to their cost-benefit analysis at the time of exerting effort), ‘bold’ agents (who remain uninformed and, on average, invest in excess), or both.

Since agents in our economy are cash-constrained, they need external finance. Hence, the entrepreneur's net return from a given project and therefore his incentives to remain strategically ignorant and invest (possibly in excess) depend on the interest rate in the economy. Naturally, this interest rate will itself be determined depending on the learning decision of all the potential borrowers. So, the interest rate set by banks and the level of entrepreneurial boldness in the economy are jointly determined in equilibrium. Assuming perfect competition between banks and a fixed (exogenous) risk-free rate, the Paper draws several conclusions. We show that there is an endogenous negative relation between the proportion of bold entrepreneurs in the economy (who make entry mistakes) and the risk-free rate. According to this, there are two reasons for which agents will undertake more investments when the credit market has a low risk-free rate. One is trivial: if the opportunity cost of investing becomes lower, the total number of individuals willing to invest increases. The other, which is the main point of the Paper, is more subtle: as the risk-free rate decreases, more agents are willing to remain strategically ignorant and blindly invest. This effect relates the state of the credit market to the number of excessive investment projects initiated. Overall, the main message of this Paper is that strategic ignorance and a bold attitude is a rational choice that avoids inefficient procrastination. In other words, overconfidence is not necessary to explain the excessive rate of failure of businesses.

The Paper also shows that under perfect competition between banks and for intermediate values of the risk-free rate, only a fraction of agents keep optimistic prospects in equilibrium, even though all of them are *ex ante* identical. This case is characterized by a population of realist and bold entrepreneurs who not only coexist in the economy but who also achieve the same expected utility. We then show that our qualitative results do not change if agents can post some outside collateral or if there is imperfect competition between banks. However, in both cases, bold entrepreneurship is less likely to occur. Interestingly, when agents have restricted collateral, competitive banks may obtain some positive profits. Last, we define entrepreneurial ability as the capacity of some individuals to select projects where the profits in case of success are highest. We demonstrate that the agents with highest ability are most likely to keep optimistic prospects and invest. In other words, there is an endogenous positive correlation between managerial ability and boldness. Overall, individuals with high entrepreneurial skills invest more than their low-skills peers, first because the expected value of their investment is higher, and second because they are more willing to remain strategically ignorant. At the same time, these high-ability types also make more entry mistakes.

1 Introduction

There is extensive evidence that the rate of failure of new businesses during their first few years is very high.[1] Camerer and Lovallo (1999) suggest three alternative explanations for this high turnover rate. The first one says that entries are profitable but only for a short period (hit-and-run strategies). According to the second one, business success is a low probability, high return event; investing is therefore profitable on average, but it is natural to observe few "winners". Last, "firms could mistakenly enter too often" (p.307). The reason commonly argued for excessive entry is simple: *intrinsic optimists*, i.e. individuals who tend to overestimate their capacity to succeed in business, self-select into being entrepreneurs. This argument was already suggested by Keynes (1936, ch.12) and it is particularly appealing given that, as De Bondt and Thaler (1995) wrote: "perhaps the most robust finding in the psychology of judgement is that people are overconfident".[2] Besides, the experimental game by Camerer and Lovallo (1999) supports the entry mistake hypothesis. Overall, there is a consensus that the first two explanations invoke standard rational arguments while the third one is based on the existence of boundedly rational individuals and the debate is centered around the plausibility of each story.

Given the importance of this question to understand the performance of markets, it is surprising to notice that scholars have not challenged the link between excessive failure rates and overconfidence. In our view, this link is not a priori so straightforward: assuming intrinsic optimism provides an explanation for excessive investment but there is no theory proving that systematic overestimation is the only way to account for entry mistakes. This work proposes a different approach to the problem. It offers an explanation to what is labeled as "entry mistakes" that does not require the assumption of intrinsic optimism.[3] More precisely, we show that rational entrepreneurs may decide to keep optimistic prospects about their chances of success, and boldly invest. This attitude leads (on average) to an excessive level of investment, and therefore to high failure rates.

[1] See e.g. Dunne, Roberts and Samuelson (1988, 1989a, 1989b).

[2] See the references there-in for general studies on overconfidence and Larwood and Whittaker (1977), Cooper et al. (1988) and Russo and Schoemaker (1992) among others for evidence of managerial overconfidence.

[3] As should be clear, we do not challenge the numerous psychological studies supporting the view that optimism is a widespread human trait. Our goal is to show that imposing this optimism is not necessary if we want to explain the observed behavior of entrepreneurs.

The paper considers a population of cash constrained agents, each of them willing to borrow capital from banks in order to undertake a risky investment. We depart from standard models by assuming that agents have dynamically inconsistent preferences (à la Strotz, 1956) so that short term events are discounted at a higher rate than long term ones.[4] The special characteristics relevant for our model are twofold. First, investment requires a current cost (in terms of effort to find a potentially valuable project, or foregone fixed outside salary) and yields a delayed benefit (profit if the project is successful). Second, individuals do not know the probability of success of their selected project, although they can learn it at no cost.

Combining the taste for immediate gratification to this temporal gap between costs and benefits of entrepreneurial activities, we first show that rational individuals may optimally keep excessively optimistic prospects about the value of their project and choose to make uninformed investments even under no cost of information acquisition. The reason is that, under time inconsistent preferences, information has two implications. On the one hand, it can as usual avoid undertaking bad projects. However, on the other hand it can also lead to a state of inefficient procrastination: the individual finds the project valuable and, at the same time, he is not willing to undertake it when the date at which the cost has to be exerted comes.[5] Overall, a necessary condition for ignorance being beneficial is that it has to avoid procrastination, i.e. it must induce an extra desire to invest. As a result, our economy can only be composed of "realist" agents (who invest according to their cost-benefit analysis at the time of exerting effort), "bold" agents (who remain uninformed and, on average, invest in excess) or both. This completes the first step towards rationalizing entry mistakes without assuming the existence of boundedly rational, intrinsic optimists.[6]

Note that the previous conclusion relies on individual investments being self financed (or, equivalently, financed at a fixed, exogenous interest rate). However, in our model agents are cash constrained, so they need external finance. Hence, the

[4]There is a well documented literature both in psychology and more recently in economics showing that individuals' discount rates are best approximated by hyperbolas rather than the traditional exponential functions. We refer the reader to Ainslie (1975), Thaler (1981) and Benzion et al. (1989) for empirical support of this theory both in animals and humans and to Ainslie (1992) or Loewenstein and Prelec (1992) for a detailed review of this literature.

[5]The paper by Carrillo and Mariotti (1997) on the value of ignorance already reaches a similar conclusion. In the present research, we use this result as a starting point.

[6]Some readers might argue that time inconsistent preferences is a form of bounded rationality. Although we do not share this opinion, the concern of the present paper is not to discuss issues related to agents' rationality but rather to explore the reasons why we might observe excessive investment.

net entrepreneur's return from a given project and therefore his incentives to remain strategically ignorant and invest (possibly in excess) depend on the interest rate in the economy. Naturally, this interest rate will itself be determined depending on the learning decision of all the potential borrowers. So, the interest rate set by banks and the level of entrepreneurial boldness in the economy are jointly determined in equilibrium. Assuming perfect competition between banks and a fixed (exogenous) risk free rate, the paper draws several conclusions. We show in Proposition 1 that *there is an endogenous negative relation between the proportion of bold entrepreneurs in the economy (who make entry mistakes) and the risk free rate.* According to this, there are two reasons for which agents will undertake more investments when the credit market has a low risk free rate. One is trivial: if the opportunity cost of investing becomes lower, the total number of individuals willing to invest increases. This effect highlights the usual relation between the state of the credit market and the number of *profitable* investment projects initiated. The other, which is the main point of the paper, is more subtle: as the risk free rate decreases, more agents are willing to remain strategically ignorant and blindly invest. This effect relates the state of the credit market to the number of *excessive* investment projects initiated. To better understand the intuition note two things. First, when the risk free rate is relatively high, competitive banks are forced to charge high interest rates because there is a substantial opportunity cost of lending. Second, when the interest rate charged by banks is high, it is relatively more costly to undertake an investment, and therefore agents have more incentives to learn their chances of success before deciding whether to apply for a loan (Lemma 2). The combination of both factors leads to the result. We want to stress that the logic behind the result is new. According to the usual view, overconfidence or intrinsic optimism pushes the individuals to become entrepreneurs, invest and possibly make some mistakes. Some authors (such as Keynes for example), even argue that optimism is necessary for undertaking a business activity, but they do not say anything about where this optimism may come from. In this paper, we claim on the contrary that strategic ignorance and a bold attitude is a rational decision that avoids inefficient procrastination. In other words, overconfidence is not necessary to explain the excessive rate of failure of businesses. The bold behavior we highlight in this paper can be interpreted as a kind of "observational optimism", since it leads agents to blindly invest and make some entry mistakes. Moreover, we predict that this observational optimism and excessive entry is more likely to occur when the credit market is in a good state. The paper also shows that under perfect competition between banks and for intermediate values of the risk free rate, then only a fraction of agents keep optimistic prospects in equilibrium, even though all of them are ex ante identical. This case is characterized by a population of realist and bold entrepreneurs who not only *coexist* in the economy but who also achieve the *same expected utility.* Propositions 2 and 3 show that our qualitative results do not change if agents can post some outside collateral or if there is imperfect competition between banks. However, in both cases bold entrepreneurship is less likely to occur. Interestingly, when agents have a restricted collateral competitive banks may obtain some positive profits. Last, we define entrepreneurial ability as the capacity of some individuals to select projects where the profits in case of success are highest. Proposition 4 shows that the agents with highest ability are most likely to keep optimistic prospects and invest. In other words, *there is an endogenous positive correlation between managerial ability and boldness.* Overall, individuals with high entrepreneurial skills invest more than their low skills peers, first because the expected value of their investment is higher, and second because they are more willing to remain strategically ignorant. At the same time, these high ability types also make more entry mistakes.

Before presenting the model, we would like to briefly review some recent papers in the behavioral corporate finance field that use managerial intrinsic optimism as a starting point. Roll (1986) proposes overconfidence (the hubris hypothesis) as an explanation for the existence of corporate takeovers which, on average, yield no gains. Manove (1997) studies competition between optimistic and realistic entrepreneurs and shows that the former may drive the latter out of the market. Manove and Padilla (1997) analyze the relationship between banks and optimistic borrowers and Heaton (1997) proposes managerial optimism as an alternative foundation for pecking order and agency cost theories. Even if these are important contributions, in our view the conceptual approach of *assuming* optimistic behavior and then deriving some predictions has two main drawbacks. First, from a methodological perspective, it is perfectly natural either to accept Bayesian inference or to challenge it. However, those papers employ a modified “quasi-bayesian” approach arguing that it fits some commonly observed patterns of human behavior. Second, optimism is certainly the simplest explanation for the tendency of entrepreneurs to over-act. Yet, as our paper points, there might be other reasons for excessive investment. Naturally, this weakens the support for the (always controversial) bounded rationality assumption.

2 The model

2.1 Preliminaries

We analyze the decision of agents to undertake an investment. Investing requires one unit of capital and one unit of effort. We denote by e the cost of exerting this effort. One can think of effort as the search cost in order to find a suitable project. Alternatively (although formally equivalent), e may represent the opportunity cost of becoming an entrepreneur and invest rather than being an employee. Agents are cash constrained. They can borrow from banks the unit of capital only for the purpose of undertaking the investment. We denote by R (≥ 1) the interest factor (i.e. one plus the interest rate) charged by banks. Furthermore, agents can also post some outside collateral C. For the time being, we assume that R and C are fixed. Investing yields some stochastic profit with a one period delay. More specifically, with probability p the investment is successful and yields benefit π. With probability $1-p$ the investment fails and yields 0 benefit. Agents do not know the probability p that their investment succeeds, but they know the probability distribution $F(p)$ with $p \in [0,1]$ from which each p is *independently* drawn.[7] We assume that the distribution satisfies the monotone hazard rate condition.

Assumption 1 $\dfrac{F(p)}{f(p)}$ *is increasing in* p.

The payoff of not investing is normalized to zero. Agents are risk neutral and have limited liability.[8] Banks observe whether the investment is a success or a failure, so profits are contractible. The debt contract offered by banks specifies a repayment R ($\leq \pi$) in case of success and the appropriation of the collateral C (if any) in case of failure. Last, we assume that agents can learn *at no cost* the probability p that their own investment is successful the period *before* investing. Note in particular that at the date of exerting effort and invest, it is no longer possible to learn the probability of success p.[9] Besides, we make the assumption that the learning decision of agents is not

[7]The analysis can easily be extended to a support $[\underline{p}, \overline{p}] \subset [0,1]$.

[8]In a previous version (available upon request), we show that all the results in the paper are reinforced if agents are risk averse.

[9]In a related paper (Carrillo and Mariotti, 1997) it is shown that the insights obtained in an infinite horizon model where individuals have time inconsistent preferences and learning is possible at every date are the same as in a three-period model where learning is possible only in the first one.

observable by banks. The timing can be summarized as follows.

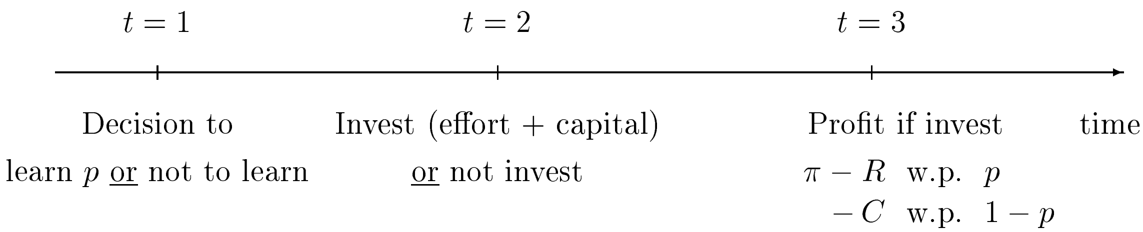

Figure 1. Timing

Our investigation departs from standard analyses in that agents have dynamically inconsistent preferences. More precisely, we assume that the period-to-period discount rate falls monotonically. For analytical tractability we use the quasi-hyperbolic discounting introduced by Phelps and Pollak (1968). Formally, period $t+s$ is discounted at a rate $\beta\delta^s$ (with $\delta \leq 1$ and $\beta \in (0,1)$) from the perspective of the agent at date t.[10] The key effect of time inconsistent preferences is that the incentives of the agent to undertake an investment from his perspective at date $t = 1$ (from now on "self-1") will be different when reconsidered one period later (i.e. by "self-2"). To focus on the interesting situation, we will assume that in the best possible scenario in which the investment succeeds for sure ($p = 1$) and the interest rate is zero ($R = 1$), self-2 finds it optimal to invest.

Assumption 2 $\beta\delta(\pi - 1) > e.$

Last, we will also assume that the intrapersonal conflict is not too acute. Indeed, given our discrete time model and the quasi-hyperbolic discount functions, the marginal rate of substitution between dates 2 and 3 from the perspective of self-1 is $1/\delta$. This means that it is not affected by the inconsistency parameter β. By contrast, from the perspective of self-2, the marginal rate of substitution between dates 2 and 3 is

[10] In recent years, this particular formalization of time inconsistent preferences has been used to study different problems. Some examples (the list is clearly not exhaustive) are: procrastination (Akerlof (1991), O'Donoghue and Rabin (1996)), saving rates (Laibson, 1996, 1997a, 1997b), addiction and self-control (Caillaud et al., 1996), excessive consumption (Carrillo, 1998) and investment under uncertainty (Brocas and Carrillo, 1998). See also Gul and Pesendorfer (1999) for an axiomatic approach to dynamic inconsistency.

$1/\beta\delta$. Therefore, when the intrapersonal conflict is very important, the agent at date 2 cares about his current payoff and completely disregards future ones, whereas self-1 internalizes both of them. In order to avoid this extreme situation, we introduce the following assumption.

Assumption 3 $\beta \in (1/2, 1)$.

At this stage we can analyze the incentives of the different selves to invest.

2.2 Intrapersonal conflict and incentives to invest

Suppose that the agent at date $t = 1$ learns his probability of success p. In that case, investing yields a positive utility from the perspective of self-1 if and only if:

$$-\beta\delta\, e + \beta\delta^2 \Big[p(\pi - R) - (1-p)C\Big] \geq 0 \;\Leftrightarrow\; p \geq p_1 \equiv \frac{C + e/\delta}{\pi - R + C} \tag{1}$$

However, when date 2 comes, self-2 chooses to invest if and only if:

$$-e + \beta\delta \Big[p(\pi - R) - (1-p)C\Big] \geq 0 \;\Leftrightarrow\; p \geq p_2 \equiv \frac{C + e/\beta\delta}{\pi - R + C} \tag{2}$$

First, note that $p_1 < p_2$. Time inconsistent preferences induce a procrastination problem in which some investments valuable from the viewpoint of date $t = 1$ are not undertaken at date $t = 2$. The idea is simple. An agent finds it profitable to exert a cost in the future in order to obtain an expected benefit one period later if the chances of success are sufficiently important (formally, if $p \geq p_1$). However, given that current payoffs are overweighed, then the agent may prefer not to invest anymore at the time at which the investment cost e has to be incurred (formally, this occurs when $p \in [p_1, p_2]$). Naturally, this is perfectly anticipated at date $t = 1$ but, in the absence of a commitment device, the agent can do nothing about it. Second, both $p_1(R, C)$ and $p_2(R, C)$ are increasing in R and C: the higher the interest factor and the collateral, the smaller the agent's expected gain of investing, and therefore the smaller the incentives to invest from the perspective of any self. Last, when β increases, the intrapersonal conflict diminishes ($\frac{\partial p_2}{\partial \beta} < 0$). The conflict between selves vanishes when $\beta = 1$ ($\lim_{\beta \to 1} p_2(\beta) = p_1$).

Given agents' risk neutrality and using (1) and (2), one can notice that if at date 1 the agent does not learn the probability of success, then investing is desirable from self-1's viewpoint if $E[p] \geq p_1$ and from self-2's viewpoint if $E[p] \geq p_2$.

2.3 Incentives to learn the value of an investment

Our next step is to analyze the incentives of each agent to acquire information about his payoff distribution, for a given pair of interest factor and collateral (R, C) set by banks.[11]

The agent's ex ante expected payoff from the perspective of date 1 if he decides to become *informed* (i.e. to learn the value of p) is given by:

$$\begin{aligned} G_i(R, C; \beta) &= \int_{p_2}^{1} \left[-\beta\delta\, e + \beta\delta^2\, p(\pi - R) - \beta\delta^2\,(1-p)C\right] dF(p) && (3) \\ &= \beta\delta \left[\delta(\pi - R + C) \int_{p_2}^{1} p\, dF(p) - (e + \delta\, C)\,[1 - F(p_2)]\right] && (4) \end{aligned}$$

Note that, for all (R, C), $G_i(R, C; \beta) > 0$. Given the intrapersonal conflict of preferences, this is not trivial a priori. However, the problem for self-1 is the tendency of self-2 to reject projects that are valuable from his perspective. This inefficiency implies zero payoff in cases where positive profits could be achieved by self-1, for instance when $p \in [p_1, p_2])$. Yet, it can never induce expected losses from self-1's perspective (i.e. time inconsistency can never imply the acceptance of bad projects). Besides, $\frac{\partial}{\partial R} G_i(R, C; \beta) < 0$ and $\frac{\partial}{\partial C} G_i(R, C; \beta) < 0$: an increase in the repayment obligation or in the posted collateral decreases the expected benefit of the investment.

If instead the agent remains *uninformed*, his ex ante expected payoff depends on whether the uninformed self-2 will invest or not. More precisely, and as stated previously, if $E[p] < p_2$ an agent who does not learn p at date 1 will not invest at date 2, in which case his payoff is zero. By contrast, if $E[p] \geq p_2$ an agent who does not learn p at date 1 strictly prefers to invest at date 2. His expected gain is therefore:

$$\begin{aligned} G_u(R, C; \beta) &= \int_{0}^{1} \left[-\beta\delta\, e + \beta\delta^2\, p(\pi - R) - \beta\delta^2\,(1-p)C\right] dF(p) && (5) \\ &= \beta\delta \left[\delta(\pi - R + C) \int_{0}^{1} p\, dF(p) - (e + \delta\, C)\right] && (6) \end{aligned}$$

Note again that $\frac{\partial}{\partial R} G_u(R, C; \beta) < 0$ and $\frac{\partial}{\partial C} G_u(R, C; \beta) < 0$ for all (R, C): increasing the repayment or the collateral also decreases the expected benefit when self-1 remains uninformed.

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[11] Note that, for simplicity, all the uncertainty in the model is reduced to one parameter: the probability of success p. We will therefore refer to "learning p" as the agent's willingness to reduce the uncertainty about the net payoff of investing.

Given (4) and (6) and since $G_i(R,C;\beta) > 0$, then not learning p dominates learning it from the perspective of self-1 if, conditional on $E[p] \geq p_2$, we have:

$$G_u(R,C;\beta) - G_i(R,C;\beta) > 0$$

that is, if:

$$\delta(\pi - R + C)\int_0^{p_2} p\,dF(p) > (e + \delta\,C)\,F(p_2) \;\Leftrightarrow\; E[p\,|\,p < p_2] > p_1$$

This result, which builds on Carrillo and Mariotti (1997), is summarized as follows.

Lemma 1 *At date $t = 1$ the agent decides not to acquire information on his probability of success if and only if the following conditions hold:*

$$E[p] > p_2(R,C) \qquad \textbf{(C1)}$$

$$E[p\,|\,p < p_2(R,C)] > p_1(R,C) \qquad \textbf{(C2)}$$

Proof. By inspection of $G_i(\cdot)$ and $G_u(\cdot)$. □

The idea is simple. Given time inconsistent preferences and the fact that costs come earlier than benefits, if $p \in [p_1, p_2]$ self-1 would like to invest, but when date 2 arrives self-2 does not want anymore. Therefore, the only potential benefit for self-1 of not learning is that it may induce self-2 to invest when the *true value* of p lies in $[p_1, p_2]$. Naturally, the cost of remaining uninformed is that self-2 may take suboptimal decisions because of his imperfect knowledge of p. In particular, he might decide to invest when the true value of p is in $[0, p_1]$. Overall, a *necessary* condition for ignorance to be optimal is that it must avoid inefficient procrastination. This is to say that, if self-1 does not learn, then self-2 must strictly prefer to invest **(C1)**. However, this condition is not sufficient. Note that, given **(C1)**, if $p \in [p_2, 1]$ it is irrelevant whether the agent learns it or not. Inequality **(C2)** simply states that, conditional on p being smaller than p_2, then the event $p \in [p_1, p_2]$ has to be relatively more likely than the event $p \in [0, p_1]$. That is ignorance must have, on average and for self-1, more benefits (investment when $p \in [p_1, p_2]$) than costs (investment when $p \in [0, p_1]$).[12]

Conditions **(C1)** and **(C2)** in Lemma 1 have been derived for given values of R and C. However, in our model the interest factor and collateral will be endogenously

[12] Including the possibility of partial learning would affect neither our intuitive arguments nor the qualitative results about the agents' incentives to acquire information.

determined in equilibrium by banks. It is therefore important to understand how the incentives of agents to learn are affected by changes in R and C. We have the following key intermediary result.

Lemma 2 *The incentives of agents to learn their true p are increasing in R and C.*

Proof. It is obvious that $E[p] - p_2(R,C)$ is decreasing in both R and C. Define

$$g(R,C;\beta) \equiv \frac{1}{\beta\delta}\left[G_u(R,C;\beta) - G_i(R,C;\beta)\right] = \int_0^{p_2} -e + \delta p(\pi - R) - \delta(1-p)C \, dF(p)$$

Noting that $p_2 = (\pi - R + C)\frac{\partial p_2(\cdot)}{\partial R}$ and $1 - p_2 = (\pi - R + C)\frac{\partial p_2(\cdot)}{\partial C}$, we get that:

$$\frac{\partial g(R,C;\beta)}{\partial R} = \delta\,(p_2 - p_1)p_2\, f(p_2) - \delta\int_0^{p_2} p\, f(p)\, dp \tag{7}$$

$$\frac{\partial g(R,C;\beta)}{\partial C} = \delta\,(p_2 - p_1)(1-p_2) f(p_2) - \delta\int_0^{p_2}(1-p) f(p)\, dp \tag{8}$$

Suppose that for some C and β, there exists $\tilde{R}(C,\beta)$ such that $g(\tilde{R}(C,\beta),C;\beta) = 0$, and that for some R and β, there exists $\tilde{C}(R,\beta)$ such that $g(R,\tilde{C}(R,\beta);\beta) = 0$. We have:

$$\left.\frac{\partial g(R,C;\beta)}{\partial R}\right|_{\tilde{R}} \propto (p_2 - p_1)p_2\,\frac{f(p_2)}{F(p_2)} - p_1$$

$$\left.\frac{\partial g(R,C;\beta)}{\partial C}\right|_{\tilde{C}} \propto (p_2 - p_1)(1-p_2)\frac{f(p_2)}{F(p_2)} - (1-p_1)$$

where "$\propto$" stands for "proportional to". Given Assumption 1, $p\,\frac{f(p)}{F(p)} < 1$ for all p. Then, we get immediately that:

$$\left.\frac{\partial g(R,C;\beta)}{\partial C}\right|_{\tilde{C}} < (1-p_1)\left[p_2\,\frac{f(p_2)}{F(p_2)} - 1\right] < 0.$$

Moreover, note that $p_1(R,C) \geq \beta p_2(R,C)$ for all $C \geq 0$. Therefore, $p_2(R,C) - 2p_1(R,C) < 0$ for all $\beta \in (1/2, 1)$. As a result:

$$\left.\frac{\partial g(R,C;\beta)}{\partial R}\right|_{\tilde{R}} < p_2 - 2p_1 < 0.$$

Since, for given C and β, $\left.\frac{\partial g}{\partial R}\right|_{\tilde{R}} < 0$, we can conclude that if we fix C then $g(R,C;\beta)$ crosses the R-axis at most once. In other words, three cases are possible: either $g(R,C;\beta)$ is always positive, or $g(R,C;\beta)$ is always negative, or there exists $\tilde{R}$ such that $g(R, C; \beta)$ is positive for all $R < \tilde{R}$ and negative for all $R > \tilde{R}$. The same conclusion applies with respect to C. $\square$

The intuition of this result is the following. First, an increase in R or in C decreases the incentives to invest at date 2 when self-1 has remained uninformed. Formally, $\partial p_2/\partial R > 0$ and $\partial p_2/\partial C > 0$. As a consequence, **(C1)** is less likely to be satisfied when R or C are high. Second, for any given probability of success, an increase in R or in C decreases the net benefit of investing. Recall that an uninformed agent invests with a higher probability than an informed one.[13] So, an increase in the repayment obligation or the collateral has a bigger (i.e. more frequent) negative impact on the expected payoff under ignorance than under learning. Hence, the higher the interest factor or the posted collateral is, the more likely that **(C2)** would not hold.

To sum up, self-1 will be more willing to learn his probability of success before investing when the residual gain of the investment is small. Still, it may be the case that for some parameter constellations self-1 strictly prefers always to learn or always to remain ignorant, independently of R and C. These cases are not very interesting, given that one of the goals of the paper is to study the endogenous interactions between the interest rate in the credit market and the agents' incentives to learn and invest. The next Lemma provides sufficient conditions on the parameters of the model such that self-1's learning decision can be affected by R and C.

Lemma 3 *If $F(\cdot)$, β, δ, π, and e are such that:*

(i) $\dfrac{E[p]}{2} < \dfrac{e}{\delta(\pi - 1)} < E\Big[p \,|\, p < E[p]\Big]$ *and* **(a)**

(ii) $\beta \in [\underline{\beta}, \overline{\beta}]$, **(b)**

then the decision of agents to acquire information depends exclusively on (R, C). More precisely, for any C there exists a function $R^*(C)$ such that, given a combination (R, C), agents learn p if $R > R^*(C)$ and remain uninformed if $R < R^*(C)$.

Proof. See Appendix 1. $\square$

First, from **(C2)** we know that ignorance can be of potential interest for agents only if the true probability of success falls in the inconsistency region (p_1, p_2) relatively

[13] Given our extreme modeling of learning, uninformed and informed agents invest with ex ante probability 1 and $1 - F(p_2)$, respectively. However, from Lemma 1, it is clear that this can be generalized to other learning technologies in which for example agents may become partially informed.

more often than below it. However, these cutoffs are endogenously determined. Part **(a)** states the formal conditions on $F(\cdot)$, δ, π and e such that p is more likely to take "intermediate" values rather than "low" ones. Condition **(a)** is therefore necessary in order to consider the possibility of remaining uninformed. Second, the incentives to learn are not monotonic in the taste for immediate gratification. For instance, when β is sufficiently small, self-1 anticipates that self-2 will not invest if he remains uninformed (i.e. $E[p] < p_2$, for all (R, C)). As a consequence, self-1 strictly prefers learning in that case. On the opposite side, if β is sufficiently high, then there is 'almost' no conflict of interests between self-1 and self-2, and again the agent also strictly prefers to learn p whatever the pair (R, C).[14] The interesting situation arises when the inconsistency parameter takes intermediate values $\beta \in [\underline{\beta}, \overline{\beta}]$.[15] Part **(b)** states that in this case the benefits from learning are mitigated, so both **(C1)** and **(C2)** may or may not hold depending on the contract (R, C) offered by banks. More specifically, there will exist a set of pairs $(R^*(C), C)$ such that agents are indifferent between being informed or uninformed. If, for a given C, R is set above (resp. below) $R^*(C)$ or, for a given R, C is set above (resp. below) $R^{*-1}(R)$, then learning (resp. not learning) becomes a strictly dominant strategy.

To sum up, when conditions **(a)** and **(b)** are not satisfied, then agents have incentives either to learn always or to remain ignorant always. Banks rationally anticipate this behavior and set the interest rate and collateral requirement accordingly. When **(a)** and **(b)** are satisfied, the pair (R, C) set by banks jointly determines (i) whether agents learn and (ii) whether agents invest conditional on the outcome of their learning decision. As we will see in the next section, the interesting feature is that R and C will be determined by banks precisely depending on the learning decision of agents. In other words, the decision of each agent whether to learn will indirectly be affected by the learning decision of all the other agents via the contract offered by banks. In order to focus on this situation, we assume for the rest of the paper that the prescriptions of Lemma 3 are satisfied.

Assumption 4 *$F(\cdot)$, β, δ, π, and e are such that conditions* **(a)** *and* **(b)** *hold.*

[14] In particular, when $\beta \to 1$, then $p_2 \to p_1$: the intra-personal conflict vanishes and learning is always desirable.

[15] See equations (A2) and (A4) in Appendix 1 for the functional forms of $\underline{\beta}$ and $\overline{\beta}$. Naturally, $[\underline{\beta}, \overline{\beta}] \subset (1/2, 1)$.

As a consequence, the utility of each agent can be written as:

$$u(R, C; \beta) = \begin{cases} G_u(R, C; \beta) & \text{if} \quad R \leq R^*(C) \\ G_i(R, C; \beta) & \text{if} \quad R > R^*(C) \end{cases} \tag{9}$$

At this point we can investigate the behavior of banks when there is perfect competition in the credit market.

2.4 The competitive credit market

We have studied the learning and investment decision of agents for a given contract (R, C) offered by banks. However, banks determine both the interest factor and the collateral anticipating their effects on the strategy of agents. We assume that there is a large number of risk neutral banks. We denote by $\bar{R}$ (≥ 1) the risk free interest factor (i.e. one plus the risk free rate) and assume that it is exogenously given. We will refer to "good times in the credit market" when the risk free rate is small and "bad times" when it is high. Banks do not observe whether a particular agent has decided to learn his probability of success, so all agents posting the same collateral can borrow at the same interest factor. Besides, the entrepreneur and the bank cannot contract at date $t = 1$ on the investment to be undertaken at date $t = 2$. Banks however observe whether the investment is a success or a failure, so the contract signed at date $t = 2$ can be contingent on the realization of profits. R and C are endogenously determined in a competitive equilibrium after the agents' learning decision.

Agents can only borrow in order to undertake the project. So, given Bertrand competition between banks, agents maximize their utility $u(R, C; \beta)$ under the constraint that banks do not make losses in expected terms. This constraint depends on whether applicants are informed or not. Denote by $(R_N(C), C)$ the pairs of competitive interest factor and collateral requirement charged by a bank to an agent who applies for a loan when it is common knowledge that the latter is uninformed. Similarly, denote by $(R_L(C), C)$ the pairs of competitive interest factor and collateral charged to an agent when it is common knowledge that he has learned his probability of success (even though his true p remains private information). Formally:

$$E[p]\, R_N(C) + (1 - E[p])\, C \;=\; \bar{R} \tag{10}$$

$$E[p \,|\, p > p_2(R_L(C), C)]\, R_L(C) + (1 - E[p \,|\, p > p_2(R_L(C), C)])\, C \;=\; \bar{R} \tag{11}$$

From (10) and (11), it is easy to see that $R_N(C) > R_L(C) > C$ for all $C \in [0, \bar{R})$, and $R_L(\bar{R}) = R_N(\bar{R}) = \bar{R}$. In fact, for any collateral requirement smaller than the repayment obligation and given **(C1)**, all uninformed agents decide to apply for the loan. By contrast, an agent who learns p may prefer not to borrow capital in order to invest if his chances of success are sufficiently small ($p < p_2(R_L(C), C)$). This *self-selection* process of informed agents diminishes the risk of not being repaid by investors, and therefore allows banks to reduce the interest factor. Overall, under incomplete but symmetric information (uninformed agents) projects have, on average, a lower profitability than under asymmetric information (informed agents). Note that this is not in contradiction with the usual results of the literature on credit, where it is argued that asymmetric information decreases the average quality of projects relative to complete information.[16]

3 Equilibria

In Sections 2.2 and 2.3 we have studied the agents' incentives to acquire information and invest conditional on the outcome of their learning decision, for a given interest factor and collateral. In Section 2.4 we have analyzed the contract offered by banks as a function of the agents' learning decision. We will now combine both the agents' and banks' behavior in order to determine simultaneously the equilibrium interest factor, collateral, and level of investment in the economy. We consider situations in which agents have respectively no collateral (Section 3.1), unrestricted collateral (Section 3.2), and restricted collateral (Section 3.3.).

3.1 Learning and investment with no collateral

We first analyze the equilibrium in this economy when agents have no collateral ($C = 0$). By abuse of notation, in this section we will call $R^* \equiv R^*(0)$, $R_N \equiv R_N(0)$, $R_L \equiv R_L(0)$ and we will drop the argument C from $p_1(\cdot)$ and $p_2(\cdot)$. Given perfect competition between banks and ex ante homogeneity of individuals, we can rewrite our problem as the maximization of the agents' utility (9) subject to the banks' break even constraints (10) and (11). Formally, we have problem **P**:

$$\mathbf{P}: \quad \max_R \; u(R;\beta) = \begin{cases} G_u(R;\beta) & \text{if} \quad R \leq R^* \\ G_i(R;\beta) & \text{if} \quad R > R^* \end{cases}$$

[16] In fact, our model yields the same predictions if we compare complete vs. asymmetric information. It can be easily shown that under complete information only projects with a probability of success greater than $p_2^*(C)$ would be financed, where $p_2^*(C) > p_2(R_L(C), C)$.

$$\begin{array}{lll} \text{s.t.} \quad E[p] \times R \geq \bar{R} & \text{if} & R \leq R^* \\ \quad\quad E[p \,|\, p > p_2(R)] \times R \geq \bar{R} & \text{if} & R > R^* \end{array}$$

which leads to the following proposition.

Proposition 1 *When $C = 0$, there exist two values $\bar{R}_1$ and $\bar{R}_2$ $(> \bar{R}_1)$ such that:*

(i) *If $\bar{R} < \bar{R}_1$, the interest factor is R_N $(= \frac{\bar{R}}{E[p]})$ and no agent learns p;*

(ii) *If $\bar{R}_1 \leq \bar{R} \leq \bar{R}_2$, the interest factor is R^* and a fraction $\alpha(\bar{R})$ of agents learn p, with $\alpha(\bar{R}_1) = 0$, $\alpha(\bar{R}_2) = 1$, and $\partial\alpha/\partial\bar{R} > 0$.*

(iii) *If $\bar{R}_2 \leq \bar{R}$, the interest factor is R_L (with $R_L = \frac{\bar{R}}{E[p \,|\, p > p_2(R_L)]}$) and all agents learn p.*

Proof. Note first that if $\bar{R} > R(\beta)$, the agent never invests. Indeed, the bank will always offer $R > \bar{R}$ in that case. Suppose that $\bar{R} \leq R(\beta)$. For a given R^*, denote by $\bar{R}_1$ and $\bar{R}_2$ $(> \bar{R}_1)$ the values such that:

$$R^* = \frac{\bar{R}_1}{E[p]} = \frac{\bar{R}_2}{E[p \,|\, p > p_2(R^*)]}$$

- $\bar{R} < \bar{R}_1 \;\Leftrightarrow\; R_N < R^*$. Given Bertrand competition, $R = R^*$ cannot be the equilibrium interest factor since it would imply benefits for banks even if no agent learns p. For all $R < R^*$ agents strictly prefer not to learn p, so the competitive equilibrium is $R = R_N$.
- $\bar{R} > \bar{R}_2 \;\Leftrightarrow\; R_L > R^*$. Then, $R = R^*$ cannot be the equilibrium interest factor since it would imply losses for banks even if all agents learn p. For all $R > R^*$ agents strictly prefer to learn p, so the competitive equilibrium is $R = R_L$.
- $\bar{R}_1 < \bar{R} < \bar{R}_2 \;\Leftrightarrow\; R_L < R^* < R_N$. In this case, $R = R^*$ is the competitive interest factor if and only if:

$$\Big[\alpha(\bar{R})E[p \,|\, p > p_2(R^*)] + (1 - \alpha(\bar{R}))E[p]\Big]\, R^* = \bar{R} \tag{12}$$

But, by definition of R^*, agents are indifferent between learning and not. Hence, for each $\bar{R} \in (\bar{R}_1, \bar{R}_2)$, there exists a value $\alpha(\bar{R}) \in (0, 1)$ that satisfies (12). □

If the risk free rate is sufficiently low ($\bar{R} < \bar{R}_1$ as in part (i)), banks do not need to impose a large interest rate to satisfy the break even constraint. In that case and by Lemma 2, the cost of ignorance (i.e. the probability of investing with expected losses from the perspective of self-1) is low. Therefore, agents prefer to remain uninformed (which solves their time inconsistency problem) and invest at $t = 2$. Overall, this case is characterized by a population of *bold entrepreneurs*. Agents are bold, in the sense that they prefer not to acquire information about their chances of success and "blindly jump into the water", a behavior that leads to excessively high failure rates. Stated differently, the level of entrepreneurial activity in the economy is such that a fraction of agents invest with negative Net Present Value: they incur "entry mistakes".[17] Given the agents' excessive willingness to invest, this conduct is observationally equivalent to optimism, so we will refer to it as "observational optimism". However it is *purely rational*, so it should not be misinterpreted as (suboptimal) intrinsic optimism. In fact, conditional on their information, individuals take the decision that maximizes their profit. Moreover, the endogenous decision to acquire pieces of news is also optimized given the intrapersonal conflict of preferences. To sum up, when the credit market is in a 'good state' boldness and entry mistakes are both observed in the economy at the aggregate level. Still, it does not require an ad hoc assumption of biased perceptions or beliefs of individuals. Instead, it is a rational decision that helps in overcoming a natural tendency to procrastinate.

When the risk free rate is sufficiently high ($\bar{R} > \bar{R}_2$ as in part (iii)), banks need to impose a high interest rate to avoid expected losses. In that case, the benefits of learning the probability of success are high relative to the costs due to time inconsistent preferences. Hence, all agents strictly prefer to know the environment they are facing and, at date 2, only a fraction $1 - F(p_2(R_L))$ of individuals decide to invest. Note that in cases (i) and (iii), the interest factor effectively charged by banks R_N and R_L are increasing in the risk free rate. The agents' expected profits of investing are therefore decreasing in $\bar{R}$.

Last, there is a whole set of values $\bar{R} \in [\bar{R}_1, \bar{R}_2]$ for which the interest rate is *fixed* and equal to R^*. When $\bar{R}$ is greater than $\bar{R}_1$ but close to it, a competitive interest factor R^* is sustainable only if almost all agents remain ignorant (weak self-selection). Similarly, when $\bar{R}$ is smaller than $\bar{R}_2$ but close to it, an interest factor R^* is sustainable only if almost every agent becomes informed (strong self-selection). By definition, when the repayment is R^* agents are indifferent between learning and not. Then, in the interval $[\bar{R}_1, \bar{R}_2]$, a change in the risk free rate leads to a change in the fraction of agents $\alpha(\bar{R})$ who learn p without affecting the repayment. Besides, since R^* is fixed, the agents' expected profit is constant in the whole interval. To sum up, this case

[17]Formally, a proportion of agents $F(p_1(R_N))$ and $F(p_2(R_N))$ invest with expected net losses from self-1's and self-2's viewpoint, respectively.

shows that bold and realist entrepreneurs may not only *coexist* in this economy but also *achieve the same expected profits* from their perspective at date 1.[18]

The most important conclusion is that, as we move towards 'good times in the credit market' ($\bar{R}$ decreases), there are two reasons for which a higher proportion of agents decide to invest. First, as usual, the opportunity cost of investing is lower: R_L (and therefore $p_2(R_L)$) decreases as $\bar{R}$ decreases (see equation (11)). This is the standard negative relation between risk free rate and number of profitable investments. But second and more importantly, agents have more incentives to remain ignorant, boldly invest and therefore incur entry mistakes. Overall, there is a *positive relation between the proportion of bold entrepreneurs in the population (measured by the amount of their excessive willingness to invest) and the state of the economy (measured by the risk free factor* $\bar{R}$*).*

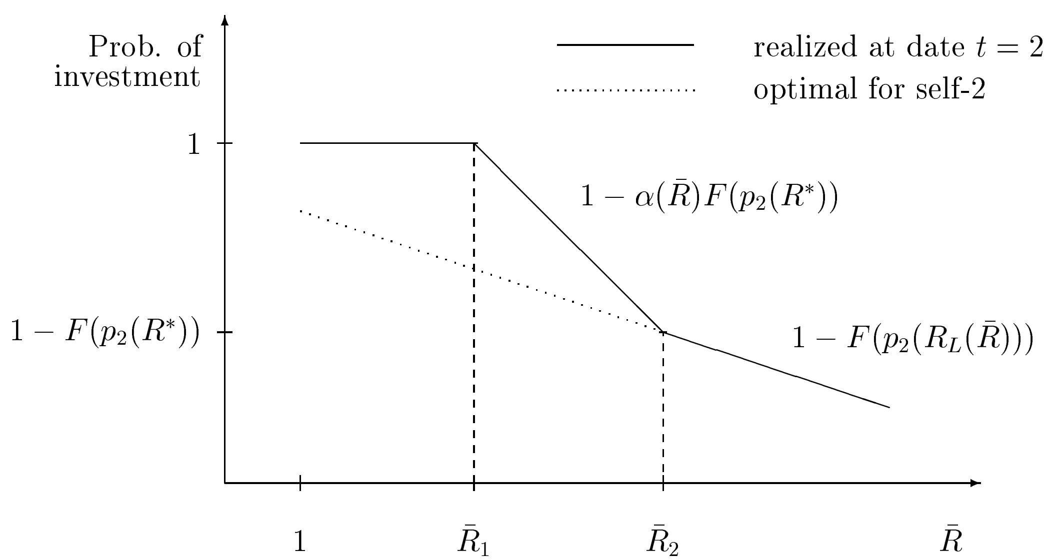

Figure 2. Level of investment in the economy.

Figure 2 depicts the level of investment in the economy as a function of the risk free rate. The negative slope of the dashed line reflects the first effect. The difference between the full line and the dashed line reflects the second effect.[19]

[18]Interestingly, from self-2's perspective, learning p is a strictly dominant strategy. However, this relies on the assumption of a three-period model and just one possibility of investment. Indeed, if p also determines the chances of success of a second investment possible in period 3 and with benefits in 4, it is no longer true that self-2 unambiguously prefers being informed.

[19]Note that the number of entry mistakes is decreasing in $\bar{R}$ also from self-1's viewpoint.

This result is very different from the conclusion reached by Keynes (and the other papers on optimism mentioned in the introduction). In his discussion of the psychological factors affecting expectations, Keynes argues that intrinsic optimism is *necessary* for business success:

> *A large proportion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation, whether moral or hedonistic or economic. [...] Thus if the animal spirits are dimmed and the spontaneous optimism falters, leaving us to depend on nothing but a mathematical expectation, enterprise will fade and die.* J. M. Keynes (1936, ch.12).

In other words, 'spontaneous optimism' (without any specification about where does it come from) breeds the entrepreneurial appetite. Our paper has the opposite logic. We claim that observing bold entrepreneurs willing to blindly invest may not be an irrational conduct. Moreover, we predict that a good state of the economy endogenously fosters boldness (or observational optimism) and therefore exhibits a higher number of entry mistakes.

Last, we would like to point out that Carrillo and Mariotti (1997) already emphasize the existence of a bias in the observed behavior of the population due to strategic ignorance. The present paper focuses on a new and very natural application, with the same intrapersonal conflict but also an interpersonal contracting problem. More importantly, there are two major novel features in our work. First, each agent's strategy endogenously depends on the strategy of the other agents. This allows us to determine the equilibrium level of boldness and entry mistakes in the economy and to perform some comparative statics. To our knowledge, this is the first study in which agents are endogenously related to each other by their time inconsistent preferences. Second, in the previously mentioned paper, agents are indifferent between acquiring information and not only in knife-edge situations. As all agents are ex ante identical and behave rationally, our work suggests that when realist and bold entrepreneurs coexist, then their average performance is the same. This also contrasts with the mainstream literature on behavioral corporate finance in which high failure rates is explained by intrinsic optimism (e.g. agents do not incorporate information in a Bayesian way, and rather overestimate systematically their chances of success). In those papers, the presence of optimists may be beneficial for the welfare of society. However, individually they systematically perform worse than their rational, realist peers (except perhaps in the long run).[20]

Remarks.

• Our result cannot be replicated by considering time consistent individuals and costly learning. Under those assumptions, if the interest factor is sufficiently small, agents invest without paying the cost of learning. Similarly, for sufficiently high interest factors, not learning and never investing is the best strategy. Then, only for intermediate values of R, agents find it profitable to pay the cost of learning before making their investment decision. In other words, the incentives to acquire information are not increasing in R (as in our Lemma 2) but rather have an inverted U-shape. Hence, contrary to our model which yields clear predictions, this approach would generate an observational bias towards boldness or towards conservatism in the population (i.e. too much or too little willingness to invest) depending on the parameters of the model.

• Suppose that the distribution is such that $E[p \mid p > p_1] > p_2$. In that case, self-1 can achieve his first best outcome by asking another individual to collect the information and to report only whether the probability of success is above or below p_1. Indeed, if the report says that $p < p_1$, self-2 does not invest at date $t = 2$, whereas if the report says that $p > p_1$ self-2 invests. Delegating the acquisition of information is therefore a way to avoid the cost of ignorance while keeping all its benefits. Naturally, when the information is about an intrinsic characteristic of the individual, delegation may not be feasible and, even when it is, it raises a problem of renegotiation-proofness. More importantly, the optimality of this strategy relies on our simple modeling. For instance, if we consider a more comprehensive learning technology (in which agents can be partially informed) or a wider set of investment alternatives (rather than just a binary decision) delegation will still improve self-1's decision. However, it will not necessarily imply that he will obtain his first best outcome.

• The coexistence of bold and realist entrepreneurs can be seen as a special case. Indeed, if we add some extra uncertainty (on the inconsistency parameter or on the opportunity cost of investing for example), then each individual will strictly prefer either to become informed or to remain ignorant. Yet, the essence of our result is that

[20] Manove (1997) argues that, in a competitive environment, optimist agents who overestimate the marginal productivity of their business may achieve a higher steady-state level of income and consumption than realist agents. However, these optimists set inefficiently high savings rates so that before reaching the steady state, their intertemporal utility is smaller than that of realists.

two "almost identical" agents may have almost the same expected utility, but radically different behavior.[21] This finding is robust to small perturbations of our setup.

• Finally, our model allows us to make a comparison by sectors of activity. It suggests that both boldness and entry mistakes are more likely to be present in sectors in which expected profits are high ($E[p]$ 'high').

3.2 Learning and investment with unrestricted collateral

In the previous section, one of the key issues for the agents' willingness to keep excessively optimistic prospects is the relatively small opportunity cost of investing. Indeed, given limited liability and no collateral, when the bad state of nature is realized agents only "lose" their cost of effort e. Allowing the use of collateral is likely to alter the incentives of agents to acquire information, and therefore the overall equilibrium of the economy. If we assume that all agents can post a collateral C ($\leq R$),[22] we get the following result.

Proposition 2 *When $C \leq R$, there exists a value $\bar{R}_0$ ($< \bar{R}_1$) such that:*

(i) *If $\bar{R} \leq \bar{R}_0$, then no agent learns p. The interest factor and collateral is any combination $(R_N(C), C)$ with $C \in [0, \bar{R}]$;*

(ii) *If $\bar{R} > \bar{R}_0$, then all agents learn p. For each $\bar{R}$, there exists an optimal combination of interest factor and collateral $(R_L(\hat{C}(\bar{R})), \hat{C}(\bar{R}))$ with $\hat{C}(\bar{R}) < \bar{R}$.*

Proof. See Appendix 2. □

Allowing to post some collateral does not affect qualitatively our results. For sufficiently low values of $\bar{R}$, agents still prefer to remain uninformed and invest. However, there are two main new effects due to the use of collateral. First, ignorance is a less likely event (for instance, learning becomes strictly optimal when $\bar{R} \in (\bar{R}_0, \bar{R}_2)$). Second, informed and uninformed agents cannot coexist anymore, except in knife-edge cases ($\alpha(\bar{R}) \notin (0, 1)$).

Posting some outside collateral increases the cost of investing and default on the payment, which in turn increases the cost of being uninformed (see (8)). Overall,

[21] Formally, suppose that the inconsistency parameter β of each agent is drawn from a known distribution. There would exist a cutoff β^* such that for all $\epsilon > 0$ the agent learns if $\beta = \beta^* + \epsilon$ and remains ignorant if $\beta = \beta^* - \epsilon$. As $\epsilon \to 0$, the utility of both types of agents converge, but not their behavior.

[22] A collateral C greater than the repayment obligation R is not enforceable by the US contract law.

agents who risk their collateral are less willing to invest blindly. Still, the intra-personal conflict does not vanish. So, for a risk free rate sufficiently low ($\bar{R} < \bar{R}_0$), ignoring some information remains the optimal strategy. This result is in contrast with the standard prediction that increasing the transfer when the investment fails does not affect negatively the incentives to keep optimistic prospects. The reason for such a prediction is that we usually assimilate bold entrepreneurs to irrational, overconfident agents who are relatively careless about payments in bad states of nature. By contrast, recall that in our work boldness is not assumed, but rather the result of a rational decision when its benefit exceeds its cost. It is therefore fairly natural that, when the anticipated costs associated to ignorance increase, this attitude becomes less attractive. Also, when $\bar{R} < \bar{R}_0$, agents are indifferent between all the combinations of interest factor and collateral that yield no profit to banks. The reason for this is technical and it should not be overemphasized.[23]

The idea that bold and realist agents can no longer coexist should not be taken too literally. Collateral adds a new dimension in which banks can compete for capturing entrepreneurs. So given an interest factor for which agents were playing a mixed strategy, there is now an optimal combination (R, C) such that one of the two alternatives (learning p or not) is strictly preferred to the other. However, this result relies crucially on our specific modeling of the uncertainty resolution. More precisely, it is due to the competition between banks on *two* dimensions (interest and collateral) together with a maximum of *two* different types of agents (uninformed and perfectly informed). If we consider gradual uncertainty resolution so that some agents can be partially informed, then individuals with different degrees of information (and therefore a different behavior) will coexist in the economy even under the inclusion of collateral.

Note also that, in this model, agents benefit from the possibility of using collateral. The logic is as usual. When agents risk their collateral, they are more likely to become informed. Given the self-selection process, this increases the average probability of success which, in turn, decreases the competitive interest factor charged by banks.

Last, the reader might argue that there is scope for discrimination. Banks could screen uninformed and informed individuals and offer to the latter a menu of contracts $(R(p), C(p))$ so that each agent would pick one according to his true probability of

[23] To be precise, by (5) and (10) and *given our stylized modeling*, the compensation in terms of reimbursement asked by banks to reduce the collateral corresponds exactly to what agents are willing to pay for it.

success (as in standard adverse selection models). We have two comments on this. First, when agents do not post collateral discriminating is not possible, since banks do not have enough tools to induce agents' self-selection (they can only fix the interest factor).[24] Second, even under collateral, discrimination would not affect our results qualitatively; in this more complicated contracting world, strategic ignorance and investment would still sometimes be the optimal way of avoiding procrastination.

3.3 Restricted collateral and imperfect competition

It is clearly unrealistic to consider that all agents have the same endowments (in terms of available collateral) and that banks are perfectly competitive. However, it also seems intuitive that the main effects highlighted in Propositions 1 and 2 in terms of incentives to ignorance and excessive investment do not depend on these two assumptions. The purpose of this section is to briefly describe the possible changes in the results when banks are not competitive and agents have restricted collateral.

Proposition 3 (i) *When there is imperfect competition between banks, agents are more likely to become informed.*

(ii) *Even under perfect competition, banks can make profits with agents who have a restricted collateral.*

Proof. (i) For simplicity, consider the case of a monopolistic credit market and no collateral. Call $v_N(R)$ and $v_L(R)$ the expected profit of a bank charging an interest factor $R \leq R^*$ (so that agents are uninformed) and $R > R^*$ (so that agents become informed) respectively. Since $\bar{R}$ is the banks' opportunity cost of lending one unit of capital, we have:

$$v_N(R) = \int_0^1 (p\,R - \bar{R})\,dF(p) \quad \text{and} \quad v_L(R) = \int_{p_2(R)}^1 (p\,R - \bar{R})\,dF(p)$$

Note that $R^* = \arg\max_{R \leq R^*} v_N(R)$ and denote by $\tilde{R} = \arg\max_{R > R^*} v_L(R)$.

If $\bar{R} < \bar{R}_1$, then $R_L < R_N < R^*$, so $v_L(R^*) > 0$ and $v_N(R^*) > 0$. The interest factor charged by banks is $\tilde{R}$ if $v_L(\tilde{R}) > v_N(R^*)$ and R^* if $v_L(\tilde{R}) < v_N(R^*)$. In the former case agents learn p and in the latter they do not. If $\bar{R}_1 < \bar{R} < \bar{R}_2$, then $R_L < R^* < R_N$, so

[24] By contrast, if p were observable to banks, they would offer a different contract to each agent. In that case, the decision to learn and to invest of each individual would be independent of the decision of others.

$v_N(R^*) < 0 < v_L(R^*)$. Then, banks charge $R = \tilde{R}$ and agents learn p. If $\bar{R}_2 < \bar{R}$, then $R^* < R_L$, so $v_L(R^*) < 0$. Banks charge $\tilde{R}$ and agents learn p if $v_L(\tilde{R}) > 0$, or there is no lending if $v_L(\tilde{R}) < 0$.

(ii) For each $\bar{R} \in (\bar{R}_0, \bar{R}_1)$, there exists a value $\bar{C}(\bar{R}) \in (0, \bar{R})$ such that $R^*(\bar{C}(\bar{R})) = R_N(\bar{C}(\bar{R}))$. From the proof of Proposition 2, we know that $-\frac{\partial R^*}{\partial C} > -\frac{\partial R_N}{\partial C} > -\frac{\partial R_L}{\partial C}$. Therefore, $\bar{C}(\bar{R}) < \tilde{C}(\bar{R})$ where recall that $\tilde{C}(\bar{R})$ is such that $R^*(\tilde{C}(\bar{R})) = R_L(\tilde{C}(\bar{R}))$. If the maximum collateral of agent k is $C_k \in [\bar{C}(\bar{R}), \tilde{C}(\bar{R}))$, then the equilibrium is $(R^*(C_k), C_k)$. Overall, agent k learns p and the bank makes positive profits. $\square$

When banks do not behave competitively, they have incentives to increase the interest for two reasons. First, the standard profit maximization motive: in order to obtain positive benefits, the interest rate must be set above the break even constraint. In a more subtle way, a higher reimbursement increases the incentives of agents to learn, which triggers off the self-selection process. Naturally, this increase in the average quality of agents applying for the loan decreases the risk of failure, which is beneficial for banks. Overall, the number of bold entrepreneurs and entry mistakes in the economy is lower when banks have monopoly power than under perfect competition.

Recall that when $\bar{R} \in (\bar{R}_0, \bar{R}_1)$, if agents have no collateral they remain uniformed, and if they have unrestricted collateral they post it, learn the probability of success, and benefit from a substantially lower interest rate (see Propositions 1 and 2). An agent with limited collateral C_k may not be able to post a collateral sufficiently high so as to be offered the competitive interest rate and still have incentives to learn his probability of success (formally, for all $C < C_k$, $g(R_L(C), C) > 0$). In that situation, he faces two options. First, to borrow at the competitive interest rate given ignorance $R_N(C_k)$, and remain uninformed. Second, to borrow at an interest rate higher than the competitive one given learning $R^*(C_k)$ $(> R_L(C_k))$, and become informed. The second option is preferable when $R_N(C_k) > R^*(C_k)$. Overall, this agent prefers to learn his probability of success even if it implies that banks obtain some profits. It is interesting to notice that this mechanism holds even though banks are perfectly competitive.

4 Entrepreneurial ability and incentives to invest

Up to now, we have considered a population of individuals with identical managerial capacity. However, it is natural to posit that agents differ in their ability to undertake (or evaluate) risky projects. This section extends our basic setup to account for differences in the agents' entrepreneurial skills. Obviously, more able individuals are likely to succeed better in their investment projects. The interesting issue is to study whether agents with high capacity to evaluate projects are more likely to remain uninformed and behave boldly or not. In other words, we want to determine the relation between ability of individuals and proportion of entry mistakes.

For simplicity, we consider an extension of the model with no collateral. Individuals are of two types. Both types have a probability of success p drawn from the same distribution $F(p)$. The difference is that, in case of success, high ability agents (in proportion $1-q$) make profit π_a while low ability agents (in proportion q) make profit π_b $(< \pi_a)$.[25] The parameters of the model $(\beta, \delta, e, F(\cdot), \pi_a, \pi_b)$ are such that Assumption 4 is satisfied for both π_a and π_b. We call $p_2^a(R) = p_2(R; \pi_a)$ and $p_2^b(R) = p_2(R; \pi_b)$. By analogy with Section 2.3, denote by R'_N, R'_{NL} and R'_L the competitive interest factor when no agent learns p, when only low ability agents learn p, and when all agents learn p, respectively. We have:

$$R'_N \times E[p] = \bar{R}, \qquad R'_{NL} \times H(p, R'_{NL}) = \bar{R}, \qquad R'_L \times J(p, R'_L) = \bar{R}$$

$$\begin{aligned} \text{where } \ H(p,R) &= \frac{(1-q)\, E[p]}{(1-q)+q(1-F(p_2^b(R)))} + \frac{q(1-F(p_2^b(R)))\, E[p \mid p>p_2^b(R)]}{(1-q)+q(1-F(p_2^b(R)))}; \\ J(p,R) &= \frac{(1-q)(1-F(p_2^a(R)))\, E[p \mid p>p_2^a(R)]}{(1-q)(1-F(p_2^a(R)))+q(1-F(p_2^b(R)))} + \frac{q(1-F(p_2^b(R)))\, E[p \mid p>p_2^b(R)]}{(1-q)(1-F(p_2^a(R)))+q(1-F(p_2^b(R)))}. \end{aligned}$$

Although the equations seem messy, they are indeed quite simple. They just represent the expected probability of success of a low and high ability agent weighted by the proportion of low and high type agents in the economy who apply for a loan, respectively. In the case of $H(\cdot)$, only the fraction q of low ability agents become informed and self-select themselves. In the case of $J(\cdot)$, there is self-selection by all agents. It is easy to check that $R'_L < R'_{NL} < R'_N$: as usual, the more agents engage in the self-selection process, the lower the competitive interest factor. Last, denote R_a^* and R_b^* the interest factor so that high ability and low ability agents are indifferent between being informed and not, respectively. That is:

$$G_u(R_a^*; \pi_a) = G_i(R_a^*; \pi_a) \qquad \text{and} \qquad G_u(R_b^*; \pi_b) = G_i(R_b^*; \pi_b)$$

[25] This way of modeling different abilities is very simplistic. Besides, one can argue that ability should influence the probability of success rather than its payoff. Still, nothing would be gained except some computational troubles if, for example, we assume that the c.d.f. of the probability of success is $F_a(p)$ for high ability agents and $F_b(p)$ for low ability ones, with $F_a(p) < F_b(p)$ for all p.

We have the following result.

Proposition 4 *There exist four values $\bar{R}_1^b < \bar{R}_2^b < \bar{R}_2^a < \bar{R}_3^a$ such that:*

(i) *If $\bar{R} < \bar{R}_1^b$, the interest factor is R'_N $(= \frac{\bar{R}}{E[p]})$ and no agent learns p;*

(ii) *If $\bar{R}_1^b \leq \bar{R} \leq \bar{R}_2^b$, the interest factor is R_b^* and only a fraction $\gamma_b(\bar{R})$ of low ability agents learn p, with $\partial\gamma_b/\partial\bar{R} > 0$;*

(iii) *If $\bar{R}_2^b < \bar{R} < \bar{R}_2^a$, the interest factor is R'_{NL} $(= \frac{\bar{R}}{H(p,R'_{NL})})$ and all the low ability agents and none of the high ability ones learn p;*

(iv) *If $\bar{R}_2^a \leq \bar{R} \leq \bar{R}_3^a$, the interest factor is R_a^* and all the low ability agents and a fraction $\gamma_a(\bar{R})$ of the high ability ones learn p, with $\partial\gamma_a/\partial\bar{R} > 0$;*

(v) *If $\bar{R}_3^a < \bar{R}$, the interest factor is R'_L $(= \frac{\bar{R}}{J(p,R'_L)})$ and all agents learn p.*

<u>Proof</u>. Following Lemma 2, note that $\frac{\partial g(\cdot)}{\partial \pi} = -\frac{\partial g(\cdot)}{\partial R} > 0$. Therefore, $R_b^* < R_a^*$ $(= R_b^* + \pi_a - \pi_b)$ and $p_2^a(R_a^*) = p_2^b(R_b^*)$. Denote by $\bar{R}_1^b$, $\bar{R}_2^b$, $\bar{R}_2^a$ and $\bar{R}_3^a$ the values such that:

$$R_b^* = \frac{\bar{R}_1^b}{E[p]} = \frac{\bar{R}_2^b}{H(p, R_b^*)} \qquad \text{and} \qquad R_a^* = \frac{\bar{R}_2^a}{H(p, R_a^*)} = \frac{\bar{R}_3^a}{J(p, R_a^*)}$$

where, given $E[p] < H(p,R) < J(p,R)$ and $R_b^* < R_a^*$, then $\bar{R}_1^b < \bar{R}_2^b < \bar{R}_2^a < \bar{R}_3^a$.

- $\bar{R} < \bar{R}_1^b \Leftrightarrow R'_N < R_b^*$. For all $R \geq R_b^*$ banks make profits. For all $R < R_b^*$, no agent learns p, so the equilibrium interest factor is R'_N.
- $\bar{R}_1^b \leq \bar{R} \leq \bar{R}_2^b \Leftrightarrow R'_{NL} < R_b^* < R'_N$. If $R > R_b^*$ only low ability agents learn p and banks make profits. If $R < R_b^*$ no agent learns p and banks make losses. In equilibrium, the interest factor is R_b^* and only a fraction $\gamma_b(\bar{R})$ of low ability agents learn p, where $\gamma_b(\bar{R}_1^b) = 0$, $\gamma_b(\bar{R}_2^b) = 1$, $\partial\gamma_b(\bar{R})/\partial\bar{R} > 0$.
- $\bar{R}_2^b < \bar{R} < \bar{R}_2^a \Leftrightarrow R_b^* < R'_{NL} < R_a^*$. The equilibrium interest factor is R'_{NL}, so that all low ability agents and none of the high ability ones become informed.
- $\bar{R}_2^a \leq \bar{R} \leq \bar{R}_3^a \Leftrightarrow R'_L < R_a^* < R'_{NL}$. By a reasoning similar to case (ii), the interest factor is R_a^*. All the low ability agents and a fraction $\gamma_a(\bar{R})$ of the high ability ones learn p, where $\gamma_a(\bar{R}_2^a) = 0$, $\gamma_a(\bar{R}_3^a) = 1$, $\partial\gamma_a(\bar{R})/\partial\bar{R} > 0$.
- $\bar{R}_3^a < \bar{R} \Leftrightarrow R_a^* < R'_L$. The interest factor is R'_L and all agents learn p. □

An increase in the benefits of a successful investment π has exactly the opposite effect of an increase in R, that is to raise the payoff in the good state of nature. Then, using the same argument as in Lemma 2 but in the opposite direction, we get that high ability agents have more incentives to forego information and boldly invest than their low ability peers.

Overall, if we interpret e as the fixed wage earned by an individual in the alternative safe occupation (i.e. as an employee), agents with high capacity are more prone to self-select themselves into becoming entrepreneurs rather than workers for two reasons. First, the trivial motive: the expected payoff of their projects is higher. But second, because they are more likely to avoid information that, due to time inconsistent preferences, would discourage some investments profitable form their perspective at date 1. To sum up, there is an endogenous positive correlation between intrinsic managerial capacity, decision to become an entrepreneur, and proportion of bold investors in the economy. At the same time, more capable managers are also more likely to commit entry mistakes. As in Proposition 1, bold and realist agents with the same ability may coexist in equilibrium. Should this happen, they necessarily achieve the same expected payoff from their perspective at date 1. Last, note that in our model there is a parallel between managerial capacity and profitability of sectors (see the last remark in Section 3.1). Ignorance and excessive investments are more likely to occur in what we can broadly define as "profitable markets". Differences in profitability may come either from differences in individual entrepreneurial ability or from sector specific differences in rates of success.

5 Concluding remarks

In this paper we have provided an explanation based on dynamic inconsistent preferences for the willingness of entrepreneurs to keep optimistic thoughts. The work does not pretend to question the existence of intrinsic optimists in the population (for which there is large evidence). It does not argue that excessive investment and high business failure rates are entirely driven by the temporal intra-personal conflict of preferences, either. Yet, we feel that deriving from preferences an attitude of agents observationally equivalent to intrinsic optimism is an important step towards a better understanding of the overall patterns of human conduct. More generally, it is sometimes unsatisfactory to adopt shortcut specifications of widely accepted human traits (such as optimism, but also confidence, self-justification, self-esteem and many other biases and behaviors extensively documented in social psychology). In our view, the conclusions reached under such assumptions may be quite different from the ones obtained if the particular trait was derived from individual preferences.

Last, we would like to point out two alleys for future research. First, it would be interesting to test whether entry mistakes are mainly the result of rational strategic ignorance and boldness or of intrinsic optimism. There are *at least* two ways of discriminating between these two alternatives. In our work, ceteris paribus, the proportion of entry mistakes over total number of investments will decrease if agents can post collateral, and it will increase if the risk free rate diminishes. By contrast, theories based on optimism suggest the opposite. Optimist individuals do not care about payoffs in bad states, so collateral only exacerbates their inefficient behavior. Similarly, in bad states for the credit market only optimists apply for loans, while in good states they are diluted in the whole population. Second, the paper highlights the negative relation between risk free rate and boldness. In a dynamic framework, one may conjecture that current boldness leads to entry mistakes and therefore has a negative impact on the future state of the economy. Overall, the economy may exhibit cycles in which periods of low interest rates and excessively high levels of entrepreneurial activity are followed by periods of high interest rates and moderate entrepreneurship.

Appendix

1. Proof of Lemma 3

The proof consists of several steps.

Step 1. Conditions for the problem to be well behaved. For all β, the probabilities are well defined, i.e. $p_1(R,C) \in (0,1)$ and $p_2(R,C) \in (0,1)$, if and only if:

$$R \leq \pi - \frac{e}{\delta\beta} = R(\beta) \tag{A1}$$

According to Assumption 2, π, e, δ and β satisfy $\beta\delta(\pi-1) \geq e$. In other words, $\beta > \tilde{\beta}$ where $\tilde{\beta} = \dfrac{e}{\delta(\pi-1)}$. In addition, for all $\beta > \tilde{\beta}$, $R(\beta) > 1$. Therefore, under Assumption 2 and provided that π, δ and e are such that $\frac{e}{\delta(\pi-1)} < 1$, the problem is well behaved for all $R \in [1, R(\beta)]$.

Step 2. Conditions under which **(C1)** is satisfied. For all C, $E[p] > p_2(R,C)$ if and only if:

$$R < \pi - \frac{e}{\beta\delta E[p]} - C\frac{1-E[p]}{E[p]} = R(C,\beta)$$

Note that $R(C,\beta)$ is decreasing in C and that $R(0,\beta) = \pi - \frac{e}{\beta\delta E(p)} < \pi - \frac{e}{\beta\delta} = R(\beta)$. As a consequence, there exists $\hat{\beta} > \tilde{\beta}$ such that $R(0,\hat{\beta}) = 1$. Besides, $R(C,\beta) < R(0,\beta) < 1$ for all $\beta < \hat{\beta}$. In other words, a necessary condition for **(C1)** to be satisfied is $\beta \geq \hat{\beta} = \frac{e}{\delta E[p](\pi-1)}$ provided that π, δ and e are such that $\frac{e}{\delta(\pi-1)} < E[p]$. Let β_1 and C_1 such that $R(C_1,\beta_1) = C_1 = 1$. By construction β_1 and C_1 are unique and $\beta_1 > \hat{\beta}$.

- for all $\beta \in [\hat{\beta}, \beta_1]$, there exists a unique $C_2(\beta) < C_1$ such that $R(C_2(\beta),\beta) = 1$ in which case **(C1)** is satisfied for all $C < C_2(\beta)$ and $R \in [1, R(C,\beta)]$;

- for all $\beta > \beta_1$, there exists a unique $C_3(\beta) > C_1$ such that $R(C_3(\beta),\beta) = C_3(\beta)$ in which case **(C1)** is satisfied for all $C < C_3(\beta)$ and $R \in [C, R(C,\beta)]$.

Step 3. Conditions under which $g(R,C;\beta) = 0$.

$$\frac{\partial g(R,C;\beta)}{\partial\beta} = \delta(\pi - R + C)f(p_2)\frac{\partial p_2}{\partial\beta}(p_2 - p_1) < 0 \quad \forall\, \beta < 1.$$

Besides, $g(R,C;1) < \delta(\pi - R + C)F(p_2)(p_2 - p_1) = 0$ since $p_2 = p_1$ when $\beta = 1$. When $\beta = \hat{\beta}$, the only pair (R,C) satisfying **(C1)** is $R = 1$ and $C = 0$ and in that case $p_2 = E[p]$. Moreover:

$$g(1,0;\hat{\beta}) = \delta(\pi-1)\int_0^{E[p]} p dF(p) - eF(E[p])$$

and sign $g(1,0;\hat{\beta}) = \text{sign}\left[E\Big[p|p<E[p]\Big] - \frac{e}{\delta(\pi-1)}\right]$. Then, we have two cases:

- $F(\cdot)$ is such that $E\Big[p|p<E[p]\Big] > \frac{E[p]}{2}$.

- if $\frac{e}{\delta(\pi-1)} \in \left[E\Big[p|p<E[p]\Big], E[p]\right]$, then $\hat{\beta} > 1/2$ and $g(1,0;\hat{\beta}) < 0$. In that case, since $g(R,C;\beta)$ is decreasing in both R and C for all $\beta > 1/2$, $g(R,C;\beta) < 0$ for all R and C and the agent always learns.

- if $\frac{e}{\delta(\pi-1)} \in \left[\frac{E[p]}{2}, E\Big[p|p<E[p]\Big]\right]$, then $\hat{\beta} > 1/2$ and $g(1,0;\hat{\beta}) > 0$. Therefore there exists $\beta > \hat{\beta}$, C and $R^*(C)$ such that $g(R^*(C),C;\beta) = 0$. More precisely, for all $\beta > \hat{\beta}$, there exists $(\tilde{R}(\beta),\tilde{C}(\beta))$ such that $(\tilde{R}(\beta),\tilde{C}(\beta)) = \text{argmin}\, g(R,C;\beta)$. Let β^* be such that $g(\tilde{R}(\beta^*),\tilde{C}(\beta^*)) = 0$. By construction, $\beta^* > \hat{\beta}$ and for all $\beta < \beta^*$, $g(R,C;\beta) > 0$, in which case the agent never learns. In addition, there also exists $\overline{\beta}$ such that $g(1,0;\overline{\beta}) = 0$, i.e. that solves:

$$\delta(\pi-1)\int_0^{\frac{e}{\delta\overline{\beta}(\pi-1)}} p dF(p) - eF\left(\frac{e}{\delta\overline{\beta}(\pi-1)}\right) = 0 \tag{A2}$$

and for all $\beta > \overline{\beta}$, $g(R,C;\beta) < 0$ and the agent always learns. Naturally, $\overline{\beta} \geq \beta^*$ by construction.

- if $\frac{e}{\delta(\pi-1)} \in \left(0, \frac{E[p]}{2}\right]$, $\hat{\beta} < 1/2$ and $g(1,0;\hat{\beta}) > 0$. Moreover,

$$g\left(1,0;\frac{1}{2}\right) = -eF\left(\frac{2e}{\delta(\pi-1)}\right) + \delta(\pi-1)\int_0^{\frac{2e}{\delta(\pi-1)}} pf(p)dp$$

and sign $g(1,0;\frac{1}{2}) = \text{sign}\, E\left[p|p<\frac{2e}{\delta(\pi-1)}\right] - \frac{e}{\delta(\pi-1)}$. It is easy to verify that $g(1,0;\frac{1}{2}) > 0$ when $\frac{e}{\delta(\pi-1)} = \frac{E[p]}{2}$. Therefore, there exist π, e and δ satisfying $\frac{e}{\delta(\pi-1)} \in (0, \frac{E[p]}{2}]$ such that $g(1,0;\frac{1}{2}) > 0$. In that situation, using the same reasoning as before, we can characterize $\overline{\beta} > \beta^* > 1/2$ such that (i) for all $\beta < \beta^*$, the agent never learns, (ii) for all $\beta > \overline{\beta}$, the agent always learns and (iii) for all $\beta \in [\beta^*, \overline{\beta}]$, there exist $R^*(C)$ such that $g(R^*(C),C;\beta) = 0$. Naturally, if e, π and δ are such that $g(1,0;\frac{1}{2}) < 0$, the agent learns for all $\beta > 1/2$ and for all R and C suitably chosen.

- $F(\cdot)$ is such that $E\Big[p|p<E[p]\Big] < \frac{E[p]}{2}$.

- if $\frac{e}{\delta(\pi-1)} \in \left[\frac{E[p]}{2}, E[p]\right]$, then $\hat{\beta} > 1/2$ and $g(1,0;\hat{\beta}) < 0$. Since $g(R,C;\beta)$ is decreasing in both R and C for all $\beta > 1/2$, $g(R,C;\beta) < 0$ for all R and C.

- if $\frac{e}{\delta(\pi-1)} \in \left[E\Big[p|p<E[p]\Big], \frac{E[p]}{2}\right]$, then $\hat{\beta} < 1/2$, $g(1,0;\hat{\beta}) < 0$ and $g(1,0;1/2) < 0$. Therefore, $g(R,C;\beta) < 0$ for all R, for all C and for all $\beta > 1/2$.

- if $\frac{e}{\delta(\pi-1)} \in \left(0; E\left[p|p < E[p]\right]\right]$, $\hat{\beta} < 1/2$ and $g(1,0;\hat{\beta}) > 0$. Here again, if π, e and δ satisfy $\frac{e}{\delta(\pi-1)} \in \left(0, E\left[p|p < E[p]\right]\right]$ and are such that $g(1,0;\frac{1}{2}) > 0$, we can determine (as before) $\overline{\beta} > \beta^* > 1/2$ such that (i) for all $\beta < \beta^*$, the agent never learns, (ii) for all $\beta > \overline{\beta}$, the agent always learns and (iii) for all $\beta \in [\beta^*, \overline{\beta}]$, there exist $R^*(C)$ such that $g(R^*(C), C; \beta) = 0$. By contrast, if e, π and δ are such that $g(1,0;\frac{1}{2}) < 0$, the agent learns for all $\beta > 1/2$ and for all R and C suitably chosen.

Step 4. Conditions for $R^*(C)$ to be the frontier between learning and not.

$$\frac{\partial R^*}{\partial C} = -\frac{\frac{\partial g(R^*(C),C)}{\partial C}}{\frac{\partial g(R^*(C),C)}{\partial R}} < 0 \tag{A3}$$

$$\frac{\partial R^*}{\partial C} = -\frac{f(p_2)(1-p_2)(p_2-p_1) - \int_0^{p_2}(1-p)dF(p)}{f(p_2)p_2(p_2-p_1) - \int_0^{p_2}(p)dF(p)} < -\frac{1-E(p)}{E(p)} = \frac{\partial R(C,\beta)}{\partial C}$$

As a consequence, a sufficient condition for $R^*(C)$ to be the frontier between learning and no learning is $g(R(0,\beta),0,\beta) < 0$, which ensures that $R^*(0) < R(0,\beta)$. Otherwise the frontier $K(C)$ is kinked and there exists $\hat{C}$ such that $R^*(\hat{C}) = R(\hat{C},\beta)$. In that case, we have:

$$K(C) = \begin{cases} R(C,\beta) & \text{if } C \leq \hat{C} \\ R^*(C) & \text{if } C > \hat{C} \end{cases}$$

Since $R(0,\beta) = \pi - e\beta\delta E(p)$, in which case $p_2 = E(p)$, there exists $\underline{\beta} \in (\beta^*, \overline{\beta})$ such that $K(C) = R^*(C)$ for all $\beta \in (\underline{\beta}, \overline{\beta})$. Besides:

$$\underline{\beta} = \frac{E[p|p < E(p)]}{E(p)} \tag{A4}$$

Naturally, the frontier is kinked when $\beta \in (\beta^*, \underline{\beta})$[26]. $\square$

2. Proof of Proposition 2

The proof consists of several steps.

Step 1. Iso-profit curves of informed and uninformed agents. Denote $R_u(C)$ and $R_i(C)$ the interest factor functions in the iso-profit curves of an uninformed and an informed agent, respectively. Formally:

$$G_u(R_u(C), C) = \bar{K} \quad \text{and} \quad G_i(R_i(C), C) = \bar{L}$$

[26]We restrict the attention to the case in which the frontier is not kinked but all our results hold also for $\beta \in (\beta^*, \underline{\beta})$

where $\bar{K}$ and $\bar{L}$ are constants. From (4) and (6), we have:

$$\begin{aligned}
\frac{\partial R_u}{\partial C} &= -\frac{\frac{\partial G_u}{\partial C}}{\frac{\partial G_u}{\partial R}} = -\frac{1-E[p]}{E[p]} \\
\frac{\partial R_i}{\partial C} &= -\frac{\frac{\partial G_i}{\partial C}}{\frac{\partial G_i}{\partial R}} = -\frac{1-E[p]-\int_0^{p_2}(1-p)\,dF(p)+(1-p_2)(p_2-p_1)f(p_2)}{E[p]-\int_0^{p_2}p\,dF(p)+p_2(p_2-p_1)f(p_2)}
\end{aligned}$$

Step 2. Iso-profit curves of banks with informed and uninformed agents. From Section 2.3., we have:

$$\begin{aligned}
\frac{\partial R_N}{\partial C} &= -\frac{1-E[p]}{E[p]} \\
\frac{\partial R_L}{\partial C} &= -\frac{\int_{p_2}^1(1-p)\,dF(p)-(R-C)\frac{\partial p_2}{\partial C}f(p_2)(E[p\,|\,p>p_2]-p_2)}{\int_{p_2}^1 p\,dF(p)+(R-C)\frac{\partial p_2}{\partial R}f(p_2)(E[p\,|\,p>p_2]-p_2)}
\end{aligned}$$

Step 3. Comparison of iso-profit slopes and banks' zero profit condition.

$$\frac{\partial R^*}{\partial C}-\frac{\partial R_u}{\partial C} \propto -A$$

where

$$A=f(p_2)(p_2-p_1)(p_2-E[p])+F(p_2)E[p]-\int_0^{p_2}pdF(p)$$

In R^*, $A \propto \frac{f(p_2)}{F(p_2)}(p_2-p_1)(p_2-E[p])+E[p]-p_1=k(E[p])$. This function is increasing in $E(p)$ and positive in $E(p)=p_2$. Therefore for all $p_2>E(p)$, $A>0$. Then, $\frac{\partial R^*}{\partial C}-\frac{\partial R_u}{\partial C}<0$. In the same lines:

$$\frac{\partial R^*}{\partial C}-\frac{\partial R_i}{\partial C} \propto -A \quad \text{and} \quad \frac{\partial R_u}{\partial C}-\frac{\partial R_i}{\partial C} \propto -A$$

As a consequence:

$$-\frac{\partial R^*}{\partial C} > -\frac{\partial R_u}{\partial C} > -\frac{\partial R_i}{\partial C} \tag{A5}$$

In addition, we have $\frac{\partial R_u}{\partial C}=\frac{\partial R_N}{\partial C}<\frac{\partial R_L}{\partial C}$. Moreover, it is easy to check that $R_L(\bar{R})=R_N(\bar{R})=\bar{R}$. Last,

$$-\frac{\partial R_i}{\partial C} > \frac{\int_{p_2}^1(1-p)\,dF(p)}{\int_{p_2}^1 p\,dF(p)} = -\left.\frac{\partial R_L}{\partial C}\right|_{(R_L(\bar{R}),\bar{R})} \tag{A6}$$

Step 4. Determination of the equilibria. Denote by $\bar{R}_0$ the value such that $R^*(\bar{R}_0)=\bar{R}_0$. Note that, given (A5), $\bar{R}_0<\bar{R}_1$.

- $\bar{R} < \bar{R}_0 \ \Leftrightarrow \ g(\bar{R}, \bar{R}; \beta) > 0$. Hence, according to (A5), $g(R_N(C), C) > 0$ and $g(R_L(C), C) > 0$ for all $C \in [0, \bar{R}]$. This implies that, in equilibrium, agents will never learn p. Given that $\frac{\partial R_u}{\partial C} = \frac{\partial R_N}{\partial C}$, every combination $(R_N(C), C)$ with $C \in [0, \bar{R}]$ yields 0 profits to banks and belong to the same iso-profit curve $G_u(R_N(C), C; \beta) = \bar{K}$.

- $\bar{R}_0 < \bar{R} < \bar{R}_2 \ \Leftrightarrow$ for each $\bar{R}$, there exists one and only one value $\tilde{C}(\bar{R}) \in (0, \bar{R})$ such that:

$$R^*(\tilde{C}(\bar{R})) = R_L(\tilde{C}(\bar{R}))$$

Given (A5), the optimal vector (R, C) compatible with no losses for banks implies: (i) learning of p, (ii) a collateral $\hat{C}(\bar{R}) \in [\tilde{C}(\bar{R}), \bar{R})$, and (iii) an interest factor $R_L(\hat{C}(\bar{R}))$. The exact position of $\hat{C}$ depends on the sign of $\frac{\partial R_i}{\partial C} - \frac{\partial R_L}{\partial C}$. In any case, given (A6), $\hat{C}(\bar{R}) < \bar{R}$. Note that $(R(\tilde{C}(\bar{R})), \tilde{C}(\bar{R}))$ is the competitive pair of interest factor and collateral if and only if $\alpha(\bar{R}) = 1$.

- $\bar{R}_2 < \bar{R} \ \Leftrightarrow \ g(R_N(C), C; \beta) < 0$ and $g(R_L(C), C; \beta) < 0$ for all $C \in [0, \bar{R}]$ by (A5). Hence, in equilibrium, agents always learn p. Again, there is an optimal collateral $\hat{C}(\bar{R}) \in [0, \bar{R})$. $\square$

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