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ABSTRACT

Positive Arithmetic of the Welfare State*

Why does the largest US welfare programme select its recipients by their age, rather than by their earnings or wealth? In a dynamic efficient overlapping generation economy with earnings heterogeneity, we analyse a welfare system composed of a within-cohort redistribution scheme and an unfunded social security system. The programme's size is determined in a bi-dimensional majoritarian election. For enough income inequality and elderly in the population, both welfare programmes are supported as a structure-induced political equilibrium of a voting game played by successive generations of voters. Social security is sustained by a voting coalition of retirees and low-income young, intragenerational redistribution by low-income young. Two features are crucial: the retirees' political power, deriving from their homogeneous voting and the intragenerational redistribution component of the social security. Therefore, to assess how changes in inequality affect the welfare state, the income distribution should be decomposed by age groups.

JEL Classification: D72, H53, H55
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NON-TECHNICAL SUMMARY

In most industrialized countries, social security represents the single largest item in social welfare expenditure and a predominant component in the government budget. In 1992 the US social security system gathered almost 76% of all cash benefits in transfers to the old, whereas unemployment, temporary disability benefits and workers' compensations obtained only 15.8% and the remaining 8.2% went to public assistance and supplemental security income. When in-kind benefits are also included, the social security system still enjoys the largest share, 58.7%.

Why does the largest social welfare scheme depend on the age of the recipients rather than on their income or wealth? More generally, what political mechanism determines the size and the composition of the social welfare expenditure? And how is the composition of the social welfare system related to the income inequality?

Our answers hinge on two simple observations: the homogeneity of the old individuals as a voting block and the existence of a significant within-cohort redistribution component in the social security system. The elderly constitute a fairly uniform group: they are old and most of them have low earnings, mainly because they have retired, although they may largely differ in their wealth. This homogeneity makes them a uniform electoral block when voting on redistribution issues: they all like social security and they all may or may not support different forms of income-based redistribution. As a result, they have a relevant political power, since they are able to cluster and shift a large amount of votes. The existence of an intragenerational redistribution element in the social security system, on the other hand, stems from the combination of a proportional payroll labour tax and a regressive benefit and it may make social security palatable to low-income young, even in the presence of other income redistribution schemes. Recent social security reforms have shown the relevance of this element for the political sustainability of these systems.

We investigate these matters in a simple model economy populated by young and old individuals. Young are heterogeneous in their working ability and thus in their labour income. Individuals take economic, as well as political decisions. Young determine their labour supply; and all agents participate in the political process. The welfare system consists of two annually budget-balanced programmes. A within-cohort redistribution scheme taxes labour income and awards a lump sum transfer to the young. An unfunded social security system imposes a payroll tax rate on labour income and pays a lump sum pension.

A political process determines the size of the two welfare programmes. The process consists of three stages: an initial proposal over the tax rates, a subsequent phase of amendments to the proposal and a final vote of the (possibly amended) proposal against the status quo. We analyse a majoritarian political-institutional arrangement in which the entire electorate has jurisdiction over the two tax rates and the initial proposal as well as the subsequent amendments has to be made one issue at a time. The final vote is at simple majority. Elections take place every period and voters are all agents alive. Moreover, current welfare policies can be modified in later periods at no cost.

We show that for a sufficiently large share of elderly in the population and enough labour income inequality, a majority of the electors supports a welfare system composed of an intragenerational transfer scheme and an unfunded social security system. In particular, the social security system is sustained by a voting coalition of elderly and low-income young, whereas the intragenerational scheme receives the votes of the low-income young only.

Old individuals strongly support the pension system. However, they oppose any intragenerational redistribution scheme (e.g. unemployment or temporary disability benefits) which does not award them any benefits and decreases the average income in the economy, due to the distortionary effects of taxation.

Young agents may benefit from within-cohort redistribution depending on their labour earning. In particular, individuals whose income is below the average income in the economy would receive a positive net transfer and thus vote in favour of the programme.

How do the young vote on the social security system? Despite the fact that they can enter the political process only to determine the current tax rate, we believe that the young expect their voting decision to have an impact on future policies. In other words, they perceive the social security scheme as a saving plan: as long as they transfer resources to the current retirees they will be rewarded with a pension in their old age. Although the average performance from contributing in social security is lower than the performance of other available assets, social security still represents the saving plan which offers the highest return to the low-income young, thanks to its intragenerational redistribution component. The low-income young therefore support the programme.

To obtain a flavour of the result, we parameterize the equilibrium welfare system to the US economy. The relative average performance of the social security system with respect to other saving schemes is set equal to 0.5 to indicate that social security pays out 50% less than private savings over the lifecycle. The degree of income inequality is summarized by the relative ability of the two median voters, which we calculate using 1992 data on earning inequality and Presidential election participation rates. The associated welfare system consists of a 2% within-cohort redistribution tax rate and a 17% social security tax rate.

Our results do not change if we investigate a different welfare system, composed of the usual social security system and of a more comprehensive income redistribution programme (e.g. public assistance and supplemental security income), which imposes a tax rate on labour income, transfers, and pensions and pays a lump sum transfer to all agents (young and old). The parameterized example delivers a 7% income redistribution tax rate and a 9% social security tax rate.

Our work suggests that the effect on each individual welfare programme (social security and intragenerational transfer) depends on the magnitude of the change in income inequality, as well as on its specific impact on the income distribution. An overall increase in income inequality which has comparable effects on both median voters' abilities would lead to an expansion in both programmes. However, an increase in dispersion localized in the lower tail of the distribution would presumably induce larger changes in the social security median voter's ability than in the intragenerational one. The final result could then be an increase in social security coupled with a constant or even decreasing intragenerational transfer. We believe that these simple considerations should be taken into account in future empirical studies.

1. Introduction

In most industrialized countries, social security represents the single largest item on the social welfare expenditure, and a predominant component in the government budget. In 1992 the US social security system gathered almost 76% of all cash bene¯ts in transfers to the old[1], as shown in Table 1. Unemployment, temporary disability bene¯ts and workers' compensations obtained 15.8% of the total, and the remaining 8.2% went to public assistance and supplemental security income. When also in-kind bene¯ts are considered, the social security system still enjoys the largest share, 58.7%, of the Federal and State budget on all bene¯ts excluding education (see Table 2).

A social welfare policy can be indexed by the characteristics of its recipients (and contributors). Pensions are awarded to individuals who have reached a minimum retirement age, who have previously contributed to the system, and who have exited the labor force. Unemployment bene¯ts and workers' compensations depend on the current and previous employment status. Public aid, medicare, and housing are mainly provided to individual with low income, regardless of their age group or employment status. Then why does the largest social welfare scheme depend on the age of the recipients rather than on their income or wealth? And why do we observe so little income redistribution among individuals of the same age group?

Since Romer (1975), Roberts (1977), and Meltzer and Richard (1981), the amount of redistribution has been related to the level of income inequality in the economy. They suggest that in democracies with unequal income distributions, redistribution policies are sustained by a majority of voters, whose income is below the average income in the economy. Building on this idea, we introduce a further charecterization of the agents, their age, to explain the contemporaneous existence of an income-based redistribution

[1] Old age, survival, disability and health insurance, railroad and public employees retirement bene¯ts, and veterans' pensions absorbed 411.8 of the total 541.9 billions dollars paid out in cash bene¯ts by the public income maintenance programs.

scheme, and an age-based transfer scheme, the social security system.

First, we notice that a large proportion of the earning poor are indeed old individuals. Using 1992 US data, Díaz-Giménez, Quadrini, and Rios Rull (1997) find that respectively 63% and 28% of the individuals in the first and second earnings quintile are older than 65 years. We will argue that these voters, the elderly, may prefer an age-based to an income-based transfer scheme, thus decreasing the support that income redistribution schemes are expected to enjoy among low-income individuals.

Like Tabellini (1990), we emphasize the intragenerational redistribution component built in the social security system. In fact, this program is known to redistribute both across and within cohort, since contributions to the social security system are proportional to the labor income (up to a maximum), whereas benefits tend to be regressive, particularly in the health insurance component (Medicaid). Boskin et al. (1987) and Galasso (1998) provide evidence supporting this view, as they show that, for a given cohort, low income families obtain larger internal rate of return from investing in social security than middle or high-income families. The existence of a within generation redistribution element in the social security system will be crucial in our analysis, as it makes social security palatable to low-income young, even in the presence of other income redistribution schemes.

Finally, low-income individuals tend to enjoy little political representation, since, as shown at tables 3 for the 1992 US Presidential elections, their voting participation rates are sensibly lower than richer voters'. The elderly, on the other hand, report the higher turn-out rates. These voting behaviors contribute to reduce the proportion of voters in favor of income redistribution transfers, and have mixed effects on the electors supporting social security.

We introduce a dynamic efficient overlapping generation economy with storage technology. Young agents differ in their working ability, and therefore in their labor income. Old individuals do not work. The welfare state consists of two annually budget-balanced programs. An (intragenerational) income redistribution scheme taxes labor income and awards a lump sum, young age transfer, whereas an unfunded social security system imposes a payroll tax rate and pays a lump sum pension. Notice that, like in Tabellini (1990), in this dynamic efficient economy, a social security system would be sustained as an equilibrium of a unidimensional majoritarian voting game only if it entails an intragenerational redistribution component.

The level of the two welfare programs, the income redistribution tax rate and the social security tax rate, are determined in a bidimensional majoritarian voting game by all agents alive at every election. Because of the bidimensionality of the issue space, the existence of a Nash equilibrium is not guaranteed. To overcome this problem, we follow Shepsle (1979) and concentrate on political equilibria induced by institutional restrictions, or structure-induced equilibria. In our political system, the entire electorate has jurisdiction over the two issues (i.e. the two tax rates), however, initial proposals and subsequent amendments can be made only one issue at a time. The final, possibly amended proposal is voted against the status quo at simple majority.

We show that, for a sufficiently large proportion of elderly in the population, and for enough income inequality, a welfare state composed of an income redistribution scheme and an unfunded social security system may arise as a structure-induced equilibrium of a majoritarian voting game. In this equilibrium, the social security system is sustained by a voting coalition of elderly and low-income young, whereas income redistribution only receives the support of the young voters whose labor income is below the average labor income in the economy[2].

The idea of a social security system which relies on a voting coalition of low-income young and retirees to obtain its political sustainability dates back to Tabellini (1990). In his model, low-income, weakly altruistic agents vote for social security since the utility

[2] The use of a more general income redistribution program, which awards benefits to young and old, instead of the exclusively intragenerational transfer mentioned above, does not change the results, as shown in section 5.

they derive from the pension their parents receive outweighs the direct cost of the social security tax, and an equilibrium with positive social security may arise. However, unlike in our model, this result is not robust to a more complete specification of the welfare state. And if an additional income redistribution scheme is introduced, the equilibrium disappears.

Our paper delivers some positive implications and a warning. An increase in the proportion of elderly in the population has opposite effects on the size of the welfare programs. On one hand, it decreases the workers to retirees ratio and thus reduces the relative profitability of the social security system as a saving scheme; on the other hand, it affects both voting coalitions by increasing the votes in favor of social security and against income redistribution. The overall impact is thus indeterminate. Additionally, our work suggests that to evaluate the effect on each welfare program of a change in income inequality it is crucial to analyze the entire extent of the change in the income distribution. In fact, an increase in dispersion localized, for example, in the lower tail of the distribution would presumably induce stronger changes in the social security coalition than in the income redistribution one. The final result could then be an increase in social security coupled with a constant or even a surprising decrease in the intragenerational transfer. We believe that these simple considerations should be taken into account in future empirical studies.

Although the within-cohort redistribution component plays a crucial role in gathering political support in favor of the social security system, as the recent discussions on reforming social security have shown, most of the theoretical literature on social security has indeed overlooked this element, with the notable exception of Tabellini (1990). The main focus has almost exclusively been on the economic and political factors related to the intergenerational redistribution aspect. Studies that in the context of a dynamic efficient economy rely on the age of the voters to explain social security are those by Browning (1975), Sjoblom (1985) and Cooley and Soares (1998) and Galasso (1999). Boldrin and

Rustichini (1995), Cooley and Soares (1998) and Galasso (1999) recognize that the institution of unfunded systems affects the allocation of resources through changes in the factor prices and thus have important implication for the wealth of the voters. Mulligan and Sala-i-Martin (1997) explain the success of the social security systems around the world with the political strength of the elderly as a group of interest. Boldrin and Montes (1998) suggest that social security schemes should not be examined in isolation, but rather be coupled with other public policy, in particular public education. Finally, Lambertini and Azariadis (1998) use Baron and Ferejohn's (1989) "closed rule" political system of recognition, proposal and voting to show that shifts of political power between the different voting coalitions that, like in our model, sustain the welfare system may explain the dynamics of the US expenditures in redistributive policies.

The paper proceeds as follows: Section 2 presents the model and the economic equilibrium. Section 3 develops the political system, introduces the concept of structured-induced equilibrium, and characterizes the equilibrium of the voting game. In section 4 we discuss the results and the related literature. Section 5 examines a more general welfare state and Section 6 concludes.

2. The Model Economy

Consider an economy with overlapping generations and a storage technology. Every period two generations are alive, we call them "Young" and "Old". Population grows at a constant rate μ. It follows that in any given period t for every young there are $1/(1+\mu)$ old.

Agents work when young, and then retire in their old age. Consumption takes place in old age only. Young individuals differ in their working ability. Working abilities are distributed on the support $[\underline{e}, \bar{e}] \subset \Re_+$, according to the cumulative distribution function $G(.)$. An agent born at time t is characterized by a level of working ability and will therefore be denoted by $e_t \in [\underline{e}, \bar{e}]$. The distribution of abilities is assumed to have mean

e_ϕ, and to be skewed, $G(e_\phi) > 1/2$.

A production function transforms labor into the only consumption good, according to the worker's ability:

$$y(e_t) = e_t n(e_t) \tag{2.1}$$

where $n(e_t)$ represents the amount of labor supplied by the agent with ability e_t. A storage technology converts a unit of today's consumption good into $1 + R$ units of tomorrow's good, $y_{t+1} = (1 + R)\, y_t$. Since there are no outside assets or flat money, all private intertemporal transfer of resources takes place through the storage technology. Then by assuming that $1 + R > 1 + \mu$ we guarantee that the economy is dynamic efficient.

Agents value young age leisure and old age consumption according to a log-linear utility function:

$$U\left(l_t; c_{t+1}^t\right) = \ln(l_t) + \beta c_{t+1}^t \tag{2.2}$$

where l is leisure, c is consumption, β represents the individual time discount, subscripts indicate the calendar time and superscripts indicate the period when the agent was born.

Young agents face the usual trade off between labor, $n(e_t)$, and leisure, $l(e_t)$, since $n(e_t) = \bar{l} - l(e_t)$, where $\bar{l}\ (> 0)$ is the total amount of disposable time, which we assume to be equal across types. Young pay payroll taxes on their labor income, receive a transfer, and save their disposable income for old age consumption. Old agents have no economic decision to take as they consume their entire income. The life time budget constraint for an agent born at time t with ability e_t is then:

$$c_{t+1}^t = [e_t n(e_t)(1 - \tau_t - \sigma_t) + T_t](1 + R) + P_{t+1} \tag{2.3}$$

where τ_t and σ_t are the payroll tax rates at time t, and T_t and P_{t+1} are respectively the young age transfer at time t, and the old age transfer at time t + 1.

Young determine their labor supply by maximizing $U\left(l_t; c^t_{t+1}\right)$ with respect to $l(e_t)$ and subject to budget constraint (2.3). We assume that the individual time discount is equal to the inverse of the interest factor, $\beta = 1/(1+R)$, so that the labor supply does not depend on the interest rate. The optimal labor supply for an ability type e_t agent is then:

$$n(e_t) = \max\left\{0; \bar{l} - \frac{1}{e_t(1 - \tau_t - \sigma_t)}\right\}. \qquad (2.4)$$

We assume that the labor supply is strictly positive for every type[3].

Because of the specification of the utility function the labor supply is only affected by changes in the tax rates and not by changes in the transfers level. In this sense, income effects play no role, whereas taxes distort labor supply decisions.

2.1. The Welfare System

We examine two social welfare instruments, an income redistribution system, and a social security (or pension) system.

The former is an intragenerational redistribution scheme which only affects young generations. In fact, all young persons benefit from a lump sum transfer, T_t, which is financed through a payroll tax, τ_t, on the labor income. Clearly, this system redistributes from rich (above mean income types) to poor (below mean income types) young. The latter scheme consists of a sequence of transfers from workers to retirees. Each worker contributes a payroll tax rate, σ_t, from her labor income, and every retiree receives a flat transfer, P_t. Every system is assumed to be individually balanced every year, so that its social expenditure has to be equal to the amount of collected taxed.

The budget constraint at time t for the income redistribution scheme is thus:

[3] This assumption amounts to imposing a restriction on the tax rates: $\tau_t + \sigma_t < 1 - 1/\bar{l}\underline{e}$.

$$T_t = \tau_t \int_{\underline{e}}^{\overline{e}} e_t n(e_t)\, dG(e_t) \quad (2.5)$$

whereas the budget constraint for the social security system is

$$P_t = \sigma_t (1+\mu) \int_{\underline{e}}^{\overline{e}} e_t n(e_t)\, dG(e_t): \quad (2.6)$$

By substituting the labor supply in (2.4) into (2.5) and (2.6), we obtain two new expressions for the welfare system budget constraints:

$$T_t(\tau_t; \sigma_t) = \tau_t \left[e_\phi \beta - \frac{1}{1 - \tau_t - \sigma_t} \right] \quad (2.7)$$

and

$$P_t(\tau_t; \sigma_t) = \sigma_t (1+\mu) \left[e_\phi \beta - \frac{1}{1 - \tau_t - \sigma_t} \right]: \quad (2.8)$$

Thus, the young age lump sum transfer displays a Laffer curve with respect to the corresponding tax rate and depends negatively on the social security payroll tax rate, which contributes to decrease the average income in the economy without increasing the benefits. Analogously, the lump sum pension displays a Laffer curve with respect to the corresponding tax rate and depends negatively on the income redistribution tax rate.

2.2. The Economic Equilibrium

The economic equilibrium can now be defined as follows

Definition 2.1. For a given sequence of social welfare tax rates, $\{\tau_t; \sigma_t\}_{t=0}^{\infty}$, an economic equilibrium is a sequence of allocations and prices, $\left\{ l(e_t); c_{t+1}^t(e_t); R \right\}_{t=0}^{\infty}$; such that:

- the consumer problem is solved for each generation, i.e. agents maximize $U\left(l_t; c_{t+1}^t\right)$ with respect to $l(e_t)$, subject to (2.3);
- the welfare budget constraints are balanced every year, and thus equations 2.5 and 2.6 are satisfied; and
- goods market clears:

$$\begin{aligned}\int_{\underline{e}}^{\bar{e}} c_t^{t-1}(e_{t-1})\, dG(e_{t-1}) &= (1+R)\int_{\underline{e}}^{\bar{e}} (1-\sigma_{t-1})\, e_{t-1} n(e_{t-1})\, dG(e_{t-1}) \\ &+ \sigma_t (1+\mu)\int_{\underline{e}}^{\bar{e}} e_t n(e_t)\, dG(e_t): \qquad (2.9)\end{aligned}$$

We identify the utility level obtained in an economic equilibrium at time t by an ability type e_t young and by an ability type e_{t-1} old agent with their indirect utility functions. For the young:

$$\begin{aligned} v_t^t(\tau_t; \tau_{t+1}; \sigma_t; \sigma_{t+1}; e_t) &= -\ln e_t - 1 - \ln(1-\tau_t-\sigma_t) + e_t\beta(1-\tau_t-\sigma_t) \\ &+ \tau_t\left[e_{\phi;t}\beta - \frac{1}{1-\tau_t-\sigma_t}\right] + \sigma_{t+1}\frac{1+\mu}{1+R}\left[e_{\phi;t+1}\beta - \frac{1}{1-\tau_{t+1}-\sigma_{t+1}}\right]: \qquad (2.10)\end{aligned}$$

For the old:

$$v_t^{t-1}(\tau_t; \sigma_t; e_{t-1}) = K(e_{t-1}) + \sigma_t\frac{1+\mu}{1+R}\left[e_{\phi;t}\beta - \frac{1}{1-\tau_t-\sigma_t}\right] \qquad (2.11)$$

where $K(e_{t-1})$ is a constant which depends on the agent's type, but not on current or future tax rates.

These indirect utility functions characterize the young and old agents' preference relations over current (and future) tax rates. Notice that the old individuals' ability type scales their utility up or down, but does not affect their preferences over the tax rates; and that young electors preferences depend both on current and future tax rates, although they only vote on current variables.

3. The Voting Game

The amount of welfare expenditures, i.e. income redistribution and social security, are decided through a political process which aggregates the agents' preferences over the two tax rates. We consider a political regime of majority voting. Elections take place every period, and voters are all agents alive. At every elections voters cast their ballots on the two current tax rates, τ and σ. Current welfare policies can be modified in later periods at no cost; moreover, there is no device to commit subsequent voters to carry on particular expenditure levels in the future.

Because of this lack of commitment, how could a social security system be sustained? Why would a majority of young (and old) individuals agree to a policy which transfers resources to current retirees if there is no guarantee that such policy will be carried on in the next periods?

If young agents expect their voting behavior to have no relevance for future choices, they should indeed vote for a zero social security tax rate, or else they would incur in a current labor tax with no future benefits. However, current electors may expect their voting decisions to have an impact on future policies. In this case, as Hammond (1975) initially suggested, if an implicit contract among successive generations of voters arises, today's young may agree on a transfer to current retirees because they expect to be rewarded with a corresponding transfer in their old age. A failure to comply with the implicit contract would be punished with no old age transfers. We choose to examine the class of voting strategies which allows this implicit contract to arise, and be sustained. In particular, as we will show in section 3.4, we consider a strategy profile which allows all young voters to cast their most preferred tax rate when the social security system is initially introduced. Later, once the system is in place, they are required either to sustain it, or, if the initial tax rate has been changed, to dismantle it.

The adoption of this strategy profile reduces the intertemporal nature of the voting game to a static one. Initial voters determine the social security tax rate, whereas future voters can only sustain or reject it, but not modify it. In this sense, all decisions collapse into the first period, when voters determine their most preferred among the tax rates which would be sustained by future voters, i.e. among the tax rates representing an equilibrium outcome of the continuation game.

However, although the use of these strategies effectively reduces the game to a static one, the bidimensionality of the issue space prevents a Nash equilibrium of this voting game from arising, unless very restrictive assumptions on the preferences of the voters are imposed[4]. The following example will illustrate this point.

Figure 1 displays the preferences of three representative voters, old, rich young, and poor young, as utility contours in the bidimensional issue space[5], $(\tau; \sigma)$. Old voters clearly support a social security scheme, and oppose any income redistribution system, τ, which decreases the average income in the economy and does not award them any benefits. The pair of tax rate which maximizes their indirect utility (eq. 2.11), i.e. their bliss point, is thus $(\tau; \sigma) = (0; \sigma^*_{old} > 0)$; and their indifference contours are represented by the dashed curves. Rich young voters, i.e. voters whose income is above the mean income in the economy, dislike both welfare schemes, to which they are net contributors. Their bliss point is the origin, $(\tau; \sigma) = (0; 0)$; and their indifference contours are the dotted lines. A poor young, e.g. a young with ability $\underline{e}$, on the other hand, is a net receiptor from both schemes. If she maximizes her indirect utility function with respect to the current tax rates, and under the assumptions that the decision of current social security tax rate binds the future tax rate, $\sigma_t = \sigma_{t+1}$, whereas the current income redistribution tax rate has no

[4] See Ordershook (1986) for an extensive review of these results.

[5] Young voters' preferences are depicted under the assumption that both tax rates remain constant over time: $\tau_t = \tau_{t+1}$, and $\sigma_t = \sigma_{t+1}$.

impact on future policies, her bliss point[6] is $(\tau, \sigma) = \left(\tau^*_{yp} > 0; \sigma^*_{yp} = 0\right)$. The poor young prefers to obtain her entire welfare transfer through the income redistribution benefit. Her preferences are represented by the continuous-line contours.

If these three agents were the only voters and had equal weights, no Nash equilibrium of the majoritarian voting game would exist, and Condorcet cycles would arise. For example, in Figure 1, point b would be preferred to point a by the poor young and the old; c would be preferred to b by the old and the rich young; and finally rich and poor young would close the cycle moving from c to a. The same result would apply to the voting game played by the entire electorate, unless a median in all directions exists[7].

To overcome this well-known problem, we follow Shepsle (1979) in analyzing voting equilibria induced by institutional restrictions, i.e. structure-induced equilibria. In the next section, we introduce our set of institutional restrictions.

3.1. The Political System

A political system describes a decision-making institution composed of the entire electorate, E. The space of alternatives, or issues, is $(\tau, \sigma) \in R^2$ subject to $\tau + \sigma \leq 1 - 1/\beta\underline{e}$, which we imposed in order for the labor supply of any young agent to be positive (see footnote 3). Individual preferences over the alternatives are derived from the indirect utility functions at (2.10) and (2.11). Notice that young electors preferences depend both on current and future tax rates, although they only vote on current policies. However, as we will discuss in section 3.4, the current choice of the social security tax rate may have an impact on future voters' decisions through an implicit social contract which links subsequent generations of electors. Current decisions on the income redistribution tax rate, on the other hand, have no relevance for future policies, as we will show in section 3.3.

[6] The condition for the bliss point to occur at $\sigma = 0$ is that $\left(e_{\chi;t} - \underline{e}\right)/e_{\chi;t} < \beta e_{\chi;t} - 1$.
[7] For a formal definition, see the appendix.

An institutional arrangement characterizes how the political system aggregates the individual preferences over the alternatives into a political outcome $(\tau^*; \sigma^*)$. An arrangement is composed of a committee system, a jurisdictional arrangement, an assignment rule, and an amendment control rule. The committee system separates the electorate, E, in committees $\{C_j\}$. The jurisdictional arrangement, J, divides the issues $(\tau; \sigma)$ into jurisdictions $\{J_k\}$. Jurisdictions are then associated to committees, according to an assignment rule, $f : C_j \rightarrow J_k$. In this way, the political system assigns the decision over a subset of the issue space, e.g. a single issue, to a particular committee. Every committee is entitled to make a proposal to change the current value of the issue (the status quo) which falls into its jurisdiction. The amendment control rule determines how proposals can be further modified (amended) by the electorate before the final stage is reached, and the (possibly amended) proposal is then voted in a majority rule, pairwise comparison against the status quo by the entire electorate.

The political system we adopt is characterized by the following arrangements[8]:

- Committee of the Whole: there exists only one committee, which coincides with the electorate, $C = \{E\}$;
- Simple Jurisdictions: each jurisdiction is a single dimension of the issue space, $J = \{\{\tau\}; \{\sigma\}\}$. In other words, one jurisdiction has the power to deliberate on the income redistribution tax rate, τ, and another one on the social security tax rate, σ.
- Every simple jurisdiction is assigned to the committee of whole, $f : E \rightarrow \{\{\tau\}; \{\sigma\}\}$.
- Germaneness Amendment Control Rule: amendments to the proposal are permitted only along the dimensions that fall in the jurisdiction of the committee. That is, if the proposal regards τ, only amendments on τ are permitted, and viceversa.

[8] See the appendix for a formal definition.

In this political system, the entire electorate has jurisdiction, i.e. it is entitled to make proposals on the two issues; however, only separately, that is, one issue at a time. The restriction that only one issue is on the floor at a time is achieved through simple jurisdictions and germaneness amendment rule, and it is needed to overcome the possible lack of a Nash equilibrium. No further restrictive jurisdictional arrangements are imposed. The choice of a committee of the whole, for example, guarantees that no subset of the electorate which constitutes a committee is effectively awarded veto power over an issue. In fact, any such committee could block any alternative to the status quo which would be preferred by a majority of the electorate, but not by a majority of the members of this committee.

We now turn to equilibria induced by our institutional arrangements.

3.2. Structure-Induced Equilibria

Since Shepsle (1979) [Theorem 4.1], we know that a sufficient condition for the existence of an equilibrium in a voting game played under the institutional arrangements described in section 3.1 is that voters' preferences are single peaked over the issue space, $(\tau;\sigma) \in R^2$.

To establish single-peakedness for our voters' preferences, it is useful to introduce some additional definitions[9]. We refer to the induced ideal point of a voter i in the j-th direction of the issue space (e.g. σ) as the point that maximizes voter i's indirect utility function along the j-th dimension (σ), while the other issue (τ) is at its status quo value. An induced ideal point for voter i on a line in the issue space maximizes her utility function on this line with respect to both issues. Finally, we define preferences to be single-peaked over a line in the issue space as follows:

Definition 3.1. Let $X = \{x \mid x = \lambda y + (1 - \lambda) z;\ y; z \in (\tau;\sigma);\ \lambda \in [0;1]\} \subseteq (\tau;\sigma)$ be the line connecting two arbitrary points y and z, in the issue space, $(\tau;\sigma)$. Preferences

[9] Formal definitions are provided in the appendix.

are single peaked on X if and only if, for all $x \in X$ and $x \neq x^{*i}$, $u_i\left[\alpha x + (1-\alpha)x^{*i}\right] > u_i\left[\beta x + (1-\beta)x^{*i}\right]$ whenever $0 \leq \alpha < \beta \leq 1$ and x^{*i} is the induced ideal point on X.

In other words, voter i's preferences are single peaked over a line in the issue space if and only if, for any point on this line on one side of voter i's induced ideal point, points closer to the induced ideal point provide higher utility.

Recall that at time t the preferences of an ability type e_t young voter over τ_t, and σ_t $(= \sigma_{t+1})$ are described by eq. 2.10; while old voters' preferences are represented by eq. 2.11. Then we can state the following proposition.

Proposition 3.2. Over the issue space $(\tau;\sigma) \in R^2$, old voters preferences are single peaked; young voters' preferences are single peaked over a line in the issue space if $N = (1+\mu)/(1+R) \geq 1/2$.

To prove single peakedness over a line in the issue space for young voters we show that $N \geq 1/2$ is sufficient to guarantee that their utility function is quasi concave in $(\tau;\sigma)$; then we apply Shepsle (1979) [Lemma 3.1] to deduce single peakedness. For old voters, since their utility is not concave, we directly apply the definition of single peakedness to eq. 2.11. A formal proof is provided in the appendix.

The next proposition characterizes the structure induced equilibrium we use.

Proposition 3.3. Let X_j^* be the set of j-th components from the induced ideal points of all voters in the direction j from the status quo x^o. For one-dimensional (simple) jurisdictions, a germaneness rule for amendments, a committee of the whole, and single peaked preferences, x^o is a structure-induced equilibrium outcome if and only if, for all j, x_j^o = median X_j^*.

Proof: Since preferences are single peaked on any line in the issue space, if x_j^o = median X_j^* then x^o defeats all points along the j-th dimension, by Black's median voter's theorem. Given simple jurisdictions and germaneness rule, issues are voted once at a time; and since x^o_j cannot be defeated by any point along any dimension j, then x^o is a structure induced equilibrium outcome, which proves sufficiency. Suppose now that x^0 is a structure induced equilibrium outcome, where $x^0 \neq x^o_j = \text{median } X^*_j \; \forall j$ along some dimension i. Since we have a committee of the whole, then x^0 would always be defeated by x^o along the i-th dimension, which proves the proposition.

Notice that the necessary condition established in this proposition relies on the use of a committee of the whole. In fact, if a jurisdiction were to be assigned to a committee which is a strict subset of the electorate, then along that jurisdiction the committee could force the electorate to choose on a restricted issue space and thus structure-induced equilibrium outcomes other than x^o could arise.

In the next two sections we calculate the median among the induced ideal points of all voters over both issues, τ and σ. In other words, we first calculate every elector's vote over the income redistribution tax rate for a given social security tax $\tau(\sigma)$, and then over the social security tax rate for a given income redistribution tax rate $\sigma(\tau)$. We order the two sets of votes and identify the median among each set of votes. By Proposition 3.3, a structure induced equilibrium outcome, $(\tau^*; \sigma^*)$, coincides with these two medians.

3.3. Voting on the Income Redistribution Tax Rate

A quick look at eq. 2.11 reveals that old generations oppose any income redistribution transfer schemes, since, due to the distortionary taxation, they reduce the average income in the economy, and thus decrease their pension benefits, while they do not provide the old with any transfer. In fact, the maximization of eq. 2.11 with respect to τ yields $\tau^*_{old} = 0$ for any positive value[10] of σ.

Young generations, on the other hand, may benefit from this intragenerational transfer scheme, depending on their ability and thus income type. For a given social security tax

[10] For $\sigma = 0$ the old's indirect utility does not depend on τ. Since the old are indifferent, we assume that $\tau^*_{old}(\sigma = 0) = 0$.

rate, $\sigma_t\,(=\sigma_{t+1})$, an ability type e_t young at time t would choose her most preferred income redistribution tax rate $\tau^*_{e,t}(\sigma)$ by maximizing eq. 2.10 with respect to τ_t. The first order condition of this problem yields:

$$(e_\phi - e_t)\beta - \frac{\tau_t}{(1-\tau_t-\sigma_t)^2} = 0.$$

And thus the optimal income redistribution tax rate, for a given σ, is

$$\tau^*_{e,t}(\sigma) = \max\left\{0;\ 1-\sigma_t + \frac{1-\sqrt{1+4(e_\phi - e_t)\beta(1-\sigma_t)}}{2(e_\phi - e_t)\beta}\right\}. \qquad (3.1)$$

Unsurprisingly, the young's most preferred income redistribution tax rate is decreasing in their income. Above average income type would vote for $\tau^* = 0$, together with the old. Poor, i.e. below average income, $e_t < e_\phi$, young vote for positive tax rates.

When voting on the income redistribution tax rate, $\tau(\sigma)$, agents can thus be ordered according to their age and income, as shown at figure 2a. Since the old generation represents a minority of the total electorate[11], the median voter on the income redistribution tax rate, hereby intragenerational median voter, is the type-m_τ young agent, who divides the electorate in halves, i.e. such that

$$G(e_{m_\tau}) = \frac{2+\mu}{2(1+\mu)}. \qquad (3.2)$$

Finally, if the median voter's ability is below the average ability, $e_{m_\tau} < e_\phi$, then $\tau^*_{m_\tau}(\sigma) > 0$, according to 3.1.

3.4. Voting on the Social Security Tax Rate

The old have again a simple choice. Since they are no longer required to contribute to the system, they vote for the social security tax rate that maximizes their current transfer,

[11] Even if we adjust for voting participation rates, retirees are still a minority, although a large and powerful one, see Mulligan and Sala-i-Martin (1997).

and thus eq. 2.11. For a given income redistribution tax rate, τ, the first order condition of their optimization problem is

$$e_\phi n(e_\phi) = \frac{\sigma_t}{(1-\tau_t-\sigma_t)^2} \tag{3.3}$$

where $n(e_\phi)$ represents the average labor supply in the economy, as by eq. 2.4. Their most preferred social security tax rate is thus:

$$\sigma^*_{old}(\tau) = 1-\tau_t-\sqrt{\frac{1-\tau_t}{e_\phi\beta}}. \tag{3.4}$$

To analyze the young agents' voting behavior, it is useful to introduce some definitions. For an ability type e_t young voter an action (or a vote), $a_{e;t}$, at time t is a social security tax rate belonging to the action space $[0;1]$. The sequence of realized tax rates until $t-1$ constitutes the history of the game at time t, h_t. For an ability type e_t young player, a strategy, $s_{e;t}$, at time t is a mapping from history into an action, $s_{e;t}: h_t \to [0;1]$. According to Proposition 3.3, the political outcome of the social security voting game at time t, $\sigma_t(\tau_t)$, is the median of the distribution of actions played by young and old voters at time t.

An ability type e_t young player at time t chooses her strategy, $s_{e;t}$, in order to maximize her indirect utility, eq. 2.10, for a given income redistribution tax rate τ, and given current and future voters' strategies. However, since each agent has zero mass, no individual player's strategy can affect the median of the distribution of actions, and therefore determine the tax rate. It follows that any strategy profile is consistent with the individual voter's optimization. To overcome this problem, we assume sincere voting, and let young voters choose their strategies as if their vote would be pivotal.

Nevertheless, young agents would not sustain social security unless an implicit contract among successive generations of voters arises. Current young would in this case agree to transfer resources to the old, since they expect to be rewarded with a corresponding transfer in their old age. Among the many strategy profiles which can induce intertemporal cooperation among subsequent generations of voters, we concentrate on stationary strategy profiles: voters' actions depend on the history of the game, but not on calendar time. Consider the following strategy profile for any ability type e_t young voter:

$$s_{e,t} = \begin{cases} \sigma^*_{e,t} & \text{if } h_t = (0, 0, ..., 0) \\ \sigma^* & \text{if } h_t = (0, ..., \sigma^*, ..., \sigma^*) \\ 0 & \text{otherwise} \end{cases} \tag{3.5}$$

where σ^* is the median among the tax rate played by young, $\sigma^*_{e,t}$, and old, σ^*_{old}, when the social security system is initially introduced. This strategy profile requires an ability type e_t young to vote an initial tax rate $\sigma^*_{e,t}$ if the social security system was not in place in the past; to sustain the tax rate σ^*, provided that σ^* has always prevailed since the introduction of the system; and to dismantle the social security scheme, $\sigma = 0$, if the initial tax rate has ever been changed. In other words, this strategy allows all young voters to cast their most preferred tax rate, $\sigma^*_{e,t}$, when the social security scheme is initially introduced; later, once the median among the actions, σ^*, has been determined, young voters are supposed to sustain it.

If an ability type e_t young voter is to play strategy 3.5, and thus to sustain a non zero social security tax rate, it will have to stem from self interest; she will have to be better off with a system σ^* in place than without it. Therefore, an existing system σ^* is sustained by all ability types e_t young voters whose indirect utility evaluated at σ^* is larger than their indirect utility evaluated at zero social security, for a given pair of income redistribution tax rate, τ_t and τ_{t+1}: $v^t_t(\tau_t, \tau_{t+1}, \sigma^*, \sigma^*, e_t) > v^t_t(\tau_t, \tau_{t+1}, 0, 0, e_t)$: If a social security system has never been introduced, each young voter sincerely votes for her most preferred tax rate, $\sigma^*_{e,t}$. Notice that in determining $\sigma^*_{e,t}$ each type e_t voter takes into account future players' strategies, and chooses her most preferred tax rate among the ones which would be sustained in the future by a majority of the voters.

An ability type e_t young voting decision, when $h_t = (0, 0, \ldots, 0)$, is equivalent to maximizing her indirect utility, eq. 2.10, with respect to the current and future social security tax rate: $\sigma_t = \sigma_{t+1} = \sigma$, and for given values of the current and future income redistribution tax rates, τ_t and τ_{t+1}. The first order condition yields:

$$\left(\frac{1+\mu}{1+R} e_\phi - e_t\right)\beta + \frac{1 - 2\tau_t - \sigma}{(1 - \tau_t - \sigma)^2} - \frac{1+\mu}{1+R} \frac{1 - \tau_{t+1}}{(1 - \tau_{t+1} - \sigma)^2} = 0 \tag{3.6}$$

If we impose $\tau_t = \tau_{t+1} = \tau$, and thus restrict our analysis to steady states, eq. 3.6 can be rewritten as

$$N e_\phi n(e_\phi) - e_t n(e_t) = \frac{\tau + N\sigma}{(1 - \tau - \sigma)^2} \tag{3.7}$$

where $N = (1+\mu)/(1+R)$ can be interpreted as the relative performance of the social security system with respect to the other available saving (storage) technology. The optimal social security tax rate for a young type e_t, given the income redistribution tax rate, τ, is then

$$\sigma^*_{e,t}(\tau) = \max\left\{0, 1 - \tau + \frac{1 - \sqrt{1 + 4\beta(N + \tau(1-N))(e_\phi N - e_t)}}{2\beta(e_\phi N - e_t)}\right\}. \tag{3.8}$$

This optimal tax rate, $\sigma^*_{e,t}(\tau)$, is clearly decreasing in the young income type, e_t, because of the within-cohort income redistribution that this scheme achieves through a combination of a proportional income tax, σ, and a lump sum old age transfer, P. In particular, for sufficiently small values of the income redistribution tax rate, $\tau \leq (1-N)/(2-N)$, only those voters whose pre-tax labor income is below a fraction N of the pre-tax average labor income in the economy, $e_t n(e_t) < N e_\phi n(e_\phi)$ (with $N < 1$), will vote for a positive social security tax; richer young will oppose the scheme.

A look at equations 3.3 and 3.7 reveals that the old always vote for a larger social security tax than the poorest young, and, therefore, than any young, since, unlike the young, they do not make any contribution to the system. Voters' preferences over social security can easily be ordered according to age, and income, as depicted at figure 2b. The median voter on the social security tax rate is the type-$m\sigma$ young who divides the electorate in halves:

$$G(e_{m\sigma}) = \frac{1}{2(1+\mu)}. \tag{3.9}$$

In other words, the median in the distribution of actions played by old and young voters is $\sigma^*(\tau) = \sigma^*_{e,t}(\tau)$ with $e_t = e_{m\sigma}$, which delivers a positive tax rate $\sigma^*(\tau)$ if $e_{m\sigma}n(e_{m\sigma}) < Ne_{\phi}n(e_{\phi})$, and $\tau \leq (1-N)/(2-N)$.

Finally, to complete the discussion of strategy 3.5, we need to make sure that once $\sigma^*(\tau) = \sigma^*_{m\sigma}(\tau)$ is initially introduced, this tax rate $\sigma^*(\tau)$ will also be sustained in future elections. This is clearly the case at steady state, because at least a majority composed by the old and the young whose ability is lower than the median voter's ability, $e_t < e_{m\sigma}$, will prefer $\sigma^*(\tau)$ to zero social security tax rate, and therefore to any other tax rate, since $\sigma = 0$ represents the best deviation from strategy 3.5.

3.5. The Equilibrium

In sections 3.3 and 3.4, we analyzed the voters' decisions over the two welfare schemes: we determined the decisive or median voter for each issue, $e_{m\tau}$ and $e_{m\sigma}$, and we calculated their most preferred tax rate, $\tau^*_{m\tau}(\sigma)$ and $\sigma^*_{m\sigma}(\tau)$. Equations 3.1 and 3.8 can indeed be interpreted as reaction functions: for a given value of the social security (income redistribution) tax rate, eq. 3.1 (3.8) pins down the income redistribution (social security) tax rate chosen by the median voter $e_{m\tau}$ ($e_{m\sigma}$). Therefore, by Proposition 3.3 the (structure-induced) equilibria of this voting game correspond to the points where these functions cross.

It is now useful to introduce a measure of the relative ability of the two median voters, $\Delta_\tau = (e_\phi - e_{m\tau})\beta = n(e_\phi)e_\phi - n(e_{m\tau})e_{m\tau}$ and $\Delta_\sigma = (Ne_\phi - e_{m\sigma})\beta$. Notice that, while

Δ_τ simply measures the difference between the average labor income in the economy and the intragenerational median voter's labor income, what is relevant in Δ_σ is the difference between the average ability in the economy weighted by the relative performance of the social security system, N, and the social security median voter's ability. This is to take into account that social security is an inferior redistributive scheme for the young, due to its inefficiency in transferring resources into the future. Finally, let Δ be equal to $\Delta_\tau(1-N) - \Delta_\sigma$, and $\bar{\Delta}_\tau$ to $\Delta_\tau - (1-N)\left(1+\sqrt{1+4\Delta_\tau}\right)/2$. The next proposition characterizes the structure-induced political equilibrium outcome of the voting game.

Proposition 3.4. There exists a structure-induced equilibrium of the voting game, with outcome $(\tau^*; \sigma^*)$, such that

(I) if $\Delta_\tau \leq 0$ and $\Delta_\sigma \leq -(1-N)$, then $\tau^* = 0$ and $\sigma^* = 0$;

(II) if $\Delta_\tau \leq 0$ and $\Delta_\sigma > -(1-N)$, then $\tau^* = 0$ and $\sigma^* = 1 + \frac{1-\sqrt{1+4N\Delta_\sigma}}{2\Delta_\sigma} > 0$;

(III) if $\Delta_\tau > 0$ and $\Delta_\sigma \leq \bar{\Delta}_\tau$, then $\tau^* = 1 + \frac{1-\sqrt{1+4\Delta_\tau}}{2\Delta_\tau} > 0$ and $\sigma^* = 0$;

(IV) if $\Delta_\tau > 0$ and $\Delta_\sigma > \bar{\Delta}_\tau$, then

$$\begin{aligned} \tau^* &= \Delta_\tau \frac{1-2N\Delta - \sqrt{1-4N\Delta}}{2\Delta^2} > 0 \\ \sigma^* &= 1 - N - \tau^* \left(2 - N - \frac{\Delta_\sigma}{\Delta_\tau}\right) > 0. \end{aligned} \tag{3.10}$$

This proposition suggests that to fully appreciate the relation between a welfare system and the labor income inequality in the economy, we need to analyze the underlining income distribution by age groups, since age, rather than income, may be the main determinant in some agents' voting decision. Therefore, the overall income distribution needs to be separated in age groups and than recomposed, as shown in Figures 2a and 2b, to take into account of the income inequality as well as of the age. In fact, although for sufficiently low and high levels of income inequality, e.g. cases I and IV, this distinction may be redundant, it nevertheless allows us to characterize intermediate situations, like cases II and III. In particular, in case II, the intragenerational median voter's ability is above the mean ability, while the social security median voter's ability is sufficiently low, and thus only the social security system is adopted. This case may arise in an economy with moderate overall income inequality and a large proportion of old voters, or in an economy where the high degree of labor income inequality is mainly due to a large share of retirees. Case III, on the other hand, presents a distribution of income with large inequality in the intragenerational voting, but only small inequality in the social security voting, and thus leads to an equilibrium with income redistribution transfers only. This case may correspond to a young, highly unequal society.

Figure 3 illustrates the reaction functions and the equilibrium in case IV, when both systems arise. A proof of proposition 3.4 is provided in the appendix.

3.5.1. An Example of Welfare System

To obtain a flavor of the result, we parameterize the equilibrium welfare system to the US economy. The returns on social security are measured by the product of the real wage growth factor and the population growth factor over a 25 year period. We set the annual real wage growth rate and the annual population growth rate respectively equal to 2% and 1.5%. The performance of the other saving schemes is indicated by the real rate of return over the same period. We set its annual rate to 6.4%. It follows that the relative performance of the social security system with respect to other saving schemes, N, is equal to 0.5, which indicates that social security pays out, on average, 50% less than private savings over the lifecycle.

The degree of income inequality is summarized by the relative ability of the two median voters, $e_{m\tau}$ and $e_{m\sigma}$. We rank the voters according to their ability and age, as in figures 2a and 2b, and then we use the 1992 Survey of Consumer Finances (SCF) data on earning inequality, and the 1992 Presidential election participation rates by age and income, to calculate the ratio of the intragenerational and the social security median voter ability to the mean ability[12]. They turn out to be respectively $e_{m\tau}/e_{\mu} = 0.99$ and $e_{m\sigma}/e_{\mu} = 0.66$. The mean ability in the economy, e_{μ}, is normalized to 1. The total amount of disposable time, $\bar{l}$, is obtained by setting the average (daily) working time equal to 8/14, and it is equal to 2.87. The relative ability of the two median voters are thus $\Delta_\tau = 0.0287 > 0$ and $\Delta_\sigma = -0.4592 > \left(\Delta_\tau = -0.48526\right)$, additionally $\Delta = 0.473$. According to Proposition 3.4, this situation corresponds to case IV, and the associated equilibrium welfare system should thus be composed of non-negative income redistribution and social security tax rates. In fact, they turn out to be respectively:

$$\tau^* = 0.019 \text{ and } \sigma^* = 0.168.$$

3.6. Equilibrium Tax Rates and Income Inequality

In this section we concentrate on a welfare system composed of both income redistribution and social security schemes, the most frequent situation, and analyze the effects of changes in income inequality on the equilibrium tax rates. Simple comparative statics show that ceteris paribus an increase in the ability of the intragenerational median voter shifts up the associated reaction function, $\tau^*_{m\tau}(\sigma)$, and thus increases the equilibrium income redistribution tax rate while decreasing the social security tax rate. Analogously, an increase in the social security median voter's ability shifts up the other reaction function, $\sigma^*_{m\sigma}(\tau)$, increases the social security tax rate, and reduces the income redistribution one. Additionally, an increase in the relative performance of the social security system as saving scheme, N, shifts up the associated reaction function, $\sigma^*_{m\sigma}(\tau)$, and thus increases the social security and decreases the income redistribution tax rate.

These results, however, are not sufficient to characterize how a change in income

[12] See the appendix for a description of the data and of the procedure to calculate the median voters' ability.

inequality would affect the equilibrium tax rates. In fact, an increase, for example, in inequality would presumably tend to decrease both median voters abilities with respect to the mean ability in the economy, i.e. Δ_τ and Δ_σ would increase, and thus would shift both reaction functions in the same direction. The analysis of the consequences on the equilibrium tax rates of such changes represents the object of the next proposition.

First, we decompose the changes in the equilibrium tax rates into the effects due to the variation in the intragenerational median voter's ability ($d\Delta_\tau$) and in the social security median voter's ability ($d\Delta_\sigma$):

$$\begin{aligned} d\tau^* &= \underset{(+)}{\frac{\partial \tau^*}{\partial \Delta_\tau}} d\Delta_\tau + \underset{(-)}{\frac{\partial \tau^*}{\partial \Delta_\sigma}} d\Delta_\sigma \\ d\sigma^* &= \underset{(-)}{\frac{\partial \sigma^*}{\partial \Delta_\tau}} d\Delta_\tau + \underset{(+)}{\frac{\partial \sigma^*}{\partial \Delta_\sigma}} d\Delta_\sigma . \end{aligned} \tag{3.11}$$

As previously noted, the direct effect of a change in the median voter's relative ability is positive, whereas the crossed effects are negative. The following lemma[13] establishes another useful result.

Lemma 3.5. For an interior solution of the voting game, $(\tau^* > 0; \sigma^* > 0)$,

$$\left| \frac{\partial \tau^*}{\partial \Delta_\sigma} \right| \leq \left| \frac{\partial \sigma^*}{\partial \Delta_\sigma} \right| .$$

The absolute value of the direct effect of a change in the social security median voter's ability is larger than the absolute value of the indirect effect. Finally, let $\eta_{\tau^*;\Delta_\sigma} = \frac{\partial \tau^*}{\partial \Delta_\sigma} \frac{\Delta_\sigma}{\tau^*}$ be the elasticity of the equilibrium income redistribution tax rate to changes in Δ_σ, and $\eta_{\sigma^*;\Delta_\sigma} = \frac{\partial \sigma^*}{\partial \Delta_\sigma} \frac{\Delta_\sigma}{\sigma^*}$ be the elasticity of the equilibrium social security tax rate to changes in Δ_σ. We can now state the following proposition, which we prove in the appendix:

Proposition 3.6. For an interior equilibrium of the voting game, $(\tau^* > 0; \sigma^* > 0)$, and

[13] A proof is in the appendix.

for positive changes of Δ_τ and Δ_σ ($d\Delta_\tau > 0$, $d\Delta_\sigma > 0$), then

i) $d\tau^* \geq 0$ and $d\sigma^* \leq 0$ if $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \leq \left|\frac{\tau^*/\sigma^*}{\eta_{\sigma^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|.$

ii) $d\tau^* \geq 0$ and $d\sigma^* \geq 0$ if $\left|\frac{\tau^*/\sigma^*}{\eta_{\sigma^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right| \leq \left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \leq \left|-\frac{1}{\eta_{\tau^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|.$

iii) $d\tau^* \leq 0$ and $d\sigma^* \geq 0$ if $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \geq \left|-\frac{1}{\eta_{\tau^*,\Delta_\sigma}} + (1-N)\frac{\Delta_\tau}{\Delta_\sigma}\right|.$

In other words, if the increase in income inequality induces a percentage increase in the measure Δ_σ of the social security median voter's ability which is sufficiently smaller than the percentage increase induced in Δ_τ (case i), then the income redistribution tax rate will increase and the social security tax rate will decrease. The opposite happens for percentage increases in Δ_σ sufficiently larger than Δ_τ (case iii). For changes in Δ_σ and Δ_τ of comparable magnitude (case ii), both tax rates increase. A numerical example will help to appreciate the magnitudes of the changes in Δ_σ and Δ_τ which lead to the three cases.

Example 3.7: We use the values of the equilibrium welfare system we parametrized to the US economy in the previous section. The changes in the tax rates can be decomposed as at eq. 3.11, and evaluated at $\tau^* = 0.019$ and $\sigma^* = 0.168$:

$$\begin{aligned} d\tau^* &= 0.728 * d\Delta_\tau - 0.134 * d\Delta_\sigma \\ d\sigma^* &= -2.165 * d\Delta_\tau + 3.008 * d\Delta_\sigma. \end{aligned}$$

Therefore, an increase in the social security tax rate associated with a decrease in the income redistribution tax rate, case (iii), occurs when the percentage change in the social security median voter's relative ability is larger than 33.9% of the corresponding change in the intragenerational median voter's relative ability, $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \geq 0.339$. Case (i), $d\tau^* \geq 0$ and $d\sigma^* \leq 0$, occurs for $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \leq 0.045$, whereas case (ii), $d\tau^* \geq 0$ and $d\sigma^* \geq 0$, takes place for intermediate values: $0.045 \leq \left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \leq 0.339$.

4. Discussion and Related Literature

In our model economy, redistributive policies are induced by age and income heterogeneity among agents. An intragenerational income redistribution policy consisting of a labor income tax and of a lump sum transfer to the young would meet the approval of the majority of the young whose labor income is below the mean labor income in the economy. The old are indi®erent whether or not to vote, since, in absence of a social security system, they are not a®ected by this intragenerational transfer. Therefore, this (labor) income-based redistributive policy would constitute a political equilibrium of a unidimensional voting game. Pure intergenerational transfer schemes, that is systems which redistribute only according to the age, on the other hand, would not arise, because the saving technology implied by any such intergenerational scheme is inferior to the alternative storage technology, $N = (1 + {}^{1}) = (1 + R) < 1$.

A hybrid redistributive scheme composed of a labor income tax and a lump sum transfer to the retiree would induce a voting coalition of old and low income young, and would thus be politically viable. The political sustainability of this scheme, which resemble a PAYG social security system hinges on the use of one instrument to achieve two type of transfers, within and across generations. However, unlike in Tabellini (1990) where the introduction of another instrument of intragenerational income redistribution breaks the old-poor young coalition apart, and thus destroys the political equilibrium, in our model a political equilibrium with social security may exist even in a bidimensional voting game, as shown in the previous section. In this case, the old remain with the poor young in the winning voting coalition which supports social security, but at the same time they team up with the rich (above mean income) young in trying to defeat any intragenerational scheme.

The intuition behind this result is straightforward. Due to the existence of a within-generation redistribution component in the social security system, low income young are willing to enter a voting coalition to support both welfare schemes, although they indeed prefer straight income redistribution to social security. For the retirees, on the other hand, age represents the main determinant in their voting decision. By contributing their voting block to promote social security and to prevent intragenerational income redistribution[14], they help to shape the two winning coalitions. On social security, the coalition is composed of retirees and poor young, and the decisive, or median, voter is a low income young, see Figure 2.b; whereas on income redistribution the decisive, or median, voter is a young agent with a higher labor income, see Figure 2.a. In this sense, the retirees' uniform voting behavior contributes to create a wedge between the abilities of the two decisive voters which is crucial to obtaining an equilibrium welfare system composed of both schemes. Analogously, the extent to which changes in the overall labor income distribution affect the equilibrium tax rate depends on their impact on this wedge between the two decisive voters, as characterized at Proposition 3.6.

The idea of a social security system which relies on a voting coalition of low-income young and retirees to obtain its political sustainability dates back to Tabellini (1990). In his overlapping generation model, heterogeneous (in income), weakly altruistic agents vote every period on the social security level. Retirees clearly sustain a scheme which awards them a pension at no cost. On the other hand, since Tabellini abstracts from voting strategies leading to implicit contracts among successive generation of voters, young voters perceive the social security scheme as a current tax not related to any future benefit. Nevertheless, because of their weak altruism, low-income young vote for a positive social security level, since the utility associated to their parents receiving a pension outweighs the direct cost of the tax. With sufficient income inequality, an equilibrium with positive social security arises. However, such equilibrium is not robust to changes in the specification of the welfare system. In particular, if a fiscal policy which achieves within

[14] In the next section, we analyze the case in which retirees choose to support some degree of income redistribution.

generation income redistribution is introduced, the equilibrium disappears.

Lambertini and Azariadis (1998) specify a more complete welfare system, consisting of intragenerational and intergenerational transfers, to account for the rapid expansion in the government redistributive expenditure of the last decades. They attribute this increase in the welfare transfers to a shift of political power between di®erent voting coalitions which, like in our model, sustain the welfare system. The main di®erence with our model lies in the voting game. Their political system follows Baron and Ferejohn's (1989) \closed rule" system: one of the three existing groups (old, skilled and unskilled young) is randomly chosen to make a policy proposal. Then the proposal is voted against the status quo at simple majority. In this setting, like in our model, a coalition formed by the old and the unskilled young is shown to support an equilibrium with positive intragenerational and intergenerational transfers. However, the economic and political mechanisms behind this result are di®erent. In our model, voting coalitions are composed of agents with similar preferences over at least one dimension of the policy space (for example, retirees and low-income young over social security). In Lambertini and Azariadis (1998), the randomly chosen agenda setter exploits her advantage by proposing her most preferred policy among those which would meet the approval of at least another group of voters. In other words, the agenda setter \buys out" the cheapest, in terms of economic policies, among the other two groups. As a result, the groups composing a voting coalition do not have to share similar preferences over even one dimension of the policy space. For example, since their intergenerational scheme entails no intragenerational redistribution component, ceteris paribus unskilled young voter would prefer not to have any intergenerational transfer.

Other work pursue the idea that social security schemes should not be analyzed in isolation, but rather coupled with other public policy. Boldrin and Montes (1998) suggest that this is the case of social security and public education. They construct a model where public education and public pensions are implemented through an intertemporal political game. Public ¯nancing of education constitutes a way for credit constrained young to borrow from the middle age generation and finance their human capital accumulation. When employed, they pay an income tax to finance current young education, and current old pensions. Finally, they receive an old age pensions. Boldrin and Montes (1998) show that such an intergenerational agreement can arise as an equilibrium of a majoritarian voting game.

5. Extensions

The adoption of an intragenerational income redistribution scheme which does not pay out any transfer to the retirees tends to favor the existence of an equilibrium with positive social security. In fact, as shown in Figure 2, the old extreme positions in their voting on the two tax rates is crucial to ensure that the median voter's ability is much lower in the social security case than in the intragenerational case. This begs the question of how our results would change if we consider a more general income redistribution scheme.

In this section we analyze the political sustainability of a welfare system composed of a general income redistribution scheme and a PAYG social security system. The latter system does not differ from the intergenerational scheme presented in the previous section. The former, however, is a comprehensive system which imposes a tax rate, τ, on labor income, transfers, and pensions, and pays a lump sum transfer, T, to all agents (young and old).

The new life time budget constraint is thus

$$c^t_{t+1} = [e_t n(e_t)(1-\sigma_t) + T_t](1-\tau_t)(1+R) + [P_{t+1} + T_{t+1}](1-\tau_{t+1}).$$

Agents have the usual log-linear utility function, and thus the associated labor supply becomes

$$n(e_t) = \max\left\{0; \beta - \frac{1}{e_t(1-\sigma_t)(1-\tau_t)}\right\}. \qquad (5.1)$$

The budget constraint of the income redistribution scheme is

$$T_t = \frac{1+\mu}{2+\mu}\tau_t \left[\int_{\underline{e}}^{\bar{e}} e_t n(e_t)(1-\sigma_t)\, dG(e_t) + T_t + \frac{T_t + P_t}{1+\mu}\right]:$$

By substituting the expression for the labor supply, eq. 5.1, the welfare system budget constraints become

$$T_t(\tau_t;\sigma_t) = \frac{1+\mu}{2+\mu}\frac{\tau_t}{1-\tau_t}\left[e\phi\beta - \frac{1}{(1-\sigma_t)(1-\tau_t)}\right]$$

and

$$P_t(\tau_t;\sigma_t) = \sigma_t(1+\mu)\left[e\phi\beta - \frac{1}{(1-\sigma_t)(1-\tau_t)}\right]:$$

The welfare system tax rates, τ_t and σ_t, are decided in a majoritarian voting game by young and old agents. We adopt the political system described in section 3.1 and thus concentrate on structure-induced equilibria.

When voting on the social security tax rate, σ, for a given level of income redistribution, τ, young and old face a similar problem to the one analyzed in the previous model. The retirees determine their optimal social security tax rate by weighting the (positive) effect on their pension against the negative effect on their income redistribution transfer induced by the distortionary taxation. Their optimal tax rate is

$$\sigma^*_{old}(\tau) = 1 - \sqrt{\frac{1-\tau+\tau/(2+\mu)}{e\phi\beta(1-\tau)^2}}:$$

Young agents choose their most preferred tax rate, $\sigma_t = \sigma_{t+1} = \sigma^*$, according to the strategy profile in eq. 3.5. They expect the current transfer of resources to the retirees to be rewarded with a corresponding pension in their old age. They also take into account the impact on the income redistribution scheme that a distortionary social security taxation introduces. The optimal social security tax rate for a type e_t young voter is

$$\sigma^*_{e,t}(\tau) = \max\left\{0, 1 + \frac{1 - \sqrt{1 + 4N\Delta_{\sigma,e}\tau\left(1 + \frac{R-\mu}{2+\mu}\right)}}{2(1-\tau)\Delta_{\sigma,e}}\right\}$$

with $\Delta_{\sigma,e} = \beta[Ne_\phi - e_t]$.

For a sufficiently large minimum ability level, $\underline{e} > \sqrt{e_\phi/\beta}$, the ordering of young and old agents on the social security voting corresponds to the one at Figure 2b. The retirees always choose a larger social security tax rate than any young, and the median voter is the type-$m\sigma$ young who splits the electorate in halves: $G(e_{m\sigma}) = \mu/2(1+\mu)$.

The introduction of a more general income redistribution system modifies the retirees' preferences over this scheme tax rate, τ. In determining their optimal income redistribution tax rate for a given level of social security, $\tau^*_{old}(\sigma)$, the old compare the positive effect on the income redistribution transfer, T, with the negative impacts due to the usual tax distortion, and to the additional taxation imposed on their pension. Their optimal choice is

$$\tau^*_{old}(\sigma) = 1 - \left[e_\phi\beta(1-\sigma)(1-(2+\mu)\sigma)\right]^{-1/2}$$

Unlike in the previous specification of the welfare system, the retirees now vote for positive level of income redistribution, provided that the social security tax rate, and thus the magnitude of the distortionary effect and of the additional taxation, is sufficiently small, $\sigma < 1/(2+\mu)$.

A type e_t young voter determines her most preferred income redistribution tax rate, $\tau^*_{e,t}(\sigma)$, by maximizing her indirect utility function with respect to this tax rate for a given value of the social security tax rate:

$$\tau^*_{e,t}(\sigma) = 1 + \frac{1 - \sigma - \sqrt{(1-\sigma)^2 + 4\beta\frac{1+\mu}{2+\mu}(1-\sigma)\left[e_\phi\frac{1+\mu}{2+\mu} - e_t(1-\sigma)\right]}}{2\beta(1-\sigma)\left[e_\phi\frac{1+\mu}{2+\mu} - e_t(1-\sigma)\right]}.$$

Notice that the young's optimal tax rate is decreasing in their income and it is positive only for ability levels below a certain threshold[15].

In order to identify the median voter on the income redistribution issue, we need to order the voters according to their age and income. For sufficiently large values of the minimum ability, $\underline{e} > \sqrt{e_\phi/\beta}$, and in absence of social security, $\sigma = 0$, the ordering, and thus the median voter's ability, corresponds to the social security voting case, as at Figure 2b. In absence of pensions, the retirees vote for the tax rate that maximizes their income redistribution transfer, which is larger than any young preferred tax rate. As we increase the social security tax rate to trace out the reaction function, $\tau^*(\sigma)$, the ordering changes. The retirees' proposed tax rate becomes lower than the poorest young optimal tax rate, and equal to some richer young's most preferred tax rate: in terms of Figure 2b the old move to the right across the spectrum of young's abilities. The median voter (ability) remains unaffected as long as her preferred tax rate is lower than the retirees'. Once the two tax rates coincide, however, any additional decrease in the retirees' most preferred income redistribution tax rate induced by increases in the social security tax rate shifts the median voter's ability to the right. The median voter's tax rate now coincides with the retirees' tax rate. Finally, as the social security tax rate increases even more, the old generation's preferred tax rate becomes very small, and the old shift almost entirely to the right of the ability distribution, as in figure 2a. The median voter is identified by the young voters who splits the electorate in halves, with the old voting "against" income redistribution. It follows that the reaction function, $\tau^*(\sigma)$, will be continuous and will be composed of three pieces, corresponding to the three different median voter's abilities.

A political equilibrium of this bidimensional voting game with positive income redistribution and social security tax rates requires the two reaction functions $\tau^*(\sigma)$ and $\sigma^*(\tau)$ to cross in an interior of the simplex $\tau + \sigma \leq 1$. Since this problem has no closed-form solution, we provide an example of an equilibrium welfare system parametrized to the

[15] $e_t < \left[(1-\sigma)\left(1 + e_\phi \frac{1+\mu}{2+\mu}\beta\right) - \frac{1+\mu}{2+\mu}\right] / \beta(1-\sigma)^2$.

1992 US data.

Like in the previous example at section 3.5.1, we set $N = 0.5$, and normalize e_{ϕ} to one. Since the ordering on the social security voting has not changed, $e_{m\sigma}/e_{\phi}$ is equal to 0.66. To determine the per capita amount of the income redistribution transfer, T, we calculate the proportion of young (or retirees) in the population: $1/(2+\mu)$. In 1992, the proportion of young in the voting population was 83.4%, which implies $\mu = 4.02$. This also pins down R, which is equal to 9, in order for $N = (1+\mu)/(1+R)$ to be equal to 0.5. The income redistribution median voter ability changes with the social security tax rate. For small values of σ, the income redistribution median voter ability coincides with the ability of the social security median voter, $e_{m\tau}/e_{\phi} = 0.66$. For intermediate values, the median voter tax rate coincides with the retirees' decision, and, for large values, its ability is equal to the intragenerational median voter of the previous model: $e_{m\tau}/e_{\phi} = 0.99$. Finally, the total amount of disposable time, $\bar{l}$, is set equal to 2.76 in order to obtain an average equilibrium (daily) working time equal to $8/14$. For this parameterization of the economy there exists a structure-induced equilibrium welfare system with positive value of income redistribution and social security tax rates which are respectively:

$$\tau^* = 0.07 \text{ and } \sigma^* = 0.09.$$

Notice that for this parametrization, there also exist two other (structure-induced) equilibria of the game, one with zero income redistribution and positive social security: $\tau^* = 0$, $\sigma^* = 0.25$; and the other with positive income redistribution and no social security: $\tau^* = 0.36$, $\sigma^* = 0$. Figure 4 displays the two reaction functions and the three (structure-induced) equilibria.

6. Concluding Remarks

Why does the largest US welfare program select its recipients by their age, rather than by their earnings or wealth? In contrast with previous literature, we suggest that the existence of a welfare system composed of a (large) PAYG social security program and an income redistribution scheme may result as the political equilibrium of a voting game played by successive generation of voters. In particular, the social security system is supported by a voting coalition of retirees and low-income young.

Two features are crucial to this result: the political power of the old, which derives from their \extreme" and uniform voting behavior; and the intragenerational redistribution component of the social security system. Unlike the young and the middle age, the elderly constitute a fairly homogeneous group. They are old, and they have zero, when retired, or low labor earnings, although they may largely di®er in their wealth. This homogeneity makes them a uniform electoral block when voting on redistribution issues: they all like social security, and they all may or may not support di®erent forms of income-based redistribution. Since they are able to cluster and shift a large amount of votes, the elderly play a crucial role in shaping the two winning coalitions, with the decisive voter on social security having a much lower labor income than the decisive voter on income redistribution, as shown at Figure 2.

The existence of a within generation redistribution element in the social security system, on the other hand, makes social security palatable to low-income young, even in the presence of other income redistribution schemes. This factor has often been overlooked by the social security literature. Its relevance for the political viability of a system is, however, crucial, as the recent social security reforms have shown. In fact, in most of these cases, the reformed systems have maintained an element of within generational redistribution, sometimes as a new, separate program ¯nanced through general taxation.

The several empirical studies which have tried to relate various measures of income inequality to the size of the different government welfare transfers have provided mixed evidence. Perotti (1996), for example, has presented evidence supporting a negative and significant relation between welfare and social security transfers and income inequality; whereas in Lindert (1997) higher inequality decreases all social expenditures[16]. Our work suggests that the effect on each individual welfare program (social security and intragenerational transfer) depends on the magnitude of the change in income inequality, as well as on its specific impact on the income distribution. An overall increase in income inequality which has comparable effects on both median voters abilities would lead to an expansion in both programs. However, an increase in dispersion localized in the lower tail of the distribution would presumably induce larger changes in the social security median voter's ability than in the intragenerational one. The final result could then be an increase in social security coupled with a constant or even decreasing intragenerational transfer. We believe that these simple considerations should be taken into account in future empirical studies. In particular, the overall income distribution, or its relevant statistics, should be decomposed by age groups, whenever age, rather than labor income, may be the main component in the agents' decisions, and then it should be reaggregated according to income and age.

Several interesting issues were left aside in this work, and deserve a more detailed analysis. We identified labor earnings homogeneity among the elderly as an important determinant of their political influence, since it makes them a uniform voting block on redistribution issues. Curiously, this earnings homogeneity can be traced back to the social security legislation, which requires the old persons to retire from working in order to receive their benefits. This link is intriguing. Is it the existence of a social security system what gives the elderly their political power[17]?

[16] Perotti measures income inequality by the share in income of the third and forth quintile. Lindert identifies inequality as the sum of the natural logaritms of the upper income gap and of the lower income gap. The upper income gap is the ratio of the average income for the top fifth to that for the third fifth. The lower income gap is the comparable measure between the third and the fifth quintiles.

[17] Mulligan and Sala-i-Martin (1997) suggest a similar explanation as they argue that the political power of the elderly derives from the low option value of their time.

Analogously, while our model contributes to shade some light on the impact that income inequality has on the political determination of a welfare system, it is not suited to analyze the long term e®ect that runs from these welfare policies to the income distribution. We believe that more research needs to be done to understand whether reduced after-tax income inequality contain the seed of its own reversal, as it tends in the long run to erode the political sustainability of the welfare state.

Table 1

1992 Cash Bene¯ts in billions of $

OASDHI	284.3
Railroad Ret.	7.3
Public Employees Ret.	103.7
Veterans' Pensions	16.5
Unemployment Ben.	37.3
Temporary Disability	4.0
Workers' Compens.	44.1
Public Assistance	22.4
SSI	22.3
TOTAL	541.9

Table 2

1992 Cash and In-Kind Bene¯ts

	Federal	State
OASDHI	416.6	-
Railroad Ret.	58.2	45.1
Public Employees Ret.	7.7	-
Veterans' Pensions	16.5	-
Unemployment Benef.	9.9	31.2
Workers Comp	3.2	40.9
Public Aid	138.7	69.2
Medicaid	32.0	37.8
Housing	17.9	2.7
TOTAL	700.7	226.9
Education	20.2	272.0

Table 3

Participation Rates by Income

18-64 Year Old Voting Population

Income (I)	1992
I < $5; 000	30:4 %
$5; 000 < I < $9; 999	34:5 %
$10; 000 < I < $14; 999	40:1 %
$15; 000 < I < $19; 999	50:6 %
$20; 000 < I < $24; 999	59:8 %
$25; 000 < I < $34; 999	68:4 %
$35; 000 < I < $49; 999	75:6 %
$50; 000 < I	79:7 %
Income not Reported	54:9 %
All Incomes	60:8 %

A. Appendix

The Political System: Our political system describes a decision-making institution which has $1+1/(1+\mu)$ members: the electorate, E. The space of alternatives is a compact subset of R^2: (τ, σ) s.t. $\tau + \sigma \le 1$. And there exists a complete, transitive binary preference relation $\succeq_i$ over all alternatives $x, y \in R^2$, $\forall i \in E$, and represented by $v_i : R^2 \to R$. Institutional arrangements differ along three dimensions: (a) committee structure; (b) jurisdiction structure; and (c) amendment structure. The first two structures follow from the definitions below.

Definition A.1 (Committee). Call the family of sets $C = \{C_j\}$ a committee system if it covers the entire electorate E. Then the committee $C = \{E\}$ is the Committee of the Whole.

Definition A.2 (Jurisdiction). Let $B = \{b_1, b_2\}$ be the orthogonal basis for R^2 where b_i is the unit vector for the i-th dimension. The family of set $J = \{J_k\}$ is a jurisdictional arrangement if it covers B. Then $J = \{\{b_1\}, \{b_2\}\}$ is a Simple Jurisdiction.

Additionally, call f the function which associate a jurisdiction with a committee, $f : C \to J_k$. In our system $f : E \to \{\{b_1\}, \{b_2\}\}$ or $f(E) = \{\{b_1\}, \{b_2\}\}$.

To define an amendment structure we need to introduce the notions of status quo, x^o, and of proposal. A status quo, x^o, represents the previous agreed level on both dimensions of the issue space. For example, at time t, $\{x^o_1, x^o_2\} = \{\tau_{t-1}, \sigma_{t-1}\}$.

Definition A.3 (Proposal). A proposal, x, is a change in x^o restricted to a single jurisdiction. The set of proposal available to the committee of the whole is

$$g(E) = \{x \mid x = x^o + \lambda_i b_i; b_i \in f(E)\} \subseteq R^2.$$

Definition A.4 (Amendment Control Rule). For any proposal $x \in g(E)$; the set $M(x) \subseteq R^2$ consists of the modifications E may make in x. $M(x)$ is said to be an amendment control rule. An amendment control rule is a Germaneness rule if $M(x) = \{x' \mid x'_i = x^o_i \text{ if } x_i = x^o_i\}$.

Definition A.5 (Induced Ideal Point). For a status quo $x^o = (x_1^o; x_2^o)$ and a jurisdiction b_j, the induced ideal point in the j-th direction for $i \in E$ is $x^{*i} = \left(x_j^{*i}; x^o_{-j}\right)$ where $x_j^{*i} = \arg\max u_i\left(x_j; x^o_{-j}\right)$. Then, x^{*i} is the induced ideal point on an arbitrary set X if $u_i(x)$ is maximized on X at $x = x^{*i}$:

Definition A.6 (Median in all Directions). In a bidimensional issue space $(\tau;\sigma)$, $\mathbf{b} = (\mathbf{b}_\tau;\mathbf{b}_\sigma)$ is a median in all direction if any line passing through $\mathbf{b}$ divided the issue space in two areas each one containing half of the electorate's ideal points.

Proof of Proposition 3.2 (I) Young voters' preferences are represented by eq. 2.10. We derive eq. 2.10 with respect to τ_t and σ $(= \sigma_t = \sigma_{t+1})$, and then we impose the stationarity condition $\tau = \tau_t = \tau_{t+1}$ to obtain the following Hessian matrix:

$$\begin{matrix} -\frac{1+\tau-\sigma}{(1-\tau-\sigma)^3} & -\frac{2\tau}{(1-\tau-\sigma)^3} \\ -\frac{2\tau}{(1-\tau-\sigma)^3} & \frac{1-3\tau-\sigma-2N+2N\tau}{(1-\tau-\sigma)^3} \end{matrix}$$

Simple algebra shows that this matrix is semi definite negative for $N \geq \frac{(1-\tau_t-\sigma_t)^2}{2[1-\sigma_t-\tau_t(\tau_t-\sigma_t)]}\left(\leq \frac{1}{2}\right)$. Then young voters' preferences are quasi concave and, by Shepsle (1979) Lemma 3.1, they are single peaked.

(II) Old voters preferences (eq. 2.11) are not concave in $(\tau;\sigma)$, thus we will establish single peakedness using definition A.7. Let $\tau = p + q\sigma$ be a line in the issue space $(\tau;\sigma)$. By definition A.6, the induced ideal point for old voters on this line is $\left(\tau^* = p + \frac{q\left(1-p-\sqrt{\frac{1-p}{e\phi\beta}}\right)}{1+q}1;\ \sigma^* = \frac{1-p-\sqrt{\frac{1-p}{e\phi\beta}}}{1+q}\right)$, for $p < 1$, and $\left(\tau^* = 0;\ \sigma^* = -\frac{p}{q}\right)$, for $p > 1$ and $q < -1$ (Clearly for $p > 1$ and $q > -1$, then $\tau + \sigma > 1$). By definition

A.7, single peakedness requires that

$$u[\alpha\tau^0 + (1-\alpha)\tau^*; \alpha\sigma^0 + (1-\alpha)\sigma^*] > u[\beta\tau^0 + (1-\beta)\tau^*; \beta\sigma^0 + (1-\beta)\sigma^*]; \tag{A.1}$$

$\forall(\tau^0;\sigma^0)$ s.t. $\tau^0 = p + q\sigma^0$, whenever $0 \leq \alpha < \beta \leq 1$. To verify eq. A.1, we substitute the values of $(\tau^*; \sigma^*)$ into $u[\alpha\tau^0 + (1-\alpha)\tau^*; \alpha\sigma^0 + (1-\alpha)\sigma^*]$, and then derive it with respect to. α. The sign of this derivative is equal to

$$\begin{cases} (1+q)\alpha\left[(1+q)\alpha(\sigma^0-\sigma^*) - 2\sqrt{\frac{1-p}{e_{\phi}\beta}}\right] & \text{for } p < -1 \\ \left(\sigma^0 + \frac{p}{q}\right)\left[e_{\phi}\beta - \frac{1-p}{\left[1+\frac{p}{q}-\alpha(1+q)\left(\frac{p}{q}+\sigma^0\right)\right]^2}\right] & \text{for } p > -1 \end{cases}$$

This sign is negative in both cases for $\alpha < 1$, which implies that the inequality at eq. A.1 holds, and that old voters preference are single peaked.

Proof of Proposition 3.4 Using equations 3.1 and 3.8, it is easy to show that these reaction functions cross only once in the simplex $\tau + \sigma \leq 1$ at $(\tau^*;\sigma^*)$. This is the only point which represents the median among the induced ideal point along both dimensions, τ and σ, and thus by Proposition 3.3 $(\tau^*;\sigma^*)$ is the only structure induced equilibrium.

If $\Delta_\tau \leq 0$ and $\Delta_\sigma \leq -(1-N)$, the reaction functions 3.1 and 3.8 are only defined on the simplex $\tau + \sigma \leq 1$ at $(\tau = 0; \sigma = 0)$. If $\Delta_\tau \leq 0$ and $\Delta_\sigma > -(1-N)$, then $\tau^*_{m\tau}(\sigma) = 0$, and thus it crosses the reaction function 3.8 on the σ axis at $\sigma^* = 1 + \left(1 - \sqrt{1+4N\Delta_\sigma}\right)/2\Delta_\sigma > 0$.

To find the condition for an interior solution, case iv, notice that for $\Delta_\tau > 0$ both reaction functions are negatively sloped, and that $\tau^*_{m\tau}(\sigma)$ has a higher intercept on the vertical, σ, axis than $\sigma^*_{m\sigma}(\tau)$. Since both reaction functions are continuous, if $\sigma^*_{m\sigma}(\tau)$ crosses the horizontal, τ, axis to the right of $\tau^*_{m\tau}(\sigma)$ there exists a political equilibrium of the voting game for $\tau^* = \Delta_\tau\left(1 - 2N\Delta - \sqrt{1 - 4N\Delta}\right)/2\Delta^2$ and $\sigma^* = 1 - N - \tau^*(2 - N - (\Delta_\sigma/\Delta_\tau))$. The condition for the reaction function $\sigma^*_{m\sigma}(\tau)$ to cross the horizontal axis to the right of $\tau^*_{m\tau}(\sigma)$ is that $\Delta_\sigma > \bar{\Delta}_\tau = \Delta_\tau - (1 - N)\left(1 + \sqrt{1 + 4\Delta_\tau}\right)/2$. If, on the other hand, $\Delta_\sigma < \bar{\Delta}_\tau$, then $\sigma^*_{m\sigma}(\tau)$ will cross the horizontal, τ, axis to the left of $\tau^*_{m\tau}(\sigma)$, and thus the equilibrium will be on the horizontal, τ, axis at $\tau^* = 1 + \left(1 - \sqrt{1 + 4\Delta_\tau}\right)/2\Delta_\tau$.

Proof of Lemma 3.5 From equation 3.10, $\frac{\partial\sigma^*}{\partial\Delta_\sigma} = \frac{\tau^*}{\Delta_\tau} - \left(2 - N - \frac{\Delta_\sigma}{\Delta_\tau}\right)\frac{\partial\tau^*}{\partial\Delta_\sigma}$ and $\frac{\partial\tau^*}{\partial\Delta_\sigma} = \frac{2}{\Delta}\tau^* + \frac{N\Delta_\tau}{\Delta^2}\left(1 - \frac{1}{\sqrt{1 - 4N\Delta}}\right)$. Since $(\partial\tau^*/\partial\Delta_\sigma) \le 0$ and $(\partial\sigma^*/\partial\Delta_\sigma) \ge 0$, thus it is sufficient to show that $-(\partial\tau^*/\partial\Delta_\sigma) \le (\partial\sigma^*/\partial\Delta_\sigma)$, which can be done from the previous two expressions and using some simple algebra.

Proof of Proposition 3.6 To prove part (iii), we use the decomposition at eq. 3.11 to write $d\tau^* \le 0$ and $d\sigma^* \ge 0$ as $\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau} \ge -\frac{(\partial\tau^*/\partial\Delta_\tau)}{(\partial\tau^*/\partial\Delta_\sigma)}$ and $\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau} \ge -\frac{(\partial\sigma^*/\partial\Delta_\tau)}{(\partial\sigma^*/\partial\Delta_\sigma)}$. From equation 3.10, $\frac{\partial\tau^*}{\partial\Delta_\tau} = \frac{\tau^*}{\Delta} - (1 - N)\frac{\partial\tau^*}{\partial\Delta_\sigma} \ge 0$ and $\frac{\partial\sigma^*}{\partial\Delta_\tau} = -\frac{\tau^*}{\Delta_\tau} - (1 - N)\frac{\partial\sigma^*}{\partial\Delta_\sigma} \le 0$. Substituting these derivatives in the previous inequality we obtain: $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \ge \left|-\frac{1}{\eta_{\tau^*;\Delta_\sigma}} + (1 - N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$ and $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \ge \left|\frac{\tau^*/\sigma^*}{\eta_{\sigma^*;\Delta_\sigma}} + (1 - N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$: Since by Lemma 3.5, $|1/\eta_{\tau^*;\Delta_\sigma}| \le |\tau^*/\sigma^* \ast \eta_{\sigma^*;\Delta_\sigma}|$, then $d\tau^* \le 0$ and $d\sigma^* \ge 0$ for $\left|\frac{d\Delta_\sigma/\Delta_\sigma}{d\Delta_\tau/\Delta_\tau}\right| \ge \left|-\frac{1}{\eta_{\tau^*;\Delta_\sigma}} + (1 - N)\frac{\Delta_\tau}{\Delta_\sigma}\right|$ which prove part (iii). We skip the proof of part (i) and (ii), which is analogous to part (iii).

B. Appendix

In this appendix the 1992 Survey of Consumer Finances (SCF) data are used to analyze some feature of the US income distribution. The data unambiguously show that earnings, income, and wealth are unequally distributed across US families. In particular, the density functions of these distributions are skewed, as they display a fat lower tail, many poor, and a thin upper tail, few rich. For example, the ratio of median to mean was equal to 0.60 for earnings, to 0.58 for income, and to 0.28 for wealth. Other inequality indicators present the same picture.

Consider the ordering in the social security voting, as shown at ¯gure 2b. The mass of retirees is placed on the left of the income distribution: they vote for a larger tax rate than the poorest young. When adjusted for the (1992 Presidential) election participation rates, they represent 19% of the actual voters. To account for the young voters, we drop the retirees (person older than 65 year) from the sample, and adjust the new earning distribution for the election participation rates. Since turnout rates are not available by earnings group, we need to combine earnings distribution data with the participation rates by income shown at table 3. Given the high correlation between income and earnings found on 1992 data by D¶ıaz-Gim¶enez, Quadrini, and Rios Rull (1997), who report a coe±cient of 0.928, we assume that there are no permutations between the income and the earnings distributions. In other words, voters with low (high) earnings are associated to voters with low (high) income and to the corresponding participation rates. After the weights of the earnings distribution have been adjusted for the di®erent participation rates, we obtain that the ratio of the social security median voter's earnings to the mean earnings in the economy is equal to $0{:}66$.

In the case of intragenerational redistribution the ordering of the voters is shown at ¯gure 2a. The retirees now vote against the transfer and are placed on the right of the income redistribution. Since in 1992 almost 13% of the elderly would receive intragenerational transfers, we subtract these individuals from the retirees and add them to the poor young. In fact, since, unlike in our theoretical model, they actually received a bene¯ts, they would presumably have voted in favor of this policy. This makes the proportion of elderly voters in the voting population drop to 16.5%. We then adjust the earning distribution to account for young voters only, as described above, and calculate the ratio of the intragenerational redistribution median voter's earnings to the mean earnings in the economy, which is equal to 0:99.

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Preferences in (τ,σ) Space

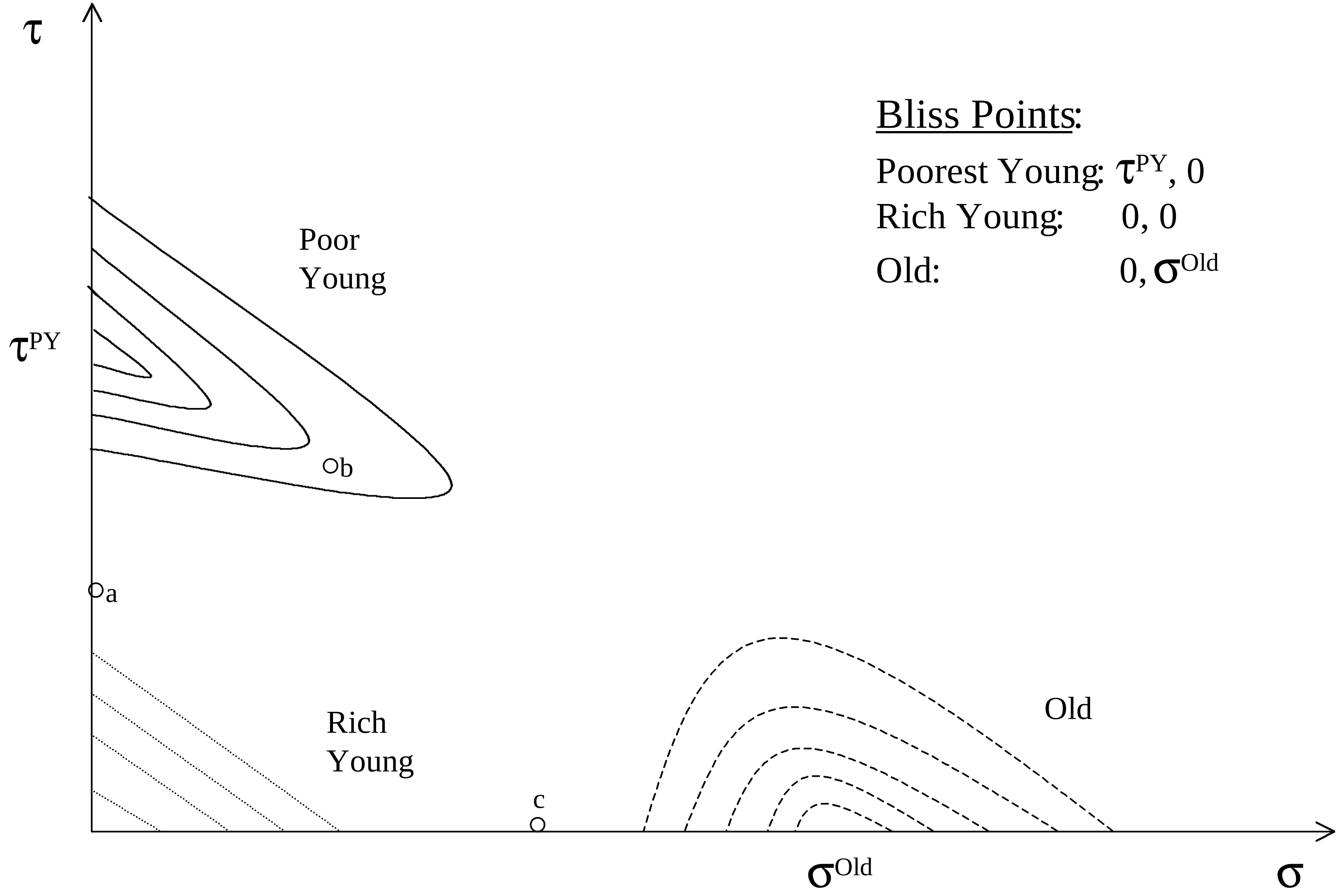
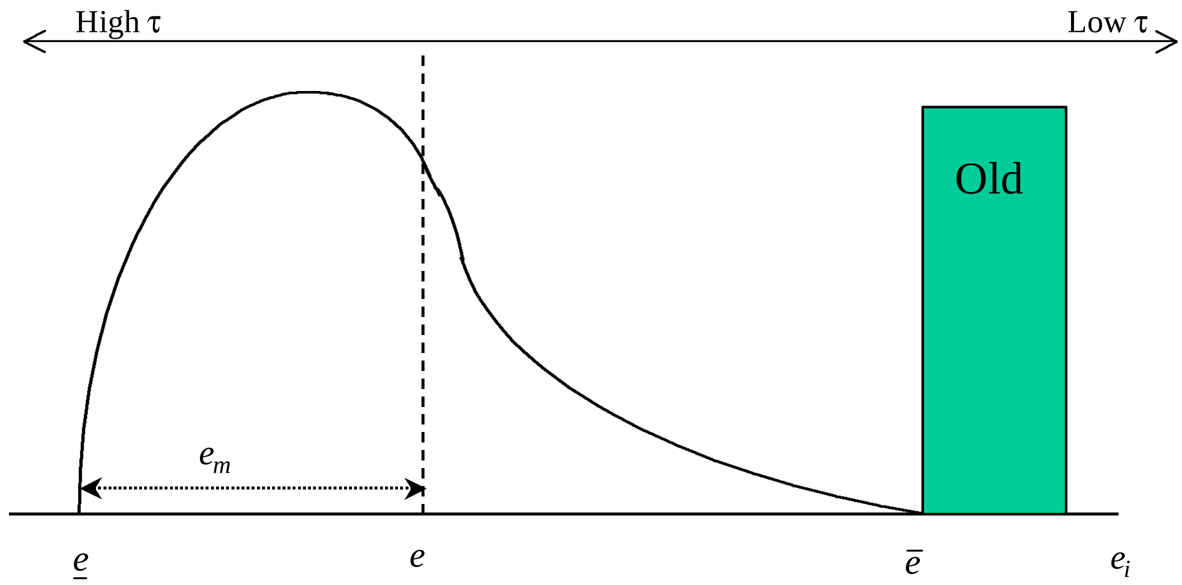


Figure 1

Voting on Intragenerational Transfers and Social Security

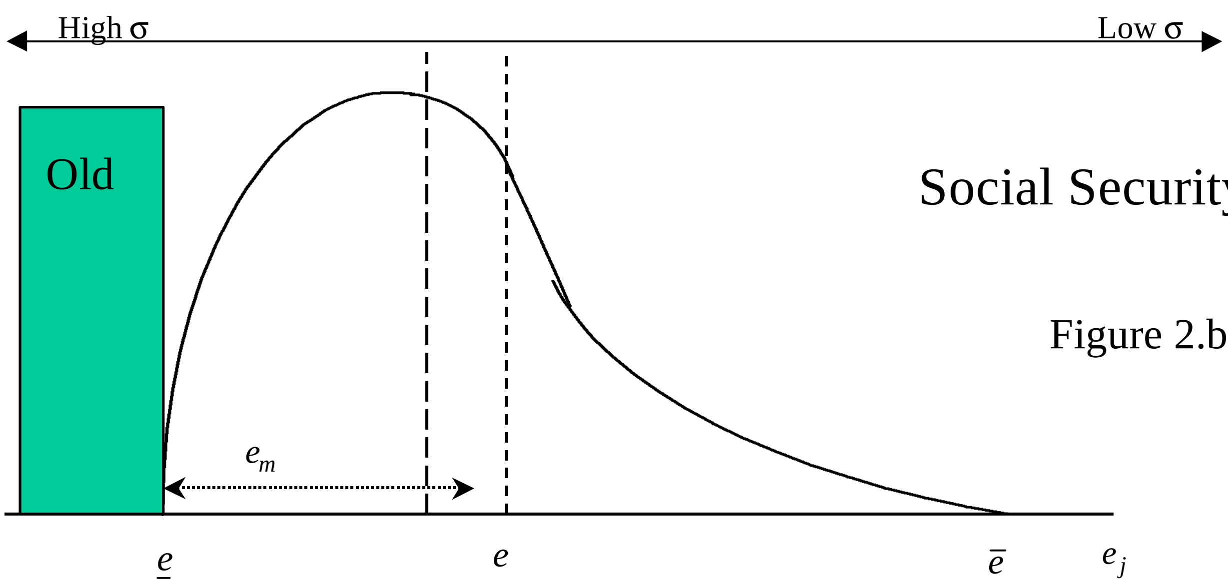
Intragenerational Transfers

Figure 2.a



Social Security

Figure 2.b



Equilibrium with Intragenerational Transfers and Social Security

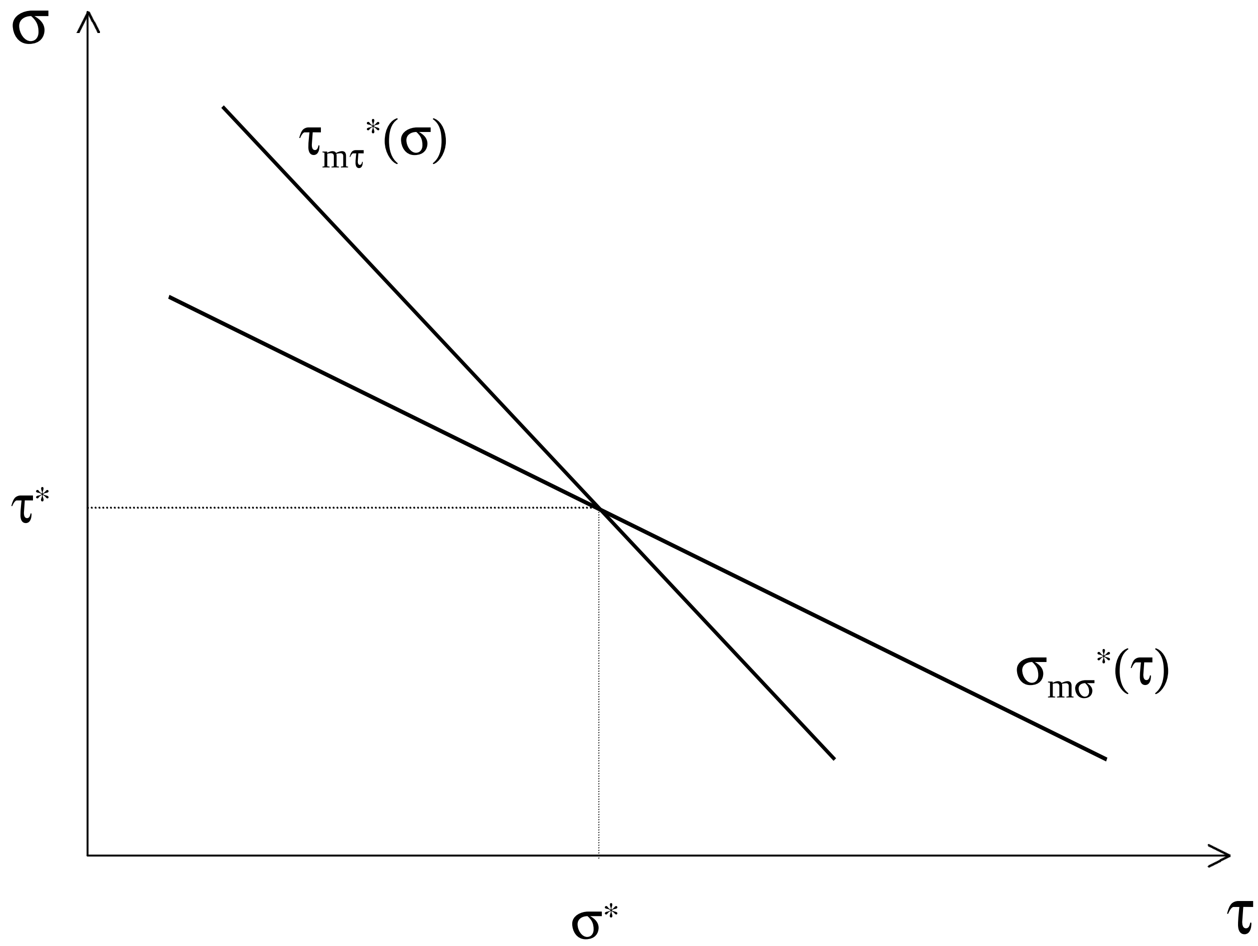


Figure 3

Equilibria with Income Redistribution and Social Security

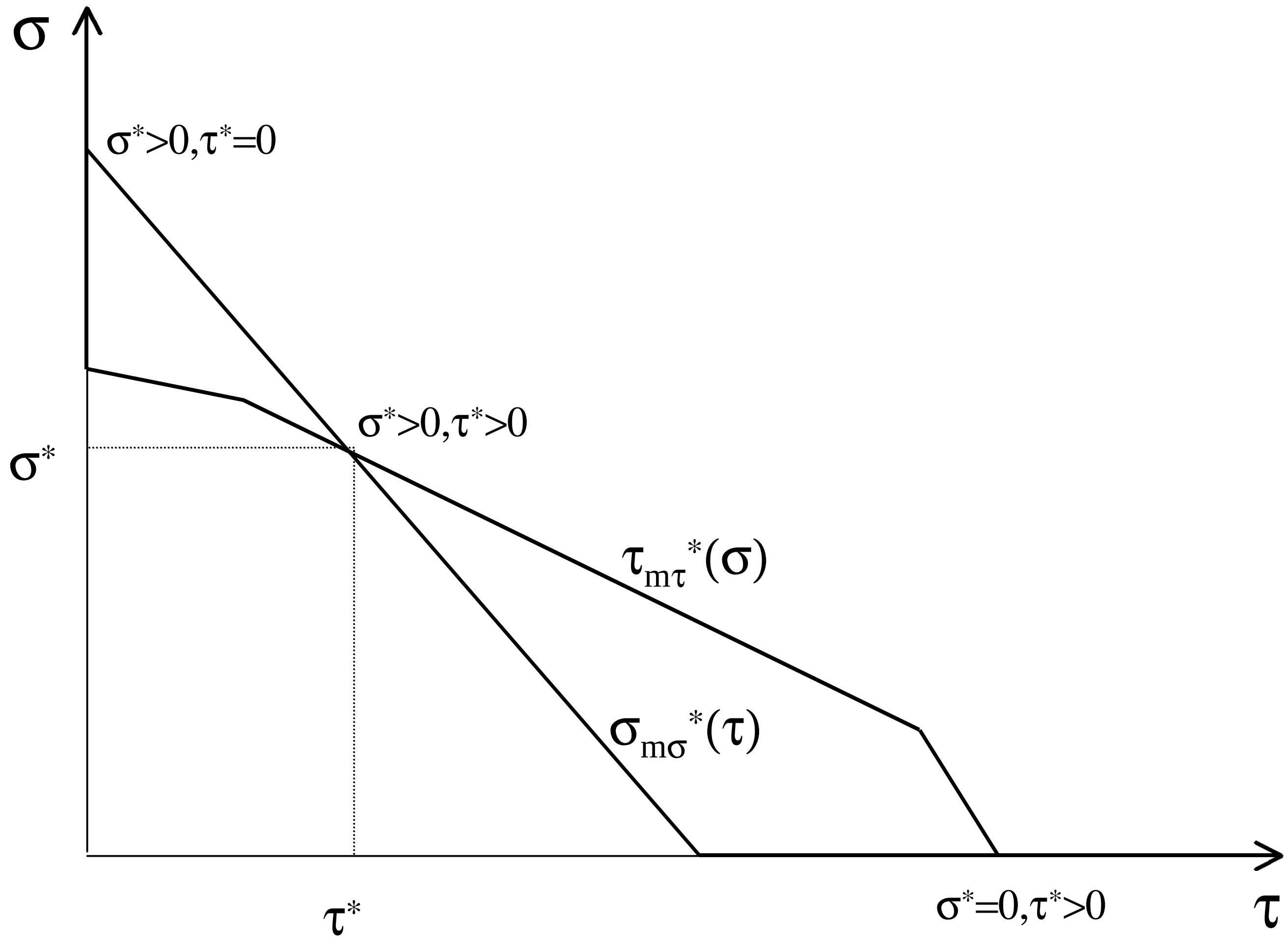


Figure 4