No. 2152

CAMPAIGN ADVERTISING AND VOTER WELFARE

Andrea Prat

PUBLIC POLICY

Centre for Economic Policy Research

ISSN 0265-8003

CAMPAIGN ADVERTISING AND VOTER WELFARE

Andrea Prat

Discussion Paper No. 2152
May 1999

Centre for Economic Policy Research
90–98 Goswell Rd
London EC1V 7RR
Tel: (44 171) 878 2900
Fax: (44 171) 878 2999
Email: cepr@cepr.org

CEPR Discussion Paper No. 2152

May 1999

ABSTRACT

Campaign Advertising and Voter Welfare*

This paper investigates the role of campaign advertising and the opportunity of legal restrictions on it. An electoral race is modelled as a signalling game with three classes of players: a continuum of voters, two candidates, and one interest group. The group has non-verifiable insider information on the candidates' valence and, on the basis of this information, offers a contribution to each candidate in exchange for a favourable policy position. Candidates spend the contributions they receive on non-directly informative advertising. This paper shows that: (1) a separating equilibrium exists in which the group contributes to a candidate only if the insider information about that candidate is positive; (2) although voters are fully rational a ban on campaign advertising can be welfare-improving; and (3) split contributions may arise in equilibrium (and should be prohibited).

JEL Classification: D72, D82, M37
Keywords: elections, campaign contributions, advertising, voter welfare, split contributions

Andrea Prat
Department of Econometrics
Tilburg University
Postbus 90153
5000 LE Tilburg
NETHERLANDS
Tel: (31 13) 466 2753
Fax: (31 13) 466 3280
Email: a.prat@kub.nl

*This paper is produced as part of a CEPR research network on The Economic Analysis of Political Institutions: Coalition Building and Constitutional Design, funded by the European Commission under the Training and Mobility of Researchers Programme (contract No ERBFMRXCT960028). I would like to thank Alex Cukierman, Eric van Damme, Mathias Dewatripont, Hans Gersbach, Jan Potters, Luca Rigotti, Gérard Roland, Aldo Rustichini, Randolph Sloof, and audiences at CORE, ECARE, Lisbon, New Orleans (1998 Public Choice Meetings), Paris

(Seminaire Fourgeaud), Rotterdam, and Tilburg for their suggestions. I am grateful to Michael Bailey, Walter Sorana, and especially John Ferejohn for their comments on an earlier version of the paper.

Submitted 4 May 1999

NON-TECHNICAL SUMMARY

In many Western democracies, political candidates are spending increasing amounts of money on various forms of advertising. This phenomenon reaches its peak in the US, where the average candidate to the senate spends $4.5 million, but it is also on the rise in several European countries, including Britain. Voters express discomfort at this mixing of money and politics and most countries have put in place restrictions on campaign giving and/or campaign spending. The issue is particularly hot, with one side appealing to freedom of expression and the other citing the excessive influence of rich lobbies. Given this state of things, it would be nice to have a model of campaign spending to evaluate alternative regulatory regimes. This paper is an attempt to fill this void and provide some initial welfare comparisons.

The problem is modeled as a game of incomplete information with three types of players: one lobby, a large number of voters, and two candidates. The candidates are evaluated by voters on two dimensions: policy and valence. Policy is announced by the candidate before the election and voters are heterogeneous with respect to their policy preferences. Valence is a set of personal characteristics of the candidate, such as charisma or ability, which benefits all voters. The main assumption of the model is that the lobby is in a better position than voters to observe valence. For instance, by talking to the candidate in private, the lobby leader forms a first-hand opinion about the personality of the candidate - something that the average voter cannot do.

The lobby, which has a different policy view than the median voter, contributes to candidates on a favor-exchange basis. The lobby leader observes the candidate's valence and then offers a monetary contribution in exchange for a favorable policy. If the candidate accepts, he spends the money on campaign advertising directed to voters. Voters are assumed to be rational and are not affected by ads, unless they get some information from them. However, it is also assumed that advertising does not provide voters with direct, verifiable information (this is in line with the experimental evidence that campaign advertising is effective even when it is completely devoid of any informational content). The idea of the paper is that rational voters may be influenced by advertising solely because it indirectly signals the candidate's valence.

Indeed, the main positive result of the paper is that there exists an equilibrium in which the lobby gives to a candidate only if it has received a positive insider signal about that candidate. Hence, campaign advertising perfectly reveals the insider signal to voters allowing them to make a more informed choice.

This could lead to conjecture that in this model campaign advertising is always beneficial to voters and that restricting it is a bad idea. However, this conjecture is wrong and the paper shows that, under reasonable parametric assumptions, banning campaign advertising (or campaign spending) is welfare improving. The intuition behind this result is simple. In the equilibrium described above, all high-valence candidates strike a deal with lobbies: in exchange for a contribution, they choose a policy that is bad for the median voter and good for the lobby (such as agricultural subsidies). All low-valence candidates do not receive money because the lobby is not willing to offer them the same amount it offers to high-valence ones. How bad a policy is a high-valence candidate willing to put up with? If he rejects the lobby's offer, voters will perceive him as a low-valence candidate. This means that the lobby can push its request up to the point when a high-valence candidate with a bad policy is perceived at the same level as a low-valence candidate with a good policy. In the limit, it is as if voters only encountered low-valence candidates with good policies. This is certainly inferior to encountering both bad and good candidates with good policies, which would be the case if advertising were banned. Notice that, if voters could commit *not* to listen to campaign advertising, they would do it. However, this commitment is clearly not credible, because once the advertising is in place, voters should get as much information from it as they can.

It is important to stress that this negative result is not due to the fact that campaign advertising is a waste of resources (printed billboards, etc). This component is not counted in the welfare measure and would constitute an additional negative element. The negative result is purely due to the bias generated in the candidates' policy choices.

This negative welfare result formalizes the idea that, in a political arena in which campaigns are fought with money, candidates become overly dependent on lobbies' contributions. This result holds also if the lobby is exogenously restricted to contributing to only one of the two candidates. We can go one step further and consider the role of *split contributions*. A split contribution occurs when the same lobby contributes to more than one candidate running in the same race. In this framework, it is shown that split contributions are unambiguously welfare decreasing. When the lobby gives to both candidates, voters have the same information they would have if the lobby had given to neither, but they have to put up with a bad policy.

Several extension of the model are considered: (1) when the candidates' policy choices are not observable; (2) when it is candidates who make offers to lobbies, rather than the converse; (3) when candidates have personal wealth they can use for campaign advertising.

1 Introduction

In electoral competitions throughout the world money is playing an increasingly important role.[1] In the last US Senate election the average candidate made campaign expenditures of \$4.5 million. Most developed countries have passed legislation to restrict campaign spending, campaign giving, or both. However, the existing regulation is generally deemed insufficient. This is true especially in the US, where the public opinion has been clamoring for years for stricter controls on campaign money.

To evaluate the opportunity of various forms of regulation, we need a model of voting with campaign advertising. Although there exists a sizeable literature on campaign contributions and interest group politics (See Morton and Cameron [28] for a survey), in none of the existing models advertising is microfounded. Typically, it is assumed that electors – or a fraction of electors – cast their vote according to an "advertising influence function," which is a mapping from campaign expenditures into vote shares. The influence function is exogenously given, not derived from assumptions on the primitives of the models. However, we cannot make welfare comparisons if we do not know how advertising affects the utility of voters. Thus, the goal of this paper is twofold. First, we develop a microfounded model of campaign advertising. Second, we use the model we have developed to evaluate the opportunity of regulation on campaign spending.[2]

In order to be plausible, a microfounded model must explain three seemingly contradictory stylized facts observed in campaign advertising:

- *It Is Paid for by Groups whose Objectives Differ from the Median Voter's Objectives.* Campaign contributions come from groups of voters whose preferences are often at odd with the preferences of the majority of voters.[3] For instance, in the US, agricultural interest groups are habitual donors. Their preferred policies – agricultural subsidies and other forms of protection to farmers – cause well documented welfare losses. Lopez and Pagoulatos [24] conduct a study on trade barriers in the US food and tobacco industry. They find that welfare losses can be up to 12.50% of domestic consumption and are positively associated with campaign contributions from

[1] For a recent cross-country survey of campaign spending and campaign regulation, see The Economist [12].

[2] Indeed, the need for a microfounded model of campaign spending is perceived in the field. See Morton and Cameron [28, p. 85], Baron [6, p. 45], and Laffont and Tirole [21, p. 634].

[3] A survey of campaign giving patterns can be found in Lehman Schlozman and Tierney [32, Chapter 10].

agricultural interest groups.

- *It Does Not Convey Hard Information.* Casual observation suggests that campaign advertising contains little direct information. Political ads are not credible. In the US, the First Amendment protects campaign advertising as free speech. Voters have no legal recourse against a candidate who broadcasts ads with misleading statements or misrepresentation of reality (such a strong protection does not apply to commercial advertising).

- *It Works.* Campaign advertising is effective. Ansolabehere and Iyengar [1] have conducted a laboratory experiment with more than 3000 residents of the Los Angeles area. Their goal was to study how political advertising on mass media affects the voters' decisions. The experimenters produced several versions of thirty-second TV ads, showed them to the subjects, and then asked the subjects to fill a questionnaire. Subjects who viewed an ad from a candidate were much more likely to vote for that candidate (exposure to a single ad increased the candidate's share of the vote by 5%). Notice that the ads contained so little hard information that they could apply to a candidate as well as to her opponent (the ads were produced in two versions that differed solely in the name of the sponsoring candidate).

Our task is to construct a model of advertising consistent with the stylized facts above. The indutrial organization literature has developed three main ways to deal with advertising: (1) advertising enters the utility function of voters (Dixit and Norman [11] and Becker and Murphy [7]); (2) advertising provides information in a direct way (See Tirole [33, Chapter 2] for a survey); and (3) advertising provides information in an indirect way (e.g. Milgrom and Roberts [27]). The first type of models is not suitable for welfare comparisons unless one can make specific assumptions on how exactly advertising modifies the consumers' utility function (this point is made in Fisher and McGowan [14]). The second type of model is certainly viable. Indeed Austen-Smith [2] has developed a model in which voters are influenced by advertising because it provides direct information about candidates' positions. However, as we have argued above, direct information transmission does not seem to be the main component of advertising. The third type of models assume that viewers are influenced by advertising not because of the message it transmits but because of the amount of money that has been spent on it. The advertiser has some information which would be of use to viewers, but she cannot communicate it in a credible way. However, if the advertiser spends enough money on costly signalling, viewers are able to infer the information in an indirect way. This paper develops a model of the third type.

The model can be sketched as follows. There are three classes of players: a continuum of voters, one interest group, and two candidates. Voters judge candidates on two dimensions: valence (e.g. ability, leadership, integrity) and policy. All voters agree on the valence dimension, but have heterogeneous preferences about policy. The interest group caters to the policy dimension of a subset of voters, but is not directly interested in the valence of candidates. The ideal policy of the median group member differs from the ideal policy of the median voter. Candidates maximize their chance of being elected.

The valence of a candidate is unknown, but there are imperfect signals about it. Some of these signals are public (candidates' records, TV debates, etc.) and some are observed by the interest group but not by voters (rumours, first-hand experience, etc.). The insider signals are non-verifiable. After observing the insider signals, the group makes an offer to each candidate. The offer consists of a monetary contribution to be spent on non-directly informative advertising and a policy to be implemented if the candidate is elected. Candidates accept or reject the group's offer. Each voter then observes the public signals, the policy choice, and the amount spent on advertising by each of the two candidates and casts a vote for one of the two candidates.

One may conjecture that in this model a ban on advertising cannot be optimal.[4] Each candidate maximizes her chance of being elected. Thus, she will accept a contribution only if it increases her chance of being perceived in a positive way by voters. Voters observe policy positions. Thus, they see any promise that a candidate makes to the interest group and they can punish a candidate who promises too much by not electing her. Therefore, if advertising occurs in equilibrium, one might conclude that voters must get more benefit in terms of indirect information than they give up in terms of policies. As is turns out, this conjecture is incorrect.

The main results of the model are:

1. *There exists a separating equilibrium in which the interest group contributes to a candidate if and only if the insider signal about that candidate is positive. A pooling equilibrium exists but does not survive the Intuitive Criterion.* The insider signal is revealed to voters through the amount of campaign advertising. In exchange for a contribution, the group obtains from the candidate a policy position that is

[4]Throughout this paper a ban on advertising and a ban on contributions produce the same effects. Thus we only refer to a ban on advertising. See Section 6 for a discussion.

favorable to the group and detrimental to the median voter. Intuitively, the group sees contributions as an investments with stochastic return: the group gets the favorable policy only if the candidate is elected. Therefore, the group prefers to contribute to a likely winner and uses the insider signal to infer the chances of the candidate's victory. A group with a good insider signal can afford a contribution that a group with a bad insider signal cannot afford.

2. *Under certain conditions, a ban on campaign advertising strictly increases the voters' welfare (which includes the group members' welfare).* Campaign contributions represent a credible threat the interest group can use against a good candidate (i.e. a candidate with a positive insider signal): in equilibrium a good candidate who rejects the group's offer is perceived by voters as bad. Through this implicit threat, the group can obtain from a good candidate a policy position that makes the median voter indifferent between a good candidate with that policy position and a bad candidate with the median voter's ideal position. This represents the candidate's participation constraint. If the ideal policy of the median group member is distant from the ideal policy of the median voter, the group wants the participation constraint to be binding. In equilibrium, it is as if voters only encountered bad candidates with the median voter's ideal position. The presence of campaign advertising brings the median voter more cost in terms of biased policy than benefit in terms of information on candidate valence. It is important to stress that the negative welfare effect is not due to the fact that advertising is a waste or real resources. The wasteful aspect of campaign advertising is not taken into account in our definition of welfare (this would be one extra reason to ban advertising). The negative welfare is due only to the policy bias that money from lobbies brings about.

3. *If the group receives equally good insider signals about the two candidates, it will contribute to both and will get favorable policies from both.* If one candidate rejects the offer, he will be perceived as bad and the other candidate will be perceived as good. This situation is particularly negative for voters: they receive useless information and they have to choose between two candidates who cater to the interest group. Indeed, it is proven that a ban on split contributions always increases the voters' welfare.

The problem is formulated in a general way. In particular, the probability distributions of signals are left in a generic form. Moreover, results are shown to be robust to modifications of the model such as the assumption that candidates make offers to the group or the assumption that policy positions are unobserved. However, results change dramatically if candidates do not receive contributions from groups but they finance campaigns out of their personal wealth. In that case a separating equilibrium need not exist.

The plan of the paper is as follows. Section 2 introduces the model. Then, for illustrative purposes, the core of the paper is divided into two main parts. Section 3 assumes that the interest group can only contribute to Candidate 1. With this restriction it is possible to prove results (1) and (2) in an intuitive way. However, the assumption that the groups can only contribute to a pre-specified candidate is unrealistic. Thus, Section 4 develops the full model in which the group can contribute to both candidates. Results (1) and (2) still hold, and, moreover, result (3) is proven. Section 5 discusses modifications of the model. Section 6 concludes.

Related Literature This paper is inspired by two strands of literature that are somewhat distant from each other: the political economy literature on campaign contributions and the industrial organization literature on advertising with rational consumers. In common with the first strand (See, among others, Baron [5, 6], Morton and Cameron [28], and Grossman and Helpman [17]), we model an electoral race as a game with three classes of players: voters, candidates, and interest groups. We adopt most of the definitions and the assumptions that are standard in the literature on campaign contributions, with three important differences: (1) all voters are rational; (2) candidates are judged on valence as well as policy; and (3) some non-verifiable signals about valence are only available to insiders.[5] The second strand includes Milgrom and Roberts [27], Kihlstrom and Riordan [20], Hertzendorf [18], and Bagwell and Ramey [4], and others. In common with the second strand we assume that what matters in advertising is the amount spent on it, not its content. Under certain conditions, an agent with non-verifiable private information is able to reveal it through non-directly informative costly signalling. However, models of commercial advertising rely on concepts – such as quantity, price, and cost – that do not find a parallel in elections. Moreover, in political advertising, the agent who pays for advertising (the interest group) does not derive a direct benefit from the actions of the receiver (voters). It is only through the interaction with the candidate that the interest group is able to gain from advertising. Thus, while the spirit is similar, our model present difficulties that are absent in the commercial advertising models.

[5]The use of a nonpolicy dimension is not new in the literature on elections with incomplete information. See for instance Cukierman [10].

Two other papers study models in which campaign advertising is non-directly informative

In Potters, Sloof, and Van Winden [30], a candidate can be of a high type or a low type. Both types benefit from being elected, but the high type benefits more (or finds advertising less expensive). Thus, the authors' rationale for campaign advertising is that good candidates have more to gain than bad candidates from being perceived as good. On the contrary, we take the agnostic viewpoint that candidates of different types benefit equally from election and face the same cost of advertising.

Gerber [15] argues that campaign advertising conveys information because it reveals the insider signals of groups. Thus, the rationale is similar to our model. However, in the separating equilibrium described by Gerber, both a group with a good candidate and a group with a bad candidate are indifferent between contributing or not contributing (we discuss this problem in Section 3 after Proposition 1.). Thus, a separating equilibrium exists only when exogenous reasons guarantee that groups with good candidates contribute and groups with bad candidates do not. On the contrary, in our separating equilibrium a group with a good candidate has a strictly higher incentive to contribute than a group with a bad candidate.[6]

Two recent papers do not tackle campaign advertising but are closely related to the present work. Grossman and Helpman [16] study political endorsements with rational voters.[7] Lohmann [23] analyzes a model of retrospective voting in which a minority of voters is (endogenously) better informed than the majority. Both these two papers and the present paper show that in equilibrium candidates choose policy positions that are biased away from the median voter. This policy bias occurs despite the fact that voters can, at least partially, observe policy positions. The reason is that a minority of voters enjoy an informational advantage and use it to extract rent from candidates in the form of favorable policies. These models are in line with the emphasis that observers of interest groups politics put on the monitoring role of groups.

[6]There are two additional differences between the present model on one side and Potters, Sloof, and Van Winden [30] and Gerber [15] on the other. First, the present model is embedded in the usual spatial competition model, while the other two rely on ad hoc assumptions. Second, the present model reaches general conclusions about voter welfare.

[7]In the present work, cheap-talk endorsements are never credible. This is because the insider signal is on valence and the group derives no direct utility from valence.

2 Model

2.1 Political Dimensions and Voters

A continuum of voters indexed with $i \in I$ must elect one of two candidates, indexed with $j \in \{1, 2\}$. The possibility of abstension is disregarded.

Each candidate is represented along two dimensions: his policy position $p_j \in \Re$ and his valence $\theta_j \in \Theta \subset \Re$. The policy dimension can be interpreted both as ideological view (position on the left-right line) or as policy stance (e.g. position on the issue of subsidies to milk producers). The valence dimension captures a set of characteristics of the candidate that are unambiguously good for voters.

Voter i is described by his preferred policy p_i, which is strictly increasing in i. Let $e \in \{1, 2\}$ denote the candidate who wins the election. The utility function of Voter i is

$$u_i(\theta_e, p_e) = \theta_e - u(p_i - p_e)$$

where $u(\cdot)$ is continuous, symmetric, and strictly increasing in $|p_i - p_e|$. If Voter i knew θ_1 and θ_2, he would vote for candidate 1 if and only if

$$\theta_1 - \theta_2 \geq u(p_i - p_1) - u(p_i - p_2) \geq 0$$

Thus, if the two candidates have identical policy positions, Candidate 1 is elected if and only if he beats Candidate 2 on the valence dimension.

2.2 Information

Voters observe policy positions p_1 and p_2 perfectly.[8] However, they cannot observe valences θ_1 and θ_2 directly. θ_1 and θ_2 are independent random variables, each of which has prior distribution $\phi(\cdot)$ defined on Θ. Priors are common knowledge.

Three signals about the valence of Candidate $j \in \{1, 2\}$ are received sequentially. First, all agents (voters, candidates, group) observe a public signal $x_j \in X \subset \Re$, which represents the candidates' historical record (for instance, it may capture the well-documented incumbency advantage). Second, only interest groups observe an insider signal $y_j \in Y \subset \Re$ which

[8]Section 5.1 will show that results do not change dramatically if voters cannot observe the policy dimension because, in equilibrium, they can infer it perfectly.

can be thought of as impressions, word-of-mouth, unproven allegations, etc.[9] The insider signal is non-verifiable. Third, all agents observe a public signal $z_j \in Z \subset \Re$ that derives from the candidate's performance during the campaign (e.g. pre-electoral TV debates). More complex signal sequences could be accomodated. The results of this paper depend uniquely on the assumption that the last public signal is received after the first insider signal.[10]

The cumulative distributions of x_j, y_j, and z_j given θ_j are, respectively, $F_x(x_j|\theta_j)$, $F_y(y_j|\theta_j)$, and $F_z(z_j|\theta_j)$. $F_x(x_j|\theta_j)$ is strictly increasing in x_j for any $\theta_j \in \Theta$, and similarly for F_y and F_z. The random variables x_1, y_1, and z_1 are assumed to be stochastically independent from x_2, y_2, and z_2. Furthermore,

Assumption 1 *For $j = 1, 2$, x_j, y_j, and z_j are mutually independent given θ_j and satisfy the Monotone Likelihood Ratio Property (MLRP).*[11]

The assumption implies that an increase in any of the three signals translates in an increase in the expected value of the valence.

Let

$$\hat{\theta}(x_j, y_j, z_j) = E(\theta_j | x_j, y_j, z_j)$$

and

$$\check{\theta}(x_j, z_j) = E(\theta_j | x_j, z_j)$$

$\hat{\theta}$ is the expected value of θ_j given the public signals and the insider signal, while $\check{\theta}$ is the expected value given the public signals only. Applying Milgrom [26, Proposition 2], if F_x, F_y, and F_z satisfy MLRP, then $\hat{\theta}(x_j, y_j, z_j)$ is strictly increasing for all $x_j \in X$, $y_j \in Y$, and $z_j \in Z$. Similarly for $\check{\theta}(x_j, z_j)$.

[9] It is assumed that candidates do not observe the insider signal. This assumption avoids the possibility – studied in a political cycle model by Rogoff [31] – that candidates signal their good type by adopting bad policies (choosing a position far away from the median voter is a costly signal). While this is an interesting question, it lies outside the scope of this paper. Of course, in a separating equilibrium candidates infer the insider signal from the group's offer.

[10] Indeed, the presence of x_j in this model is not necessary for most results.

[11] A cumulative distribution function $F(\cdot|\cdot)$ satisfies MLRP if its p.d.f. $f(\cdot|\cdot)$ is such that, for every $s' > s$ and every $t' > t$

$$\frac{f(s'|t')}{f(s'|t)} > \frac{f(s|t')}{f(s|t)}$$

A simplifying assumption we make throughout the paper is that y_j is a binary signal: $Y = \{0, 1\}$. the insider signal is either good or bad. This assumption will allow us to characterize the incentive-compatibility constraints in a simple way.

The assumption that the interest group has information on candidates that voters do not have seems to correspond to reality. Lehman Schlozman and Tierney [32, Chapter 10] describe how Washington lobbies decide whether and how much they should contribute to a candidate. Before deciding, a typical interest group would collect all kind of intelligence – formally and informally – about the prospective beneficiary. If the group is considering a large contribution, the candidate will usually meet face to face with a group representative. Moreover, lobbyists have frequent opportunity of exchanging nonverifiable information (political gossip) with government officials, journalists, and other insiders.

2.3 Voters' Choice

Let $\tilde{\theta}_j \in \Re$ represent the expected value of θ_j, conditional on the voters' information at the moment of the vote.[12] Given $\tilde{\theta}_j$, Voter i votes for Candidate 1 if and only if

$$\tilde{\theta}_1 - \tilde{\theta}_2 - u(p_i - p_1) + u(p_i - p_2) \geq 0$$

Let m be the median voter's ideal policy: $m \in \Re$ is the unique solution to $\int_{p_i<m} di = \int_{p_i>m} di$. The proof of the following is immediate:

Lemma 1 *Candidate 1 is elected if and only if*

$$\tilde{\theta}_1 - \tilde{\theta}_2 - u(m - p_1) + u(m - p_2) \geq 0$$

A candidate is elected if and only if he is preferred by the median voter. The median voter evaluates candidates on how high their expected valences are and on how close to the median voter's ideal position their policies are.

2.4 Candidates

The only goal of a candidate is to win the election. He derives no direct utility from policy or valence. While his valence is given, Candidate j chooses his policy position p_j, which is publicly observable.

[12] This paper abstracts from the problem of the heterogeneity of information among voters and its aggregation. All voters observe the same signals and hold the same beliefs. Section 6 discusses this assumption.

Consider the policy choice of Candidate j. Given Lemma 1, for any voters' belief on valence and for any policy chosen by the other candidate $(-j)$, j maximizes his election chances by choosing $p_j = m$:

Lemma 2 *For any distribution of probability over $\tilde{\theta}_j$ and $\tilde{\theta}_{-j}$ and for any p_{-j}, $p_j = m$ is a best response.*

Lacking any other influence, both candidates should choose the median voter's ideal policy.

2.5 Interest Group

An interest group leader acts as the representative of a subset of the voters regarding the policy dimension. The subset has mass μ and median member $g > m$. The group leader, G, maximizes the policy component of the utility of the median group member. The interest group is therefore not directly interested in the valence of candidates.[13] G can make contributions to candidates 1 and 2, denoted respectively with A_1 and A_2. The group's payoff is assumed to be separable in contributions and policy. The payoff to G if e is elected is $-\mu u(g - p_e) - A_1 - A_2$.[14]

G announces a desired policy p^* and then she can make an offer A_1^* to 1 and an offer A_2^* to 2. Each candidate can accept or reject the offer. If he accepts, he receives a campaign contribution $A_j = A_j^*$ and commits to implementing p^* if elected. If he rejects, he receives $A_j = 0$ but he is free of choosing any policy position. If the candidate accepts the contribution, he can use it for not directly informative campaign advertising.[15]

[13] If the group represents a subset of voters, one may think it should care about both policy and valence. However, there are two reasons to believe that the group should be more concerned about policy. The first is that there can exist an agency problem between the group members and the group leader. Suppose that, while outcomes on the policy dimension can be contracted upon, outcomes on the valence dimension are hard to measure and to verify. Then, the group leader only has an incentive to perform on the policy line. The second reason has to do with the free-riding problem. If voters have identical preferences over valence but disagree over policy, one can expect that subsets of policy-homogeneous voters will have more incentives to pool resources to influence policy rather than to enhance valence.

[14] A_1 and A_2 do not enter the other players' utilities. Thus, the assumption that they enter G's utility function in a linear way and with unitary coefficients is without loss of generality.

[15] This model assumes that a candidate can credibly commit to implement p^* if elected. It is mostly an open question – outside the scope of this paper – why the candidate should live up to its pre-electoral promises to interest groups (See however Austen-Smith [3] for self-enforcing agreements in which the candidate credibly promises his group 'access' to the policy-making process in exchange for a contribution).

2.6 Campaign Advertising

As campaign advertising occupies a central role in the analysis, it is worth spelling out all the assumptions that are made about it. As we discussed in the Introduction, there are three main economic theories of advertising: (1) Viewers are not rational, or, equivalently, advertising modifies preferences (Dixit and Norman [11] or Becker and Murphy [7]); (2) Viewers are rational and advertising provides direct, verifiable information (Such as Butters [8] – See Tirole [33, Chapter 2] for a survey); (3) Viewers are rational but advertising only provides indirect, nonverifiable information (Kihlstrom and Riordan [20] or Milgrom and Roberts [27]). The first theory in unsatisfactory because it does not allow for welfare comparisons. The second theory does not seem appropriate because the experimental evidence collected by Ansolabehere and Iyengar [1] indicates that campaign advertising is highly effective even when it is devoid of anydirect informational content.

This paper uses the third theory. The idea is that advertising is not important because of the direct message that it conveys but because of the money that is spent on it. Advertising is an expensive signal, readily observed by all viewers. The fact that the advertiser is able and willing to spend money on advertising signals something to viewers.

The theory of non-directly informative advertising is, however, quite abstract. To let the reader understand that it can indeed describe campaign advertising, we now present an 'almost realistic story' that fits this theory. The results of this paper apply – but are not restricted to – this particular story.

There exists one for-profit TV station that all voters watch. Candidates can use the TV station in two ways: they can communicate information to the news for free and they can buy time for commercials at a constant unitary rate.[16]

It is crucial to distinguish between verifiable information and nonverifiable information. If a candidate has information that is verifiable – i.e. facts supported by hard evidence – then he can communicate it to the news service who will take care of broadcasting it to voters. Clearly, each candidate will communicate all the information that is to his favor or his opponent's detriment. This guarantees that all relevant verifiable information is communicated through the news service. Without loss of generality, we can include all the verifiable information in the public signals x_1 and x_2 and focus on the nonverifiable

[16] In reality, candidates use other forms of advertising besides TV commercials, such as: direct mailing, newspaper advertising, or leaflet distribution. However, the bulk of advertising budgets goes to TV. For instance, in the 1992 US presidential elections, Clinton and Bush spent respectively 2/3 and 3/4 of their budgets on television ads (Source: Kaid and Holtz-Bacha [19]).

information captured by y_1 and y_2.

The communication of nonverifiable information is more complex. Clearly, it cannot occur through newscast (because it would not be credible and it would expose the TV station to the risk of lawsuits). However, it could occur – indirectly – through paid commercials. Following Milgrom and Roberts [27], we do not attempt to model the psychological process through which viewers are influenced by advertising. The content of the ad is not important as long as it identifies the candidate who is paying for it. We can assume that ads only mention the name of the candidate and some vague, nonlegally-binding statements about him (like the ads used in the experiment by Ansolabehere and Iyengar [1]).

Voters are affected by the number of ads they see (. In particular, the beliefs of voters on the valences of the two candidates, $\tilde{\theta}_1$ and $\tilde{\theta}_2$, are functions of the number of TV ads bought by the two candidates. The crucial assumption that we make with regard to advertising is that voters are not systematically fooled. This assumption pins down the model by requiring beliefs to be consistent in equilibrium. Thus, in a separating equilibrium,

$$\tilde{\theta}_j = \hat{\theta}(\theta_j | x_j, y_j, z_j).$$

This of course does not imply that candidates *cannot* use advertising to fool voters. Indeed, a low-quality candidate could always pass for a high-quality candidate if he spends enough money. Likewise, a high-quality candidate would be mistaken for a low-quality one if he is stingy. However – as we will see – in equilibrium, the incentive is there for the high-quality candidate to spend more than the low quality candidate.[17]

3 When Only One Candidate Can Receive Contributions

This model takes into account interactions on three levels: (a) How a candidate influences voters' beliefs through advertising, (b) How the group exchanges contributions for favorable policies, and (c) How candidates compete with each other for contributions. The general

[17] This model can be criticized on the ground that advertising amounts to money burning, and money could be 'burnt' in better ways than by giving it to TV stations. For instance, the candidate could make a donation to a charity. However, for the signalling to work, the act of money burning must be publicly observable and must not bring direct benefits to the burner. Clearly, nothing is better observable than TV ads. Also, other forms of money burning may bring unobserved benefit to the burner. For instance, a charitable donation can lead voters to suspect some exchange of favors or some other hidden deal between the candidate and the charity.

model is quite complex. For illustrative purposes, it is useful to fully explore (a) and (b) before including (c). This section makes the temporary assumption that the group can only contribute to Candidate 1 and that there is no uncertainty about the valence of Candidate 2.

Assumption 2 *(i) G can only make offers to Candidate 1 and (ii)* $\theta_2 \equiv 0$.

Assumption 2 is maintained throughout this section and will be dropped in Section 4.

To summarize, the electoral race is represented as follows:

Game 1 *The players are: voter* $i \in I$, *candidate* $j \in \{1, 2\}$, *and interest group* G. *The game consists of four stages:*

1. *Nature: Nature chooses* $\theta_1 \in \Theta$, *which remains unknown to all players.* $\theta_2 \equiv 0$. $x_1 \in X$ *is realized and becomes common knowledge among all players.*

2. *Insider Stage:* G *observes* $y_1 \in \{0, 1\}$, *selects* p^*, *and offers* A_1^* *to 1. 1 accepts or rejects. If he rejects, then he makes advertising expenditure* $A_1 = 0$ *but he is free to set* p_1. *If he accepts, then* $A_1 = A_1^*$ *and* $p_1 = p^*$. $A_2 \equiv 0$ *and 2 is free to set* p_2.[18]

3. *Public Stage:* $z_1 \in Z$ *is realized. Voters observe* p_1, p_2, A_1, *and* z_1. *For* $i \in I$, *Voter* i *votes for either* 1 *or* 2. *Let* e *denote the candidate that receives the higher number of votes and let* $-e$ *denote the other candidate.*

4. *Payoff Distribution:* θ_1 *is revealed. Voter* i *receives* $\theta_e - u(p_e - p_i)$. e *receives 1 and* $-e$ *receives 0.* G *receives* $u(g - p_e) - A_1$.

The players' action sets are: $p^* \in \Re$ *and* $A_1^* \in [0, \infty)$ *for* G*; {"accept", "reject"} and* $p_1 \in \Re$ *(if "reject") for* 1*;* $p_2 \in \Re$ *for 2; and* $e_i \in \{1, 2\}$ *for* i.

[18] Here we assume that G makes offers in the time window after she observes y_j but before she observes z_j. If she made offers before or after this time window, it can be shown that advertising is never credible. Therefore, even if G is free to choose when to make a contribution, she will only make it in the 'credible' time window. With a more complex information structure (for instance, one with a continuous sequence of both public and insider signals) the credible time window could cover the whole campaign.

3.1 Equilibrium under Advertising Ban

As a benchmark, consider the case in which advertising (or campaign giving) is prohibited by law ($A_1 \equiv 0$). By Lemma 2, candidates will set $p_1 = p_2 = m$. By Lemma 1, Candidate 1 will be elected if and only if x and z are such that $\check{\theta}(x, z) \geq 0$. Let $\bar{z}^P(x)$ be the unique solution to $\check{\theta}(x, \bar{z}^P(x)) = 0$ for all $x \in X$. $\bar{z}^P$ is strictly decreasing in x. It is immediate to see that under an advertising ban $p_j = m$ for $j = 1, 2$ and $e = 1$ if and only if x and z are such that $z > \bar{z}^P(x)$.

Under an advertising ban, candidates cannot do anything to influence the voters' beliefs over the valence dimension. Their optimal strategy is to cater to the median voter on the policy dimension.

3.2 Equilibrium with Advertising

In a separating equilibrium, campaign advertising fully reveals y. This section proves existence of a separating equilibrium. Before stating the main results, some notation must be introduced.

Let $\bar{z}(x_1, y_1, p_1)$ denote the unique value of z_1 for which

$$\hat{\theta}(x_1, y_1, z_1) + u(m - p_1) - u(0) = 0$$

If we suppose that x_1 has been realized, that voters believe y_1, that 1 chooses p_1, and that 2 chooses $p_2 = m$ (a consequence of Lemma 2), then $\bar{z}(x_1, y_1, p_1)$ represents the lowest realization of z_1 at which 1 is elected. $\bar{z}$ is strictly decreasing in x_1, z_1 and strictly increasing in p_1 for $p_1 > m$.[19]

If both candidates choose m, G's payoff is certainly $-\mu u(g - m)$. Let

$$\Pi_y(p^*) = \mu \Pr\left[\hat{\theta}(x, 1, z) - u(p^* - m) - u(0) \geq 0 \middle| x, y\right] \left[u(g - m) - u(g - p^*)\right] \qquad (1)$$

$\Pi_y(p)$ is the expected payoff of G net of $-\mu u(g - m)$, given that 1 has accepted policy p^*. $\Pi_y(p)$ is gross of the contribution A_i^*, which will be determined shortly. To avoid confusion between candidate subscripts and realizations of y, let $\Pi_H = \Pi_1$ and $\Pi_L = \Pi_0$ as in 'high type' and 'low type'.

$\Pi_H(p^*)$ represents the gross expected payoff for G if her candidate is a high-type and voters believe her candidate is a high type. On the other hand, $\Pi_L(p^*)$ represents the gross expected payoff for G if her candidate is a low-type and voters believe her candidate is a high type.

[19] As we have assumed that $g > m$, it is obvious that 1 will never choose $p_1 < m$.

Proposition 1 *Let $\bar{p}$ be the largest p such that*

$$\bar{z}(x_1, 1, p) \leq \bar{z}(x_1, 0, m).$$

There exists a sequential equilibrium of Game 1 as follows:

(i) Voters' beliefs: For any $p_1 \in (0, \infty)$

$$\tilde{\theta} = \begin{cases} \hat{\theta}(x, 1, z) & \text{if } A_1 \geq \Pi_L(p^*) > 0 \\ \hat{\theta}(x, 0, z) & \text{otherwise} \end{cases}$$

(ii) Voters' choice: $e_i = 1$ if and only if $\tilde{\theta} - u(p_i - p_1) + u(p_i - p_2) > 0$. 1 is elected if and only if $\tilde{\theta} - u(m - p_1) + u(m - p_2) > 0$

(iii) Group's offer: G offers $p^* = p_{\max}$ and

$$A_1^* = \begin{cases} \Pi_L(p_{\max}) & \text{if } y_1 = 1 \\ 0 & \text{if } y_1 = 0 \end{cases},$$

where

$$p_{\max} = \mathit{argmax}_p \Pi_H(p) - \Pi_L(p)$$

subject to $p^* \leq \bar{p}$.

(iv) Candidate 1 accepts A_1^* if and only if $A_1^* \geq \Pi_L(p^*)$ and $p^* \leq \bar{p}$. If he rejects, $p_1 = m$.

(v) Candidate 2 sets $p_2 = m$.

Let us discuss Proposition 1 by examining the equilibrium behavior of voters, candidates, and the interest group one at a time.

Voters believe that Candidate 1 has given a high insider signal y_1 only if advertising is above a certain threshold. This threshold is exactly the gross profit of G if the candidate is of low quality. As we shall see shortly, this belief is correct in equilibrium because a high-quality candidate is able to reach the threshold and a low-quality candidate is not. Given their beliefs, voters compute the expected value of Candidate 1's valence as if they knew the insider signal and vote accordingly. The median voter is decisive (By Lemma 1).

Candidate 1 knows that he should either collect enough campaign contributions to reach the threshold and be perceived as a high-quality candidate or he should just collect no contributions and cater to the median voter. Hence, Candidate 1 rejects all contributions below the threshold. If Candidate 1 receives an offer above the threshold, he must still weigh the benefit of being perceived as a high-quality candidate against the cost of deviating from the median voter's ideal policy. This determines the participation constraint $p^* \leq \bar{p}$. The policy $\bar{p}$ makes the candidate indifferent between adopting $\bar{p}$ and being perceived as high-quality or adopting m and being perceived as low-quality. If G asks for a policy to the right of $\bar{p}$ then the candidate rejects the offer. Hence, a candidate accepts G's offer only if the money is above the threshold and the policy is not too extreme.

Candidate 2 does not play an interesting role. As he cannot receive contributions, it is a dominant strategy for him to choose the median voter's ideal policy (See Lemma 2).

The interest group G realizes that Candidate 1 rejects all contributions below the threshold. She has to decide if Candidate 1 is 'worth' the threshold contribution. But the threshold is set to exactly offset the expected benefit of financing a low-quality candidate. Thus, G offers the threshold contribution if and only if she has received a good insider signal about Candidate 1. This closes the equilibrium, because it proves that voters' beliefs are correct: a candidate who spends more than the threshold is a high-quality candidate.

The interest group knows that Candidate 1 rejects policies to the right of $\bar{p}$. She can decide to ask for $\bar{p}$ or anything between m and $\bar{p}$. This depends on whether the maximum of $\Pi_H(p) - \Pi_L(p)$ is to the right or to the left of $\bar{p}$. The more extreme G is, the more likely it is that she will go all the way and ask for $\bar{p}$. Instead, a centrist group may be satisfied with a policy to the left of $\bar{p}$.

The existence of a separating equilibrium is guaranteed by the fact (established in the proof of Proposition 1) that for any $p > m$,

$$\Pi_H(p) > \Pi_L(p). \tag{2}$$

The interest group uses y_1 to forecast z_1. Given θ_1, y_1 and z_1 are independent. However, because θ is unknown, y_1 is positively correlated with z_1. If the interest group receives a high insider signal, she expects that the candidate will produce a high public signal later on. Hence, a candidate with a high insider signal is – everything else equal – more likely to be elected than a candidate with a low insider signal.

Given (2), it is possible to find a threshold A^* such that $\Pi_H(p) > A^* \geq \Pi_L(p)$. Then, the interest group is willing to contribute A^* if and only she has received a high insider signal. This intuition appears to be very robust and goes beyond the particular assumptions made in this section. Sections 4 and 5 will show that a separating equilibrium continues to exist with a variety of different assumptions.

Some remarks are in order:

1. *If we assumed that y_1 is perfectly informative, then Condition (2) would not hold.* Suppose that y_1 is perfectly informative. Then, if voters know y_1, they can infer θ_1. Then, in a separating equilibrium, voters decide based only on y_1 and they do not look at z_1. If voters believe $y_1 = 1$, a group with a high-type candidate has the same expected payoff as a group with a low-type candidate. Thus, Condition (2) does not hold.[20]

2. *Both the case $p_{\max} < \bar{p}$ and the case $p_{\max} = \bar{p}$ are possible.* One may wonder whether it could be the case that Candidate 1's participation constraint is always binding or never binding. However, both cases are possible depending on the functional forms chosen. A numerical example in Section 7.2 proves this point. The participation constraint is not binding when g is close to m. $\bar{p}$ is independent of g. So for any $\bar{p}$, if g is small enough, $p^* < \bar{p}$. On the other hand, the participation constraint is binding when g is high and z_j is more informative than y_j. Hold fixed the precision of y and increase the precision of z: voters become less interested in learning the signal y and Candidate 1 has less to gain from being revealed as $y_1 = 1$ rather than $y_1 = 0$. However, G would still like to have a policy position close to g.

3. *The equilibrium of Proposition 1 is the only sequential equilibrium that survives the Intuitive Criterion.* Signalling games are plagued by a large number of sequential equilibria. The present game makes no exception and one can find several other sequential equilibria besides the one in Proposition 1. In particular, there exists a pooling equilibrium in which voters' beliefs do not depend on advertising and, therefore, Candidate 1 has no reason to advertise. In that equilibrium Candidate 1 chooses the median voter's ideal policy and rejects any offer from G. However, Appendix 7.3 uses the Intuitive Criterion introduced by Cho and Kreps [9] and shows that any pure-strategy sequential equilibrium besides the one in Proposition 1 is based on implausible out-of-equilibrium beliefs. For instance, the reason why the zero-advertising pooling equilibrium is not robust is that there always exists a level of contributions that, if believed, is profitable if the candidate is high-quality and not profitable if the candidate is low-quality. If voters observe such a level they must draw the conclusion that the candidate is high-quality.

[20]Still, when y_1 is perfectly informative, a separating equilibrium can exist. However, this equilibrium is both arbitrary and brittle, as Section 7.5 of the Appendix shows.

3.3 Welfare

The ex-post voter welfare is[21]

$$\begin{aligned} W(e,p_e) &= \int_{i\in I}[\theta_e - u(p_i - p_e)]di \\ &= \theta_e - \int_{i\in I} u(p_i - p_e)di \end{aligned}$$

Does this definition of welfare include the utility of lobby members? Let us assume that lobby members are voters and not, for instance, foreign citizens. The direct utility that lobby members derive from policy and valence is definitely counted in our definition. On the other hand, the disutility that lobby members incur because of the cost of campaign contributions – which G presumably collects from members' dues – is not counted. Hence, our case against campaign advertising does *not* hinge on the argument that campaign advertising is a waste or real resources.

Let us assume that, for $i \in I$, p_i is symmetrically distributed around m. Recall that $u(\cdot)$ is symmetric around p_i. Then, as is well known, W is maximized when the ex-post welfare of the median voter is maximized. Thus, from now on we focus on $w = \theta_e - u(p_i - p_e)$, which can be expressed as a function of e and p_1:

$$w(e,p_1) = \begin{cases} \theta_1 - u(m - p_1) & \text{if } e = 1 \\ -u(0) & \text{if } e = 2 \end{cases}$$

In the separating equilibrium of Proposition 1,

$$w(e,p_1) = w_S(e,y_1,p^*) = \begin{cases} \theta_1 - u(m - p^*) & \text{if } e = 1 \text{ and } y_1 = 1 \\ -u(0) & \text{otherwise} \end{cases} \tag{3}$$

while in the equilibrium under advertising ban,

$$w(e,p_1) = w_P(e) = \begin{cases} \theta_1 - u(0) & \text{if } e = 1 \\ -u(0) & \text{if } e = 2 \end{cases}$$

Let the expected voter welfare be the expected payoff for m after x is realized but before y and z are observed.[22] Under a separating equilibrium, the expected voter welfare is $\bar{w}_S(p^*) = E_\theta(w_S(e,y_1,p^*)|x)$, while under the advertising ban, $\bar{w}_P = E_\theta(w_P(e)|x)$. Let us first consider the case in which the Participation Constraint binds, that is $p^* = \bar{p}$.

[21] As G is made of a subset of voters, the payoff of group members is already included in the voter welfare.

[22] The welfare analysis holds a fortiori if the expected voter welfare is defined as the expected payoff *before* x is realized.

Lemma 3 *if* $p^* = \bar{p}$*, the separating equilibrium yields the same voting outcome as an equilibrium under advertising ban in which voters use the rule*

$$e = 1 \Leftrightarrow \check{\theta}(x, z) \geq b(x)$$

where $b(x) > 0$ *for all* $x \in X$*, instead of using the rule*

$$e = 1 \Leftrightarrow \check{\theta}(x, z) \geq 0$$

Thus, $\bar{w}_S(\bar{p}) < \bar{w}_P$*.*

Lemma 3 examines the case in which Candidate 1's participation constraint is binding. In that case, the expected voter welfare under an advertising ban is strictly higher than the expected voter welfare with advertising. What is the intuition behind this result?

Given the definition of $\bar{p}$, if the participation constraint is binding, Candidate 1 is exactly indifferent between: (a) being revealed as $y_1 = 1$ and choosing p^*; or (b) being revealed as $y_0 = 0$ and choosing m. He is indifferent because he is elected under (a) if and only if he is elected under (b). If the candidate were more likely to be elected under (a) than under (b), the participation constraint would not be binding, while if he were more likely to be elected under (b) than under (a), he would reject G's offer. In equilibrium, if the insider signal is good, case (a) occurs, while, if the insider signal is bad, case (b) occurs. Notice however that case (b) is equivalent to the following scenario: G does not exist; voters observe the insider signal directly; the insider signal happens to be bad. As Candidate 1 is elected under (a) if and only if he is elected under (b), voters behave as if they always had a 'bad' Candidate 1.

In general G uses its insider signal to extract rent from voters in the form of biased policy. If the participation constraint is binding, it means that G has pushed the policy bias to the point at which, in the eyes of voters, there is no difference between a good candidate with a biased policy and a bad candidate with an unbiased policy. Thus, it is as if voters encountered only bad candidates with unbiased policies. However, in an equilibrium under advertising ban, voters encounter only 'average' candidates (that is, candidates for whom the insider signal can be good or bad). Under an advertising ban, candidates always adopt an unbiased policy. Thus, voters are strictly better off under an advertising ban than under a separating equilibrium.

Let us now characterize the general case in which the participation constraint may or may not be binding:

Proposition 2 *for any* $x \in X$*, there exists a* $k \in (0, \bar{p})$ *such that* $\bar{w}_S(p_{\max}) < \bar{w}_P$ *when* $p_{\max} \in (k, \infty)$*,* $\bar{w}_S(p_{\max}) = \bar{w}_P$ *when* $p_{\max} = k$*, and* $\bar{w}_S(p_{\max}) > \bar{w}_P$ *when* $p_{\max} \in (0, k)$*, .*

If the participation constraint is not binding, G still extracts all the rent she can extract, but, in doing so, she leaves some informational rent to voters. Now it is not anymore as if voters encountered only bad candidates with unbiased policies. By the fact that the expected voter welfare is continuous and strictly decreasing in p^*, there exists a policy $k \in (0, \bar{p})$ such that if $p^* = k$ voters are indifferent between prohibiting advertising and allowing advertising. If the goal of G or the information structure is such that $p^* > k$, voters would like advertising to be banned. If, on the other hand, $p^* < k$, advertising is beneficial.

4 When Both Candidates Can Receive Contributions

The previous section relied on the assumptions that the valence of Candidate 2 is known and that G can only contribute to Candidate 1. This section removes both assumptions.

4.1 Modifications to the Model

Assumption 2 is substituted with

Assumption 3 *G can make an offer to each candidate. Offers are simultaneous and secret.*

G can try to win the favors of both candidates. The assumption that offers are simultaneous excludes the possibility that G makes an offer to one candidate, waits for his reply, and then makes an offer to the other candidate. While it may be more realistic, this possibility is outside the scope of this paper. The assumption that offers are secret means that A_j^* is not observed by Candidate $-j$. This is to avoid the possibility that G could pre-commit to financing only one of the two candidates.

Let us make the following simplifying assumptions on the primitives of the model:

Assumption 4 *(i)* $x_1 \equiv x_2 \equiv 0$*; (ii)* $\phi(\theta_j)$ *is symmetric around the mean; (iv) voters do not observe* z_1 *and* z_2 *but only* $z = z_1 - z_2$*; (v) G does not observe* y_1 *and* y_2 *but only* $y = y_1 - y_2$*.*

Part (i) of the assumption eliminates the incumbent advantage and makes candidates equal until the insider signal is observed. Parts (ii) guarantees the symmetry of the problem. Parts (iv) and (v) assume that voters and groups can only observe the differences between signals and not the absolute value of signals. Although Assumption 4 does not appear to be central to the results that are going to be presented, it is useful because it leads to a simple characterization of the participation constraints for candidates.

With Assumption 4, it is possible to rewrite the problem in terms of differences rather than absolute values. The domains of y and z are respectively $\tilde{Y} = \{-1, 0, 1\}$ and $\tilde{Z} = \{z_1 - z_2 | \forall z_1 \in Z, \forall z_2 \in Z\}$.

Taking into account Assumptions 3, Assumptions 4, and the new definitions, Game 1 becomes:

Game 2 *The players are: voter $i \in I$, candidate $j \in \{1, 2\}$, and interest group G. The game consists of four stages:*

1. *Nature: Nature chooses $\theta_1 \in \Theta$ and $\theta_2 \in \Theta$, which remain unknown to all players.*

2. *Insider Stage: G observes $y \in \{-1, 0, 1\}$; announces p^*; and makes offers $A_1^* \geq 0$ and $A_2^* \geq 0$. Candidate j does not observe A_{-j}^*. If candidate j accepts the offer, then he must set $p_j = p^*$. If he rejects, he is free to decide p_j.*

3. *Public Stage: $z \in \tilde{Z}$ is realized. Voters observe p_1, p_2, A_1, A_2, and z. For $i \in I$, Voter i votes for either 1 or 2. Let e denote the candidate that receives the higher number of votes.*

4. *Payoff Distribution: θ_1 and θ_2 are revealed. Voter i receives $\theta_e - u(p_e - p_1)$. e receives 1 and $-e$ receives 0. G receives $-u(g - p_e) - A_1 - A_2$.*

The players' action sets are: For G, $p^* \in \Re$ and $A_j^* \in [0, \infty)$ with $j = 1, 2$; For Candidate $j \in \{1, 2\}$, {"accept", "reject"} and $p_j \in \Re$ (if "reject"); For Voter $i \in I$, $e_i \in \{1, 2\}$.

Let H indicate Candidate 1 and L indicate Candidate 2 if $y = 1$ and viceversa if $y = -1$. Let M (as in 'medium quality') indicate both candidates if $y = 0$.

4.2 Equilibrium

Let us redefine $\hat{\theta}$ as

$$\hat{\theta}(y,z) = E(\theta_1 - \theta_2 | y, z)$$

$\hat{\theta}(y,z)$ is strictly increasing in y and z. Notice that, as the voters' utility is linear in valence, voters gain nothing by basing their decisions on both $E(\theta_1|y,z)$ and $E(\theta_2|y,z)$ rather than only on $E(\theta_1 - \theta_2|y,z)$.

As in the previous section, let the gross expected profit of the interest be Π_y. However, now four possibilities turn out to be of interest: Π_H (G has a deal with only one candidate, who is high-quality and is perceived as high-quality); Π_M (G has a deal with only one candidate, who is medium-quality and is perceived as high-quality); Π_L (G has a deal with only one candidate, who is low-quality and is perceived as high-quality); Π_{all} (G has a deal with both candidates – and it is irrelevant what quality the two candidates are and are perceived). Formally,

$$\Pi_y(p) = \mu \Pr\left[\hat{\theta}(1,z) - u(p^* - m) - u(0) \geq 0 \middle| y\right][u(g-m) - u(g-p^*)]$$

and $\Pi_H(p^*) = \Pi_1(p^*)$, $\Pi_M(p^*) = \Pi_0(p^*)$, and $\Pi_L(p^*) = \Pi_{-1}(p^*)$. Also,

$$\Pi_{\text{all}}(p^*) = \mu[u(g-m) - u(g-p^*)$$

$\Pi_{\text{all}}(p^*)$ does not depend on the quality of candidates because, if G has a deal with both candidates, then p^* is implemented for sure independently of who wins the election.

Let $\underline{z}(y, p_1, p_2)$ be the unique solution to

$$\hat{\theta}(y,z) - u(m-p_1) + u(m-p_2) = 0$$

It is easy to see that $\underline{z}$ is continuous and increasing in $-y$, p_1, $-p_2$. Given y, p_1, $-p_2$, $\underline{z}$ is the minimal z at which Candidate 1 wins. Candidate 1 minimizes z while Candidate 2 maximizes it.

Proposition 3 *Let $\underline{p}$ be the unique p for which $\underline{z}(1,p,m) = 0$.*[23]*. For any $p \in \Re$, let $\alpha(p) = \Pi_{all}(p) - \Pi_H(p)$. There exists a sequential equilibrium of Game 2 as follows:*

(i) Voters' beliefs:

$$\tilde{\theta} = \begin{cases} \hat{\theta}(1,z) & \textit{if } A_1 \geq \alpha(p^*) \textit{ and } A_2 < \alpha(p^*) \\ \hat{\theta}(0,z) & \textit{if } \max(A_1,A_2) < \alpha(p^*) \textit{ or } \min(A_1,A_2) \geq \alpha(p^*) \\ \hat{\theta}(-1,z) & \textit{if } A_1 < \alpha(p^*) \textit{ and } A_2 \geq \alpha(p^*) \end{cases}$$

[23] $\underline{p}$ may be higher than, equal to, or lower than $\bar{p}$

(ii) *Voters' choice:* $e_i = 1$ *if and only if* $\tilde{\theta} - u(p_i - p_1) + u(p_i - p_2) > 0$. *1 is elected if and only if* $\tilde{\theta} - u(m - p_1) + u(m - p_2) > 0$.

(iii) *Group's offer: For* $y \in \{-1, 0, 1\}$, $p^* = \underline{p}_{\max}$, *and*

(a) *If* $y = -1$, $A_1^* = 0$ *and* $A_2^* = \alpha(\underline{p}_{\max})$;

(b) *If* $y = 0$, $A_1^* = A_2^* = \alpha(\underline{p}_{\max})$;

(c) *If* $y = 1$, $A_1^* = \alpha(\underline{p}_{\max})$ *and* $A_2^* = 0$.

where

$$\underline{p}_{\max} = argmax_p 2\Pi_H(p) - \Pi_{all}(p),$$

subject to $p \leq \underline{p}$.

(iv) *Candidates' acceptance: for* $j = 1, 2$, *Candidate* j *accepts* A_j^* *if and only if* $A_j^* \geq \alpha(p^*)$ *and* $p^* \leq \underline{p}$. *If* j *rejects,* $p_j = m$.

Voters' beliefs are simple. There exists an advertising threshold $\alpha(p^*)$. If one candidate reaches the threshold and the other one does not, voters think that the former is better than the latter. If both candidates reach the threshold, voters think that they are equally good.

Candidate j accepts G's offer only if the offer is above the threshold. If it is below, clearly, the offer is of no use. If it is above, the candidate accepts it only if, in addition, the policy requested satisfies the participation constraint $p^* \leq \underline{p}$. An important 'trick' is that the participation constraint is independent of the value of y. This is due to Assumption 4. In a more general case, the participation constraint would depend on y, and the equilibrium would be much more complex.

Interest group G may face two cases: $y \neq 0$ or $y = 0$. In the first case, the threshold $\alpha(p^*)$ is set at exactly the level that makes G indifferent between contributing to both candidates or contributing only to the better one. In equilibrium, she contributes only to the better candidate. Instead, if $y = 0$, the expected profit of contributing to only one candidate is strictly lower than the expexted profit of contributing to both and G contributes to both.

Two remarks are in order:

1. The equilibrium of Proposition 3 is not the only separating equilibrium in the two-candidate case. There are two ways in which a fully separating equilibrium is achieved

and they differ in the case $y = 0$. When the two candidates are equal, G may make deals with both candidates or with none of them. The former case occurs in Proposition 3 and can be labelled the *Split-Contribution Equilibrium* (a split contribution is when an interest group contributes to both candidates). The latter case is possible as well and is called the *No-Split-Contribution Equilibrium*.

The No-Split-Contribution Equilibrium is analyzed in Section 7.4. It is robust to the Intuitive Criterion. However, we argue that a no-split-contribution equilibrium is defeated (in the sense of Mailath, Okuno-Fujiwara, and Postlewaite [25]) by a split-contribution equilibrium. We show that, for any y, the interest group has a higher profit under the Split-Contribution Equilibrium. As the interest group is the Sender in this signalling game, we can expect the Split-Contribution Equilibrium to be focal.

2. One may wonder if there could be a separating equilibrium in which, if $y \neq 0$, G makes an infinitesimal contribution to H and a zero-contribution to L. This equilibrium could be seen as an endorsement à la Grossman and Helpman [16]. If G could commit to contribute to exactly one candidate, such equilibrium would indeed exist. However, this possibility is excluded in the present model by the assumption that offers are secret. G has no way of committing to contribute to only one candidate. The only way to ensure that G does not make two contributions when $y \neq 0$ is the respect of the incentive-compatibility constraint $A_j^* \geq \alpha(p^*)$.

4.3 Split Contributions

It is worth spelling out the following:

Corollary 1 *In the Split-Contribution Equilibrium, if $y = 0$, G offers a contribution to both candidates, both candidates accept, and $\underline{p}_{\max}$ is implemented for sure. If Candidate j rejected the contribution, voters would believe that j is low-quality and $-j$ is high-quality.*

When $y = 0$, G has an implicit threat against both candidates. If one of the candidates rejects the contribution, only the other candidate will advertise and voters will perceive the candidate who advertises as H and the candidate who does not advertise as L.[24]

[24]In the present model, when $y = 0$, candidates are indifferent between the situation in which both advertise and the hypothetical situation in which neither advertises. However, we could assume that each candidate derives a small, but positive, utility from catering to voters. Then, candidates would strictly prefer the situation in which neither advertise to the situation in which both advertise. However, in

Split contributions are a tool G uses to extract rent from voters. Would a ban on split contributions (assuming that it is feasible) be optimal from the point of view of voters?

Proposition 4 *A ban on split contributions always increases the ex-ante voter welfare.*

If split contributions are banned, full revelation of y will still occur but, when $y = 0$, neither candidate will advertise and candidates will select m rather than $\underline{p}$. Voters will be better off and G will be worse off. Thus, a ban on split contributions always increases the voter welfare.

4.4 Voter Welfare

What happens to voter welfare if advertising is banned altogether? The answer is analogous to the answer in the case in which G can only contribute to Candidate 1. A formal statement is superfluous. If the participation constraint of the candidates is binding, that is if $\underline{p}_{\max} = \underline{p}$, then by an argument analogous to Lemma 3, a ban on advertising certainly increases voter welfare. If the participation constraint is not binding, then a ban on advertising may or may ot be optimal according to how close $\underline{p}$ is to m.

5 Discussion and Extensions

This section tackles some important aspects of campaign advertising that were disregarded in the previous sections. For ease of exposition, we refer to the simpler model of Section 3.

5.1 Unobservable Policy Choice

The model has assumed that, before the election, voters observe policies p_1 and p_2 that candidates are going to adopt if elected. Suppose on the contrary that p_1 and p_2 are unobservable. For the rest, let us consider an electoral race as in Game 1.

There is, however, one problem with a model with unobservable policy and office-seeking candidates. Unless the candidate has a deal with the interest group, he is perfectly indifferent among policy positions. To sidestep this indeterminacy, let us assume that candidates pursue two goals: election and the maximization of the median voter's welfare.

equilibrium, it would still be a dominant strategy for candidate j to accept G's offer. This equilibrium is Pareto-inefficient from the point of view of candidates. If candidates could commit not to accept contributions, they would be better off.

But the second goal is infinitely less important that the second. Thus, the ex-post utility of Candidate j is 0 if he is not elected and $1 - k(p_j - m)^2$ if he is elected, where k is a strictly positive parameter. The main result of this section deals with the case in which k tends to zero.[25]

In the previous section voters had beliefs only on valence. Now that also policy is unobserved, voters' beliefs relate to both valence θ and policy p. Hence, let $\tilde{p}$ indicate beliefs on the policy adopted by candidate 1.

Proposition 5 *Let $k \to 0^+$. If $\bar{p} > g$, there exists a separating equilibrium as follows:*

(i) Voters' beliefs:

(a) If $A^* = 0$, then $\tilde{p} = m$ and $\tilde{\theta} = \hat{\theta}(x, 0, z)$;

(b) If $0 < A^* < \Pi_L(g)$, then $\tilde{p} = g$ and $\tilde{\theta} = \hat{\theta}(x, 0, z)$;

(c) If $A^* \geq \Pi_L(g)$, then $\tilde{p} = g$ and $\tilde{\theta} = \hat{\theta}(x, 1, z)$.

(ii) Voters' choice: $e_i = 1$ if and only if $\tilde{\theta} - u(p_i - p_1) + u(p_i - p_2) > 0$. 1 is elected if and only if $\tilde{\theta} - u(m - p_1) + u(m - p_2) > 0$

(iii) Group's offer: G offers $p^* = g$ and

$$A_1^* = \begin{cases} \Pi_L(g) & \text{if } y_1 = 1 \\ 0 & \text{if } y_1 = 0 \end{cases}.$$

(iv) Candidate 1 accepts A_1^* if and only if $A_1^* \geq \Pi_L(g)$. If he rejects, $p_1 = m$.

(v) Candidate 2 sets $p_2 = m$.

If $\bar{p} < g$, there exists no separating equilibrium.

With unobservable policy, G takes all the advantage she can from the candidate by asking for her ideal policy g. Voters realize that a candidate who advertises is going to implement g. Thus, even if voters do not observe p_1, they can anticipate it perfectly. If the candidate advertises, $p_1 = g$. If he does not advertise, $p_1 = m$ (because of the infinitesimal concern for policy).

[25] The assumption that the candidate cares directly about the median voter's welfare may be motivated by the fact that the he has policy preferences or an (unmodeled) concern for re-election.

The next question is whether candidate 1 should accept G's offer. If he does, voters perceive him as high-quality but they also understand that he 'sold out'. If g is not too high, the benefit of being perceived as high-quality offsets the damage of selling out. If g is high, the reverse is true. The cutoff is exactly the participation constraint $\bar{p}$ discussed in Section 3. This is intuitive because at $g = \bar{p}$, the candidate is exactly indifferent between $(p_1 = m, y = 0)$ and $(p_1 = g, y = 1)$. Hence, if $g < \bar{p}$, there exists a separating equilibrium, while with a higher g the equilibrium disappears because any deal with G makes the candidate worse off.

The case $g > \bar{p}$ corresponds to a political system with a very extreme interest group. The median voter punishes anyone who associates with such extremists. An example is the tobacco industry in the US, whose ideal policies seem to be hated by the median voter. A candidate who is caught receiving tobacco money is stygmatized by the media and by his opponents. Thus, in the recent election cycles many candidates have made a point of not accepting contributions from tobacco interests.

On the welfare side, prohibiting campaign contributions has no effect if $g > \bar{p}$ (because there are no contributions to start with). If instead $g < \bar{p}$, the results for the case in which policy is observed carry on to the present case. Proposition 2 holds as stated, except that $p_{\max}$ is substituted with g. Hence, there are three cases according to whether g is low, medium, or high. In the first, contributions should be legal. In the second, they should be forbidden. In the third, it is indifferent.

5.2 The Candidate Makes Offers

The model has given all the bargaining power to G by assuming that G can make candidates a take-it-or-leave-it offer. Let us consider the opposite case. It will be shown that results change dramatically if policy is observable and are almost unchanged if policy is unobservable.

Let us modify Game 1 by assuming that 1 asks G for contribution A_1^* in exchange for policy p^* and A_1^*. G accepts or rejects. The following is immediate.

Proposition 6 *If the candidate makes offers and p_1 is observable, there exists a separating equilibrium in which 1 offers $p^* = m + \epsilon$, where ϵ is positive and infinitesimal, and asks for $A_1^* = \Pi_L(m + \epsilon)$. G accepts if and only if $y_1 = 1$.*

For any p^*, 1 can ask G for a contribution that G can afford only if $y = 1$. To maximize the chance of election, 1 sets p^* as low as possible. Revelation occurs at an infinitesimal cost. Thus,

Corollary 2 *If the candidate makes offers and* p_1 *is observable, an advertising ban is never optimal.*

However, this result relies on the perfect observability of p_1. If, on the contrary, we assume that p_1 is unobservable, we have

Proposition 7 *Suppose the candidate makes offers and* p_1 *is unobservable. Let* P *be the lowest solution to* $\Pi_H(P) = \Pi_L(g)$. *If* $P > \bar{p}$, *there exists no separating equilibrium. If* $P < \bar{p}$, *there exists a separating equilibrium in which 1 offers* $p^* = P$ *and asks for* $A_1^* = \Pi_L(g)$. *G accepts if and only if* $y = 1$.

The equilibrium in Proposition 7 is identical to the equilibrium in Proposition 5 except that $P < g$ (The proof too is similar and is omitted). As the candidate, rather than G, has the bargaining power, he will choose the lowest p^* that satisfies the incentive-compatibility constraint. However, in general, P is not infinitesimal: P can take any value in $(0, \bar{p}]$. Thus, it is easy to see that Proposition 2 holds as stated, except that $p_{\max}$ is replaced with P. Therefore:

Corollary 3 *If the candidate makes offers and* p_1 *is unobservable, a ban on advertising can be optimal.*

5.3 Self-Financing Candidates

This model has assumed the only source of campaign funds for candidates are group contributions. Sometimes, however, candidates have personal wealth they can spend on campaign advertising.[26] One may conjecture that the results for group-financed campaigns extend readily to self-financed campaigns. As it will be seen, this conjecture is not granted. In particular, with self-financed candidates, a separating equilibrium may not exist.

Once again, reconsider Game 1 but assume that the interest group is not there. The only players are the two candidates and the voters. Candidate 1 observes a signal about his own valence, y_1, and has limitless personal funds, which he can use for campaign advertising. As before, $A_1 \in [0, \infty)$ is the amount spent on advertising. The candidate

[26] Recent examples of large-scale self-financed campaigns include Ross Perot in the US, Silvio Berlusconi in Italy, and Bernard Tapie in France. In 1995, a candidate to the US Senate, Michael Huffington, spent over 30 million dollars of mostly personal funds on his campaign. For the record, he lost.

wants to be elected. Of course, he also dislikes spending his own money. So, his payoff is $1 - A_1$ if he is elected and $-A_1$ if he is not elected.

To simplify things – as in Section 3 – the other candidate plays almost no role. His valence is known and normalized to zero.

In the absence of an interest group offering money, the Median Voter's Theorem holds. By Lemma 2, it is a dominant strategy for both candidates to select the median voter's ideal policy m. Hence, we can without loss of generality assume that $p_1 = p_2 = m$. Candidates compete only on valence (and only on Candidate 1's valence). The description of the game is considerably simpler than Game 1:

Game 3 *The players are: voters $i \in I$ and candidates $j \in \{1, 2\}$. The game consists of four stages:*

1. *Nature: Nature chooses $\theta_1 \in \Theta$, which remains unknown to all players. $\theta_2 = 0$.*

2. *Insider Stage: Candidate 1 observes $y_1 \in \{0, 1\}$ and selects $A_1 \in [0, \infty)$.*

3. *Public Stage: $z_1 \in Z$ is realized. Voters observe A_1 and z_1. For $i \in I$, Voter i votes for either 1 or 2. Let e denote the candidate that receives the higher number of votes and let $-e$ denote the other candidate.*

4. *Payoff Distribution: θ_1 is revealed. Voter i receives $\theta_e - u(p_e - p_i)$. If $e = 1$, 1 receives $1 - A_1$ and 2 receives 0. If $e = 2$, 1 receives $-A_1$ and 2 receives 1.*

The players' action sets are: $A_1 \in [0, \infty)$ for 1 and $e_i \in \{1, 2\}$ for i. 2 makes no decision.

Proposition 8 *A separating equilibrium of Game 3 may not exist.*

A separating equilibrium may not exist in a self-financed campaign but always exists in a group-financed campaign. Why is it? In a group-financed campaign, if G does not contribute, she gets 0 independently of y. On the other hand, if she contributes, she gets a higher expected profit from a high-quality candidate than from a low quality candidate. That is why there always exists a level of contribution that only a group with a high-quality candidate is willing to sustain. In a self-financed campaign, a high-quality candidate may have less to gain from being perceived as high-quality than a low-quality candidate has. Hence, there may not exist a level of expenditure that a high type is willing to sustain and a low type is not. If such a level does not exist, there cannot be a separating equilibrium.

An empirical study of French campaign contributions by Palda and Palda [29] has found that expenditures by group-financed candidates are more effective than expenditures from self-financed candidates. This finding is consistent with the present model. If some voters are aware that a given candidate is using his personal wealth to finance his campaign, they are inclined to mistrust the signal coming from his campaign advertising. If instead voters know that the candidate has received money from lobbies, they take conspicuous expenditures as a quality stamp by insiders.

The model could be further extended by considering candidates with two sources of finance: lobbies' contributions and personal wealth. Then, a candidate who uses personal wealth signals that he could not receive money from lobbies. If voters know the source of funding, this should further decrease the effectiveness of self-financed expenditures.

5.4 Existence of Optimal Mechanisms

Voters face a tradeoff between renouncing the insider information (advertising ban) and putting up with bad policies (equilibrium with advertising). Can voters avoid this tradeoff altogether? Is there a mechanism through which G reveals her information at a minimum cost for voters?

It is easy to see that many such mechanisms exist. For instance, suppose campaign contributions are banned and G is asked to name one of the two candidates. If the named candidate is actually elected, then G receives \$1 (or an infinitesimal shift on the policy line). If the named candidate is not elected, G receives nothing. With this mechanism, G will name the candidate which has given her the highest insider signal (if $x \neq 0$, the odds of the bet need to be modified in order to offset the ex-ante advantage). Such mechanism induces truthful revelation of y at an infinitesimal cost.

However, mechanisms of this type are unrealistic for two reasons. First, they are not robust to collusion between G and the candidates: G could offer a candidate to name him in exchange for a favorable policy. In order to achieve collusion-proofness, G should be promised at least as much as she would get under the equilibrium with advertising, but that defeats the purpose of those mechanisms. Second, these mechanisms assume that voters and interest groups can make agreements. It is difficult to see how an unorganized mass of voters can be so organized to coordinate on a mechanism choice and to make credible commitments.

6 Conclusion

An electoral race with campaign advertising has been modeled as a signaling game with one interest group, two candidates, and a continuum of fully rational voters. Two versions of the model have been developed. In the first version, the interest group can only contribute to a pre-specified candidate. The main results are that: (1) a separating equilibrium exists and (2) under certain conditions, the voters' welfare is higher under an advertising ban than under the separating equilibrium. In the second version of the model, the group can contribute to both candidates. The main results are similar to the first version except that, if the insider signals about the two candidates are of equal quality, then the group will make split contributions. Prohibiting split contributions strictly increases the voter welfare.

Campaign advertising is a complex issue. Many aspects that have been left out by the present may, in the future, be addressed within a similar framework:

First, the model has assumed that only one interest group is active. It would be important to extend the model to several groups in competition with each other. This could be done in a common agency framework (See for instance Grossman and Helpman [17]). A conjecture is that the negative welfare effects of campaign advertising disappear if interest groups are symmetrically distributed around the median voter.[27]

Second, the model has assumed that the amount spent on advertising is perfectly observable by all voters. In a more realistic framework (like Hertzendorf [18]), advertising expenditures translate in a probability distribution over the number of TV ads each voter will watch.

Third, in this model voters have heterogeneous preferences but they are assumed to have homogeneous information: x and z are the same for all voters. The model could be extended to include heterogeneous voters information, which will provide a link with the literature on information aggregation in elections (e.g. Lohmann [22] or Feddersen and Pesendorfer [13]).

Lastly, in this model a ban on advertising produces the same effect as a ban on contri-

[27] However, in reality interest groups do not seem to be symmetrically distributed around the median voter, but their median member appears to be more affluent and advantaged than the median voter. Lehman Schlozman and Tierney [32, p. 87] conduct a comprehensive survey on US groups and conclude that: "In terms of skew, organization members are drawn disproportionately from the ranks of upper-status individuals – those with high levels of income, education, and occupational prestige.[. . .] Surprisingly, in spite of the appearance of new groups representing the previously underrepresented, the imbalance of the pressure community seems to have become more pronounced in the recent years."

butions. In practice, there are important differences.[28] First, while advertising restrictions can be enforced, the experience of several countries shows that restrictions on campaign contributions are often disregarded or dodged. Second, contributions can be spent in a variety of ways, which give different signals to different voters. Thus, a ban on advertising does not necessarily make contributions useless to candidates. Third, campaign advertising is an expression of political opinion. Thus, restrictions on it can be seen as restrictions on free expression and may be unconstitutional. The first argument supports restrictions on advertising, the last two arguments point in favor of restrictions on contributions. More detailed models should be developed with the goal of comparing the effects of the two types of restrictions.

References

[1] Stephen Ansolabehere and Shanto Iyengar. *Going Negative: How Political Advertisements Shrink and Polarize the Electorate*. The Free Press, New York, 1996.

[2] David Austen-Smith. Interest groups, campaign contributions, and probabilistic voting. *Public Choice*, 54:123–139, 1987.

[3] David Austen-Smith. Campaign contributions and access. *American Political Science Review*, 89:566–581, 1995.

[4] Kyle Bagwell and Garey Ramey. Coordination economies, advertising, and search behavior in retail markets. *American Economic Review*, 84(3):498–517, 1994.

[5] David P. Baron. Service-induced campaign contributions and the electoral equilibrium. *Quarterly Journal of Economics*, 104:45–72, 1989.

[6] David P. Baron. Electoral competition with informed and uninformed voters. *American Political Science Review*, 88(1):33–47, 1994.

[7] Gary S. Becker and Kevin S. Murphy. A simple theory of advertising as good or bad. *Quarterly Journal of Economics*, 108, 1993.

[28] The US has chosen the road of regulating contributions but letting candidates spend freely. European countries, instead, tend to focus on spending. For instance, in Britain individual candidates are not allowed to run TV ads.

[8] G. Butters. Equilibrium distributions of sales and advertising prices. *Review of Economic Studies*, 44:465–491, 1977.

[9] In-Koo Cho and David Kreps. Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102(2):179–221, 1987.

[10] Alex Cukierman. Asymmetric information and the electoral momentum of public opinion polls. *Public Choice*, 70, 1991.

[11] Avinash Dixit and Victor Norman. Advertising and welfare. *Bell Journal of Economics*, 9(1):1–17, 1978.

[12] The Economist. Politicians for rent, 1997. 8 February.

[13] Timothy Feddersen and Wolfgang Pesendorfer. Voting behavior and information aggregation in elections with private information. *Econometrica*, 65(5):1029–1058, 1997.

[14] Franklin M. Fisher and John J. McGowan. Advertising and welfare: Comment. *Bell Journal of Economics*, 10:726–727, 1979.

[15] Alan Gerber. Rational voters, candidate spending, and incomplete information: A theoretical analysis with implications for campaign finance reform. unpublished, 1996.

[16] Gene M. Grossman and Elhanan Helpman. Competing for endorsements. Discussion paper, Centre for Economic Policy Research, 1996.

[17] Gene M. Grossman and Elhanan Helpman. Electoral competition and special interest policies. *Review of Economic Studies*, 63:265–286, 1996.

[18] Mark N. Hertzendorf. I'm not a high-quality firm but I play one on TV. *RAND Journal of Economics*, 24(2):236–247, 1993.

[19] Lynda Lee Kaid and Christina Holtz-Bacha. *Political Advertising in Western Democracies: Parties and Candidates on Television*. SAGE, 1995.

[20] Richard E. Kihlstrom and Michael H. Riordan. Advertising as a signal. *Journal of Political Economy*, 92:427–450, 1984.

[21] Jean-Jacques Laffont and Jean Tirole. *A Theory of Incentives in Procurement and Regulation*. MIT Press, Cambridge and London, 1993.

[22] Susanne Lohmann. Information aggregation through costly political action. *American Economic Review*, 84(3):518–530, 1994.

[23] Susanne Lohmann. Electoral incentives, informational asymmetries, and the policy bias toward special interest groups. mimeo, UCLA, 1997.

[24] Rigoberto A. Lopez and Emilio Pagoulatos. Trade protection and the role of campaign contributions in U.S. food and tobacco industries. *Economic Inquiry*, 34:237–248, 1996.

[25] George J. Mailath, Masahiro Okuno-Fujiwara, and Andrew Postlewaite. Belief-based refinements in signalling games. *Journal of Economic Theory*, 60(2):241–276, 1993.

[26] Paul Milgrom. Good news and bad news: Representation theorems and applications. *Bell Journal of Economics*, 12(2):380–391, 1981.

[27] Paul Milgrom and John Roberts. Price and advertising signals of product quality. *Journal of Political Economy*, 94:796–821, 1986.

[28] Rebecca Morton and Charles Cameron. Elections and the theory of campaign contributions: A survey and critical analysis. *Economics and Politics*, 4(1):79–108, 1992.

[29] Filip Palda and Kristian Palda. The impact of campaign expenditures on political competition in the french legislative elections of 1993. *Public Choice*, 94:157–174, 1998.

[30] Jan Potters, Randolph Sloof, and Frans van Winden. Campaign expenditures, contributions and direct endorsements: The strategic use of information and money to influence voter behavior. *European Journal of Political Economy*, 13:1–31, 1997.

[31] Kenneth Rogoff. Equilibrium political business cycles. *American Economic Review*, 80(1):21–49, 1990.

[32] Kay Lehman Schlozman and John T. Tierney. *Organized Interests and American Democracy*. Harper and Row, New York, 1986.

[33] Jean Tirole. *The Theory of Industrial Organization*. MIT Press, Cambridge, Mass., 1988.

7 Appendix

7.1 Proofs

Proof of Proposition 1 A preliminary result is useful:

Lemma 4 *For any $p^* > m$, $\Pi_H(p^*) > \Pi_L(p^*)$.*

Proof: Let $\Gamma(t|x,y) = \Pr(z \le t|x,y)$. Then

$$\Pi_y(p^*) = (1 - \Gamma(\bar{z}(x,1,p^*)|x,y))\mu[u(g-m) - u(g-p^*)] \tag{4}$$

Let $K(\theta_1|x_1,y_1)$ be the posterior distribution of θ_1 given x_1 and y_1. $K(\cdot|\cdot,\cdot)$ satisfies FOSD. Thus, for every strictly increasing function $\gamma(\cdot)$, $\int_\Theta \gamma(\theta)dH(\theta_1|x_1,y_1)$ is strictly increasing in x_1 and y_1. But, because MLRP implies that $F_z(z_1|\theta_1)$ is a strictly decreasing function of θ_1, then

$$\Gamma(z_1|x_1,y_1) = \int_\Theta F_z(z_1|\theta_1)dH(\theta_1|x_1,y_1)$$

is a strictly decreasing function on x_1 and y_1 (that is, Γ satisfies FOSD). Thus, for any $z_1 \in Z$, $\Gamma(z_1|x_1,1) < \Gamma(z_1|x_1,1)$. Therefore, $\Pi_H(p^*) > \Pi_L(p^*)$ for any $p^* > 0$. □

To prove Proposition 1, it suffices to show that (i) is consistent with (ii)-(v) and that each of the points in (ii)-(v) are best responses given the other points.

(i) Given (iii) and (iv), if $y_1 = 0$, then G offers 0 (and Candidate 1 rejects), while, if $y_1 = 1$, then G offers p^* and $\Pi_L(p^*)$ (and Candidate 1 accepts). Hence, voters are correct in believing that y_1 if and only if $A_1 \ge \Pi_L(p^*)$.

(ii) See Lemma 1.

(iii) *Step 1: Suppose that G has decided to ask for p^*. Then, the following is a best-response contribution for G:*

$$A_1^* = \begin{cases} \Pi_L(p^*) & \text{if } y_1 = 1 \\ 0 & \text{if } y_1 = 0 \end{cases}.$$

Proof of Step 1: From (iv), G knows that Candidate 1 accepts only if $A^* \ge \Pi_L(p^*)$. Hence, for any p^*, we can restrict w.l.o.g. the attention to $A^* \in \{0, \Pi_L(p^*)\}$. Let π denote the net expected profit of G (gross payoff minus contribution). There are two cases: $y = 0$ and $y = 1$. If $y = 0$ and $A^* = 0$, then $\pi = 0$. If $y = 0$ and $A^* = \Pi_L(p^*)$, then $\pi = \Pi_L(p^*) - \Pi_L(p^*) = 0$. Hence, $A^* = 0$ is a (weak) best response if $y = 0$. If $y = 1$ and $A^* = 0$, then $\pi = 0$. If $y = 1$ and $A^* = \Pi_H(p^*)$, then $\pi = \Pi_H(p^*) - \Pi_L(p^*)$. By Lemma 4, $\Pi_H(p^*) - \Pi_L(p^*) > 0$. Hence, $A^* = \Pi_L(p^*)$ is a best response if $y = 1$.

Step 2: Given Step 1, $p_{\max}$ is the optimal p^*:

Proof of Step 2: When $y_1 = 0$, the choice of p^* is irrelevant. When $y_1 = 1$, G selects p^* in order to maximize $\Pi_H(p^*) - \Pi_L(p^*)$.

(iv) The first part of (iv) is immediate because, given (i) and (ii), contributions below $\Pi_L(p^*)$ are useless. The second part requires comparing the probability of election if 1 accepts the contribution with the probability of election if he rejects. The probability of election is a monotonically decreasing function of the cutoff $\bar{z}$. Hence, 1 minimizes $\bar{z}$. The cutoff if 1 accepts is $\bar{z}(x_1, 1, p^*)$, while if he rejects it is $\bar{z}(x_1, 0, m)$. By the definition of $\bar{p}$, if $p^* \leq \bar{p}$, then $\bar{z}(x_1, 1, p^*) \geq \bar{z}(x_1, 0, m)$, while, if $p^* > \bar{p}$, then $\bar{z}(x_1, 1, p^*) < \bar{z}(x_1, 0, m)$. To minimize $\bar{z}$, 1 accepts the offer if and only if $p^* \leq \bar{p}$.

(v) See Lemma 2.

Proof of Lemma 3: In the separating equilibrium, if $y_1 = 0$, Candidate 1 is elected if and only if $z_1 \geq \bar{z}(x_1, 0, m)$. If $y_1 = 1$, Candidate 1 is elected if and only if $z_1 \geq \bar{z}(x_1, 1, p^*)$. If $p^* = \bar{p}$, the definition of $\bar{p}$ implies $\bar{z}(x_1, 1, p^*) = \bar{z}(x_1, 0, m)$. Thus, e=1 if an only if $z \geq \bar{z}(x, 0, m)$, irrespective of whether $y = 1$ or $y = 0$. Thus, e=1 if an only if $\hat{\theta}(x, 0, z) \geq 0$. Let $b(x) = \check{\theta}(x, \bar{z}(x, 0, m)) - \hat{\theta}(x, 0, \bar{z}(x, 0, m))$. Then, $e = 1 \Leftrightarrow \check{\theta}(x, z) \geq b(x)$.

As the rule $e = 1 \Leftrightarrow \check{\theta}(x, z) \geq 0$, is optimal by defintion within the set of rules which use only x and z, it follows that $\bar{w}_S(\bar{p}) < \bar{w}_P$.

Proof of Proposition 2: Given Lemma 3 and the fixed-point theorem, it is sufficient to prove the two following claims:

(i) $\bar{w}_S(p^*)$ is strictly decreasing and continuous in p^*;

(ii) Let $\bar{w}_S(m) = \lim_{p^* \to m^+} \bar{w}_S(p^*)$. Then, $\bar{w}_S(m) > \bar{w}_P$.

To prove Claim 1, recall (3) and notice that $w_S(1, 1, p^*)$ is strictly decreasing and continuous in p^* and $w_S(0, 0, p^*) = w_S(0, 1, p^*) = w_S(1, 0, p^*)$ is constant in p^*. Suppose $y_1 = 1$: then the expected voter welfare is $\bar{w}_S(p^*|y_1 = 1) \equiv \max\{w_S(1, 1, p^*), w_S(0, 1, p^*)\}$, which is strictly decreasing in p^*. Suppose $y_1 = 0$: then the expected voter welfare is $\bar{w}_S(p^*|y_1 = 0) \equiv \max\{w_S(1, 0, p^*), w_S(0, 1, p^*)\}$, which is constant in p^*. As

$$\bar{w}_S(p^*) = \Pr(y_1 = 1)\bar{w}_S(p^*|y_1 = 1) + \Pr(y_1 = 0)\bar{w}_S(p^*|y_1 = 0)$$

and the event $y_1 = 1$ has positive probability which does not depend on p^*, the claim is proven.

To prove Claim 2, suppose that, if $p^* \to m^+$. Then, under the separating equilibrium, when $y = 1$, $p_1 \to p^m$. As $p_1 = m$ when $y_1 = 0$ in all cases, the ex post welfare under a separating equilibrium becomes

$$w(e, m) = \begin{cases} \theta_1 & \text{if } e = 1 \\ 0 & \text{if } e = 2 \end{cases}$$

which is identical to the ex post welfare under the pooling equilibrium. However, under the separating equilibrium voters know x, y, and z while under a pooling equilibrium voters only know x and z. Because y is informative,

$$E_\theta[\max_{e \in \{1,2\}} E(w(e, m)|x, y, z)] > E_\theta[\max_{e \in \{1,2\}} E(w(e, m)|x, z)]$$

which proves Claim 2.

Proof of Proposition 3 Two preliminary results are useful:

Lemma 5 *In Game 2 the following inequalities hold for any $p > m$: (i) $\Pi_{all}(p) > \Pi_H(p) > \Pi_M(p) > \Pi_L(p)$; (ii) $\Pi_H(p) + \Pi_L(p) > \Pi_{all}(p)$; (iii) $2\Pi_M(p) > \Pi_{all}(p)$.*

Proof: The first inequality of (i) is obvious. The other three inequalities in (i) are due to MLRP and can be proven analogously to Lemma 4. Part (ii) is proven by observing that $\Pi_H(p) + \Pi_L(p) > \Pi_{\text{all}}(p)$ is equivalent to

$$\begin{aligned} & \Pr\left[\hat{\theta}(1,z) - u(p-m) - u(0) \geq 0 \middle| 1\right] + \Pr\left[\hat{\theta}(1,z) - u(p-m) - u(0) \geq 0 \middle| -1\right] \\ > \ & \Pr\left[\hat{\theta}(0,z) - u(p-m) - u(0) \geq 0 \middle| 1\right] + \Pr\left[\hat{\theta}(0,z) - u(p-m) - u(0) \geq 0 \middle| -1\right] \\ = \ & 1, \end{aligned}$$

which is immediate. Part (ii) is proven by observing that $2\Pi_M(p) > \Pi_{\text{all}}(p)$ is equivalent to

$$\begin{aligned} & 2\Pr\left[\hat{\theta}(1,z) - u(p-m) - u(0) \geq 0 \middle| 0\right] \\ > \ & 2\Pr\left[\hat{\theta}(0,z) - u(p-m) - u(0) \geq 0 \middle| 0\right] = 1. \end{aligned}$$

□

Lemma 6 *For any $y \in \{-1, 0, 1\}$ and any $z \in \tilde{Z}$, $\hat{\theta}(y,z) = -\hat{\theta}(-y,-z)$.*

Proof: This Lemma appears straightforward because of the entirely symmetric nature of the problem. Yet, a formal proof is included for completeness. Let

$$g_y(y|\theta_1,\theta_2) = \sum_{y_1+y_2=y} f(y_1|\theta_1)f(y_2|\theta_2)$$

and

$$g_z(z|\theta_1,\theta_2) = \int_Z f_z(z_1|\theta_1)f_y(z-z_1|\theta_2)dz_1$$

It is easy to see that the p.d.f.'s g_y and g_z are antisymmetric in θ_1 and θ_2, that is

$$g_y(y|\theta',\theta'') = g_y(-y|\theta'',\theta') \tag{5}$$

for any $y \in \tilde{(Y)}$ and any $\theta', \theta'' \in \Theta$ (and similarly for z). Also, it follows immediately from Assumption 1 that g_y and g_z satisfy the Monotone Likelihood Ratio Property with respect to θ_1 and $-\theta_2$.

Thus,

$$\hat{\theta}(y,z) = \frac{\int_\Theta \int_\Theta (\theta_1 - \theta_2) f_y(y,\theta_1,\theta_2) f_z(z,\theta_1,\theta_2)\phi(\theta_1)\phi(\theta_2)d\theta_1 d\theta_2}{\int_\Theta \int_\Theta f_y(y,\theta_1,\theta_2) f_z(z,\theta_1,\theta_2)\phi(\theta_1)\phi(\theta_2)d\theta_1 d\theta_2}$$

and

$$-\hat{\theta}(-y,-z) = \frac{\int_\Theta \int_\Theta (\theta_2 - \theta_1) f_y(-y,\theta_1,\theta_2) f_z(-z,\theta_1,\theta_2)\phi(\theta_1)\phi(\theta_2)d\theta_1 d\theta_2}{\int_\Theta \int_\Theta f_y(-y,\theta_1,\theta_2) f_z(-z,\theta_1,\theta_2)\phi(\theta_1)\phi(\theta_2)d\theta_1 d\theta_2}$$

By recalling Part (ii) of Assumption 4, applying (5), and switching θ_1 with θ_2, we have $-\hat{\theta}(-y,-z) = \hat{\theta}(y,z)$. □

We can now prove Proposition 3 by showing that point (i) is a consistent belief and the other points are best-responses.

(i) There are three cases: $y = 1$, $y = 0$, and $y = -1$. If $y = 1$, then by (iii) $A_1^* = \alpha(p^*)$ and $A_2^* = 0$ and by (iv) Candidate 1 accepts and Candidate 2 rejects. Hence, $\tilde{\theta} = \hat{\theta}(1, z)$ is consistent. If $y = 0$, then by (iii) $A_1^* = A_2^* = \alpha(p^*)$ and by (iv) both candidates accept. The case $y = -1$ is symmetric to $y = 1$ and is omitted.

(ii) See Lemma 1.

(iii) *Step 1: Suppose that G has decided to ask for p^*. Then, the following is a best-response contribution for G:*

(a) If $y = -1$, $A_1^* = 0$ and $A_2^* = \alpha(p^*)$;

(b) If $y = 0$, $A_1^* = A_2^* = \alpha(p^*)$;

(c) If $y = 1$, $A_1^* = \alpha(p^*)$ and $A_2^* = 0$.

Proof of Step 1: Given (iv), G can restrict w.l.o.g. her attention to contributions $(A_1^*, A_2^*) \in \{0, \alpha(p^*)\}^2$. Hence, for a given y, there are four strategies:

(a) $A_1^* = A_2^* = 0$.

(b) $A_1^* = \alpha(p^*), A_2^* = 0$.

(c) $A_1^* = 0, A_2^* = \alpha(p^*)$.

(d) $A_1^* = A_2^* = \alpha(p^*)$.

Let π denote the net expected profit of G – that is the difference between Π and A. If $y = 1$, the net profits for each of the four strategies above are:

$$\begin{aligned} \pi_a &= 0; \\ \pi_b &= \Pi_H(p^*) - \alpha(p^*) = 2\Pi_H(p^*) - \Pi_{\text{all}}(p^*); \\ \pi_c &= \Pi_L(p^*) - \alpha(p^*) = \Pi_H(p^*) + \Pi_L(p^*) - \Pi_{\text{all}}(p^*); \\ \pi_d &= \Pi_{\text{all}}(p^*) - 2\alpha(p^*) = 2\Pi_H(p^*) - \Pi_{\text{all}}(p^*). \end{aligned}$$

From Lemma 5, $\pi_b > \pi_a$, $\pi_b > \pi_c$, and $\pi_b = \pi_d$. Hence, (b) is a (weak) best response when $y = 1$. Next, if $y = 0$, we have

$$\begin{aligned} \pi_a &= 0; \\ \pi_b &= \Pi_M(p^*) - \alpha(p^*) = \Pi_H(p^*) + \Pi_M(p^*) - \Pi_{\text{all}}(p^*); \\ \pi_c &= \Pi_M(p^*) - \alpha(p^*) = \Pi_H(p^*) + \Pi_M(p^*) - \Pi_{\text{all}}(p^*); \\ \pi_d &= \Pi_{\text{all}}(p^*) - 2\alpha(p^*) = 2\Pi_H(p^*) - \Pi_{\text{all}}(p^*). \end{aligned}$$

From Lemma 5, $\pi_d > \pi_a$ and $\pi_d > \pi_b = \pi_c$. Hence, (d) is a best response when $y = 0$. The case $y = -1$ is symmetric to $y = 1$ and is omitted.

Step 2: Given Step 1, $\underline{p}_{\max}$ is the optimal p^*.

Proof of Step 2: G chooses p^* to maximize net expected profit π. If $y \neq 1$, $\pi = \pi_c = 2\Pi_H(p^*) - \Pi_{\text{all}}(p^*)$. If $y = 0$, $\pi = \pi_d = 2\Pi_H(p^*) - \Pi_{\text{all}}(p^*)$. Hence, independently of y, G selects p^* in order to maximize $2\Pi_H(p^*) - \Pi_{\text{all}}(p^*)$.

(iv) Let us focus on candidate 1. The analysis for Candidate 2 is symmetric. Given (i), it is a best response for 1 to reject any $A_1^* < \alpha(p^*)$. By (iii), 1 receives an offer $A_j^* = \alpha(p^*)$ if $y = 1$ or $y = 0$. Hence, there will be a participation constraint for $y = 0$ and a participation constraint for $y = 1$. Next, we show that the two participation constraints are identical and that they they correspond to $p^* \leq \underline{p}$.

If $y = 1$, only Candidate 1 is made an offer. Hence, by (i), if he accepts, voters believe $y = 1$, while, if he rejects, voters believe $y = 0$. Thus, 1 accepts if he prefers $(y = 1, p_1 = p^*, p_2 = m)$ to $(y = 0, p_1 = m, p_2 = m)$, that is

$$\underline{z}(1, p^*, m) \leq \underline{z}(0, m, m) \tag{6}$$

If $y = 0$, both candidates are made offers. Candidate 1 knows that 2 will accept. Hence, by (i), if he accepts, voters believe $y = 0$ and, if he rejects, voters believe $y = -1$. It is optimal for 1 to accept if he prefers $(y = 0, p_1 = p^*, p_2 = p^*)$ to $(y = -1, p_1 = m, p_2 = p^*)$, that is

$$\underline{z}(0, p^*, p^*) \leq \underline{z}(-1, m, p^*) \tag{7}$$

However, notice that $\underline{z}(0, p, p) = 0$ for any p and, by Lemma 6,

$$\underline{z}(y, p', p'') = -\underline{z}(-y, p'', p')$$

for any p', p'' and y. Then, both (7) and (6) can be rewritten as $\underline{z}(1, p^*, m) \leq 0$, which is true if $p^* \leq \underline{p}$. Hence, independently of y, Candidate 1 accepts if and only if $p^* \leq \underline{p}$.

As the participation constraint of the candidate is independent of y, a candidate does not need to know y to decide whether to accept the contribution or not. If Candidate 1 is made an offer, by (iii) he knows that $y \in \{0, 1\}$ and his acceptance rule is $p^* \leq \underline{p}$ whether $y = 0$ or $y = 1$. The same holds for 2.

Proof of Proposition 4: Under a ban on split contributions, the No-Split-Contribution Equilibrium (NSCE) of Proposition 10 in Section 7.4 will still be a sequential equilibrium of Game 2. Let us compare SCE with NSCE. If $y \neq 0$, the outcome for voters is identical in both equilibria. If $y = 0$, 1 is elected in one equilibrium if and only he is elected in the other (when $z > 0$) but in the SCE $p_1 = p_2 = \underline{p}$ while in the NSCE $p_1 = p_2 = m$. Thus, if $y = 0$, the voter welfare is strictly higher under NSCE. Therefore, the ex-ante welfare (before y is realized) is higher in NSCE than in SCE.

Proof of Proposition 5: Let us begin with the case $\bar{p} > g$.

(i) Consistent with (iii) and (iv).

(ii) Lemma 1.

(iii) Given (iv), G can restrict her attention w.l.o.g. to $A^* \in \{0, \Pi_L(g)\}$. Let π denote the net expected payoff, that is the gross expected payoff minus the contribution. There are two cases: $y_1 = 0$ and $y_1 = 1$. If $y_1 = 0$ and $A^* = 0$, then $\pi = 0$. If $y_1 = 0$ and $A^* = \Pi_L(g)$, then $\pi = \Pi_L(g) - \Pi_L(g) = 0$. Hence, $A^* = 0$ is a (weak) best response when $y_1 = 0$. if $y_1 = 1$ and $A^* = 0$, then $\pi = 0$. If $y_1 = 0$ and $A^* = \Pi_L(g)$, then $\pi = \Pi_H(g) - \Pi_L(g)$. By Lemma 4, $\Pi_H(g) - \Pi_L(g) > 0$. Hence, $A^* = \Pi_H(g)$ is a best response when $y_1 = 1$. By (iv), Candidate 1's acceptance does not depend on p^*. Hence, G should always set $p^* = g$.

(iv) Given (i), Candidate 1 will reject all $0 < A^* < \Pi_L(g)$. If $A^* = 0$, the infinitesimal concern for policy induces the candidate to choose $p_1 = m$. If $A^* \geq \Pi_L(g)$ and Candidate 1 accepts G's offer, his election cutoff moves from $\bar{z}(x, 0, m)$ to $\bar{z}(x, 1, g)$. If $g < \bar{p}$, then $\bar{z}(x, 0, m) > \bar{z}(x, 1, g)$ and the candidate should accept the offer.

(v) Lemma 2.

If, instead, $\bar{p} < g$, suppose for contradiction that a separating equilibrium exists. In the separating equilibrium, a candidate with $y = 1$ chooses $p^* = g$. But, because $\bar{p} < g$, $\bar{z}(x, 0, m) < \bar{z}(x, 1, g)$ and a candidate with $y = 1$ should reject G's offer.

Proof of Proposition 8: Let $\tilde{y}$ be voters' beliefs on y_1. Candidate 1 is elected if and only if $\hat{\theta}(\tilde{y}, z_1) \geq 0$. The probability of election of 1 conditional on y_1 and $\tilde{y}$ is

$$R(\tilde{y}, y_1) = \Pr(\hat{\theta}(\tilde{y}, z_1) \geq 0 | y_1) = \Pr(z_1 \geq \bar{z}(\tilde{y}) | y_1).$$

Let $\Gamma(z_1|y_1)$ be the cumulative distribution of z_1 given y_1. The probability of election of 1 can be rewritten as $R(\tilde{y}, y_1) = \Gamma(\bar{z}(\tilde{y})|y_1)$.

It is easy to see that, if a separating equilibrium exists, it takes the form: If $y_1 = 0$, $A_1 = 0$; If $y_1 = 1$, $A_1 = A^* > 0$. In a separating equilibrium, only Candidate 1 undertakes A^*. For this to be true it is necessary that

$$R(1,1) - A^* \geq R(0,1); \tag{8}$$

$$R(0,1) - A^* \leq R(0,0). \tag{9}$$

If (8) fails, a candidate with $y_1 = 1$ does not undertake A^*. If (9) fails, a candidate with $y_1 = 0$ does undertake A^*. An A^* satisfying both (8) and (9) exists if and only if

$$R(1,1) - R(1,0) \geq R(0,1) - R(0,0). \tag{10}$$

We now see by means of a numerical example that (10) may or may not hold. Let $\theta_1 \in \{0, 1\}$ with $\Pr(\theta_1 = 1) = k$ with $0 < k < 1$.

Instead of the usual $\theta_2 = 0$, let $\theta_2 = \frac{1}{2}$.[29] As usual, $Y \in \{0, 1\}$. Let

$$\Pr(y_1 = 1|\theta_1) = \begin{cases} \frac{3}{4} & \text{if } \theta_1 = 1 \\ \frac{1}{4} & \text{if } \theta_1 = 0 \end{cases}$$

[29] Instead, one could assume $\theta_2 = 0$ and $\theta_1 \in \{-0.5, 0.5\}$. The results would not change.

Let $Z \in [0,1]$ with

$$f_{z_1}(z_1|\theta) = \begin{cases} 2z_1 & \text{if } \theta_1 = 1 \\ 2(1-z_1) & \text{if } \theta_1 = 0 \end{cases}$$

By Bayes' Theorem,

$$\hat{\theta}(y_1, z_1) = \frac{k(1+2y_1)z_1}{(3-2y_1)(1-z_1) + k(2y_1 + 4z_1 - 3)}.$$

By solving $\hat{\theta}(\tilde{y}, z_1) = 0$,

$$\bar{z}(\tilde{y}) = \frac{(1-k)(3-2y_1)}{3 - 2k - 2y_1 + 4ky_1}.$$

Also,

$$\Gamma(z_1|y_1) = \frac{z_1((3-2y_1)(2-z_1) + k(4y_1 + 4z_1 - 6))}{3 - 2k - 2y_1 + 4ky_1}.$$

and, by using $R(\tilde{y}, y_1) = \Gamma(\bar{z}(\tilde{y})|y_1)$,

$$R(1,1) - R(1,0) - (R(0,1) - R(0,0)) = \frac{384(k-1)^2 k^2 (2k-1)}{(2k-3)^3(1+2k)^3},$$

which is positive if $\frac{1}{2} < k < 1$ and negative if $0 < k < \frac{1}{2}$. Hence, Condition (10) holds if $\frac{1}{2} \leq k < 1$ and does not hold if $0 < k < \frac{1}{2}$.

7.2 Example for the Participation Constraint

Let us assume that $u(\cdot) = \frac{1}{\lambda}|\cdot|$ where $\lambda > 0$. Thus, $u(p_i - p_e)$ is decreasing at a constant rate up to p_i and increasing at a constant rate after p_i. Of course, the same applies to $\mu u(g - p_e)$. Let us also assume that $m = 0$.

Also, $\theta_1 \in \{0,1\}$ with $\Pr(\theta_1 = 0) = \Pr(\theta_1 = 0)$; $X = [0,1]$ and $Z \in [0,1]$ with

$$f_x(x_1|\theta) = \begin{cases} 2x & \text{if } \theta_1 = 1 \\ 2(1-x_1) & \text{if } \theta_1 = 0 \end{cases}$$

and

$$f_z(z_1|\theta) = \begin{cases} 2z & \text{if } \theta_1 = 1 \\ 2(1-z_1) & \text{if } \theta_1 = 0 \end{cases}$$

Instead of the usual $\theta_2 = 0$, let $\theta_2 = \frac{1}{2}$. As usual, $Y \in \{0,1\}$. Let

$$f_y(y_1|\theta_1) = \begin{cases} \frac{1}{2}[1 + \rho(2y_1 - 1)] & \text{if } \theta_1 = 1 \\ \frac{1}{2}[1 - \rho(2y_1 - 1)] & \text{if } \theta_1 = 0 \end{cases}$$

where $\rho \in (0,1)$ represents the precision of y_1. Then,

$$\begin{aligned} \hat{\theta}(x_1, y_1, z_1) &= \Pr(\theta_1 = 1|x_1, y_1, z_1) \\ &= \frac{f_x(x_1|\theta_1 = 1) f_y(y_1|\theta_1 = 1) f_z(z_1|\theta_1 = 1)}{f_x(x_1|\theta_1 = 0) f_y(y_1|\theta_1 = 0) f_z(z_1|\theta_1 = 0) + f_x(x_1|\theta_1 = 1) f_y(y_1|\theta_1 = 1) f_z(z_1|\theta_1 = 1)} \\ &= \frac{x_1 z_1 [1 + \rho(2y_1 - 1)]}{(1-x_1)(1-z_1)[1 - \rho(2y_1 - 1)] + x_1 z_1 [1 + \rho(2y_1 - 1)]} \end{aligned}$$

To find Γ, let

$$\begin{aligned}\gamma(z_1|x_1, y_1) &= \frac{1}{2} f_z(z_1|\theta_1 = 1)\Pr(\theta_1 = 1|x_1, y_1) + \frac{1}{2} f_z(z_1|\theta_1 = 0)\Pr(\theta_1 = 0|x_1, y_1) \\ &= \frac{z_1 x_1[1 + \rho(2y_1 - 1)] + (1 - z_1)(1 - x_1)[1 - \rho(2y_1 - 1)}{(1 - x_1)[1 - \rho(2y_1 - 1)] + x_1[1 + \rho(2y_1 - 1)]}\end{aligned}$$

Then,

$$\begin{aligned}\Gamma(z_1|x_1, y_1) &= \int_0^{z_1} \gamma(t|x_1, y_1)dt \\ &= \frac{2(1 - x_1)z_1[1 - \rho(2y_1 - 1)] + z_1^2(2x_1 - 1 - \rho + 2y_1\rho}{1 + \rho - 2x_1\rho - 2y_1\rho + 4x_1y_1\rho}\end{aligned}$$

Recall that $\bar{z}(x_1, y_1, p^*)$ is the unique z_1 that solves

$$\lambda[\hat{\theta}(x_1, y_1, z_1) - \frac{1}{2}] - p^* = 0$$

Then,

$$\bar{z}(x_1, y_1, p^*) = \frac{-1 + x_1 - 2p^*\lambda + 2p^*x_1\lambda + \rho - x_1\rho + 2p^*\lambda\rho - 2p^*x_1\lambda\rho}{-1 - 2p^*\lambda + 4p^*x_1\lambda + \rho - 2x_1\rho + 2p^*\lambda\rho} \tag{11}$$

Let $\bar{p}$ be the unique p^* that solves

$$\bar{z}(x_1, 1, p^*) = \bar{z}(x_1, 0, 0)$$

Then,

$$\bar{p} = \frac{\rho}{\lambda(1 + \rho^2)}$$

(the fact $\bar{p}$ is independent of x_1 is a feature of this example, but is not true in general).

By (4),

$$\Pi_y(p^*) = (1 - \Gamma(\bar{z}(x, 1, p^*)|x, y))\mu[u(g - m) - u(g - p^*)]$$

Thus,

$$\Pi_H(p^*) - \Pi_L(p^*) = -\frac{8p^*(x_1 - 1)^2x_1^2(4(p^*)^2\lambda^2 - 1)\mu\rho(\rho^2 - 1)}{[-1 + (1 - 2x_1)^2\rho^2][-1 + \rho - 2x_1\rho - 2x_1\rho + 2\rho\lambda(2x_1 - 1 + \rho)]^2}$$

$p_{\max}$ cannot be found analytically, but, given some parameter values, it can be computed. For instance, let us assume that $g = 1$, $\lambda = 1$, and $\mu = 1$. Suppose $\rho = 0.3$. Then,

$$\begin{aligned}\bar{p} &= 0.275 \\ p_{\max} &= 0.326\end{aligned}$$

Suppose instead that $\rho = 0.5$. Then,

$$\begin{aligned}\bar{p} &= 0.4 \\ p_{\max} &= 0.358\end{aligned}$$

Thus, we have shown that both the case $\bar{p} > p_{\max}$ and the case $\bar{p} < p_{\max}$ are possible.

7.3 Other Equilibria

There exist other sequential equilibria besides the one in Proposition 1. However – as this section shows – all the others fail the Intuitive Criterion of Cho and Kreps [9].

The concept of sequential equilibrium imposes no restriction on out-of-equilibrium beliefs. The intuitive Criterion requires that the receiver (in this case, voters), if confronted, with an out-of-equilibrium action asks himself which type of sender benefits from such an action.[30]

One technical difficulty lies in the fact that in the signalling game under consideration there is not just one sender and one receiver. Both the interest group and Candidate 1 can be viewed as senders. We overcome this discrepancy by assuming that G and 1 can talk to eachother and agree on (self-enforcing) deviations. Hence, deviations are subject to the constraint that they benefit both G and 1.

For simplicity, we restrict the attention to pure-strategy equilibria, but results are not expected to change in the general case.

Proposition 9 *The equilibrium of Proposition 1 is the only pure-strategy sequential equilibrium of Game 1 that survives the Intuitive Criterion.*

Proof: The restriction to pure strategies implies that sequential equilibria are either separating or pooling (i.e. there are no semipooling equilibria).

We first show that the equilibrium of Proposition 1 is the only separating equilibrium that survives the Intuitive Criterion. Notice that in a separating equilibrium $A_1 = 0$ if $y_1 = 0$. This is because a candidate who is revealed as low quality rejects any $p^* \neq m$. Hence, the only offer that he can receive is zero. Next, we exclude the possibility that $A^* < \Pi_L(p_1)$ if $y = 1$. If this were the case, an interest group with $y = 0$ would find it profitable to offer A^* as well, which is a contradiction.

We are then left with separating equilibria in which $A^* \geq \Pi_L(p_1)$ if $y = 1$ and we need to apply the Intuitive Criterion. In this context, there are two types of senders: G with $y = 0$ (let us call her G_0) and G with $y = 1$ (G_1). A separating equilibrium fails the Intuitive Criterion if there exists a deviation that is profitable for G_1 if voters believe that G_1 is the deviator but is not profitable for G_0 (under any voters' belief). In a separating equilibrium in which $A^* > \Pi_L(p_1)$, such a deviation exists: suppose that G_1 offers $A' = \Pi_L(p_1) + \epsilon$ and that voters believe the deviator is G_1. This deviation is profitable for G_1 (who gets $\Pi_H(p_1) - \Pi_L(p_1) - \epsilon$) but it is not profitable for G_0 (who gets $-\epsilon$). We conclude that the only separating equilibrium that survives the Intuitive Criterion has $A^* = \Pi_L(p_1)$.

We now show that all pooling equilibria fail the Intuitive Criterion. In a pooling equilibrium, A_1 may be zero or strictly positive.

In pooling equilibrium with $A_1 = 0$, voters believe that a candidate with $A_1 = 0$ is of average quality (y is equally likely to be 0 or 1). This pooling equilibrium fails the Intuitive criterion if there exists a a deviation that is profitable for G_1 if voters believe that G_1 is the deviator but is not profitable for G_0. In this equilibrium, the expected payoff of G is zero and the election cutoff for Candidate 1 is $\bar{z}^P(x, m)$. Consider instead the following deviation. Let $\bar{p}^d$ be the unique $p > 0$ such that $\bar{z}(x, 1, p) = \bar{z}^P(x, m)$.

[30] For a detailed application of the Intuitive Criterion to signalling games with money burning, see Milgrom and Roberts [27].

Consider the deviation $p^* = p^d$ with $0 < p^d < \bar{p}^d$ and $\Pi_L(p^d) < A_1^* < \Pi_H(p^d)$. Such deviation is profitable to G if and only if $y = 1$. 1 accepts the offer because it strictly increases his chances of elections.

Lastly, it is immediate to see that a pooling equilibrium with $A_1 > 0$ fails the Intuitive Criterion because there exists a credible and profitable deviation (for both types) to $A_1 = 0$. $\square$

7.4 No-Split-Contribution Equilibrium

The electoral game of Section 4 has another separating equilibrium besides the one characterized in Proposition 3:

Proposition 10 *Let $\underline{p}$ be defined as in Proposition 3. The following is a sequential equilibrium of Game 2:*

(i) Voters' beliefs:

$$\tilde{\theta} = \begin{cases} \hat{\theta}(1, z) & \text{if } A_1 \geq \Pi_M(p^*) \text{ and } A_2 < \Pi_M(p^*) \\ \hat{\theta}(0, z) = \check{\theta}(z) & \text{if } \max(A_1, A_2) < \Pi_M(p^*) \text{ or } \min(A_1, A_2) \geq \Pi_M(p^*) \\ \hat{\theta}(-1, z) & \text{if } A_1 < \Pi_M(p^*) \text{ and } A_2 \geq \Pi_M(p^*) \end{cases}$$

(ii) Voters' choice: $e_i = 1$ if and only if $\tilde{\theta} - u(p_i - p_1) + u(p_i - p_2) > 0$. 1 is elected if and only if $\tilde{\theta} - u(m - p_1) + u(m - p_2) > 0$

(iii) Group's offer: For $y \in \{-1, 0, 1\}$, $p^* = P$ and

(a) If $y = -1$, $A_2^* = \Pi_M(P)$ and $A_1^* = 0$;

(b) If $y = 0$, $A_1^* = A_2^* = 0$;

(c) If $y = 1$, $A_1^* = \Pi_M(P)$ and $A_2^* = 0$.

where

$$P = argmax_p \Pi_H(p) - \Pi_M(p)$$

subject to $P \leq \underline{p}$.

(iv) Candidates' acceptance: for $j = 1, 2$, Candidate j accepts A_j^* if and only if $A_j^* \geq \Pi_M(p^*)$ and $p^* \leq \underline{p}$. If j rejects, $p_j = m$.

Proof: Points (i), (ii), and (iv) are shown analogously to Proposition 3. Point (iii) is shown in two steps:

Step 1: If p^* is held fixed, then the following is an optimal strategy for G:

(i) If $y = -1$, $A_2^* = \Pi_M(p^*)$ and $A_1^* = 0$;

(ii) If $y = 0$, $A_1^* = A_2^* = 0$;

(iii) If $y = 1$, $A_1^* = \Pi_M(p^*)$ and $A_2^* = 0$.

Proof of Step 1: Given (iv), G can restrict w.l.o.g. her attention to contributions $(A_1^*, A_2^*) \in \{0, \Pi_M(p^*)\}^2$. Hence, for a given y, there are four stretegies:

(a) $A_1^* = A_2^* = 0$.

(b) $A_1^* = \Pi_M(p^*), A_2^* = 0$.

(c) $A_1^* = 0, A_2^* = \Pi_M(p^*)$.

(d) $A_1^* = A_2^* = \Pi_M(p^*)$.

If $y = 1$,

In addition to the inequalities used for Proposition 3, the following inequality is useful: $\Pi_M(p^*) \geq \frac{1}{2}\Pi_{\text{all}}(p^*)$. This inequality if one thinks that $\Pi_{\text{all}}(p^*) - \Pi_M(p^*)$ must be equal to the gross expected profit of a candidate who is M and is revealed as bad.

Let π denote the net expected profit of G. If $y = 1$, the net profits for each of the four strategies above are:

$$\begin{aligned}
\pi_a &= 0; \\
\pi_b &= \Pi_H(p^*) - \Pi_M(p^*); \\
\pi_c &= \Pi_L(p^*) - \Pi_M(p^*); \\
\pi_d &= \Pi_{\text{all}}(p^*) - 2\Pi_M(p^*).
\end{aligned}$$

As π_b is greater than π_a, π_c, and π_d, (b) is a best response when $y = 1$. Next, if $y = 0$, we have

$$\begin{aligned}
\pi_a &= 0; \\
\pi_b &= \Pi_M(p^*) - \Pi_M(p^*) = 0; \\
\pi_c &= \Pi_M(p^*) - \Pi_M(p^*) = 0; \\
\pi_d &= \Pi_{\text{all}}(p^*) - 2\Pi_M(p^*).
\end{aligned}$$

As $\pi_a = \pi_b = \pi_c$ and $\pi_a > \pi_d$, (b) is a best response when $y = 0$. The case $y = -1$ is symmetric to $y = 1$ and is omitted.

Step 2: Given Step 1, P is the optimal p^*.

Proof of Step 2: Immediate. □

In any separating equilibrium, voters must be able to tell whether candidates are of the same quality ($y = 0$) or one is better than the other ($y \neq 0$). This differentiation can be achieved in two ways. In the Split-Contribution Equilibrium, if $y = 0$ both candidates receive a contribution and if $y \neq 0$ only the better candidate spends money. In the No-Split-Contribution Equilibrium, if $y = 0$ neither candidate receives a contribution and if $y \neq 1$ only the better candidate receives money. Thus, the difference lies in the case $y = 0$.

In the Split-Contribution Equilibrium, the threshold is $\Pi_H - \Pi_{\text{all}}$, while in the other equilibrium it is $\Pi_H - \Pi_M$. The first threshold is obviously lower than the second one. That is why, when $y = 0$, G is willing to contribute to both candidates in the Split-Contribution Equilibrium but not in the other one.

Which of the two equilibria is more plausible? In the comparison between two separating equilibria, the Intuitive Criterion of Cho and Kreps [9] is difficult to apply. We can try with the notion of *undefeated equilibria* proposed by Mailath, Okuno-Fujiwara, and Postlewaite [25]. The notion of undefeated equilibrium is very general but for our purpose it is enough to focus on a signaling game with a Sender with two types and a Receiver. Suppose the game has two sequential equilibria, E_1 and E_2, and that both types of sender have a higher payoff in E_1 than E_2. Then, we say that E_1 *defeats* E_2 and we expect that E_2 will not arise. The reason is simple. If we are in E_2 and the Sender sends a message m that is an out-of-equilibrium message for E_2 but is an equilibrium message for E_1, then the Receiver should think that the Sender is playing E_1 rather than E_2. As E_1 is better than E_2 for both types, then both types are willing to use m and E_2 does not arise.

Here, we have two types of Senders, G with $y = 0$ and G with $y \neq 0$ (the cases $y = 1$ and $y = -1$, being perfectly symmetric, can be treated in the same way). We say that the Split-Contribution Equilibrium defeats the No-Split Contribution Equilibrium if both types of G are better off in a Split-Contribution Equilibrium. A difference between Mailath, Okuno-Fujiwara, and Postlewaite and the present model is that here the role of senders is shared between G and the candidates. Hence, we also require that candidates are not worse off in a Split-Contribution Equilibrium.

Proposition 11 *The Split-Contribution Equilibrium defeats the No-Split-Contribution Equilibrium.*

Proof: From Proposition 10, the expected payoff of G under NSCE is 0 if $y = 0$ and $\Pi_H - \Pi_M$ if $y \neq 0$. From the proof of Proposition 3, the expected payoff under SCE is $2\Pi_H - \Pi_{\text{all}}$ for any y. By Lemma 5, $2\Pi_H - \Pi_{\text{all}} > \Pi_H - \Pi_M$ (because $\Pi_H + \Pi_M > 2\Pi_M > \Pi_{\text{all}}$). Obviously, $2\Pi_H - \Pi_{\text{all}} > 0$. The expected payoff for G is strictly higher under SCE than under NSCE for any y. $\square$

G always prefers the Split-Contribution Equilibrium. If voters and candidates know that, they should anticipate that G will play according to the Split-Contribution Equilibrium. Then the No-Split-Contribution Equilibrium is unlikely to arise.

7.5 Perfectly Informative y_1.

In this model y_1 is not completely informative. Suppose on the contrary that y_1 were perfectly informative (or that G observes θ_1 as in Gerber [15]). For instance: $\theta_1 \in \{0, 1\}$ and $y_1 = \theta_j$. Then, $\hat{\theta}(x_1, y_1, z_1) = y_j$ and, from (1), one can see that $\Pi_H(p) = \Pi_L(p)$. There would exist a separating equilibrium.[31] However, it would be quite arbitrary. In equilibrium, by (4), $A^*(p^*) = \Pi_H(p^*) = \Pi_L(p^*)$, so that G would be indifferent between contributing and not contributing *both* when she has observed $y_1 = 0$ and when she has observed $y_1 = 1$.

To illustrate the brittleness of such a separating equilibrium, assume that G must pay an infinitesimal amount ϵ to observe y_1. Because ϵ is a sunk cost, it does not influence $\Pi_y(p^*)$. G's net expected payoff would be $r(p^*) \equiv [\Pi_H(p^*) - \Pi_L(p^*)] \Pr(y_1 = 1) - \epsilon$. If Y_1 is perfectly informative, $\Pi_H(p^*) = \Pi_L(p^*)$ and $r(p^*) = -\epsilon$. G would not observe y_1 and the separating equilibrium would not exist. If on the contrary y_1 is not perfectly informative, then $\Pi_H(p^*) \gg \Pi_L(p^*)$ and $r(p^*) > 0$, so that G would pay ϵ to observe y_1.

[31] I thank Randolph Sloof for pointing out the existence of a separating equilibrium in this case.