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Kamal Saggi and Nikolaos Vettas

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Centre for Economic Policy Research
90–98 Goswell Rd
London EC1V 7RR
Tel: (44 171) 878 2900
Fax: (44 171) 878 2999
Email: cepr@cepr.org

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ABSTRACT

On Intrabrand and Interbrand Competition: The Strategic Role of Fees and Royalties*

We examine oligopolistic markets with both intrabrand and interbrand competition. We characterize equilibrium contracts involving a royalty (or wholesale price) and a fee when each upstream firm contracts with multiple downstream firms. Royalties control competition between their own downstream firms at the expense of making them passive against rivals. When we endogenize the number of downstream firms, we find that each upstream firm chooses to have only one downstream firm. This result is in sharp contrast to previous literature where competitors benefit by having a larger number of independent downstream firms under only fixed fee payments. We discuss how allowing for contracts that involve both fees and per-unit payments dramatically alters the strategic incentives.

JEL Classification: L13, L14, L22, L42
Keywords: intrabrand competition, strategic contracting, two-part tariffs, royalties

Kamal Saggi
Department of Economics
Southern Methodist University
Dallas
TX 75275–0496
USA
Tel: (1 214) 768 3274
Fax: (1 214) 768 1821
Email: ksaggi@mail.smu.edu

Nikolaos Vettas
The Fuqua School of Business
Duke University
Durham
NC 27708–0120
USA
Tel: (1 919) 660 7756
Fax: (1 919) 684 2818
Email: nikos.vettas@duke.edu

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NON-TECHNICAL SUMMARY

In many market transactions firms do not sell their products directly to final consumers. Vertical market relations include licensing of technology to manufacturers, franchising, manufacturers selling their products through retailers and firms that supply parts or intermediate products to other firms. In this paper we explore how strategic considerations shape vertical contracts in oligopolistic markets.

We consider two-part tariffs, contracts that specify a fixed fee and a per-unit payment that we call royalty (in our framework, this per unit payment can also be viewed as a wholesale price). Such payment schemes are widely prevalent in vertical relations between 'upstream' firms (e.g. producers) and 'downstream' firms (e.g. retailers). The paper focuses on two key strategic decisions of upstream firms: the number of independent downstream firms and the terms of the contracts. In the model, oligopolistic competition is both 'intrabrand' and 'interbrand'. Specifically, we allow each of the upstream oligopolists to be associated with an arbitrary number of (oligopolistic) downstream firms. Thus, our analysis nests as special cases two situations that have been examined in past work, the case of a monopolist contracting with multiple downstream firms, as well as upstream oligopolists each contracting with a single downstream firm.

When choosing royalty rates, an upstream firm has to balance two opposing incentives. It prefers its downstream firms to be more 'passive' against one another (passive meaning that they choose a lower output level), but it would like them to be committed to more 'aggressive' behaviour against rival downstream firms. In equilibrium, the larger the number of its own downstream firms relative to that of rival firms and the higher the degree of differentiation between products, the higher the royalty rate chosen by an upstream firm. More generally, our analysis implies that higher competition between rival upstream firms and lower competition between downstream firms contracting with the same upstream firm is expected to decrease royalty rates.

Building on the characterization of competition with an arbitrary number of downstream firms, we then endogenize the number of downstream firms. We find that each upstream firm has a strategic incentive to minimize the number of its independent downstream divisions. In equilibrium, each upstream firm chooses to have only one downstream firm. This result is in sharp contrast to previous work showing that, when only fixed fee payments can be used, upstream firms prefer to increase the number of their downstream firms for strategic reasons. In particular, in related previous work, a higher number of downstream divisions represents commitment to more aggressive behaviour,

a ‘divide and conquer’ strategy. We show how allowing for contracts that involve both fees and per-unit payments dramatically alters the strategic incentives. While an upstream firm can give its downstream firms the right marginal incentives to produce the desired target output level, a larger number of downstream firms implies that, for any royalty rate, the rival has a stronger incentive to behave strategically. This result may have important implications for the study of vertical markets. The insight that a larger number of downstream divisions implies a stronger strategic position is often used when trying to explain ‘divisionalization’ policies of firms. Since, in addition to fees, per-unit payments are often used in markets, our analysis shows that these arguments require qualifications.

We also consider a number of modifications of the basic model. When firms choose royalties (and fees) sequentially rather than simultaneously, we find that our main result has to be modified slightly. Upstream firms have a strict incentive to minimize the number of their downstream firms when, in the subsequent stage, they pick their royalties simultaneously or before their rival, while when they choose royalty rates after their rival their profit does not depend on the number of their downstream firms. It is crucial, however, that in the sequencing of strategic choices, the choice of royalties follows that of the number of firms. We consider alternative scenarios such as fee-only contracts or royalties chosen before (or simultaneously with) the number of firms. Then we find that, as long as royalties are viewed as given at any level that is not ‘too low’, the strategic incentive for commitment via a higher number of firms is present. Also, in this case, it is shown that as products become more differentiated, a lower number of downstream firms, instead of a higher, becomes profitable.

Finally, we examine the equilibrium when only royalties can be charged (and no fixed fees). Then the result that both upstream firms prefer to minimize the number of their downstream firms is no longer true. In fact, assuming that one upstream firm is choosing to have a single downstream firm, the other upstream firm has an incentive to increase its number of firms without end. It follows that for firms to have a strategic incentive to decrease the number of their downstream divisions, contracts have to involve both fixed fees and per-unit payments, not just one of these instruments. We also discuss the implications of this observation for comparing the royalty-only and the fee-and-royalty cases.

1. Introduction

In a large number of market transactions, firms do not sell their products directly to final consumers. Vertical market relations are therefore important to understand. Such relations include licensing of technology to manufacturers, franchising, manufacturers selling their products through retailers, and firms that supply parts or intermediate products to other firms. In this paper we explore how strategic considerations shape vertical contracts in oligopolistic markets. We consider two-part tariffs, contracts that specify a fixed fee and a per-unit payment which we call royalty.[1] Such payment schemes are widely prevalent in vertical relations. The paper focuses on two key strategic decisions of "upstream" firms: the number of independent "downstream" firms and the terms of the contracts.

A central feature of the analysis is that we allow oligopolistic competition to be both *intra*brand and *inter*brand. Specifically, we characterize fee-and-royalty contracts (and downstream competition) when each of the upstream oligopolists is associated with an arbitrary number of (oligopolistic) downstream firms. Past work on strategic contracting has developed important insights for the case of a monopolist contracting with multiple downstream firms and for upstream oligopolists each contracting with a single downstream firm.[2] Our analysis nests these two situations as special cases.[3]

Given the characterization of competition with an arbitrary number of downstream firms, we then move to the main focus of the paper by endogenizing the number of downstream firms. Our main result is that each upstream firm prefers to *minimize* the number of its independent downstream firms. In equilibrium, each upstream firm chooses to have only one downstream firm. This result is in sharp

[1] In our framework, the per unit payment can be also viewed as a wholesale price.

[2] Or, more generally, contracting with non-strategic downstream firms.

[3] Bonanno and Vickers [2], p.264, refer to the first of these two cases as (1,n) and the second as (m,1). Our analysis contributes to the understanding of the (m,n) case.

contrast to previous work showing that upstream firms prefer to have a *higher* number of downstream firms for strategic reasons. In particular, Baye, Crocker, and Ju [1] have shown that, in a two-stage game where upstream firms choose the number of their downstream divisions and quantity competition follows in the downstream market, there is an incentive to increase the number of downstream divisions since this practice represents a commitment to more aggressive downstream behavior, a "divide and conquer" strategy.[4] The reason why our implications are so diametrically different from previous work on the strategic choice of downstream divisions is that such work has considered the case of only fee payments from the downstream to the upstream firms. We show how allowing for contracts that involve both fees and per-unit payments dramatically alters the strategic incentives. This result may have important implications for the study of vertical markets. The insight offered by previous work, that a larger number of downstream divisions implies a stronger strategic position, is often used when trying to explain "divisionalization" policies of firms. Since, in addition to fees, per-unit payments are often used in markets, our analysis shows that these arguments require important qualifications.[5]

Section 2 presents the basic model. Formally, we consider a three-stage game. Two upstream firms choose the number of their independent (i.e. maximizing own profit) downstream firms and then choose their royalty rates (and fees). In the last stage, downstream firms compete in quantities. Demand is parameterized in a way that allows us to consider how competition changes as we move from perfect substitutes to upstream monopoly. As a first step, in Section 3 we take the number of

[4] The term is from Polasky [22] who, like Corchón [4], also demonstrates this strategic benefit of a larger number of downstream firms. In Baye, Crocker, and Ju [1] an equilibrium with a finite number of firms is found by specifying a cost for each additional division. This point is also related to the literature showing that, for strategic reasons, horizontal mergers may not be profitable (see e.g. Salant, Switzer, and Reynolds [25]).

[5] In fact, our result only requires that firms have the *ability* to pick royalty rates after they have chosen their downstream firms, not necessarily that they choose non-zero royalty rates in equilibrium.

downstream firms as given and examine the last two stages of the game. The goal is to characterize contracts and competition with an arbitrary number of downstream firms per upstream and, in particular, examine how behavior depends on the number of downstream competitors. When choosing royalty rates, an upstream firm has to balance two opposing incentives. On the one hand, it prefers its downstream firms to be more *passive* against one another. On the other hand, it would like them to be committed to more *aggressive* behavior against rival downstream firms. In equilibrium, the larger the number of its own downstream firms relative to that of rival firms and the higher the degree of differentiation between products, the higher the royalty rate chosen by an upstream firm. More generally, higher competition between rival upstream firms and lower competition between downstream firms contracting with the same upstream firm is expected to decrease royalty rates.[6]

To see the key tension in this framework, it is useful to start with the intuition from previous work that examines the two extreme situations mentioned earlier. Consider first an upstream monopoly that contracts with multiple oligopolistic downstream firms. This case represents pure intrabrand competition. Then the upstream firm can extract the entire downstream monopoly profit by using a combination of a fee and a royalty. The royalty forces the downstream firms to internalize the horizontal

[6] To illustrate this implication, consider a software or microchip producer who supplies multiple computer manufacturers and faces little competition for his product. Our analysis implies that, to the extent that the supplier has the bargaining power, he would tend to offer contracts that involve relatively high royalties, with the goal of reducing the competition between computer manufacturers. In contrast, to shift to a different industry, consider a fast-food company which faces severe competition from other fast-food companies and has franchising agreements with a number of (downstream) sellers. In this case, we expect that the franchising agreements will tend not to put much weight on royalties, because the fast-food company wants to strengthen the strategic position of its franchisees relative to rival franchisees. This would be especially true in the case of a newcomer into the industry which has only a small number of franchises and is competing with established firms with a larger number of franchisees. Empirically, while fees and royalties are widely used, there are indeed differences with some contracts specifying relatively higher royalties and other higher fees.

externality that exists among them and raises the market price from the oligopoly to the monopoly level, while the fee transfers the monopoly profit upstream. Essentially, the upstream firm shifts the downstream firms' reaction functions and makes them more passive against one another. This intuition is central in Dixit [6], Mathewson and Winter [18], and other work that examines optimal contracts and "minimally sufficient instruments" to replicate vertical integration outcomes.[7] Next consider pure upstream (or interbrand) competition: two upstream firms each contracting with a single downstream. In contrast to the previous case, now, under Cournot competition, each upstream competitor would like its downstream firms to be more aggressive. It follows that the equilibrium involves non-positive royalties.[8] This result is known from the "strategic contracting and delegation" literature which includes, for example, Brander and Spencer [3] in the context of international trade policy, and Bonanno and Vickers [2], Fershtman and Judd [8], McGuire and Staelin [21], Sklivas [26] and Vickers [28] on delegation and managerial incentives. The central insight of this research is that competing principals may have a (unilateral) incentive to make their agents commit to more aggressive behavior (and, in our framework, this commitment could take place through the specification of lower royalties). In out framework, with multiple downstream competitors for each upstream firm, forces related to both intrabrand as well as interbrand competition are responsible for shaping vertical contracts.[9]

The main result of the paper is in Section 4. It is shown that, anticipating how competition will be in the royalty and the downstream stage, each upstream firm prefers to minimize the number of its downstream firms. In particular, this incentive is present regardless of the number of rival downstream firms and thus in the unique

[7] See Katz [13] for a survey on vertical contracts; in particular, for downstream price competition as a negative externality, see pp.678-679.

[8] These correspond to wholesale unit prices lower than the upstream cost.

[9] When taking the number of downstream firms as given, our analysis and the spirit of results are related to Dixit [7] who explores aspects of trade policy with multiple competitors.

subgame perfect equilibrium each upstream firm has only one downstream.[10] As noted above, in Baye, Crocker, and Ju [1] and related analyses there is a strategic incentive to increase the number of downstream firms. In our framework, competition and firms' marginal incentives in the downstream market can be controlled through the choice of either the number of firms or royalties. But why is it preferable from a strategic viewpoint to help downstream firms commit to more aggressive behavior by choosing low royalties instead of (also) choosing a larger number of firms? As shown in the analysis, an upstream firm can give its downstream firms the right marginal incentives to produce the desired target output level. However, a larger number of downstream firms implies that, for any royalty rate, an upstream firm's brand (that is, aggregate) reaction function becomes more aggressive and thus the rival has a stronger incentive to behave strategically.[11]

To gain a precise understanding of the strategic interaction between royalties and the number of firms, we also consider a number of modifications of the basic model. In Section 5 we consider the case where firms choose royalties (and fees) sequentially rather than simultaneously. We find that our main result has to be modified only slightly. More precisely, upstream firms have a strict incentive to minimize the number of their downstream firms when, in the subsequent stage, they pick their royalties simultaneously or before their rival, while when they choose royalty rates after their rival their profit does not depend on the number of their downstream firms. It is crucial, however, for the main result, that the choice of royalties follows that of the number of firms. In Section 6 we consider alternative scenarios such as fees only or

[10] This is true regardless of the degree of differentiation between products (as long, of course, as there is some degree of substitution).

[11] That, under some conditions, it may be desirable for an upstream firm to restrict the number of its downstream is also a feature of some previous work including Kamien and Tauman [12] and Katz and Shapiro [14] on licencing an innovation. There is a fundamental difference between such approaches and ours since they consider a single upstream firm whereas upstream competition is critical for our results.

royalties chosen before (or simultaneously with) the number of firms. Then, with some qualification, the logic of previous work (Baye, Crocker and Ju [1]) applies and each firm prefers a higher number of firms. In other words, we find that as long as royalties are viewed as given at any level that is not "too low" (not just at zero), the strategic incentive for commitment via a higher number of firms is present. This result requires products to be close enough substitutes: we also generalize previous work that considers only fees by allowing the degree of differentiation between products to vary (as products become more differentiated a lower number of downstream firms, instead of a higher, becomes profitable).

In addition to royalties not being fixed when the number of firms is chosen, it is also critical that contracts involve not only royalties but fees as well. Section 7 explores the equilibrium when only royalties can be charged. Then the result that both upstream firms prefer to minimize the number of their downstream firms is no longer true. In fact, assuming that one upstream firm is choosing a single downstream firm, the other upstream firm has an incentive to increase its number of firms without end. We also compare the royalty-only and the fee-and-royalty cases. For a fixed number of downstream firms, upstream firms may be better off with royalty-only contracts than with fee-and-royalty contracts, the intuition being that competition with fee-and-royalty contracts implies royalty rates that are too low from the viewpoint of joint profit maximization. If now firms could choose the type of their contracts, they may find themselves in a prisoners' dilemma situation (with both firms better-off under royalty-only contracts but having a unilateral incentive to pick fee-and-royalty contracts). This result is already present in previous work examining similar situations.[12] By endogenizing the number of downstream firms our analysis allows

[12] In particular, Gal-Or [10] shows that, under price competition in the downstream market, upstream firms may benefit from a commitment to not using a fee or other "vertical restraints". Under these circumstances, firms choose higher wholesale prices, which facilitates sustaining higher retail prices. In our model, with quantity competition between firms contracting with different upstream, these incentives are reversed, and thus our analysis is different. Whereas our results also include

us to see that this prisoners' dilemma structure may not exist, since fee-and-royalty contracts imply an incentive to minimize the number of downstream firms. As noted above, this incentive is not present under royalty-only contracts.

As is clear from the above discussion, this paper is related to a number of important literatures. In particular, it creates a link between work on strategic contracting and work exploring the choice of downstream divisions. In addition to the ones mentioned above, there are a number of other related papers and, while space does not allow us to review all of them, we discuss here the most relevant. Our paper is related to Rey and Stiglitz [23] who show that vertical restraints, which may affect intrabrand competition, can be used to reduce interbrand competition. They consider exclusive territories which, like picking a single downstream firm, eliminate downstream competition. However, the two analyses are very different. First, Rey and Stiglitz [23] do not examine oligopolistic intrabrand competition, they only compare perfect competition and exclusive territories. As a result, they do not analyze the role of the number of downstream competitors which is at the heart of our approach. Second, they consider downstream price competition and the strategic motives are very different. They show that with exclusive territories and prices being strategic complements one's rival retailers have an incentive to increase price (as a response to one's higher price and that of one's retailers). As a result, there is a lower perceived elasticity of demand and higher prices.[13] In contrast, our result that a lower number of downstream firms is desirable requires that both fees and royalties can be used; when only royalties can be used, the result does not hold and the strategic incentives are actually

that firms may benefit when fees cannot be used, in our case this result is not true when each upstream firm has only one downstream, but relies on the existence of multiple downstream firms. Also, in Gal-Or [10] each upstream firm contracts with only one downstream, whereas we examine competition between (multiple) downstream firms. See also Rey and Stiglitz [23] (Proposition 3) for a related result.

[13] See also Slade [27] for related empirical work that emphasizes the interaction between exclusive-dealing clauses and strategic factors.

reversed. In other related work, Kühn [15] shows that wholesale prices can be used as a commitment to relax competition between upstream firms. He considers fully nonlinear wholesale schedules and finds that (with constant marginal costs) the number of downstream firms is irrelevant for strategic contracting.[14] In contrast, our analysis, under fee-and-royalty contracts, assigns a critical role to the number of downstream firms. The difference is due to the fact that with fee-and-royalty contracts upstream firms can only control the position of the downstream firms' reaction functions but with fully nonlinear contracts the slope can be controlled as well. While fee-and-royalty contracts are, of course, restrictive in that the marginal price does not vary with quantity, they are very common in many markets.[15] For this reason, and also since much of the previous work to which we want to compare our results considers fee-and-royalty contracts, it is important to understand the strategic role of the number of competitors under such contracts.

It may be important to discuss at this point why upstream firms may be concerned with affecting the downstream firms' behavior through the design of vertical contracts instead of directly determining the final product prices. The reason is that this practice may not be an available option. In particular, firms are not allowed to engage in such "resale price maintenance," which is *per se* illegal, under U.S. antitrust laws.[16] Of course, in some vertical structures firms attempt to indirectly determine prices through the use of "suggested prices" or other means.[17] Downstream firms

[14] With increasing marginal costs, however, an increase in the number of retailers reduces equilibrium prices (see pp.55-56).

[15] In general, the class of contracts that can be written should depend on the informational structure. See Rey and Tirole [24] for informational assumptions underlying two-part tariffs. An upstream firm may be able to only observe the amount of the product supplied to a downstream but not the actual quantity sold to final consumers. Then, if marginal prices are not equal for all quantities, downstream firms may set up a secondary market for the product.

[16] See Ippolito [11] for an analysis of evidence from litigation.

[17] The Monopolies and Mergers Commission in the U.K. has recently decided to ban the use of

may try to avoid following such suggestions if they are not in their individual interest and may succeed in doing so if monitoring is costly.[18]

Our analysis demonstrates a strategic incentive to decrease the number of independent downstream divisions. Of course, in reality there are additional considerations implying that firms may choose to contract with more than one downstream firm. Such considerations may be, for example, related to the technology of production or distribution, the information structure, or legal restrictions. The model analyzed here helps highlight a strategic effect that contributes towards shaping vertical relations. More generally, to simplify the model and focus on the purely strategic motives, we abstract from a number of issues that are, without doubt, important for vertical contracts but have been analyzed elsewhere. In particular, we do not examine the implications of uncertainty. Allowing for other such factors may, of course, modify the results. For example, a royalty may be desirable for risk-sharing or incentives reasons even if it is not desirable for strategic reasons. Previous work has examined such other factors; the focus of our study is on strategic incentives.[19]

"recommended retail prices" by electric-appliance makers. See *The Economist*, May 31, 1997, p.53.

[18] In general, the ability to set prices may be left in the hands of downstream firms for the additional reason (left unmodelled in this analysis) that they may have better information about demand.

[19] Several other important factors that affect contract design have been examined in the literature. They include (adverse) selection, risk-aversion, and incentives (moral hazard). See, for example, Mathewson and Winter [19] for a theoretical analysis from an agency viewpoint, and Lafontaine [16] for an overview of theoretical results as well as an empirical study. In particular, one of the findings in this last paper, that empirically fees and royalties are not negatively related, is inconsistent with many other models but is consistent with our framework: higher royalties make the downstream market less competitive and thus the fees that can be extracted are not necessarily lower. See also Lafontaine and Slade [17] for a discussion of empirical regularities in vertical contracts.

2. The basic model

We consider two upstream firms, each of which may be contracting with multiple downstream firms.[20] We denote the two upstream firms by A and B. The number of downstream firms associated with upstream firm i is $n_i \geq 1$, $i = A, B$.

To simplify the analysis, we assume that production cost is zero both at the upstream and the downstream stage. As long as unit production costs are constant, this assumption is without loss of generality. To capture the idea that the product sold by the downstream firms contracting with one upstream firm may be differentiated from the product sold by firms contracting with a different upstream, we assume that the market demand function is as follows:

$$p_i = a - Q_i - sQ_j, \qquad i, j = A, B, \tag{1}$$

where

$$Q_i = \sum_{k=1}^{n_i} q_i^k, \qquad i = A, B$$

denotes the aggregate output of all the downstream firms selling product i (each downstream firm contracting with upstream firm i produces quantity $q_i^k, k = 1, ...n_i$ and sells at a price p_i). The parameter $s \in [0, 1]$ in the demand function measures the degree of substitution between the two products. Clearly, when $s = 1$ the two products are perfect substitutes whereas when $s = 0$ they are perfectly differentiated.

We study a three stage game. In the first stage, the two upstream firms simultaneously choose the number of their downstream firms, $n_i \geq 1$, $i = A, B$. Next, the two upstream firms (simultaneously) make a take-it-or-leave-it offer to each of their downstream firms that specifies a pair (f_i, r_i), where f_i is a fixed fee (independent of sales level) and r_i is a royalty rate per unit sold.[21] Thus, the total payment (to an

[20] The case of more than two upstream firms represents a straightforward extension of the model.

[21] Placing all the bargaining power in the hands of upstream firms may represent the fact that there is a competitive supply of potential downstream firms. This assumption is standard in this

upstream firm) of a firm that produces q units equals

$$f_i + r_i q.$$

The outside option of each downstream firm is normalized to zero. We assume that the same contract is offered to all of downstream firms associated with a given upstream firm.[22] Finally, in the third stage of the game, firms that accept the contract compete in quantities in the final market. In particular, firms in the downstream market compete in quantities. At this stage the possibility of differentiation, as measured by s, is also allowed.

We consider the subgame perfect equilibrium of this game. Before proceeding to the analysis, a few remarks are in order. First, note that this formulation admits as special cases situations that have been studied in the literature. If $n_A = n_B = 1$ (only one downstream firm for each upstream) we have the case of pure interbrand competition. This has been the focus of much of the strategic contracting literature.[23] If, on the other hand, we have $s = 0$ (or $n_j = 0$) then there is only intrabrand competition. Furthermore, the second stage of the basic game can be slightly modified to isolate the role royalties and fees play in transferring profits (from downstream to upstream firms). In particular, later in the paper we compare scenarios which differ from the basic model in that firms are able to use only one of the instruments. The case where only fixed fees are used corresponds to the literature where the only strategic variable is the number of downstream firms, as in Baye, Crocker and Ju [1].

Second, the timing adopted here (choosing downstream firms before royalties) reflects a situation where it is more costly to change the number of downstream

class of models.

[22] In other words, we do not allow discrimination in the form of offering different contracts to different firms. In reality, there is indeed uniformity observed regarding contracts of the same firm - see, for example, Lafontaine and Slade [17], pp.15-16. In some cases, legal restrictions also contribute to this wholesale-price uniformity.

[23] See the introduction for a discussion and references.

firms than to change royalties (or wholesale prices) and appears to be a natural assumption in many markets. Later in the paper we discuss in detail the implications of alternative timing assumptions, with the number of downstream firms chosen after the royalty rates or with both the number of firms and royalties chosen simultaneously (see Section 6).

Finally, we assume quantity competition in the downstream market. With respect to this modelling choice, both the usual criticism of quantity competition models ("firms choose prices") as well at its usual defences (including capacity precommitment) apply. As known from the strategic contracting literature, is does matter whether the strategic variables in the downstream market are prices or quantities, since this determines if the downstream reaction functions are increasing or decreasing.[24] In our framework, quantity competition represents a simple formulation that allows us to capture in a convenient way the tension that a firm may want its downstream firms to be committed to more passive behavior against its own firms but more aggressive against rivals. This tension has implications for the design of contracts. Therefore it seems appropriate that in the following analysis we adopt the quantity competition formulation of the downstream market. Moreover, the quantity competition assumption is adopted in previous work (cited above) on the strategic choice of downstream divisions, and maintaining this assumption facilitates the comparison of our results to previous results.

3. Analysis: competition with given numbers of downstream firms

We first take the number of downstream firms, n_A and n_B, as given and proceed to analyze the choice of contracts and downstream quantity competition. In other words, we analyze the last two stages of the game described above. The analysis

[24] A price competition formulation would imply higher royalties than our formulation.

here is, of course, a necessary step before one can study the choice of number of downstream firms. Furthermore, the analysis of competition with a given number of downstream is also of independent interest. In some markets, the downstream firms may not fully control the number of their downstream firms and this number may reflect a number of different factors. While such factors are outside the present model it is nevertheless interesting to study competition when there are multiple upstream firms that contract with multiple downstream firms.

3.1. Equilibrium royalties and quantities

Suppose that downstream firms have accepted contracts that involve royalty rates r_i. The fees specified in the contracts clearly do not affect the downstream firms output decision; they only determine whether they want to operate in the market or not. Profit (net of royalty payments) for a downstream firm m that accepts a contract involving a royalty rate r_i equals:

$$(a - Q_i - sQ_j - r_i)q_i^m, \qquad i, j = A, B. \tag{2}$$

where again Q_i denotes the aggregate quantity produced by all downstream firms that contract with firm i, $i = A, B$. Taking the quantities produced by all other firms as given, the reaction function of downstream firm m is

$$q_i^m(Q_{i,-m}, Q_j; r_i) = \frac{1}{2}(a - Q_{i,-m} - sQ_j - r_i), \tag{3}$$

where $Q_{i,-m} \equiv \sum_{k=1,k\neq m}^{n_i} q_i^k$ denotes the aggregate quantity produced by all other downstream firms contracting with firm i except for firm m . Then, we can solve the system of downstream reaction functions (3) for the equilibrium output levels (as functions of the number of firms and royalty rates)

$$q_i = \frac{a - r_i - n_j(a(s-1) + r_i - sr_j)}{1 + n_i + n_j + n_i n_j(1 - s^2)}. \tag{4}$$

The above expression represents equilibrium behavior in the third stage of the game (taking as given the number of firms and royalty rates). The next step is to determine the equilibrium royalty rates. Substituting the output levels (4) into (2), we obtain the downstream profits. Since the upstream firms have all the bargaining power and can make take-it-or-leave-it offers, they extract the entire residual profit of downstream firms by choosing a fee equal to their after-royalty profit. Therefore, taking as given the royalty rate of firm j, the objective of upstream firm i is to choose its royalty rate to maximize the aggregate profit of its downstream firms:

$$\underset{r_i}{Max} \;\; \pi^i(r_i, r_j; n_i, n_j) = n_i p_i q_i,$$

where per-firm quantities are given by (4) and prices by (1). This optimization generates the following royalty reaction functions of upstream firms:

$$r_i(r_j; n_i, n_j) = \frac{[n_j((s^2-1)n_i+1) - n_i + 1]\,[n_j(a(s-1) - sr_j) - a]}{2(n_j+1)[(1-s^2)n_j+1]n_i}, \qquad i, j = A, B. \tag{5}$$

Solving the system of the two royalty reaction functions we obtain:

Proposition 1. *With two upstream firms, each contracting with* n_i, $i = A, B$ *downstream firms, the equilibrium royalty rates are*

$$r_i^* = r(n_i, n_j) = \frac{a[2 - s + n_i(2 - s^2 - s)][n_i - 1 - n_j(s^2 n_i - n_i + 1)]}{n_i[n_i n_j(s^4 - 5s^2 + 4) + (n_i + n_j)(4 - 3s^2) + 4 - s^2]}, \; i, j = A, B. \tag{6}$$

This expression appears somewhat complicated - a discussion and intuition are provided below. The equilibrium royalty captures a central tension in our analysis. Charging a positive royalty makes downstream firms more passive and has two conflicting effects. On the one hand, passive behavior is desirable in order to control competition with other downstream firms contracting with the *same* upstream firm. On the other hand, it is undesirable from the point of view of competing with firms contracting with a different upstream firm. Both of these effects need to be considered in order to determine whether positive royalties will be used.

Before proceeding to examine the properties of the equilibrium, it is instructive to provide an illustration of the equilibrium construction using "brand reaction functions". This approach clarifies the intuition, gives a "cleaner" proof for the results above, and is also useful in the remainder of the paper.

3.2. Exposition using brand reaction functions

Consider again the reaction function of a given downstream firm m, as given by (3). This represents, of course, the optimal output for firm m taking the output levels of *all* other firms as given. Notice that each of these reaction functions has slope (relative to Q_j) equal to $s/2$. Adding (3) over m and rearranging we obtain the *brand reaction function* for i:

$$Q_i(Q_j) = \frac{n_i}{n_i + 1}(a - r_i - sQ_j), \qquad i, j = 1, 2 \tag{7}$$

This function indicates the aggregate output of all of firm i's downstream firms given the output of all of firm j's downstream firms.

Remark 1. *The brand reaction function for firm i has slope $sn_i/(n_i+1)$. Thus, for a given output Q_j of firm j, the higher is n_i the higher the aggregate output of firm i's downstream firms. In addition, decreasing the royalty r_i generates a* parallel *upward shift of firm i's brand reaction function.*

Figure 1 illustrates how firm A's brand reaction function changes when the number of A's downstream firms increases from n_A to n'_A. As n_A increases, this function rotates and its slope $sn_i/(n_i+1)$ increases (with Q_j graphed against Q_i, the reaction function becomes flatter). The intuition here is that downstream firms do not internalize the effect of their output decisions on other downstream firms, including the ones selling the same brand. As n_A increases, the effect of not internalizing this externality becomes stronger and downstream firms behave more aggressively. Note that this effect underlies the result that (when royalties are not used) a larger number of downstream firms offers a stronger strategic position (e.g. as in Baye, Crocker, and Ju [1]).

Returning to the construction of equilibrium, it is clear that firm i, would like its downstream firms to produce an output level that maximizes their *total* profit. Thus, while choosing its royalty rate, r_i, firm i acts like a Stackelberg leader along j's brand reaction function (taking r_j as given). In other words, it wants its downstream firms to collectively produce output Q_i that solves

$$\max_{Q_i} \{[a - Q_i - sQ_j(Q_i)]Q_i\}. \tag{8}$$

or, substituting firm j's optimal response,

$$\max_{Q_i} \left\{ [a - Q_i - \frac{sn_j}{n_j + 1}(a - r_j - sQ_i)]Q_i \right\}.$$

The first-order condition (with respect to Q_i) can be written as:[25]

$$a - \frac{sn_j}{n_j + 1}(a - r_j) = \frac{2[(1 - s^2)n_j + 1]}{n_j + 1} Q_i$$

Solving for Q_i, we obtain the Stackelberg leader's output level:

$$Q_i^{SL} = \frac{[a + s(r_j - a)]n_j + a}{2[(1 - s^2)n_j + 1]}. \tag{9}$$

Substituting into firm j's reaction function, we obtain the Stackelberg follower output level:

$$Q_j^{SF} = \frac{n_j}{n_j + 1} \left[a - r_j - sQ_i^{SL} \right] =$$

$$= \frac{n_j}{n_j + 1} \left\{ \frac{[(2 - s - s^2)n_j + 2 - s]a - [(2 + s - 2s^2)n_j + 2]r_j}{2[(1 - s^2)n_j + 1]} \right\}. \tag{10}$$

Now, for a given r_j, firm i wants to choose a royalty rate r_i that will make its brand reaction curve pass through the point (Q_i^{SL}, Q_j^{SF}). In other words, it chooses r_i such that

[25] It is easy to check that the second-order conditions for a maximum are satisfied.

$$Q_i^{SL} = \frac{n_i}{n_i + 1}\left[a - r_i - sQ_j^{SF}\right]. \tag{11}$$

Substituting (9) and (10) into (11) and solving for r_i we obtain the reaction functions (5) of the upstream firms in terms of royalties. The only remaining step is to solve the system of these reaction functions to obtain the equilibrium royalty rates given by (6). This exposition via brand reaction functions helps clarify the strategic effects in the final two stages of the game.

Before examining the equilibrium royalties in detail, it is useful to discuss the monotonicity of the royalty reaction functions. It is easy to see that these can be *either increasing or decreasing, depending on the numbers of downstream firms.* Moreover, it is possible that one of these functions is decreasing while the other is increasing. Specifically, standard calculations show that:

Remark 2. *(Monotonicity of royalty reaction functions): The royalty r_i is decreasing in r_j if and only if*

$$n_i < n_j[(s^2 - 1)n_i + 1] + 1. \tag{12}$$

3.3. Benchmarks: the cases of pure intrabrand and pure interbrand competition

There are three critical parameters in (6): n_A, n_B, and s. To isolate the role of each parameter, consider first the case where products of the two upstream firms are perfectly differentiated ($s = 0$). This corresponds to the case of pure intrabrand competition.

Remark 3. (Pure intrabrand competition) *When $s = 0$, we have*

$$r_i^*(s = 0) = \frac{a(n_i - 1)}{2n_i}, \qquad i = A, B. \tag{13}$$

As has been shown in past work, an upstream monopolist charges a positive royalty if and only if the number of downstream firms exceeds one. A positive royalty serves to effectively raise the downstream firms' marginal cost and makes them internalize the horizontal externality that exists among them. It is easy to check that the optimal contract for the upstream firm drives the downstream market to the monopoly level (with price $a/2$) and allows the upstream firm to obtain exactly the monopoly profit ($a^2/4$). It is also easy to see that the optimal royalty decreases in the number of downstream firms. In the extreme case with only one downstream firm, the upstream firm charges no royalties: extracting a per-unit payment from a downstream monopolist simply leads to "double marginalization". Finally, as $n_i \to \infty$ we have $r^* \to a/2$, so that as the number of downstream firms increases to infinity, the optimal royalty equals the monopoly price.

Next turn to the other extreme case, that of pure interbrand competition. Now each of the two upstream firms signs an exclusive contract with a single downstream firm: $n_i = n_j = 1$. Also, to focus the discussion, assume no differentiation ($s = 1$). It is easy to see that in this case equation (6) yields:

Remark 4. (Pure interbrand competition) *When $n_i = n_j = 1$ and $s = 1$, we have*

$$r_i^*(n_i = n_j = 1;\ s = 1) = -a/5, \qquad i = A, B. \tag{14}$$

If negative royalties (that is, subsidies) are not possible, the equilibrium involves zero royalties (and fees equal to the per-firm Cournot profit).

This represents a standard result in the strategic contracting literature (see the Introduction).[26] In this case, the upstream firms would not only avoid charging a royalty but they would actually like to subsidize their downstream firms per unit of output sold, if this were possible. Note that, since in this model there is an analogy between royalties and wholesale prices, a subsidy simply means charging wholesale prices below upstream cost. If subsidies are not possible, then the prediction is that only fees will be specified in the contracts. The basic intuition is that, by charging

[26] More generally, when $n_i = n_j = 1$ and $s \in [0, 1]$, we have $r_i = s^2 a/(s^2 - 2s - 4)$.

a lower royalty rate, an upstream firm reduces the unit cost of the downstream firm it is associated with and as a result the downstream firm obtains a stronger strategic position against its rival. At a more technical level, decreasing the royalty rates generates an outward shift in the downstream firms' reaction functions.[27]

The above results say that royalties are positive in the case of pure intrabrand competition whereas they are negative under pure interbrand competition. Why? If there is a single upstream firm, it would like downstream firms to be *passive* (against one another) so that the total downstream profit is maximized whereas when there are two upstream firms, strategic considerations dictate that the downstream firms be more *aggressive* against the other downstream competitor.

3.4. Further analysis of the equilibrium contracts

When $s > 0$ downstream competition across the firm types matters and this competition is most intense when $s = 1$. The case of homogenous products ($s = 1$) represents an additional benchmark for our analysis and is also useful for better understanding our model since analytical expressions are quite transparent in this case .

When $s = 1$, from (4) we obtain the quantities in the downstream market:

$$q_i = \frac{a - r_i - n_j(r_i - r_j)}{n_i + n_j + 1}. \tag{15}$$

Similarly, the expressions for the Stackelberg leader and follower outputs in equations (9) and (10) can be simplified:

$$Q_i^{SL} = \frac{a + n_j r_j}{2}, \qquad \text{and} \qquad Q_j^{SF} = \frac{n_j}{n_j + 1}\left[\frac{a - r_j(n_j + 2)}{2}\right].$$

[27] Note that total output is decreasing (and market prices are increasing) in royalty rates and therefore upstream firms benefit if they are not allowed to use negative royalties. While competition forces each firm to be aggressive and subsidize its downstream firm, both firms would be better off with a zero royalty than with the equilibrium (negative) rate. Specifically, compared to a zero rate, a royalty equal to $-a/5$ decreases the market price form $a/3$ to $a/5$ and each upstream firm's total profit from $a^2/9$ to $2a^2/25$.

Also, from (5), the royalty reaction functions become

$$r_i(r_j; n_i, n_j) = \frac{(n_i - n_j - 1)(r_j n_j + a)}{2(n_j + 1)n_i}. \tag{16}$$

Therefore, when $s = 1$, condition (12) can be simplified to $n_i < n_j + 1$. Further, from (6) we have

Remark 5. *If* $s = 1$

$$r_i^*(s = 1) = \frac{a(n_i - n_j - 1)}{n_i(n_A + n_B + 3)}, \qquad i, j = A, B \tag{17}$$

so that firm A employs a positive royalty if and only if $n_A > n_B + 1$. *Furthermore, when* $n_A > n_B + 1$ *firm B does not employ a positive royalty.*

The intuition for this result is that a positive royalty is justified only when the incentive to soften downstream competition among its *own* contracting firms is stronger than the incentive to provide them a better strategic position against the other downstream firms.[28]

In particular, from (6) it is easy to see that we can extend the intuition of the two upstream and two downstream firms case (presented in Remark 4) as follows:

Remark 6. *When* $n_A = n_B$ *and* $s = 1$ *the two firms would like to subsidize their respective downstream firms.*

Continuing the analysis of the $s = 1$ case, it is useful to report here the equilibrium downstream quantities and price

$$q_i^*(s = 1) = \frac{a(n_j + 1)}{n_i(n_A + n_B + 3)}, \qquad i, j = A, B \tag{18}$$

[28] This analysis suggests an empirically testable implication, that dominant upstream firms (in the sense of having a higher number of downstream firms) are likely to charge relatively higher royalty rates and lower fixed fees than their smaller competitors.

and

$$p_i^*(s=1) = p^* = \frac{a}{(n_A + n_B + 3)}, \qquad i = A, B \tag{19}$$

From (18) one obtains $q_A^* > q_B^* \Leftrightarrow n_A < n_B$. From (19) note that the final price only depends on the total number of downstream firms in the market, $(n_A + n_B)$, and not on how these firms are allocated between the two upstream rivals.

To isolate the role played by the degree of product differentiation (s), consider again the case where $n_A = n_B = n$. From (6) we obtain:

Remark 7. *When $n_A = n_B = n$ the royalty charged by each upstream firm equals:*

$$r^* = \frac{a[n^2(1-s^2)-1]}{n[n(2+s-s^2)+(s+2)]}. \tag{20}$$

Thus $r^* > 0$ if and only if $n^2(1-s^2) > 1$.

From the above result, note that, as s increases, n must also increase for the royalty rate to be positive. In the limit when s goes to 1, optimal royalties are never positive. The intuition is that the more similar the downstream products, the stronger must be the competitive externality between firms that contract with a single upstream firm for the latter to charge a positive royalty.

3.5. Monotonicity and limit behavior

We now examine how equilibrium royalties vary with the number of downstream firms. First, we find that a firm's royalty decreases with the number of rival downstream firms, regardless of the magnitude of s. To see this, it is enough to differentiate r_i^* from (6) with respect to n_j. We obtain

$$\frac{\partial r_i^*}{\partial n_j} = \frac{-2as^2(n_i+1)[(s^2+s-2)n_i+s-2][(s^2-1)n_i-1]}{n_i[(s^4-5s^2+4)n_in_j-(3s^2-4)(n_i+n_j)-s^2+4]^2} < 0. \tag{21}$$

Regarding the monotonicity with respect to own firms, the result is not unambiguous. While numerical examples indicate that royalties typically increase in the number of own firms, this result is not generally true. In particular, when $s = 1$, from (17) we obtain

$$\frac{\partial r_i^*}{\partial n_i} = \frac{a[n_j(n_j + 2n_i + 4) + n_i(2 - n_i) + 3]}{n_i^2(n_i + n_j + 3)^2}. \tag{22}$$

It is easy to see, for example, that when $n_j = 1$ this expression is positive for $n_i = 5$ and negative for $n_i = 6$. Figure 2 illustrates that, as n_i increases, r_i is initially negative, increases to positive levels and then decreases converging to zero from above. We can summarize these results as follows.

Remark 8. *Equilibrium royalty of firm i (i) decreases as the number of rival downstream firms (n_j) increases, and (ii) can either increase or decrease in the number of own firms (n_i).*

Given the above discussion, it is interesting to examine the limit behavior of equilibrium royalty rates and profits. As Figure 2 shows, when $s = 1$, $r_i^* \to 0$ as $n_i \to \infty$. More generally, from (6) we find that, as the number of own downstream firms increases, equilibrium profit converges to

$$\lim_{n_i \to \infty} \pi_i^* = \frac{a^2(s^2 + s - 2)^2[1 + (1 - s^2)n_j](n_j + 1)}{[n_j(s^4 - 5s^2 + 4) + 4 - 3s^2]^2}.$$

Notice that, in the limit, profit is positive if $s < 1$. If $s = 1$ both firm i's royalty and profit tend to zero as $n_i \to \infty$.

4. Equilibrium number of downstream firms

Thus far we have taken the number of downstream firms as fixed. In this section we examine the choices of n_A and n_B by the upstream firms (given that equilibrium choices of royalties and quantities will follow).

Substituting the equilibrium royalty rates (6) into the downstream quantities and prices, we obtain the upstream profits $\pi^i(n_i, n_j) =$

$n_i p_i(r_i^*(n_i,n_j), r_j^*(n_i,n_j)) q_i(r_i^*(n_i,n_j), r_j^*(n_i,n_j))$, $i = A, B$. Then firm i chooses its number of downstream firms to solve

$$\max_{n_i} \{ n_i p_i(r_i^*(n_i,n_j), r_j^*(n_i,n_j)) q_i(r_i^*(n_i,n_j), r_j^*(n_i,n_j)) \}.$$

Substitution and differentiation with respect to n_i yields

$$\frac{\partial \pi^i(n_i,n_j)}{\partial n_i} = -\frac{2s^3a^2(n_j+1)[(2-s^2-s)n_j+2-s][1+(1-s^2)n_j][(2-s^2-s)n_i+2-s]}{[n_in_j(4+s^4-5s^2)+(n_i+n_j)(4-3s^2)+4-s^2]^3} < 0 \tag{23}$$

for all $0 < s \leq 1$.[29] It follows that each upstream firm wants to minimize its own downstream firms, regardless of the number of firms chosen by its rival. The unique equilibrium then is for each firm to pick only one downstream firm. To summarize:

Proposition 2. *If $s > 0$, in equilibrium we have $n_A^* = n_B^* = 1$, so that each upstream firm chooses to have only one downstream. Moreover, it is strictly optimal for firm i to have only one downstream firm, regardless of the number of downstream firms chosen by firm j.*

Note that, if there is no substitution among the two products, we know from the analysis in the previous section that:[30]

Remark 9. *If $s = 0$, any number of downstream firms n_i^* is optimal for firm i, $i = A, B$ (and i can always achieve the monopoly profit).*

To illustrate the effect of increasing the number of downstream firms consider the following example. Suppose that $s = 1$ and $n_B = 1$. Then assume that n_A increases from 1 to 2. It is easy to calculate the relevant royalties, quantities, prices, and profits from the expressions given above. Table 1a reports how the equilibrium changes as

[29] Since $0 < s \leq 1$, the following hold: (i) $2 - s > 2 - s^2 - s > 0$; (ii) $4 + s^4 - 5s^2 > 0$ and (iii) $4 - s^2 > 4 - 3s^2 > 0$. These inequalities imply that the term in parentheses is strictly positive. Also the second order conditions guarantee that we have a maximum.

[30] This can be also checked directly, since then the derivative in (23) becomes zero for $s = 0$.

n_A increases, and Figure 3a provides a diagrammatic illustration. We find that, as n_A increases, firm A's royalty increases (from $-a/5$ to zero) and firm B's royalty decreases (from $-a/5$ to $-a/3$). The total output produced by firm A's downstream firms decreases and that of firm B's increases. The price decreases. Finally, firm A's total profit decreases while that of firm B increases. As a result (when $n_B = 1$) firm A would not like to increase the number of its firms from $n_A = 1$ to $n_A = 2$.

The intuition for the result is as follows. In our framework, either a higher number of downstream competitors or a lower royalty rate represents a commitment to more aggressive behavior in the downstream market. So the question is why it is preferable from a strategic viewpoint to choose low royalties instead of (also) choosing a larger number of firms. As a first step, assume for a moment that the rival's number of firms and royalty are fixed. Then for an upstream firm that is choosing its number of firms and royalty rate there are infinite combinations of the two instruments that (given the rival's behavior) maximize profit. More precisely, in this case an upstream firm would simply choose a combination of number of firms and royalties that would make its brand reaction function (7) intersect the rival's reaction function at the Stackelberg point (9). In fact, it is easy to see that since a brand reaction curve can shifts out either as a result of a higher n_i or a lower r_i, a larger n_i implies that a higher r_i is optimal (as a function of r_j).[31] However, and contrary to the assumption above, in the game the rival's number of firms and royalty are not fixed. While, when choosing its number of downstream, an upstream firm takes the rival number of downstream firms as given, it does consider the effect on the rival's royalty choice. In particular, a larger number of downstream firms implies that, for any royalty rate, an upstream firm's brand reaction function becomes steeper and, as a result, the rival has a stronger incentive to behave strategically (by choosing a lower royalty rate). Since it is desirable to have higher rival royalty rates, each firm has an incentive to

[31] Note, however, that this does *not* imply that a higher n_i leads to a higher *equilibrium* r_i since its level also depends on r_j.

choose to lower number of downstream competitors.[32]

5. Sequential choices of royalties

To examine the robustness of our main result (Proposition 2), we now consider the case where the two firms choose their royalty rates *sequentially*. To simplify the exposition, we assume that $s = 1$. Given the number of firms and the royalty rates, the output of a downstream firm associated with upstream firm i is again given by (15). As before, the total profit of firm A (after it collects both fees and royalties) if it chooses royalty r_A and B chooses r_B is $\pi^A(r_A, r_B) = pn_Aq_A = (a - n_Aq_A - n_Bq_B)n_Aq_A$ and maximization gives rise to the reaction functions (16).

Now, instead of solving the system of these two reaction functions, as we did before when royalties were chosen simultaneously by the two firms, we have to allow for the fact that the first mover (say firm A) anticipates how B's royalty choice depends on its own choice. Thus, firm A solves

$$\max_{r_A} \pi^A(r_A, r_B(r_A)). \tag{24}$$

Substitution of (16) into the profit expressions yields

$$\pi^A(r_A, r_B(r_A)) = \pi^A(r_A, \frac{(n_B - 1 - n_A)(n_Ar_A + a)}{2n_B + 2n_An_B}) = \frac{n_A(-n_Ar_A - 2r_A + a)(n_Ar_A + a)}{4(n_A + 1)^2}. \tag{25}$$

[32] One may ask if, given the incentive to minimize the number of downstream firms, upstream firms may also prefer to become vertically integrated. It is easy to see that, while both firms are better-off under vertical integration, they do not have a *unilateral* incentive to become vertically integrated. To see this point, observe that a vertically integrated structure corresponds, in our model, to zero royalties. It should be clear, then, that both firms are better-off with zero royalties than with the royalties $(-a/5)$ chosen in the equilibrium of our game (see footnote 27). However, by construction we know that, given that its rival behaves as prescribed in equilibrium, an upstream firm does not want to become vertically integrated (we know this since the best response to $r_j = -a/5$ is $r_i - a/5$ and not $r_i = 0$). Similarly, it is easy to check that, given its rival is vertically integrated, an upstream firm prefers to be vertically separated and change a negative royalty than to be integrated.

Note that n_B drops out from the expression: firm A's profit when it is a leader with respect to the choice of royalties does not depend on the number of B's downstream firms. Maximization implies that the leader's royalty is:

$$r_A^L = \arg\max_{r_A} \pi^A(r_A, r_B(r_A)) = -\frac{a}{n_A(n_A+2)}. \tag{26}$$

Substituting into B's reaction function we obtain the follower's royalty

$$r_B^F = r_B(r_A^L) = a\frac{n_B - 1 - n_A}{2n_B(n_A+2)}. \tag{27}$$

Clearly we have:

Remark 10. *The leader's royalty, r_A^L, is (i) negative and (ii) increasing in the number of own firms, n_A.*

It is also easy to see that $\partial r_B^F/\partial n_A = -a(1+n_B)/2n_B(n_A+2)^2 < 0$ and thus:

Remark 11. *The follower's royalty, r_B^F, decreases in the number of firms of the leader.*

Now we can calculate the equilibrium profit for the leader, $\pi^A(r_A^L, r_B^F)$. Substituting we obtain:

$$\pi^A(r_A^L, r_B^F) = \frac{a^2}{4(n_A+2)}. \tag{28}$$

Similarly, we can calculate the follower's profit

$$\pi^B(r_A^L, r_B^F) = \frac{a^2(n_A+1)}{4(n_A+2)^2}. \tag{29}$$

Thus, we have:

Proposition 3. *The Stackelberg leader's profit is decreasing in the number of its own firms. The Stackelberg follower's profit is also decreasing in the number of the leader's firms. Both the Stackelberg leader's and the Stackelberg follower's profit does not depend on the number of the follower's firms.*

The proof is obvious for the leader from (28). For the follower, from (29) we have $\partial \pi^B(r_A^L, r_B^F)/\partial n_A = -a^2 n_A / 4\,(2+n_A)^3 < 0$.

In particular, note that the Stackelberg leader's profit goes to zero as the number of its own firms goes to infinity.[33]

The analysis of the sequential royalty choice, together with the simultaneous royalty case analyzed earlier, imply the following result.

Proposition 4. *An upstream firm is strictly worse-off with a larger number of downstream firms (of its own) when both fixed fees and royalty rates are used, regardless of whether the firm chooses its royalty rate simultaneously with or before its upstream rival's royalty rate. An upstream firm's profit does not depend on its number of downstream firms when its royalty rate is chosen after its upstream rival's royalty rate.*

We can also compare the royalty rates under sequential choices to the rates from the simultaneous choices, r^*, given by (17).

Proposition 5. *Compared to the simultaneous choice of royalties and for a given number of firms, the follower (firm B) always charges a higher royalty. The leader (firm A) charges a higher (lower) royalty if and only if n_B is greater (smaller) than $n_A + 1$.*

To prove this result observe that the leader's profit is $\pi^A(r_A, r_B(r_A))$ and

$$\left.\frac{d\pi^A}{dr_A}\right|_{r_A = r_A^L} = \frac{\partial \pi^A}{\partial r_A} + \frac{\partial \pi^A}{\partial r_B}\frac{\partial r_B}{\partial r_A}. \tag{30}$$

In a Nash equilibrium (in royalties) each firm takes the other firm's royalty rate as given ($\partial r_B/\partial r_A = 0$) and so we have $\partial \pi^A/\partial r_A = 0$. As long as π^A has a unique maximum (this holds with linear demand), this implies that $r_A^L > r^* \Leftrightarrow \partial r_B/\partial r_A > 0$.

[33] In contrast, recall that when an upstream firm faces no competition ($s = 0$) then, even with an infinite number of downstream firms, the upstream firm can achieve the monopoly profit.

However, from (12) we know that $\partial r_B/\partial r_A > 0 \Leftrightarrow n_B > n_A + 1$. It follows that $r_A^L > r^* \Leftrightarrow n_B > n_A + 1$. Now compare r_B^F to r^*. From above, when $\partial r_B/\partial r_A > 0$ then $r_A^L > r^*$. But, then, $r_B^F = r_B(r_A^L) > r_B(r^*) = r^*$. When, on the other hand, $\partial r_B/\partial r_A < 0$ then $r_A^L < r^*$. But, in this case we also have $r_B^F = r_B(r_A^L) > r_B(r^*) = r^*$. So we always have $r_B^F > r^*$. Finally, note that when $n_B = n_A + 1$ we have $\partial r_B/\partial r_A = 0$ and so $r_A^L = r^* = r_B^F$.[34]

A special case of interest is the following:

Remark 12. *When $n_B = n_A$, and in particular when each upstream has only one downstream, under sequential choice of royalties, the leader chooses a lower royalty rate while the follower chooses a higher royalty relative to their respective royalties under simultaneous choice.*

6. Royalties before the number of downstream competitors

To better understand the strategic incentives in our framework and, in particular, the role royalties play in influencing the choice of number of downstream firms, we now examine some further modifications of our basic model. In particular, we examine situations where the choice of royalty rates does not follow that of the number of downstream firms. This could be either because royalties cannot be used at all, or because royalties must be chosen before or at the same time as the number of competitors.

Suppose first that royalties cannot be used and that only fixed fees can be used to transfer profit upstream. In the absence of royalties, the only strategic choice of upstream firms is the number of downstream firms. When $s = 1$, this case corresponds to the literature on strategic choice of downstream divisions discussed above (e.g. Baye, Crocker, and Ju [1]) and is presented here for comparison purposes. Naturally, our conclusion is the same as in that work, that there is an incentive to have a higher

[34] These inequalities can be also easily checked with direct calculations using (26), (27) and (17).

number of downstream units (and, without some cost for adding downstream firms, an equilibrium fails to exist). Thus the strategic incentives are reversed when royalties cannot be used. We further examine the fee-only case when $s < 1$. Not surprisingly, the result that a larger number of downstream firms is preferred does not hold as the degree of differentiation between the two products increases.

The fee-only case can be viewed as having royalties fixed at rate zero. We generalize the result from this case and show that the strategic incentive to have a higher number of downstream firms (and the non existence of equilibrium) is present when royalties are fixed at any level that is not "too low". In particular, *for given royalty rates* that are not so low that the relevant brand reaction function is beyond the Stackelberg point, each firm has an incentive to act strategically and have more downstream firms than the rival. Thus, when our basic model is changed so that royalties are chosen first and the number of firms second or, alternatively, so that fee-and-royalty contracts and numbers of firms are chosen simultaneously, we find that, subject to the important qualification noted above, the strategic incentives are reversed. We conclude that it is crucial for our main result that per-unit payments be chosen *after* the number of firms. We also explore firms' behavior when royalties are low in the sense described above. The formal analysis and results follow.

6.1. Only fees

At the fee stage, both firms simply extract the downstream profits of their respective downstream firms (with per firm profit equal to $p_i q_i$). The expressions for downstream output choices are as before (see (4)) except that we need to substitute $r_i = 0$. Consider the choice of number of firms. Notice that now the upstream firms cannot control the marginal incentives of their downstream firms. Taking n_j as given, upstream firm i chooses n_i to maximize $\pi_i^{r=0} = n_i p_i q_i$. The first order condition is:

$$\frac{\partial \pi_i^{r=0}}{\partial n_i} = \frac{a^2(1 + n_j - sn_j)^2(1 + n_j - n_i - n_i n_j + s^2 n_i n_j)}{(1 + n_j + n_i + n_i n_j - s^2 n_i n_j)^3}.$$

Setting this term equal to zero gives the reaction function of firm i

$$n_i = \frac{1+n_j}{1+(1-s^2)n_j}, \qquad i,j = A,B. \tag{31}$$

In particular, when $s = 1$ the reaction function of firm i is

$$n_i = n_j + 1, \qquad i,j = A,B. \tag{32}$$

Clearly, the two reaction functions are upward sloping and parallel to each other: each firm wants to have one more downstream firm than its rival.[35] It follows that there is no equilibrium when products are perfectly homogenous ($s = 1$). To illustrate the consequences of increasing the number of downstream firms when only fees can be used, consider again a simple example we discussed in the fee and royalty case. Suppose, with $s = 1$ and $n_B = 1$, that n_A increases from 1 to 2. The results are reported in Table 1b. It is easy to see that as its number of firms increases, A's total profit increases (while that of firm B decreases). Thus, firm A has an incentive to increase the number of its firms from $n_A = 1$ to $n_A = 2$.

Consider now the case where $s = 0$: the two products are perfectly differentiated. Then, as can be seen from (31), the optimal choice of each firm is to pick a single downstream firm in order to extract monopoly profit. This is intuitive: under upstream monopoly, when only fees can be used, the firm would prefer a single downstream since increasing the number of downstream firms simply decreases total downstream profit. When $0 < s < 1$, solving the system given by (31) we find that there is a symmetric equilibrium where the optimal number of firms of each type equals

$$n^* = \frac{1}{\sqrt{1-s^2}}. \tag{33}$$

As is obvious, as s approaches zero, n^* goes to 1 whereas as s approaches 1, n^* goes to infinity. In general, as products become more homogenous, each firm wants to increase its number of downstream firms. As noted above, when $s = 1$, our analysis

[35] Note that this is the same expression reported in Corchón [4] and Polasky [22].

is almost identical to Baye, Crocker and Ju [1] where the only difference is that they specify a cost for each additional downstream firm and thus find an equilibrium with a finite number of divisions.

6.2. Choosing number of downstream competitors after or at the same time as royalties

Suppose now that firms choose royalty rates and numbers of downstream firms simultaneously. To simplify the exposition, we present the results for the case of $s = 1$.[36] Each firm solves the following problem:

$$\max_{r_i,\, n_i} \; n_i p_i(r_i, r_j) q_i(r_i, r_j).$$

It will be shown that for royalties that are not "too low", the incentive to have a higher number of firms than the rival (that we saw before) is also present in this case. Thus to have an equilibrium it is required that the royalties specified are low enough. To see this point, note that to have an equilibrium the number of firms, n_i, must be optimal for firm i given the two royalty rates and the rival firm's number of firms. Thus, the first order condition of firm i with respect to n_i must hold.[37] Solving the first order condition, we find that, given r_i, r_j, and n_j, the optimal value of n_i is

$$n_i^* = \frac{(n_j + 1)(a + r_j n_j)}{a + n_j r_j - 2n_j r_i - 2r_i}. \tag{34}$$

Using the above equation, it is easy to check that *when royalties are nonnegative* we have $n_i^* > n_j + 1$ and thus

$$n_i^* > n_j. \tag{35}$$

But, clearly, this condition cannot hold simultaneously for both firms (it cannot be that $n_i > n_j$ and $n_j > n_i$). Further, note that (34) must be also satisfied in

[36] As above, when $s \to 0$ the incentive to have a smaller number of downstream firms dominates.

[37] We have $\partial \pi_i^{r,n} / \partial n_i = -(a - r_i - n_j r_i + n_j r_j)(a n_i - 2r_i n_i - 2n_i n_j r_i + n_i n_j r_j - a - a n_j - n_j r_j - r_j n_j^2)/(1 + n_i + n_j^2)$

an equilibrium of the game where royalties are chosen before the number of firms. Therefore:

Remark 13. *When royalties have been chosen at a nonnegative level, an equilibrium in the number of downstream firms cannot exist (we could only have an equilibrium if we allow for an infinite number of downstream firms). Similarly, there cannot exist an equilibrium with nonnegative royalties when royalties and the number of firms are chosen simultaneously.*

Thus, with nonnegative royalties when contracts are chosen at the same time as the number of firms, or when royalties are chosen before the number of firms, there is an incentive to increase the number of downstream firms (and this results in a non-existence of equilibrium). More generally, we show that the strategic incentive to increase the number of firms is present not only for zero royalties but also for any fixed royalty rates that are not too low. Specifically, the key observation is that, unless r_i is so low that (given r_j and n_j) firm i's brand reaction function is already beyond the Stackelberg point, i has an incentive to choose $n_i^* > n_j$.

To see more precisely how the royalty rates affect the choice of the number of firms, we proceed as follows. To simplify the exposition, focus on symmetric equilibria. So suppose that $r_i = r_j = r$. With equal royalties, and using (34), (35) becomes $n_i^* = (n_j+1)(a+rn_j)/(a-rn_j-2r) > n_j$ which is true if and only if $r > -a/n_j(3+2n_j)$ (and $r < a/(n_j+2)$). The key point here is that firm i wants to have a higher (respectively: lower, equal) number of downstream firms than firm j if and only if the royalty r is above (below, equal to) $-a/n_j(3+2n_j)$. What is the intuition for this result? As discussed above, firm i wants to make its brand reaction function pass through the Stackelberg point on j's brand reaction function. In particular, (5) gives the royalty that makes the reaction function go through the Stackelberg point. Imposing symmetry and solving (5) for r we find the same critical level as above, that when $r = -a/n_j(3+2n_j)$ the reaction function passes exactly through the Stackelberg point when $n_i^* = n_j = n$. Thus, if $r > -a/n_j(3+2n_j)$ firm i needs a higher than n_j number of firms to make its reaction curve go through the Stackelberg point. If, on the other hand, $r < -a/n_j(3+2n_j)$, to make its reaction curve go through the Stackelberg point, firm i chooses $n_i^* < n_j$. Finally, when $r = -a/n_j(3+2n_j)$ the optimal is $n_i^* = n_j$. This reasoning allows us to construct symmetric equilibria as follows. Consider (34) and impose symmetry also with respect to the number of firms to obtain $n = (n+1)(a+rn)/(a-rn-2r)$ or

$$rn(3+2n) + a = 0. \tag{36}$$

Having both firms choose a royalty rate and number of firms that satisfy (36) constitutes an equilibrium. Note that, as discussed above, (36) requires that the royalty rates are negative. In particular the royalty rates have to satisfy $r = -a/n(3+2n) < 0$. For example $n_A = n_B = 1$ and $r_A = r_B = -a/5$ is an equilibrium of the game where royalties and the number of firms are chosen simultaneously. Asymmetric equilibria could be also constructed.

We can summarize as follows.

Proposition 6. *Suppose that $s = 1$. When royalties and the number of downstream firms are chosen at the same time, there is no equilibrium with nonnegative royalties (since, with such royalties, each firm has an incentive to have a higher number of firms than its rival) but there is a multiplicity of equilibria with negative royalties. When royalties are chosen before the number of downstream firms, if nonnegative royalties are chosen in the subsequent stage each firm would like to have a higher number of downstream firms than its rival and, thus, there is no equilibrium.*

7. Royalty-only contracts and comparison

Our final modification of the basic model is to allow firms to use only royalties. Thus, now, in the second stage we consider linear contracts. The main result here is that removing the firms' ability to transfer part of the profit upstream in the form of a fixed fee changes the strategic incentives and each firm prefers to increase its number of downstream firms, as long as the rival number of firms are not too large.

7.1. Equilibrium with royalty-only contracts

First, both firms simultaneously choose their number of downstream firms. Next, they choose their royalty rates and finally downstream firms compete in the product market. At the royalty stage, n_i and n_j are already determined, and taking r_j as given, firm i solves the problem

$$\max_{r_i} \; n_i r_i q_i(r_i, r_j) \tag{37}$$

where the output decisions are as before (see (4)). Note that the objective function indicates that only a part of downstream profit can be transferred upstream via the royalties (while under fee-and-royalty contracts the entire profit can be transferred). The first order conditions for the choice of royalties are given by

$$\frac{n_i(a - 2r_i - asn_j + an_j - 2n_j r_i + sr_j n_j)}{1 + n_i + n_j - n_i n_j + s^2 n_i n_j} = 0, \qquad i, j = A, B.$$

Solving these first order conditions yields the royalty rates

$$r_i^{f=0} = \frac{a[2(1 + n_i) + n_j(2 - s) + (2 - s - s^2)n_i n_j]}{4(n_i + 1) + n_j(4n_i - s^2 n_i + 4)}, \qquad i, j = A, B.$$

Next consider the choice of number of firms. Each upstream firm solves the problem

$$\max_{n_i} \; \{n_i r_i^{f=0} q_i(r_i^{f=0}, r_j^{f=0})\}.$$

After appropriate substitutions and when $s = 1$, the total upstream profit (maximand) can be rewritten as

$$\pi_i^{f=0} = \frac{a^2 n_i(n_j + 1)(2n_i + n_j + 2)^2}{[4n_i + 4n_j + 3n_i n_j + 4]^2 \, [n_i + n_j + 1]} \tag{38}$$

While the analytical expressions for the above problem are rather complicated, it is easy to show that there exists no symmetric equilibrium in this game. We have

$\partial\pi_i^{f=0}/\partial n_i = a^2(n_j+1)(2n_i+n_j+2)[n_i(10n_jn_i+8n_i-3n_j^3+11n_j^2+30n_j+16)+4n_j^3+16n_j^2+20n_j+8] \ / \ [4n_i+4n_j+3n_in_j+4]^3 \, [n_i+n_j+1]^2$. It is easy to check from this expression that when n_j is small, $\partial\pi_i^{f=0}/\partial n_i > 0$ for all n_i and that when n_j is big enough, $\partial\pi_i^{f=0}/\partial n_i < 0$ for all $n_i > 1$.[38] This may suggest that there exists a symmetric equilibrium, in which each upstream firm selects a 'medium' number of downstream firms. It turns out that this is not the case. To see this, impose symmetry on the above first order condition and write

$$\left.\frac{\partial\pi_i^{f=0}}{\partial n_i}\right|_{n_i=n_j=n} = \frac{-a^2(n+1)(n^3-9n^2-12n-4)}{(2n+1)^2(n+2)^3(3n+2)}$$

While the above first order condition admits a positive root at approximately $n = 10.21$, inspection of the second order condition reveals that this root picks out a local minimum for both firms. In fact, it is straightforward to show that at $n_j = 10.21$, firm i can benefit from increasing its number of firms. Thus, there is no positive root of the above first order condition at which profits are maximized.

Proposition 7. *Suppose that $s = 1$ and that contracts can specify only royalties. Then if an upstream firm chooses to have only one downstream firm, the other upstream firm's best response would be to have an infinite number of downstream firms. Furthermore, there exists no symmetric equilibrium in which both firms pick a finite number of downstream firms.*

To illustrate the effect of an increase in the number of downstream firms when fees cannot be used, consider again an example we discussed above. Suppose that $s = 1$, $n_B = 1$ and n_A increases from 1 to 2. The results of this change are reported in Table 1c and illustrated graphically at Figure 3b. We find that, in contrast to the

[38] Notice that in the expression for $\partial\pi_i^{f=0}/\partial n_i$ the denominator is positive, as well as all terms of the numerator with the possible exception of $(4-3n_i)n_j^3$. For n_j large enough, this term dominates and $\partial\pi_i^{f=0}/\partial n_i$ has the same sign as $(4-3n_i)$. It is easy to show that, as $n_j \to \infty$, it is optimal to have $n_i = 1$ (as this gives higher profit than $n_i = 2$ and $\partial\pi_i^{f=0}/\partial n_i < 0$ for $n_i \geq 2$).

fee-and-royalty case, now as n_A increases both firms royalty rates decrease (where firm A's royalty decreases less than that of B). The total output produced by firm A's downstream firms increases and that of firm B's decreases. The price decreases. Finally, firm A's total profit increases while that of firm B's decreases. As a result, in this case, firm A would like to increase the number of its firms from $n_A = 1$ to $n_A = 2$.

The intuition is as follows. When firms can only choose royalty rates, the incentive to increase the number of downstream firms in order to assume a more aggressive posture against the rival is *reinforced* by the incentive to induce competitive behavior in the downstream market by increasing the number of downstream firms, as long as the number of rival firms are small (which is the case in this example).[39] Consequently, firm A wants to proliferate the number of its downstream firms without end, so long as firm B does not have too many firms.

The above proposition does not rule out asymmetric equilibria. In fact, we can show that when firm A chooses an infinite number of downstream firms it is optimal for firm B to have one downstream firm and, conversely, when B chooses to have one downstream firm it is optimal for A to have and infinite number. Since this is not the main focus of the paper, we do not expand more on this issue. The important point we want to emphasize here is that under royalty-only contracts, if one firm picks one or only a few downstream firms, then its rival wants to proliferate its downstream firms without end.[40]

[39] We can show that $\partial(\partial\pi_i^{f=0}/\partial n_i)/\partial n_j < 0$ i.e., the marginal incentive to increase your own firms is strictly decreasing in the number of firms of your rival.

[40] The possibility of asymmetric equilibria in which both firms pick a finite number of firms cannot be ruled out, though we have been unable to find such equilibria.

7.2. Comparison

Now we turn to a comparison of the cases of royalty-only and fee-and royalty contracts. The comparison for a given number of downstream firms, can be summarized as follows. While royalty-only contracts allow downstream firms to keep part of their profits, upstream firms (as well as downstream) may be better off with royalty-only contracts. The reason is that, under fee-and-royalty contracts, royalties are too low from the viewpoint of maximization of joint upstream profits and making fees infeasible makes firms choose higher royalty rates. Indeed, under some conditions, upstream firms may be in a prisoners' dilemma when choosing the type of their contracts since both could be better-off if they could commit to royalty-only contracts, but it is a dominant strategy for each to choose a fee-and-royalty contract. This observation has been already made in the literature, in very similar frameworks.[41] The new point here is that when the number of downstream firms is endogenous (and precedes the choice of contracts) then a restriction to royalty-only contracts is not expected to yield higher profit for upstream firms. In other words, the prisoners's dilemma structure vanishes when the number of firms is endogenized. The reason is that, as shown above, while under royalty only contracts, firms prefer to increase the number of downstream firms, with fee-and-royalty contracts they prefer to have only one.

Some formal results follow. First, we compare royalty-only and fee-and-royalty contracts when the number of downstream firms is given. We show that the two upstream firms may make themselves better-off by "tying their hands": if both firms charge only royalties, and no fixed fees, their profits may increase. To see the intuition, observe that, under fee-and-royalty contracts, in equilibrium competition induces firms to choose royalty rates that are too low. By way of illustration, if we suppose $s = 1$ and $n_i = n_j (= n)$ then the (symmetric) royalties that maximize joint upstream profits are $a(2n - 1)/4n$ which are higher than the equilibrium contracts

[41] See, in particular, Gal-Or [10] and Rey and Stiglitz [23].

$-a/n(2n+3)$ from (17).[42] Removing fees from the set of strategic variables allows the firms to jointly commit to less aggressive behavior (higher royalties) and this may lead to higher profits (in some, but not all, cases).

This intuition is confirmed as follows. We compare maximized profits of the upstream firms under the two different contracts. Consider the case when $s = 1$, when downstream competition is at its strongest. Straightforward but lengthy calculations show that the ratio of maximized profits under the two-instrument contract to profits under the royalty-only contract is given by

$$\rho \equiv \frac{\Pi_i^{f,r}}{\Pi_i^{f=0}} = \frac{[2-s+n_i(2-s^2-s)][(n_j(s^2-1)-1][4(n_i+1)+n_j(4n_i-s^2n_i+4)]^2[\Phi]}{n_i[n_j(s^2n_i+sn_i-2n_i+s-2)-2(n_i+1)]^2[\Psi]}$$

where

$$\Phi \equiv n_j[s^2n_i - 1 - n_i] - [1+n_i]$$

and

$$\Psi \equiv \left[n_j[s^4n_i - 5s^2n_i + 4n_i - 3s^2 + 4] + n_i[4-3s^2] + 4 - s^2\right]^2.$$

To gain some insight into the above ratio of profits, suppose $n_A = n_B = n$ and $s = 1$. In this case, ρ becomes

$$\rho = \frac{(1+2n)}{n}\left[\frac{n+2}{2n+3}\right]^2.$$

It is easy to show that ρ is decreasing in n and we can state the following result.

Remark 14. *Firms may enjoy higher profits under royalty-only contracts than under fee-and-royalty contracts. If $n_A = n_B = n$ and $s = 1$, while the two-instrument contract yields higher profits when $n = 1$, the royalty-only contract dominates when n is equal to 2 or higher. As s becomes smaller, the two-instrument contract becomes more profitable relative to a royalty-only contract, since controlling downstream competition becomes less important. In the limit when $s = 0$, firms always make higher profits with the two-instrument contract.*

[42] Of course, when $s = 0$, there is no downstream competition across the firm types so that the royalties charged by upstream firms also maximize joint profits.

Thus, under some conditions, the upstream (as well as the downstream) firms may be better-off in a world where royalty-only contracts can be used as compared to a world where both fees and royalties can be used. This observation suggests a game where upstream firms first choose the type of their contracts (royalty-only or fee-and-royalty) and then compete in these contracts. It is straightforward to show that, although both firms could be better off if they choose royalty-only contracts, each has a dominant strategy (in the first stage) to choose a two-instrument contract. Thus, the two upstream firms may find themselves in a prisoners' dilemma when choosing the nature of their vertical contracts. Note that, previous work (Gal-Or [10] and Rey and Stiglitz [23]) has demonstrated this prisoners' dilemma feature of contracts' choices under downstream price competition. We show here that this result also holds under quantity competition but it is crucial to that each upstream firm has more than one downstream. In particular, it is required that the number of downstream firms per upstream firm is two or higher; if they each just have one downstream firm then this result does not hold.

The above result, however, keeps the number of downstream firms fixed. By making this number an endogenous choice of the upstream firms, we see that the picture may change dramatically and the prisoners' dilemma feature described above may not be present. The reason should be clear by now. Suppose that we consider the same enlarged game where firms can choose their contract type, but now also allow them to choose the number of their downstream firms before they compete in contracts. From the above analysis we know that in equilibrium each upstream firm chooses a single downstream under fee-and-royalty contracts, whereas when only royalties can be used and a competitor has a low number of downstream firms there is an incentive to increase indefinitely the number of downstream firms. As a result, when the number of firms is endogenized, it is problematic to state that competition in two-instrument contracts leads to lower profit than competition in royalty-only contracts and the strategic incentives are modified.

8. Conclusion

This paper examines strategic interactions in vertically related markets where both intrabrand and interbrand competition may be present. We focus on contracts specifying a fixed fee and a per-unit payment (royalty). Concerning strategic implications, the critical point is that royalties affect the behavior of downstream firms (by altering their marginal costs) while fees transfer residual profits upstream. We characterize equilibrium contracts when each upstream firm contracts with multiple downstream and show how strategic interactions both at the upstream and the downstream level affect the design of these contracts. The basic tension is that higher royalties help control competition among own downstream firms while making them less aggressive against rival downstream firms.

Our main contribution lies in using the analysis of contracts as a building block to endogenize the number of downstream firms (when competition under fee-and-royalty contracts follows). In contrast to previous work that has emphasized the strategic benefit from having a larger number of downstream firms (when contracts specify only a fixed fee), we show that the strategic incentives are reversed when, in addition to fees, upstream firms can also choose per-unit payments. In particular, under such contracts, each firm prefers to minimize the number of its downstream firms. Our analysis is not purely of theoretical interest: contracts that specify both fees and royalties are frequently used in the real world. Thus, the statement that a larger number of retailers is preferred for strategic reasons is not generally true and requires qualification.

To fully understand the interaction between royalties and the number of downstream competitors we have considered a number of alternative structures. We find that the main result is not sensitive to whether royalties are chosen simultaneously or sequentially. As long as royalties are chosen after the number of firms, a larger number of firms is not desirable. However, it is crucial that royalties are chosen after the number of firms: we find that if they are chosen before or simultaneously with the number of firms, then (under some conditions) the incentive to increase the number of downstream firms reappears. Similarly, it is important for the main result that contracts involve not only royalties but also fees.

Our model shares with most of the work in this area the feature that upstream firms can commit to their contracts and that these contracts are observable. This is an important assumption in our framework: its main implication is that, at the level of downstream competition, all firms know the terms of all contracts. Uncertainty about contracts would represent uncertainty about the true reaction functions of competitors. In this paper we assume that public commitment is possible at the stage of announcing contracts. The implications of relaxing this assumption have been explored in other work.[43]

To highlight the strategic motives and to facilitate comparison with the existing literature on strategic contracting, we have kept the model as simple as possible. Clearly, a number of factors that play an important role in vertically related markets, including factors related to imperfect information, are not captured in this model. Such factors could lead to different conclusions regarding the equilibrium contracts. Further, we have employed a simple parameterization with linear demand (and costs). This formulation allows us to highlight the strategic motives in a clear way and we expect the main implications to be more generally true. Nevertheless, alternative formulations may result in modified conclusions.

[43] See, for example, Fershtman and Kalai [9] on unobservable strategic contracting. A related but different issue is whether the commitment problem faced by an upstream firm that licenses to multiple downstream firms can be solved using nondiscrimination clauses. See, for example, DeGraba and Postlewaite [5] who find that such clauses are effective with linear prices and McAfee and Schwartz [20] who find that they are not effective under two-part tariffs (both papers consider a monopolist upstream).

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n_A	n_B	r_A	r_B	q_A	$Q_A = n_A q_A$	$q_B = Q_B$	p	π^A	π^B
1	1	$-\frac{a}{5}$	$-\frac{a}{5}$	$\frac{2a}{5}$	$\frac{2a}{5}$	$\frac{2a}{5}$	$\frac{a}{5}$	$\frac{2a^2}{25}$	$\frac{2a^2}{25}$
2	1	0	$-\frac{a}{3}$	$\frac{a}{6}$	$\frac{a}{3}$	$\frac{a}{2}$	$\frac{a}{6}$	$\frac{a^2}{18}$	$\frac{a^2}{12}$

1a: Fee and royalty contracts

n_A	n_B	r_A	r_B	q_A	$Q_A = n_A q_A$	$q_B = Q_B$	p	π^A	π^B
1	1	-	-	$\frac{a}{3}$	$\frac{a}{3}$	$\frac{a}{3}$	$\frac{a}{3}$	$\frac{a^2}{9}$	$\frac{a^2}{9}$
2	1	-	-	$\frac{a}{4}$	$\frac{a}{2}$	$\frac{a}{4}$	$\frac{a}{4}$	$\frac{a^2}{8}$	$\frac{a^2}{4}$

1b: Fees only

n_A	n_B	r_A	r_B	q_A	$Q_A = n_A q_A$	$q_B = Q_B$	p	π^A	π^B
1	1	$\frac{a}{3}$	$\frac{a}{3}$	$\frac{2a}{9}$	$\frac{2a}{9}$	$\frac{2a}{9}$	$\frac{5a}{9}$	$\frac{2a^2}{27}$	$\frac{2a^2}{27}$
2	1	$\frac{7a}{22}$	$\frac{3a}{11}$	$\frac{7}{44}a$	$\frac{7}{22}a$	$\frac{9}{44}a$	$\frac{21}{44}a$	$\frac{49}{484}a^2$	$\frac{27}{484}a^2$

1c: Royalty-only contracts

TABLE 1. Numerical example: n_A increases from 1 to 2 (with $n_B = 1$ and $s = 1$)

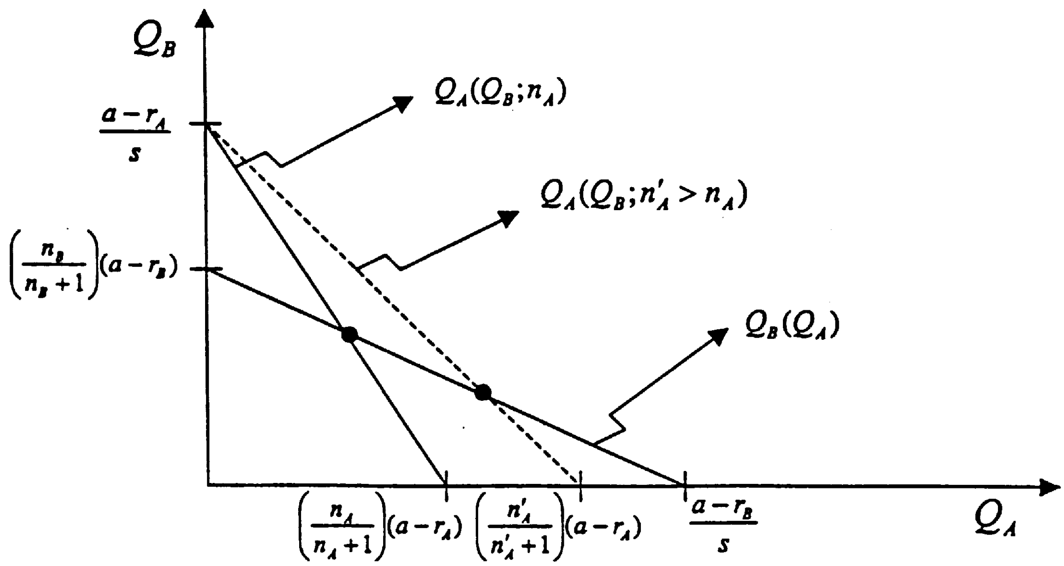


FIGURE 1. Brand reaction functions

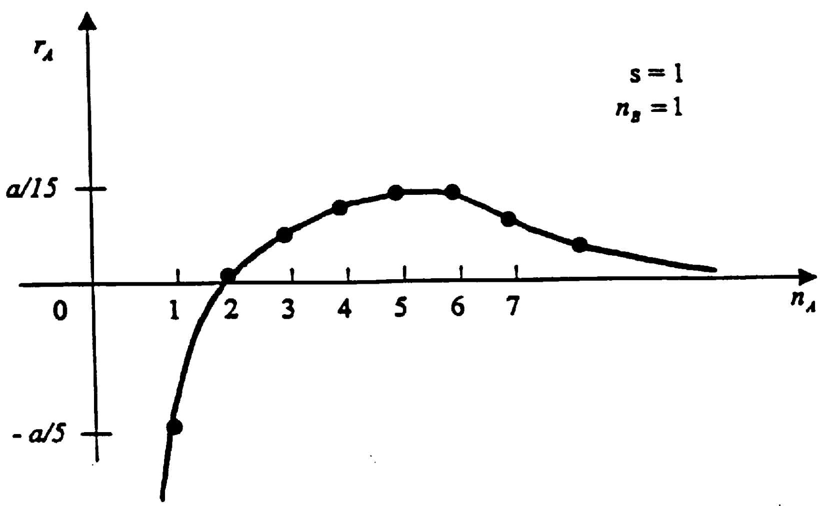
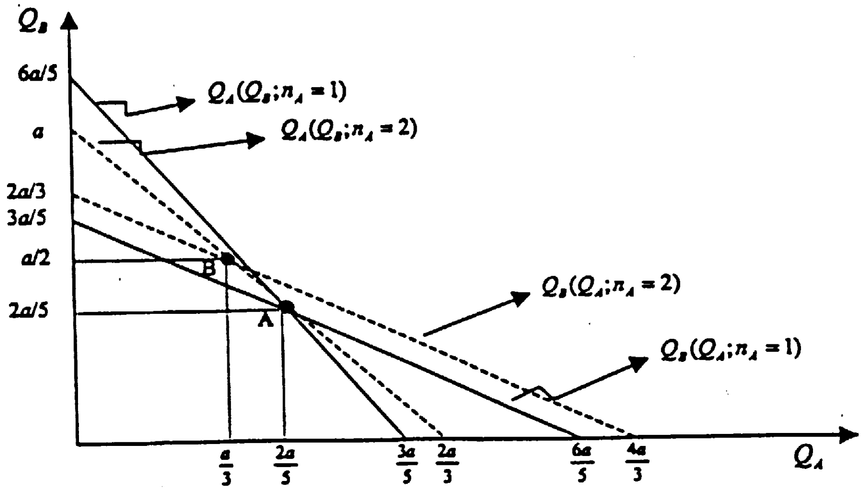
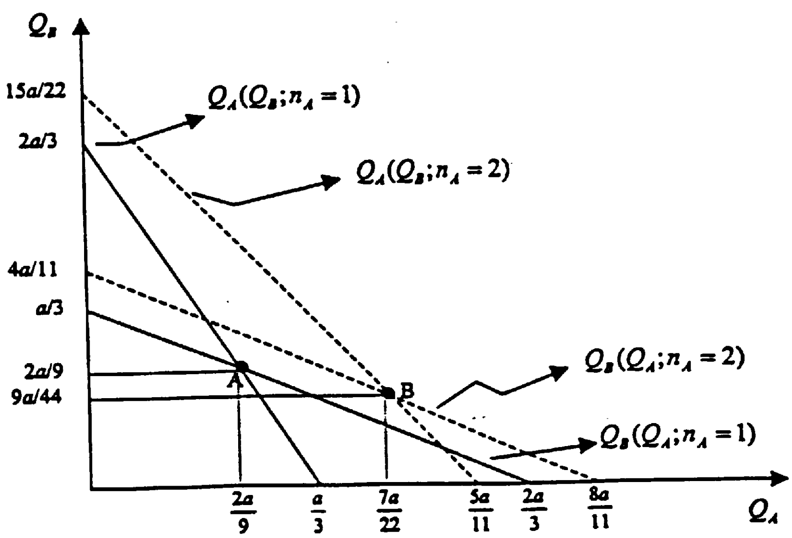


FIGURE 2. Equilibrium royalty as a function of own number of firms



3a. Fee and royalty contracts



3b. Royalty-only contracts

FIGURE 3. Comparison of equilibria when n_A increases from 1 to 2 (with $n_B = 1$ and $s = 1$)