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ABSTRACT

Project Evaluation and Organizational Form*

In situations of imperfect testing and communication, as suggested by Sah and Stiglitz (1986), organizational forms can be identified with different rules of aggregating evaluations of individual screening units. In this paper, we discuss the relative merits of polyarchical organizations versus hierarchical organizations in evaluating cost reducing R&D projects when individual units' decision thresholds are fully endogenous. Contrary to the results of Sah and Stiglitz, we find that the relative merit of an organizational form depends on the curvature of the screening functions of the individual evaluation units, rather than the pool of potential projects. We find that for certain parameters organizations would want to implement asymmetric decision rules across screening units. This allows us to derive sufficient conditions for a polyarchy to dominate a hierarchy. We also find conditions for which the cost curves associated with the two organizational forms cross each other. In this case the optimal organizational form will depend on product market conditions and on the 'lumpiness' of cost reducing R&D.

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NON-TECHNICAL SUMMARY

As argued by Sah and Stiglitz (1986) organizations differ in the process of identifying and approving research projects. Even in the absence of incentive problems the statistical search properties for socially valuable projects may vary substantially across different organizational forms. Their theory builds on the two hypotheses that information of each individual concerning a project's value is imperfect and that communication of (imperfect) information is fallible.

If one identifies hierarchies with organizations that sample projects sequentially on several levels, and approve projects only when they have been approved on each hierarchy level, one finds that such organizations will be very conservative. As argued by Sah and Stiglitz (1986) organizations differ in the process of approving projects altogether. An alternative organizational form that approves a project provided a single reviewer finds it worthwhile, will accept far more projects. This form is dubbed a polyarchy by Sah and Stiglitz. As long as individuals' observations are exogenously given and cannot be strategically adjusted, each organizational form has its relative advantages and disadvantages. The hierarchy will often erroneously reject worthwhile projects, while seldom risking acceptance of unworthy projects, while a polyarchy will rarely reject worthwhile projects, while often accepting unworthy projects. According to Sah and Stiglitz, the social choice between different organizational forms should therefore depend on the relative social valuations of the error of erroneously accepting a bad project and the error of erroneously rejecting a good project, and, accordingly, the pool of underlying projects.

In this paper, we extend the Sah and Stiglitz analysis in order to discuss the relative merits of polyarchical organizations versus hierarchical organizations in evaluating cost-reducing R&D projects when individual units' decision thresholds are fully endogenous. We find that firms with different organizational forms will typically want to use different thresholds. The hierarchy typically will relax thresholds while the polyarchy will sample more strictly. Accordingly, the value of individual observations will differ across organizations. Also, in general, it is no longer true that a hierarchy with the same number of reviewers will be more conservative than the polyarchy and vice versa.

Contrary to the results of Sah and Stiglitz, we find that the relative merit of an organizational form depends on the curvature of the screening functions of the individual evaluation units, rather than the pool of potential projects. We find that for certain parameters organizations would want to implement asymmetric decision rules across screening units. In this case, the polyarchy is more flexible in fine-tuning the value of each individual observation relative to the hierarchy, while the hierarchy prefers to rely on a single observation. This allows us to derive sufficient conditions for a polyarchy to dominate a hierarchy. This constellation highlights an interesting feature of the Sah and Stiglitz approach, in which the value of information may not increase in the number of observations.

We also find conditions for which the cost curves associated with the two organizational forms cross each other. In this case the optimal organizational form will depend on product market conditions and on the 'lumpiness' of cost-reducing R&D. When desired success probabilities are low the hierarchy is the cost minimizing organization, while it is the polyarchy when desired success probabilities are high.

Project Evaluation and Organizational Form

1. Introduction

When firms search for new products or ideas they need to develop judgements about the likelihood of success. If these judgements are not perfectly accurate it may be desirable to ask different individuals to evaluate the idea and provide independent assessments. These assessments can then be used to decide whether, or not, to pursue the product or idea in question. If all assessments resemble each other, an overall decision will be easily reached. If there is disagreement, however, the overall decision will depend on the nature of the aggregation rule used by the organization.

In this paper we focus on the case where firms must evaluate (potentially) cost-reducing R&D projects. Following Sah and Stiglitz (1986,1988), we assume that individual reviewers cannot communicate perfectly their evaluation of a given project. They can only express whether or not they believe that the project exceeds a pre-specified measure of quality. We will refer to these minimum quality standards as "thresholds". An organization can then be seen as a set of review units capped by a "strategic" unit which sets the thresholds and decides how to aggregate the assessments of the reviewers. Two such aggregation rules are considered. In a hierarchy, unanimous approval by the review units is required for the R&D project to be approved and carried out. On the contrary, a polyarchy would pursue any project approved by at least one of its units.

The assumption of limited communication seems to be reasonable. Individual reviewers may well develop sophisticated assessments of the project at hand but the sheer complexity of the task combined with differences in the skills and backgrounds of reviewing and strategic unit may hamper the effective communication of such detailed appraisals. Also it may be difficult to articulate "gut feelings" about the profitability of a project. Alternatively, incentive reasons may obscure public statements by researchers who may feel uneasy about revealing areas in which their knowledge is rather imprecise.

For the type of cost-reducing R&D projects that we consider we show that the performance of an organization can be summarized by a "cost function" $C(q)$ which gives the expected cost of approving a worthwhile project with probability q. We can then compare polyarchies and hierarchies by ranking their corresponding cost functions. To achieve this, we depart from Sah and Stiglitz (1986, 1988) by allowing the strategic unit to set different thresholds to different review units. This extra flexibility allows the organization to affect the informational value of an observation communicated to the strategic unit. Typically, the informational value of an individual observation differs across organizational forms. We find situations, where a hierarchy prefers a single observation to two observations, while the polyarchy will always select two observations. This can be implemented by an asymmetric choice of thresholds, where the hierarchy selects one threshold which never binds. While the hierarchy will always loosen thresholds when additional observations are possible, and hence reduce the informational content of the communicated signals, the polyarchy will always tighten thresholds and thus increase the informational content of the communicated signals. This implies that the polyarchy always prefers more observations, while the hierarchy has to trade off the gains from an additional observation against the costs of reduced informativeness of the individual observations. Hence, whenever the costs of reduced informativeness exceed the gains from additional observations, the hierarchy prefers a single observation. Whenever this happens the cost function of a polyarchy lies everywhere below the cost function of the hierarchy. This will occur when the distribution of signals received by the review units has a decreasing likelihood ratio and when signals are not too informative.

Whenever the hierarchy prefers to use both observations, both, polyarchies and hierarchies, would choose the same threshold for all review units so that Sah and Stiglitz's assumption is actually verified. Still we can extend their results by showing that, for our cost-reducing R&D projects, the cost functions associated with hierarchy and polyarchy must cross at least once. This suggests that the optimal organizational form depends on the desired level of q and thus on market conditions. Moreover, a polyarchy must be more efficient than a hierarchy for high levels of q while the opposite must be true for low levels of q.

The effect of threshold choice on the informativeness of signals distinguishes this literature on communication of coarse information quite sharply from the literature that allows perfect communication of signals. The assumption of limited communication may seem reasonable in situations when projects have to be evaluated in a rather short time period. When the period between successive evaluations is long enough, however, the organization may wish to make later evaluations dependent on the outcomes of earlier ones. In a context of career concerns Meyer (1991, 1992) shows that organizational form may serve to enhance communication when communication is limited. She shows that sequential project evaluations typically dominate simultaneous evaluations since the second reviewer can condition on information generated by the first reviewer. Furthermore, she shows that biasing sequential screens may be good for incentive reasons. Also Melumad, Mookherjee and Reichelstein (1990) discuss incentive effects of organizational form when communication is limited. Aghion and Tirole (1994) analyze endogenous communication in organizations when complete contracts are infeasible and the allocation of control rights matters. In this paper we want to concentrate purely on the informational differences associated with different decision rules. Therefore, we completely abstract from incentive effects, treating review units rather mechanically as truthful information generating devices. Also we do not consider sequential organizations. Rather, we concentrate on the relative performance of simultaneous decision rules.

The paper is organized as follows. In section 2 we present the market environment, the screening processes, and the stochastic environment faced by the firm. In section 3 we obtain conditions under which polyarchies and hierarchies choose interior or corner solutions for their thresholds. We use this result in section 4 to rank hierarchies and polyarchies according to their cost functions and discuss how the choice of organizational form might depend on the firm's external environment. Section 5 presents parametric examples and section 6 concludes.

2. The Model

We will first describe the market environment in which the firm operates. We will then turn to the internal organization of the firm and to a precise specification of the stochastic environment.

Market Environment

Consider a single firm which has the option to conduct cost reducing research. The outcome of the research effort is uncertain. However, the firm may hire experts, who will develop some imperfect judgement about the project's likelihood of success. If the project is successful the firm can reduce marginal costs of production to zero. If the project is unsuccessful, research expenses are lost and production continues at the current marginal costs $\bar{c} > 0$. The cost of carrying out the project is assumed to be fixed and is equal to $F > 0$.

After the initial research period a market game is played. Denote the market payoff $R(c) \geq 0$. Furthermore, define as q the probability that the firm ends up with a good project, i.e. the probability that the firm ends up with marginal costs of 0. Let $C(q)$ be the cost of carrying out the approved project. The firm is assumed to be risk neutral. Its optimization problem now reads:

$$\max_q \quad qR(0) + (1-q)R(c) - C(q) \tag{1}$$

Architecture of the Firm and the Screening Process

The firm is viewed as consisting of a strategic (policy-setting) unit and two screening units. Screening units $i = 1, 2$ have to evaluate potential research projects. The result of their screening activities are two imperfect signals $\tilde{y}_i, i = 1, 2$ of a project's quality. Based on its own signal each unit decides whether or not to recommend the project for adoption. The recommendation is the only information passed on to the strategic unit. The decision to recommend a project is based on a decision rule A_i.

Different organizational forms are identified with rules that aggregate individual decisions. When unanimity is required to implement the project we refer to the organisation as a *hierarchy.* When the project requires only one vote of approval we shall call the organization a *polyarchy* (see figure 1).

It should be noted that our definition of a polyarchy implies some form of coordination, which excludes the duplication of projects. In our setting the project will be adopted by the organization only after individual decisions are aggregated.[1]

The strategic unit selects an organizational form and determines the decision rules $A_i, i = 1, 2$ to maximize the firm's expected profits. While a general rule would specify precisely the set of signals for which adoption should occur, we concentrate on *threshold decision rules.* A screening unit will vote for adoption, whenever $A_i = \{\tilde{y}_i \mid \tilde{y}_i \leq T_i\}$. T_i is called a threshold and can be thought of as corresponding to the internal "hurdle rates" used by most US firms.

Screening units are assumed to do their prescribed tasks rather mechanically: they observe their signals and only report whether or not they meet the assigned thresholds. We abstract from incentive issues by assuming that the quality of the signal $\tilde{y}_i$ does not depend on the effort exerted by the screening agent and that the welfare of the screening unit is independent of its report.

In order to compare our two organizational forms, some kind of normalization is necessary. One could explicitly introduce reviewing costs and let each organization decide how many of the available projects to review. Rather, we decided to ignore review costs and to normalize the number of projects reviewed to one. This approach has the advantage that each organizational form gets the same number of independent signals. In the concluding section we discuss the implications of our findings for alternative normalizations.

[1] Such a view seems reasonable, when the organization is interpreted as a firm or a committee. When economic systems are compared, as in Sah, Stiglitz (1986), presumably one would interpret each screening unit as a firm that could adopt the project on its own. In this case duplication of projects will occur in the case of polyarchies. This cost would not apply to hierarchies.

We take the signal received by a single review unit to be a one-dimensional random variable distributed on the interval $[0, 1]$ with density $g(y)$ and distribution function $G(y)$. The conditional density of a good project, given the observation y, is denoted as $f(y)$. Hence, the a priori success probability of the project, denoted by $p > 0$, can be determined as $p = \int_0^1 f(y)g(y)dy$.

The conditional densities $h_0(y)$ of observing $\tilde{y} = y$ given that the project is really good, and $h_c(y)$ of observing $\tilde{y} = y$ when the project is actually bad, are of particular interest. Denote their respective cumulative distributions by $H_0(y)$ and $H_c(y)$.

Between f, g and h_0 the following relation (Bayes' law) holds:

$$h_0(y) = \frac{1}{p} f(y) g(y) .$$

Given a decision rule A_i, the probability $\hat{q}_i$ that a single unit i accepts the project is equal to

$$\begin{aligned} \hat{q}_i &= \int_{y \in A_i} dG(y) \\ &= pH_0(T_i) + (1-p)H_c(T_i) . \end{aligned}$$

The probability of unit i accepting a good project q_i is determined by:

$$\begin{aligned} q_i &= p \int_{y \in A_i} h_0(y)\, dy \\ &= p\, H_0(T_i) . \end{aligned}$$

Organizational design becomes trivial when both screening units observe perfectly correlated signals; in this case they always agree. We are however interested in the case where observation errors across screening units are independent.

Obviously the signals cannot be independent themselves, since the underlying project is either good or bad.

Assumption: Independent Observation Errors

The joint conditional distributions of $(\tilde{y}_1, \tilde{y}_2)$ given $\tilde{c}$ can be written $H_0(y_1, y_2) = H_0(y_1)H_0(y_2)$ and $H_c(y_1, y_2) = H_c(y_1)H_c(y_2)$.

Finally we discuss the meaning of signals. We assume that low realizations of $\tilde{y}$ can be taken as an indication of low costs, and hence constitute good news, while high realizations are rather bad news. This is formalized as:

Definition: Monotone Likelihood Ratio Property (MLRP)

Let $h_0(y) > 0$ and $h_c(y) > 0$ and let $h_0(y)$ and $h_c(y)$ be differentiable for $0 < y < 1$. Furthermore, let $H_0(0) = H_c(0) = 0$. The monotone likelihood property (MLRP) is satisfied when

$$\frac{h_0'(y)}{h_0(y)} \leq \frac{h_c'(y)}{h_c(y)} \quad , 0 < y < 1 \quad . \tag{2}$$

For part of the analysis we shall be interested in the case where $h'_0(y) \leq 0$ and $h'_c(y) \geq 0$. This assumption implies the validity of the monotone likelihood ratio property.

3. Optimal Organizational Structures

As shown in equation (1) the profitability of a firm is uniquely determined by q, the probability of carrying out a good project and $C(q)$, the expected cost of developing a good project with probability q. In order to compare polyarchies and hierachies we need to derive their corresponding cost functions $C^H(q)$ and $C^P(q)$. These functions will usually not be the same. For identical thresholds, the polyarchy will accept both good and bad projects with a higher probability than the hierarchy (Sah and Stiglitz 1986). The polyarchy's thresholds could of course be lowered to yield the same probability of acceptance of a good project as the hierarchy but then the two organizations would still differ in the probability of acceptance of a bad project.

This section provides an analysis of these cost functions with threshold decision rules. The strategic unit defines threshold levels $T_i \in [0, 1]$ for $i = 1, 2$ and then decides whether overall approval by the firm requires acceptance by both units, which represents the case of an hierarchical order, or by a single unit only, as in the case of a polyarchy.

In the case of the hierarchy, for given thresholds T_i, $i = 1, 2$, the probability of acceptance of a good project is the probability that the given project is good, p, times the probability an organization accepts the project, conditional on the project being good.

$$\begin{aligned} q^H(T_1, T_2) &= p \int_{y \leq T_1} h_0(y) dy \int_{y \leq T_2} h_0(y) dy \\ &= p\, H_0(T_1)\, H_0(T_2) \end{aligned}$$

Likewise in the case of the polyarchy the probability of accepting a good project is determined by:

$$\begin{aligned} q^P(T_1, T_2) &= p \left(1 - \left(1 - \int_{y \leq T_1} h_0(y) dy\right) \left(1 - \int_{y \leq T_2} h_0(y) dy\right)\right) \\ &= p\,(1 - (1 - H_0(T_1))\,(1 - H_0(T_2))) \end{aligned}$$

The probability of accepting a project includes the possibility of erroneously accepting a bad project. This reads in the case of a hierarchy as:

$$\hat{q}^H(T_1, T_2) = p\, H_0(T_1)\, H_0(T_2) + (1 - p)\, H_c(T_1)\, H_c(T_2)$$

Likewise in the case of the polyarchy the project is accepted, if either screening unit accepts, or alternatively, if both units do not reject. So the probability of acceptance is:

$$\hat{q}^P(T_1,T_2) = p\Big(1 - (1-H_0(T_1))(1-H_0(T_2))\Big)$$
$$+ (1-p)\Big(1 - (1-H_c(T_1))(1-H_c(T_2))\Big)$$

We are now in a position to derive the cost functions $C^H(q)$ and $C^P(q)$. Suppose the strategic unit would like the organization to accept a good project with probability q. The cost associated with this requirement consists of the erroneous acceptance of bad projects. The probability of an erroneous acceptance will depend on the choice of T_1 and T_2. The cost of achieving success probability q is defined by the choice of (T_1, T_2) that minimizes erroneous adoptions. So it is the solution of an optimization problem.

A firm organized as a hierarchy solves

$$C^H(q) := \min_{T_1,T_2} \Big[(q + (1-p)\, H_c(T_1)\, H_c(T_2))\, F \;\mid\; p\, H_0(T_1)\, H_0(T_2) = q\Big] \quad \textbf{(H)}$$

Likewise, a firm organized as a polyarchy solves

$$C^P(q) := \min_{T_1,T_2} \Big[(q + (1-p)\,(1 - (1-H_c(T_1))(1-H_c(T_2))))\, F \;\mid$$
$$p\,(1 - (1-H_0(T_1))\,(1-H_0(T_2))) = q\Big] \quad \textbf{(P)}$$

The concept of *log concavity* and *log convexity* will prove useful in characterizing different regimes for organizational form.

Definition

A function $h: D \rightarrow D$, where $D \subset I\!R$ is called *log concave* if and only if $ln\, h(x)$ is concave for all $x \in D$. The function $h: D \rightarrow D$ is called *log convex* if and only if $ln\, h(x)$ is convex.

Differentiable log concave and log convex functions exhibit some useful properties, which we shall summarize in the following lemma.

Lemma 1

Let $h(x) \geq 0$ be differentiable for $x \in D$, where $D \subset I\!R$ is compact.

a) The function $h(.)$ is (strictly) log concave if and only if $h''(x)h(x) - (h'(x))^2 < 0$ for all $x \in D$. It is (strictly) log convex if and only if $h''(x)h(x) - (h'(x))^2 > 0$ for all $x \in D$.

b) If the function $h(.)$ is concave it is also log concave.

c) If the function $h(.)$ is log convex it is also convex.

d) Let $k : D \to D$ and $k(x) = h(1-x)$. Then $h(.)$ is log concave (log convex) if and only if $k(.)$ is log concave (log convex).

e) Let $k : D \to D$ be the inverse function of h, i.e. $k = h^{-1}$. Furthermore, $h(x) > 0$ and $h'(x) > 0$ for $x \in \text{int } D$.

Then $h(.)$ is log concave if and only if $h''(x) > -\frac{k'(x)}{x}$, for all x

And $h(.)$ is log convex if and only if $h''(x) < -\frac{k'(x)}{x}$, for all x

f) Let $x \leq 0$. Then $h(exp(x)) : [-\infty, 0] \to [0, 1]$ is log concave if $h''(x)h(x) - (h'(x))^2 < \frac{-1}{x}h(x)h'(x)$ for all x and $k(1 - exp(x)) : I\!R_{\leq 0} \to [0, 1]$ is log concave if $k''(x)k(x) - (k'(x))^2 < \frac{1}{1-x}k(x)k'(x)$ for all x.

$h(exp(x)) : [\infty, 0] \to [0, 1]$ is log convex if $h''(x)h(x) - (h'(x))^2 > \frac{-1}{x}h(x)h'(x)$ for all x and $k(1 - exp(x)) : [\infty, 0] \to [0, 1]$ is log convex if $k''(x)k(x) - (k'(x))^2 > \frac{1}{1-x}k(x)k'(x)$ for all x.

Proof:

Straightforward differentiation establishes the above results. In case e) observe that $h(k(x)) = x$ implies $h'(k(x))k'(x) = 1$ and (by differentiating again) $h''(k(x))(k'(x))^2 + h'(k(x))k''(x) = 0$. Application of a) yields the result.

Q.E.D.

Define the conditional success probability for a good project $z := \frac{q}{p}$. We are now in a position to discuss the solutions to the optimization problems **(H)** and **(P)**.

<u>Result 1: Hierarchy</u>

a. If $H_c(H_0^{-1}(e^z))$ is log concave in z, the solution to **(H)** *is a corner solution with $(1-T_1)(1-T_2) = 0$.*

b. If $H_c(H_0^{-1}(e^z))$ is log convex in z, the solution to **(H)** *is uniquely determined and symmetric, i.e. $T_1 = T_2$.*

Proof:

With $z = \frac{q}{p} \in [0,1]$, the hierarchy's planning problem is equivalent to

$$\min_{T_1,T_2} \left[(1-p) H_c(T_1) H_c(T_2) F \mid H_0(T_1) H_0(T_2) = z \right]$$

Equivalently,

$$\min_{T_1,T_2} \left[H_c(H_0^{-1}(exp(lnH_0(T_1)))) H_c(H_0^{-1}(exp(lnH_0(T_2)))) \mid \right.$$
$$\left. lnH_0(T_1) + lnH_0(T_2) = ln(z) \right]$$

or

$$\min_{S_1,S_2} \left[H_c(H_0^{-1}(exp(S_1))) H_c(H_0^{-1}(exp(S_2))) \mid S_1 + S_2 = ln(z) \right]$$

where $S_i = lnH_0(T_i))$. Substitute $A(S_i) := H_c(H_0^{-1}(exp(S_i)))$. This is a monotonic transform. It is readily established that along the cost curve $\frac{dS_2}{dS_1} = -\frac{A'(S_1)A(S_2)}{A(S_1)A'(S_2)}$. Differentiating again and observing that $A'(S) > 0$ we get $\frac{d^2S_2}{dS_1^2} > 0$ iff $A''(S_i)A(S_i) - (A'(S_i))^2 < 0$ for $i = 1,2$. Likewise, it follows that $\frac{d^2S_2}{dS_1^2} < 0$ iff $A''(S_i)A(S_i) - (A'(S_i))^2 > 0$ for $i = 1,2$.

So, by lemma 1a, the optimization problem **(H)** attains a corner solution with $(1-T_1)(1-T_2)=0$ when $A(S)$ is log concave and **(H)** attains an interior solution with $T_1 = T_2$ when $A(S)$ is log convex.

Q.E.D.

The proof generalizes easily to the case of $N > 2$ reviewing units with a hierarchical decision rule. The same applies the the case of the polyarchical decision rule.

Result 2: Polyarchy

a. If $1 - H_c(H_0^{-1}(1-e^z))$ is log concave in z, the solution to **(P)** *is uniquely determined and symmetric, i.e., $T_1 = T_2$.*

b. If $1 - H_c(H_0^{-1}(1-e^z))$ is log convex in z, the solution to **(P)** *is a corner solution with $(1-T_1)(1-T_2)=0$.*

Proof:

The logic of the proof is the same as for result 1. The firm's minimization problem is

$$\min_{T_1,T_2}\left[1-(1-H_c(T_1))\,(1-H_c(T_2))\mid((1-H_0(T_1))\,(1-H_0(T_2)))\;=\;1-z\right]$$

Or, equivalently,

$$\max_{S_1,S_2}\Big[\,(1-H_c(H_0^{-1}(1-exp(S_1))))\;(1-H_c(H_0^{-1}(1-exp(S_2))))\;\mid$$
$$S_1\;+\;S_2\;=\;ln(1-z)\Big]$$

where the monotonic transformation $S_i = ln(1-H_0(T_i))$ has been made. Defining $A(S_i) = 1 - H_c(H_0^{-1}(1-\exp(S_i)))$ we have

$$\max_{S_1, S_2} \left[A(S_1)A(S_2) \mid S_1 + S_2 = ln(1 - z) \right]$$

This problem has an interior solution if the isocost curve is convex to the origin and corner solutions if the isocost curve is concave to the origin. As for result 1, it is easily shown that convexity of the isocost curve obtains iff $A(S_i)$ is log concave while concavity obtains iff $A(S_i)$ is log convex.

Q.E.D.

The conditions for the potential selection of corner solutions under the two organizational forms are of particular interest, since precisely this phenomenon is ruled out in the analysis of Sah and Stiglitz (1986). The next corollary illustrates the conditions for corner solutions in terms of the primitive distribution functions.

<u>**Corollary**</u>

Let $H_0(0) = H_c(0)$ and $h_0(y) > 0$, $h_c(y) > 0$ and differentiable for $0 < y < 1$. Then

*a) **(H)** attains corner solutions when*

$$\frac{h_c(y)}{H_c(y)} - \frac{h_c^{'}(y)}{h_c(y)} > \frac{h_0(y)}{H_0(y)} - \frac{h_0^{'}(y)}{h_0(y)} \quad , \quad 0 < y < 1$$

*b) and **(P)** attains corner solutions when*

$$\frac{-h_c(y)}{1 - H_c(y)} - \frac{h_c^{'}(y)}{h_c(y)} > \frac{-h_0(y)}{1 - H_0(y)} - \frac{h_0^{'}(y)}{h_0(y)} \quad , \quad 0 < y < 1$$

Proof:

The proof is by direct evaluation of the conditions for corner solutions in Result 1.a and Result 2.b. Observe that the (first) derivative of $\ln(H_c(y))$ where $y = H_0^{-1}(e^z)$ can be written as $\frac{H_0(y)}{H_c(y)} \frac{h_c(y)}{h_0(y)}$. Likewise the (first) derivative of $\ln(1 - H_c(y))$ where $y = H_0^{-1}(1 - e^z)$ can be written as $\frac{1-H_0(y)}{1-H_c(y)} \frac{h_c(y)}{h_0(y)}$. The conditions of a) and b) then follow by differentiating again and checking for concavity and convexity respectively.

Q.E.D.

The hierarchy attains corner solutions when the relative slopes $\frac{h_c'(y)}{h_c(y)}$ and $\frac{h_0'(y)}{h_0(y)}$ are sufficiently close. In other words, corner solutions occur when the conditional distribution functions $H_0(y)$ and $H_c(y)$ are relatively close, or signals are not very informative (see figure 2).

The condition for corner solutions for a polyarchically structured organization imply that $\frac{h_c'(y)}{h_c(y)} - \frac{h_0'(y)}{h_0(y)} < 0$ whenever the signal $\tilde{y}$ is informative. Hence, under MLRP, the polyarchical firm will never select corner solutions.

4. Properties of Optimal Organizations

First note that the monotone likelihood ratio property implies the convexity of $H_c(H_0^{-1}(z))$.

Lemma 2

Under the monotone likelihood ratio property the function $H_c(H_0^{-1}(z))$ is convex in $z \in [0,1]$.

Proof:

Observe that $D_z H_c(H_0^{-1}(z)) = LR^{-1}(H_0^{-1}(z))$ and $D_z^2 H_c(H_0^{-1}(z)) \geq 0$ iff $\frac{h_0'(H_0^{-1}(z))}{h_0(H_0^{-1}(z))} \leq \frac{h_c'(H_0^{-1}(z))}{h_c(H_0^{-1}(z))}$

Q.E.D.

An immediate consequence of lemma 2 is that the function $1 - H_c(H_0^{-1}(1 - exp(z)))$ is log concave. Therefore, we find that under the monotone likelihood ratio property the polyarchy will always select an interior solution.

Result 3

Under the monotone likelihood ratio property the polyarchy will always select an interior solution $T_1^P = T_2^P$.

Proof:

We show that under MLRP the function $1 - H_c(H_0^{-1}(1 - e^z))$ is log concave. According to Result 2 this implies that the polyarchy selects an interior solution.

To do this define the auxiliary function $l(z) := H_c(H_0^{-1}(z))$. This function has the following derivatives

$$l'(z) = \frac{h_c(H_0^{-1}(z))}{h_0(H_0^{-1}(z))}$$

$$l''(z) = \frac{h_c(H_0^{-1}(z))}{h_0^2(H_0^{-1}(z))}\left(\frac{h_c'(H_0^{-1}(z))}{h_c(H_0^{-1}(z))} - \frac{h_0'(H_0^{-1}(z))}{h_0(H_0^{-1}(z))}\right)$$

Accordingly, $\frac{l''(z)}{l'(z)} - \frac{l'(z)}{l(z)} > \frac{1}{1-z}$ is equivalent to

$$\frac{h'_c(z)}{h_c(z)} - \frac{h'_0(z)}{h_0(z)} > \frac{h_c(z)}{H_c(z)} + \frac{h_0(z)}{1-z}$$

Since the right hand side is always positive, according to lemma 2, MLRP implies this relation. Hence, by application of lemma 1.f we find that $1 - l(1 - e^z)$ is log concave.

Q.E.D.

In the case of the hierarchy a similar result does not obtain. The next lemma provides an interpretation of the condition for interior solutions for hierarchies.

Lemma 3

$H_c(H_0^{-1}(exp(z)))$ *is log convex in* z *if and only if:*

$$\frac{h_c'(z)}{h_c(z)} - \frac{h_0'(z)}{h_0(z)} > \frac{h_c(z)}{H_c(z)} - \frac{h_0(z)}{z} \quad , 0 < z < 1 . \qquad (3)$$

Proof:

Using the auxiliary function $l(z)$ of the proof of Result 3 one finds that $\frac{l''(z)}{l'(z)} - \frac{l'(z)}{l(z)} > \frac{-1}{z}$ for all z iff

$$\frac{h'_c(z)}{h_c(z)} - \frac{h'_0(z)}{h_0(z)} > \frac{h_c(z)}{H_c(z)} - \frac{h_0(z)}{z} \quad , 0 < z < 1$$

According to lemma 1.f this implies that $l(e^z)$ is log convex.

Q.E.D.

The condition for an interior solution of the hierarchy's problem is restrictive, since (3) is not implied by MLRP. As a result, with MLRP two possibilities arise. Either the hierarchy chooses a corner solution, in which case it is (weakly) dominated by the polyarchy, or it chooses an interior solution.

Result 4: Dominance of the Polyarchy

When $H_c(H_0^{-1}(exp(z)))$ is log concave in $z \in I\!R_{\leq 0}$ under the monotone likelihood ratio property the polyarchy has lower costs than the hierarchy, i.e. $C^P(q) \leq C^H(q)$.

Proof:

According to result 3 the polyarchy selects an interior solution, while according to lemma 3 and result 2 the hierarchy selects a corner solution. This corner solution is a feasible choice for the polyarchy.

Q.E.D.

This situation differs markedly from the analysis of Sah and Stiglitz (1986), since it demonstrates that with an endogenous choice of thresholds, organizations may select asymmetric thresholds within a given organizational form. Sah and Stiglitz impose symmetric solutions.

Interestingly, the polyarchy is the dominant form of organization whenever MLRP is satisfied, and when $H_c(H_0^{-1}(exp(z)))$ is log concave in z. This condition is more likely to be met, when signals are not very informative. In such situations a hierarchy may be too conservative. Therefore, the organization has to restrict itself to a single observation, in order to meet the desired probability of success q. This result may seem surprising since the hierarchy selects to ignore information from further sampling even though there is no explicit cost of sampling. In fact, this result is driven by the coarseness of information communicated by the review units. To see this, consider the following situation. Suppose product market characteristics are such that the organization wants to implement success probability q_0. If there was a single review unit it would select a threshold level T_0 such that $q(T_0) = q_0$. How would an additional review unit affect the organizations payoffs under each organizational form?

According to result 4, a hierarchy would select either a corner solution with a corresponding threshold T_0, or an interior solution with threshold T^H such that $q^2(T^H) = q_0$. In the latter case the hierarchy has to relax the hurdle rate $T^H > T_0$ for each review unit to compensate for the increased likelihood of rejecting a good project under two successive reviews. This reduces the tightness of screening and hence the informational value of each observation. So employing a second review unit has the advantage of acquiring an additional (independent) observation, but it has the potential cost of a lower informational value of each observation. If the latter effect dominates the hierarchy prefers to rely on a single observation. If the former effect dominates the hierarchy will use both evaluations and implement a symmetric solution. The loss in information value of a single observation is particularly damaging, when signals are not very informative. This can be seen for example for the case when $h_c(y) = 1$ and $\frac{h'_0(y)}{h_0(y)}$ is close to zero (figure 3.a). When $\frac{h'_0(y)}{h_0(y)}$ is rather negative the signal may still remain fairly informative even when its tightness is reduced (figure 3.b).

In the case of the polyarchy thresholds T^P are chosen symmetrically such that $(1-q(T^P))^2 = q_0$. This implies $T^P < T_0$. Hence, the polyarchy increases the tightness of filtering when additional observations are used. This actually means that the polyarchy gains both, from additional observations *and* from a higher value of each observation. This explains why the polyarchy will never select a corner solution.

Result 4 demonstrates that organizational choice will typically depend on the curvature of the likelihood ratio. When signals are more informative, i.e. $H_c(H_0^{-1}(exp(z)))$ is log convex, both organizations will select (symmetric) interior solutions. This means that the assumption of symmetric thresholds made by Sah and Stiglitz (1986) is justified and their conclusions remain valid. Indeed we can extend their results by presenting conditions under which $C^P(q)$ and $C^H(q)$ must cross at least once. We also obtain some limit results on the ranking of the two cost functions for sufficiently high and sufficiently low values of q.

<u>Result 5: Crossing Cost Curves</u>

When $H_c(H_0^{-1}(exp(z)))$ is log convex and when the MLRP is satisfied, both organizations select interior solutions. Furthermore, there are $\underline{q} > 0$ and $\bar{q} < p$ such that $C^H(q) < C^P(q)$ for $0 < q \leq \underline{q}$ and $C^H(q) > C^P(q)$ for $p > q \geq \bar{q}$.

Proof:

Under the conditions of result 5 both organizations will choose interior solutions. These are uniquely determined and symmetric, i.e. $T_1^H = T_2^H$ and $T_1^P = T_2^P$. (This follows from the fact that log convexity/concavity are imposed globally). So the cost functions can be written as:

$$C^H(q) = (q + (1-p)A^2(\sqrt{\frac{q}{p}}))\,F$$

$$C^P(q) = (q + (1-p)(1 - (1 - A(1 - \sqrt{1 - \frac{q}{p}}))^2))\,F \quad ,$$

where $A(z) = H_c(H_0^{-1}(z))$.

Obviously, $C^H(0) = C^P(0) = 0$ and $C^H(p) = C^P(p) = F$. We shall demonstrate that the two organizational forms exhibit different marginal behaviour in the limits. Define

$$B^H(z) := A^2(\sqrt{z})$$

$$B^P(z) := 1 - (1 - A(1 - \sqrt{1-z}))^2$$

First consider the marginal behaviour for small z, i.e. $z \rightarrow 0$. By application of l'Hospital's rule one finds:

$$\begin{aligned}
\lim_{z\rightarrow 0} D_z B^H(z) &= \lim_{z\rightarrow 0} \frac{1}{\sqrt{z}} D_z A(\sqrt{z})\, A(\sqrt{z}) \\
&= D_z A(0)\, \frac{\lim_{z\rightarrow 0} D_z A(\sqrt{z})}{\lim_{z\rightarrow 0} D_z \sqrt{z}} \\
&= (D_z A(0))^2
\end{aligned}$$

$$\begin{aligned}
\lim_{z\rightarrow 0} D_z B^P(z) &= \lim_{z\rightarrow 0} \left(1 - (1 - A(1 - \sqrt{1-z}))^2\right) \\
&= \lim_{z\rightarrow 0} \frac{1}{\sqrt{1-z}} (1 - A(1 - \sqrt{1-z})\, D_z A(1 - \sqrt{1-z}) \\
&= D_z A(0)
\end{aligned}$$

According to lemma 2 the function $A(z)$ is convex for $z \in [0,1]$. Therefore, $A : [0,1] \rightarrow [0,1]$ implies $D_z A(0) < 1$. Hence, there is a $\underline{z} > 0$, such that $B^H(z) < B^P(z)$ for $0 < z \leq \underline{z}$.

The reverse is true as $z \rightarrow 1$. In this case we find:

$$\lim_{z\rightarrow 1} D_z B^H(z) = D_z A(1)$$

$$\begin{aligned}
\lim_{z\rightarrow 1} D_z B^P(z) &= \lim_{z\rightarrow 1} \left(1 - (1 - A(1 - \sqrt{1-z}))^2\right) \\
&= D_z A(1) \lim_{z\rightarrow 1} \frac{1 - A(1 - \sqrt{1-z})}{\sqrt{1-z}} \\
&= (D_z A(1))^2
\end{aligned}$$

Again, because of convexity of $A(z)$ the derivative $D_z A(1) > 1$. So the cost function has a steeper slope in case of the polyarchy. Therefore, in a sufficiently small neighbourhood of $q = p$ we find $C^P(q) < C^H(q)$.

Q.E.D.

The intuition behind this result relies on the fact that for small q, T^H and T_0, the threshold level of a hypothetical firm with a single review unit, are fairly close to zero, relative to T^H. Hence, both a single review firm and a polyarchy tend to be rather restrictive in their evaluations. In this case the additional gains through tighter observations and additional filtering are rather small as compared to the gains that can be achieved by additional evaluations with signals that are less tight. In this case the hierarchy is less conservative than the polyarchy.

Likewise, when q is large, T_0 and T^H are close to one. In this case, these two organizational forms do little filtering and basically accept all projects. Hence, the value of a second evaluation is rather small for the hierarchy, and smaller than tighter filtering by independent review units, as in the case of the polyarchy.

According to result 5, for bounded and strictly decreasing signal densities we can find situations, in which firms may prefer the hierarchical organization, when they choose a (rather) low conditional success probability $\frac{q}{p}$, while they will prefer a polyarchical organization, when their desired conditional success probability $\frac{q}{p}$ is (rather) large. A direct consequence of this is that the optimal organizational form depends on the level of q that the strategic unit wants to achieve. This in turn will depend on the precise shape of the payoff function $R(c)$, i.e., on *market conditions*. For example, the same firm may prefer one or the other organizational form, independently of the a priori success probability p. All else equal, the firm is more likely to seek a high level of q when the cost reduction expected from a good project (i.e. $\bar{c}$) is large. In that sense, one would expect polyarchical review processes to emerge in industries where cost-reducing innovations are "lumpy".

At the points of intersection of $C^H(q)$ and $C^P(q)$, typically, the firm's cost curve $\min(C^H(q), C^P(q))$ is not differentiable (see figure 4). Under MLRP, for example, both $C^H(q)$ and $C^P(q)$ are convex functions. This implies that the set of acceptance probabilities that is potentially chosen by the firm may be non-convex. This non-differentiability also implies potentially drastic reactions in the optimal organizational form with respect to small changes in product market conditions (see figure 5). Indeed, if one were to consider a two-stage game where firms choose their organizational forms and threshold before reviewing a project. observe their new marginal cost, and compete in prices, one would easily obtain multiple equilibria, some of them asymmetric.

5. Parametric Examples

Results 4 and 5 provide sufficient conditions for dominance of the polyarchical form or for crossing cost curves. In this section we shall provide examples. demonstrating that these possibilites do actually occur for some classes of distribution functions. First we shall provide a class of distribution functions. for which the polyarchy is the dominant organizational form and then we shall provide an example with crossing cost curves.

Example 1

Consider the following distribution functions:

$$H_0(y) := y^a \quad , a < 1$$

$$H_c(y) := y^b \quad , b > 1$$

In this case, $h_0(y) = ay^{a-1}$ is declining and $h_c(y) = by^{b-1}$ is increasing in y. So MLRP is satisfied. In this case

$$B^H(z) = z^{\frac{b}{a}}$$

$$B^P(z) = 1 - (1 - \sqrt{1-z})^{2\frac{b}{a}}$$

It is readily verified that $B^H(z)$ is log concave, but that $B^H(exp(z))$ is neither log convex nor log concave. Consequently, the hierarchy is indifferent in the number of screens it uses. On the other hand $B^P(exp(z))$ is log concave. In accordance with result 3 the polyarchy will employ both filters.

Example 2

In this example let $H_0(y) = \frac{4y}{3+y}$ and $H_c(y) = y$. Hence, $h_0(y) = \frac{12}{(3+y)^2}$ and $h_c(y) = 1$ which implies $h'_0(y) < 0$ and $h_c(y) = 0$. So MLRP is satisfied.

It is readily verified that $A(exp(z))$ is log convex. [2]

6. Concluding Comments

The anaysis so far has concentrated on situations where the firm can finance the project completely by internal funds. When external funding is necessary, of course, the analysis needs to be extended to include the funding costs in the firm's decision problem (1).

In many situations the available funds, however, are predetermined by the internal budget allocation within an institution. For example, national science foundations may have little discretion about varying the amounts of grants. Rather they decide about the allocation to individual researchers, while the budget is determined on a political level. This situation with pre-determined budgets provides an alternative normalization where firms are normalized by the number of projects they can implement (i.e. grants they can provide).

In our context, one can ask the question what success probability q could be achieved by either organizational form, when the available funds are restricted to $\bar{C}$ on an ex ante basis. Given that firms under either organizational form consider all projects (i.e. one), our results suggest that under conditions of result 4 the polyarchy would attract projects of higher quality. Under conditions of result 5 the answer depends on the size of the allocated budget. When the ex ante available funds are fairly restricted, i.e. $\bar{C}$ is sufficiently small, the hierarchy would implement better projects. When funds are available sufficiently generously, i.e. $\bar{C}$ is sufficiently close to $C^P(p)$, the polyarchy would seem to attract the better projects.

[2] One finds, for example, $C^H(.1) < C^P(.1)$ (and $C^H(.9) < C^P(.9)$), while $C'^H(.99) > C'^P(.99)$.

These findings contrast with Gersbach and Wehrspohn (1997) who analyze the same question in a different context. They allow institutions to look at different numbers of projects, such that the expected number of projects implemented are identical across organizations. For exogenous and identical thresholds across review units and organizational forms they find that the hierarchy will screen projects more tightly (as pointed out by Sah and Stiglitz, 1986) and, consequently, that it will evaluate more projects. Accordingly, in their framework the hierarchy always performs better. Our analysis suggests that endogenizing thresholds might modify the results of Gersbach and Wehrspohn. By requiring higher thresholds the polyarchy might actually act more conservatively in some instances than the hierarchy, and hence sample more projects. In this case the argument of Gersbach and Wehrspohn is just reverted.

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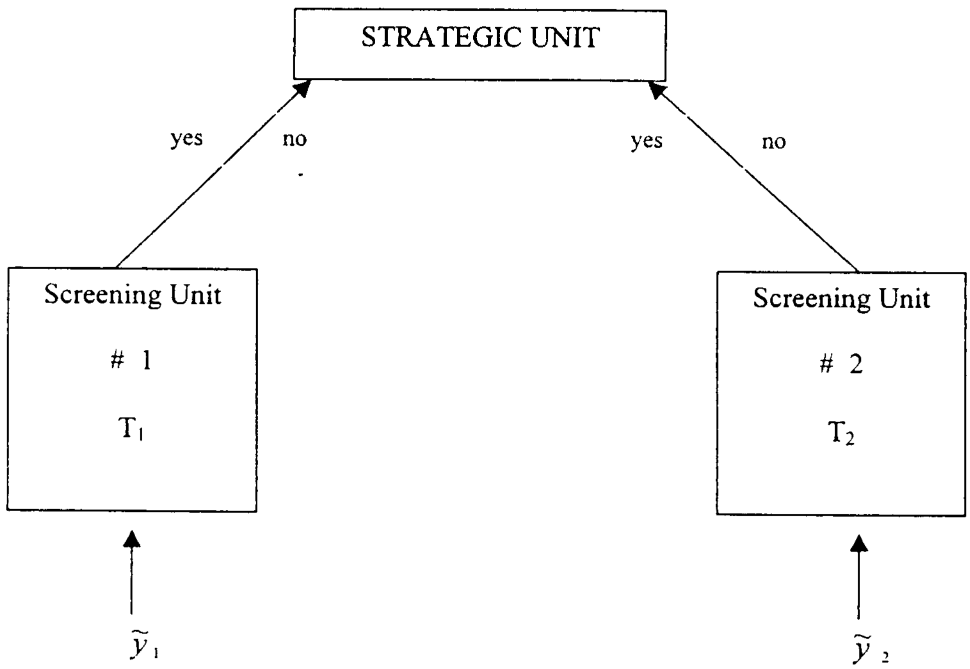
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no
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Screening Unit
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Screening Unit
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FIGURE # 1

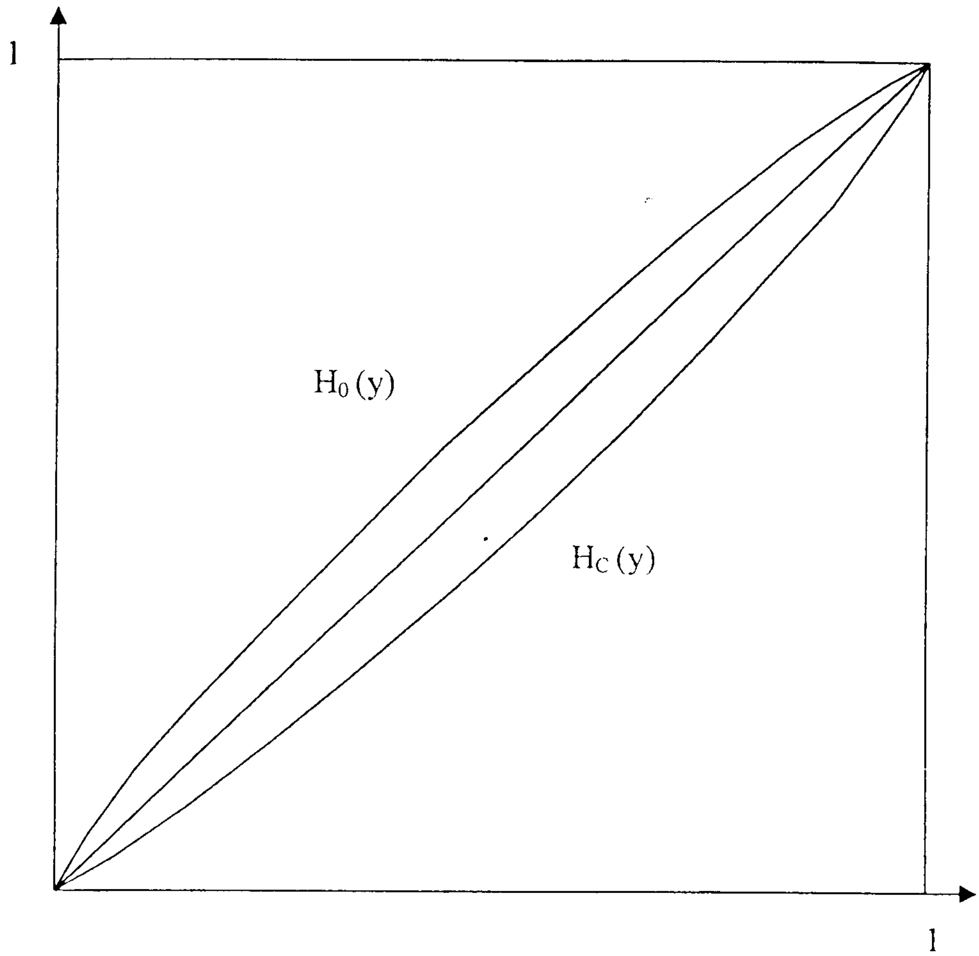

Figure 2

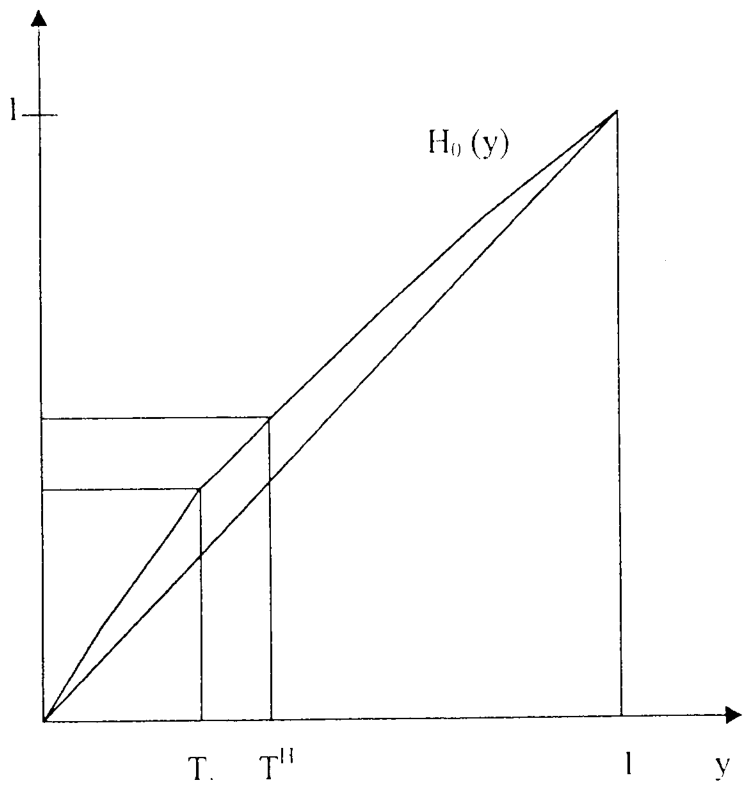

Figure 3a

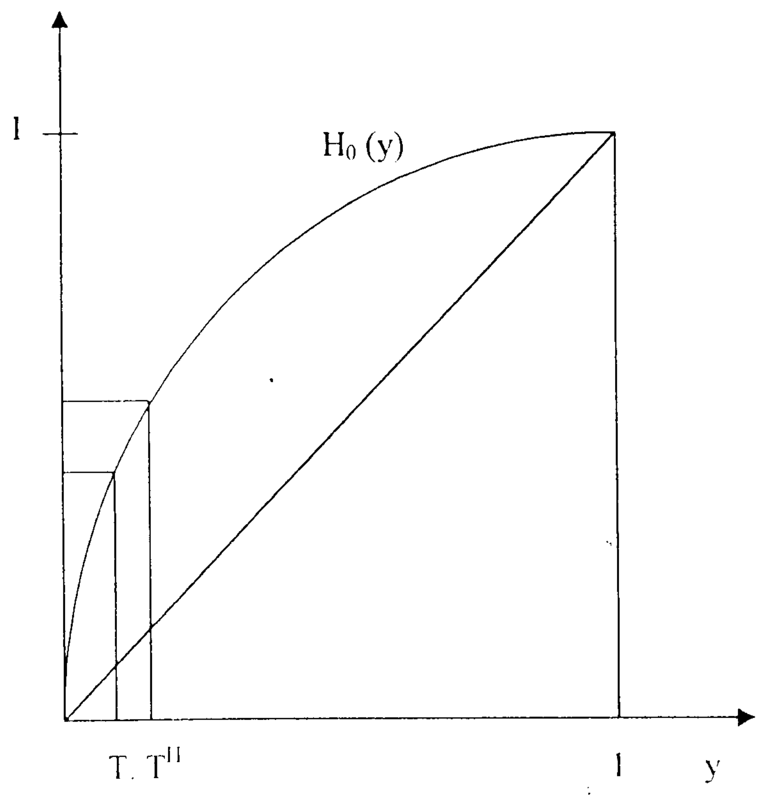

Figure 3b

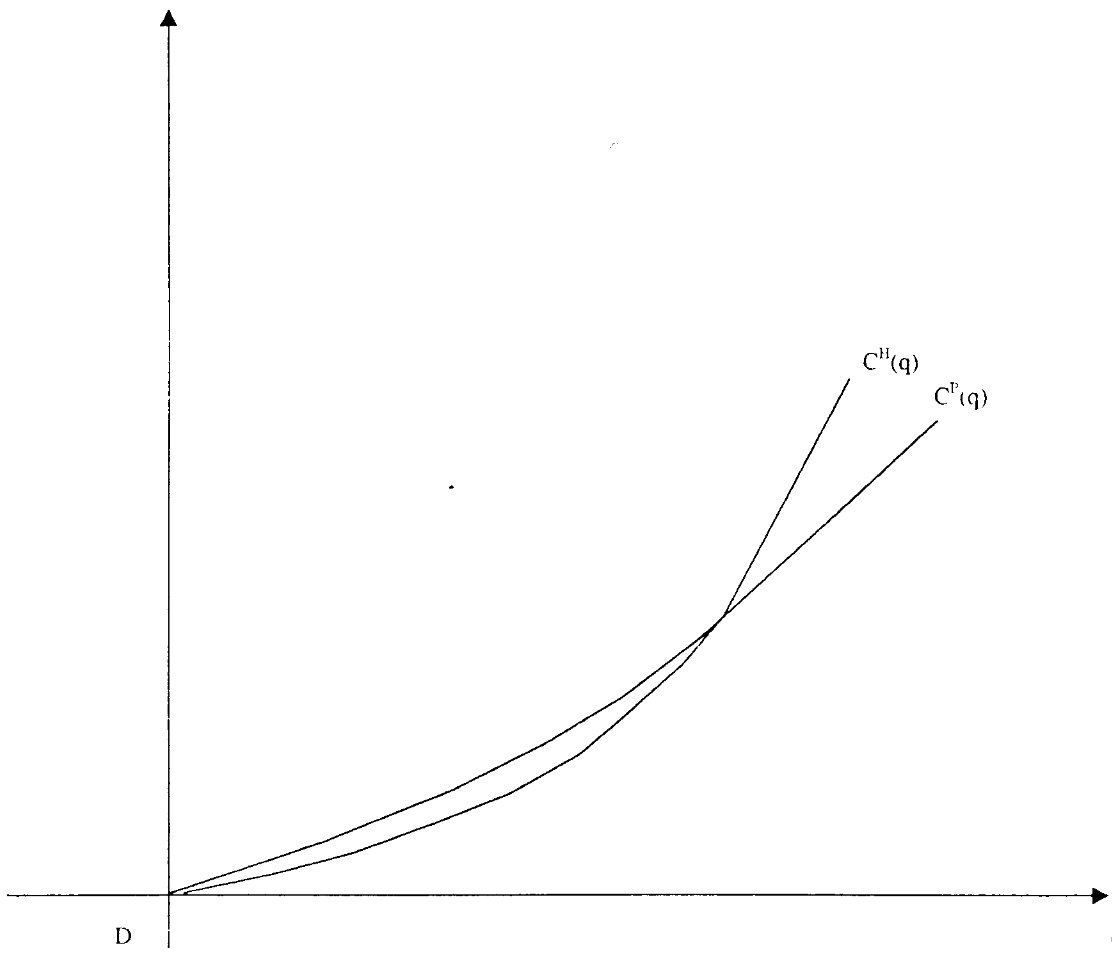

FIGURE # 4

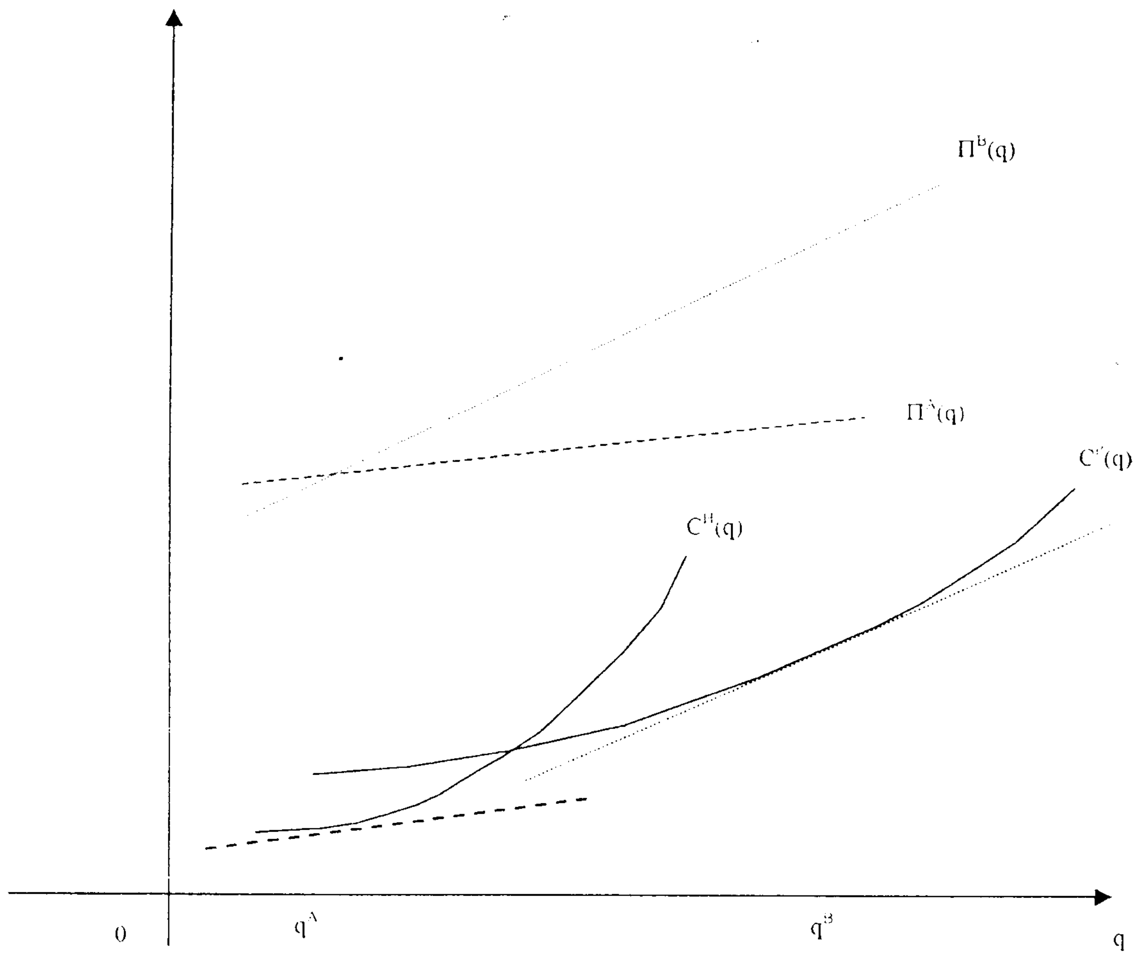

FIGURE # 5