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ABSTRACT

IQ, Social Mobility and Growth*

Intelligent agents may contribute to higher technological growth if assigned appropriate positions in the economy. These positive effects on growth are unlikely to be internalized on a competitive labour market. The allocation of talent depends on the relative award the market assigns to intelligence versus other individual merits, which will also influence intergenerational social mobility. To illustrate this, we present an endogenous growth model where each agent can choose to be a worker or an entrepreneur. The reward to entrepreneurs is an endogenous function of the abilities they have been endowed by nature as well as of the amount of knowledge and other social assets they inherit from their parents. When growth is low, the equilibrium in the labour market implies that the reward to entrepreneurs depends more on social assets than on intelligence. This gives children of entrepreneurs a large *ex-ante* advantage over children of workers when working as entrepreneurs, which will cause low intergenerational social mobility and an inefficient allocation of human resources and, consequently, low growth. Conversely, there is also a stable equilibrium with high growth which mitigates the inefficiencies generated by the labour market and implies high intergenerational social mobility.

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NON-TECHNICAL SUMMARY

Individuals are not born equal. Society endows different individuals with different capabilities. Upbringing, i.e. the family into which the individual is born, affects the capacity to respond adequately to the problems economic agents face. Nature is another cause of differences between individuals, some agents being more intelligent than others. Using economic jargon, we may say that an individual is born with two types of assets – innate and social. Holding other things constant, innate and social assets both increase expected lifetime earnings of an individual.

The distribution of innate and social assets among individuals is not independent between generations. In the game of allocating intellectual capacity, Mother Nature stacks the cards in favour of individuals with gifted parents. We can call this the genetic heritage. Similarly, the upbringing of one's offspring provides a powerful mechanism for transferring social advantages between generations. This is the social heritage.

In this paper, we will assume that genetic heritage is weaker than social heritage. In other words, an individual's amount of innate assets depends less on their parents, and more on chance, than does their amount of social assets at birth. More specifically, we assume that individual intelligence shows a relatively low degree of correlation between generations, while the social advantages that come with a particular upbringing are highly determined by the parents' social position. Social mobility will then depend on whether the social sorting mechanism emphasizes traits and abilities determined by innate assets or by social assets. If intelligence is important for an individual's social position, social mobility will be high. If the individual's upbringing, determined by their parents, is more important, social mobility will be high instead.

Our first goal is to demonstrate that economic mechanisms determine the relative importance of innate capabilities and social heritage in the assignment of economic roles to individuals. We will show that this mechanism is affected by the growth rate of the economy. For this purpose, we construct a stylized economy where each individual chooses whether to become an entrepreneur or a worker. Workers will be paid a common wage, determined on a competitive labour market. Entrepreneurs, on the other hand, will receive rewards that depend on their ability to take the correct action in difficult situations. This choice has no barriers – individuals are free to choose the option that gives maximum expected lifetime utility, based on their innate and social assets.

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An entrepreneur's ability to determine which action is the best increases both with the amount of social and innate assets. The relative importance of these two types of assets will depend on growth, however. Low growth means that the world changes slowly – that the right actions yesterday are still the right ones today. We assume that parents who were entrepreneurs themselves, have learned the optimal behaviour of an entrepreneur and can transfer this information to their children, who will find it useful provided that the world has not changed much since their parents were entrepreneurs. The children of entrepreneurs will then have a large *ex-ante* advantage over children of workers, when trying to reach entrepreneurial positions.

If, on the other hand, the rate of growth is high, we expect the economic environment to change rapidly. The information that parents working as entrepreneurs acquire regarding how to be a successful entrepreneur then depreciates quickly. Consequently, the children of entrepreneurs will not enjoy as large an advantage over children of workers as in the low growth case. Intelligence becomes more important, however. Intelligence can be defined as 'the ability to learn or understand or to deal with new or trying situations' (from Merriam-Webster's Collegiate, Tenth Edition, from Encyclopedia Britannica On-Line Edition). When growth is high, the world changes more between generations, and thus the environment is 'new and trying' for everybody. In this case, intelligence is a more important determinant of individual success.

Our second goal is to show the previous mechanism not only implies that social mobility and growth are positively correlated, but that it may also help us understand why two societies that start with the same amounts of factors of production may show radically different steady-state growth rates and social structures.

The accumulation of aggregate knowledge in the economy may cause indefinite growth. The creation of knowledge entails substantial externalities, since ideas created by intelligent people in new situations can be used by other people and adapted to other situations. But the likelihood of an intelligent individual creating an idea depends on their position in society. Thus, a society that allocates clever and innovative individuals to positions where they produce large externalities should have larger growth rates than one placing average individuals in these positions. This is the main point in Murphy *et al.*, who also show empirical evidence for the hypothesis that talented individuals are more important for growth if they work as engineers rather than as lawyers. Similarly, Baumol uses historical evidence to support the idea that growth increases if society manages to direct more entrepreneurial talent to productive rather than to rent-seeking activities. In some contrast to these

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papers, we will focus on a vertical dimension. We assume that individuals are workers or entrepreneurs. The more intelligent the entrepreneurs, the more new ideas are created and the higher is growth. A high degree of intergenerational social mobility is then necessary to ensure an allocation of talents which fosters long-run growth.

The main result of the paper is that high growth and high intergenerational social mobility interacts and cause a feedback mechanism. High (low) growth increases (decreases) the intellectual demands on entrepreneurs and managers. This improves (worsens) the sorting efficiency of the labour market and implies that future generations of entrepreneurs and managers have a higher (lower) level of intelligence. This in turn leads to higher (lower) growth. The model will produce multiple stationary state equilibria. Some with low growth and no intergenerational social mobility and one with high growth and high mobility.

Consider two *ex-ante* identical societies, 'Richland' and 'Poorland'. They have access to the same resources (both human and physical), but for historical reasons they have different social structures. The entrepreneurial class of Poorland mainly consists of the sons and children of previous entrepreneurs. From an intellectual point of view, they are a random sample of society's entire population, and consequently have average amounts of innate assets. Thus, they are not very innovative, and do not change the world substantially. Nevertheless, they confront economic challenges and learn from these. They can explain to their children what actions were the best to take in their working life. The advantage this brings about to the children of the entrepreneurs is sufficient to give them the upper hand – they will become the entrepreneurs of the next generation. Consequently, the intelligence of the entrepreneurial class of Poorland will remain on an average level. Poorlandians will have little or no growth for generations to come.

In Richland, the situation is different; the entrepreneurs are the most intelligent individuals in society and innovate and generate growth. They thus make the world change rapidly, and the information that they can pass on to their children thus depreciates so fast that it is of little or no value. The next generation of entrepreneurs will thus be formed by the intellectually gifted and the people of Richland will enjoy consistent high growth.

The situation in both Richland and Poorland will be stable – small exogenous perturbations in growth or average IQ among entrepreneurs will not be sufficient to make the economy switch from high to low growth or *vice versa*. A radical shock to the intellectual demands on individuals in positions with large

growth externalities could transform Poorland into Richland, however. Such a shock could be an opening to foreign trade. Nevertheless, the shock could reduce output in the short run, before the improved social sorting has become effectual.

1 Introduction

Individuals are not born equal. Society endows different individuals with different abilities. Upbringing, i.e., in which family the individual is born, affects her ability to respond adequately to the problems faced by economic agents. Other differences between individuals are due to nature – some agents are, for example, more intelligent than others. Using economic jargon, we may say that an individual is born with two types of assets – innate and social assets. Holding other things constant, innate and social assets both increase the expected lifetime earnings of an individual.

The distribution of innate and social assets among individuals is not independent between generations. In the game of allocating intellectual ability, mother nature stacks the cards in favor of individuals with gifted parents. We can call this the genetic heritage. Similarly, the upbringing of one's offspring provides a powerful mechanism for transferring social advantages between generations. This is the social heritage.

In this paper, we will assume that genetic heritage is weaker than social heritage. In other words, an individual's amount of innate assets depends less on her parents, and more on chance, than does her amount of social assets at birth. More specifically, we assume that individual intelligence shows a relatively low degree of correlation between generations, while the social advantages that come with a particular upbringing are highly determined by the parents' social position. Social mobility will then depend on whether the social sorting mechanism emphasizes traits and abilities determined by innate assets or by social assets. If intelligence is important for an individual's social position, social mobility will be high. If the individual's upbringing, determined by her parents, is more important, social mobility will instead be high.

Our first goal is to demonstrate that economic mechanisms determine the relative importance of innate abilities and social heritage when individuals are assigned economic roles in society. We will show that this mechanism is affected by the growth rate of the economy. For this purpose, we construct a stylized economy where each individual chooses whether to become an entrepreneur or a worker. Workers will be paid a common wage, determined on a walrasian labor market. Entrepreneurs, on the other hand, will receive rewards depending on their ability to take the correct action in difficult situations. There are no barriers to

this choice – individuals are free to choose the option that gives maximum expected lifetime utility, based on their innate and social assets.

An entrepreneur's ability to determine which action is the best increases both with her amount of social and of innate assets. The relative importance of these two types of assets will, however, depend on growth. Low growth means that the world changes slowly – that the right actions yesterday are still the right ones today. We assume that parents who were entrepreneurs themselves, have learned the optimal behavior of an entrepreneur and can transfer this information to their children, who will find it useful provided that the world has not changed much since their parents were entrepreneurs. The children of entrepreneurs will then have a large ex-ante advantage over children of workers, when trying to reach entrepreneurial positions.

If, on the other hand, the rate of growth is high, we expect the economic environment to change rapidly. The information regarding how to be a successful entrepreneur acquired by parents working as entrepreneurs, then depreciates quickly. Consequently, the children of entrepreneurs will not enjoy as great an advantage over children of workers as in the low growth case. On the other hand, intelligence becomes more important. Intelligence can be defined as *“the ability to learn or understand or to deal with new or trying situations”*¹. When growth is high, the world changes more between generations, and thus the environment is “new and trying” for everybody. Intelligence is then a more important determinant of individual success.

Our second goal is to show that the previous mechanism does not only imply that social mobility and growth are positively correlated, but may also help us understand why two societies that start with the same amount of factors of production may show radically different steady state growth rates and social structures.

There is a growing body of literature which shows that the rates of growth do not seem to converge, but to diverge towards a bimodal distribution.² Such a bimodal distribution can be the prediction of endogenous growth models with multiple equilibria (poverty traps). These models generally require an accumable factor of production with non-diminishing returns and the accumulation of this factor must be subject to some externality. One such

¹Merriam-Webster's Collegiate, Tenth Edition, from Encyclopedia Britannica On-Line Edition

²See Pritchett [12] and Jones [3] for recent surveys.

factor is aggregate knowledge in the economy since ideas created by intelligent people in new situations can be used by other people and adapted to other situations. But, the likelihood of an intelligent individual creating an idea depends on her position in society. Thus, a society which allocates intelligent and innovative individuals to positions where they produce large externalities should have larger growth rates than one which places average individuals in these positions. This is the main point in Murphy et al. [10]. They also show empirical evidence for the hypothesis that talented individuals are more important for growth if they are engineers rather than lawyers. Similarly, Baumol [1] uses historical evidence to support the idea that growth increases if society manages to direct more entrepreneurial talent to productive rather than to rent-seeking activities. In some contrast to these papers, we will focus on a vertical dimension. We will assume that individuals are workers or entrepreneurs. The more intelligent the entrepreneurs are, the more new ideas are created and the higher is growth. A high degree of intergenerational social mobility is then necessary to insure an allocation of talents which fosters long run growth.

Most of the literature dealing with growth, social mobility and income distribution has focused on the effects of financial market imperfections on human capital accumulation (Galor and Zeira [6] and Benabou[2], for instance). The central issue in our paper – how growth affects the sorting efficiency of the labor market – has not been considered in this line of literature. Galor and Tsiddon[7] is the paper closest to ours.³ By defining two different types of technological change, major technological breakthroughs (“inventions”) and gradual technological progress (“innovation”), they model the effects of technological change on intergenerational mobility. In contrast to our paper, however, they *assume* that “inventions” increase the relative return of ability and that “innovation” has the opposite effect. Intergenerational social mobility increases and social sorting becomes more efficient after an invention, which produces a “burst” of economic growth, followed by more innovation and a return to lower growth and less efficient social sorting. This produces cycles in growth and mobility. In contrast to Galor and Tsiddon [7], we focus on long-run growth. More importantly, we endogenize the relative returns to the two types of human capital

³We were not aware of their paper until after completing the first version of ours, in the summer of 1997.

and intergenerational social mobility as equilibrium outcomes on the labor market.

Empirical evidence of the relationship between growth and social mobility is scarce, but seems consistent with our results. Eriksson and Goldthorpe [8] provide empirical findings which are consistent with the conventional wisdom that there is a jump in intergenerational social mobility at some point in the development of economies. They also construct an index of intergenerational social mobility for 9 countries. Average intergenerational mobility in The Netherlands, France, Germany, Italy and the U.K. is lower than in Sweden, Japan, the U.S. and Australia. The average long-run growth rate also seems to have been lower in the former group. The average growth rate per year between 1870 and 1979 was 1.77% per year in the former group versus 2.43 in the latter. If the somewhat exceptional case of Australia is removed from the latter group, the difference becomes greater.⁴

Our paper is structured in the following way: Section 2 describes the basic model with growth exogenous, which allows us to analyze the social sorting mechanism as a function of growth. Section 3 endogenizes growth by introducing a link between the allocation of innate assets, generated by the social sorting mechanism, and growth. Section 4 summarizes and concludes.

2 A Model of Human Resource Allocation

2.1 Entrepreneurs and Workers

In each discrete time period, there is a continuum of mass 1 of individuals, indexed by $i \in [0, 1]$. Each individual lives one period only, and their common utility function is logarithmic.⁵

Each individual chooses whether to be a worker or an entrepreneur. If she chooses to be a worker, she gets the known market wage at time t , denoted w_t . If she chooses to be an entrepreneur, she creates a firm and is the residual claimant to firm profits.

An entrepreneur has to make two decisions. First, she has to choose how many workers to hire. Second, she has to take an entrepreneurial decision $a \in R$. The task of the

⁴Our own calculations from Maddison [9].

⁵The degree of risk aversion is not important for the results. The logarithmic utility function facilitates the exposition. In appendix C, we present a model with risk neutral agents which produces qualitatively similar results.

entrepreneur is to set a as close to an unobservable stochastic variable x_t as possible. The larger the distance between a and x_t , the lower are the profits in t . More specifically, the profits of the firm are:⁶

$$\Pi = e^{-(x_t - a)^2} \left(2 A_t l^{\frac{1}{2}} - w_t l \right). \quad (1)$$

We can think of x_t as the “best way” of running a firm, which is certainly a multi-dimensional object in the real world. To simplify, we assume that it is uni-dimensional, however. Profits are clearly maximized ex-post if $a = x_t$. However, no individual knows the value of x_t ex-ante. Furthermore, individuals differ in their beliefs about x_t , although we assume that all agents have rational expectations. Below, we will describe how these expectations are formed. Now, consider an individual i who believes that x_t is normally distributed, with mean $\mu(i)$ and variance $\frac{1}{P(i)}$. $P(i)$ is thus the precision of i 's beliefs. In other words, $\frac{1}{P(i)}$ is the expected (squared) error of an entrepreneur when running a firm.

It is straightforward to show that all entrepreneurs will hire $l = \left(\frac{A_t}{w_t}\right)^2$ workers, regardless of their beliefs. The best action does, of course, depend on beliefs and will be $a = \mu(i)$. An entrepreneur's utility will be stochastic with an expected value given by

$$V^e(i) = E \log(\Pi) = 2 \log(A_t) - \log(w_t) - \frac{1}{P(i)}. \quad (2)$$

which, of course, increases in the precision $P(i)$.

If the individual instead chooses to be a worker, her utility will be certain and equal to $\log(w_t)$, independent of her beliefs about, and the realization of, x_t .

If $w_t \geq A_t$, it is obvious that nobody will choose to be an entrepreneur. For lower wages, an individual with precision $P(i)$ chooses to be an entrepreneur if

⁶The somewhat peculiar profit function used is not important for the results, but greatly simplifies the algebra. The model in appendix C uses a profit function where the entrepreneurial decision affects gross production.

$$P(i) > \frac{1}{2(\log A_t - \log w_t)} \equiv z_t. \quad (3)$$

Thus, z_t is the threshold precision such that an individual is indifferent between being an entrepreneur and a worker. This threshold is a monotonously increasing function of the wage. When deriving the equilibrium conditions of the model below, we use z_t rather than the wage, which simplifies the notation considerably. Note also that the labor demand can be written

$$l_t = e^{\frac{1}{z_t}} \quad (4)$$

which is decreasing in z_t and thus in the wage.

If (3) holds with equality, the agent is clearly indifferent between being an entrepreneur and being a worker. If her precision is smaller than z_t , she chooses to be a worker.

2.2 Information and Intelligence

Now assume that x_t follows the stochastic process

$$x_t = \sqrt{\rho}x_{t-1} + \epsilon_t \quad (5)$$

where ϵ_t is white noise with a variance equal to σ . σ can be considered as an index of the flow rate of new ideas and technological innovations. If σ is high, the flow is high, and the "best way" of running a firm thus changes quickly. A high level of σ thus implies that the intrinsic difficulty of being an entrepreneur is high. Similarly, ρ measures at what rate the "best way" decays. Holding σ constant, a higher value of ρ increases the informational value in period t of knowing the "best way" in $t - 1$, since that piece of information says more about how to be a successful entrepreneur than it does if ρ is low. At present, we let σ be exogenous, and later, we will make it endogenous.

The expected profits of an entrepreneur depend on the precision in her information

about x_t . We assume that each entrepreneur observes the value of x_t after she has decided a . She will thus learn ex-post which action would have been the best. This is of no importance to her, but it can be important to her descendants. We assume that the children of entrepreneurs (CoE for short) know the realization of x_{t-1} . On the other hand, the children of workers (CoW for short) only know the unconditional distribution of x_t , determined by (5).⁷ The extra knowledge given to CoE is what we called “social assets” in the introduction.⁸ The fact that there is no market for knowledge about the “best way” of running a firm in the previous period is, of course, crucial for our results. We consider these social assets as *embodied* human capital rather than a tradable piece of information. We thus implicitly assume that such knowledge can only be transferred through the (slow) process of upbringing in the parental household.⁹

Individuals also differ due to their having different amounts of innate assets. Individual innate assets determine individual intelligence, which we now want to define. Intelligence certainly consists of many different traits: perception, creativity, memory, reasoning and the ability to grasp and process information, for example. A basic requirement of any reasonable definition of intelligence is, however, that a more intelligent person on average tends to do better than a less intelligent person in new situations about which they have been given the same information. More specifically, if two individuals with identical pay-off functions are confronted with new situations about which they have the same prior information, the most intelligent person is expected to do better. In addition, the definition of intelligence should also imply that intelligence is stable over time and cannot be transferred between individuals.

Given these requirements, differences in intelligence could be modeled in two ways. We might assume that intelligent individuals process whatever information they might

⁷We could make their knowledge conditional on the x observed by the last of her ancestors to be an entrepreneur, but this would certainly complicate the analysis without adding any qualitative change.

⁸A different, but clearly related, social heritage is modeled in Sjögren [4], where it is assumed that an individual knows her ability in the trade of her parents but is unsure of her ability in other occupations.

⁹In this extremely stylized model, “social assets” are just the knowledge of a particular number x_t , i.e., something that could in theory be bought and sold. In reality, it is inconceivable that the knowledge and the experience that is acquired by growing up in “the right” family could be bought and sold at a perfect market. In our view, this is not because this kind of knowledge cannot be represented by numbers associated with particular stochastic variables. Rather it is because human limitations imply that this knowledge is so complex (multidimensional) that it can only be transferred if the individual grows up in the right circumstances. It is only for simplicity that we represent this by a univariate variable.

have *better* than less gifted individuals (for example, by adding a random error to the less intelligent individual's decision). We have, however, followed a second strategy. We assume a perfect processing ability, but in order to generate differences in expected pay-offs, we assume that the level of intelligence is determined by the precision in an unbiased private signal about the world.¹⁰ Each individual rationally combines her private information with information received from other sources. Rational Bayesian updating with normally distributed signals implies that posterior beliefs are normally distributed with a precision equal to the sum of private and public precision. Thus, given identical parents, the person with the most informative private information set (i.e., the most intelligent person) will always have the highest expected pay-off. The private information set is not transferable between individuals, it is invariant over time and it cannot be affected by the individual (for example through training or education). We should note that in a situation with full common information, intelligence is of no importance. The more difficult a situation, the more important is intelligence, which seems to be well in line with the dictionary definition of intelligence quoted in the introduction.

We now proceed by making the simplification that individuals are either intelligent or stupid.¹¹ If they are intelligent, they receive an unbiased signal on x_t that is distributed as a normal with variance α ($\alpha < 1$).¹² If they are stupid, they get another unbiased signal, but with a variance equal to one. We also assume that the IQ of the parents is completely uncorrelated with the IQ of their children.¹³ q intelligent agents are born in each period.

All individuals in the economy belong to one of four types: CoE and intelligent, CoE and stupid, CoW and intelligent and CoW and stupid (see table 1). Clearly, the intelligent CoE (with a precision $P_{CoE,intelligent} = \frac{1}{\sigma} + \frac{1}{\alpha}$) are always the best suited to be entrepreneurs. Similarly, the stupid CoW make the worst entrepreneurs (with a precision $P_{CoW,stupid} =$

¹⁰These two representations of intelligence are observationally equivalent. It could be objected that to model intelligence as processing ability would imply bounded rationality, if rationality is understood as *perfect* processing ability. We have thus chosen the second modeling strategy. In any case, our choice is not restrictive and should not be interpreted as if we think we know what intelligence *really* is.

¹¹The model in appendix C assumes a continuum of IQ levels. The results do not change, but we can no longer provide analytical results.

¹²We could assume that the action of an entrepreneur is multidimensional. We could then let intelligence affect the quality of decisions along some particular dimensions while social heritage affects the quality along some other dimensions. Even if this adds some realism, it would not change any of the results.

¹³Again this is for simplicity. As long as this correlation is smaller than unity, the qualitative results would hold.

<i>TYPE</i>	<i>Number at t</i>	<i>Precision</i>
CoE, intelligent	$m_{t-1} q$	$\frac{1}{\sigma} + \frac{1}{\alpha}$
CoE, stupid	$m_{t-1} (1 - q)$	$\frac{1}{\sigma} + 1$
CoW, intelligent	$(1 - m_{t-1}) q$	$\frac{1-\rho}{\sigma} + \frac{1}{\alpha}$
CoW, stupid	$(1 - m_{t-1}) (1 - q)$	$\frac{1-\rho}{\sigma} + 1$

Table 1: Precision of different agents

$\frac{1-\rho}{\sigma} + 1$). Now consider the two intermediate groups – the stupid CoE and the intelligent CoW. Which of these will be the best entrepreneurs depend on two factors; the intrinsic difficulty of being an entrepreneur, σ , and the rate of decay of the information about how to be a good entrepreneur, ρ . For low values of σ and/or high values of ρ , the stupid CoE are better fitted for entrepreneurial tasks than the intelligent CoW, while the opposite applies for high values of σ and/or low values of ρ :

$$P_{CoE, stupid} > P_{CoW, intelligent} \quad \text{iff} \quad \frac{\sigma}{\rho} < \frac{\alpha}{1 - \alpha} \quad (6)$$

The intuition is straightforward. For low values of $\frac{\sigma}{\rho}$, the information given by entrepreneurs to their children is quite accurate in the sense that little has changed between the two periods. In a stagnant world, it is thus a great advantage to have parents in entrepreneurial positions. Under such circumstances, a stupid CoE will be able to make good decisions. If $\frac{\sigma}{\rho}$ is large on the other hand, the information that entrepreneurs transfer to their children is not of much use and thus, intelligence is a more important determinant of the expected entrepreneurial success.

2.3 Equilibrium conditions

The model has two equilibrium conditions. First, given individual career choices, the labor market must clear. Second, given the wage established in the labor market, each individual chooses the the career that maximizes her expected utility.

Consider first the labor market equilibrium. This is a price of labor, which we express in terms of z_t , and an amount of entrepreneurs, m_t , such that labor supply equals labor demand. For a given number of entrepreneurs at t , labor supply is completely inelastic and equal to $(1 - m_t)$. Labor demand is a function of z_t , since each entrepreneur will hire $e^{\frac{1}{z_t}}$ workers. In equilibrium, the number of workers demanded must equal the fixed supply of labor, i.e.,

$$\begin{aligned} m_t e^{1/z_t} &= 1 - m_t \\ \Rightarrow m_t &= \frac{1}{1 + e^{\frac{1}{z_t}}} \equiv SD(z_t) \\ SD'(z_t) &> 0. \end{aligned} \tag{7}$$

This establishes a (positive) relationship between z_t (the wage) and the number of entrepreneurs.

The second equilibrium condition is that m_t equals the number of agents with a precision higher than or equal to z_t , so that all agents choose the job which maximizes their individual expected utility. This condition also establishes a relation between m_t and z_t , which we will denote $m_t = M(z_t)$. At very low wage levels, everybody prefers to be entrepreneurs. Increasing the wage implies that group after group will come to prefer being workers. The stupid CoW are always the first to do this and the intelligent CoE are the last. Which of the intermediate groups comes first is determined by whether (6) is satisfied or not. If $\frac{\sigma}{\rho} < \frac{\alpha}{1-\alpha}$, the intrinsic difficulty of being a manager is relatively low and the value of knowing the previous period's "best way" is high, which means that the intelligent CoW are the first to prefer to be workers as the wage increases. In either case, $M(z)$ is a step function where each step occurs at the wage where a category of individuals is indifferent between the two career choices. Formally, $M(z)$ is given by

$$m_t = M(z_t) = \begin{cases} \text{If } \frac{\sigma}{\rho} < \frac{\alpha}{1-\alpha} \Rightarrow \begin{cases} 0 & \text{if } z_t > P_{CoE,intelligent} \\ [0, m_{t-1}q] & \text{if } z_t = P_{CoE,intelligent} \\ m_{t-1}q & \text{if } P_{CoE,intelligent} > z_t > P_{CoE,stupid} \\ [m_{t-1}q, m_{t-1}] & \text{if } z_t = P_{CoE,stupid} \\ m_{t-1} & \text{if } P_{CoE,stupid} > z_t > P_{CoW,intelligent} \\ [m_{t-1}, q + m_{t-1}(1-q)] & \text{if } z_t = P_{CoW,intelligent} \\ q + m_{t-1}(1-q) & \text{if } P_{CoW,intelligent} > z_t > P_{CoW,stupid} \\ [q + m_{t-1}(1-q), 1] & \text{if } z_t = P_{CoW,stupid} \\ 1 & \text{if } P_{CoW,stupid} > z_t \end{cases} \\ \text{If } \frac{\sigma}{\rho} \geq \frac{\alpha}{1-\alpha} \Rightarrow \begin{cases} 0 & \text{if } z_t > P_{CoE,intelligent} \\ [0, m_{t-1}q] & \text{if } z_t = P_{CoE,intelligent} \\ m_{t-1}q & \text{if } P_{CoE,intelligent} > z_t > P_{CoW,intelligent} \\ [m_{t-1}q, q] & \text{if } z_t = P_{CoW,intelligent} \\ q & \text{if } P_{CoW,intelligent} > z_t > P_{CoE,stupid} \\ [q, q + m_{t-1}(1-q)] & \text{if } z_t = P_{CoE,stupid} \\ q + m_{t-1}(1-q) & \text{if } P_{CoE,stupid} > z_t > P_{CoW,stupid} \\ [q + m_{t-1}(1-q), 1] & \text{if } z_t = P_{CoW,stupid} \\ 1 & \text{if } P_{CoW,stupid} > z_t \end{cases} \end{cases} \quad (8)$$

Clearly, (7) and (8) together are necessary and sufficient for equilibrium. The relations between m_t and z_t given by SD and M are depicted in figure 1. The function $SD(z_t)$ monotonically increases from zero and converges asymptotically to $\frac{1}{2}$.¹⁴ $M(z_t)$ decreases in steps, from one to zero. This insures the existence of an (unique) equilibrium (m_t and w_t) at t for any possible m_{t-1} , σ and ρ . Note also that $M(z_t)$ depends on m_{t-1} , since the height of some of the steps depends on the number of CoE, which is determined by m_{t-1} .

It is now convenient to make two definitions:

Definition 1 Let $m_w(\sigma)$ denote the number of entrepreneurs such that the equilibrium

¹⁴Clearly the equilibrium number of entrepreneurs cannot be higher than $\frac{1}{2}$, because each of them hires $\left(\frac{A_t}{w_t}\right)^2 (> 1)$ workers.

labor market wage makes the intelligent CoW indifferent between career choices:

$$m_w(\sigma) = SD(P_{CoW,intelligent}(\sigma, \rho)) = SD\left(\frac{1-\rho}{\sigma} + \frac{1}{\alpha}\right)$$

Definition 2 Let $m_e(\sigma)$ denote the number of entrepreneurs such that the equilibrium labor market wage makes the stupid CoE indifferent between career choices:

$$m_e(\sigma) = SD(P_{CoE,stupid}(\sigma)) = SD\left(\frac{1}{\sigma} + 1\right)$$

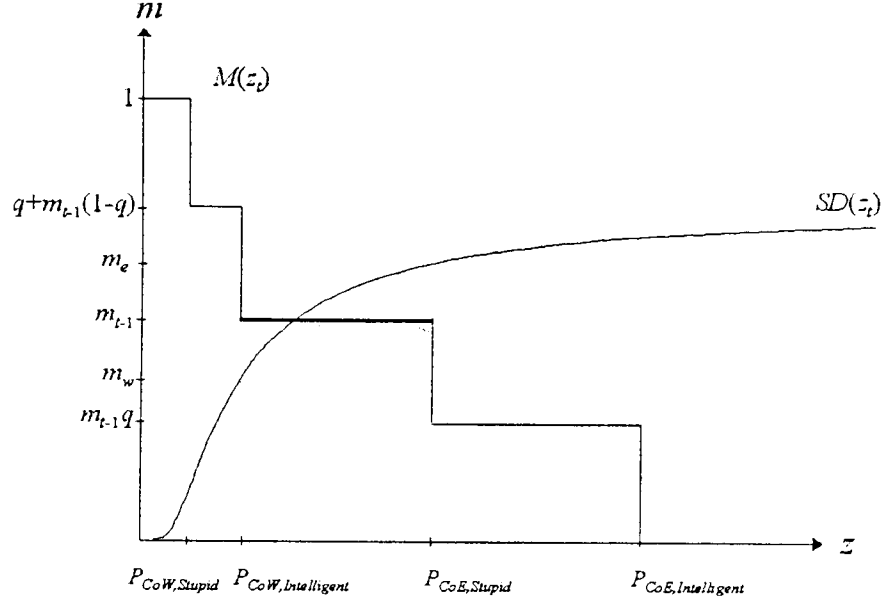
The two cases discussed above, i.e., whether intelligent CoW or stupid CoE are better entrepreneurs, can now be expressed in terms of m . If $m_w(\sigma) > m_e(\sigma)$, then social assets are more important than intelligence, in the sense that a stupid CoE is more suited to be an entrepreneur than an intelligent CoW. Since stupid CoE make better entrepreneurs than intelligent CoW, they can accept a larger number of managers than intelligent CoW, before the resulting equilibrium labor market wage becomes so high that they prefer to be workers. If $m_w(\frac{\sigma}{1-\rho}) < m_e(\sigma)$, the opposite is true. It is straightforward to see that

$$m_w(\sigma) > m_e(\sigma) \iff \sigma > \frac{\alpha\rho}{1-\alpha}. \quad (9)$$

2.4 Steady State

Let us now focus on the steady state in the model. In a steady state equilibrium, we require the number of entrepreneurs to be constant: $m_{t-1} = m_t = m$. Consider first the case when social assets are more important than intelligence. This happens when $\sigma < \rho\frac{\alpha}{1-\alpha}$. Then, $m_e > m_w$, reflecting that stupid CoE are better managers than intelligent CoW for σ in this range and they thus tolerate a larger number of managers before the resulting equilibrium wage makes them prefer to become workers. The two equilibrium relations $m_t = SD(z_t)$ and $m_t = M(z_t)$ for this case are depicted in figure 1. We see that one segment of $M(z_t)$ (marked with a thicker line) equals m_{t-1} . In a steady state equilibrium, $SD(z_t)$ must thus cross $M(z_t)$ at that segment. If $SD(z_t)$ crosses $M(z_t)$ at any other segment, the resulting equilibrium value of m_t differs from m_{t-1} .

Figure 1: An equilibrium if $\sigma \leq \frac{\alpha\rho}{1-\alpha}$.



For any value of m_{t-1} such that $SD(z_t)$ crosses $M(z_t)$ at the segment where it equals m_t , but for these values of m_{t-1} only, the resulting equilibrium is a steady state. At all these equilibria, the equilibrium wage is lower than $P_{CoE,Stupid}$, so that all CoE are entrepreneurs. Furthermore, the equilibrium wage is higher than $P_{CoW,intelligent}$, so that all CoW are workers. Mobility is nil and we have a society which is stratified in self-reproducing castes. Expressed formally, we have:

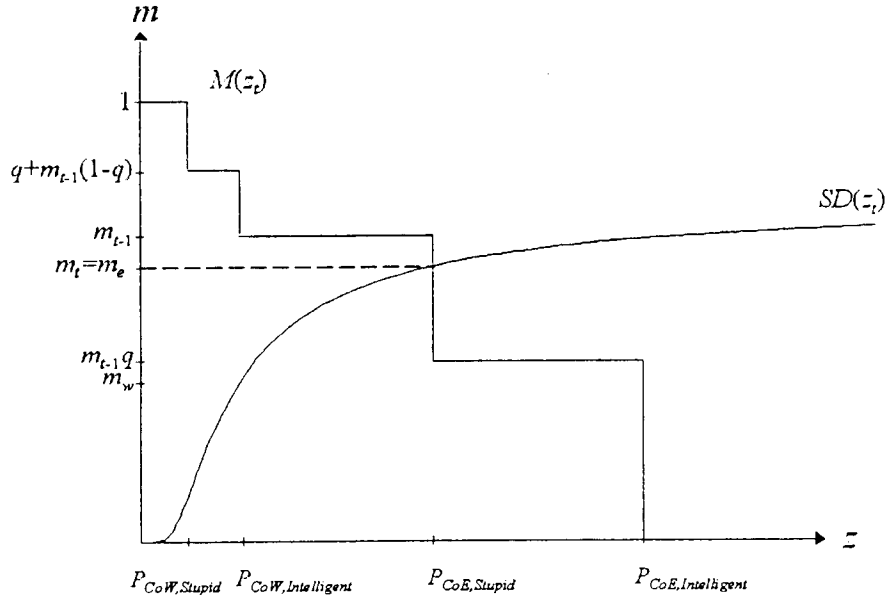
Result 1 *If $\sigma \leq \rho \frac{\alpha}{1-\alpha}$, m and z are a Steady State equilibrium iff $m \in [m_w(\sigma), m_e(\sigma)]$ and $m = SD(z)$.*

Consider the dynamical stability of steady states with no social mobility. First, we note that for any $m_{t-1} \in (m_w, m_e)$, a small deviation in m_t simply moves the steady state to the new value of m . Then, consider an $m_{t-1} > m_e$, which is thus outside the steady state region. This situation is depicted in figure 2, where we see that the equilibrium value of m_t is now m_e , unless $qm_{t-1} > m_e$. The value $m_t = m_e$ is a steady state. Similarly, but not depicted; if $m_{t-1} < m_w$, but $q + (1-q)m_{t-1} > m_w$, the equilibrium value of m_t is m_w ,

which is a steady state.¹⁵ The conclusion is thus:

Result 2 *If $\sigma < \rho \frac{\alpha}{1-\alpha}$, any value of m_{t-1} in a neighborhood of the set of steady state values of m , produces an equilibrium value of m_t , which is a steady state. The resulting steady state is the endpoint of the set of steady state values closest to m_{t-1} .*

Figure 2: Out of steady state when if $\sigma \leq \frac{\alpha\rho}{1-\alpha}$.

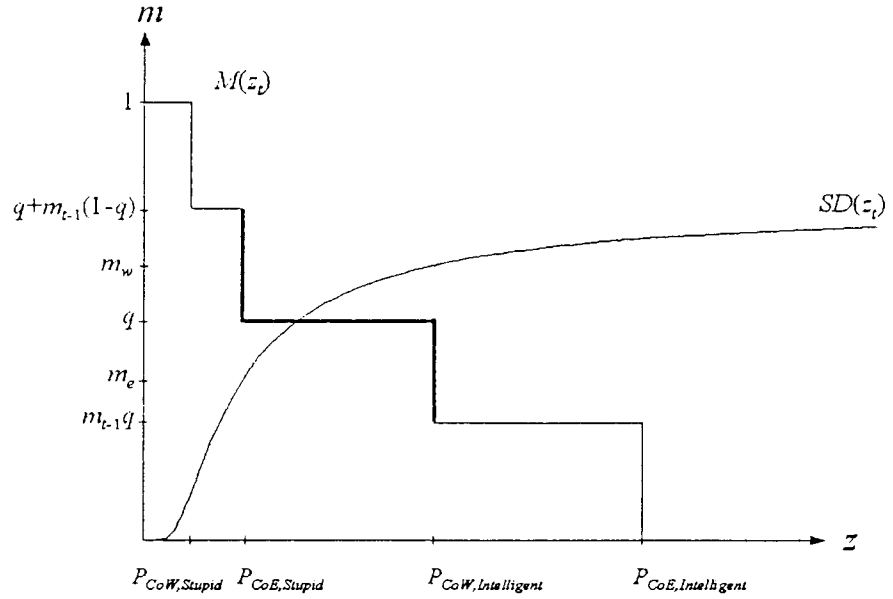


Let us now turn to the case when $\sigma > \rho \frac{\alpha}{1-\alpha}$. In this case, the intelligent CoW are better entrepreneurs than the stupid CoE, so $m_w > m_e$. This situation is depicted in figure 3. In a steady state, the wage can clearly not exceed the wage that makes the intelligent CoW indifferent between being entrepreneurs and a workers. Otherwise, only the intelligent CoE would be entrepreneurs in the next period, i.e., $m_t = m_{t-1}q < m_{t-1}$. Furthermore, the wage in a steady state cannot be lower than the wage that makes the stupid CoE indifferent. Otherwise, *all* the CoE *and* the intelligent CoW would become entrepreneurs in the next period, i.e., $m_t = q(1 - m_{t-1}) + m_{t-1} > m_{t-1}$. Graphically, this means that a steady state equilibrium cannot occur at the thin segments of $M(z_t)$ in figure 3.

¹⁵For larger deviations from the steady state, the equilibrium number of managers clearly moves in the direction towards the steady state region, but a steady state may not be achieved in one period only.

We now have three cases, depending on the value of the parameter q , i.e., the share of intelligent individuals in the whole economy. The first case arises if the total share of intelligent individuals is lower than m_w and higher than m_e . This case is depicted in figure 3, where we see that the steady state equilibrium is at the point where $SD(z_t)$ crosses $M(z_t)$ at the horizontal thick segment where $q = M(z_t)$. This is clearly the unique steady state equilibrium in this case. All intelligent but no stupid individuals are entrepreneurs, i.e., $m = q$ and $z = SD^{-1}(q)$.

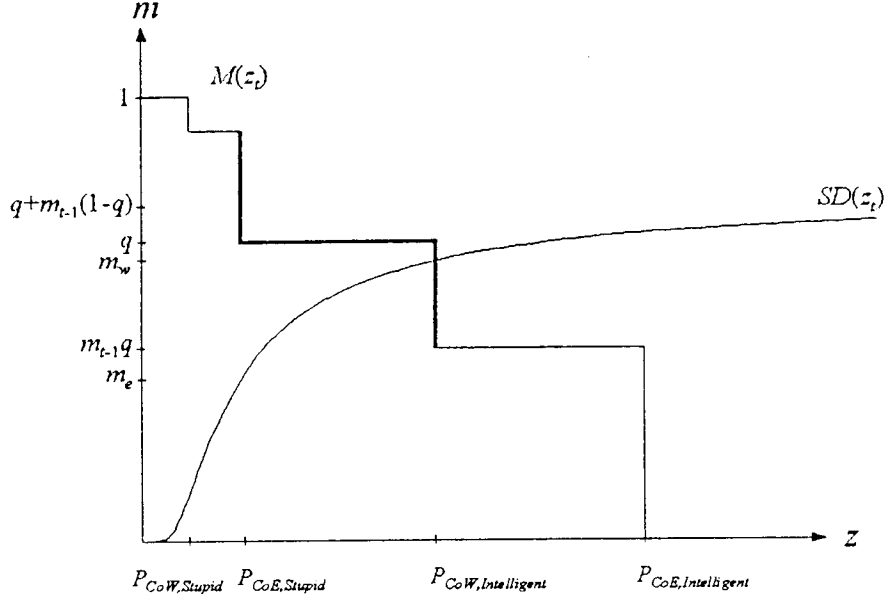
Figure 3: An equilibrium if $\sigma \geq \frac{\alpha\rho}{1-\alpha}$.



The second case arises if the parameters are such that $q > m_w$. In this case, $SD(z_t)$ crosses $M(z_t)$ at the vertical segment above $P_{CoW,intelligent}$, as depicted in figure 4. This equilibrium replicates itself and is the unique steady state. In this case, the share of intelligent people is so large that not all of them can become entrepreneurs in equilibrium. The wage is thus $P_{CoW,intelligent}$, which makes intelligent CoW indifferent between the career choices. Some of them become workers and some entrepreneurs.

Last, the third case arises if the share of intelligent people is so small that $q < m_e$, as depicted in figure 5. In this case, some stupid individuals will also be entrepreneurs. The

Figure 4: An equilibrium if $\sigma \geq \frac{\alpha\rho}{1-\alpha}$ and q is high.



wage is $P_{CoW,Stupid}$, so that stupid CoW are indifferent between the careers and some of them become workers and some become entrepreneurs.

Our conclusions for the case when $\sigma > \rho \frac{\alpha}{1-\alpha}$ can now be summarized as follows:

Result 3 *If $\sigma > \rho \frac{\alpha}{1-\alpha}$, there is only one steady state equilibrium for each value of σ , with m and z given by:*

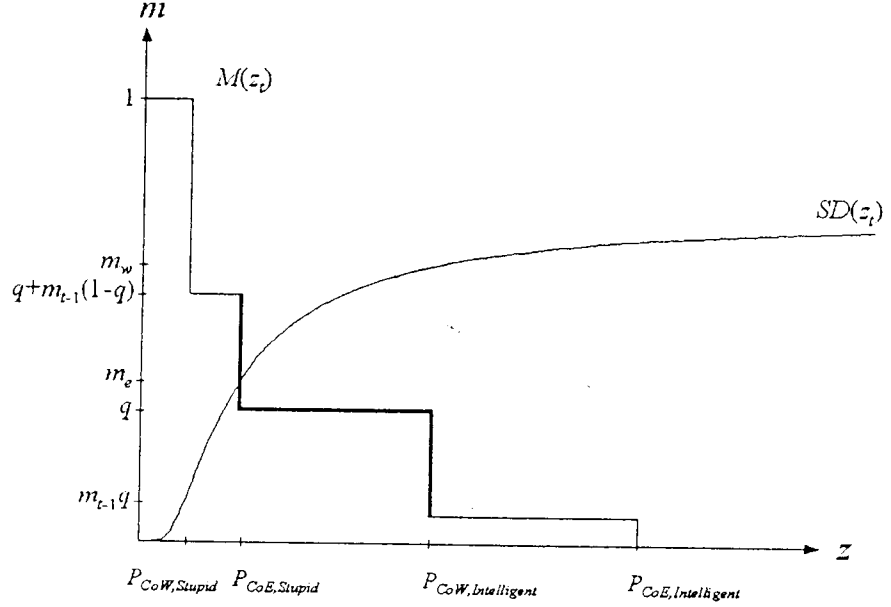
$$(m, z) = \begin{cases} (m_w(\sigma), SD^{-1}(m_w(\sigma))) & \text{If } q > m_w(\sigma) \\ (q, SD^{-1}(q)) & \text{If } m_w(\sigma) \geq q \geq m_e(\sigma) \\ (m_e(\sigma), SD^{-1}(m_e(\sigma))) & \text{If } m_e(\sigma) > q \end{cases}$$

Proof in appendix A.

The intuition behind our result is straightforward. When $\frac{\sigma}{\rho}$ is large, it is difficult to be an entrepreneur and the inherited information depreciates fast, so that intelligent individuals have the edge. All intelligent, but no stupid, individuals will become entrepreneurs unless

- there are so many intelligent individuals that if they were all entrepreneurs, the wage

Figure 5: An equilibrium if $\sigma \geq \frac{\alpha\rho}{1-\alpha}$ and q is high.



paid to the (scarce) workers would be so high that the intelligent CoW would be unwilling to become entrepreneurs, or if

- there are so few intelligent individuals that if they were the only entrepreneurs, there would be so many workers that the equilibrium wage would be so low that the stupid CoE would prefer to be entrepreneurs.

Let us analyze the stability of the steady states. Start with the first case, depicted in figure 3. It is clear that the equilibrium value of m_t is the steady state, whatever the value of m_{t-1} . This steady state is thus not only stable, it is achieved immediately, regardless of the initial share of managers. Then, proceed with the second case when $q > m_w$. In figure 4, we see that any value of m_{t-1} such that $m_w > m_{t-1}q$ results in an equilibrium value of m_t equal to m_w , which is the steady state. Last, when $m_e > q$, figure 5 shows that any m_{t-1} such that $q + m_{t-1}(1 - q) > m_e$ results in an equilibrium value of m_t equal to m_e , which is the steady state. The conclusion is thus:

Result 4 *If $\sigma \leq \rho \frac{\alpha}{1-\alpha}$, any value of m_{t-1} in a neighborhood of the steady state value of m , produces an equilibrium value of m_t that is the steady state.*

2.5 Managerial IQ

2.5.1 Managerial IQ as a function of σ

In the previous subsection, we established that there is a corresponding set of steady state equilibrium values of m for each value of $\frac{\sigma}{\rho}$. Now, let us take a closer look at this correspondence. Since we intend to endogenize σ below, we analyze how the set of equilibrium values of m , and the corresponding intelligence of managers, vary with σ .¹⁶

The upper left panel of figure 6 depicts m_w and m_e as functions of σ . As σ increases, it becomes more difficult to be an entrepreneur. A higher wage is then required to make intelligent CoW and stupid CoE indifferent between the career choices. Since a higher equilibrium wage requires a lower share of managers, both m_w and m_e are falling in σ . Both functions achieve their maximum values at $\frac{1}{2}$ as σ approaches zero. As σ increases, m_w and m_e asymptotically converge to $\frac{1}{1+e\alpha}$ and $\frac{1}{1+e}$ respectively.

Consider first values of $\sigma < \rho \frac{\alpha}{1-\alpha}$. In this range of σ , intelligence is irrelevant for social sorting. As we know, m_e here exceeds m_w . In this range for σ , all values of m such that $m_e \geq m \geq m_w$ constitute an equilibrium, which is represented by the shadowed area between the two curves in the bottom left panel. The resulting mass of intelligence, denoted IQ , and the average intelligence, denoted \overline{IQ} , are depicted in the two panels to the right. Since intelligence is irrelevant for sorting, $IQ = mq$ and \overline{IQ} are equal to q , i.e., the average intelligence of the population.

When $\sigma > \rho \frac{\alpha}{1-\alpha}$, $m_e < m_w$ and the equilibrium value of m is unique, as shown in proposition 3. As σ increases from the point where m_w and m_e cross, there is a jump in social mobility. All intelligent individuals are entrepreneurs and if $q < m_w$, also a share of the stupid CoE. IQ thus jumps from qm_e to q . The number of entrepreneurs is given by m_e since the stupid CoE must be indifferent between the two career choices.¹⁷ As σ increases further, the number of stupid CoE who become entrepreneurs falls and \overline{IQ} thus increases.

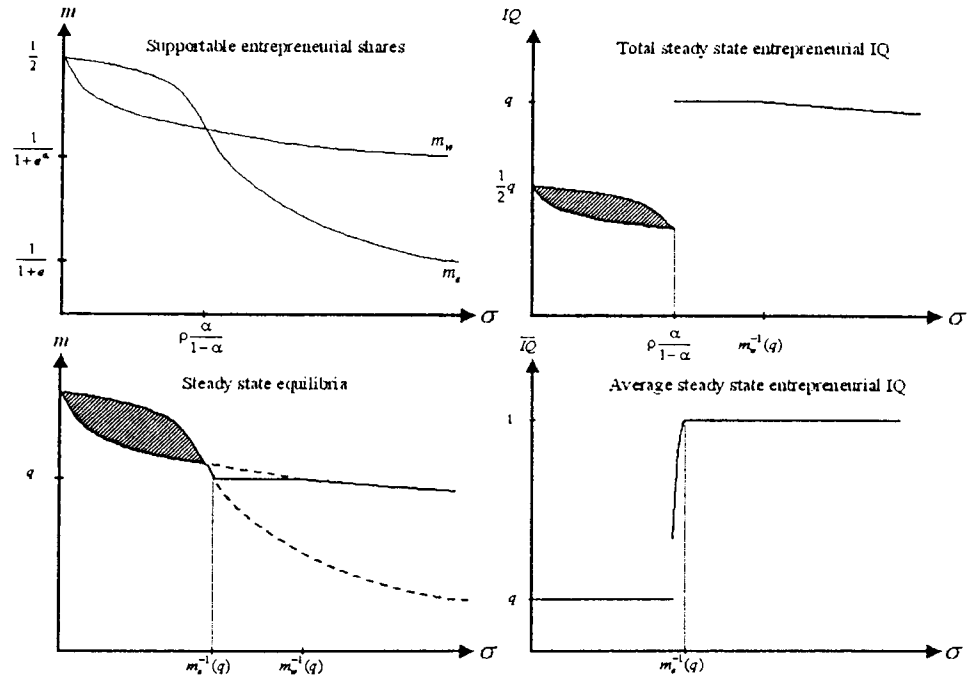
¹⁶We do the same exercise for variations in ρ in appendix B.

¹⁷In the depicted case, $q < m_w$ at the point where $m_w = m_e$. If the opposite is true, the steady state value of m is given by m_w when $\sigma > \rho \frac{\alpha}{1-\alpha}$, $IQ = m_w$ and $\overline{IQ} = 1$.

As long as some stupid CoE choose to be entrepreneurs, the number of managers will equal m_e , which falls as σ continues to increase. The average intelligence among entrepreneurs then increases, as seen in the right bottom panel. Eventually, no stupid CoE prefers to be an entrepreneur and only intelligent people are entrepreneurs. From this value of σ , denoted $m_e^{-1}(q)$, $m = q = IQ$ and $\overline{IQ} = 1$.

As σ continues to increase, the value of σ for which $m_w = q$ (denoted $m_w^{-1}(q)$) is eventually reached, unless $q < \frac{1}{1+e^\alpha}$. For larger values of σ , all intelligent CoE, but only some intelligent CoW are entrepreneurs. The intelligent CoW must thus be indifferent between career choices, so $m = m_w$. Since no stupid individual is an entrepreneur, \overline{IQ} is unity, but IQ is falling as the number of intelligent CoW in entrepreneurial positions decline.

Figure 6: Correspondences between the steady state equilibrium share of managers, IQ , \overline{IQ} and σ .



Our conclusions regarding the correspondence between the steady state equilibrium values of m , IQ and σ can now be expressed as follows:

Result 5

- Let $S = S(\sigma)$ denote the correspondence between the set of values of m which can be sustained as a steady state equilibrium, and σ . Then, $S(\sigma) = [m_e(\sigma), m_w(\sigma)]$ if $\sigma < \frac{\alpha\rho}{1-\alpha}$ and a single point otherwise.
- $S(\sigma)$ is a non-increasing correspondence of σ , in the sense that both $\max\{m|m \in S\}$ and $\min\{m|m \in S\}$ are non-increasing and continuous functions of σ .
- Let $IQ = IQ(\sigma)$ denote the correspondence between the set of values of IQ that can be sustained as a steady state equilibrium and σ . For $\sigma < \frac{\alpha\rho}{1-\alpha}$, $IQ(\sigma) = qS(\sigma)$. For $\sigma > \frac{\alpha\rho}{1-\alpha}$, $IQ = \min\{q, m_w\}$.

Two mechanisms create a link between the allocation of talented individuals and managerial difficulty in our model. The first mechanism is that when it becomes more difficult to be an entrepreneur, a smaller share of the population becomes entrepreneurs. *Ceteris paribus*, higher managerial difficulty reduces the expected pay-off to entrepreneurs for everybody. This would, in general, lead to a smaller number of individuals choosing to become entrepreneurs which leads to a smaller stock of intelligence among entrepreneurs, but to higher average intelligence. This mechanism is responsible for the non-increasing regions of the correspondence $IQ(\sigma)$.

The second mechanism comes into the picture due to the fact that everybody does not begin life with the same amount of social assets. As the difficulty of being an entrepreneur increases, the *relative* advantage of having a father who was an entrepreneur decreases. Expressed in a more general way: if other individual characteristics than intelligence are important for entrepreneurial rewards, their relative importance would decrease as managerial difficulty increases. The intelligence of the individual then becomes more important in determining occupational choice, and *both* the average and the total stock of intelligence among entrepreneurs increase. If intelligence shows a relatively small degree of correlation between parents and their offspring relative to other relevant individual characteristics, intergenerational social mobility increases. This mechanism is responsible for the jumps in IQ and in intergenerational social mobility that occur at $\sigma > \frac{\alpha\rho}{1-\alpha}$.

3 Mobility and Endogenous Growth

We have now established some relationships between the intelligence of entrepreneurs and σ , when the latter is exogenous. In this section, we endogenize growth and σ , i.e., the flow rate of new ideas and technologies. We will abstract from growth which is driven by the accumulation of physical capital of a constant quality, and instead focus on growth caused by increases in the stock of knowledge. First, we model how growth affects the difficulty of the entrepreneur's problem, and then, we let the growth rate be determined by entrepreneurial intelligence.

3.1 Managerial difficulty as a function of growth

In the introduction, we argued that it is reasonable to assume that the speed at which the economic environment changes is affected by the rate of growth. Let us now define how the entrepreneurial problem changes with growth. At t , entrepreneurs have to choose an action as close as possible to x_t when the level of technological advancement of the firms is characterized by A_t . For each level of technology, A_t , there is a "best entrepreneurial decision", $x(A_t)$. Conditional on knowing $x(A_{t-1})$, the closer two technologies A_t and A_{t-1} are, the easier it is to find an action close to $x(A_t)$. Using the notation above, i.e., σ denoting the variance of x_t conditional on knowing x_{t-1} , we assume

$$\sigma = \sigma\left(\frac{A_t}{A_{t-1}}\right), \quad (10)$$

where $\sigma(\cdot)$ is an increasing function. In terms of (5), this means that σ is increasing in the distance between the old and the new technologies. Furthermore, in a steady state where the size and the composition of entrepreneurs are constant and only the latest technology is adopted, the gross growth rate of the economy between t and $t - 1$ is given by the ratio $\frac{A_t}{A_{t-1}}$. Thus, the difficulty of using technology A_t , conditional on knowing how to use A_{t-1} , is an increasing function of the steady state growth rate between t and $t - 1$.

3.2 Externalities and growth

Now turn to the growth externality produced by intelligent individuals. In Murphy et al. [10], it is assumed that the best practice in the previous period becomes the commonly used technology in the next period. The larger the (finite) number of intelligent entrepreneurs, the larger is the average growth rate. We use a very similar idea. Since we have a continuum of entrepreneurs, the mechanism must be slightly modified. Otherwise, the practice of the next period would always be the best possible one, independently of the share of intelligent entrepreneurs. We simply postulate the rate of growth as some increasing function of the total number of intelligent entrepreneurs at $t - 1$.¹⁸ Let IQ_{t-1} denote the number of intelligent entrepreneurs in period $t - 1$. We then assume that

$$\frac{A_t}{A_{t-1}} = G(IQ_{t-1}), \quad (11)$$

where G is some increasing function.¹⁹

As we will see, the exact characteristics of G , apart from the non-negative derivative, are generically not of qualitative importance for our results. Let us, however, give a specific example of G . Assume, for instance, that technological improvements are caused by the total number of entrepreneurial ideas produced in the previous period. Furthermore, suppose that an entrepreneur finds a new idea with a probability which depends on her intelligence. For simplicity, assume that only intelligent entrepreneurs can contribute to the accumulation of new ideas and that the productivity increase due to a successful idea is a constant. Let the probability that an intelligent entrepreneur finds an idea be denoted p .

The increase in productivity due to a successful idea is assumed to be a pure externality.

¹⁸We have also investigated the case when growth is a function of average intelligence among entrepreneurs. In this case, we also get multiple steady state equilibria, but the high growth steady state is unstable. This is easily understood – when growth increases with average intelligence, an increase in σ incurs a reduction in the number of entrepreneurs. This always has a non-negative effect on average intelligence – thus causing even more growth and less entrepreneurs.

¹⁹Note that we assume that also a stupid CoE, who chooses to be an entrepreneur, always adopts the newest available technology, even though she knows perfectly well how to use the “old” technology. If we allow stupid CoE to choose the old technology, they will do so if, and only if, the noise associated with the new technology is sufficiently high relative to the increase in productivity it incurs. The necessary and sufficient condition for this is: $\sigma(g) < \frac{\log g^2}{1 - \log g^2}$. Thus if we allow this choice, the model result would be slightly different unless $\sigma(\cdot)$ is a function such that this restriction is satisfied. This issue is examined in appendix D.

As in Murphy et al. [10], we create the externality by assuming that successful ideas become public information the period after they are discovered.²⁰ Since each individual has measure zero, we also assume that each idea has measure zero. The amount of successful ideas is then equal to the number of intelligent entrepreneurs who find an idea.²¹ The function G then has the very simple form

$$G(IQ_{t-1}) = p IQ_{t-1}. \quad (12)$$

3.3 Multiple Equilibria

The possibility of multiple equilibria is now easily seen. From (10), we have that σ_t is an increasing function of $\frac{A_t}{A_{t-1}}$. Define $g_t \equiv \frac{A_t}{A_{t-1}}$ and note that g_t is equal to the steady state growth rate of output. Furthermore, g_t is equal to $G(IQ_{t-1})$, as given by (11) (or (12)). We can then write

$$\sigma_t = \sigma(G(IQ_{t-1})) \equiv f(IQ_{t-1}) \quad \text{with} \quad f'(\cdot) > 0. \quad (13)$$

The function $\sigma_t = f(IQ_{t-1})$ can be inverted to $IQ_{t-1} = f^{-1}(\sigma_t)$. In figure 7, we superimpose this inverted function on the graph of the correspondence that defines the mapping between σ and the steady state equilibrium values of IQ which we derived in section 2.5. We see that if $f^{-1}(\sigma)$ crosses $IQ(\sigma)$ both to the right and to the left of the cut off, we will have multiple endogenous growth steady state equilibria with different levels of growth and intergenerational social mobility. Given the shape of the correspondence, with a non-increasing initial range and a large upward jump followed by another non-increasing part, we are not obliged to impose strong additional restrictions on $f^{-1}(\sigma_t)$ in order to insure the existence of both a high and a low growth equilibrium.

One equilibrium (**B**) is characterized by a high σ . This induces an entrepreneurial class

²⁰Without changing the results, we could let the entrepreneur enjoy a share of the social return to the idea as long as some externality remains. Similarly, we could let the time lag between the creation and general adoption of an idea be smaller than a generation by introducing an overlapping generation structure.

²¹This amount is conveniently non-stochastic.

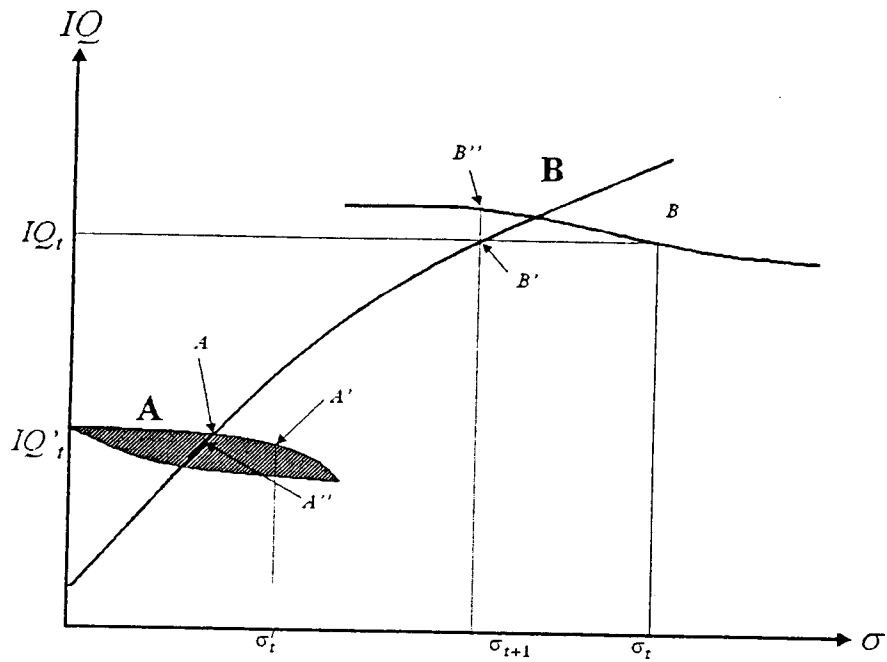


Figure 7: Multiple steady state equilibria.

formed by intelligent agents and a high level of intergenerational social mobility. The world changes rapidly in this equilibrium and the information received from parents is thus of small relative value which means that only intelligent individuals are entrepreneurs. This, in turn, implies a large flow of innovations, which makes the task of the new generation of entrepreneurs difficult, and the high level of intelligence among entrepreneurs is thus replicated.

There is also a set of steady state equilibria with low growth and no mobility. This set of equilibria is given by the segment of $f^{-1}(\sigma_t)$ inside the shaded area, A, in figure 7. Only children with parents in entrepreneurial positions become entrepreneurs in these equilibria. The children of entrepreneurs are more able entrepreneurs than children of workers, due to the informational advantage of having parents who used to be entrepreneurs. But, the entrepreneurs are mediocre from an intellectual viewpoint. Thus, they do not innovate and hence, the economy changes slowly. The information entrepreneurs can pass on to their children is of sufficiently high relative value for them to prefer to be entrepreneurs, even if their intelligence is low.

At any of the low growth equilibria, there are more entrepreneurs than in the high growth equilibrium, since low growth translates into relatively easy entrepreneurial tasks. Nevertheless, the total amount of brain power in entrepreneurial tasks is higher in the high growth equilibrium, since high growth restricts the access to entrepreneurial jobs to the intelligent individuals.

Let us now analyze the dynamic stability of the equilibria. First, recall that in results 2 and 4, we showed that for small variations in m_{t-1} around the steady state, equilibrium m_t equals the steady state value of m . This implies that the correspondences in figure 6 do not only depict the steady state equilibrium values of m and IQ , but also the equilibrium values of m_t and IQ_t for any value of m_{t-1} that is close to the steady state equilibrium for a particular value of σ . In other words, we can use the graphs in figure 6 to analyze out-of-steady-state dynamics in a neighborhood of the steady state.

Now, consider the high growth equilibrium in **B**. Let us assume that there is an exogenous (small) increase in σ to σ_t as depicted in figure 7. We assume that the shock is small, so the steady state equilibrium number of managers when σ is exogenously set to σ_t is close to the number of managers in **B**. Then, result 4 implies that the equilibrium in t occurs at B which gives IQ_t . This, in turn, determines σ_{t+1} from the relation $\sigma_{t+1} = f(IQ_t)$ at B' . The equilibrium in $t + 1$ occurs at B'' .

Result 6 *Let IQ^* and σ^* denote the high growth steady state value of IQ and σ , such that $\sigma^* > \rho \frac{\alpha}{1-\alpha}$. Let $IQ'(\sigma^*)$ denote the slope of the correspondence $IQ(\sigma)$ in the range where $IQ(\sigma)$ is a single number.²² Then if $IQ'(\sigma^*)f'(IQ^*) > -1$, the high growth steady state is locally stable. Furthermore:*

- *if the high growth steady state occurs at the horizontal portion of $IQ(\sigma)$, a small exogenous shock to σ around the steady state has no effect on IQ in the same period and in the period after the shock, the economy returns to the steady state, and*
- *if the high growth steady state occurs at the downward sloping portion of $IQ(\sigma)$, IQ and σ oscillate around the steady state after a small shock to σ .*

²² $IQ'(\sigma^*)$ equals zero or $\frac{\partial m_w(\sigma)}{\partial \sigma}$. The latter can easily be calculated from definition 1.

Proof: Around the steady state, we can approximate

$$\sigma_{t+1} - \sigma^* \approx (IQ_t - IQ^*)f'(\sigma^*) \approx (\sigma_t - \sigma^*)IQ'(\sigma^*)f'(\sigma^*), \quad (14)$$

so $IQ'(\sigma^*)f'(IQ^*) > -1$ implies that $\|\sigma_{t+1} - \sigma^*\| < \|\sigma_t - \sigma^*\|$.

Now, consider the low-growth set of steady states, given by the line segment of $f^{-1}(\sigma)$ in the shaded area of IQ at A. For any interior point on this line, a small exogenous shift in σ_t has no effect on m_t and thus not on IQ_t . This is the case, since $m_e > m_{t-1} > m_w$ so all CoE strictly prefer to be entrepreneurs and all CoW strictly prefer to be workers. Now consider a small exogenous increase in σ_t from a steady state at an endpoint of the line of steady states, for example, $m_{t-1} = m_e$ (point A in figure 7). A small increase in σ to σ'_t leads to an equilibrium in t at A' , with $m_t < m_{t-1}$ and a corresponding IQ'_t . The system is in steady state in $t+1$ at the new steady state A'' , with a smaller share of entrepreneurs and lower growth. The opposite adjustment would occur if a small negative shock to σ occurs at the steady state where $m_{t-1} = m_w$. The conclusion is thus

Result 7 *In a low growth steady state where $\sigma < \rho \frac{\alpha}{1-\alpha}$, a small exogenous shock to σ_t*

- *moves the economy to a new steady state in $t+1$ with $m_{t+1} < m_{t-1}$ and with lower growth if $m_{t-1} = m_e$ and the shock is positive,*
- *moves the economy to a new steady state in $t+1$ with $m_{t+1} > m_{t-1}$ and with higher growth if $m_{t-1} = m_w$ and the shock is negative, and*
- *has otherwise no effect on IQ , growth and m .*

4 Conclusions

The model we have presented is very stylized. Nevertheless, we think it describes important real world mechanisms which relate growth, social mobility and the demands put on individuals in different social positions. Intelligent individuals produce externalities by creating ideas and new ways of doing things. The extent to which society can take advantage of this depends on the efficiency of the social sorting mechanism. It seems very unlikely that the

full social value of such externalities are captured by those producing them. A walrasian labor market will then assign jobs in an inefficient manner.

In a stagnant economy, individuals who are not intelligent but happen to be born in an entrepreneurial household, will enjoy a great advantage in the competition for the best (entrepreneurial) jobs. In such circumstances, the labor market will be particularly inefficient in assigning roles. According to the market, the best entrepreneurs are those maximizing today's production, without taking their contribution to future growth into account. Individuals with small abilities to contribute to growth are more likely to fill positions where the growth externality could have been produced in a low growth economy than in a high growth economy. Thus, the effects interact; lack of social mobility causing low growth, and low growth causing lack of social mobility.

The inefficiencies of the labor market are mitigated when the growth rate is higher. The reason is that large growth rates reduce the social transmission of advantages, thus making individuals compete at face value and basing their merits more on their intellectual ability. Growth is produced by intellectual ability, so the winners in the job market are those producing growth. As before, both effects interact, social mobility causing growth, and growth causing mobility.

Two useful, but certainly unrealistic, simplifications in the model is that the intelligence level can only take two values and that there are only two types of jobs. This implies that intergenerational social mobility jumps when intellectual demands on entrepreneurs have increased to the point where intelligent children of workers are more suited to be entrepreneurs than stupid children of entrepreneurs. Below and above this point, higher intellectual demands lead to a smaller number of intelligent people in entrepreneurial positions. The jump causes a distinct division between the two types of endogenous growth steady states.

In reality, there is a large range of jobs with different intellectual demands and intelligence is continuously distributed in the population. Allowing for this in the model would, of course, change the relation between the degree of efficiency in the allocation of intelligent individuals and entrepreneurial difficulty. We conjecture that instead of having *one* jump in intergenerational social mobility, there could be a multiplicity of ranges of growth where

mobility increases rapidly. In these ranges, higher intellectual demands lead to more intelligent people in jobs with a high potential for growth externalities. In other regions, higher intellectual demands could lead to a small number of intelligent people in these positions. This may increase the number of possible steady state endogenous growth rates. We believe that our numerical results for models with continuous IQ support this conjecture.

In our model, history determines the equilibrium of a particular economy. Nevertheless, the model allows for some policy recommendations: a country that suffers from low growth and low social mobility should take measures to increase social mobility by making entrepreneurial positions more challenging. A reasonable way of achieving this is by opening the economy to trade and to foreign influences. One could interpret such an opening of an economy in the same manner as an increase in σ , i.e., as an increase in the inherent difficulty of being an entrepreneur. At first, the consequences might be negative, since the existing entrepreneurs will fail to make the right decisions. However, even if real wages fall, new opportunities will open for brilliant individuals for whom there was no other option than the lower end of the ladder in the closed economy. These individuals will eventually succeed and produce economic growth. Here, the positive effect of trade on growth is due to the uncertainty that trade may cause regarding the way a firm should be managed – not to comparative advantages. Uncertainty and its associated short run costs may result in a long run change in social and economic structures which is beneficial to society.

There are certainly other ways of producing growth miracles. We have abstracted from issues dealing with human capital accumulation and imperfect financial markets, which is emphasized in most of the previous literature on the topic. Such factors surely play important roles in determining the level of social mobility enjoyed by society, and they seem likely to enforce the mechanisms discussed in this paper.

A Steady State for large values of σ

If $\sigma \geq \frac{\alpha q}{1-\alpha}$ it is clear that in steady state z_t has to be such that:

$$P_{CoE, stupid} \leq z_t \leq P_{CoW, intelligent}$$

- If $m_w(\sigma) \geq q \geq m_e(\sigma)$, it is clear that there is a unique steady state equilibrium at $m = q$ and $z = SD^{-1}(q)$.
- If $q > m_w(\sigma)$, the the curves SD and M cross at a value of m lower than q . Consequently, if there is a steady state equilibrium, it must be at a wage such that the intelligent CoW are indifferent between career choices: $z = P_{CoW, intelligent}$. $m_w(\sigma)$ is the only number of entrepreneurs that produce this wage as a labor market equilibrium. A necessary and sufficient condition for the existence of a steady state equilibrium is then that a $\gamma \in [0, 1]$ exists such that:

$$m_w(\sigma) = qm_w(\sigma) + \gamma q(1 - m_w(\sigma)).$$

Clearly,

$$\gamma = \frac{m_w(\sigma)}{1 - m_w(\sigma)} \frac{1 - q}{q} > 0,$$

and

$$\gamma \leq 1 \iff m_w(\sigma) \leq q$$

Thus, a steady state equilibrium exists and it is unique.

- If $m_e(\sigma) > q$, the the curves SD and M cross at a value of m larger than q . Consequently, if there is a steady state equilibrium, it has to be at a wage such that the stupid CoE are indifferent between career choices: $z = P_{CoE, stupid}$. $m_e(\sigma)$ is the only number of entrepreneurs that produce this wage as a labor market equilibrium. A necessary and sufficient condition for the existence of steady state equilibrium is then that a $\delta \in [0, 1]$ exists such that:

$$m_e(\sigma) = qm_e(\sigma) + q(1 - m_e(\sigma)) + \delta m_e(\sigma)(1 - q)$$

Clearly,

$$\delta = \frac{m_e(\sigma) - q}{m_e(\sigma)(1 - q)} < 1,$$

and

$$\delta \geq 0 \iff m_e(\sigma) \geq q.$$

Thus, a steady state equilibrium exists and it is unique.

B Managerial IQ as a function of ρ

Consider an exercise similar to the one in section 2.5. Here ρ is varied between zero and unity. A higher ρ implies that the entrepreneurial task becomes easier for the CoE (because they then have better inherited information), but not for the CoW. Thus, at high levels of ρ , the social sorting mechanism should assign more weight to social than to innate assets, i.e., social background is more important than intelligence. Note that m_e does not depend on ρ since only σ affects the difficulty of being a manager for CoE. On the other hand, m_w is a decreasing function of ρ since the unconditional variance of $x = \frac{\sigma}{1-\rho}$, i.e., the difficulty of being a manager, increases in ρ for CoW.

At $\rho = \frac{1-\alpha}{\alpha}\sigma$, $m_e = m_w$, provided that $\frac{1-\alpha}{\alpha}\sigma < 1$. For values of ρ above this threshold, stupid CoE have an advantage over intelligent CoW. Thus, any value of m such that $m_e \geq m \geq m_w$ is a steady state equilibrium in which all CoE, but no CoW, become entrepreneurs.

For values of ρ below $\frac{1-\alpha}{\alpha}\sigma$, $m_w > m_e$. Then, if q is larger than m_e only intelligent people, but not all of them, become entrepreneurs. In order to make some intelligent CoW willing to become workers, m must equal m_w . As ρ decreases from $\frac{1-\alpha}{\alpha}\sigma$, the number of intelligent CoW who become entrepreneurs increase and equals $\min\{q, m_w\}$. A second case arises if $q < m_e$, such that there is a scarcity of intelligent people. In this case, some of the stupid CoE must be willing to be entrepreneurs, so in this case, $m = m_e$ for all values of $\rho < \frac{1-\alpha}{\alpha}\sigma$.

C An alternative model

Here we describe a model closely resembling the model in section 2. There are, however, three main differences which make this model impossible to solve analytically. First, instead of having only two levels of intelligence, we assume that the intelligence levels, denoted q , are continuous with a distribution represented by a distribution function $F(q)$. As above, there is an individual signal on x_t with a precision given by q_i .

Second, we assume that the entrepreneurial error affects output rather than profits. The profit function of a firm is thus

$$\pi = \alpha^{-1} e^{-\frac{(x-a)^2}{2}} A l^\alpha - w l. \quad (15)$$

Let the precision in the beliefs about x by an individual with intelligence i and with a parent who had the occupation $j \in \{w, e\}$ be denoted $P(i, j)$. Solving for the maximum of expected profits over l , we get that the labor demand in a firm run by an individual of type i, j firm is given by

$$l_d(w/A, q_i, j; \sigma) = \left(\sqrt{\frac{1}{1 + P(i, j)^{-1}} \frac{A}{w}} \right)^{\frac{1}{1-\alpha}} \quad (16)$$

and expected profits

$$E\pi(A, w/A, q_i, j; \sigma) = A \left(\sqrt{\frac{1}{1 + P(i, j)^{-1}} \frac{A}{w}} \right)^{\frac{\alpha}{1-\alpha}}. \quad (17)$$

Compared to the model in the main text, labor demand by an individual firm thus depends on the intelligence of the particular entrepreneur.

Third, we assume risk neutrality. Then, each individual chooses to become a worker (an entrepreneur) if the wage is higher (lower) than the expected profits. The threshold level of IQ is determined by the condition that the expected profit equals the wage, which makes the (risk-neutral) agent indifferent between the two choices.

$$\pi(A, w/A, q_i, j; \sigma) = A \left(\sqrt{\frac{1}{1 + P(i, j)^{-1}} \frac{A}{w}} \right)^{\frac{\alpha}{1-\alpha}} = w. \quad (18)$$

Solving this for the threshold precision $\bar{P}(w/A)$ yields

$$\bar{P}(w/A) = \left(\left(\frac{w}{A} \right)^{\frac{1-\alpha}{\alpha}} - 1 \right)^{-1}. \quad (19)$$

We can then find the two threshold intelligence levels, denoted \bar{q}_e and \bar{q}_w

$$\bar{q}_e(w/A; \sigma) = \bar{P}(w/A) - \frac{1}{\sigma}, \quad (20)$$

and

$$\bar{q}_w(w/A; \sigma) = \bar{P}(w/A) - \frac{1-\rho}{\sigma}. \quad (21)$$

Now consider the labor market. The supply of workers is the number of entrepreneurs' children with an IQ lower than \bar{q}_m and the number of workers' children with an IQ lower than \bar{q}_w . This means that the aggregate labor supply in period t is

$$L_s(w/A, m_{t-1}; \sigma) = m_{t-1} F(\bar{q}_e) + (1 - m_{t-1}) F(\bar{q}_w). \quad (22)$$

We also have that

$$m_t = m_{t-1} (1 - F(\bar{q}_m)) + (1 - m_{t-1}) (1 - F(\bar{q}_w)). \quad (23)$$

The aggregate labor demand is given by

$$L^d(w/A, m_{t-1}; \sigma) = m_{t-1} \int_{\bar{q}_a}^{\infty} l_d(w/A, q_i, e; \sigma) dF(q) + (1 - m_{t-1}) \int_{\bar{q}_m}^{\infty} l_d(w/A, q_i, w; \sigma) dF(q).$$

In a steady state equilibrium, it is required that

$$m_t = m_{t-1} \tag{24}$$

$$L^d(w/A, m_{t-1}; \sigma) = L^s(w/A, m_{t-1}; \sigma).$$

The two equations in (24) together define a steady state value of m and a corresponding steady state level of w/A .

Now let us specify some parameters in order to illustrate the behavior of the model. We have used $\alpha = 0.5, \rho = 0.5$ and set $F(q) = q$, so that $q \in [0, 1]$. The results are depicted in figure 8. The top left panel shows the cut-off level of intelligence such that all individuals with an IQ lower than that level prefer to be workers. We see that for low enough σ , all CoE choose to become entrepreneurs and all CoW to become workers – intergenerational mobility is zero. As σ increases, innate assets become relatively more important. The cut-off levels of intelligence for the two groups thus become closer and approach the same level at around 0.55. The bottom left panel shows the number of entrepreneurs that can be sustained in a steady state equilibrium. For low values of σ , there is a multiplicity of equilibria for the same reason as in the model in the main text; here there is a range of wages such that neither the CoE nor the CoW want to do anything else than their parents. Any level of m that produces a wage within this range is a steady state equilibrium. Above the level of σ where social mobility starts to become operative, there is a single steady state equilibrium for each level of σ .

The increase in intergenerational social mobility that an increase in σ brings about, increases the average IQ among entrepreneurs. This is seen in the bottom right panel of figure 8. The total amount of IQ among entrepreneurs has a shape very similar to the one depicted in figure 6. For low values of σ , only IQ levels at or below 0.25 are sustainable. At the point of σ where social mobility becomes operative, IQ increases quickly, since sorting becomes more efficient. Then IQ starts falling slowly, reflecting that it becomes more difficult for everybody to be an entrepreneur, so the share of entrepreneurs falls. Here, as in the model in the main text, two mechanisms working in opposite directions are creating a non-monotonous relation between σ and IQ. The first is responsible for the downward slope for low and high levels of σ . The other creates an intermediate range where IQ increases rapidly, but not discontinuously, as in the model in section 2.

The correspondence between σ and IQ depicted in figure 8 is clearly very similar to the correspondence derived in section 2. We could then use the former instead of the latter in the endogenous growth model in section 3 and produce very similar results.

D Voluntary adoption of the new technology

If we assume that individuals have to decide not only *if* to become entrepreneurs, but also *which* technology to adopt, the problem only becomes slightly more complicated.

The new problem is straightforward for the CoW, since they have no information regarding the technology in the previous period. Both technologies have the same level of complexity and they will thus always adopt the latest available technology.

For CoE there is a non-trivial decision to make. On one hand, they know how to use the old technology. On the other, they know that the new technology has better productive potential. CoE who adopt the new technology (A_t), instead of the old (A_{t-1}), expect the utility

$$V^n = 2 \log A_t - \log w_t - \frac{1}{P(i)},$$

but if they instead use the old technology, about which they have infinite precision, they obtain

$$V^o = 2 \log A_{t-1} - \log w_t.$$

Clearly, if a CoE decides to be an entrepreneur, she adopts the new technology iff

$$P(i) \geq \frac{1}{\log \left(\frac{A_t}{A_{t-1}} \right)^2} = \frac{1}{\log g_t^2}$$

Whatever σ , the precision of intelligent agents is larger than the precision of stupid ones, which means

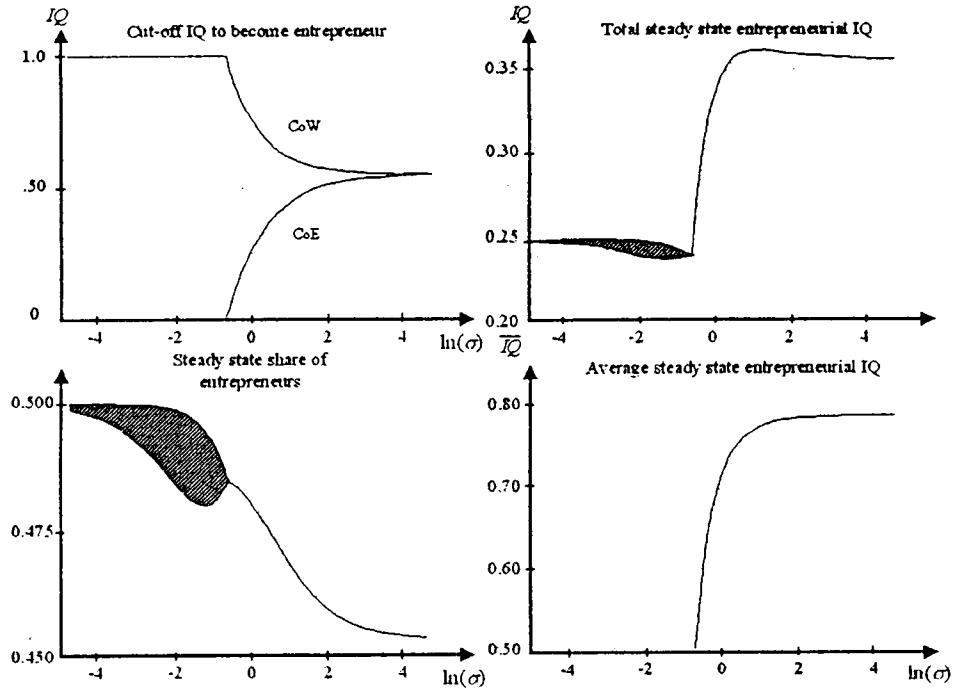


Figure 8: Growth Steady States

that they will adopt the new technology more eagerly. All CoE will adopt it if

$$P_{DOM, stupid} > \frac{1}{\log g^2}$$

or

$$\sigma(g) < \frac{\log g^2}{1 - \log g^2} \quad (25)$$

As long as $\sigma(\cdot)$ is such that (25) holds, all entrepreneurs use the new technology. The results will then be identical to the ones exposed in the paper.

References

- [1] Baumol, William J., (1990), "Entrepreneurship: Productive, Unproductive and Destructive", *Journal of Political Economy*, vol. 98:5, 893-921.
- [2] Benabou, Roland. (1996), "Equity and Efficiency in Human Capital Investment: The Local Connection". *Review of Economic Studies*, 63(2), 237-64.
- [3] Jones, Charles I., (1997) "On the Evolution of the World Income Distribution". *Journal of Economic Perspectives*, Vol. 11, 19-36.
- [4] Sjögren, Anna, (1997), "The Effects of Redistribution on Occupational Choice and Intergenerational Mobility: Does Wage Equality Nail the Cobbler to His Last?", mimeo, Stockholm School of Economics.
- [5] Lucas, Robert E. (1993) "Making a Miracle". *Econometrica*, 61(2), 251-72

- [6] Galor, Oded and Zeira, Josep, (1993) "Income Distribution and Macroeconomics". *Review of Economic Studies*, 60(1), 35-52.
- [7] Galor, Oded and Daniel Tsiddon, (1997), "Technological Progress, Mobility and Economic Growth", *American Economic Review*.
- [8] Eriksson, Robert and John Goldthorpe, (1992), *The Constant Flux: A Study of Class Mobility in Industrial Societies*, Oxford Clarendon Press.
- [9] Maddison, Angus, (1982), *Phases of Capitalist Development*, Oxford University Press, Oxford, England.
- [10] Murphy, Kevin M., Andrei Scheifer and Robert W. Vishny, (1991), "The Allocation of Talent: Implications for Growth", *Quarterly Journal of Economic*, Vol. 106:2, 503-530.
- [11] Thomas Piketty, (1995), "Social Mobility and Redistributive Politics", *Quarterly Journal of Economics*, 110(3), 551-84.
- [12] Pritchett, Lant (1997) "Divergence, Big Time". World Bank Policy Research Working Paper No. 1522.

