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ABSTRACT

Foreign Exchange Intervention, Policy Objectives and Macroeconomic Stability*

Within a simple model of monetary policy for an open economy, we study how foreign exchange intervention may be used to condition agents' beliefs of the objectives of the policy-makers. In contrast to cheap talk, foreign exchange intervention guarantees a unique equilibrium. Foreign exchange intervention does not bring about a systematic policy gain, such as an increase in employment or a reduction in the inflationary bias. It can, however, stabilize the national economy, for it drastically reduces the fluctuations of employment and output. Foreign exchange intervention is profitable, but a trade-off exists between these profits and the stability gain it brings about. Finally, an important normative conclusion of our analysis is that foreign exchange intervention and monetary policy should be kept separate, in that a larger stability gain is obtained when these two instruments of policy-making are under the control of different governmental agencies.

JEL Classification: D82, G14 and G15
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NON-TECHNICAL SUMMARY

Recent empirical research on foreign exchange intervention suggests that sterilized purchases and sales of foreign currencies on the part of the monetary authorities influence market expectations on exchange rates. Despite the general opinion that foreign exchange intervention may represent an independent instrument of policy-making, there is no consensus among practitioners and researchers on what it can achieve.

In this respect, in this Paper we show how foreign exchange intervention may be instrumental in stabilizing the economy, reducing the volatility of employment and output, whenever there is uncertainty over the objectives of the policy-makers. In particular, if private investors are uncertain of the motives of the policy-makers, they possess an incentive to manipulate market beliefs. While simple announcements will generally lack credibility, this can be acquired if an expensive signalling mechanism is employed. Indeed, a point that we make in this Paper is that foreign exchange intervention permits the acquisition some degree of credibility, since the central bank stakes its own wealth in support of its signals.

To highlight the signalling role of foreign exchange intervention and to present its macro-level effects, the model we employ combines the classical macroeconomic approach to monetary policy with the more recent market microstructure approach to exchange rates. Thus, our contribution offers a first formal analysis of the interplay between the microstructure of the foreign exchange market and its macro-level implications.

In particular, the market microstructure framework we propose for the market for foreign exchange presents three important features: (i) it focuses on the informational role of the order flow, as recent empirical research (Lyons, 1995, and Evans and Lyons, 2001) shows that the flow of orders in the foreign exchange market moves exchange rates; (ii) it allows identifying a clear link between the intervention operations of central banks, agents' expectations, exchange rates and employment; (iii) it produces a series of testable implications which are consistent with a large body of empirical evidence concerning the statistical relations between intervention operations, exchange rates and monetary aggregates.

Then, we see that the policy-makers will try to exploit the signalling role of foreign exchange intervention to misrepresent their objectives and obtain a policy gain, in the form of an increase in the employment level and/or a reduction in the inflationary bias. While in equilibrium their attempt is frustrated, the policy-makers will, however, benefit from foreign exchange intervention, even though not in the way they hoped for, as this helps stabilize the economy by reducing the volatility of the employment level.

Our analysis also indicates that the central bank profits from its intervention operations in the foreign exchange market. Indeed, we see that there exists a trade-off between the profits the central bank will make and the stability gain foreign exchange intervention provides. Moreover, different institutional arrangements concerning the control of policy-making instruments produce important differences with respect to the effects of foreign exchange intervention. In particular, we find that full coordination of monetary policy and foreign exchange intervention will result in larger profits at the expense of a more volatile employment level.

Therefore, a normative implication of our analysis is that foreign exchange intervention and monetary policy should be kept separate to reduce the instability of the economy. This is a quite novel and important conclusion, in that the institutional arrangements governing these two instruments of policy-making differ from country to country and have been completely overlooked in the existing monetary policy literature.

Introduction

In a recent survey by Neely (2000) most central bankers agreed with the thesis that foreign exchange intervention has an impact on the market for foreign exchange. This thesis is reinforced by some recent empirical research, notably Dominguez and Frankel (1993a, 1993b), Fischer and Zurlinden (1999), Payne and Vitale (2000), on the effectiveness of foreign exchange intervention, which suggests that sterilised purchases and sales of foreign currencies on the part of central banks influence exchange rates. However, there is no consensus among practitioners and researchers on what foreign exchange intervention can achieve.

Thus, in Neely's survey 47 percent of the respondents claimed that foreign exchange intervention is aimed at resisting short-term trends, 22 percent suggested that its main goal is to eliminate misalignements from fundamental values, while the rest indicated different, unspecified reasons for intervention. Likewise, in the academic arena several theses have been suggested: foreign exchange intervention may indicate future changes in the monetary policy (Mussa (1981)), new target levels for the exchange rates (Bhattacharya and Weller (1997), Vitale (1999)), and may also be used to reduce market instability and smooth exchange rate movements (Dominguez and Frankel (1993a)).

We go beyond the analysis of the simple effects of foreign exchange intervention in the market for foreign exchange and look at its more general macroeconomic implications. We argue that foreign exchange intervention may be instrumental in stabilising a national economy, as it can be an effective channel through which policy makers can *signal* their objectives and hence reduce agents' uncertainty on future policy decisions. In this way foreign exchange intervention is shown to represent an independent instrument of policy making.

To highlight the signalling role of foreign exchange intervention and present its macro-level effects, we propose a model which combines the classical macroeconomic approach to monetary policy with the more recent market microstructure approach to exchange rates. Indeed, a recent popular view among researchers, notably Frankel *et al.* (1996) and Lyons (2001), is that the traditional asset market approach fails to recognise the role that the structure of the foreign exchange market, with its trading protocols and organisation, plays in the determination of exchange rates.

In this respect our contribution offers a first formal analysis of the interplay between the microstructure of the foreign exchange market and its macro-level implications. In particular, the market microstructure framework we propose for the market for foreign exchange presents three important features: i) it focuses on the informational role of the order flow, as recent empirical research (Lyons (1995) and Evans and Lyons (2001)) shows that the flow of orders in the foreign exchange market *moves* exchange rates; ii) it allows identifying a clear *link* between the intervention operations of central banks, agents' expectations, exchange rates and employment; iii) it produces a series of testable implications which are consistent with a large body of empirical evidence concerning the statistical relations between intervention operations, exchange rates and monetary aggregates.

Through this market microstructure framework we are then able to analyse the macro-level effects of foreign exchange intervention. In particular, within a monetary policy model *à la* Barro and Gordon we see that the monetary authorities attempt to raise employment above its natural level with unexpected monetary shocks. If individual agents are rationale and fully understand the motives of the policy makers, this attempt completely fails whilst an inflationary bias emerges. On the other hand, if there is incomplete information on their objectives, the policy makers might be able to stimulate the economy or reduce the inflationary bias if they can manage to manipulate agents' beliefs and mis-represent their objectives.

In practice, manipulating agents' beliefs may be difficult: i) precise public announcements on the policy objectives will lack credibility, as the policy makers possess an incentive to lie, and cannot be used; ii) imprecise statements can instead be credibly employed, but since different announcements are *per se* all the same, in the sense that they are all equivalently *inexpensive*, multiple signals of different informative quality can be used at the same time. On the contrary, we see that this problem of the multiplicity of the equilibria does not emerge if a potentially *expensive* signalling mechanism is employed. Indeed, we show that foreign exchange intervention i) permits obtaining some degree of credibility, since the central bank stakes its own wealth in support of its signals, and ii) delivers a uniquely informative message.

Once more, while the policy makers will try to exploit this signalling channel to mis-represent their objectives and obtain a policy gain, in equilibrium their attempt to eliminate the inflationary bias and simultaneously stimulate the domestic economy is frustrated. And yet, the policy makers will benefit from foreign exchange intervention, even though not in the way they hoped for, as this helps stabilising the economy by reducing the volatility of the employment level.

Our analysis also shows that in equilibrium the policy makers exploit their informational advantage and gain profits from their intervention operations. However, we see that there exists a trade off between these profits and the stability gain foreign exchange intervention brings about. This conclusion is reinforced when we compare different institutional arrangements concerning the division of power over monetary policy and foreign exchange intervention. We find in fact that full coordination of these two instruments of policy making will result in larger profits at the expenses of a more volatile employment level.

Therefore, a normative implication of our analysis is that foreign exchange intervention and monetary policy should be kept separated to reduce the instability of the economy. This is a quite novel and important conclusion, in that the institutional arrangements governing these two instruments of policy making differ from country to country and have been completely overlooked in the existing monetary policy literature.

The paper is organised as follows. In Section I we present a simple model of monetary policy for an open economy. We discuss the usual problem of time-consistency and show that under incomplete information the policy makers may benefit from the manipulation of the agents' beliefs. In Section II we discuss in details the practice and the motivation of foreign exchange intervention and present a market microstructure analytical framework for the analysis of the foreign exchange market. In the following Section we characterise the equilibrium of the model when foreign exchange intervention and monetary policy fall under the jurisdiction of two separate governmental bodies. We then show the stabilising effects of foreign exchange intervention on the employment level. In Section IV we extend the analysis to the case in which a unique central authority controls both foreign exchange intervention and monetary policy and consider some comparative analysis exercises. In Section V we propose some concluding remarks. In the Appendix we present brief proofs of the Propositions of the paper.

I. Monetary Policy in an Open Economy

Let now introduce a simplified model of monetary policy for an open economy as a basic set-up for the analysis of the relation between foreign exchange intervention, monetary policy, and macroeconomic stability.[1]

A. A Basic Set-Up

Let us consider two countries, a domestic economy and a foreign one, and the corresponding nominal exchange rate. We assume that the nominal exchange rate reacts to monetary injections in the home country according to the following formulation:

$$s_t = m_t, \tag{1}$$

where s_t indicates the log of the nominal exchange rate between the home and foreign country, i.e. the units of domestic currency required to purchase one unit of the foreign one, and m_t denotes the log of the money supply in the home country in period t. Not only a monetary injection can create inflation and depreciate the national currency, but it might also stimulate the domestic economy increasing national employment and output. In particular, we have:

$$n_t = m_t - w_t, \tag{2}$$

[1] This model is a special case of the analytical framework proposed by Canzoneri and Henderson (1991). Details of its derivation can be obtained from the author on request.

where n_t denotes the log of the level of employment and w_t denotes the log of the nominal wage in the domestic economy. In period t the nominal wage is negotiated before the monetary injection takes place. The wage setters will set it equal to the expected value of the money supply,

$$w_t = m_t^e, \tag{3}$$

in an attempt to stabilise the domestic employment level. Then, the domestic monetary authorities might attempt to push employment beyond its natural level, normalised to 0, via unexpected monetary shocks and inflation.

As common in the monetary policy literature, we suppose that the natural level of employment is suboptimal due to some distortion in the economy. Moreover, unexpected inflation may also be desirable *per se* as it guarantees a seignorage tax and reduces the cost of servicing government debt. In practice, we suppose that there is an incentive on the part of the government to surprise the wage setters and inflate the economy. However, the authorities also pursue the complementary goal of stabilising the price level or, equivalently, the nominal value of the domestic currency.

In order to capture the tension between these two complementary goals, we assume that in the home country the term of office of the monetary authorities is confined to one period and that they will then minimise the expected value of the following loss function:

$$\mathcal{L}_m = (n_1 - \bar{n})^2 + (s_1 - \bar{s})^2, \tag{4}$$

where $\bar{n}$ and $\bar{s}$ represent optimal values for the employment level and the exchange rate.

The particular choice of the monetary authorities's loss function is inconsequential for the results of our analysis. We could use alternative specifications, such as those put forward by Barro and Gordon (1983), Canzoneri (1985), Cukierman and Meltzer (1986), where the nominal exchange rate is replaced by the inflation rate or the price level. The conclusions of our analysis would not change.[2]

Moreover, we can easily justify the particular choice we have made. Indeed, many small or developing countries, notably Argentina, Mexico and other Latin American countries, use or have used the exchange rate as a nominal *anchor* to stabilise their domestic economies. The recent experience of the European Monetary System also suggests that even large industrialised economies have used this nominal anchor to fight inflation. In addition, Japan and the United States have tried to target the value of the dollar in the 1980s in order to stabilise their terms of trade.[3] Finally,

[2]It is not difficult to prove that simple linear mappings exist between the nominal variables in Canzoneri and Henderson's analytical framework. This means that we could easily reformulate our loss function in terms of the price level.

[3]Agenor and Montiel (1996) offer detailed accounts of the stabilisation programmes attempted in Latin America

Great Britain may decide to join EMU in the near future. In this case the British government would probably desire to target the nominal exchange rate of the pound before a conversion rate were negotiated.

B. Incomplete Information and the Manipulation of Agents' Beliefs

When the loss function of the monetary authorities, $\mathcal{L}_m$, is common knowledge, we can easily find the values for the money supply and the nominal wage in a *discretionary* equilibrium:

$$m_1 \;=\; w_1 \;=\; \bar{s} \;+\; \bar{n}. \tag{5}$$

Therefore, the equilibrium value of $\mathcal{L}_m$ is

$$\mathcal{L}_m \;=\; \mathbf{l}_c \;\equiv\; 2\,\bar{n}^2. \tag{6}$$

As usual, under discretion the condition of time consistency induces an *inflationary bias*, $\bar{n}$, whilst the monetary authorities are totally unsuccessful in their attempt to raise the employment level. In this case the monetary authorities would be better off if they could commit to a non-inflationary policy where the monetary supply were fixed at $m_1 = \bar{s}$. Still, they would not be able to stimulate the economy, as n_1 would remain at its natural level zero, but at least the exchange rate would converge to the target level, $\bar{s}$.

In this context foreign exchange intervention cannot play any role, *unless* we consider some form of asymmetric information. Indeed, we can claim that in general the monetary authorities possess various forms of private information. In practice, this often stems from incomplete information on their preferences, as economic agents might not know what levels of employment and inflation the monetary authorities prefer or there can be uncertainty on their *type*. For instance, in Barro (1986) and Cukierman and Meltzer (1986), agents do not know how strongly the monetary authorities are committed to fighting inflation.

Here we follow Stein (1989) in assuming that there is incomplete information on the target level for the exchange rate, $\bar{s}$. In particular, we assume that the wage setters do not know the exact value of the target level, $\bar{s}$. However, $\bar{s}$ is drawn from a Normal distribution with mean $\bar{s}^e$ and variance Σ at the beginning of the term of office. On the contrary, no uncertainty exists on the target value for the employment level, $\bar{n}$, while the distribution of $\bar{s}$ is common knowledge and so is the form of the loss function.

We can justify this particular formulation of incomplete information on the basis of past experience. In fact, in the 1980s reference values were set for the exchange rates of the main currencies

in the 1970s and 1980s, while Giavazzi and Pagano (1988) discuss the anchoring of France and Italy to the German monetary policy in the 1980s through the European ERM. Funabashi (1988) discusses the attempts of the G-5 to target the exchange rates of the dollar, the deutsche mark and the yen in the 1980s.

by the G-5, but were neither officially endorsed nor publicly announced.[4] Moreover, we could also argue that the British government would desire to target the pound/euro nominal exchange rate before joining EMU, without having the possibility of announcing its future plans.

Under this new assumption we find that the wage setters are not capable of predicting the exact value of the monetary injection. In a discretionary equilibrium the nominal wage and the money supply are now as follows:

$$w_1 = \bar{s}^e + \bar{n}, \tag{7}$$
$$m_1 = \bar{s} + \bar{n} - \frac{1}{2}(\bar{s} - \bar{s}^e), \tag{8}$$

where $\bar{s}^e$ denotes the expectation of the target level conditional on the information of the wage setters in period 1.

The prediction error of the wage setters affects the equilibrium value of the monetary authorities' loss function, $\mathcal{L}_m$, which now becomes:

$$\mathcal{L}_m = \mathbf{l}_i \equiv \mathbf{l}_c - \mathbf{g}, \quad \text{where}$$
$$\mathbf{g} \equiv 2(\bar{s} - \bar{s}^e)\bar{n} - \frac{1}{2}(\bar{s} - \bar{s}^e)^2.$$

The extra term, $\mathbf{g}$, denotes the *gain* from the ignorance of the wage setters, measured by the prediction error $\bar{s} - \bar{s}^e$. Thus, since $\mathbf{g}$ can be positive, if the monetary authorities possessed some instrument to manipulate the wage setters' beliefs, they could potentially reduce the equilibrium value of the loss function, $\mathbf{l}_i$.

In practice, it may be difficult to manipulate the wage setters' beliefs. In particular, precise announcements on the target level for the spot rate, $\bar{s}$, will generally lack credibility. Suppose, in fact, that the monetary authorities make an announcement on the target level, $\bar{s}$, before the nominal wage is negotiated and that such announcement is believed by the wage setters. This amounts to assuming that the monetary authorities can freely set the expected value of the target level, $\bar{s}^e$. Then, if the monetary authorities announce the true value, $\bar{s}$, we return to the complete information world, where the best outcome they might hope for corresponds to the full commitment one.

However, there is nothing that prevents the monetary authorities from lying. Indeed, they could announce the following false target level for the exchange rate, $\bar{s} - 2\bar{n}$. Assuming that the wage

[4] In particular, according to Funabashi (1988), at the Plaza meeting in 1985 unofficial reference levels were set close to 215 for the yen-dollar rate and 2.60 for the deutsche mark-dollar rate. Likewise, at the Louvre meeting in 1987, when the dollar had experienced a large devaluation since the Plaza, new reference levels were fixed at 153.5 and 1.825 respectively. These figures were repeatedly adjusted in the following year but never announced.

setters believe such statement, $\bar{s} - 2\bar{n}$ replaces $\bar{s}^e$ in equations (7) and (8), so that the employment level and the nominal wage are now $\bar{n}$ and $\bar{s}$ and the value of the monetary authorities' loss function is zero. In other words, if the wage setters wrongly believed the monetary authorities' announcement, these would be capable of stimulating the economy and targeting the nominal exchange rate in the desired way.

Only in the special case in which $\bar{n}$ were equal to zero the monetary authorities could credibly announce the true value of the target level for the spot rate, $\bar{s}$. They would not have an incentive to lie, as the two goals of stimulating the economy and stabilising the nominal exchange rate would not be in conflict. For $\bar{n} \neq 0$, on the contrary, the monetary authorities possess an incentive to mis-represent their preferences and a precise statement on their part could not be believed by the wage setters.

While, as suggested by Stein (1989), some imprecise statements could still be used to convey credible signals to the market, we focus on an alternative mechanism of communication based on foreign exchange intervention. Indeed, operations in the foreign exchange market by a central bank may be used to "buy credibility" and convey signals to the market, in that the central bank stakes its own capital in support of the signal. We are now going to see how this is possible: first we will see how foreign exchange intervention can be modelled and then we will study its macroeconomic consequences.

Before introducing foreign exchange intervention, though, we conclude this Section by deriving the unconditional variance of the employment level and the unconditional expectation of the policy-maker losses in the equilibrium *without* intervention. These values will then be used as yardsticks to study the effects of foreign exchange intervention. Simple calculations give the following:

$$\mathrm{Var}_{\emptyset}\left[n_1\right] = \frac{1}{4}\,\Sigma, \tag{9}$$

$$E_{\emptyset}\left[\mathcal{L}_m\right] = 2\,\bar{n}^2 + \frac{1}{2}\,\Sigma, \tag{10}$$

where henceforth the subscript $\emptyset$ refers to the equilibrium without foreign exchange intervention.

II. Foreign Exchange Intervention

According to Mussa (1981) operations in the foreign exchange market by a central bank may be used to signal future changes in the monetary policy. Purchases (sales) of foreign exchange should signal a forthcoming monetary contraction (expansion) more effectively than a simple announcement, because the central bank stakes its own capital in support of the future policy. In fact, when a sale of foreign assets is followed by a monetary expansion that forces a devaluation of the domestic currency, the central bank incurs in a net loss.

In our context we argue that the policy makers might attempt to manipulate the wage setters' expectations of the target level, $\bar{s}$, and of the money supply, m_1, before the nominal wage, w_1, is set, by buying or selling the foreign currency in the foreign exchange market. In order to show how this "signalling channel" can be exploited we need to introduce an analytical framework for foreign exchange intervention. To do so let us see how foreign exchange intervention operates.

A. The Practice of Foreign Exchange Intervention

The practice of foreign exchange intervention depends on the complex and evolving structure of the market for foreign exchange. This is a fragmented dealership market, where bilateral direct inter-dealer transactions, conducted on the phone or via electronic communication systems, coexist with brokered ones, mediated by brokerage firms or via electronic brokerage services. Thus, when undertaking an intervention operation, a central can either trade via a broker or place individual orders directly with one or more dealers.

In the past the Federal Reserve would prefer the latter route when maximum visibility was desired, while the former would be favoured whenever secrecy was required. These days, however, electronic limit order books have largely replaced brokerage firms and hence the Federal Reserve splits its activity between EBS, the main electronic brokerage service in the United States, and direct trading with many different dealers. Other central banks use similar practices.

Despite the massive dimension of the foreign exchange market, its fragmented structure allows a central bank to operate with individual dealers and influence their quotes with transactions of small size, as the activity of single dealers remain within limited size. Furthermore, as Chaboud and LeBaron (2001) find for the intervention conducted by the Federal Reserve, operating through different routes a central bank is able to provoke a wave of inter-dealer transactions that quickly spreads news of intervention in the market. The diffusion of news of intervention is so rapid that newswire services would report intervention activity in the space of few minutes.

However, no bank official makes a public statement announcing the *exact* size of any intervention operation. Moreover, despite the recent consolidation brought about by electronic brokerage services, such as EBS and Reuters 2000-2, the market for foreign exchange remains fragmented and opaque. This means that, since most transactions are the result of private bilateral meetings between sellers and buyers, single dealers cannot observe *all* market orders and prices cannot immediately incorporate *all* private information contained in individual trades. This implies that news of intervention will diffuse in the market with some noise, in the sense that the *average* dealer will imprecisely estimate its exact amount.

Indeed, reports of central bank intervention usually appear in the press. However, as shown by Klein (1993) and Hung (1997), they are often imprecise: sometimes central banks intervene and no report appears on financial newspapers or reports appear and intervention operations have not occurred.[5] Moreover, quantities are rarely indicated and when this happens figures are way out from the actual ones and generally smaller. Moreover, Chaboud and LeBaron (2001) find evidence that *per se* these reports *neither* increase trading volume in the foreign exchange market *nor* influence exchange rates, but that only actual intervention operations *do*.

B. A Market Micro-Structure Framework for the Foreign Exchange Market

To capture these features of foreign exchange intervention in a formal model we suppose that the term of office is divided in two stages: 0 and 1. At the beginning of stage 0 $\bar{s}$ is realised. Again we assume that the unconditional distribution of $\bar{s}$ is normal with mean $\bar{s}^e$ and variance Σ and that this information is common knowledge. At the beginning of stage 1 the nominal wage is set. After observing the value of w_1, the monetary authorities fix the money supply, m_1.

Now, let us assume that the central bank may intervene in the market for foreign exchange both in stage 0, after the target level, $\bar{s}$, is realised, and in stage 1, after the nominal wage and the money injection have been selected. In this market a single (representative) dealer trades the foreign currency with a group of clients, which comprises the central bank and some liquidity traders. The organisation of trading is the following:

In both stages the dealer calls an auction for the foreign currency. In auction τ ($\tau = 0, 1$) the central bank and the liquidity traders anonymously place their respective market orders, I_τ and u_τ. While the central bank will place its order to manipulate market expectations, the liquidity traders will trade for liquidity reasons and will place unpredictable market orders, normally distributed with mean 0 and variance σ_u^2. Customers' orders are eventually batched and the total market

[5] Thus, Klein finds that in the 1980s the likelihood of intervention being reported when it actually occurred was 72 percent, while the likelihood of intervention actually occurring when it was reported was 88 percent. Hung finds that 40 percent of intervention operations conducted by the Federal Reserve was not reported in newspaper or newswire services in the same period.

order, $x_\tau = I_\tau + u_\tau$, is passed to the dealer, which then fixes the exchange rate and clears all the orders.

The structure of the foreign exchange market clearly does not correspond to that of the auction market *à la* Kyle (1985) described here, but there are several reasons that can justify its use in the present context. First, this framework is elegant and powerful, as simple analytical solutions are easily derived and have intuitive interpretations. Second, crucial characteristics of the market are naturally defined and the effects of foreign exchange intervention on the market performance, agents' beliefs and macroeconomic variables can be easily established. Finally, this batch framework captures the most important aspect of the foreign exchange market: its *lack* of transparency.

Thus, saying that in this framework the representative dealer cannot see the individual orders, I_0 and u_0, but just the total one, x_0, is equivalent to claiming that the *average* dealer in the market for foreign exchange cannot directly observe *all* the intervention operations of a central bank. In other words, the random orders the liquidity traders place can be considered as a proxy for the opaqueness of the foreign exchange market that prevents a precise estimation of the size of the intervention operations.

To establish the effect of these intervention operations we assume that the representative dealer is risk-neutral and that potential price competition from other dealers will force him to break-even, inducing a semi-strong form efficiency condition for the exchange rate. Thus, the exchange rate will be set equal to the expected fundamental value of the foreign currency, f, given the information the dealer obtains from the order flow, x_τ:[6]

$$s_\tau \;=\; E[f \,|\, x_\tau], \quad \tau = 0, 1.$$

For simplicity we assume that the dealer and the wage setters share the same prior on the value of the target level, $\bar{s}$. In addition, the dealer can observe the nominal wage and the money supply before fixing the spot rate in stage 1, s_1. On the other hand, both the wage setters and the monetary authorities can observe the exchange rate fixed by the dealer in stage 0, s_0, before selecting the nominal wage and the money supply in stage 1. In this way, at the end of stage 0 the dealer and the wage setters will share the same expectations of the spot rate target level, $\bar{s}$. Finally, to *isolate* the signalling role of foreign exchange intervention, we assume that all purchases and sales of the foreign currency by the central bank are automatically *sterilised* through compensatory open market operations, as it is generally the case in the practice of intervention.[7]

Under these assumptions, the fundamental value of the foreign currency corresponds to the

[6] We suppose that the two stages are close in time, so that we can disregard any discounting.

[7] Henceforth for foreign exchange intervention we will mean sterilised intervention operations in the market for foreign exchange.

equilibrium exchange rate after the nominal wage and the money supply have been fixed:

$$f = m_1.$$

However, the money supply, m_1, depends on the nominal wage set by the wage setters in stage 1. This on turn is function of their expectations on the target level, $\bar{s}$. Now by trading in stage 0, the central bank will be able to influence the expected value of $\bar{s}$ and hence the same fundamental value, f. This is an important conclusion, because it is commonly argued that, in the absence of portfolio balance effects, sterilised foreign exchange intervention cannot alter the fundamental value of a foreign currency.[8]

C. The Objectives of Foreign Exchange Intervention

Now, since in stage 1 all uncertainty on the fundamental components of the exchange rate is resolved, foreign exchange intervention will not be useful and the central bank will concentrate its activity in stage 0. To determine what is the optimal intervention strategy in stage 0 we need to analyse the decision process which leads to intervention and its ultimate goals.

In this respect we need to stress that this decision process varies from country to country. In particular, in Japan the Minister of Finance is *sole* responsible for intervention in the market for foreign exchange, while the Bank of Japan acts *only* as an agent, carrying out the intervention operations selected by the Minister of Finance through its Foreign Exchange Fund Special account. In the United States instead intervention operations are the result of the initiative of the Treasury, in consultation with the Federal Reserve, and are carried out by the Federal Reserve in New York. This will split the cost of intervention with the Treasury drawing funds in equal shares from its SOMA (System Open Market Account) account and the Treasury's ESF (Exchange Stabilization Fund) account.

On the contrary, before the launch of the euro, in Germany the Minister of Finance would select the appropriate exchange rate regime. However, within that regime the Bundesbank had full jurisdiction over foreign exchange intervention and would use its own funds to finance it. With the launch of the euro these features have changed: in September 2000 the Ministers of Finance in Euroland called for some action before the European Central Bank intervened to sustain the value of the euro. However, afterwards the European Central Bank claimed, through its president, to have intervened of its own choice.

Given these different practices and the monetary independence that central banks possess in these countries, we can say that there exist important differences with respect to the division of

[8]Interestingly this fundamental value might be *indeterminate* if either no signalling equilibrium exists or multiple signalling equilibria prevail in stage 0.

responsibility over monetary policy and foreign exchange intervention. Then, we will be able to span these various institutional arrangements by considering two extreme cases: 1) that of *coordination* in which a single authority, such as the Bundesbank in the past, has full control on both policy instruments and coordinate them according to a unique set of goals; 2) that of *separation* in which the two policy instruments fall under the jurisdiction of two different governmental bodies, such as the Minister of Finance and the Bank of Japan in Japan, which will then choose their policies according to possibly different sets of goals.

To determine which are these sets of goals consider that foreign exchange intervention can be expensive. Authorities which are responsible for foreign exchange intervention will be accountable for the possible losses intervention brings about and will take them into account when selecting operations in the market for foreign exchange. Thus, in the case of *separation* while the monetary authorities will care *only* for the macroeconomic implications of their decisions, the governmental body that controls foreign exchange intervention will *also* be concerned with the profits and losses intervention generates. On the contrary, in the case of *coordination* the potential cost of foreign exchange intervention might also affect the monetary policy. Then, we assume that:

- In the case of *separation*, $\mathcal{S}_e$:

  1. The monetary authorities do not care for the cost intervention and choose the money supply, m_1, in order to minimise the expected value of the loss function, $\mathcal{L}_m$, given in (6).

  2. The governmental body controlling foreign exchange intervention, conventionally denoted as the central bank, shares information with the monetary authorities and chooses its intervention operation, I_0, in order to minimise the expected value of a modified loss function, $\mathcal{L}_b$, which captures both the macroeconomic objectives of the policy makers and the cost of intervention. This modified loss function is defined as follows:

$$\mathcal{L}_b \;=\; (n_1 \,-\, \bar{n})^2 \,+\, (s_1 \,-\, \bar{s})^2 \,+\, 2\,Q\,\mathcal{C}, \tag{11}$$

     where Q is a positive constant measuring the weight attached to the cost of intervention, $\mathcal{C}$, which is equal to $(s_0 - s_1)I_0$.[9]

- In the case of *coordination*, $\mathcal{C}_o$, the central authority selects both I_0 and m_1 in order to minimise the expected value of a modified loss function, $\mathcal{L}_a$, which respects definition (11).

[9] In effect, since we use natural logs, the difference $s_1 - s_0$ measures the log of the gross rate of return on the foreign currency between stage 0 and 1, $\ln(1+R)$. For R small we can use the approximation $\ln(1+R) \approx R$, so that $\mathcal{C}_o$ is an approximation of the cost of intervention.

Again, in the definition of the modified loss function we assume that its form and the value of Q are common knowledge.[10] Since comparing these two different institutional arrangements might have important normative implications we desire to analyse both. However, for exposition reasons we first study the separation case. We postpone the analysis of the coordination one to Section IV.

III. Equilibrium under Separation

We now study the consequences of foreign exchange intervention under separation: first, we will study the sequential equilibrium of the model by characterising the unique signalling equilibrium that exists in the market for foreign exchange in stage 0; second, we will study the macroeconomic implications of this equilibrium.

A. Signalling Equilibrium under Separation

To pin down the sequential equilibrium of this model consider that in stage 0 the dealer and the wage setters will obtain some information on the target level, $\bar{s}$, from the activity of intervention of the central bank. Then, at the end of stage 0 they will update their prior belief, $\bar{s}^e$, and obtain a new expected value for the target level, $\bar{s}_0^e$. Hence, the optimal money supply and nominal wage set in stage 1 will be different from those derived in Section II insofar the unconditional expectation for the target level, $\bar{s}^e$, differs from the conditional one, $\bar{s}_0^e$, as now equations (7) and (8) are replaced by the following:

$$w_1 = \bar{s}_0^e + \bar{n}, \tag{12}$$

$$m_1 = \bar{s} + \bar{n} - \frac{1}{2}(\bar{s} - \bar{s}_0^e). \tag{13}$$

In Figure 1 we present a time line summary of the structure of the model under separation.

By backward induction we can determine the optimal intervention strategy of the central bank and pricing policy of the dealer in stage 0 and hence complete the analysis of the sequential equilibrium of the model. Indeed, we can say that in the stage 0 the central bank and the dealer play a signalling game: the former places a market order, i.e. it intervenes in the foreign exchange market, in an attempt to manipulate the expectations of the dealer and minimise the expected value of its loss function, $\mathcal{L}_b$; the latter seeks, instead, to extract all possible information on the target level, $\bar{s}$, contained in the order flow he observes, x_0, in an attempt to correctly price the foreign currency.

[10]The particular value taken by Q might well be very small, in that when intervening the policy makes are not principally motivated by speculative purposes and may have access to large foreign exchange reserves. However, they could not sustain the political burden of infinite losses, a condition which is captured in the model by a value of Q strictly greater than zero.

Then, in the appendix we prove the following Proposition, that characterises the unique signalling equilibrium that exists in this market.

Proposition 1 *Under separation, there exists a unique signalling equilibrium in the market for foreign exchange in stage 0. In this equilibrium the central bank sends a signal on the target level,* $\bar{s}$*, with an intervention operation,* I_0*, while the dealer rationally updates his prior belief on the target level,* $\bar{s}$*, using all the information contained in the order flow he observes,* x_0*. The market order of the central bank and the expected target level of the dealer are:*

$$\begin{aligned} I_0 &= \beta_{\mathcal{S}_e} (\bar{s} - \bar{s}^e) - \theta_{\mathcal{S}_e} \bar{n}, \\ \bar{s}_0^e &= \bar{s}^e + \lambda_{\mathcal{S}_e} (x_0 + \theta_{\mathcal{S}_e} \bar{n}), \end{aligned}$$

where $\beta_{\mathcal{S}_e}$ *is the unique positive root of the following equation*

$$\sigma_u^2 (\sigma_u^2 Q + \Sigma \beta_{\mathcal{S}_e}) = Q \Sigma^2 \beta_{\mathcal{S}_e}^4, \tag{14}$$

and $\lambda_{\mathcal{S}_e}$ *and* $\theta_{\mathcal{S}_e}$ *are positive coefficients given in the Appendix.*

B. Foreign Exchange Intervention and Signalling

Proposition 1 proposes some interesting conclusions.

• Through its order the central bank can influence the expectations of the dealer and the wage setters and hence the exchange rate, s_0 and s_1, the nominal wage, w_1, and the employment level, n_1. The exact size of its order, I_0, need not be publicly announced and, indeed, any public statement on its value would be equivalent to one regarding the target level, as in equilibrium I_0 is a simple function of $\bar{s}$. This means that, as already established for the objectives of the policy makers, we can rule out precise statements on the intervention operation of the central bank as they would be equally not credible.[11]

• While Stein's work suggests that imprecise statements on the objectives of the policy makers could be useful and credible, signalling equilibria based on *cheap talk* are plagued by the problem of their multiplicity. This problem does not emerge when foreign exchange intervention is employed. This is because, differently from public statements, foreign exchange intervention represents a potentially *costly* instrument of communication. In effect, one should observe that only if Q is strictly larger than 0, i.e. only if the central bank cares for the cost of this instrument, it "means what it says" and we can have a proper equilibrium.

[11] Indeed, in the fall of 1985 the G-5 heavily intervened in the foreign exchange markets to signal the reference levels set at the Plaza meeting for the exchange rates between the dollar, the deutsche mark and the yen, without precise announcements on these values or on the intervention activity.

• Not always the central bank "tells the true". In fact, its market order contains two parts: a "true" signal component, $\beta_{\mathcal{S}_e}(\bar{s}-\bar{s}^e)$, that indicates the prediction error of the dealer and the wage setters, and a "false" signal term, $-\theta_{\mathcal{S}_e}\bar{n}$, through which the central bank tries to manipulate the expectations of the wage setters and stimulate the economy. Then, there can be instances in which the central bank buys (sells) the foreign currency when this is overvalued (undervalued), i.e. when $\bar{s} < \bar{s}^e$ ($\bar{s} > \bar{s}^e$). This means that at times foreign exchange intervention might be *inconsistent* with the monetary policy. In this respect, it is at least reassuring that, as documented by Dominguez (1992) and Kaminsky and Lewis (1996), in several occasions in the 1980s intervention operations in the United States were not coherent with subsequent open market operations.

• The dealer and the wage setters rationally update their beliefs using the information contained in the order flow. However, since they cannot directly observe the central bank's market order, I_0, they on average underestimate its size, as it is usually the case when intervention events are reported in the press.[12] Moreover, foreign exchange intervention and the exchange rate are not consistent when the central bank purchases (sells) foreign currency, $I_0 > 0$ ($I_0 < 0$), and the rest of the dealer's clients sells (buys) large quantities, $u_0 < 0$ ($u_0 > 0$) with $|u_0| > |I_0|$. Once again, this feature of our analysis is coherent with those many events in the 1980s and 1990s in which after an intervention operation the exchange rate did not move in the direction of intervention.

• Foreign exchange intervention more than *halves* the uncertainty of the dealer and the wage setters on the target level, $\bar{s}$. In fact, let $\Sigma_{0|\mathcal{S}_e}$ denote the variance of the target level conditional on the information contained in the order flow, x_0. Then, applying the projection theorem for normal distributions, we find that this *efficiency coefficient* is:

$$\Sigma_{0|\mathcal{S}_e} \equiv \operatorname{Var}[\bar{s} \mid x_0] = (1 - \lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})\,\Sigma,$$

where $(1-\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e}) \in (0, 1/2)$ and Σ is the unconditional variance of the target level, $\bar{s}$, i.e. the level of uncertainty on the value of $\bar{s}$ in the absence of intervention. This result has important stabilising consequences for the economy. Also notice that the *liquidity coefficient*, $\lambda_{\mathcal{S}_e}$, measures the transaction costs imposed by the dealer in the market for foreign exchange. These transaction costs are a function of the adverse selection that the dealer faces. We will in fact see that through its intervention activity the central bank gains positive profits at the expenses of the dealer. Then, the dealer will be able to break-even only by charging a transaction fee to all his clients.

[12] Thus, let I_0^e denote the expected intervention size given the information the dealer and the wage setters extract from the order flow, x_0. Then, we can prove that given the information possessed by the central bank, the absolute expected value of I_0^e is smaller than the actual size of intervention, i.e. $|E[I_0^e \mid I_0]| < |I_0|$.

C. Policy Gain and Macroeconomic Stability

We have seen that the central bank's market order, I_0, contains both a "true" signal component, $\beta_{\mathcal{S}_e}(\bar{s}-\bar{s}^e)$, and a "false" signal one, $-\theta_{\mathcal{S}_e}\bar{n}$, through which the central bank tries to manipulate the expectations of the wage setters in an attempt to stimulate the economy. However, this attempt is frustrated: the dealer and the wage setters are capable of filtering out the "false" signal component, $-\theta_{\mathcal{S}_e}\bar{n}$, and hence extract the relevant information from the order flow, x_0, to properly estimate the target level, $\bar{s}$. Indeed, the unconditional expectations of the exchange rate and the employment level correspond to the equilibrium values that prevail when the central bank does *not* intervene in the foreign exchange market. Formally one can prove that $E_\emptyset[s_1] = E_{\mathcal{S}_e}[s_1]$ and $E_\emptyset[n_1] = E_{\mathcal{S}_e}[n_1]$.

This suggests that there is *no* systematic policy gain from foreign exchange intervention, because the central bank is able neither to *fool* the wage setters and stimulate the economy, nor to *reduce* the misalignement of the exchange rate. One might then be drawn to the conclusion that foreign exchange intervention is useless, as it fails to achieve the goals it is aimed for. However, by reducing the volatility of the employment level, n_1, foreign exchange intervention contributes to stabilise the economy and reduce the expected losses of the policy makers. This is formally stated in the following Proposition.

Proposition 2 *Under separation, when the central bank intervenes in the market for foreign exchange: 1) the unconditional variance of the employment level more than halves and 2) the unconditional expected losses of the monetary authorities fall, in that:*

$$\mathrm{Var}_{\mathcal{S}_e}[n_1] \ < \ \frac{1}{2}\mathrm{Var}_\emptyset[n_1], \qquad E_{\mathcal{S}_e}[\mathcal{L}_m] \ < \ E_\emptyset[\mathcal{L}_m].$$

In practice, by reducing the uncertainty of the wage setters on the objectives of the policy makers, the central bank drastically reduces the probability of extreme events, in which the wage setters greatly overestimate or underestimate the target level for the exchange rate and in which the employment level dramatically swings away from its natural level. Then, given the *convexity* of the loss function, $\mathcal{L}_m$, the monetary authorities benefit from the reduction in the volatility of the employment level and in the probability of extreme outcomes.[13]

Notice that if the policy makers could commit to reveal the true target level, $\bar{s}$, the fluctuations in the employment level induced by the uncertainty on their objectives could be completely eliminated and there would be *no* role for foreign exchange intervention. Indeed, we can easily see that under

[13] While to our knowledge there is no empirical investigation on the effects of foreign exchange intervention on employment, an indirect confirmation of Proposition 2 is some evidence by Hung (1997) and Dominguez (1998) that foreign exchange intervention increases exchange rate volatility, as we can prove that within this model a byproduct of the intervention activity is an increase in the unconditional variance of the exchange rate.

true revelation, $\mathcal{R}_e$, foreign exchange intervention is not required and the expected losses of the monetary authorities are *smaller* than those obtained when the central bank intervenes in the market for foreign exchange under discretion, $E_{\mathcal{R}_e}[\mathcal{L}_m] < E_{\mathcal{S}_e}[\mathcal{L}_m]$. This suggests that the policy makers had better commit to truly reveal their policy objectives than trying to manipulate agents' beliefs through foreign exchange intervention.

However, while desirable, true revelation of the policy makers' objectives is not implementable, as the policy makers possess an incentive to lie. On the other hand, the sequential equilibrium with foreign exchange intervention that we have characterised is fully time consistent. Therefore, through Proposition 2 we can argue that, when a commitment technology does not exist, intervention operations can supplement the lack of credibility of the policy makers and represent a useful policy instrument.

In addition to the stability gain foreign exchange intervention also induces speculative profits for the central bank. This is not in contradiction with the claim that since it is potentially expensive foreign exchange intervention: i) buys credibility and ii) guarantees a unique signalling equilibrium. Indeed, two operations of different size will imply two different, even if possibly negative, intervention costs. Then, the central bank will be induced to discriminate and pass to the dealer a unique informative market order.

However, while in some instances the central bank will actually incur in intervention losses, on average it gains profits exploiting its superior information. In fact, leaving aside the "false" signal component, $-\theta_{\mathcal{S}_e}\bar{n}$, we can see that the central bank will buy (sell) the foreign currency to signal a greater (smaller) than expected target level for the spot rate. That is, it will buy (sell) when the foreign currency is undervalued (overvalued). Then, since $\bar{s}$ and $\bar{n}$ are uncorrelated, on average the central bank makes some profits, as formally stated in this Proposition.

Proposition 3 *Under separation, the unconditional expected profits the central bank gains from trading in the foreign exchange market, $E_{\mathcal{S}_e}[\pi]$, are positive and increasing in: i) the volume of liquidity trading in the market for foreign exchange, measured by σ_u^2, and ii) the transaction costs the dealer charges, as measured by the liquidity coefficient, $\lambda_{\mathcal{S}_e}$, i.e.:*

$$E_{\mathcal{S}_e}[\pi] \;=\; \frac{1}{2}\lambda_{\mathcal{S}_e}\,\sigma_u^2.$$

This result is coherent with: i) Friedman's prescription that foreign exchange intervention should be equivalent to stabilising speculation, and hence profitable, and ii) some recent empirical results, notably Sweeney (1997, 2000), which show that, at least in the medium run, the risk-adjusted profits central banks obtain when intervening in the foreign exchange market are positive.

IV. Equilibrium under Coordination

Let us now consider the case in which there exists a unique central authority, which operates in both the foreign exchange and monetary markets. We first determine the new sequential equilibrium of the model, by characterising the unique signalling equilibrium that exists in the market for foreign exchange in stage 0. We will then study the macroeconomic implications of this new equilibrium and compare them to those prevailing under separation.

A. Signalling Equilibrium under Coordination

With respect to the equilibrium under separation, the central authority will now consider that changes in the money supply in stage 1 affects the cost of intervention, $\mathcal{C}$, and the loss function $\mathcal{L}_a$. Then, solving for the discretionary equilibrium in stage 1, we find that the equilibrium values for the nominal wage, w_1, and the money supply, m_1, are equal to:

$$w_1 = \bar{s}_0^e + \bar{n} + Q\, I_0^e, \tag{15}$$

$$m_1 = \bar{s} + \bar{n} + Q\, I_0 - \frac{1}{2}(\bar{s} - \bar{s}_0^e) - \frac{Q}{2}(I_0 - I_0^e), \tag{16}$$

where I_0^e indicates the expected value of the intervention operation in the market for foreign exchange, given the information the dealer and the wage setters possess at the end of stage 0.

Equation (16) indicates that when monetary and foreign exchange intervention are coordinated the latter will have a feed-back effect on the former through *two* different channels. First, we have the usual expectations channel, in that the signal the central authority passes to the dealer in stage 0 conditions the wage setters' expectations of the future money supply and hence the nominal wage, w_1. This will then induce the central authority to revise the optimal value of the money supply, as indicated by the fourth component of equation (16). This term also appears under separation, as indicated by equation (13).

Second, a new channel emerges through the cost of intervention. In fact, the value of the money supply fixed in stage 1, m_1, determines the exchange rate in stage 1, s_1, and hence the cost of intervention, $\mathcal{C}$. Such cost will alter the choice of the money supply, m_1, and the same nominal wage, w_1. Indeed, with respect to the formulation obtained under separation, we have two extra terms in equation (16), QI_0 and $Q(I_0^e - I_0)/2$, function respectively of the actual value of intervention and its unexpected component.

The presence of this second channel of transmission will also affect the value of the central authority's losses, $\mathcal{L}_a$. This is equal to:

$$\mathcal{L}_a = \mathbf{l}_i \equiv \mathbf{l}_c - \mathbf{g}_a,$$

where the gain from manipulation is now given by the following expression:

$$\mathbf{g}_a \equiv \left\{ 2\,\Delta\,\bar{n} \;+\; 2Q\,(\Delta \,-\, \bar{n})\,I_0 \;-\; \frac{1}{2}\,\Delta^2 \;-\; Q^2\,I_0^2 \right\},$$

$$\text{with} \qquad \Delta \;\equiv\; (\bar{s} \,-\, \bar{s}_0^e) \;+\; Q\,(I_0 \,-\, I_0^e).$$

Once again, the term Δ can be considered a measure of the ignorance of the wage setters, as it is a function of their prediction errors with respect to the intervention operation and the target level. Since $\mathbf{g}_a$ can be positive for some values of Δ, the central authority might attempt to exploit the wage setters' ignorance and manipulate their beliefs in order to reduce the value of its losses. Then, proceeding as in Section III we can determine that a new unique signalling equilibrium will prevail in the market for foreign exchange, as established in the following Proposition.

Proposition 4 *Under cooperation, there exists a unique linear signalling equilibrium in stage 0, in which the central authority sends a signal on the target level, $\bar{s}$, with an intervention operation, I_0, while the dealer rationally updates his prior beliefs on the target level, $\bar{s}$, and the intervention operation, I_0, using all the information contained in the order flow he observes, x_0. The market order of the central authority and the dealer's expected values for the target level and the intervention operation are:*

$$\begin{aligned} I_0 &= \beta_{\mathcal{C}_o}\,(\bar{s} \,-\, \bar{s}^e) \,-\, \theta_{\mathcal{C}_o}\,\bar{n}, \\ \bar{s}_0^e &= \bar{s}^e \,+\, \lambda_{\mathcal{C}_o}\,(x_0 \,+\, \theta_{\mathcal{C}_o}\,\bar{n}), \\ I_0^e &= \beta_{\mathcal{C}_o}\,\lambda_{\mathcal{C}_o}\,(x_0 \,+\, \theta\bar{n}) \,-\, \theta_{\mathcal{C}_o}\bar{n}, \end{aligned}$$

where $\beta_{\mathcal{C}_o}$ is the unique positive root of the following equation

$$\sigma_u^2\,(\sigma_u^2\,Q \,+\, \Sigma\,\beta_{\mathcal{C}_o}) \;=\; 2\,Q\,\Sigma^2\,\beta_{\mathcal{C}_o}^4, \tag{17}$$

and $\lambda_{\mathcal{C}_o}$ and $\theta_{\mathcal{C}_o}$ are positive coefficients given in the appendix.

Proposition 4 confirms the first claim of our analysis: insofar we can employ an expensive instrument of communication, such as foreign exchange intervention, a unique signalling equilibrium exists. Indeed, we argue that this is a robust result, irrespective of either the specific arrangements concerning the control of foreign exchange intervention or the details of the monetary policy model. Likewise, whilst we can again prove that foreign exchange intervention does not produce a systematic policy gain, in the form of an increase in employment or a reduction in the inflationary bias, we still observe a significant reduction in the uncertainty of the wage setters on the objectives of the policy makers. In particular, the conditional variance of the target level, s_1, given the information the wage setters possess at the end of stage 0, $\Sigma_{0|\mathcal{C}_o}$, is always smaller than $(\sqrt{2}/(\sqrt{2}+1))\Sigma$, with a reduction of at least 40% from the unconditional value, Σ.

However, comparing equations (14) and (17) one can immediately see that the value of β, the intensity of trading against the prediction error of the dealer and the wage setters, $\bar{s} - \bar{s}_0^e$, is smaller in the case of coordination than in that of separation, in that $\beta_{\mathcal{C}_o} > \beta_{\mathcal{S}_e}$. This means that the central authority will now place a less informative market order. This result has important implications for the macroeconomic consequences of foreign exchange intervention.

B. Coordination, Intervention Profits and Stability Gain

Comparing equations (14) and (17) we can obtain the following results:

Proposition 5 *With respect to the equilibrium under separation, coordination of foreign exchange intervention and monetary policy will result in: i) less information on the target level,* s_1*, revealed through foreign exchange intervention, in that* $\Sigma_{0|\mathcal{C}_o} > \Sigma_{0|\mathcal{S}_e}$*; ii) a more volatile economy, as the unconditional variance of the employment level,* n_1*, is larger under the former institutional arrangement,* $\text{Var}_{\mathcal{C}_o}[n_1] > \text{Var}_{\mathcal{S}_e}[n_1]$*; iii) and larger unconditional expected profits for the central bank,* $E_{\mathcal{C}_o}[\pi] > E_{\mathcal{S}_e}[\pi]$.

This Proposition outlines an existing trade off between the stability gain that a more informative signal may induce and the profits that a more conservative intervention strategy brings about. When foreign exchange intervention and monetary policy are coordinated more weight is attached to the cost of intervention, so not surprisingly in this case larger expected profits are obtained at the expenses of a less stable economy. However, a very important and surprising conclusion of our analysis is that we can *no* longer claim that under cooperation a stability gain from foreign exchange intervention is always obtained.

Indeed, for small values of Q, the weight attached to the cost of intervention, and σ_u^2, the volume of noise trading in the foreign exchange market, the unconditional variance of the employment level, $\text{Var}_{\mathcal{C}_o}[n_1]$, is smaller than the equivalent value for the equilibrium without intervention, $\text{Var}_{\emptyset}[n_1]$. However, we can conceive extreme scenarios in which intervention operations in the market for foreign exchange actually *increase* the volatility of the employment level. As an example, assume that $Q = (\Sigma/\sigma_u^2)^{1/2}$. Then, it is not difficult to prove that $\text{Var}_{\mathcal{C}_o}[n_1] = \Sigma/2$, which is actually *twice* the volatility of the employment level in the equilibrium without intervention, i.e. $\text{Var}_{\mathcal{C}_o}[n_1] = 2\text{Var}_{\emptyset}[n_1]$.

To explain this potential perverse effect of foreign exchange intervention let us return to equation (16), which contains two extra terms, QI_0 and $Q(I_0^e - I_0)/2$, with respect to the equilibrium under separation. The latter will now appear in the equilibrium expression of the employment level, n_1, and represents an extra source of variability for the employment level. This source turns out to be prominent when either the volume of liquidity trading in the foreign exchange market, σ_u^2, or the weight the central authority attaches to the cost of intervention, Q, is large.

In particular, consider large values of σ_u^2, the volume of liquidity trading in the market for foreign exchange that we take to represent its lack of transparency. Large values of σ_u^2 will force the central authority to employ more aggressive trading strategies in its attempt to manipulate the wage setters' beliefs. Then, given the larger volume of noise in the market for foreign exchange, the prediction error of the dealer and the wage setters, $I_0 - I_0^e$, will increase, inducing a more volatile monetary shock, $m_1 - m_1^e$, and employment level, n_1. Likewise, in the latter scenario, when a larger weight is attached to the cost of intervention, the central authority augments the volatility of the monetary policy and the employment level to gain larger profits. Indeed, its profits, π, are a *convex* function of the monetary shock, $m_1 - m_1^e$. Then, via Jensen's inequality we know that a more volatile monetary shock will deliver larger expected profits.

These can be considered extreme scenarios, as we may argue that in reality the actual values of Q and σ_u^2 are small. Yet, they reveal a dangerous risk in maintaining monetary policy and foreign exchange intervention under the jurisdiction of the same governmental body. This is an important conclusion, in that the specific institutional arrangements that govern these two policy instruments differ among countries and have so far been overlooked. Instead, our analysis clearly indicates that the division of power over monetary policy and foreign exchange intervention is important and should be carefully considered. For this very reason we now turn to a brief comparative analysis exercise.

C. Comparative Analysis

We now discuss some interesting comparative statics results, investigating the characteristics and properties of the equilibria as we move Q, the weight attached to the cost of intervention, and σ_u^2, the volume of liquidity trading in the market for foreign exchange. Then, with respect to changes in the value of Q we can establish as follows.

Proposition 6 *Both under coordination and separation, the conditional variance of the target level ($\Sigma_{0|\mathcal{C}_o}$ and $\Sigma_{0|\mathcal{S}_e}$), the unconditional variance of the employment level ($\mathrm{Var}_{\mathcal{C}_o}[n_1]$ and $\mathrm{Var}_{\mathcal{S}_e}[n_1]$), the unconditional profits of foreign exchange intervention ($E_{\mathcal{C}_o}[\pi]$ and $E_{\mathcal{S}_e}[\pi]$) and the unconditional losses of the policy makers ($E_{\mathcal{C}_o}[\mathcal{L}_a]$ and $E_{\mathcal{S}_e}[\mathcal{L}_m]$) are all increasing in Q, the weight attached to the cost of intervention.*

Both under coordination and separation, the conditional variance of the target level, the unconditional variance of the employment level, alongside with the unconditional expected profits of intervention and the unconditional expected losses of the policy makers converge to limit values when either $Q \uparrow \infty$ or $Q \downarrow 0$. In particular:

1. *For $Q \uparrow \infty$: i) $\Sigma_{0|\mathcal{C}_o} \uparrow (\sqrt{2}/(\sqrt{2}+1))\Sigma$, while $\mathrm{Var}_{\mathcal{C}_o}[n_1]$, $E_{\mathcal{C}_o}[\pi]$ and $E_{\mathcal{C}_o}[\mathcal{L}_a]$ all converge to infinite; ii) $\Sigma_{0|\mathcal{S}_e} \uparrow (1/2)\Sigma$, $\mathrm{Var}_{\mathcal{S}_e}[n_1] \uparrow (1/8)\Sigma$, $E_{\mathcal{S}_e}[\pi] \uparrow (1/4)(\sigma_u^2\Sigma)^{1/2}$ and $E_{\mathcal{S}_e}[\mathcal{L}_m] \uparrow (1/4)\Sigma + 2\bar{n}^2$.*

2. *For $Q \downarrow 0$: $\Sigma_{0|\Xi}$, $\mathrm{Var}_\Xi[n_1]$ and $E_\Xi[\pi]$ all converge to 0, while $E_\Xi[\mathcal{L}_a]$ converges $2\bar{n}^2$, when i) $\Xi = \mathcal{C}_o$ and ii) $\Xi = \mathcal{S}_e$.*

A graphical representation of this Proposition may help interpreting its results. Thus, in Figure 2 various characteristics of the equilibrium under coordination and separation are presented against different values of Q.[14] Then, we immediately see that Proposition 6 underlines the existing trade off between the profitability of intervention and the stability gain it brings about. Indeed, under both institutional arrangements, for a larger weight attached to the cost of intervention, larger expected profits are obtained by reducing the informational content of the intervention operation, at the expenses of a more volatile employment level.

While these monotonicity results are quite intuitive, the limiting behaviour of the characteristics of the equilibrium is more surprising. In particular, when nearly no weight is attached to the cost of intervention, $Q \downarrow 0$, the residual uncertainty of the dealer and the wage setters nearly vanishes, as $\Sigma_{0|\Xi} \downarrow 0$ for $\Xi = \mathcal{C}_o$ and $\Xi = \mathcal{S}_e$, so that in the limit we approach a set of equilibria which are nearly *equivalent*, even if not *equal*, to that prevailing when the policy makers commit to the true revelation of the target level, $\bar{s}$, and do not undertake foreign exchange intervention.

In practice, this shows a discontinuity, as for Q *exactly* null we would not have a signalling equilibrium in the market for foreign exchange. Indeed, values of Q very close to zero are unrealistic, as a very small weight attached to the cost of intervention would imply a willingness on the part of the policy makers to consume all their reserves, an hypothesis that many would find hard to accept.

Proposition 6 also restates a fundamental difference between the two institutional arrangements we have already discussed. In particular, in the equilibrium under separation there are *always* benefits from foreign exchange intervention, in terms of both larger macroeconomic stability and smaller losses for the policy makers. This is no longer the case under coordination, as we see that for some values of Q both the unconditional variance of the employment level and the expected losses of the central authority are *larger* when foreign exchange intervention is undertaken than when it is not, i.e. $\mathrm{Var}_{\mathcal{C}_o}[n_1] > \mathrm{Var}_\emptyset[n_1]$ and $E_{\mathcal{C}_o}[\mathcal{L}_a] > E_\emptyset[\mathcal{L}_a]$. Therefore, under these circumstances this instrument of policy making might even be counterproductive.[15]

[14] In the construction of the panels presented in this Figure and the following one the actual values of the coefficients that characterise the equilibria, specifically β_Ξ and λ_Ξ for $\Xi = \mathcal{C}_o$ and $\Xi = \mathcal{S}_e$, are obtained numerically.

[15] To be absolutely precise, under separation, when the central bank just intervenes to profit from its superior

Proposition 7 completes our comparative statics analysis with respect to changes in the value of σ_u^2. Once more, Figure 3 may help interpret its results more effectively.

Proposition 7 *Both under coordination and separation, the conditional variance of the target level ($\Sigma_{0|\mathcal{C}_o}$ and $\Sigma_{0|\mathcal{S}_e}$), the unconditional variance of the employment level ($\mathrm{Var}_{\mathcal{C}_o}[n_1]$ and $\mathrm{Var}_{\mathcal{S}_e}[n_1]$) and the unconditional profits of foreign exchange intervention ($E_{\mathcal{C}_o}[\pi]$ and $E_{\mathcal{S}_e}[\pi]$) are all increasing in σ_u^2, the volume of liquidity trading in the market for foreign exchange.*

Both under coordination and separation, the conditional variance of the target level, the unconditional variance of the employment level, alongside with the unconditional expected profits of intervention and the unconditional expected losses of the policy makers converge to limit values when $\sigma_u^2 \uparrow \infty$ or $\sigma_u^2 \downarrow 0$. In particular:

1. *For $\sigma_u^2 \uparrow \infty$: i) $\Sigma_{0|\mathcal{C}_o} \uparrow (\sqrt{2}/(\sqrt{2}+1))\Sigma$, while $\mathrm{Var}_{\mathcal{C}_o}[n_1]$ and $E_{\mathcal{C}_o}[\pi]$ all converge to infinite; ii) $\Sigma_{0|\mathcal{S}_e} \uparrow (1/2)\Sigma$, $\mathrm{Var}_{\mathcal{S}_e}[n_1] \uparrow (1/8)\Sigma$, $E_{\mathcal{S}_e}[\pi] \uparrow (1/4)(\sigma_u^2\Sigma)^{1/2}$ and $E_{\mathcal{S}_e}[\mathcal{L}_m] \uparrow (1/4)\Sigma + 2\bar{n}^2$.*
2. *For $\sigma_u^2 \downarrow 0$: $\Sigma_{0|\Xi}$, $\mathrm{Var}_{\Xi}[n_1]$ and $E_{\Xi}[\pi]$ all converge to 0, when i) $\Xi = \mathcal{C}_o$ and ii) $\Xi = \mathcal{S}_e$. Moreover, $E_{\mathcal{S}_e}[\mathcal{L}_m] \downarrow 2\bar{n}^2$, while $E_{\mathcal{C}_o}[\mathcal{L}_a] \downarrow (2+Q)\bar{n}^2$.*

Proposition 7 shows some intuitive conclusions: a larger volume of liquidity trading, or equivalently a less transparent market for foreign exchange, reduces the dealer's ability to extract information from his order flow and the efficiency of the market, while augmenting the profitability of intervention and the volatility of the employment level. Moreover, a more transparent market also implies smaller expected losses for the monetary authorities. Therefore, we can advocate the use of foreign exchange intervention during periods of higher transparency of the foreign exchange market.

Finally, notice that Proposition 7 also reinforces our original claim that the lack of transparency is fundamental for the effectiveness of foreign exchange intervention. Indeed, even for $\sigma_u^2 \downarrow 0$ we approach a set of equilibria which are nearly *equivalent* to that in which the policy makers commit to truly reveal the target level, $\bar{s}$, suggesting that in a perfectly transparent market we could not have a signalling equilibrium with foreign exchange intervention.

information, i.e. for $Q \uparrow \infty$, the monetary authorities will not benefit at all from foreign exchange intervention, i.e. in the limit $E_{\mathcal{S}_e}[\mathcal{L}_m] = E_{\emptyset}[\mathcal{L}_m]$. Indeed, while foreign exchange intervention reduces the volatility of the employment level, it also increases that of the exchange rate. Whilst the former effect would in general dominate the latter, when $Q \uparrow \infty$ the two will exactly offset each other.

V. Concluding Remarks

Recently growing interest has emerged for i) the microstructure of the market for foreign exchange and ii) the impact of foreign exchange intervention on market sentiment and exchange rates. Within this strand of the literature, we have investigated the signalling role of foreign exchange intervention and its macroeconomic implications, employing an analytical framework which combines a simple monetary policy model *à la* Barro-Gordon with a market microstructure framework for the foreign exchange market.

At the microstructural level this framework allows to: i) isolate the informative role of the order flow in the market for foreign exchange, ii) identify a clear link between intervention operations, agents' expectations and exchange rates, and iii) reproduce a series of stylesed facts that concern the statistical relations between intervention operations, monetary aggregates and exchange rates. At the macro level it allows to show how foreign exchange intervention can be exploited to i) address the lack of credibility of the policy makers and ii) reduce the instability of employment and output.

In particular, assuming that the policy makers possess some private information on the objectives of their monetary policy, we see that they can benefit from the manipulation of the wage setters' beliefs, increasing the employment level and/or reducing the inflationary bias. In these circumstances precise announcements on the objectives of the monetary policy will lack credibility, whilst this can be acquired if an expensive signalling device is employed.

In this respect, foreign exchange intervention represents a genuine signalling device, because the central bank stakes its own capital in support of its signals. Indeed, since it is potentially expensive foreign exchange intervention also possesses an edge with respect to *cheap talk*. In fact, while for the former instrument of policy making a unique equilibrium exists, when imprecise announcements are employed multiple equilibria generally emerge.

However, if foreign exchange intervention can "buy" credibility, this cannot be "spent" to acquire a systematic policy gain, as originally sought by the policy makers. Indeed, while in the first place intervention activity is aimed at stimulating the economy and taming the inflationary bias, none of these goals is achieved. Foreign exchange intervention will instead benefit the policy makers through a *second* moment effect, in that, when conducted by an autonomous institution, it reduces the volatility of the employment level and hence helps stabilising the economy. This is an orthogonal conclusion to the contribution of Rogoff (1985), who suggests that a conservative independent central bank is capable of reducing the inflationary bias due to the lack of credibility of the policy makers, but at the cost of a more unstable economy.

The reduction in the amplitude of the fluctuations of the employment level is also accompanied by positive profits from the intervention activity. However, a trade off between these two effects exists. Indeed, when the policy makers attach a larger weight to the cost of intervention, less information will be revealed and more volatile the employment level will be. This trade off also implies that the division of responsibility over monetary policy and foreign exchange intervention influences the effectiveness of these policy instruments. In other words, a normative conclusion of our analysis is that the specific institutional arrangements concerning the control of foreign exchange intervention and monetary policy are not inconsequential and should not be overlooked.

VI. Appendix

Proof of Proposition 1.
Assume first that $I_0 = \beta_{\mathcal{S}_e}(\bar{s} - \bar{s}^e) - \theta_{\mathcal{S}_e}\bar{n}$, where $\beta_{\mathcal{S}_e}$ and $\theta_{\mathcal{S}_e}$ are non-negative constants. Then, given the assumption of normality for $\bar{s}$ and u_0, we can apply the projection theorem for normal distributions and find that:

$$\bar{s}_0^e \equiv E[\bar{s} \mid x_0] = \bar{s}^e + \lambda_{\mathcal{S}_e}(x_0 + \theta_{\mathcal{S}_e}\bar{n}),$$
$$\text{where} \quad \lambda_{\mathcal{S}_e} = \frac{\beta_{\mathcal{S}_e}\Sigma}{\beta_{\mathcal{S}_e}^2\Sigma + \sigma_u^2}.$$

Suppose, then, that the dealer updates his expectations of the target level for the spot rate according to the following expression $\bar{s}_0^e = \bar{s}^e + \lambda_{\mathcal{S}_e}(x_0 + h)$, where h is some constant. Inserting this expression for the expected target level, $\bar{s}_0^e = \bar{s}^e + \lambda_{\mathcal{S}_e}(x_0 + h)$, in equations (12) and (13), considering that $s_0 = w_1$ and $s_1 = m_1$, we find relations between the market order of the central bank, the spot rates in stages 0 and 1, and the employment level in stage 1. Then, plugging the new expressions for s_0, s_1, n_1 in the modified loss function (11), taking its expectation with respect to the central bank's information and minimising with respect to I_0, we obtain, after some tedious but simple algebra, that the central bank's optimal order is:

$$I_0 = \beta_{\mathcal{S}_e}(\bar{s} - \bar{s}^e) - \theta_{0|\mathcal{S}_e}\bar{n} - \beta_{\mathcal{S}_e}\lambda_{\mathcal{S}_e}h,$$
$$\text{where} \quad \beta_{\mathcal{S}_e} = \frac{1}{\lambda_{\mathcal{S}_e}}\frac{\lambda_{\mathcal{S}_e} + Q}{\lambda_{\mathcal{S}_e} + 2Q}, \quad \theta_{0|\mathcal{S}_e} = \frac{2}{\lambda_{\mathcal{S}_e} + 2Q}.$$

Notice that to have a Nash equilibrium the central bank's market order and the expectations of the dealer must be consistent. This is the case if two conditions are met. First, the constants h, $\theta_{0|\mathcal{S}_e}$ and $\beta_{\mathcal{S}_e}$ are such that: $h = \theta_{0|\mathcal{S}_e}\bar{n} + \beta_{\mathcal{S}_e}\lambda_{\mathcal{S}_e}h$. This implies that: $h = \theta_{\mathcal{S}_e}\bar{n}$ with $\theta_{\mathcal{S}_e} = 2/Q$. Second, $\beta_{\mathcal{S}_e}$ and $\lambda_{\mathcal{S}_e}$ solve simultaneously the system of equations:

$$\beta_{\mathcal{S}_e} = \frac{1}{\lambda_{\mathcal{S}_e}}\frac{\lambda_{\mathcal{S}_e} + Q}{\lambda_{\mathcal{S}_e} + 2Q}, \qquad \lambda_{\mathcal{S}_e} = \frac{\beta_{\mathcal{S}_e}\Sigma}{\beta_{\mathcal{S}_e}^2\Sigma + \sigma_u^2}. \tag{18}$$

Substituting the expression for $\lambda_{\mathcal{S}_e}$ into that for $\beta_{\mathcal{S}_e}$ we obtain equation (14). This presents two solutions, one positive and one negative. However, it is not difficult to see that only the positive one satisfies the second order condition of the central bank's optimisation problem, $\lambda_{\mathcal{S}_e}(\lambda_{\mathcal{S}_e} + 2Q) > 0$. To complete the prove notice that we have characterised the unique *linear* equilibrium.

Proof of Proposition 2.

First, consider that from the expression for $\lambda_{\mathcal{S}_e}$ in (18) we see that the liquidity coefficient has maximum for $\beta_{\mathcal{S}_e} = \bar{\beta} \equiv (\sigma_u^2/\Sigma)^{1/2}$ equal to $\bar{\lambda} \equiv (\Sigma/\sigma_u^2)^{1/2}/2$. Second, consider that $n_1 = m_1 - w_1$. From equations (12) and (13) we find that: $n_1 = (\bar{s} - \bar{s}_0^e)/2$. Thus, inserting the expression for the expected target level, s_0^e, given in Proposition 1, we find that:

$$n_1 \;=\; \frac{1}{2}\Bigg((1 - \lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})\,(\bar{s} \,-\, \bar{s}^e) \,-\, \lambda_{\mathcal{S}_e}\,u_0\Bigg). \tag{19}$$

Then, if we take the unconditional variance of n_1 when the central bank intervenes in the foreign exchange market, we find the following:

$$\mathrm{Var}_{\mathcal{S}_e}[n_1] \;=\; \frac{1}{4}\Bigg((1 \,-\, \lambda_{\mathcal{S}_e}\,\beta_{\mathcal{S}_e})^2\,\Sigma \,+\, \lambda_{\mathcal{S}_e}^2\sigma_u^2\Bigg). \tag{20}$$

Considering the expression for $\beta_{\mathcal{S}_e}$ in (18), we find that: $1-\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e} = Q/(\lambda_{\mathcal{S}_e}+2Q) < 1/2$. Finally, noticing that $\lambda_{\mathcal{S}_e}$ is smaller than $\bar{\lambda}$, we find that $\mathrm{Var}_{\mathcal{S}_e}[n_1] < \Sigma/8$. This completes the proof of the first part of the statement, as the unconditional variances of n_1 when the central bank does not intervene is $\Sigma/4$. As for the second part of the statement, consider that $s_1 = m_1$. Then, combining equation (13) with the expression for the expected target level, s_0^e, given in Proposition 1, we find that: $s_1 = \bar{s}+\bar{n}+[(1-\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})(\bar{s}-\bar{s}^e)-\lambda_{\mathcal{S}_e}u_0]/2$. Thus, given this expression and that for the employment level, one can easily check that: $E_{\mathcal{S}_e}[\mathcal{L}_m] = 2\mathrm{Var}_{\mathcal{S}_e}[n_1] + 2\bar{n}^2$. The equivalent expected loss function when the central bank does not intervene can be calculated repeating the same steps for $\lambda_{\mathcal{S}_e} = 0$. This gives: $E_{\emptyset}[\mathcal{L}_m] = \Sigma/2 + 2\bar{n}^2$. Then, given the result for the variance of the employment level, we find that: $E_{\mathcal{S}_e}[\mathcal{L}_m] < E_{\emptyset}\,[\mathcal{L}_m]$.

Proof of Proposition 3.

Consider first the change in the spot rate between stage 0 and 1. This is given by the following expression: $s_1 - s_0 = m_1 - n_1 = [(1-\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})(\bar{s} - \bar{s}^e) - \lambda_{\mathcal{S}_e}u_0]/2$. Then, considering equation (19) and given that $I_0 = \beta_{\mathcal{S}_e}(\bar{s} - \bar{s}^e) - \theta\bar{n}$, we find that: $\pi = \pi_1 + \pi_2$, where $\pi_1 = (1-\lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})\beta_{\mathcal{S}_e}(\bar{s} - \bar{s}^e)^2/2$ and π_2 is linear function of the cross products of $(\bar{s}-\bar{s}^e)$, $\bar{n}$ and u_0. Given the assumption of independence, the unconditional expected value of π is:

$$E_{\mathcal{S}_e}[\pi] \;=\; \frac{1}{2}\beta_{\mathcal{S}_e}\,(1 \,-\, \lambda_{\mathcal{S}_e}\,\beta_{\mathcal{S}_e})\,\Sigma \;=\; \frac{1}{2}\lambda_{\mathcal{S}_e}\,\sigma_u^2, \tag{21}$$

where the last equality is obtained considering the definition of $\lambda_{\mathcal{S}_e}$ in (18).

Proof of Proposition 4.

The proof is analogous to that of Proposition 1. However, since the algebra involved is quite cumbersome, we give some details. Hence, assume again that $I_0 = \beta_{\mathcal{C}_o}(\bar{s} - \bar{s}^e) - \theta_{\mathcal{C}_o}\bar{n}$, where $\beta_{\mathcal{C}_o}$ and $\theta_{\mathcal{C}_o}$ are non-negative constants. Then, given the assumption of normality for $\bar{s}$ and u_0, we can apply the projection theorem for normal distributions and find that:

$$\begin{aligned}
\bar{s}_0^e \;&\equiv\; E[\bar{s} \mid x_0] \;=\; \bar{s}^e \,+\, \lambda_{\mathcal{C}_o}\,(x_0 \,+\, \theta_{\mathcal{C}_o}\,\bar{n}),\\
I_0^e \;&\equiv\; E[I \mid x_0] \;=\; -\theta_{\mathcal{C}_o}\bar{n} \,+\, \lambda_{\mathcal{C}_o}\,(x_0 \,+\, \theta_{\mathcal{C}_o}\,\bar{n}),\\
&\quad\text{where}\quad \lambda_{\mathcal{C}_o} \;=\; \frac{\beta_{\mathcal{C}_o}\,\Sigma}{\beta_{\mathcal{C}_o}^2\,\Sigma \,+\, \sigma_u^2}.
\end{aligned}$$

Suppose, then, that $\bar{s}_0^e = \bar{s}^e + \lambda_{\mathcal{C}_o}(x_0 + h)$ and $I_0^e = -h + \lambda_{\mathcal{C}_o}(x_0 + h)$, where h is some constant. Inserting these expression in equations (15) and (16), considering that $s_0 = w_1$ and $s_1 = m_1$. Plugging the new expressions for s_0, s_1, n_1 in the modified loss function (11), taking its expectation with respect to the central authority's information and minimising with respect to I_0, we obtain, after some long but simple algebra, that the central bank's optimal order is:

$$
\begin{aligned}
I_0 &= \beta_{\mathcal{C}_o}(\bar{s} - \bar{s}^e) - \theta_{\mathcal{C}_o}\bar{n} - \beta_{\mathcal{C}_o} a_{\mathcal{C}_o} h, \\
\text{where} \qquad \beta_{\mathcal{C}_o} &= \frac{a_{\mathcal{C}_o} + 2Q}{a_{\mathcal{C}_o}(a_{\mathcal{C}_o} + 4Q) + 2Q^2}, \qquad \theta_{0|\mathcal{C}_o} = 2\beta_{\mathcal{C}_o}, \\
a_{\mathcal{C}_o} &= \lambda_{\mathcal{C}_o} + Q(\mu_{\mathcal{C}_o} - 1), \quad \text{and} \quad \mu_{\mathcal{C}_o} = \beta_{\mathcal{C}_o}\lambda_{\mathcal{C}_o}.
\end{aligned}
$$

The condition of consistency for the central authority's market order and the expectations of the dealer are: first, the constants h, $\theta_{0|\mathcal{C}_o}$ and $\beta_{\mathcal{C}_o}$ are such that: $h = \theta_{0|\mathcal{C}_e}\bar{n} + \beta_{\mathcal{C}_o} a_{\mathcal{C}_o} h$. This implies that: $h = \theta_{\mathcal{C}_o}\bar{n}$ with $\theta_{\mathcal{C}_o} = (a_{\mathcal{C}_o} + 2Q)/(Q(a_{\mathcal{C}_o} + Q))$. Second, $\beta_{\mathcal{C}_o}$ and $\lambda_{\mathcal{C}_o}$ solve simultaneously the system of equations:

$$
\beta_{\mathcal{C}_o} = \frac{a_{\mathcal{C}_o} + 2Q}{a_{\mathcal{C}_o}(a_{\mathcal{C}_o} + 4Q) + 2Q^2}, \qquad \lambda_{\mathcal{C}_o} = \frac{\beta_{\mathcal{C}_o}\Sigma}{\beta^2_{\mathcal{C}_o}\Sigma + \sigma^2_u}. \tag{22}
$$

Substituting the expression for $\lambda_{\mathcal{C}_o}$ into that for $\beta_{\mathcal{C}_o}$ after some massage we obtain the following equation:

$$
\sigma^2_u(1 + Q\beta_{\mathcal{C}_o})(\sigma^2_u Q + \Sigma\beta_{\mathcal{C}_o}) = 2Q(1 + Q\beta_{\mathcal{C}_o})\Sigma^2\beta^4_{\mathcal{C}_o}.
$$

Notice that the obvious solution $\beta_{\mathcal{C}_o} = -1/Q$ can be proved to violate the second order condition of the central authority optimisation problem, $a_{\mathcal{C}_o}(a_{\mathcal{C}_o} + 4Q) + 2Q^2 > 0$. Then, simplifying we obtain equation (17). This presents two solutions, one positive and one negative. However, only the positive one satisfies the second order condition.

Proof of Proposition 5.
Both under separation and coordination the conditional variance for the target level, $\bar{s}$, can be obtained from the projection theorem for normal distributions. We have that $\Sigma_{0|\Xi} = (1 - \lambda_\Xi\beta_\Xi)\Sigma$, where $\Xi = \mathcal{C}_o$ or $\mathcal{S}_e$. Since $\partial\lambda_\Xi\beta_\Xi/\partial\beta_\Xi = 2\sigma^2_u\lambda_\Xi/(\beta^2_\Xi\Sigma + \sigma^2_u) > 0$ and $\beta_{\mathcal{C}_o} < \beta_{\mathcal{S}_e}$, we find that: $\Sigma_{0|\Xi}$ is larger for $\Xi = \mathcal{C}_o$. Notice also that from equation (17) we see that $\beta_{\mathcal{C}_o} \geq (1/\sqrt[4]{2})\bar{\beta}$, so that $\Sigma_{0|\mathcal{C}_o} \leq (\sqrt{2}/(\sqrt{2}+1))\Sigma$.

Through the same argument proposed for the proof of Proposition 2 we can see that under coordination the unconditional variance of the employment level is as follows:

$$
\mathrm{Var}_{\mathcal{C}_o}[n_1] = \frac{1}{4}\left(1 + Q\beta_{\mathcal{C}_o}\right)^2\left((1 - \lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o})^2\Sigma + \lambda^2_{\mathcal{C}_o}\sigma^2_u\right). \tag{23}
$$

In comparing equations (20) and (23) consider that: i) $(1-\lambda_\Xi\beta_\Xi)$ is larger for $\Xi = \mathcal{C}_o$; ii) λ_Ξ is decreasing in β_Ξ for $\beta_\Xi > \bar{\beta}$; iii) $\beta_{\mathcal{S}_e} > \beta_{\mathcal{C}_o}$ and $\beta_{\mathcal{S}_e} \geq \bar{\beta}$; iv) for $Q < \bar{Q} \equiv (\Sigma/\sigma^2_u)^{1/2}$ $\beta_{\mathcal{C}_o} > \bar{\beta}$. Then, it is immediate to see that $\mathrm{Var}_\Xi[n_1]$ is larger for $\Xi = \mathcal{C}_o$ when $Q < \bar{Q}$. In the case in which $Q \geq \bar{Q}$, consider that $\forall Q$: i) $\beta_{\mathcal{C}_o} \geq (1/\sqrt[4]{2})\bar{\beta}$ and ii) $\lambda_{\mathcal{S}_e} \leq \bar{\lambda}$. Then, it is not difficult to see that: $(1 + Q\beta_{\mathcal{C}_o})^2\lambda^2_{\mathcal{C}_o}\sigma^2_u \geq ((\sqrt[4]{2}+1)^2/(\sqrt{2}+1)^2)\Sigma > \Sigma/4 \geq \lambda^2_{\mathcal{S}_e}\sigma^2_u$, so that even in this case $\mathrm{Var}_\Xi[n_1]$ is larger for $\Xi = \mathcal{C}_o$.

Through the same argument proposed for the proof of Proposition 3 we can see that under coordination the unconditional expected profits of the central authority are:

$$
E_{\mathcal{C}_o}[\pi] = \frac{1}{2}(1 + Q\beta_{\mathcal{C}_o})\beta_{\mathcal{C}_o}(1 - \lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o})\Sigma = \frac{1}{2}(1 + Q\beta_{\mathcal{C}_o})\lambda_{\mathcal{C}_o}\sigma^2_u. \tag{24}
$$

Now, in comparing equations (21) and (24) consider first of all that under coordination separation the unconditional expected profits of the central bank presents upper bound: $E_{\mathcal{S}_e}[\pi] = (\sigma^2_u\Sigma)^{1/2}/4$. Then, consider that when $Q < \bar{Q}$, $\bar{\beta} < \beta_{\mathcal{C}_o}$. Moreover, notice that under coordination for all Q, $1 - \lambda_{\mathcal{C}_o}\beta_{\mathcal{C}_o} > 1/2$. Then, it is immediate to see that for $Q < \bar{Q}$, $E_\Xi[\pi]$ is larger for $\Xi = \mathcal{C}_o$. In the case in which $Q \geq \bar{Q}$, consider that $\forall Q$: i) $\beta_{\mathcal{C}_o} \geq (1/\sqrt[4]{2})\bar{\beta}$; and ii) $\beta_{\mathcal{S}_e} \geq \bar{\beta}$ and $\lambda_{\mathcal{S}_e} \leq \bar{\lambda}$. Then, it is not difficult to see that: $(1 + Q\beta_{\mathcal{C}_o})\lambda_{\mathcal{C}_o}\sigma_u^2 \geq ((\sqrt[4]{2}+1)/(\sqrt{2}+1))(\sigma_u^2\Sigma)^2 > (\sigma_u^2)\Sigma/2 \geq \lambda_{\mathcal{S}_e}\sigma_u^2$, so that even in this case $E_\Xi[\pi]$ is larger for $\Xi = \mathcal{C}_o$.

Proof of Proposition 6.
Let us consider the separation case and define the implicit function $F(\beta_{\mathcal{S}_e}, Q, \sigma_u^2) \equiv \sigma_u^2(\sigma_u^2 Q + \Sigma\beta_{\mathcal{S}_e}) - Q\Sigma^2\beta_{\mathcal{S}_e}^4$. Using inequalities obtained from equation (14) and the fact that $\beta_{\mathcal{S}_e} > \bar{\beta}$, one can then check that: $\partial F/\partial\beta_{\mathcal{S}_e} < 0$ and $\partial F/\partial Q < 0$. Thus, from the implicit function theorem one concludes that $\partial\beta_{\mathcal{S}_e}/\partial Q < 0$. Then, considering that $1 - \lambda_\Xi\beta_\Xi = \sigma_u^2/(\beta_\Xi^2\Sigma + \sigma_u^2)$ one immediately sees $\partial\Sigma_{0|\mathcal{S}_e}/\partial Q > 0$. Likewise, one should notice that $(1 - \lambda_\Xi\beta_\Xi)^2\Sigma + \lambda_\Xi^2\sigma_u^2 = \sigma_u^2\Sigma/(\beta_\Xi^2\Sigma + \sigma_u^2)$. Then, it is immediate to see that $\partial\mathrm{Var}_{\mathcal{S}_e}[n_1]/\partial Q > 0$. For the profits, notice that, since $\beta_{\mathcal{S}_e} > \bar{\beta}$, $\partial\lambda_{\mathcal{S}_e}/\partial Q > 0$. Then, since $E_{\mathcal{S}_e}[\pi] = \lambda_{\mathcal{S}_e}\sigma_u^2/2$, the result is immediate. For the losses of the monetary authorities, notice that $E_{\mathcal{S}_e}[\mathcal{L}_m] = 2\mathrm{Var}_{\mathcal{S}_e}[n_1] + 2\bar{n}^2$, so the result is also immediate.

Let us now consider the coordination case. Defining from equation (17) the new implicit function, $G(\beta_{\mathcal{S}_e}, Q, \sigma_u^2) \equiv \sigma_u^2(\sigma_u^2 Q + \Sigma\beta_{\mathcal{S}_e}) - 2Q\Sigma^2\beta_{\mathcal{S}_e}^4$, we have very similar steps to show that $\partial\Sigma_{0|\mathcal{C}_o}/\partial Q > 0$ and $\partial\mathrm{Var}_{\mathcal{C}_o}[n_1]/\partial Q > 0$. (to be completed)

We now turn to the asymptotic properties. For $Q \uparrow \infty$, it is immediate to see from equations (14) and (17) that: $\beta_{\mathcal{S}_e} \downarrow \bar{\beta}$, while $\beta_{\mathcal{C}_o} \downarrow \sqrt[4]{2}\bar{\beta}$. Then, all the limits follow immediately. On the contrary for $Q \downarrow 0$ in both cases $\beta_\Xi \uparrow \infty$ and once again the asymptotic results follow suit.

Proof of Proposition 7.
We can proceed as in the proof of Proposition 6, considering that $\partial F/\partial\sigma_u^2 > 0$. Thus, from the implicit function theorem one concludes that $\partial\lambda_{\mathcal{S}_e}/\partial\sigma_u^2 < 0$, and since $\beta_{\mathcal{S}_e} > \bar{\beta}$, one immediately finds $\partial\beta_{\mathcal{S}_e}/\partial\sigma_u^2 > 0$, then since $1 - \lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e} = Q/(\lambda_{\mathcal{S}_e} + 2Q)$, one sees that: $\partial\Sigma_{0|\mathcal{S}_e}/\partial\sigma_u^2 > 0$. As for the derivatives of the conditional variance of n_1 and the expected value of $\mathcal{L}_m$ just notice that, after some tedious algebra, one can prove that: $\partial[\lambda_{\mathcal{S}_e}^2\sigma_u^2]/\partial\sigma_u^2 = [(3\sigma_u^2\beta_{\mathcal{S}_e}\lambda_{\mathcal{S}_e}\Sigma)(\beta_{\mathcal{S}_e}^2\Sigma - \sigma_u^2)]/[(\beta_{\mathcal{S}_e}^2\Sigma + \sigma_u^2)(4Q\Sigma\beta_{\mathcal{S}_e}^3 - \sigma_u^2)]$, which is positive, as $\beta_{\mathcal{S}_e}$ respects (14) and is larger than $\bar{\beta}$. Since $(1 - \lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})^2$ is increasing with σ_u^2, this is sufficient to show that: $\partial\mathrm{Var}_{\mathcal{S}_e}[n_1]/\partial\sigma_u^2 > 0$ and $\partial E_{\mathcal{S}_e}[\mathcal{L}_m]/\partial\sigma_u^2 > 0$. Finally, for the expected value of π notice that we have seen that: $\partial\beta_{\mathcal{S}_e}/\partial\sigma_u^2 > 0$ and $\partial(1 - \lambda_{\mathcal{S}_e}\beta_{\mathcal{S}_e})/\partial\sigma_u^2 > 0$. Then, it is immediate to see that: $\partial E_{\mathcal{S}_e}[\pi]/\partial\sigma_u^2 > 0$.

The proof for the signs of the corresponding derivatives in the case of coordination is analogous. While for the asymptotic results consider that as for the proof of Proposition 6 we find that for $\sigma_u^2 \uparrow \infty$, $\beta_{\mathcal{S}_e} \to \bar{\beta} \uparrow \infty$, while $\beta_{\mathcal{C}_o} \to (1/\sqrt[4]{2})\bar{\beta} \uparrow \infty$. Then, all the limits follow immediately. On the contrary for $\sigma_u^2 \downarrow 0$ in both cases $\beta_\Xi \downarrow 0$ and once again the asymptotic results can be immediately shown.

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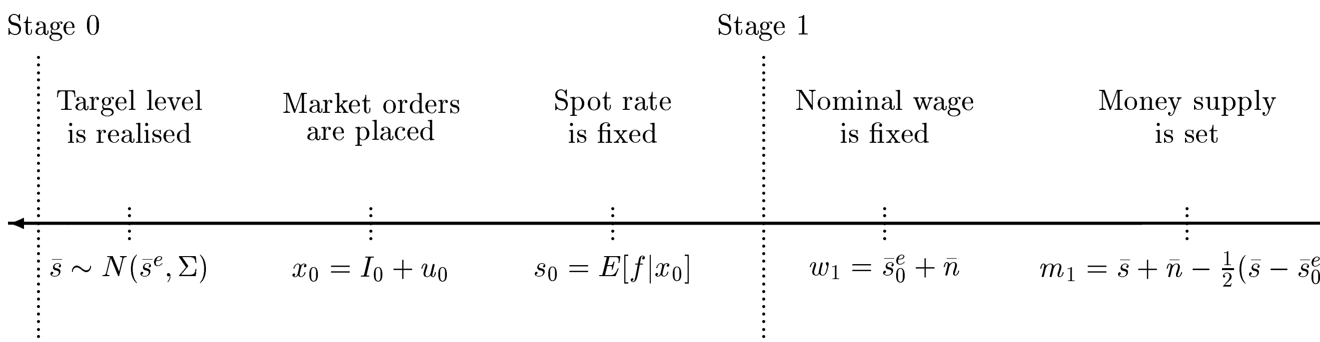
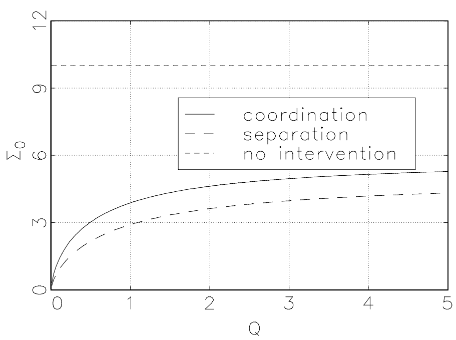
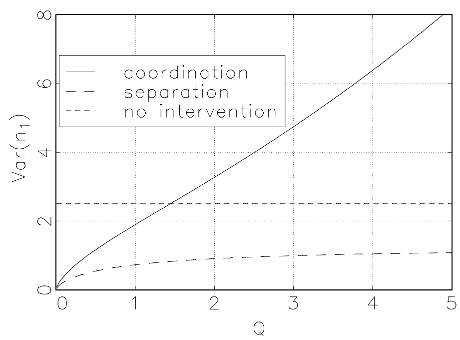


Figure 1: The time line representation of the model under separation

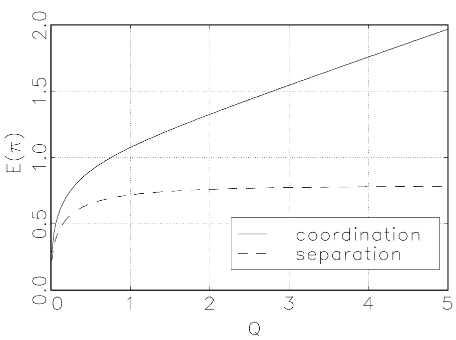
Figure 2: Characteristics of the equilibrium as functions of Q.



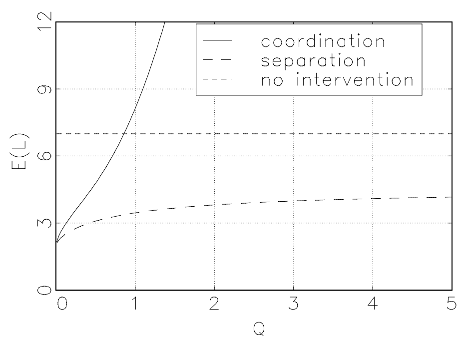
(a) Efficiency



(b) Employment Volatility



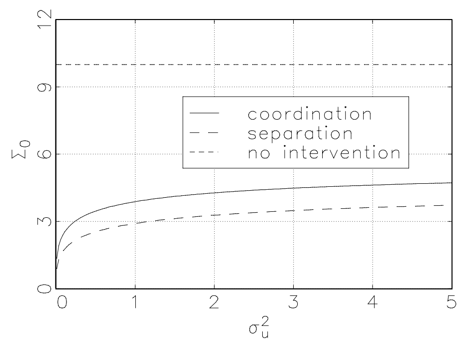
(c) Expected Profits



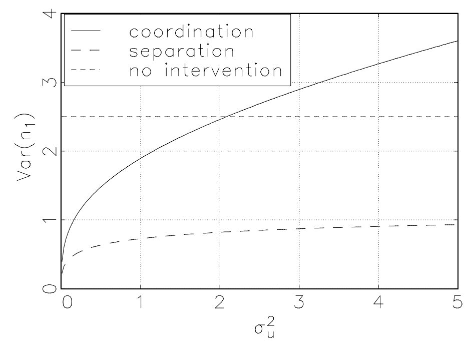
(d) Expected Losses

Notes: $\Sigma = 10$, $\sigma_u^2 = 1$. The continuous line represents the equilibrium value of the coefficient in the case of *coordination*. The long dash line represents the same coefficient in the case of *separation*, while the short dash line refers to the coefficient value in the absence of foreign exchange intervention. The efficiency coefficients, the unconditional variances of the employment level, alongside with the unconditional expected profits of intervention and the losses of the monetary authorities converge to limit values for $Q \uparrow \infty$, both in the case of coordination and separation. In particular, in the former case $\Sigma_{0|\mathcal{C}_o} \uparrow (\sqrt{2}/(\sqrt{2}+1))\Sigma = 5.8579$; while in the latter, $\Sigma_{0|\mathcal{S}_e} \uparrow (1/2)\Sigma{=}5$. Moreover, in the former case $\text{Var}_{\mathcal{C}_o}[n_1]$, $E_{\mathcal{C}_o}[\pi]$ and $E_{\mathcal{C}_o}[\mathcal{L}_a]$ all converge to infinite, while in the latter: $\text{Var}_{\mathcal{S}_e}[n_1] \uparrow (1/8)\Sigma = 1.25$, $E_{\mathcal{S}_e}[\pi] \uparrow (1/4)(\sigma_u^2\Sigma)^{1/2} = 0.7906$ and $E_{\mathcal{S}_e}[\mathcal{L}_m] \uparrow (1/4)\Sigma + 2\bar{n}^2 = 4.5$. Likewise limits exist for $Q \downarrow 0$. In particular, $\Sigma_{0|\mathcal{C}_o}$ and $\Sigma_{0|\mathcal{S}_e}$ converge to 0 and so do $\text{Var}_{\mathcal{C}_o}[n_1]$, $\text{Var}_{\mathcal{S}_e}[n_1]$, $E_{\mathcal{C}_o}[\pi]$ and $E_{\mathcal{S}_e}[\pi]$. Finally, $E_{\mathcal{S}_e}[\mathcal{L}_m] \downarrow 2\bar{n}^2 = 2$ and so does $E_{\mathcal{C}_o}[\mathcal{L}_a]$.

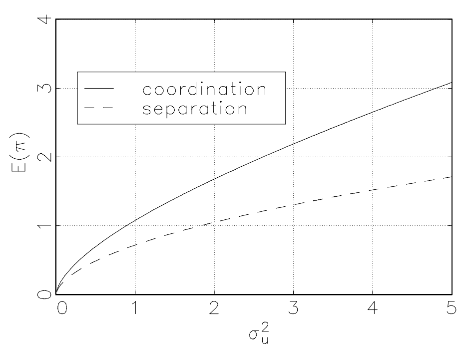
Figure 3: Characteristics of the equilibrium as functions of σ_u^2.



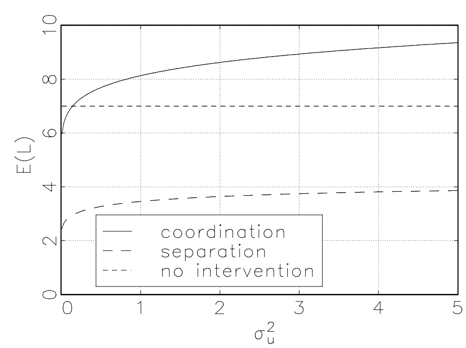
(a) Efficiency



(b) Employment Volatility



(c) Expected Profits



(d) Expected Losses

Notes: $\Sigma = 10$, $Q = 1$. The continuous line represents the equilibrium value of the coefficient in the case of *coordination*. The long dash line represents the same coefficient in the case of *separation*, while the short dash line refers to the coefficient value in the absence of foreign exchange intervention. The efficiency coefficients, the unconditional variances of the employment level, alongside with the unconditional expected profits of intervention and the losses of the monetary authorities converge to limit values for $\sigma_u^2 \uparrow \infty$, both in the case of coordination and separation. In particular, in the former case $\Sigma_{0|\mathcal{C}_o} \uparrow (\sqrt{2}/(\sqrt{2}+1))\Sigma = 5.8579$; while in the latter, $\Sigma_{0|\mathcal{S}_e} \uparrow (1/2)\Sigma{=}5$. Moreover, in the former case $\mathrm{Var}_{\mathcal{C}_o}[n_1]$, $\mathrm{Var}_{\mathcal{C}_o}[s_1]$ and $E_{\mathcal{C}_o}[\pi]$ all converge to infinite, while in the latter: $\mathrm{Var}_{\mathcal{S}_e}[n_1] \uparrow (1/8)\Sigma = 1.25$, $E_{\mathcal{S}_e}[\pi] \uparrow \infty$ and $E_{\mathcal{S}_e}[\mathcal{L}_m] \uparrow (1/4)\Sigma + 2\bar{n}^2 = 4.5$. Likewise limits exist for $\sigma_u^2 \downarrow 0$. In particular, $\Sigma_{0|\mathcal{C}_o}$ and $\Sigma_{0|\mathcal{S}_e}$ converge to 0 and so do $\mathrm{Var}_{\mathcal{C}_o}[n_1]$, $\mathrm{Var}_{\mathcal{S}_e}[n_1]$, $E_{\mathcal{C}_o}[\pi]$ and $E_{\mathcal{S}_e}[\pi]$. Finally, $E_{\mathcal{S}_e}[\mathcal{L}_m] \downarrow 2\bar{n}^2 = 2$, while $E_{\mathcal{C}_o}[\mathcal{L}_a] \downarrow [1 + (1+Q)^2]\bar{n}^2 = 5$.