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# EMPIRICAL ANALYSIS OF LIMIT ORDER MARKETS

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# ABSTRACT

## Empirical Analysis of Limit Order Markets*

We analyse order placement strategies in a limit order market, using data on the order flow from the Stockholm Stock Exchange. Traders submitting market or limit orders trade off the order price against both the execution probability and the winner's curse risk associated with different order choices. The optimal order strategy is characterized by a monotone function, which maps the liquidity demand of the investors into their order choice. We develop and implement a semiparametric test of this monotonicity property, and find no evidence against the monotonicity property for buy orders or sell orders. We do find evidence against the hypothesis that the trader's decision to be a buyer or a seller depends only on the trading profits available in the limit order book. We estimate that traders submitting market buy orders have private valuations that exceed the asset value by 2.3% on average and receive an average pay-off of at least 1.8% of the asset value. Traders submitting limit buy orders at the price below the best ask quote have private valuations between 0.1% and 2.3% above the asset value, and earn an average pay-off of between 0.3% and 1.8% of the asset value. Although the distribution of liquidity demand does not depend on conditioning information, conditioning information helps us to predict the composition of the order flow in our data. These findings imply that variation in the composition of the order flow can be explained by empirical variation in the relative profitability of alternative order choices and movements in the common value of the asset.

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# NON-TECHNICAL SUMMARY

In this Paper, we develop and test a model of optimal order placement in a computerized limit order market. Our data set is obtained from the Stockholm Stock Exchange and contains detailed data, which allow us to approximate the information available to the traders at the time of order submissions, and to track the eventual outcomes of all orders. We use the data on traders' order placement choices together with the outcomes of these choices to estimate the expected pay-offs from alternative trading strategies. Theory imposes testable restrictions on the relative pay-offs of alternative order choices, which we test in our data. We use our empirical estimates of the optimal trading strategy and the conditional distribution of the traders' order entry decisions to make inferences on the traders' liquidity demands in the market. This allows us to decompose variation in the order flow into changes in trading opportunities and changes in the distribution of liquidity demand.

A computerized limit order market resembles a continuous double auction market, which has been studied extensively by economists. The practical importance of this market structure is growing as many financial markets have adopted computerized limit order books, while others are evaluating the merits of introducing limit order books into their market architecture. The Stockholm Stock Exchange completed the introduction of a computerized limit order market in 1990. The computerized trading system SAX (Stockholm Automated Exchange) is very similar to other electronic limit order markets, such as the Paris Bourse.

In a limit order market, buyers and sellers can submit orders of two types. A market order executes immediately at the most attractive price posted by previous limit orders. A limit order specifies a particular price, and is a promise to trade at that price. Unexecuted limit orders enter the electronic limit order book where they are stored until executed or cancelled. In most limit order markets there are no floor traders, market makers or specialists with special quoting obligations or trading privileges. The most important source of liquidity is the queue of unfilled limit orders in the order book. There is a trade-off between the order price and the probability that the limit order transacts, which is reflected as an implicit cost for immediacy or liquidity. In addition, the underlying value of the asset generally moves stochastically, and since a trade involves a previously submitted limit order being transacted against a newly submitted market order, trades typically involve a market order submitted by a trader who has better information than the trader who previously submitted the limit order. Therefore, limit orders may be exposed to adverse selection, which we refer to as winner's curse.

Overall, the order placement problem involves trade-offs between the order price, the probability of execution, and the winner's curse. These trade-offs

form the basis of much of the theoretical literature on the choice between limit and market orders. Our contribution is to estimate these trade-offs directly in our data set, and to test whether the traders' observed order placement decisions can be rationalized by these trade-offs that our model characterizes. We also use the observed order choices made by the traders to make inferences about their demands for liquidity.

Our empirical approach is as follows. The theoretical model characterizes the optimal order submission strategy in terms of the conditional probability that different limit orders transact, along with the conditional winner's curse associated with different limit orders. We use non-parametric methods to estimate conditional execution probabilities and winner's curse terms and use these to construct estimates of the optimal order submission strategy. The theoretical model imposes a monotonicity restriction on non-linear functions of the conditional execution probabilities and associated winner's curse terms. We form tests of these monotonicity restrictions in our data set. Combining the estimates of the optimal order strategy with the actual choices made by the traders, we infer properties of the distribution of traders' liquidity demands.

Our empirical analysis uses data on all orders submitted for one of the most actively traded stocks on the Stockholm Stock Exchange, Telefon AB LM Ericsson. We find that the conditional probability that a limit order eventually transacts decreases as the limit order price distances from the best quotes. We do not reject the monotonicity property implied by traders' optimization over limit and market orders when we analyse buy and sell decisions separately. Thus, traders appear to behave in a way that can be rationalized by the trade-offs between the order price, the conditional probability that the order transacts, and the winner's curse. Our monotonicity test rejects the implication that the trader's choice to be a buyer or a seller of the asset is based only on the profits available in the limit order book. Our empirical estimates imply that the average market order buyer earns an average pay-off of at least 2 kronor (SKr) per share, or roughly 1.8% of the average value of Ericsson from the order submission, while the corresponding pay-offs for the average market order seller are at least 1.6 SKr per share, approximately 1.5% of the average value of the asset. Traders submitting limit orders have less extreme private valuations than traders submitting market orders and earn a smaller average pay-off from their order submissions. Overall, our empirical work provides evidence that limit orders are submitted by traders with an active interest in trading who earn positive pay-offs from their order submissions. Our estimates of liquidity demand distribution reveal little evidence that conditioning information affects distribution trends. Since the conditioning information helps predict the distribution of order submissions, our estimates imply that changes in the trading opportunities – rather than changes in liquidity demand – drive much of the empirical variation in the distribution of order submission strategies.

Our empirical findings have two important implications for empirical studies that measure the performance or pay-off on different order placement strategies. First, they imply that the trading opportunities available in the market influence traders' order placement strategies. Second, limit orders are submitted by traders who have an active interest in trading, that is, traders with a non-zero demand for immediacy. Thus, in order to accurately measure the pay-off on a given order strategy it is crucial to account for the information that the traders have when they make their order entry decisions, and to consider the valuations of these traders following a given order strategy. These considerations are likely to be relevant for any market institutions that include a limit order book in their design. Glosten (1994) and Seppi (1997) derive equilibrium limit order books in environments where limit orders are submitted by perfectly competitive traders with a zero liquidity demand, implying that the marginal limit order earns zero expected profits. In our data, we cannot reject the null hypothesis that the average pay-off of the most aggressive limit order is zero. Traders submitting the most aggressive buy limit orders have negative private valuations for the asset and traders submitting aggressive sell orders have strictly positive private valuations for the asset. Traders submitting less aggressive limit orders receive strictly positive average pay-offs and the average private valuations of these traders are non-zero. On average, limit orders are submitted by traders with an active interest in trading, who earn positive pay-offs from their order submissions. Overall, the results are consistent with the arguments in Glosten (2000) that limit orders need not be submitted by marginal traders earning zero profits.

# 1 Introduction

In limit order markets, the order flow determines trading activity in the market, and the composition of the order flow generally depends on the number and nature of unfilled orders in the limit order book. In this paper, we develop and test a model of optimal order placement in a computerized limit order market. Our data set is obtained from the Stockholm Stock Exchange, and contains detailed data which allow us to approximate the information available to the traders at the time of order submissions, and to track the eventual outcomes of all orders. We use the data on traders' order placement choices together with the outcomes of these choices to estimate the expected payoffs from alternative trading strategies. Theory imposes testable restrictions on the relative payoffs of alternative order choices which we test in our data. We use our empirical estimates of the optimal trading strategy and the conditional distribution of the traders' order entry decisions to make inferences on the traders' liquidity demands in the market, allowing us to decompose variation in the order flow into changes in trading opportunities and the changes in the distribution of liquidity demand.

A computerized limit order market resembles a continuous double auction market, which has been studied extensively by economists. The practical importance of this market structure is growing as many financial markets have adopted computerized limit order books, while others are evaluating the merits of introducing limit order books into their market architecture. For example, Domowitz (1993) documents that approximately 35 financial markets in 16 different countries contain elements of limit order mechanisms in their designs. Recently, the New York Stock Exchange has debated the benefits of adopting elements of a consolidated limit order book into its design (New York Stock Exchange Market Structure Report, (2000)).

In a limit order market, buyers and sellers can submit orders of two types. A market order executes immediately at the most attractive price posted by previous limit orders. A limit order specifies a particular price, and is a promise to trade at that price. Unexecuted limit orders enter the limit order book where they are stored until executed or canceled. There is a trade-off between the order price and the probability that the limit order transacts, which is reflected as an implicit cost for immediacy or liquidity. In addition, the underlying value of the asset generally moves stochastically, and since a trade involves a previously submitted limit order being transacted against a newly submitted market order, trades typically involve a market order submitted by a trader who has better information than the trader who previously submitted the limit order. Therefore, limit orders may be exposed to adverse selection which we refer to as winner's curse. Overall, the order placement problem involves trade-offs between the order price, the probability of execution, and the winner's curse.

These trade–offs form the basis of much of the theoretical literature on the choice between limit and market orders.[1] Our contribution is to estimate these trade–offs directly in our data set, and to test whether the traders' observed order placement decisions can be rationalized by these trade–offs that our model characterizes. We also use the observed order choices made by the traders to make inferences about their demands for liquidity.

Our empirical approach is as follows. The theoretical model characterizes the optimal order submission strategy in terms of the conditional probability that different limit orders transact, along with the conditional winner's curse associated with different limit orders. We use nonparametric methods to estimate conditional execution probabilities and winner's curse terms and use these to construct estimates of the optimal order submission strategy. The theoretical model imposes a monotonicity restriction on nonlinear functions of the conditional execution probabilities and associated winner's curse terms. We form tests of these monotonicity restrictions in our data set. Combining the estimates of the optimal order strategy with the actual choices made by the traders, we infer properties of the distribution of traders' liquidity demands.

The trade–offs that determine the optimal order strategy are similar to the familiar trade–offs in auction settings. In auctions, bidders trade off the probability of winning the object against the price that they pay and the value of the object, conditional upon winning the auction. Elyakime,

[1] Demsetz (1968) studies the costs of immediacy in an environment where buyers and sellers arrive at different times. Cohen, Maier, Schwartz, and Whitcomb (1981) theoretically analyze a trader's optimal choice between market and limit orders. Biais, Martimort and Rochet (2000), Foucault (1999), Glosten (1994), O'Hara and Oldfield (1986), Parlour (1998), Parlour and Seppi (2001), Rock (1996), and Seppi (1997), analyze prices, trading volumes and efficiency in financial markets with limit order books. Domowitz and Wang (1994) solve for the stationary distribution of the order book and the waiting time to order execution in a model with exogenous arrival rates of market and limit orders using queuing theory.

Laffont, Loisel and Vuong (1994) and Guerre, Perrigne, and Vuong (2000) and Laffont and Vuong (1996) show how to construct nonparametric estimators of the optimal bidding strategy in different private value auction settings. In these papers, empirical estimates of the optimal bidding strategies are used to invert for bidders' valuations to estimate the distribution of valuations, assuming that the empirical bidding strategy is monotone. These authors do not develop empirical tests of the monotonicity property of the optimal bidding strategy.

There are important differences between the financial limit order market that we study and the auction environments analyzed previously. The limit order market is a sequential market, in which traders choose order prices from a discrete set. In financial markets, we expect the underlying value of the asset to be stochastic. Since trades involve a previously submitted limit order transacting with a newly submitted market order, limit orders may be exposed to changes in the underlying asset value. The order book, which is the queue of unfilled limit orders, is a stochastic process. Together with the stochastic value of the asset, this implies that the expected payoffs and the optimal strategy may depend on information available to the traders when they submit their orders. Our econometric techniques deal with the time series nature of the data and allow for conditioning information.

There has been previous empirical work studying limit orders. Using data on limit and market orders from the Paris Bourse, Biais, Hillion and Spatt (1995), find evidence that traders submit more market orders when the order book is relatively full and more limit orders when the order book is relatively empty. This behavior is consistent with order placement strategies that depend on the trading opportunities offered in the limit order book.[2]

Our empirical analysis uses data on all orders submitted for one of the most actively traded stocks on the Stockholm Stock Exchange, Telefon AB L.M. Ericsson. We find that the conditional

[2] Harris and Hasbrouck (1996), and Handa and Schwartz (1996) analyze the profitability of alternative order placement strategies in different market conditions. Lo, MacKinlay, and Zhang (2001) estimate several econometric models of limit order execution times. Kavajecz (1999) presents empirical evidence on the interaction between the specialist and the limit order book on the New York Stock Exchange and Ready (1999) provides a theoretical and empirical analysis of the interaction between the limit order book and the specialist's use of stopped orders on the New York Stock Exchange. Goldstein and Kavajecz (2000) document dramatic shifts in traders' willingness to submit limit orders in the limit order book during extreme market movements in the New York Stock Exchange. Sandås (2001) estimates a model of competitive market making in a limit order market.

probability that a limit order eventually transacts is decreasing in the distance of the limit order price from the best quotes. We do not reject the monotonicity property implied by traders' optimization over limit and market orders when we analyze buy and sell decisions separately. Thus, traders appear to behave in a way that can be rationalized by the tradeoffs between the order price, the conditional probability that the order transacts, and the winner's curse. Our monotonicity test rejects the implication that the traders' choice to be a buyer or a seller of the asset is based only on the profits available in the limit order book. Our empirical estimates imply that the average market order buyer earns an average payoff of at least 2 Kronor (SKr) per share, or roughly 1.8% of the average value of Ericsson from the order submission, while the corresponding payoffs for the average market order seller are at least 1.6 SKr per share, approximately 1.5% of the average value of the asset. Traders submitting limit orders have less extreme private valuations than traders submitting market orders and earn a smaller average payoff from their order submissions. Overall, our empirical work provides evidence that limit orders are submitted by traders with an active interest in trading who earn positive payoffs from their order submissions.

Our estimates of the distribution of liquidity demand reveal little evidence that conditioning information changes the distribution of liquidity demand. Since the conditioning information helps predict the distribution of order submissions, our estimates imply that changes in the trading opportunities, rather than changes in liquidity demand drive much of the empirical variation in the distribution of order submission strategies.

The next section of the paper provides a brief qualitative and quantitative description of the Stockholm Stock Exchange. In section 3 we present the theoretical model, deriving its testable restrictions. Section 4 contains an empirical implementation of the model and the final section concludes. All proofs and regularity conditions for our econometric estimators are contained in the Appendices.

## 2 Description of the Market and the Data

This section describes the relevant institutional details about the Stockholm Stock Exchange for the time period we study, December 1991 to March 1992, and some stylized facts of the data that help motivate our subsequent analysis. During this time period, the Stockholm Stock Exchange was a nonprofit organization, with a board of directors representing the listing firms, intermediaries and the Swedish government. The Exchange was incorporated in 1993, and it merged with the OM Group, a for–profit derivatives exchange in 1998.

In 1990 the Stockholm Stock Exchange completed the introduction of a computerized limit order market. The computerized trading system SAX (Stockholm Automated Exchange) is very similar to other electronic limit order markets, such as the Paris Bourse and the Toronto Stock Exchange. In the Stockholm Stock Exchange, there are no floor traders, market makers or specialists with special quoting obligations or trading privileges. The most important source of liquidity is the queue of unfilled limit orders in the order book. Trading in this system is continuous from 10 A.M. to 2:30 P.M. and the opening price at 10 A.M. is determined by a call auction. Investors submit market or limit orders to the electronic limit order book through brokers.

All order prices are required to be multiples of a fixed minimum price unit, referred to as the tick size. When prices are below 100 SKr, the tick size is 1/2 SKr and when prices exceed 100 SKr, the tick size is 1 SKr. During the sample period $1 was roughly equal to 6.25 SKr. The order size is must be an integer multiple of a round lot, with a typical round lot size of 100 shares.

Limit orders are stored in the centralized computer system and automatically executed as they cross with incoming market orders. Thus, all trading in the limit book is between market and limit orders. Limit orders in the order book are prioritized first by price and then by time of submission. If an incoming market order is for a smaller quantity than that available at the best quote in the book, then it will trade in full at a price equal to the best quote. If the market order cannot be filled completely at the best quotes, it will walk up the book until it is either filled in full, or no more limit orders remain. In the absence of designated market makers, it is possible for the limit order book to be empty on one or both sides of the market. When this occurs, it is not possible for a market order to obtain immediate execution and so the market order is converted into a limit order and added to the order book at a price one tick away from the current best quote. Similarly, the remaining quantity of a large market order that exhausts the available liquidity in the book before it is filled in full is converted to a limit order at the last trade price. A limit order can be canceled at any time at no extra cost. Traders can also submit limit orders with only a portion of the order quantity displayed in the order book. These orders are known as hidden orders. The hidden part of a limit order has lower priority than all displayed limit orders at the same order price in the limit order book.

Only exchange member firms can enter orders in the SAX system. A member firm trades both as a broker on behalf of customers, and as a dealer on his own behalf.[3] During the time period we study there were a total of 24 exchange member firms. These firms include all major Swedish banks as well as most brokerage firms that actively trade Swedish securities. The major Swedish banks have extensive retail branch networks, and act as brokers for customers throughout Sweden. We will refer to the member firms as brokers.

The brokers are directly connected to the SAX system. They observe in real time the total order quantities available at each price level in the order book for a given stock. In addition they observe, for every price level in the book, the codes for each of the brokers with outstanding orders. This information is updated almost instantaneously after order submissions or cancellations. Short delays of up to one minute may arise, for example, just after the opening, or just before the closing of the market when activity levels tend to be high. Investors, who are not directly connected to the SAX system, can obtain information about the five best bid and ask price levels, and the corresponding order quantities through information vendors, such as Reuters or Telerate. Investors can submit orders directly to the limit order book through exchange members.

Until January 1st, 1993, the Stockholm Stock Exchange was the only authorized marketplace for equity trading in Sweden. Many of the listed companies were also cross listed on foreign exchanges during this period. Trading in London on the international Stock Exchange Automated

[3]Commissions are negotiable. Commissions were around 0.5% depending on the level of service for this time period. There is also a fixed exchange fee per order of less than one dollar.

Quotation and in the U.S. on the National Association of Securities Dealers Automated Quotation system accounted for a significant fraction of the turnover for many Swedish firms.[4] Trading is not completely automated since trades can be settled, subject to some quantity restrictions, outside the electronic system either during normal trading hours or during after-hours trading.[5]

Our data set, provided by the Stockholm Stock Exchange, consists of order and trade records obtained directly from the SAX system. The order records is a list of new order submissions, changes in outstanding orders and order cancellations. The trade records list the actual transactions in the market chronologically. The data set contain sufficient information to reconstruct individual order histories. The sample period is the 59 trading days between December 3, 1991, and March 2, 1992.

We use the following procedure to construct the individual order histories from the order flow and trade files. Each limit order submitted to the SAX system receives a unique tracking number, and subsequent changes in the outstanding order quantity are recorded using the same tracking number. We combine this information with data on transactions to determine whether a change in the order quantity was due to a trade or a cancellation. This process allows us to reconstruct complete transaction and cancellation histories for limit orders, and to reconstruct the order book information available to the brokers and the traders in the marketplace at any point during our sample period.

Our data set is very detailed, but there are some important limitations. First, we cannot distinguish the trades that a broker makes on his own behalf from those he makes for his customers. Second, the investors in our data set are essentially anonymous, since we can only identify the broker submitting the order. Therefore, we cannot link orders submitted by the same investors at *different times*, for example, as part of a dynamic order placement strategy. Third, we do not

[4] Trading abroad was particularly attractive due to the transaction tax levied on equity trades in Sweden. In 1991 the transaction tax was equal to 0.5%. The tax was abolished on December 1, 1991, before the start of our sample period.

[5] For the most active stocks, a trade of 100-500 round lots can be settled outside the SAX system if the transaction price is within the bid-ask spread. This rule allows brokers to cross customer orders in-house. Trades of more than 500 round lots can be settled at a price outside the bid-ask spread. These orders do not respect the priority rules within the order book, nor do they interact with the orders in the book. All off–exchange trades arranged during regular trading hours must be reported to the exchange within 5 minutes. This information is disseminated to market participants. Trades made during after-hours trading must be reported to the exchange before the opening of the market the following day.

observe directly whether an order includes a hidden order quantity component or not. In our data set we are able to infer *ex post* that an order must have involved some hidden quantities only if the displayed proportion of the hidden order is executed in full. In these situations, we will observe a subsequent order quantity increase with the same tracking number as the original order when the hidden order quantity is added to the outstanding order quantity. In our sample, hidden orders appear to be used very infrequently, at least for the orders that fully execute.

Limit orders typically are good until canceled and so the time that a limit order remains in the order book is random. A censoring problem arises in our data since some orders submitted in our sample remain outstanding at the end of our sample period. To minimize the effects of censoring on our analysis, we do not use orders submitted during the last two days of our sample in our subsequent empirical work. We follow each order in our sample for two trading days after it was placed. Only 2.8% of the limit orders submitted by the traders remain in the system for more than two trading days and 62.3% of these limit orders are eventually canceled. We also discard orders submitted during the first three minutes of the trading day. This ensures that our data reflects only continuous trading. These filtering rules leave us with 20,760 observations of individual market and limit orders submitted to the SAX system.

Table 1 reports descriptive statistics on the daily trading activity for Ericsson. The tick size varies between 1/2 SKr and 1 SKr since the price fluctuates below and above 100 SKr. The average daily return for the stock is -0.22%, but due to positive overnight returns, the stock had a 10.05% total return over the period we study. The standard deviation, maximum and minimum returns indicate that the stock returns have been relatively volatile during this period.[6] Thus, limit orders submitted over this period can be subject large adverse price changes. There are 24 brokers that trade shares in Ericsson over the sample period. Nine brokers have market shares of more than 5% and the most active one has a market share of 11.3%. The third row of the table reports statistics on the number of active brokers, defined as brokers who make at least one trade on a given day.

[6] For comparison, the standard deviation of daily returns on Ericsson cross-listed on the National Association for Security Dealers Automated Quotation system is equal to 2.5% for the five year period 1989 to 1993, computed using data from the Center for Research in Securities Prices. Using the same data source, we find a daily return standard deviation of 3.1% over the sample period we study.

On average, 19 different brokers trade the shares on any given day. Sorting the brokers by their trading volume, the top 3 brokerage firms each transact 10% to 11% of the total trading volume in Ericsson, and the next 7 brokerage firms each transact 5% to 9% of the total trading volume. The numbers are almost identical for order submissions. Figure 1 provides the cumulative market share of the dealers based on trading volume. The Herfindahl index based on trading volume is 691 and the Herfindahl index based on order submissions is 641.[7]

The daily trading volume in the limit order system is reported on the fourth row. Corresponding descriptive statistics are reported for orders crossed internally by brokers, block trades during regular trading hours, after-hours trading, and the total trading volume, respectively, on the following four rows. The 1991 annual report from the Stockholm Stock Exchange (Stockholms Fondbörs Årsrapport (1991)) reports that during 1991, the turnover rate of the stock was 38%. All our subsequent analysis concentrates of order submissions and trades in the electronic limit order book.

Table 2 provides sample statistics on the order flow. The first row presents the number of market and limit orders submitted. For both the buy and sell sides, most of the orders are market orders, and there are more buy than sell orders. The second line of the table reports averages of the fill ratios for different limit orders where the fill ratio measures the fraction of the limit order quantity that was eventually traded. We consider a time horizon of two trading days, so that we measure the fraction of the limit order quantity traded within two trading days. The fill ratio for market orders is, by definition, equal to one, if sufficient liquidity is available. If the limit order book is empty, then it is not possible to submit a market order of any size. This never occurs in our data set. The fill ratios reported in the table show the unconditional trade–off between the probability that the order transacts and the order price in the data, since the fill ratio for limit orders drops monotonically the more favorable the submitted limit order price is. The final row of the table gives estimates of the average time–to–fill for limit orders. These sample averages suggest that, conditioning on the order being filled, more aggressively priced limit orders take longer to fill than less aggressively priced limit orders. Given the stochastic movement in trading prices, the

[7] The Herfindahl index is computed as $\sum_{\text{dealers}}(\%\text{ market share of dealer})^2$.

average time–to–fill suggests that aggressively priced limit orders are more exposed to changes in the underlying value of the asset than less aggressively priced orders.

Table 3 gives summary statistics on the order quantities across eight order price categories. The average order quantity across categories ranges from 15 to 36 round lots. We reject the null of equal means across categories (chi-squared=486.30, 7 d.f., p-value $< 0.001$). We also reject the null hypothesis that the average order quantity is the same for buy and sell orders (chi-squared=261.95, 1 d.f., p-value $< 0.0001$), the null hypothesis that all sell orders have the same average quantity (chi-squared=33.16, 3 d.f., p-value $< 0.001$) and the null hypothesis that the buy orders all have the same average quantity (chi-squared=124.57, 3 d.f., p-value $< 0.001$). For both buy and sell orders, the highest order size is for limit orders at two ticks away from the quotes, and the lowest order size is for the three tick away orders. On average, market orders are for smaller quantities than limit orders.

The second column of Table 3 reports the standard deviation of the order quantity within each price category. These numbers indicate a lot of variation in the order quantities submitted. The third column reports the medians of the distribution, and the medians are uniformly smaller than the means, indicating that the distribution is right-skewed. The final four columns give the minimum, first and third quartiles and the maximum order quantity for each choice, providing additional evidence on the skewness of the quantity distribution.

Table 4 contains information on the limit order book over our sample period. The first six rows of the table provide information on the size of the order queue at the best 3 buy and sell price quotes. The average market order size is roughly 20 round lots, so typically the quantities at the best quotes equal about 9 incoming market orders. There is a large variation in the order book quantities ranging from one to more than one thousand round lots. For an investor who demands immediacy, the cumulative quantities available in the order book are more relevant than the quantities offered at different price levels in the book. The seventh through the tenth rows of the table provide the cumulative order quantity for the two and three best bid and ask quotes. An empty limit order book does not occur in our sample and so the book can always satisfy an immediate demand for immediacy, provided it is not too large. In our sample, twelve orders walk up the book, each going one tick beyond the best quote.

We provide information on the price quotes in the last six rows of the table. The median, the first and third quartiles indicate that the three best price quotes on the bid and ask sides are spaced one tick apart most of the time. Overall, the main characteristic of the limit order book is that the order prices tend to cluster tightly around the midquote, and that the bid–ask spread is relatively constant at one tick.

A limit order submitted to this market may be either fully or partially executed or canceled. Figure 2 is a plot of the the sample survivor function for limit orders. This function is defined as the probability that a limit order remains outstanding for $t$ periods or more. In calculating the survivor function, we account for partial executions by giving weight to each observation that equals the proportion of the original order quantity executed or canceled at that time. A separate survivor function is plotted for limit orders submitted between 10:00 A.M. and 11:00 A.M. (solid line), 11:00 A.M. and 1:00 P.M. (dashed line), and 1:00 P.M. through 2:30 P.M. (dash-dot line). Roughly one in ten limit orders submitted between 1:00 P.M. and 2:30 P.M. survive for more than one hour, while more than three out of ten limit order submitted during the first hour of trading do so. Overall, only 4.26% of the limit orders last for more than one trading day, and the survivor fraction drops to 2.84% for two trading days and to 1.65% for three trading days. The expected fill ratio drops quickly as the time an order remains outstanding increases. A limit order remaining in the book for more than one trading day has an expected fill ratio of 0.43 whereas an order remaining in the book for three days has an expected fill ratio of only 0.28.

The bottom two plots in Figure 2 show the cumulative distribution function for order fill and cancellation times. In calculating these sample distribution functions we use the same weighting scheme described above to handle partial fills and cancellations. The distribution of the time–to–fill indicates that for all limit orders, 90% of the fills occur within three hours after order submission. It is evident from the plot of the distribution of order cancellations that many order cancellations are made close to the end of the trading day.

Biais, Hillion, and Spatt (1995) document that traders' order submission decisions depend on the order book in the Paris Bourse. In order to determine whether order placement decisions depend on conditioning information in our data, we estimate two ordered probit models for the choice between order types, one for buy orders and one for sell orders. Table 5 contains definitions of the conditioning variables that we use. The estimated coefficients and associated standard errors are reported on the two first rows of each panel of Table 6. We reject the null hypotheses that all coefficients are jointly equal to zero for both the buy and sell model. Rows 3 through 6 of each panel provide the marginal effects of a change in one of the explanatory variables on the different choice probabilities, evaluated at the median values of the conditioning information. The marginal effects suggest that these variables pick up systematic variation in the traders' order placement decisions.

Overall, our data shows that there is an unconditional trade–off between the limit order price, the unconditional probability that the order executes within two days and the time it takes for the limit order to be filled. The state of the limit order book changes empirically and this information is useful at predicting the composition of the order flow. In the next section, we describe the theoretical model of order submissions into the limit order book that we will use to interpret this data.

# 3 Theoretical Model

We assume that at time $t$, one potential trader arrives in the market and, for a limit amount of time, has the opportunity to receive some gains from trading the stock. We assume this window of opportunity is determined by factors outside of this model, such as positions taken in other stocks. Once a trader arrives in the market, he observes the limit order book and can decide to enter an order into the system. An order can either be a market order or a limit order. Because the trader observes the current limit order book when submitting his order, he can determine the price that results in immediate execution, and we will refer to such an order as a market order. We use the decision indicator variables $d^s_{kt}$, for $k = 0, 1, \ldots, \mathcal{K}$, $d^b_{lt}$ for $l = 0, 1, \ldots, \mathcal{L}$ and denote the trader's decision at time $t$. We let $q_t$ denote the size of the order submitted by the trader and assume that $q_t$ is chosen exogenously to the order price. It is possible to consider quantity choice in this environment, and in Appendix C, we characterize the optimal decision rule when both order quantity and order price are choice variables. We assume that $\mathcal{K} < \infty$ and that $\mathcal{L} < \infty$ so that the trader chooses from a finite set of order prices. If the trader submits a market sell order, then $d^s_{0t} = 1$, and the order price is equal to the best bid quote. If the trader submits a market buy order, then $d^b_{0t} = 1$ and the order price equals the best ask quote. If $d^s_{kt} = 1$, then the trader submits a limit sell order at the price $k$ ticks above the current best bid quote, correspondingly if $d^b_{lt} = 1$, then the trader submits a limit buy order at the price $l$ ticks below the current best ask quote. If the trader does not submit any order at time $t$, then $d^s_{kt} = 0$ for all $k$ and $d^b_{lt} = 0$ for all $l$.

All traders are assumed to be risk neutral, and choose order submission strategies to maximize their expected utility. At time $t$, we assume that a trader arrives with valuation $v_t$ per share for the asset. We assume that once a trader enters an order, at a random time in the future the investor's surplus from the order will go to zero and that the investor will cancel the order when the opportunity to receive a surplus disappears. The maximum life of the order is bounded with probability one. Following Tauchen and Pitts (1983) and Foucault (1999), we decompose $v_t$ into two components:

$$v_t = y_t + u_t. \tag{1}$$

The random variable $y_t$ represents the common value of the asset at time $t$; one interpretation is that it is equal to the markets' expectation of the liquidation value of security. The common value is a stochastic process, and changes randomly over time as the market learns new information. With this interpretation, $y_t$ is is a martingale relative to the market's information information set. That is,

$$\forall t,\ \ y_t = E_t\left[y_{t'}\right],\ \ \forall t' > t, \tag{2}$$

where the subscript $t$ refers to conditioning on the market's information set at time $t$. Since $y_t$ is stochastic, traders who enter into the market in the future will have an informational advantage relative to current traders regarding the common value of the asset. In this way, limit orders are exposed to adverse selection or the winner's curse in our model.

We refer to the random variable $u_t$ as the private component of the trader's valuation. The private component of traders' valuation is drawn independently and identically across traders from the continuous distribution

$$\text{Prob}\left( u_t \leq u \right| \text{Information at } t) = G_t\left(u\right), \tag{3}$$

with continuous density $g_t(\cdot)$. This distribution is conditional on information available at time $t$. We interpret $u_t$ as a measure of the traders' demand for immediacy or liquidity. Traders with extreme values of $u_t$ have a high desire to trade the asset immediately, and traders with private values close to zero have no particular reason to trade, unless the limit order book presents them with profits from doing so. Once a trader enters the market, his private valuation does not vary until the time that the order is canceled, while the common value portion of his valuation moves stochastically as new information arrives. If a trader submits an order to buy or sell the asset, he must pay a fixed cost of $c$ per share. This cost is the same for all types of orders submitted.

Suppose that a trader with valuation $v = y + u$ submits a buy order of size $q$, at a price $p_l$, $l$ ticks away from the ask quote, so that $d_l^b = 1$. Here, we drop the $t$ subscripts for brevity. Define $d\tilde{Q}_\tau$ as the number of shares of the order that transact $\tau$ periods from the time that the order is submitted. If the surplus goes to zero $\tau'$ periods from the order entry time, then $d\tilde{Q}_\tau = 0$ for all $\tau \geq \tau'$. We let $\bar{\mathcal{T}} < \infty$ be the maximum possible life of the order.

The payoff that the trader receives from a purchase of $d\tilde{Q}_\tau$ shares of the security in $\tau$ periods at price $p_l$ is equal to

$$d\tilde{Q}_\tau\left(\tilde{y}_\tau + u - p_l\right) = d\tilde{Q}_\tau\left(v - p_l\right) + d\tilde{Q}_\tau\left(\tilde{y}_\tau - y\right),$$

where $\tilde{y}_\tau$ is the common value of the security in $\tau$ periods. The term $d\tilde{Q}_\tau\left(v - p_l\right)$ is equal to the payoff that a trade of size $d\tilde{Q}_\tau$ would earn upon immediate execution at price $p_l$. The term $d\tilde{Q}_\tau\left(\tilde{y}_\tau - y\right)$ is equal to the number of shares transacted in $\tau$ periods multiplied by the change in the common value, and captures the winner's curse to which the order is exposed. Including the cost of submitting the order, $qc$, the realized payoff from submitting the order is equal to:

$$U(p_l, q, \text{buy}) = \sum_{\tau=0}^{\bar{\mathcal{T}}} d\tilde{Q}_\tau \,(v - p_l) + \sum_{\tau=0}^{\bar{\mathcal{T}}} d\tilde{Q}_\tau \,(\tilde{y}_\tau - y) - qc. \tag{4}$$

The expected payoff to the trader from submitting the order is equal to the expected value of equation (4), conditional on the traders' information when they enter the market,

$$\begin{aligned} E_t\left[U(p_l, q, \text{buy})|v\right] &= E_t\left[\sum_{\tau=0}^{\bar{\mathcal{T}}} d\tilde{Q}_\tau \,(v - p_l)\middle| q\right] + E_t\left[\sum_{\tau=0}^{\bar{\mathcal{T}}} d\tilde{Q}_\tau \,(\tilde{y}_\tau - y)\middle| q\right] - qc \\ &= qE_t\left[\sum_{\tau=0}^{\bar{\mathcal{T}}}\left(\frac{d\tilde{Q}_\tau}{q}\right)\middle| q\right](v - p_l) + qE_t\left[\sum_{\tau=0}^{\bar{\mathcal{T}}}\left(\frac{d\tilde{Q}_\tau}{q}\right)(\tilde{y}_\tau - y)\middle| q\right] - qc \\ &= q\,\psi_{lt}^b(q)\,(v - p_l) + q\,\xi_{lt}^b(q) - q\,c, \end{aligned} \tag{5}$$

where,

$$\psi_{lt}^b(q) \equiv E_t\left[\sum_{\tau=0}^{\bar{\mathcal{T}}}\left(\frac{d\tilde{Q}_\tau}{q}\right)\middle| q\right]$$

is the expected fill ratio for the order, defined as the expected fraction of the order that eventually transacts up to the maximum lifetime of the order, $\bar{\mathcal{T}}$, conditional on the information that the trader has at the time of submission and the order price chosen, $p_l$. If the order is a market order, then the expected fill ratio, $\psi_{0t}^b(q)$, equals one by definition.

The first term in the trader's expected payoff, $q\,\psi_{lt}^b(q)\,(v - p_l)$, is equal to the expected number or shares that will eventually transact multiplied by the current surplus per share for certain execution of the order at price $p_l$. This quantity measures the trade–off in the order submission problem between the order price and the expected number of shares transacted.

The term,

$$\xi_{lt}^b(q) \equiv E_t\left[\sum_{\tau=0}^{\bar{\mathcal{T}}}\left(\frac{d\tilde{Q}_\tau}{q}\right)(\tilde{y}_\tau - y)\middle| q\right],$$

is the winner's curse associated with the order. This measures the covariance of changes in the common value of the asset with the fraction of the order that transacts; it measures the risk of the common value moving against the limit order when it transacts against an incoming market order. The final term in the expected payoff, $q\,c$, is the cost of submitting the order. This cost is the same for all order prices, and is proportional to the order size. The expected payoff to a trader choosing a sell order of size $q$ at price $p_k$ is defined similarly.

The trader chooses the order submission strategy which maximizes his expected payoffs. Conditional on the trader's information set and the trader's order quantity $q$, the trader chooses the price of the order to submit to solve,

$$\max_{\{d^s_{kt}\in\{0,1\}\}_{k=0}^{\mathcal{K}},\{d^b_{lt}\in\{0,1\}\}_{l=0}^{\mathcal{L}}} \sum_{k=0}^{\mathcal{K}} d^s_{kt} E_t \left[U(p_k, q, \text{sell})|v\right] + \sum_{l=0}^{\mathcal{L}} d^b_{lt} E_t \left[U(p_l, q, \text{buy})|v\right], \tag{6}$$

subject to the constraint that at most one price is chosen. Let $d^{*s}_{kt}(v,q)$ and $d^{*b}_{lt}(v,q)$ be the optimal strategy. If the trader finds it optimal not to submit any order, then $d^{*s}_{kt}(v,q) = 0$ for $k = 0, \dots, \mathcal{K}$ and $d^{*b}_{lt}(v,q)$ for $l = 0, \dots, \mathcal{L}$.

Lemma 1 provides an important monotonicity property of the optimal order submission strategy for traders who enter the market with valuation $v = y + u$, who desire to trade $q$ units of the asset, and face a limit order book providing trading opportunities summarized by $\{\psi^s_{kt}(q), \xi^s_{kt}(q)\}_{k=0}^{\mathcal{K}}$ and $\{\psi^b_{lt}(q), \xi^b_{lt}(q)\}_{l=0}^{\mathcal{L}}$. The result follows from a revealed preference argument.

**Lemma 1** *Suppose that a buyer with valuation $v$ for $q$ shares optimally submits a buy order at price $l \geq 0$ ticks below the ask quote, so that $d^{*b}_{lt}(v,q) = 1$.*

1. *A buyer with valuation $v' > v$ for $q$ shares submit a buy order at a price $l'$ ticks below the ask quote such that the expected fill ratio is higher at $l'$ than at $l$:*

$$\psi^b_{l't}(q) \geq \psi^b_{lt}(q). \tag{7}$$

2. *If the expected fill ratios are strictly decreasing in the distance between the limit order price and the best ask quote, i.e. $l < l+1$ implies that $\psi^b_{lt}(q) > \psi^b_{l+1t}(q)$, $l = 0, \dots, \mathcal{L}-1$, then the buyer with valuation $v' > v$ chooses a price weakly closer to the ask quote:*

$$\psi^b_{l't}(q) \geq \psi^b_{lt}(q) \text{ and } l' \leq l. \tag{8}$$

*Similar results hold on the sell side.*

Given that the common value $y_t$ is fixed at time $t$, we can compare the order strategy of two traders with the same $y_t$, and different private values, $u_t$. Buyers with higher values of $u_t$ submit orders with higher expected fill ratios and higher prices. Conversely, the lower the buyers' $u_t$, the lower the expected fill ratio chosen, and the lower the order price chosen. It is in this sense that $u_t$ measures the trader's demand for liquidity. A similar result holds on the sell side; sellers with lower valuations choose order prices leading to higher expected fill ratios and lower prices.

Lemma 1 implies that we can partition the set of valuations into intervals in which all traders whose valuations lie within that interval and who have the same order quantity submit orders at the same price. The indifference valuations which define these intervals can be solved for explicitly. Define functions $\theta^b_{ll't}(q)$ as the valuation of a trader who is indifferent between submitting a buy order order at price $p_l$ and a buy order at price $p_{l'}$, $l' > l$,

$$\theta^b_{ll't}(q) = p_l + \frac{(p_l - p_{l'})\,\psi^b_{l't}(q) + \left(\xi^b_{l't}(q) - \xi^b_{lt}(q)\right)}{\psi^b_{lt}(q) - \psi^b_{l't}(q)}. \tag{9}$$

Similarly, the valuation of a seller indifferent between submitting an order at prices $p_k$ and $p_{k'}$, $k' > k$ is given by

$$\theta^s_{kk't}(q) = p_k - \frac{(p_{k'} - p_k)\,\psi^s_{k't}(q) + (\xi^s_{kt}(q) - \xi^s_{k't}(q))}{\psi^s_{kt}(q) - \psi^s_{k't}(q)}. \tag{10}$$

We refer to these functions as threshold valuations. The valuation of a trader who is indifferent between submitting a limit sell order at price $p_k$ and not entering any order is given by

$$\theta^s_{k\emptyset t}(q) = p_k - \left(\frac{\xi_{kt}(q) + c}{\psi^s_{kt}(q)}\right), \tag{11}$$

with a similar definition for $\theta^b_{l\emptyset t}(q)$ on the buy side.

An implication of the monotonicity of the optimal order submission strategy is that the indifference valuations associated with prices that are chosen by the traders must be monotonically increasing.

**Lemma 2** *Assume that the expected fill ratios are decreasing in the distance from the best quotes.*

1. *Let $p_{l_1} > p_{l_2} > \ldots > p_{l_L}$ denote a set of buy prices ordered so that $p_{l_1}$ is equal to the best ask*

*quote. If each of these prices is optimally chosen by some trader who trades q units at t, then*

$$\theta^b_{l_1 l_2 t}(q) > \theta^b_{l_2 l_3 t}(q) > \ldots > \theta^b_{l_{L-1} l_L t}(q) > \theta^b_{l_L \emptyset t}(q). \tag{12}$$

*A similar result holds on the sell side.*

2. *If the trader chooses to be a buyer or a seller of $q$ units based on the trading opportunities available in the limit order book, and the sell prices $p_{k_i}$, $i = 1, \ldots, K$ and the buy prices $p_{l_j}$, $j = 1, \ldots, L$ are optimal for some trader $t$ submitting quantity $q$, then*[8]

$$\theta^b_{l_1 l_2 t}(q) > \theta^b_{l_2 l_3 t}(q) > \ldots > \theta^b_{l_L \emptyset t}(q) > \theta^s_{k_K \emptyset t}(q) > \ldots > \theta^s_{k_1 k_2 t}(q). \tag{13}$$

Figure 3 provides an example where the thresholds on the buy side do not satisfy the monotonicity restriction. Here, the winner's curse for all buy limit orders is equal to zero, and the expected fill ratios are

$$\psi^b_0(1) = 1 > \psi^b_1(1) = 0.7 > \psi^b_2(1) = 0.6,$$

where the order size is equal to one and we drop the $t$ subscripts to reduce notational clutter. The conditional fill ratios are monotonically decreasing in the distance from the market order. The market order price is equal to 100, and the price tick is 1. The horizontal axis in the graph is the valuation of the trade, and the vertical axis is the expected utility for submitting various orders. The light solid line (—) is the expected utility of a trader submitting a market buy order, the dashed (- - -) line is the expected utility from submitting a limit buy order at 99 and the dash-dot line (-.-) is the expected utility from submitting a limit buy order at 98. The dark solid line is the upper envelope of the expected utility for each choice, and a submitting a limit order at 99 is not optimal for a trader with any possible valuation. The thresholds are given by

$$\theta^b_{01}(1) = 100 + \frac{(1)(0.7)}{0.3} = 102.33,$$

$$\theta^b_{12}(1) = 99 + \frac{(1)(0.6)}{0.1} = 105.00.$$

[8]Equation (13) holds if there is a set of traders who find it optimal not to submit any order. If instead all traders find it optimal to submit an order, then the terms $\theta^b_{l_L \emptyset t}(q)$ and $\theta^s_{k_K \emptyset t}(q)$ in equation (13) are replaced by the valuation of the trader who is indifferent between submitting a buy order at $p^b_{l_L}$ and a sell order at $p^s_{k_K}$.

In this example, $\theta^b_{01}(1) < \theta^b_{12}(1)$, so that no trader finds it optimal to submit a limit order at a price 99. This example show the restrictions that optimization imposes on the conditional fill ratios: Observing a trader submitting limit order at 99 cannot be rationalized by the model. In this case, our model would predict that traders either submit market orders, or limit orders at 98, depending of their valuation. If a trader's valuation is exceeds

$$\theta^b_{02}(1) = 100 + \frac{(2)(0.6)}{0.4} = 103.00,$$

then a market order is optimal, and a limit order at 98 is optimal if the valuation is lower than 103.00.

In this example, there is a range of conditional fill ratios for which the model cannot rationalize traders submitting a limit order at 99. Holding the fill ratio for a market order equal to one and the fill ratio for a limit order at 98 equal to 0.6, a limit order at 99 can be rationalized if and only if

$$\theta^b_{12}(1) = 99 + \frac{(1)(0.6)}{\psi^b_1(1) - 0.6} < \theta^b_{01}(1) = 100 + \frac{(1)(\psi^b_1(1))}{1 - \psi^b_1(1)}. \tag{14}$$

For fill ratios of a limit order at 99 satisfying $0.60 < \psi^b_1(1) < 0.75$, the fill ratios are monotonically decreasing in distance from the market order and inequality (14) is not satisfied, and if $0.75 < \psi^b_1(1) < 1$, then (14) holds. In the above example, the winner's curse terms are all equal to zero, but with non zero winner's curse terms, inequality (14) is modified accordingly.

Let $0 = k_1 < k_2 < \ldots < k_K$ index prices that are optimal for some seller, let $0 = l_1 < l_2 < \ldots < l_L$ index optimal prices for some buyer and assume that the buy versus sell decision is determined by the trading opportunities in the limit order book. Then, the optimal decision rule is given by

$$\begin{aligned}
d^{*s}_{0t}(v,q) &= \begin{cases} 1, & v \leq \theta^s_{k_1 k_2 t}(q), \\ 0, & \text{else}, \end{cases} \\
d^{*s}_{k_i t}(v,q) &= \begin{cases} 1, & v \in \left(\theta^s_{k_{i-1} k_i t}(q), \theta^s_{k_i k_{i+1} t}(q)\right], \; i = 2, \ldots, K-1, \\ 0, & \text{else}, \end{cases} \\
d^{*b}_{l_j t}(v,q) &= \begin{cases} 1, & v \in \left(\theta^b_{l_{j+1} l_j t}(q), \theta^b_{l_j l_{j-1} t}(q)\right], \; j = 2, \ldots, L-1, \\ 0, & \text{else}, \end{cases} \\
d^{*b}_{0t}(v,q) &= \begin{cases} 1, & v \geq \theta^b_{l_1 l_2 t}(q), \\ 0, & \text{else}. \end{cases}
\end{aligned} \tag{15}$$

We now describe the implications of the optimal order placement strategy for the conditional probability that different limit and market orders are observed. At time $t$, traders' valuations are given by $v_t = y_t + u_t$, with $u_t \sim G_t(\cdot)$, where $G_t(\cdot)$ is a continuous distribution and $y_t$ is common knowledge at time $t$. Using the optimal order placement strategy given above, the conditional probability that we observe a market sell order at $t$ is equal to

$$\begin{aligned}
\Pr\left(\text{Market sell at } t \,|\text{Information at } t\,\right) &= \Pr\left(v_t \leq \theta^s_{k_1k_2t}(q)|\text{Information at } t\right) \\
&= \Pr\left(y_t + u_t \leq \theta^s_{k_1k_2t}(q)\right|\text{ Information at } t\left.\right) \\
&= G_t\left(\theta^s_{k_1k_2t}(q) - y_t\right). \qquad (16)
\end{aligned}$$

Similarly, the probability that any particular limit sell order is submitted at $t$

$$\Pr\left(d^{*s}_{k_it}(v,q) = 1\middle|\text{ Information at } t\right) = G_t(\theta^s_{k_ik_{i+1}t}(q) - y_t) - G_t(\theta^s_{k_{i-1}k_it}(q) - y_t), \qquad (17)$$

with similar expressions for buy market and limit orders. If the direction of the trade does not depend on the trading opportunities in the book, then the conditional choice probabilities in equations (16) through (17) can be modified accordingly.

Figure 4 depicts the optimal order submission strategy for a state where traders find it optimal to submit market and limit orders up to two ticks away from the bid and ask prices. Here, we plot the trader's private valuation for the asset, $u$, against the price chosen. The upper curve of the plot gives the density of the distribution of private valuations, $g(\cdot)$. The thresholds minus the common value, $\theta^s_{kk'}(q) - y$ and $\theta^b_{ll'}(q) - y$ partition the private valuations into intervals, and traders within each interval make the same order choice. For example, a trader with a private valuation less than $\theta^s_{01}(q) - y$ finds it optimal to submit a market sell order. A trader with a valuation between $\theta^s_{01}(q) - y$ and $\theta^s_{12}(q) - y$ finds it optimal to submit a limit sell order at the price directly above the bid quote in the limit order book and the mass of traders who submit a limit sell order at two ticks above the bid quote is given by the area under the density of private valuations in the area marked by diagonal lines in the plot. Traders with valuations equal to $\theta^s_{2\emptyset}(q) - y$ are indifferent between submitting a sell limit order at the $2^{nd}$ highest price above the bid quote and not entering any order. Similarly, traders with valuations equal to $\theta^b_{2\emptyset}(q) - y$ are indifferent between submitting a limit buy order at the second highest limit price below the ask quote and not submitting any order. Traders with valuations between $\theta^s_{2\emptyset}(q) - y$ and $\theta^b_{2\emptyset}(q) - y$ do not find it optimal to submit an order.

To summarize, we have provided a characterization of the optimal order submission strategy for a risk neutral trader who has one chance to submit an order of an exogenously determined size. Using a revealed preference argument, we have shown that this decision problem implies a monotonicity restriction on nonlinear functions of conditional fill ratios and the winner's curse associated with alternative limit and market orders. We have also shown how the solution to this decision problem relates the distribution of valuations to the conditional probabilities of observing different order choices. We now discuss some of the important assumptions in this model in more detail.

In our model, traders evaluate each order entry decision individually. That is, in the trader's objective function, equation (4), the traders receive a surplus of zero if the order does not execute within the traders' window of opportunity. Consequently, the effects of any future order entry decisions are ignored in making the current order entry decision. The monotonicity restriction in Lemma 1 and a characterization of the optimal strategy similar to that in Lemma 2 still hold if there is a fixed, nonzero continuation value when the order does not execute. In this case, the indifference valuations in equations (10) through (11) must be adjusted for this continuation value. The lemmas also apply to the overall probability of executing over the entire trading period in a dynamic trading context, where cancellation and submission is possible. Our results do not apply to each individual order submission if the trader can withdraw and resubmit orders. More generally, we do not completely model traders' incentives to cancel orders once they are entered into the order book, but we can incorporate state dependent cancellation policies. For example, we can allow the conditional probability of cancellation to depend on the distance between the order price and the common value during the time that the order is in the limit book.

We analyze the order price choice of the trader, *conditional* on the order quantity. In Appendix C, we allow for endogenous order quantity, characterizing the optimal order price and quantity when traders choose order size based on trading profits. A natural way to model an investor's demand for securities is to derive it from a portfolio choice problem, where the investor chooses optimal holdings of the securities, subject to an intertemporal budget constraint. The solution to this portfolio problem leads to the quantity choice of the trader, and would depend upon the investor's preferences, wealth, current asset holdings and the price of all available assets.

Bertsimas and Lo (1998) study the optimal dynamic order submission problem of an institutional investor who desires to trade a fixed number of shares in a single asset over a predetermined time period, facing an exogenous price impact function and a bid–ask spread. They show that to minimize overall transactions costs, the optimal strategy is to split the total quantity over time. Hall and Rust (1999) study the optimal trading strategy of an intermediary facing an exogenous price and demand process subject to an inventory constraint. They solve for the optimal order quantity of the intermediary, and show how it depends on the inventory level and the stochastic processes followed by spot price and customer demands. Although Bertsimas and Lo (1998) and Hall and Rust (1999) do not consider the choice between market and limit orders, their results suggest that the trading opportunities offered in the limit order book would partially determine order size choices. In this case, our assumption that quantity is exogenous to the price choice would likely be incorrect. However, the trade–offs we model between the price submitted, the conditional fill ratio and the winner's curse would still be important for determining the choice between market and limit orders in these environments.

Our model also abstracts from possible differences between the orders submitted by brokers for individual investors and the orders submitted by the brokers themselves. We also ignore the effects of the broker's market power in our analysis. These choices are motivated by our data: We do not observe the identity of the investors, and on average, there are 19 active brokers trading shares of Ericsson each day, and the Herfindahl index for the dealers' shares of order submissions is 641.

Finally, we assume that $y_t$ is common knowledge to all traders at time $t$. This assumption can be relaxed. For example, a trader at time $t$ may know that common value at time $t' \neq t$. In this case, the conditional expectations in equations (5) through (6) should be conditioned on the trader's information and the results in Lemmas 1 and 2 still continue to hold with these new definitions. The conditional choice probabilities is equations (16) through (17) need to be modified to allow for the distribution of private information across the traders.

# 4 Empirical Implementation

The estimation and hypothesis tests undertaken here are based on a time series of decision indicators for these orders submitted, the execution and cancellation histories for the orders and the order book at the time of order submissions, constructed as in Section 2 above. We first provide our estimates the common value, and then we show how we estimate the conditional fill ratios and conditional winner's curse risk terms to form estimates of the threshold valuations. We then describe and implement tests of the theoretical monotonicity restrictions, concluding with our estimates of the private value distributions.

In our model, the common value is a martingale relative to the market's information set. This implies that the common value has a unit root. In order to form an estimate of the common value, we assume that there is a factor, $f_t$, such that the common value is linear in this factor,

$$y_t = Bf_t, \tag{18}$$

and that the factor follows a non-stationary process. To estimate $B$, we assume that the bid quote at time $t$, is cointegrated with the common value, so that

$$\begin{aligned} p^s_{0t} &= y_t + \varepsilon_t \\ &= Bf_f + \varepsilon_t, \end{aligned} \tag{19}$$

where $p^s_{0t}$ is the bid quote at time $t$ and $\varepsilon_t$ is a stationary process. This assumption implies that we can use a cointegrating regression between $p^s_0$ and the factor to estimate $B$. Engle and Granger (1987) show that this estimator of $B$ is super–consistent. Letting $\hat{B}$ denote the estimate of $B$ obtained by the cointegrating regression, our estimate of the common value is

$$\hat{y}_t = \hat{B}f_t. \tag{20}$$

We use minute-by-minute observations of the value of the OMX market index as our factor series.[9] The OMX index is a value weighted index of the 30 most traded companies on the Stockholm Stock Exchange. Table 7 provides the results from the cointegration analysis. The first column of the first four rows of the table report the results from a Dickey–Fuller test for a unit root in the bid quote, the ask quote, the midquote which is equal to the average of the bid and ask quotes, and the value of the OMX index in our data. The test fails to reject the unit root null. The final three columns report the results from estimating a cointegrating regression between the OMX market index and the bid, the ask and the midquotes. The regression coefficient obtained in the cointegrating regression is similar across the choice of dependent variable, the bid, the ask or the midquote. The final row reports the Engle–Granger (1987) test for cointegration. These statistics provide evidence that the bid, the ask and the midquotes are cointegrated with the OMX index.

We now turn to estimating the threshold valuations. These functions depend on the conditional fill ratios and winner's curse terms for each order, at each information set. To estimate the conditional fill ratios and winner's curse terms, we assume that the traders' have rational expectations about the conditional fill ratios and conditional winner's curse for each order, and that their conditioning information can be captured by a low dimensional set of state variables. We approximate traders' expectations using nonparametric regressions of the realized execution history of each of the orders onto a set of state variables. In these nonparametric estimates, we condition on a vector of 4 variables, denoted by $\Omega_t$ and the order quantity, denoting the conditioning information by the vector $X_t = (q_t, \Omega_t)$. Table 5 contains definitions of the conditioning variables and we now discuss our choice of conditioning variables in detail.

We expect that the conditional fill ratio is lower for a limit order of a given size when the queue of unfilled limit orders in the order book is longer. A lower expected fill ratio may also be associated with a longer expected time until the order is filled. This would make the order relatively more exposed to changes in the common value through the winner's curse. We capture these effects by conditioning on two measures of the length of the order book queue. We use the total number

[9] We have also experimented with including the US/SKr exchange rate and Swedish interest rates as factors. These variables do not have much explanatory power in the cointegrating regressions.

of shares offered in the order book within one tick and three ticks of the midquote, respectively. These measures provide a rough characterization of the length of the order book queue near the best quotes, as well as further away from the quotes. The order size itself is also likely to have a similar effect. For example, we expect that on average, it will take a longer time for larger limit order to be filled, holding everything else constant.

Trading activity in financial markets tends to be clustered in time. Engle and Russell (1998) and Engle (2000) document that on the New York Stock Exchange, periods of high activity are likely to be followed by periods of high activity and vice versa for slow periods. A limit order submitted in a period of high activity may be more likely to be filled within a given time interval. This effect may be counteracted if more traders submit limit orders to take advantage of the higher expected fill ratios. We account for such interactions by conditioning on both the level of activity and the length of the order book queue, measuring activity by trading volume over the previous ten minutes.

Shifts in the conditional volatility of the common value directly affect the relative payoffs from different limit order strategies via the winner's curse. If the volatility is expected to be high, a trader may rationally expect the winner's curse to be greater. High trading activity tends to be is associated with changes in the volatility of the security. By including both measures of volatility and trading volume we capture how interactions between these measures affect trading opportunities. We measure volatility by the standard deviation of the changes in the OMX market index over the previous sixty minutes or the number of minutes elapsed since the market's open. Our assumptions for the common value series imply that the volatility of the index is perfectly correlated with the volatility of the common value series.

The conditional fill ratios are computed as a nonparametric regression of realized fill ratios on information known at the time of order submission. Define the variable

$$\frac{d\tilde{Q}^s_{kt+\tau}}{q}\left(X_t\right),$$

equal to the fraction of the sell limit order that is transacted at time $t+\tau$, that was submitted at time $t$ when the state variables were $X_t$ at a price $k$ ticks away from the market order. Let

$J\left[h_T^{-1}\left(X_{t'}-X_t\right)\right]$ be a kernel function where $h_T$ is the bandwidth associated with each argument.[10] The nonparametric estimate of $\psi_k^s(X_t)$ is computed using the kernel estimator,

$$\hat{\psi}_k^s(X_t) \equiv \frac{\sum_{t'=1,t'\neq t}^{T}\left(\sum_{\tau=1}^{\bar{\mathcal{T}}}\frac{dQ^s_{kt'+\tau}}{q}(X_{t'})\right)J\left(h_T^{-1}(X_{t'}-X_t)\right)}{\sum_{t'=1,t'\neq t}^{T}J\left(h_T^{-1}(X_{t'}-X_t)\right)}, \tag{21}$$

with a similar definition on the buy side. Given the empirical evidence in Figure 2 that almost all limit orders remain in the limit order book for less than two days, we set the maximum life of the order, $\bar{\mathcal{T}}$ in equation (21), equal to two days.

Our maintained assumption is that the conditional fill ratios are monotonically decreasing in the distance between the limit price and the best quotes. To test the monotonicity of the fill ratios, define

$$DF \equiv E\left[I(X_t\in\bar{X})\left(\psi_{l_1}^b(X_t)-\psi_{l_2}^b(X_t),\ldots,\psi_{k_1}^s(X_t)-\psi_{k_2}^s(X_t)\right)\otimes z_t^{++}\right], \tag{22}$$

where $z_t^{++}>0$ are functions of the vector $X_t$, and $I(X_t\in\bar{X})$ is a trimming indicator for the set $\bar{X}$ in the interior of the support of $X_t$.[11] Here, $k_i<k_{i+1}$ index prices chosen by the traders with positive probability in our data. Monotonicity of the conditional fill ratios imply the null hypothesis:

$$H_0:\ \ DF>0.$$

To test this hypothesis, we form the vector

$$\widehat{DF}_T=\frac{1}{T}\sum_{t=1}^{T}\left\{I(X_t\in\bar{X})\left(\hat{\psi}_{l_1}^b(X_t)-\hat{\psi}_{l_2}^b(X_t),\ldots,\hat{\psi}_{k_1}^s(X_t)-\hat{\psi}_{k_2}^s(X_t)\right)\otimes z_t^{++}\right\}. \tag{23}$$

In Appendix B, we provide regularity conditions under which $\sqrt{T}\left(\widehat{DF}_T-DF\right)$ converges in distribution to a normal random variable, and we provide the asymptotic variance-covariance matrix. Wolak (1989) derives a test statistic for a local test of $H_0$:

$$M_{DF}=\min_{\{a|a\geq 0\}}T(\widehat{DF}_T-a)\mathcal{A}^{-1}(\widehat{DF}_T-a)', \tag{24}$$

[10]We provide the required properties for $J(\cdot)$ and the bandwidth sequence in Appendix B.

[11]The trimming indicator is used to simplify the asymptotic distribution. See Ahn and Manski (1993) for details.

and shows that under the null hypothesis, $M_{DF}$ converges in distribution to the weighted sum of chi-squared variables,

$$Pr(M_{DF} \geq r) = \sum_{i=0}^{dim(M_{DF})} \Pr[\chi_i^2 \geq r] \mathrm{w}(dim(M_{DF}), dim(M_{DF}) - i, \mathcal{A}), \tag{25}$$

where $\chi_i^2$ is a chi–squared variable with $i$ degrees of freedom, $dim(M_{DF})$ is the rank of the asymptotic variance covariance matrix and $\mathrm{w}(dim(M_{DF}), dim(M_{DF}) - i, \mathcal{A}),\ i = 0, \ldots, dim(M_{DF})$ are a set of weights which depend on the asymptotic variance–covariance matrix. Wolak (1989) describes a Monte Carlo method for calculating these weights.

Table 8 reports the results of the monotonicity tests of the conditional fill ratios. The test is computed using the thresholds for the market and one tick away limit order, the one and two tick away limit orders and the two and three tick away limit orders, for both the buy and sell sides. Each row in the table reports the point estimates of the unconditional differences in fill ratios multiplied by state variables, associated standard errors and p–values for the null of monotonicity of the conditional fill ratios for different order choices.[12] Each column corresponds to a different state variable. The final row of the table reports the $M_{DF}$ test described above for each state variable and all choices, and the final column of the table reports the test statistic across each choice. All of the point estimates are strictly positive and none of these statistics reject the null hypothesis of monotonicity of the conditional fill ratios. These tests provide no evidence against monotonicity of the conditional fill ratios.

Applying our assumption that the common value is linear in the factor, equation (20), and the assumption that $X_t$ measures traders' information,

$$\begin{aligned} \xi_k^s(X_t) &\equiv E\left[\sum_{\tau=0}^{\bar{\mathcal{T}}} \left(\frac{d\tilde{Q}_{kt+\tau}^s}{q}\right) (\tilde{y}_{t+\tau} - y_t) \middle| X_t\right] \\ &= BE\left[\sum_{\tau=0}^{\bar{\mathcal{T}}} \left(\frac{d\tilde{Q}_{k\tau}^s}{q}\right) \tilde{\Delta}_\tau f_t \middle| X_t\right], \end{aligned} \tag{26}$$

[12]The standard errors are computed with 50 lags using the method described in Appendix B to capture the overlap in the errors in the fill ratios between orders submitted at different times. The empirical results are robust to changes in the lag length. The asymptotic p-values for the monotonicity tests are computed using 10,000 Monte Carlo simulations.

where $\tilde{\Delta}_\tau f_t \equiv \tilde{f}_{t+\tau} - f_t$. Our estimate of the conditional expectation of the fraction filled times the change in the factor is

$$\hat{E}\left[\sum_{\tau=0}^{\bar{\mathcal{T}}} \frac{d\tilde{Q}_{t+\tau}^k}{q} \tilde{\Delta}_\tau f_t \middle| X_t\right] \equiv \frac{\sum_{t'=1,t'\neq t}^{T} \left(\sum_{\tau=0}^{\bar{\mathcal{T}}} \frac{d\tilde{Q}_{t'+\tau}^k}{q}(X_{t'})\tilde{\Delta}_\tau f_{t'}\right) J\left(h_T^{-1}(X_{t'} - X_t)\right)}{\sum_{t'=1,t'\neq t}^{T} J\left(h_T^{-1}(X_{t'} - X_t)\right)}. \tag{27}$$

The estimate of the conditional winner's curse, $\hat{\xi}_k^s(X_t)$, is formed by substituting the estimate of $\hat{B}$ from the cointegrating regression and the kernel estimator, (27) into equation (26) above.

We form estimates of the threshold valuations as

$$\hat{\theta}_{kk'}^s(X_t) = p_{kt} - \frac{(p_{k't} - p_{kt})\,\hat{\psi}_{k'}^s(X_t) + \left(\hat{\xi}_k^s(X_t) - \hat{\xi}_{k'}^s(X_t)\right)}{\hat{\psi}_k^s(X_t) - \hat{\psi}_{k'}^s(X_t)}, \tag{28}$$

where $p_{kt}$ is the $k$ tick away price at time $t$. We form a similar estimator for the buy side. As long as $\psi_k^s(X_t) - \psi_{k'}^s(X_t) > 0$, then $\theta_{kk'}^s(X_t)$ is a continuous function of the conditional fill ratios and conditional winner's curse terms, and so consistency of our estimators for the conditional fill ratios and winners curse terms imply that $\hat{\theta}_{kk'}^s(X_t)$ is a consistent estimator

We use our estimators for the threshold valuations, equation (28), to form a test statistic for the theoretical monotonicity restrictions in equations (12) and (13) of Lemma 2. These restrictions are that if the conditional fill ratios are monotonically decreasing in the distance of the limit order price from the best bid, then the thresholds must form a monotonic sequence.

In order to test the monotonicity hypothesis for the thresholds, we define

$$D\theta \equiv E\left[I(X_t \in \bar{X})\left(\theta_{l_1l_2}^b(X_t) - \theta_{l_2l_3}^b(X_t), \ldots, \theta_{k_2k_3}^s(X_t) - \theta_{k_1k_2}^s(X_t)\right) \otimes z_t^{++}\right], \tag{29}$$

where $p_{l_i}$ and $p_{k_j}$ are prices chosen with strictly positive probability by the traders in our data. The null hypothesis that the thresholds form a monotone sequence implies that

$$H_0: \;\; D\theta > 0.$$

To test this hypothesis, we form

$$\widehat{D\theta}_T = \frac{1}{T}\sum_{t=1}^{T}\left\{I(X_t \in \bar{X})\left(\hat{\theta}_{l_1l_2}^b(X_t) - \hat{\theta}_{l_2l_3}^b(X_t), \ldots, \hat{\theta}_{k_2k_3}^s(X_t) - \hat{\theta}_{k_1k_2}^s(X_t)\right) \otimes z_t^{++}\right\}, \tag{30}$$

In Appendix B, we provide conditions under which $\sqrt{T}\left(\widehat{D\theta_T} - D\theta\right)$ converges in distribution to a normal random variable and provide the asymptotic variance–covariance matrix. We form a similar test statistic to $M_{DF}$ in equation (24) above to test the monotonicity hypothesis.

Table 9 reports estimates of the average threshold differences. The first panel reports the average of the threshold differences multiplied by positive state variables, with associated asymptotic standard errors and p-values for the null that the differences are positive reported below each estimate.[13] Each column uses a different positive state variable, including a constant, the logarithm of the size of the order, the depth at the best quotes, the depth at the second best quotes, lagged trading volume and volatility of the common value. The final column reports the $M_{D\theta}$ statistic for each threshold difference for all the state variables jointly, with associated asymptotic p-values reported in parenthesis.

The estimates of the threshold differences are positive for all buy orders, and the asymptotic p-values do not reject the null hypothesis of monotonicity, both individually for each decision and state variable, and jointly across all state variables. On the sell side, the point estimates of the threshold differences between a three and two tick limit order and a two and one tick limit order, $E\left[(\theta^s_{23}(X_t) - \theta^s_{12}(X_t)) \otimes z^{++}\right]$, is negative for all the state variables except the depth at the best quotes. However, the test statistics do not reject the null hypothesis of monotonicity. The estimates of the average threshold differences between a one and a two tick sell order, and a market sell and one tick away limit order, $E\left[(\theta^s_{12}(X_t) - \theta^s_{01}(X_t)) \otimes z^{++}\right]$, sell are strictly positive, and fail to reject the null hypothesis of monotonicity. The point estimates of the threshold difference associated with the most aggressive buy order relative to the most aggressive sell order, $E\left[(\theta^b_{23}(X_t) - \theta^s_{23}(X_t)) \otimes z^{++}\right]$, are negative when multiplied by all state variables. The univariate p-values and joint $M_{D\theta}$ statistics all reject the hypotheses that $E\left[(\theta^b_{23}(X_t) - \theta^s_{23}(X_t)) \otimes z^{++}\right] > 0$.

The bottom panel of the table reports the joint $M_{D\theta}$ statistics for the buy side decisions, the sell side decisions and the buy and sell side decisions together, with associated asymptotic p-values reported below the coefficients. Considering the buy or sell sides separately, we do not test the

[13] The standard errors are computed as described in Appendix B using the Newey and West (1987) procedure with 50 lags and the asymptotic p-value for the $M_{D\theta}$ statistic is computed using the simulation method given in Wolak (1989) with 10,000 simulation trials.

restriction that $E\left[(\theta^b_{23}(X_t) - \theta^s_{23}(X_t)) \otimes z^{++}\right] > 0$. We do not reject the null hypothesis that the thresholds are monotone when we test the buy and sell sides separately. The final two rows of the table test the monotonicity of all thresholds simultaneously, including the restrictions when the traders chooses to be a buyer or a seller endogenously. We reject the monotonicity restriction using these statistics. In Appendix C, we develop and implement tests of the monotonicity implications of the traders' choosing their order size solely based on the trading opportunities in the limit order book. The tests are reported in Table C1, and they reject the monotonicity hypothesis of endogenous quantity choice for the market versus limit order decision. Overall, Table 9 shows that our data is consistent with the theoretical trade–offs we consider between market and limit orders, conditional the order being a buy or a sell order. We do reject the hypothesis that the trading opportunities in the limit order book are the only determinant of the decision to be a buyer or a seller, as well as the hypothesis that trading opportunities solely drive the order sizes of the traders.

Table 10 reports unconditional averages of various estimated quantities from our model. The first column of the table reports unconditional averages of the conditional fill ratios. For limit orders, the average fill ratios are monotonically decreasing in the distance from the quotes. The winner's curse estimates in the second column reveal that, on average, more aggressive limit orders face more severe adverse selection. The third column provides the expected value of the asset *conditional* on execution. This is computed by dividing the conditional winner's curse terms by the conditional fill ratios, and the table contains the unconditional average of this measure. These estimates imply that on average, executing more aggressive limit orders is associated with larger changes in the common value than executing less aggressive limit orders.

The fourth column of the table reports the expected payoff per share received by traders with a valuation equal to the indifference valuation associated with each order type. This is computed by substituting our estimates of the indifference valuations, the conditional fill ratios, conditional winner's curse terms and the common value into equation (5), dividing by the order quantity and computing the average of these estimates for various order choices. We do not subtract the order entry cost of $c$ per share in these computations. Traders submitting the limit order closest to the ask for the buy side receive an expected payoff of between 0.33 SKr and 1.98 SKr per share, while sellers closest to the bid receive a surplus of between 0.10 SKr and 1.6 SKr per share. The average payoff is decreasing in the distance from the best quotes, with traders submitting 3 tick away limit buys receiving an average payoff of -0.046 SKr per share, and sellers receiving an average payoff of 0.11 SKr per share, with neither estimate statistically significantly different from zero. Given that traders would not enter an order unless the expected payoffs adjusted for the order entry cost is positive, these estimates imply that the average order entry cost per share for orders, $c$, is in the range of 0 SKr to 0.11 SKr per share.

The final column of the table reports the average private value of the traders evaluated at the thresholds associated with various orders adjusted for a day effect with 57 day dummies. On average, buyers submitting market orders have private valuations more than 2.5 SKr per share and sellers submitting market orders have private valuations less than -2 SKr per share. Using the asymptotic standard errors, we reject the null hypothesis that that these averages are equal to zero. Buyers submitting the most aggressive limit buy orders have private valuations on average of at least -1.06 SKr per share and sellers choosing the most aggressive limit orders have private valuations of less than 0.3 SKr per share. We do not reject the null hypothesis that the average private valuations of the most aggressive sellers is equal to zero, and we reject the null hypothesis that the average private valuations of the most aggressive buyers is different from zero. The theoretical model implies that the average private values at the thresholds should be monotonically decreasing as the order price decreases and so the averages in the fifth column of the table should be decreasing as we move down the column. This monotonicity holds for the first three rows, the buy orders, but the averages are not monotonically decreasing between the three tick sell order and the three tick buy order. For the three tick sell order and below, the average private valuations again are monotonically decreasing. These results are consistent with the monotonicity statistics in Table 9 discussed above.

From equations (16) and (17),

$$E\left[\sum_{i=0}^{I} d^{*s}_{k_i}(v, X_t)\middle| X_t\right] = G\left(\theta^s_{k_I k_{I+1}}(X_t) - y_t\middle| X_t\right), \;\; I = 0, 1, \ldots, K-1, \tag{31}$$

where $k_1 = 0 < k_{i+1} < \ldots < k_K$ index prices chosen by some seller at information set $X_t$. We observe the choices made by the sellers in our data, and using our consistent estimates of $\hat{\theta}^s_{k_i k_{i+1}}(X_t)$ and the common value, equation (31) implies that a nonparametric regression of the sum of the decision indicators onto the estimated thresholds minus the estimated common values provides a consistent estimator of the private value distribution at the appropriate points. Although the common value, $y_t$ is nonstationary, our assumption that the common value and the bid price are cointegrated implies that $\theta^s_{k_i k_{i+1}}(X_t) - y_t$ is stationary. We form a similar estimator of the valuation distribution on the buy side.

Figure 5 plots the estimated cumulative distribution of the per share private valuations for the buyers and sellers. The estimates are formed from a kernel regression of the cumulative choice indicators onto the thresholds minus the common value as in equation (31).[14] The valuation distributions estimated using this procedure are plotted with a dashed line (- - -) for the buyers and a dash–dot line (-.-) for the sellers in Figure 5. We estimate the sell distribution function over the range of probabilities equal to approximately 0.25 to 0.95 for the sell side and approximately 0.05 through 0.65 for the buy side. Our estimates of the cumulative distributions depend on the conditional choice probabilities observed in the data according to equation (31) and so the estimated probability ranges reflect the conditional choice probabilities in the data. The overlap in the valuation distributions on the buy and sell sides in Figure 5 are due to the non–monotonicities in the thresholds between the buy and sell sides reported in Table 9. The range of the sell valuations is between -4 SKr and +4 SKr, while for the buy side, the range is between -6 SKr and +2 SKr. On the buy side, approximately 90% of the probability mass is for positive private valuations, while for the sell side, approximately 90% of the mass of the private valuations is negative.

In order to investigate if the distribution of private values depends on conditioning information, we compute the cumulative distribution functions of private values, conditional on the state variables used in the estimation of the thresholds. That is, for each of our state variables, we divide the sample into two sub–samples, one sub-sample when the state variable is above its median, and the other is when it is below its median. We then estimate equation (31) for each of the sub–samples, using kernel regressions. The resulting cumulative distribution functions, conditional on order size

[14] We provide the kernel function and the bandwidths in Appendix B.

are provided in Figure 6. Overall, the distributions do not vary much with order size. The buy side distribution for large and small order sizes in Figure 6 shows that buyers with orders sizes above the median tend to have relatively lower demand for immediacy than buyers with orders sizes below the median. Figure 7 plots the cumulative distribution functions for buyers and sellers, conditional on volatility above and below its median value. The distributions do not change appreciably for changes in volatility. Similar, unreported, results are obtained conditioning on the other three state variables. These estimates provide evidence that the distributions of private values, or demand for immediacy, do not vary with the conditioning information. Since the results in Table 6 show that our state variables are useful in predicting the composition of the order flow, our estimates of the private value distributions imply that variation in the common value and the relative profitability of market and limit orders drives variation in the distribution of order choices, rather than variation in the traders' demand for immediacy.

## 5 Conclusions

In this paper, we characterize the optimal order placement strategy for traders in a limit order market, starting from the standard trade-offs between the order price, the probability of transacting, and the winner's curse. A revealed preference argument shows that the optimal order strategy is a monotone function of a trader's demand for liquidity. We develop and implement a semiparametric test of this monotonicity property. We find no evidence against this monotonicity restriction when we consider buyers and sellers individually. When we combine buyers and sellers, we do find evidence against monotonicity. This is evidence that buyers and sellers consider the theoretical trade–offs we model when making their order entry decisions, but that the traders' decision to be a buyer or a seller of the asset itself does not only depend on expected profits available in the limit order book. Similarly, we document that the traders' order quantity choices depend on factors outside of the profits available in the limit order book.

We also document predictable variation in the order flow and use our estimates of the optimal order submission strategy to estimate the distribution of the traders' demand for immediacy. Our estimates reveal little evidence that these distributions depend on conditioning information, implying that variation in the common value and the relative profitability of market and limit orders drives much of the predictable variation in the distribution of order choices. Traders submitting market buy orders have private valuations that on average equal 2.3% of the average asset price and receive an average payoff that is at least 1.8% of the asset value. Traders submitting limit buy orders the next price below the best ask quote have private valuations that are between 0.1% and 2.3% of the asset price and they earn an average payoff between 0.3% and 1.8% of the asset value.

Our empirical findings have two important implications for empirical studies that measure the performance or payoff on different order placement strategies. First, they imply that the trading opportunities available in the market influence traders' order placement strategies. Second, limit orders are submitted by traders who have an active interest in trading, that is, traders with a non-zero demand for immediacy. Thus, in order to accurately measure the payoff on a given order strategy it is crucial to account for the information that the traders have when they make their order entry decisions, and to consider the valuations of these traders following a given order strategy. These considerations are likely to be relevant for any market institutions that include a limit order book in their design.

Glosten (1994) and Seppi (1997) derive equilibrium limit order books in environments where limit orders are submitted by perfectly competitive traders with a zero liquidity demand, implying that the marginal limit order earns zero expected profits. In our data, we cannot reject the null hypothesis that the average payoff of the most aggressive limit order is zero. Traders submitting the most aggressive buy limit orders have negative private valuations for the asset and traders submitting aggressive sell orders have strictly positive private valuations for the asset. Traders submitting less aggressive limit orders receive strictly positive average payoffs and the average private valuations of these traders are non zero. On average, limit orders are submitted by traders with an active interest in trading, who earn positive payoffs from their order submissions. Overall, the results are consistent with the arguments in Glosten (2000) that limit orders need not be submitted by marginal traders earning zero profits.

We use the estimated order placement strategies together along with the observed order choices to infer characteristics of the traders' demand for immediacy. This provides an important building block for welfare calculations. Glosten (2000) advocates the use of welfare calculations that account for the preferences of all groups of traders to evaluate market institutions. We do not carry out welfare calculations, but our results do provide some indication of how important it is to consider all traders in calculating the efficiency of the trading mechanism. For example, traders submitting buy limit orders close to the quotes on average have non zero private valuations and earn a positive payoff. Traditional measures of the efficiency of the trading mechanism such as the bid-ask spread ignore this by implicitly assuming that traders submitting the limit orders have zero private valuations.

Our model makes several strong assumptions: Traders evaluate each order entry decision individually, we do not model the incentives to cancel orders, we model the order price decision, conditional on the size of the order and all traders agree on the common value at each point in time. Nonetheless, we believe that our model is reasonable to apply to our data for several reasons. First, the nature of the data makes it very difficult to determine the dynamic trading strategy followed by traders, since we do not observe the identity of the retail investors in our data, and the resubmissions after cancellations. Second, the basic trade–offs in our model will be important in more elaborate dynamic environments. We test, and do not reject, the empirical restrictions implied by our model. Although we reject the restrictions implied by endogenous quantity choice in our data, there is little empirical evidence that the distributions of private valuations themselves depend on the order size. Finally, our model can be modified to allow for private information. We do not reject the model in its current form, and so this extension is left for future research.

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# A Proofs

**Proof of Lemma 1**

Given $d^{*b}_{lt}(v,q)=1$, for $v$

$$q\left(\psi^b_{lt}(q)\left(v-p_{lt}\right)+\xi^b_{lt}\right)-qc\geq q\left(\psi^b_{l't}(q)\left(v-p_{l'}\right)+\xi^b_{l't}(q)\right)-qc,$$

and for $v'$, $d^{*b}_{l't}(v',q)=1$

$$q\left(\psi^b_{l't}(q)\left(v'-p_{l'}\right)+\xi^b_{l't}(q)\right)-qc\geq q\left(\psi^b_{lt}(q)\left(v'-p_{l}\right)+\xi^b_{lt}(q)\right)-qc.$$

Multiplying the second inequality by $-1$, adding and rearranging yields:

$$\left(\psi^b_{lt}(q)-\psi^b_{l't}(q)\right)\left(v-v'\right)\geq 0, \tag{A1}$$

Proving the first part. If the conditional fill ratio is monotone in distance from the ask, then equation (A1) implies that $l'\geq l$. The proof for the sell side is symmetric.

■

**Proof of Lemma 2**

The first part of the lemma follows from the monotonicity of the optimal strategy established in Lemma 1. To show the second part of the lemma, we show that if a trader with valuation $v$ submits a buy order, then for traders with valuations $v'>v$, it is also optimal to submit a buy order. Let $k$ be an arbitrary sell order, and suppose that $d^{*b}_{lt}(v,q)=1$. Then,

$$\begin{aligned}
\psi^b_{lt}(q)\left(v'-p_{lt}\right)+\xi^b_{lt}(q)-qc &> \psi^b_{lt}(q)\left(v-p_{lt}\right)+\xi^b_{lt}(q)-qc\\
&\geq \psi^s_{kt}(q)\left(p_{kt}-v\right)-\xi^s_{kt}(q)-qc\\
&\geq \psi^s_{kt}(q)\left(p_{kt}-v'\right)-\xi^s_{kt}(q)-qc.
\end{aligned} \tag{A2}$$

The first line follows because $v'>v$ and the second line follows because a trader with valuation $v$ finds submitting a buy order at $l$ optimal and the third line follows because $v'>v$. Symmetric arguments hold on the sell side. Thus, there exists $\overline{v}\geq\underline{v}$ such that all traders with values $v>\overline{v}$ find it optimal to submit buy orders and all trader with values below $\underline{v}$ find it optimal to submit sell orders. Monotonicity of the associated thresholds follows from Lemma 1. ■

# B Econometric Appendix

In this appendix, we briefly describe the asymptotic properties for the estimators used in the monotonicity tests. We start with a general description of the estimators. Our dataset consists of a sequence of state variables, $X_t$, the decision indicators, $d^s_{kt}$, $k=0,...,K$, $d^b_{lt}$, $l=0,...,L$, the realized fills for each order, and realized product of the fills times the changes in the factor for each order. Let $w_t$ be the vector of variables whose conditional expectations we compute. We define the conditional expectations functions

$$C^s_k(X)\equiv E[w|X,d^s_k=1] \tag{B1}$$

with a similar definition for $C_l^b(X)$ and let $C(X) \equiv \left(C_1^s(X), C_2^s(X), \ldots, C_2^b(X), C_1^b(X)\right)$ be the vector of conditional expectations. The object to be estimated depends on the vector valued function $\rho(C(X), X)$. Define

$$\varrho \equiv E\left[I(X_t \in \bar{X})\rho(C(X_t), X_t)\right], \tag{B2}$$

where $I(X_t \in \bar{X})$ is a trimming indicator for the set $\bar{X}$ in the interior of the support of $X_t$. Our estimator for $\varrho$ is

$$\hat{\varrho}_T \equiv \frac{1}{T}\sum_{t=1}^{T} I(X_t \in \bar{X})\rho(\hat{C}(X_t), X_t), \tag{B3}$$

where $\hat{C}(X_t)$ is estimated using a nonparametric kernel regression.

For the tests described in the text, the vector of conditional expectations is equal to

$$C(X_t) \equiv \left(\psi_1^s(X_t), E\left[\sum_{\tau=0}^{\bar{\mathcal{T}}}\left(\frac{d\tilde{Q}_{1\tau}^s}{q}\right)\tilde{\Delta}_\tau f_t \middle| X_t\right], \ldots, \psi_1^b(X_t), \ldots\right).$$

For testing monotonicity of the conditional fill ratios,

$$\rho(C(X_t), X_t) \equiv \begin{pmatrix} \psi_0^s(X_t) - \psi_1^s(X_t), \ldots, \psi_{K-1}^s(X_t) - \psi_K^s(X_t), \\ \psi_0^b(X_t) - \psi_1^b(X_t), \ldots, \psi_{L-1}^b(X_t) - \psi_L^b(X_t) \end{pmatrix} \otimes z_t^{++},$$

where $z_t \in X_t$. For testing monotonicity of the thresholds, we define the composite function

$$\rho(\theta(C(X_t), X_t), X_t) \equiv \begin{pmatrix} \theta_{12}^s(X_t) - \theta_{01}^s(X_t), \ldots, \theta_{K-1K}^s(X_t) - \theta_{K-1K-2}^s(X_t), \\ \theta_{LL-1}^b(X_t) - \theta_{L-1L-2}^b(X_t), \ldots \theta_{01}^b(X_t) - \theta_{12}^b(X_t) \end{pmatrix} \otimes z_t^{++},$$

where $z_t \in X_t$, and

$$\theta(C(X_t), X_t) \equiv \left(\theta_{01}^s(X_t; C(X_t)), \ldots \theta_{01}^b(X_t; C(X_t))\right),$$

with

$$\theta_{01}^s(X_t) = p_{0t} - \frac{(p_{0t} - p_{1t})\,\psi_{1t}^b(X_t) + -BE\left[\sum_{\tau=0}^{\bar{\mathcal{T}}}\left(\frac{d\tilde{Q}_{1\tau}^s}{q}\tilde{\Delta}_\tau f_t\right)\middle| X_t\right]}{1 - \psi_{1t}^b(X_t)},$$

and so on.

For the quantity monotonicity tests in Appendix C,

$$\rho(C(X_t), X_t) \equiv \begin{pmatrix} \frac{E[Q_{1t+\bar{\mathcal{T}}}^b H_l(q_{lt}^*)|\Omega_t]}{E[H_1(q_{lt}^*)|\Omega_t]} - \frac{E[Q_{l't+\bar{\mathcal{T}}}^b H_2(q_{l't}^*)|\Omega_t]}{E[H_2(q_{l't}^*)|\Omega_t]}, \ldots, \\ \frac{E[Q_{l't+\bar{\mathcal{T}}}^b H_1(q_{l't}^*)|\Omega_t]}{E[H_1(q_{l't}^*)|\Omega_t]} - \frac{E[Q_{l''t+\bar{\mathcal{T}}}^b H_2(q_{l''t}^*)|\Omega_t]}{E[H_2(q_{l''t}^*)|\Omega_t]}, \ldots \end{pmatrix} \otimes z_t^{++},$$

where $X_t = (\Omega_t, q_t)$, $Q_{l't+\bar{\mathcal{T}}}^b$ is the quantity of the order filled, $H_1(\cdot)$, $H_2(\cdot)$ are weighting functions, $z_t^{++}$ is formed from element of $\Omega_t$ and the vector of conditional expectations $C(X_t)$ is defined accordingly.

Under the regularity conditions provided below, the results in Robinson (1989) and Ahn and Manski (1993)[15] imply that $\sqrt{T}\,(\hat{\varrho}_T - \varrho)$ converges in distribution to a normal random vector with covariance matrix defined below.

**A1** The data, $X_t, w_t, d^s_{kt}, k = 0, \dots, K, d^b_{lt}, l = 0, \dots, L$ are absolutely regular and the $\beta$-mixing coefficient is $o(j^{-\nu})$. For a definition of absolute regularity and the $\beta$-mixing coefficient, see Robinson (1989). We also require that

$$\sup_{X \in \bar{X}} \|\rho(C(X), X)\|^{\varphi} < \infty,$$

where $\nu > 1 + \frac{2}{\varphi - 2}$.

**A2** (a) There are M state variables. The distribution of the conditioning variables, $X_t$ has Lebesgue density $\pi(\cdot)$ which is bounded and at least M+1 times differentiable, with the first M+1 derivatives bounded.

(b) The realized fill ratios and winner's curse terms have bounded support.

(c) The conditional expectations $C^s_k(X)$ and $C^b_l(X_t)$ are M+1 times differentiable with bounded derivatives.

(d) The conditional choice probabilities, $\alpha^s_k(X) = \text{Prob}(d^s_k = 1|X)$ is M+1 times differentiable with bounded derivatives. A similar restriction holds on the buy side. The function $\pi(X)\alpha^s_k(X)$ satisfies the condition

$$\inf_{X \in \bar{X}} \pi(X)\alpha^s_k(X) > 0$$

for $k = 0, \dots, K$, similarly for the buy side. In particular, this implies that the conditional choice probability is strictly positive.

**A3** (a) The partial derivatives satisfy

$$\sup_{X_t \in \bar{X}} \left\| \frac{\partial \rho(C(X_t), X_t)}{\partial C(X_t)} \right\| < \infty.$$

(b) There is an $R < \infty$ such that the cross partial derivatives satisfy

$$\sup_{X_t \in \bar{X}} \left\| \frac{\partial^2 \rho(C(X_t), X_t)}{\partial C(X_t) \partial C(X_t)'} \right\| < R.$$

**A4** Define the matrix of expected derivatives as

$$\mu(X) \equiv E\left[ \frac{\partial \rho(C(X), X)}{\partial C(X)} \middle| X \right],$$

with generic element $\mu_{ij}(X)$. These functions satisfy

$$\frac{\mu_{ij}(X)}{\alpha^s_k(X)} < \infty,$$

and are M+1 times differentiable with bounded derivatives.

[15] Ahn and Manski (1993) consider an environment with *i.i.d.* data. The uniform consistency results from Collomb and Härdle (1986) regarding the kernel estimators applied in Ahn and Manski (1993) continue to apply in our time-series environment.

**A5** Define the vector of error terms $\epsilon^s_{kt} = d^s_{kt}[w_t - C^s_k(X_t)]$, with a similar definition for $\epsilon^b_{lt}$ and define $\epsilon_t = \sum_k \epsilon^s_{kt} + \sum_l \epsilon^b_{lt}$. There exists a positive semi-definite matrix $\mathcal{C}$ such that $\sup_{X_t \in \bar{X}} \lim_{J\to\infty} \sum_{ll=-LL}^{LL} E[\epsilon_{t-ll}\epsilon'_{t+ll}|X_t] < \mathcal{C}$.

**A6** The estimator of the conditional expectations is

$$\hat{C}^s_k(X_t) \equiv \frac{\sum_{t'=1,t'\neq t}^{T} w_{t'} d^s_{kt'} J\left(h_T^{-1}(X_{t'} - X_t)\right)}{\sum_{t'=1,t'\neq t}^{T} J\left(h_T^{-1}(X_{t'} - X_t)\right)}.$$

(a) The bandwidth sequence is such that $Th_T^{2(M+1)} \to \infty$, $T^{1-2\kappa}h_T^{2M} \to 0$ as $T \to \infty$ for some $\kappa > 0$.

(b) The kernel function $J(\cdot)$ is bounded and symmetric around zero, $\int J(z)dz = 1$ and $\int |z|^{2(M+1)}J(z)dz < \infty$. There exists $\gamma > 0$ and $c < \infty$ such that $J(\cdot)$ satisfies the Lipschitz condition that $|J(z) - J(z')| \leq c|z - z'|^\gamma$ for all $z, z' \in \mathcal{R}^M$.

(c) The first M moments of $J(\cdot)$ are zero.

Define

$$\eta_t = \rho(C(X_t), X_t) - \varrho,$$

and the vector

$$e_t \equiv \left(\frac{\epsilon^s_{0t}}{\alpha^s_0(X_t)}, \ldots, \frac{\epsilon^s_{Kt}}{\alpha^s_K(X_t)}, \frac{\epsilon^b_{0t}}{\alpha^b_0(X_t)}, \ldots, \frac{\epsilon^b_{Lt}}{\alpha^b_L(X_t)}\right).$$

Then,

$$\mathcal{A} = \lim_{LL\to\infty} \sum_{ll=-LL}^{LL} E\left[(\eta_{t-ll} + \mu(X_{t-ll})'e_{t-ll})(\eta_{t+ll} + \mu(X_{t+ll})'e_{t+ll})'\right]. \tag{B4}$$

We estimate $\eta_t$ with $\rho(\hat{C}(X_t), X_t) - \hat{\varrho}_T$, and $e_t$ is formed using a second stage kernel estimator for the conditional choice probabilities along with the residuals from the nonparametric estimators to estimate the conditional expectations $C(X_t)$. These residuals are then used in a Newey and West (1987) procedure to form an estimator for $\mathcal{A}$.

The conditional winner's curse risk terms depend both on the expectation of changes in the factors times executions, and on the factor loading are estimated with a cointegrating regression, $B$. Since the thresholds are linear in these coefficients, the super–consistency of the cointegrating regression implies that the asymptotic distribution is unaffected by pre–estimating $B$. See De Jong (2001) for details.

In implementation, we use independent Gaussian product kernels for in forming estimates of the conditional expectations, with bandwidths $4 \times 1.06 \times \hat{\sigma}(X_{it})T^{\frac{1}{2\times 5+2}}$ for the conditional fill ratios and terms in the winner's curse functions, and bandwidths $4 \times 1.06 \times \hat{\sigma}(\Omega_{it})T^{\frac{1}{2\times 4+2}}$ for the expected quantities. Here, $X_t = (X_{1t}, \ldots, X_{5t})$ and $\Omega_t = (\Omega_{1t}, \ldots \Omega_{4t})$ are the conditioning variables, with $\hat{\sigma}(X_{it})$ and $\hat{\sigma}(\Omega_{it})$ the associated sample standard deviations. For the monotonicity tests discussed in the text, we trim the outer 5% of the observations according to

$$(X_t - \bar{X})cov(X_t)^{-1}(X_t - \bar{X})',$$

where $cov(X_t)$ is the covariance matrix of the conditioning information and $\bar{X}$ is the sample mean, leaving us with 19,732 observations. For the quantity monotonicity test, we trim the outer 8% according to a similar criteria, leaving us with 19,103 observations.

In estimating the kernel regressions for the distribution of liquidity using equation (31), we use a univariate Gaussian kernel, with bandwidth equal to $2\hat{\sigma}(\hat{\theta})T^{1/5}$, where $T$ is the number of observations and $\hat{\sigma}(\hat{\theta})$ is the standard deviation of the estimated thresholds minus the common value.

# C Endogenous Quantity Choice

This appendix allows for endogenous quantity choice, and derives and implements test of the resulting implications. We will drop the $t$ subscript in this appendix, to reduce notational clutter. Suppose $q$ is a chosen optimally , and let $q\psi_l^b(q)$ and $q\xi_l^b(q)$ denote the expected quantity transacted and the total winner's curse, if the trader with valuation $v = y + u$ submits a buy order $l$ ticks above the bid price of size $q$. The expected utility from this choice is

$$U_l^b(y+u,q) \equiv q\psi_l^b(q)(y+u-p_l) + q\xi_l^b(q) - qc, \tag{C1}$$

and the trader maximizes this expression by choosing $(p_l, q)$. Without loss of generality, we normalize $y = 0$ for the remainder of this appendix.

For each price $p_l$, define

$$q_l^*(u) \equiv \arg\max_q U_l^b(u,q),$$

as the optimal quantity choice for a trader with valuation $u$, who is required to choose a price $l$ ticks away from the market. We assume that $q_l^*(u)$ is the uniquely defined interior solution.

The revealed preference arguments used in establishing the monotonicity of the conditional fill ratios when quantity is exogenous to prices also apply when quantity choice is endogenous.

**Lemma C1** *If $p_l, q_l^*$ maximizes equation (C1) for $u$ and $p_{l'}, q_{l'}^*$ maximizes equation (C1) for $u'$, then*

$$\left(q_l^*\psi_l^b(q_l^*) - q_{l'}^*\psi_{l'}^b(q_{l'}^*)\right)(u-u') \geq 0. \tag{C2}$$

*If $u \neq u'$ and $p_l, q*_l$ uniquely maximizes equation (C1) for $u$, then the inequality above is strict.*

**Proof:** We prove the second part of the lemma since the first part is proved the same way. If $(p_l, q_l^*)$ is uniquely optimal for $u$,

$$q_l^*\psi_l^b(q_l^*)(u-p_l) + q_l^*\xi_l^b(q_l^*) - q_l^*c > q_{l'}^*\psi_{l'}^b(q_{l'}^*)(u-p_{l'}) + q_{l'}^*\xi_{l'}^b(q_{l'}^*) - q_{l'}^*c,$$

and since $(p_{l'}, q_{l'}^*)$ is optimal for $u'$,

$$q_{l'}^*\psi_l^b(q_{l'}^*)(u'-p_{l'}) + q_{l'}^*\xi_{l'}^b(q_{l'}^*) - q_{l'}^*c \geq q_l^*\psi_l^b(q_l^*)(u'-p_l) + q_l^*\xi_l^b(q_l^*) - q_l^*c.$$

The result follows from subtracting the second equation from the first and rearranging. ■

To derive a testable restriction on endogenous quantity choice, we make the following additional regularity assumption.

**Assumption C1** *Suppose that $p_l, q_l^*$ is optimal for $u$, and $p_{l'}, q_{l'}^*$ is optimal for $u'$. Without loss of generality, assume that*

$$q_l^*\psi_l^b(q_l^*) - q_{l'}^*\psi_{l'}^b(q_{l'}^*) \geq 0.$$

*Then, assume that*

$$\left[q_l^*(\ddot{u})\psi_l^b(q_l^*(\ddot{u})) - q_{l'}^*(\ddot{u})\psi_{l'}^b(q_{l'}^*(\ddot{u}))\right] \geq 0, \ \forall \ddot{u}. \tag{C3}$$

This assumption says that prices can be ranked uniformly. Specifically, if the expected quantity filled is higher for $p_l, q_l^*$ than for $p_{l'}, q_{l'}^*$, then for any private valuation, the expected quantity transacted when traders are constrained to price $p_l$ exceeds the expected quantity transacted when traders are constrained to price $p_{l'}$.

The next lemma implies that when Assumption C1 holds, if $p_l$ is optimal for both traders with private values $u_1$ and $u_3$, then $p_l$ is optimal for all $u \in (u_1, u_3)$.

**Lemma C2** *Let $u_1 > u_2 > u_3$ be given and suppose that $p_{l_1}, q^*_{l_1}(u_1)$ is uniquely optimal for $u_1$ with $p_{l_2}, q^*_{l_2}(u_2)$ and $p_{l_3}, q^*_{l_3}(u_3)$ optimal for $u_2$ and $u_3$ respectively. If $p_{l_1} \neq p_{l_2}$ and Assumption C1 holds, then $p_{l_1} \neq p_{l_3}$.*

**Proof:** From the envelope theorem, for an arbitrary price, $p_i$ and valuation,

$$\frac{\partial}{\partial u} U_i \left(u, q^*_i\left(u\right)\right) = q^*_i(u)\psi^b_i\left(q^*_i\left(u\right)\right). \tag{C4}$$

Since $u_1 > u_2 > u_3$,

$$\begin{aligned}
U_{l_3}(u_3, q^*_{l_3}(u_3)) &\geq U_{l_2}(u_3, q^*_{l_2}(u_3)) \\
&= U_{l_2}\left(u_2, q^*_{l_2}(u_2)\right) - \int_{u_3}^{u_2} \frac{\partial}{\partial u} U_{l_2}\left(u, q^*_{l_2}\left(u\right)\right) du \\
&\geq U_{l_1}\left(u_2, q^*_{l_1}\left(u_2\right)\right) - \int_{u_3}^{u_2} \frac{\partial}{\partial u} U_{l_2}\left(u, q^*_{l_2}\left(u\right)\right) du \\
&= U_{l_1}\left(u_3, q^*_{l_1}(u_3)\right) + \int_{u_3}^{u_2} \frac{\partial}{\partial u} U_{l_1}\left(u, q^*_{l_1}\left(u\right)\right) du - \int_{u_3}^{u_2} \frac{\partial}{\partial u} U_{l_2}\left(u, q^*_{l_2}\left(u\right)\right) du \\
&= U_{l_1}\left(u_3, q^*_{l_1}\left(u_3\right)\right) + \int_{u_3}^{u_2} \left[q^*_{l_1}(u)\psi^b_{l_1}\left(q^*_{l_1}\left(u\right)\right) - q^*_{l_2}(u)\psi^b_{l_2}\left(q^*_{l_2}\left(u\right)\right)\right] du \\
&> U_{l_1}\left(u_3, q^*_{l_1}\left(u_3\right)\right).
\end{aligned}$$

The first line follows since $p_{l_3}, q^*_{l_3}(u_3)$ is optimal for the trader with valuation $u_3$, the second line follows from the fundamental theorem of calculus, the third line follows again from optimality, and the fourth line follows from the fundamental theorem of calculus. The fifth line follows from the envelope condition, equation (C4). Since $u_1 > u_2$, Lemma C1 implies that

$$q^*_{l_1}(u_1)\psi^b_{l_1}(q^*_{l_1}(u_1)) > q^*_{l_2}(u_2)\psi^b_{l_1}(q^*_{l_2}(u_2)),$$

and Assumption C1 implies that

$$q^*_{l_1}(u)\psi^b_{l_1}(q^*_{l_1}(u)) > q^*_{l_2}(u)\psi^b_{l_1}(q^*_{l_2}(u)), \ \forall u,$$

proving the result. ∎

Lemma C2 implies that the threshold characterization for price choice continues to hold with endogenous quantity choice under Assumption C1. That is, there is a set of thresholds, $\theta^b_{l_i l_{i+1}}$ for $i = 0, 1, \ldots L$ such that all traders with valuations $u \in [\theta^b_{l_i l_{i+1}}, \theta^b_{l_{i+1} l_{i+2}})$ will optimally choose the same order price.

**Lemma C3** *Let $u_1, u_2, u'_1, u'_2$ be four valuations, and assume that traders with valuations $u_1$ and $u_2$ find it optimal to pick price $p_l$ and a traders with valuations $u'_1$ and $u'_2$ find it optimal to pick price $p_{l'}$, where $l' > l$. Then if assumption Assumption C1 holds,*

$$\left[q^*_l(u_1)\psi^b_l\left(q^*_l(u_1)\right) - q^*_{l'}(u'_1)\psi^b_{l'}\left(q^*_{l'}(u'_1)\right)\right]\left[\psi^b_l\left(q^*_l(u_2)\right)q^*_l(u_2) - q^*_{l'}(u'_2)\psi^b_{l'}\left(q^*_{l'}(u'_2)\right)\right] \geq 0. \tag{C5}$$

*Suppose that the expected fill ratio is monotonically related to the distance from the ask quote and $q^*_l(u_1) = q^*_{l'}(u'_1) = q^*$. Then $\forall u$ such that $p_l$ is optimal and $\forall u'$ such that $p_{l'}$ is optimal,*

$$q^*_l(u)\psi^b_l\left(q^*_l(u)\right) - q^*_{l'}(u')\psi^b_{l'}\left(q^*_{l'}(u')\right) > 0. \tag{C6}$$

**Proof:** By Lemma C1 and Lemma C2, $u_1, u_2$ and $u_1', u_2'$ belong to non-overlapping intervals of the real line, since the traders choose different order prices. This implies the first part of the lemma. If $u_1$ and $u_1'$ optimally chose the same quantity, and the fill ratio is monotonically decreasing in distance from the quote, then $l' > l$ implies that

$$q^* \psi_l^b (q^*) > q^* \psi_{l'}^b (q^*),$$

and substituting into equation (C5) proves the second part of the result. ■

Lemma C3 says that the expected quality filled for all traders optimally submitting an order at one price is uniformly different from the expected quality filled for all traders optimally submitting orders at an alternative price. The second part of the lemma says that if the fill ratios are monotonically decreasing in the distance from the market price, and if there is any overlap in the quantities submitted, then the expected quantity filled is higher the closer the order is to a market order.

We are interested in using the implications of Lemma C3 to test for endogenous quantity choice in our data. The next lemma shows that convex combinations of the expected quantity filled conditional on the distance the limit price is from the spread must line up. We present the result for the case where fill ratios decrease in the distance from the spread and there is some overlap in the quantities across order choices

**Lemma C4** *Suppose that at each point in time there exists some $q_t$ such that $p_l, q_t$ is optimal for some $u_{1t}$, and $p_{l'}, q_t$ is optimal for some $u_{2t}$, and $\psi_{lt}^b (q_t) q_t > \psi_{l't}^b (q_t) q_t$. Let $U_t$ denote the set of valuations who find $p_l$ optimal to time $t$ and let $U_t'$ the set of valuations who find $p_{l'}$ optimal at time $t$. Then,*

$$E\left[\int_{U_t} q_l^*(u)\psi_{lt}^b (q_l^*(u))\, h_{lt}(q_l^*(u))du - \int_{U_t'} q_{l'}^*(u')\psi_{lt}^b (q_{l'}^*(u'))\, h_{l't}(q_l^*(u'))du' \middle| \mathit{Info\ at\ t}\right] > 0, \quad \text{(C7)}$$

*where $h_{lt}(\cdot)$ and $h_{l't}(\cdot)$ are such that $h_{lt}(\cdot) > 0$, $\int_U h_{lt}(q_{lt}^*(u))du = 1$, similarly for $h_{l't}(\cdot)$.*

**Proof:** From the previous lemma, $\forall u \in U_{lt}$, $\forall u' \in U_{l't}$,

$$\left[q_l^*(u)\psi_{lt}^b (q_l^*(u)) - q_{l'}^*(u')\psi_{l't}^b (q_{l'}^*(u'))\right] > 0. \quad \text{(C8)}$$

Integrating over an the density $h_{lt}(\cdot)$, for $u$ and using the condition that $\int_U h_{lt}(u)du = 1$,

$$\left[\int_{U_{lt}} q_l^*(u)\psi_{lt}^b (q_l^*(u))\, h_{lt}(u)du - q_{l'}^*(u')\psi_{l't}^b (q_{l'}^*(u'))\right] > 0, \quad \text{(C9)}$$

Integrating equation (C9) with respect to $h_{l't}(\cdot)$ and taking expectations yields the result. ■

We construct $h_{it}(u)$, $i = 1, 2$ empirically as follows. Let $H_i(q) : R^+ \to R^+$ satisfy

$$E\left[H_i (q_l^*(u))\middle| \Omega_t, d_{lt}^{b*} = 1\right] < \infty,$$

where we are are now conditioning on the state variables, $\Omega_t$. Define $h_{it}(u)$ as

$$h_{it}(u) \equiv \frac{H_i (q_{lt}^*(u))\, g(u)}{E_t\left[H_{it} (q_{lt}^*(u))\middle| \Omega_t, d_{lt}^{b*} = 1\right]}. \quad \text{(C10)}$$

To implement a test of restriction (C7), define

$$DQ \equiv E\left[ I(\Omega_t \in \bar{\Omega}) \begin{pmatrix} \frac{E[Q^b_{lt+\bar{T}} H_1(q^*_{lt})|\Omega_t]}{E[H_1(q^*_{lt})|\Omega_t]} - \frac{E[Q^b_{l't+\bar{T}} H_2(q^*_{l't})|\Omega_t]}{E[H_2(q^*_{l't})|\Omega_t]}, \ldots, \\ \frac{E[Q^b_{l't+\bar{T}} H_1(q^*_{l't})|\Omega_t]}{E[H_1(q^*_{l't})|\Omega_t]} - \frac{E[Q^b_{l''t+\bar{T}} H_2(q^*_{l''t})|\Omega_t]}{E[H_2(q^*_{l''t})|\Omega_t]}, \ldots \end{pmatrix} \otimes z^{++}_t \right], \tag{C11}$$

where $z^{++}_t > 0$ is a function of the information set $\Omega_t$, $I(\Omega_t \in \bar{\Omega})$ is a trimming indicator and $Q^b_{lt+\bar{T}}$ is the quantity of the order that is eventually filled, conditional on the price choice $l$. Equation (C7) and the law of iterated expectations implies the null hypothesis

$$H_0: \ DQ > 0.$$

We use the techniques described in Appendix B to form an estimator for equation (C11), and perform a version of the monotonicity test described in the text.

The results of the monotonicity test for quantity are in Table C1. Here, we use the weighting functions

$$H_1(q) \equiv 11 - \min(10, q)$$

and

$$H_2(q) \equiv \min(10, q).$$

$H_1(\cdot)$ puts more weight on low quantities than on high quantities submitted and and $H_2(\cdot)$ puts relatively more weight on high quantities than on low quantities. The standard errors are computed as described in Appendix B using the Newey and West (1987) procedure with 50 lags. Each column uses a different positive state variable, including a constant, the depth at the best quotes, the depth at the second best quotes, lagged trading volume and volatility of the common value. The final column of the table reports the $M_{DQ}$ statistic for each threshold difference for all the state variables jointly, with associated asymptotic p-values reported in parenthesis. The asymptotic p-values for the $M_{DQ}$ statistics are computed using the simulation method given in Wolak (1989) with 10,000 simulation trials. The monotonicity test rejects the monotonicity restriction implied by endogenous quantity choice for the decision between a one tick away limit order and a market order, for both the buy and sell sides using all state variables. The test does not reject the monotonicity restriction considering one versus two tick, and two versus three tick limit orders on both the buy and sell sides, for all state variables. The monotonicity restriction is also rejected when considering all decisions jointly, for all state variables.

Table 1: **Daily Trading Activity**

| | Average | Std.Dev. | Min. | Max. |
|---|---|---|---|---|
| Daily closing mid-quote | 110.15 | 9.19 | 89.00 | 127.00 |
| Daily open-to-close return (percent) | -0.22 | 2.06 | -5.29 | 4.12 |
| Daily number of active brokers | 19.28 | 2.28 | 14 | 23 |
| | | | | |
| Daily Trading Volume in millions of SKr | | | | |
| SAX system | 38.77 | 19.53 | 11.58 | 114.88 |
| Internal crosses | 12.20 | 9.04 | 1.48 | 57.51 |
| Block trades (10 A.M. -2:30 P.M. ) | 0.39 | 0.92 | 0.00 | 5.45 |
| After-hours (2:30 P.M. and later) | 4.66 | 6.15 | 0.00 | 28.26 |
| Total trading volume | 56.02 | 29.52 | 13.06 | 201.28 |
| | | | | |
| Daily number of SAX orders (10:03 A.M.-2:30 P.M.) | | | | |
| All orders | 364.23 | 141.13 | 128 | 733 |
| Limit orders | 212.18 | 77.58 | 73 | 408 |
| Market orders | 152.04 | 67.14 | 55 | 330 |

This table reports summary statistics on the daily trading activity of Ericsson. The daily open–to–close returns are calculated using the mid-quotes. The number of active brokers for each trading day is defined as the number of brokers who made at least one trade.

Table 2: **Order Flow**

| | Limit Buy Orders ticks from best ask | | | Market buy | Limit Sell ticks from best bid | | | Market sell |
|---|---|---|---|---|---|---|---|---|
| | $\geq 3$ | 2 | 1 | | $\geq 3$ | 2 | 1 | |
| Total number of orders | 893 | 992 | 4225 | 6031 | 563 | 800 | 3212 | 4044 |
| Average | 0.12 | 0.33 | 0.68 | 1.00 | 0.13 | 0.28 | 0.63 | 1.00 |
| fill ratio | (0.01) | (0.01) | (0.01) | | (0.01) | (0.02) | (0.01) | |
| Average time–to–fill | 172.33 | 73.94 | 24.55 | | 170.95 | 84.21 | 18.31 | |
| (minutes) | (14.86) | (5.22) | (0.94) | | (18.73) | (7.92) | (0.83) | |

This table reports descriptive statistics for the order flow in Ericsson. There are a total of 20,760 order submissions. The average size, fill ratio and time–to–fill with corresponding standard errors are reported for eight order categories. The fill ratio is defined as the fraction of the originally submitted limit order quantity that is traded within two trading days from the order submission. Likewise the time–to–fill reflects the average time until a limit order is filled ignoring possible fills that occur later than two trading days after the order was submitted.

Table 3: **Order Quantity**

| | | | | | Quartiles | | |
|---|---|---|---|---|---|---|---|
| | Average | Std.Dev. | Median | Min. | First | Third | Max. |
| Order Quantities in round lots (100's of Shares) | | | | | | | |
| Market buy | 19.72 | 39.21 | 6.00 | 1.00 | 2.00 | 28.00 | 859.00 |
| 1 tick buy | 25.14 | 37.60 | 10.00 | 1.00 | 2.00 | 50.00 | 500.00 |
| 2 tick buy | 29.63 | 44.58 | 10.00 | 1.00 | 3.00 | 50.00 | 500.00 |
| 3 tick buy | 14.65 | 32.28 | 4.00 | 1.00 | 1.00 | 13.00 | 400.00 |
| 3 tick sell | 23.73 | 91.29 | 10.00 | 1.00 | 2.00 | 30.00 | 2000.00 |
| 2 tick sell | 37.08 | 44.87 | 20.00 | 1.00 | 5.00 | 50.00 | 300.00 |
| 1 tick sell | 36.08 | 49.48 | 20.00 | 1.00 | 5.00 | 50.00 | 500.00 |
| Market sell | 30.73 | 48.90 | 11.00 | 1.00 | 3.00 | 50.00 | 808.00 |

This table reports descriptive statistics for the order quantities submitted at different prices. A limit buy order submitted at one tick below the current best ask quote is labeled as a 1 tick buy order. Sell orders are categorized symmetrically relative to the best bid quote. There are a total of 20,760 order submissions used in the computations, and the units are in round lots, which consist of 100 shares.

Table 4: **Order Books**

| | | | | | Quartiles | | |
|---|---|---|---|---|---|---|---|
| | Average | Std.Dev. | Median | Min. | First | Third | Max. |
| Order Book Quantities in Round Lots (100's of Shares) | | | | | | | |
| 3rd Ask | 169.8 | 105.0 | 175.9 | 1.0 | 25.0 | 296.0 | 1061.1 |
| 2nd Ask | 260.8 | 214.0 | 203.0 | 1.0 | 93.0 | 430.0 | 1161.0 |
| 1st Ask | 200.9 | 147.0 | 194.1 | 1.0 | 44.0 | 325.0 | 1314.0 |
| 1st Bid | 185.9 | 143.0 | 173.8 | 1.0 | 38.0 | 310.0 | 1504.2 |
| 2nd Bid | 242.9 | 190.0 | 217.7 | 1.0 | 86.0 | 350.0 | 1504.2 |
| 3rd Bid | 167.9 | 115.0 | 190.9 | 1.0 | 38.0 | 251.0 | 1355.2 |
| Cumulative Order Book Quantities in round lots (100's of Shares) | | | | | | | |
| 1st+2nd+3rd Ask | 631.5 | 564.0 | 368.8 | 13.0 | 295.0 | 962.0 | 1935.6 |
| 1st+2nd Ask | 461.7 | 401.0 | 294.9 | 2.0 | 205.3 | 694.0 | 1809.0 |
| 1st+2nd Bid | 428.8 | 359.0 | 310.2 | 5.0 | 194.1 | 597.0 | 2176.9 |
| 1st+2nd+3rd Bid | 596.8 | 505.5 | 396.0 | 15.0 | 296.0 | 804.0 | 2730.4 |
| Distance between Order Book Quotes and the mid–quote in ticks | | | | | | | |
| 3rd Ask | 2.68 | 2.50 | 0.56 | 2.50 | 2.50 | 2.50 | 9.50 |
| 2nd Ask | 1.58 | 1.50 | 0.29 | 1.50 | 1.50 | 1.50 | 6.50 |
| 1st Ask | 0.53 | 0.50 | 0.13 | 0.50 | 0.50 | 0.50 | 3.00 |
| 1st Bid | -0.53 | -0.50 | 0.13 | -3.00 | -0.50 | -0.50 | -0.50 |
| 2nd Bid | -1.56 | -1.50 | 0.30 | -9.00 | -1.50 | -1.50 | -1.50 |
| 3rd Bid | -2.66 | -2.50 | 0.68 | -19.00 | -2.50 | -2.50 | -2.50 |

Descriptive statistics for the order books. The statistics in the table are computed for each order book observed in the market immediately prior to an order submission. There are a total of 20,760 observations.

Table 5: **Description of the Conditioning Variables**

| Variable | Description |
|---|---|
| Order Quantity | the logarithm of the size of the order submitted at $t$ ($q_t$) |
| Depth Measure 1 | the logarithm of the total number of shares offered in the order book within one tick of the mid-quote |
| Depth Measure 2 | the logarithm of the total number of shares offered in the order book within 3 ticks of the mid-quote |
| Trading Volume | the logarithm of the cumulative number of shares transacted during the time interval $[t - 10 \text{ minutes}, t)$ |
| Common Value Volatility | the logarithm of one plus the standard deviation of the OMX market index returns over the minimum of 60 minutes and the number of minutes since the open. The std.dev. is normalized by multiplying by the square root of the number of minutes per trading day. |

Table 6: **Ordered Probit Analysis for Order Choices**

| | Qty | $\text{Depth}_1$ | $\text{Depth}_3$ | Volume | Volatility |
|---|---|---|---|---|---|
| | Buy Order Choices (N=12,141) | | | | |
| Coefficient | -0.0424 | 0.0715 | 0.0505 | 0.0339 | -0.0941 |
| | (0.0071) | (0.0037) | (0.0166) | (0.0052) | (0.0269) |
| $\partial Pr(\text{market})/\partial X_j$ | -0.0169 | 0.0285 | 0.0202 | 0.0135 | -0.0375 |
| $\partial Pr(\text{1 tick buy})/\partial X_j$ | 0.0071 | -0.0119 | -0.0084 | -0.0056 | 0.0157 |
| $\partial Pr(\text{2 tick buy})/\partial X_j$ | 0.0042 | -0.0071 | -0.0051 | -0.0034 | 0.0094 |
| $\partial Pr(\text{3 tick buy})/\partial X_j$ | 0.0056 | -0.0094 | -0.0067 | -0.0045 | 0.0124 |
| | Sell Order Choices (N=8,619) | | | | |
| Coefficient | 0.0194 | -0.0840 | 0.1223 | -0.0152 | 0.1762 |
| | (0.0086) | (0.0047) | (0.0292) | (0.0061) | (0.0326) |
| $\partial Pr(\text{market})/\partial X_j$ | -0.0077 | 0.0334 | -0.0486 | 0.0061 | -0.0701 |
| $\partial Pr(\text{1 tick sell})/\partial X_j$ | 0.0031 | -0.0136 | 0.0198 | -0.0025 | 0.0285 |
| $\partial Pr(\text{2 tick sell})/\partial X_j$ | 0.0022 | -0.0096 | 0.0140 | -0.0017 | 0.0202 |
| $\partial Pr(\text{3 tick sell})/\partial X_j$ | 0.0024 | -0.0102 | 0.0148 | -0.0018 | 0.0213 |

Estimation results from an ordered probit model of order submissions for buy and sell orders. The estimated coefficients and standard errors are reported on the first two rows of each panel. A chi-squared test statistics for the null hypothesis that all coefficients are jointly equal to zero rejects the null for both models: For buy orders, $\chi^2(5) = 569.88$, p-value=0.0000 and for sell orders, $\chi^2(5) = 386.09$, p-value=0.0000. The last four rows of each panel report the marginal effects of a change in one of the explanatory variable on the choice probabilities, evaluated at the mean values of the conditioning information.

Table 7: **Cointegration Results for Common Value Estimation**

| | Unit root test (lag length=10) | Cointegrating Regression Bid Quote | Cointegrating Regression Ask Quote | Cointegrating Regression Mid-Quote |
|---|---|---|---|---|
| Bid Quote | -1.107 | | | |
| | (0.712) | | | |
| Ask Quote | -1.081 | | | |
| | (0.723) | | | |
| Mid-Quote | -1.084 | | | |
| | (0.722) | | | |
| OMX | -0.888 | 0.3676371 | 0.362978 | 0.365308 |
| | (0.793) | (0.000946) | (0.000928) | (0.000937) |
| $R^2$ | | 0.8791 | 0.8806 | 0.8800 |
| Cointegration Test | | -3.671 | -3.619 | -3.619 |
| (lag length=10) | | (<0.025) | (<0.025) | (<0.025) |
| Number of Observations | 20,760 | | | |

Estimation of common value series. The first column reports unit root tests for the time series of best ask and bid quotes, the mid-quotes, and the OMX market index. All series are demeaned. The unit root test is an augmented Dickey-Fuller t-test and p-values are reported below each t-statistic in parenthesis. The OLS factor regressions results are reported for three different demeaned price series: the best bid quote, the best ask quote, and the mid-quote series. The estimated coefficient on the demeaned OMX market index is reported for each price series with the standard error in parenthesis. The cointegration test is an augmented Engle-Granger test. P-values are reported below the test statistics in parenthesis.

Table 8: **Test of Monotonicity of the Conditional Fill Ratios**

| | Conditioning Variables | | | | | | |
|---|---|---|---|---|---|---|---|
| | Constant | Qty | $\text{Depth}_1$ | $\text{Depth}_3$ | Volume | Volatility | $M_{DF}$ |
| market buy | 0.3169 | 2.2381 | 3.2107 | 3.6707 | 2.9025 | 0.2065 | 0 |
| - 1 tick buy | (0.0140) | (0.0982) | (0.1475) | (0.1640) | (0.1293) | (0.0120) | – |
| | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9813 |
| 1 tick buy | 0.3569 | 2.4758 | 3.5872 | 4.1368 | 3.2755 | 0.2322 | – |
| - 2 tick buy | (0.0225) | (0.1497) | (0.2349) | (0.2637) | (0.2045) | (0.0180) | 0 |
| | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9851 |
| 2 tick buy | 0.2079 | 1.3777 | 2.0194 | 2.3816 | 1.9200 | 0.1442 | – |
| - 3 tick buy | (0.0241) | (0.1599) | (0.2453) | (0.2778) | (0.2228) | (0.0192) | 0 |
| | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9833 |
| 2 tick sell | 0.1141 | 0.7802 | 1.1307 | 1.3180 | 1.0643 | 0.0772 | 0 |
| - 3 tick sell | (0.0210) | (0.1505) | (0.2150) | (0.2428) | (0.2027) | (0.0171) | – |
| | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9813 |
| 1 tick sell | 0.4220 | 2.8414 | 4.2224 | 4.8811 | 3.8804 | 0.2759 | 0 |
| - 2 tick sell | (0.0197) | (0.1395) | (0.2065) | (0.2299) | (0.1839) | (0.0167) | – |
| | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9854 |
| market sell | 0.3586 | 2.5461 | 3.6560 | 4.1570 | 3.2655 | 0.2266 | 0 |
| - 1 tick sell | (0.0142) | (0.1035) | (0.1489) | (0.1658) | (0.1284) | (0.0112) | – |
| | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9845 |
| | Joint $M_{DF}$ Statistic | | | | | | |
| All choices | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0.9824 | 0.9835 | 0.9828 | 0.9840 | 0.9838 | 0.9840 | 1.0000 |

This table reports the point estimates, asymptotic standard errors in parenthesis, inequality statistics and p-values for average differences in the fill ratios across order choices multiplied by positive state variables. The state variables are a constant, Qty: the logarithm of the order size, $\text{Depth}_1$: the logarithm of one plus the number of shares offered in the book within one tick of the mid-quote, $\text{Depth}_3$: the logarithm of one plus the number of shares offered in the book within 3 ticks of the mid-quote, Volume: the logarithm of one plus the number of shares traded over the past 10 minutes, Volatility: the logarithm of one plus an estimate of the volatility of the market index over the last hour. We ensure that all state variables are strictly positive by replacing them with 0.00001 if they are zero.

Table 9: **Monotonicity Tests of the Thresholds**

| | Conditioning Variables | | | | | | Joint |
|---|---|---|---|---|---|---|---|
| | Constant | Qty | $\text{Depth}_1$ | $\text{Depth}_3$ | Volume | Volatility | $M_{D\theta}$ |
| $E\left[(\theta^b_{01}(X_t) - \theta^b_{12}(X_t)) \otimes z_t^{++}\right]$ | 2.45 | 2.98 | 2.48 | 1.07 | 2.71 | 0.79 | 0 |
| | (0.16) | (0.21) | (0.29) | (0.11) | (0.25) | (0.08) | – |
| | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.81 |
| $E\left[(\theta^b_{12}(X_t) - \theta^b_{23}(X_t)) \otimes z_t^{++}\right]$ | 1.17 | 1.42 | 1.10 | 0.51 | 1.42 | 0.37 | 0 |
| | (0.17) | (0.20) | (0.16) | (0.10) | (0.31) | (0.07) | – |
| | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.79 |
| $E\left[(\theta^b_{23}(X_t) - \theta^s_{23}(X_t)) \otimes z_t^{++}\right]$ | -1.37 | -1.66 | -1.15 | -0.59 | -1.64 | -0.42 | 7.84 |
| | (0.50) | (0.65) | (0.64) | (0.34) | (0.70) | (0.21) | – |
| | 0.01 | 0.01 | 0.04 | 0.03 | 0.01 | 0.02 | 0.01 |
| $E\left[(\theta^s_{23}(X_t) - \theta^s_{12}(X_t)) \otimes z_t^{++}\right]$ | -0.03 | -0.043 | 0.10 | -0.01 | -0.01 | -0.02 | 0.01 |
| | (0.55) | (0.71) | (0.68) | (0.36) | (0.75) | (0.22) | – |
| | 0.48 | 0.48 | 0.56 | 0.50 | 0.49 | 0.47 | 0.76 |
| $E\left[(\theta^s_{12}(X_t) - \theta^s_{01}(X_t)) \otimes z_t^{++}\right]$ | 2.39 | 2.90 | 2.86 | 1.08 | 2.65 | 0.77 | 0.00 |
| | (0.14) | (0.19) | (0.37) | (0.09) | (0.20) | (0.07) | – |
| | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.83 |
| | Joint $M_{D\theta}$ Statistic | | | | | | |
| Buy side | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.81 | 0.79 | 0.78 | 0.81 | 0.79 | 0.81 | 0.98 |
| Sell side | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 |
| | 0.78 | 0.80 | 0.77 | 0.81 | 0.81 | 0.75 | 0.98 |
| Buy and | 86.84 | 80.69 | 60.82 | 61.09 | 39.03 | 52.10 | 96.17 |
| sell together | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

This table reports the point estimates, asymptotic standard errors in parenthesis, inequality statistics and p-values for average differences in the thresholds across order choices multiplied by positive state variables. The state variables are a constant, Qty: the logarithm of the order size, $\text{Depth}_1$: the logarithm of one plus the number of shares offered in the book within one tick of the mid-quote, $\text{Depth}_3$: the logarithm of one plus the number of shares offered in the book within 3 ticks of the mid-quote, Volume: the logarithm of one plus the number of shares traded over the past 10 minutes, Volatility: the logarithm of one plus an estimate of the volatility of the market index over the last hour. We ensure that all state variables are strictly positive by replacing them with 0.00001 if they are zero.

Table 10: **Unconditional Averages**

| Choice | Fill Ratio | Winner's Curse | $E[\Delta y \vert \text{transaction}]$ | Expected Utility | Private Value |
|---|---|---|---|---|---|
| 1 tick buy | 0.680 | 0.008 | 0.012 | 1.976 | 2.561 |
| | (0.012) | (0.022) | (0.03) | (0.119) | (0.119) |
| 2 tick buy | 0.325 | -0.111 | -0.344 | 0.334 | 0.113 |
| | (0.019) | (0.051) | (0.205) | (0.057) | (0.083) |
| 3 tick buy | 0.11 | -0.175 | -1.477 | -0.046 | -1.062 |
| | (0.014) | (0.078) | (1.675) | (0.044) | (0.126) |
| 3 tick sell | 0.106 | 0.100 | 0.925 | 0.111 | 0.309 |
| | (0.015) | (0.055) | (0.088) | (0.142) | (0.451) |
| 2 tick sell | 0.226 | 0.139 | 0.618 | 0.104 | 0.342 |
| | (0.018) | (0.058) | (0.089) | (0.061) | (0.097) |
| 1 tick sell | 0.633 | 0.008 | 0.013 | 1.612 | -2.047 |
| | (0.013) | (0.019) | (0.029) | (0.096) | (0.105) |

This table reports unconditional averages of the conditional fill ratios, conditional winner's curse terms, the expected value of changes in the common value *conditional* upon execution, the expected utility of a trader with a valuation equal to the threshold valuation, and the private valuation of a trader equal to the threshold valuation. The average private values are adjusted with 57 daily dummies. Asymptotic standard errors are reported in parenthesis, computed using 50 lags.

Table C1: **Monotonicity Test for Endogenous Quantity Choice**

| | Conditioning Variables | | | | | |
|---|---|---|---|---|---|---|
| | Constant | $\text{Depth}_1$ | $\text{Depth}_3$ | Volume | Volatility | $M_{DQ}$ |
| market buy | -2.1977 | -22.633 | -25.452 | -20.277 | -1.4669 | 43.478 |
| - 1 tick buy | (0.36026) | (3.6908) | (4.2958) | (3.7116) | (0.22289) | – |
| | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 tick buy | 0.23944 | 2.4716 | 2.7776 | 2.2383 | 0.1596 | 0 |
| - 2 tick buy | (0.234) | (2.4657) | (2.7762) | (2.0433) | (0.16682) | – |
| | 0.8469 | 0.84192 | 0.84147 | 0.86333 | 0.83065 | 0.9696 |
| 2 tick buy | -0.58599 | -6.0676 | -6.8083 | -5.4203 | -0.38355 | 1.0633 |
| - 3 tick buy | (0.63728) | (6.8904) | (7.3568) | (6.6418) | (0.37196) | – |
| | 0.17891 | 0.18927 | 0.17737 | 0.20722 | 0.15123 | 0.64566 |
| 2 tick sell | -0.093693 | -0.98649 | -1.1007 | -0.84441 | -0.057292 | 0.047564 |
| - 3 tick sell | (0.4296) | (4.6019) | (5.0603) | (4.2403) | (0.32669) | – |
| | 0.41368 | 0.41513 | 0.41391 | 0.42108 | 0.43039 | 0.93189 |
| 1 tick sell | 0.15166 | 1.5924 | 1.7796 | 1.3606 | 0.085663 | 0 |
| - 2 tick sell | (0.35889) | (3.694) | (4.2404) | (3.7398) | (0.37632) | – |
| | 0.6637 | 0.6668 | 0.66264 | 0.642 | 0.59004 | 0.9668 |
| market sell | -1.1197 | -11.534 | -12.971 | -10.342 | -0.75039 | 11.166 |
| - 1 tick sell | (0.33555) | (3.5688) | (3.9917) | (3.2573) | (0.25884) | – |
| | 0.0004 | 0.0006 | 0.0006 | 0.0007 | 0.0019 | 0.0101 |
| | Joint $M_{DQ}$ Statistic | | | | | |
| All choices | 54.696 | 55.231 | 52.049 | 44.649 | 57.039 | 62.201 |
| | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

This table reports the point estimates, asymptotic standard errors in parenthesis, inequality statistics and p-values for the quantity monotonicity test in equation (C11). The test is based on moment conditions of the form:

$$E\left[I(\Omega_t \in \bar{\Omega})\left(\frac{E[Q^b_{lt+\bar{T}}H_1(q^*_{lt})|\Omega_t]}{E[H_1(q^*_{lt})|\Omega_t]} - \frac{E[Q^b_{l't+\bar{T}}H_2(q^*_{l't})|\Omega_t]}{E[H_2(q^*_{l't})|\Omega_t]}\right) \otimes z_t^{++}\right] > 0,$$

where

$$H_1(q) \equiv 11 - \min(10, q)$$

$$H_2(q) \equiv \min(10, q).$$

The state variables are a constant, $\text{Depth}_1$: the logarithm of one plus the number of shares offered in the book within one tick of the mid-quote, $\text{Depth}_3$: the logarithm of one plus the number of shares offered in the book within 3 ticks of the mid-quote, Volume: the logarithm of one plus the number of shares traded over the past 10 minutes, Volatility: the logarithm of one plus an estimate of the volatility of the market index over the last hour. We ensure that all state variables are strictly positive by replacing them with 0.00001 if they are zero.

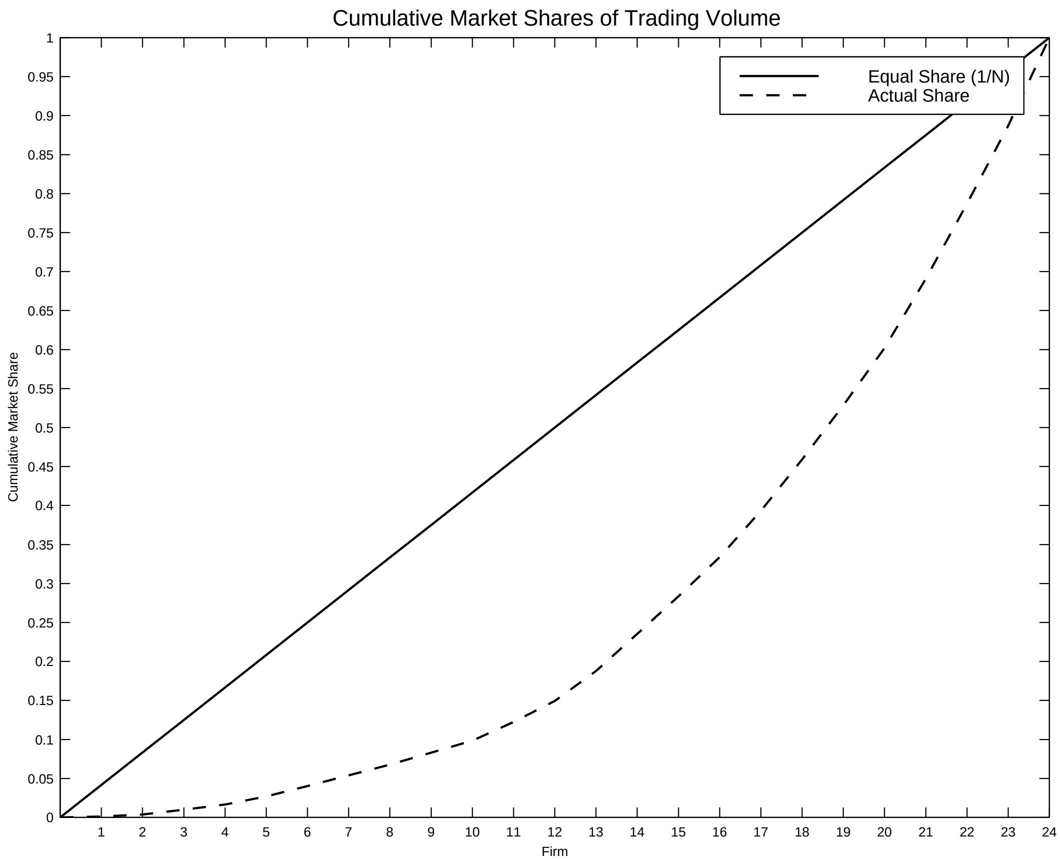


Figure 1: Dealer market shares. The dashed line represents the cumulative market shares of the dealers based on the total trading volume in our sample. The solid line plots the cumulative market shares in the hypothetical situation where the dealers have equal market shares.

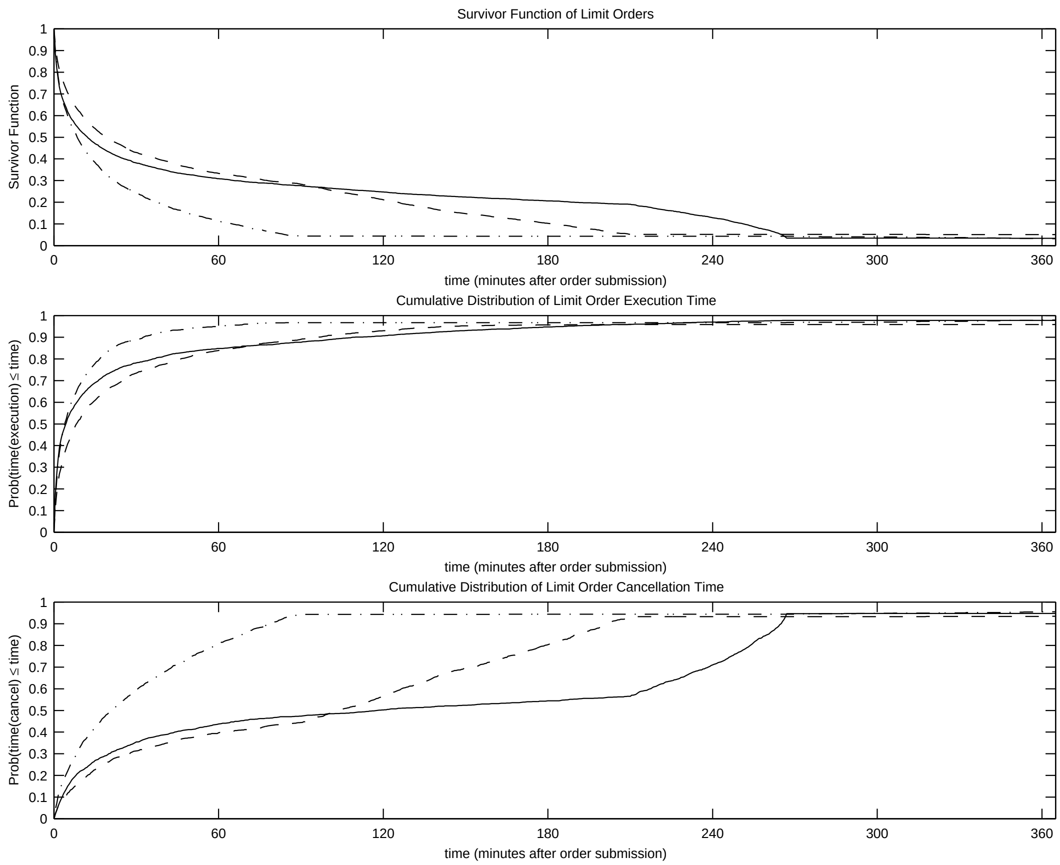


Figure 2: The top graphs give the survivor function for limit orders. The middle and the bottom graphs show the cumulative distribution functions for the order fill and cancellation times. All three functions are computed for orders submitted between 10:03 A.M. and 11:00 A.M. (solid line), 11:00 A.M. and 1:00 P.M. (dashed line ), and 1:00 P.M. – 2:30 P.M. (dash-dot line). There are a total of 11,760 orders with 4448, 3945, and 3367 in the three sub-groups. The survivor and distribution functions are calculated by assigning a weight to each observation that is proportional to the fraction of the order quantity executed or canceled. Limit orders submitting during the last two trading days in our sample or further than 5 SKr from the midquote are not used in these calculations.

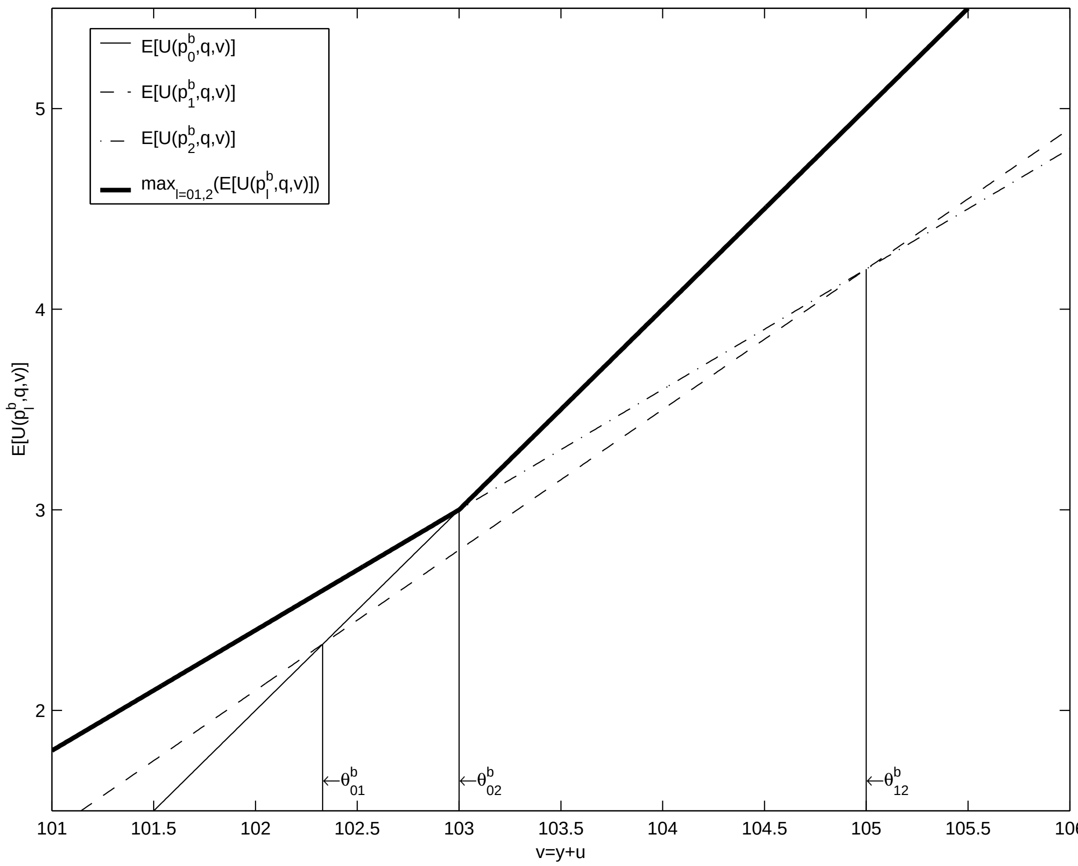


Figure 3: This figure provides an example where the thresholds do not satisfy the monotonicity restriction, but where the expected fill ratios for limit orders are monotonically decreasing farther away from the market price. The expected fill ratios are $\psi_0^b(1) = 1$, $\psi_1^b(1) = 0.7$, $\psi_2^b(1) = 0.6$, the tick size is 1 SKr, the market price is 100 SKr and all the picking off risk terms are equal to zero. We plot expected utility as a function of his valuation conditional on submitting a buy market order (–), a limit buy order one tick below the ask quote (- -), and a limit buy order two ticks below the ask quote (- .). The solid dark line is the upper envelope of the expected utility for each choice.

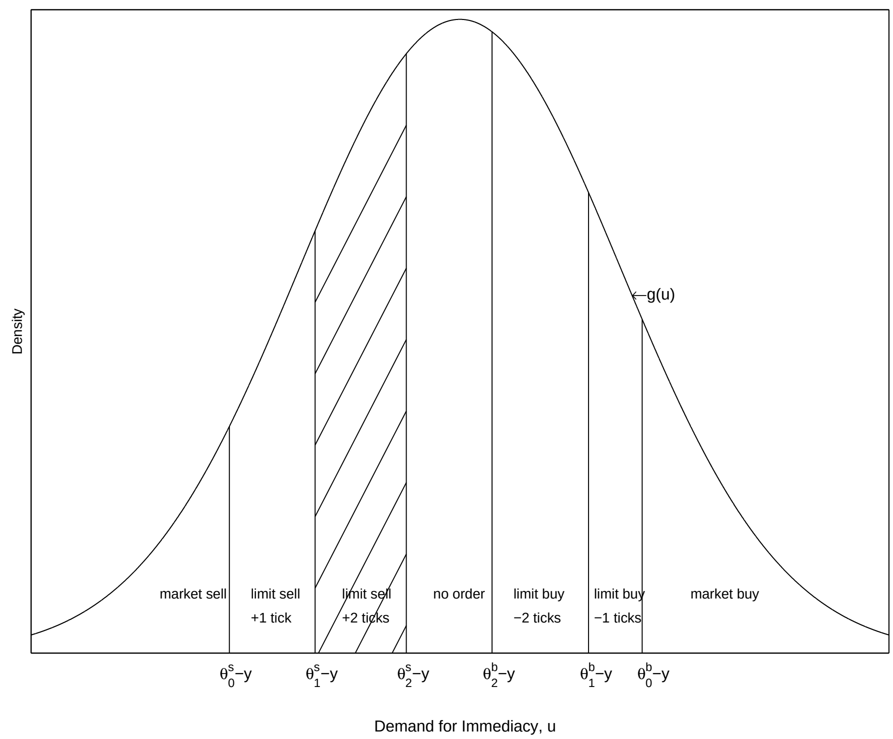


Figure 4: The graph illustrates how the probabilities of observing different order choices are determined in our model. There are two types of limit orders that a trader would consider submitting. The threshold valuations are computed using equations (9) through (11). The probability of observing a given order choice is given by the area under the probability distribution function between two adjacent threshold valuations. For example, the probability of observing a limit sell order at two ticks from the best bid quote is given by the probability that the trader has a demand for immediacy $u$ that falls in the interval $[\theta^s_{01} - y, \theta^s_{12} - y)$, which corresponds to the area with diagonal lines in the graph.

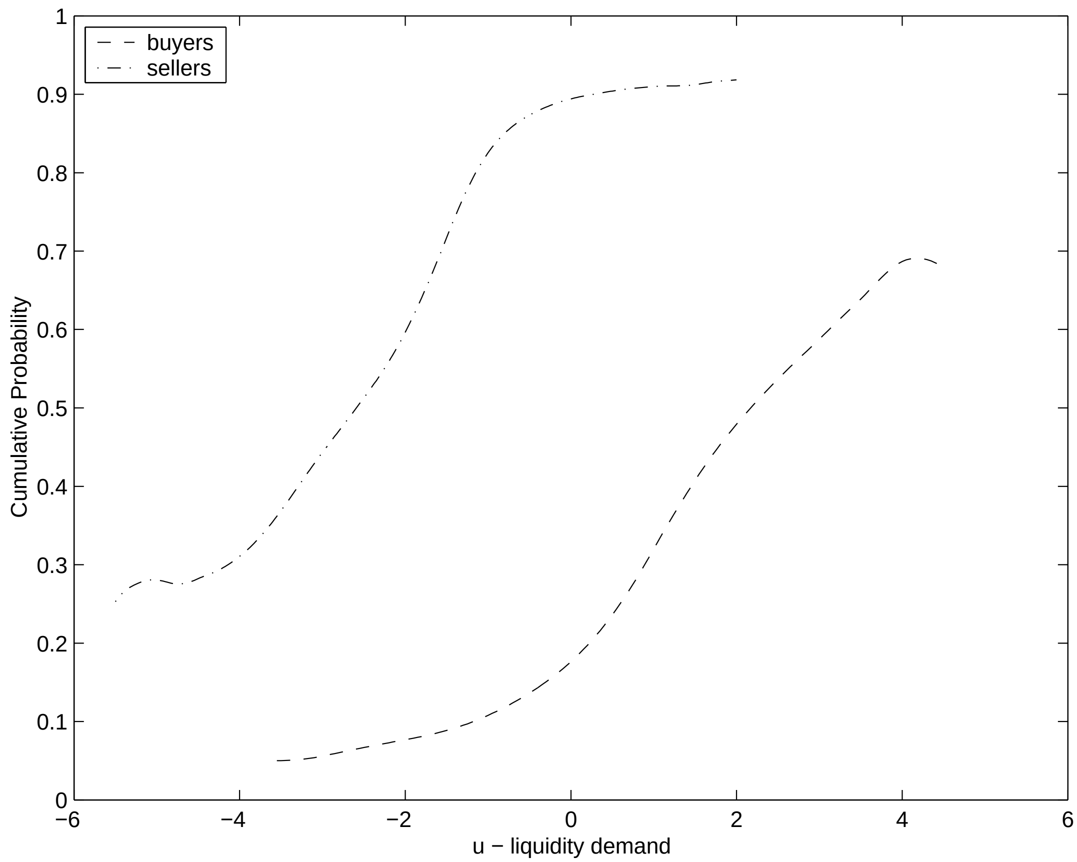


Figure 5: Nonparametric estimates of the cumulative distribution of demand for immediacy. The estimates are based on kernel regression estimates of equation (31) for all buy and sell choices, using a Gaussian kernel with bandwidth equal to $2\hat{\sigma}(\hat{\theta})(T)^{1/5}$, where we have $T = 67,194$ buy observations, $T = 47,424$ sell observations, and $\hat{\sigma}(\hat{\theta})$ is the standard deviation of the estimated thresholds less the common value. The dashed line (- - - ) is the buyer liquidity demand distribution and the dash dot line (-.-) is the seller liquidity demand distribution.

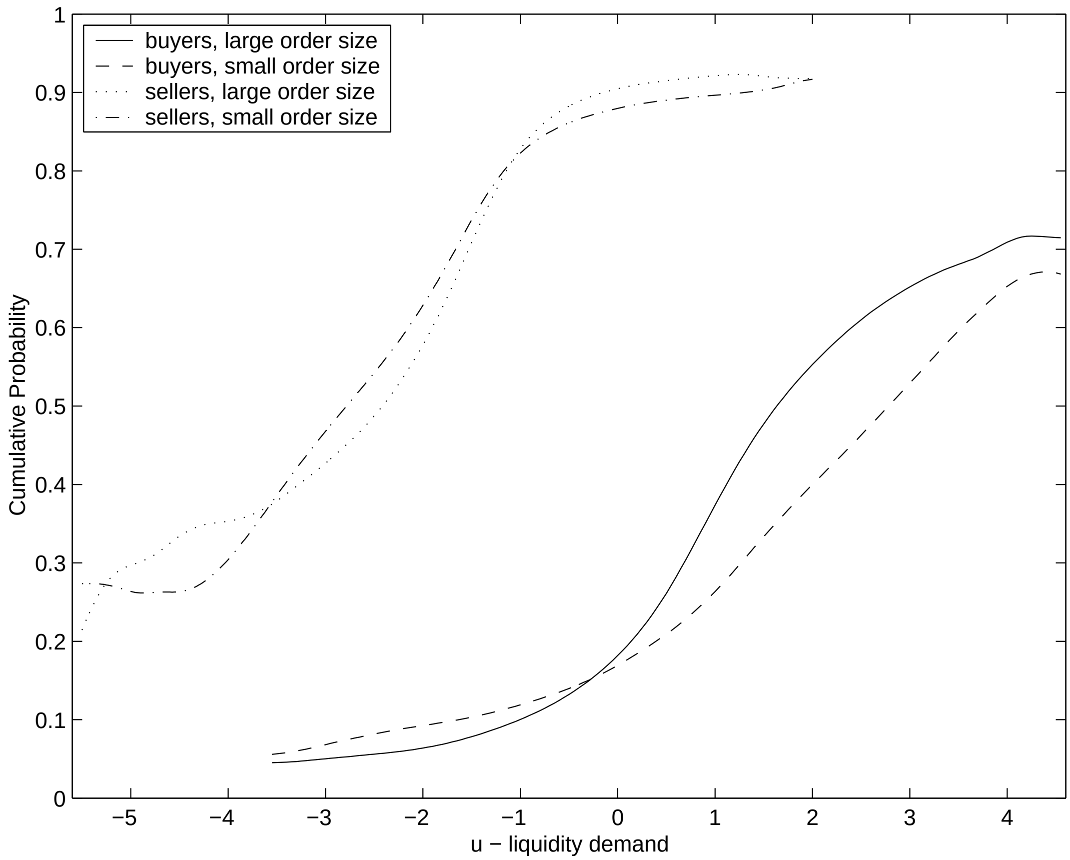


Figure 6: Nonparametric estimates of the cumulative distribution of the demand for immediacy, conditional on the order size. The estimates are based on a kernel regression estimate of equation (31) using a Gaussian kernel with bandwidth equal to $2\hat{\sigma}(\hat{\theta})(T)^{1/5}$, where $T$ is the number of observations, and $\hat{\sigma}(\hat{\theta})$ is standard deviation of the estimated thresholds less the common value. The solid line (—) is buyers with order quantities above the median order size and the dashed line (- - -) is buyers with order quantities below the median order size. The dotted line (. . .) is sellers with order quantities larger than the median and the dash dot line (- . -) is sellers with order quantities smaller than the median.

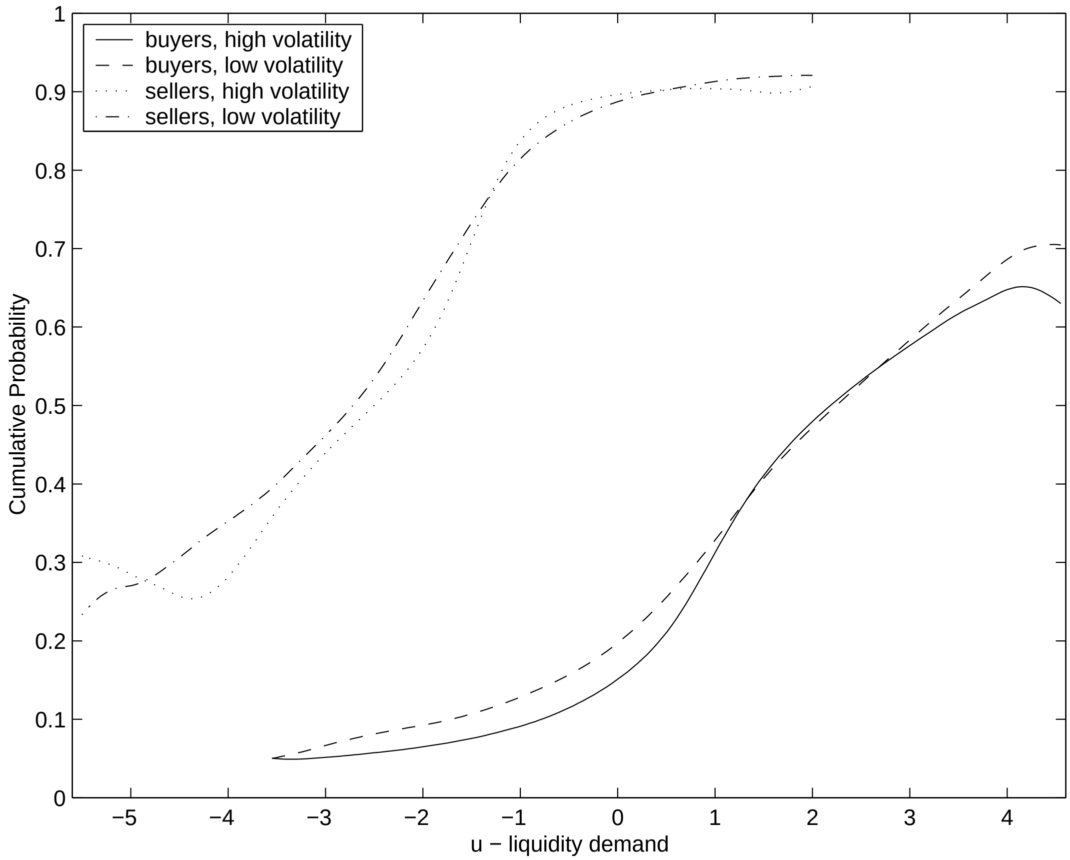


Figure 7: Nonparametric estimates of the cumulative distribution of the demand for immediacy, conditional on the volatility of the common value. The estimates are based on a kernel regression estimate of equation (31) using a Gaussian kernel with bandwidth equal to $2\hat{\sigma}(\hat{\theta})(T)^{1/5}$, where $T$ is the number of observations, and $\hat{\sigma}(\hat{\theta})$ standard deviation of the estimated thresholds less the common value. The solid line (—) is buyers when volatility is above the median order size and the dashed line (- - -) is buyers when common value volatility is below the median order size. The dotted line (. . .) is sellers when common value volatility is larger than the median and the dash dot line (- . -) is sellers when common value volatility is smaller than the median.