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ABSTRACT

Portfolio Choice and Liquidity Constraints*

We study the infinite horizon model of household portfolio choice under liquidity constraints and revisit the portfolio specialization puzzle for impatient consumers with access to riskless and risky assets. We consider a labour income process that allows us to decompose the consumption and portfolio effects of permanent and transitory shocks to labour income and show their interaction with liquidity constraints and their relative importance in producing precautionary effects and the portfolio specialization result. We show why the puzzle has proved robust for a number of model variations attempted in the literature, and argue that positive correlation between earnings shocks and stock returns is unlikely to provide a plausible resolution. We then offer an alternative explanation for observed stock-holding patterns and the slow emergence of an equity culture. Specifically, we find that relatively small, fixed, stock market entry costs are sufficient to deter households from participating in the stock market. Such entry costs could arise, for example, from informational considerations, sign-up fees and investor inertia.

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NON-TECHNICAL SUMMARY

The recent emergence of an 'equity culture' among households in the United States and in Europe has stimulated research in generalizing the single asset saving model to allow for portfolio choice between risky and riskless financial assets when households face borrowing restrictions and cannot insure against earnings risk. The saving behaviour of the median household is reasonably well approximated by the 'buffer stock' model, which assumes that the median household is impatient and shows that it typically holds a few weeks or months worth of assets in order to buffer consumption from earnings shocks. When extended to allow for portfolio choice between a riskless and a risky asset parameterized to match the properties of US equity returns, the model implies (counterfactually) that the household should invest its entire portfolio in stocks. This implication has proved robust to reasonable changes in parameter configurations and preference specifications. Heaton and Lucas (2000) show that positive correlation between stock returns and shocks to labour income (or income from business ownership) discourage portfolio specialization, and offer this as an explanation of why small savers do not hold only stocks in their portfolios. Based on empirical estimates of such correlations, however, low education households should invest more heavily in the stock market while college graduates and entrepreneurs should abstain from stock holding. Portfolio data exhibit instead a strong positive correlation between education level and stock holding.

In this Paper, we offer an alternative resolution to the portfolio specialization puzzle and observed stock-holding patterns. Specifically, we find that relatively small, fixed, stock market entry costs are sufficient to deter impatient households from participating in the stock market, while much larger costs are needed to deter wealthier segments of the population from participating. Such entry costs could arise from informational considerations, sign-up fees, and investor inertia, and they are consistent with observed differences in the stock-holding tendencies of these groups.

Intuitively, when households face earnings risk, there is tension between the desire to accumulate wealth as a buffer and the desire to limit exposure to stock-holding risk. In addition, impatient households also prefer more current consumption at the expense of future well-being. When borrowing constraints are present, households hold limited amounts of assets and quite often find themselves constrained with zero holdings of stocks and bonds. The opposite is true for more patient, wealthier households. Thus, the gain from entering the stock market is very small for the former and much larger for the latter households. As a result, a small initial entry cost can deter an impatient household from entering the stock market, while wealthier households require much larger entry costs to be deterred. The finding that small costs can deter stock holding is also consistent with the observation that the recent

emergence of an ‘equity culture’ among households came in response to the proliferation and aggressive advertising of mutual funds as well as to systematic education of workers regarding retirement accounts.

1 Introduction

Moral hazard and adverse selection problems have prevented the emergence of markets that insure households against idiosyncratic earnings risk. Such market incompleteness has stimulated substantial research interest in models of precautionary saving.[1] Following Deaton (1991) and Carroll (1992, 1997), a subset of this literature has focused on the interaction between the precautionary saving motive and liquidity constraints (the buffer stock saving model). The evidence adduced by Gourinchas and Parker (1999) and Ludvigson and Michaelides (forthcoming) is supportive of the buffer stock saving model as a plausible alternative to the classic Permanent Income Hypothesis in explaining consumption dynamics.[2]

The recent emergence of an "equity culture" among a sizeable proportion of households in the United States and in major European countries has stimulated research in generalizing the single asset saving model to allow for portfolio choice between risky and riskless financial assets under conditions of nondiversifiable earnings risk.[3] An important subset of the portfolio literature has shown that borrowing and short sales constraints can significantly affect household portfolio choice.[4] In contrast to its success in explaining median saving

[1] See, for example, Attanasio, Banks, Meghir and Weber (1999), Carroll and Samwick (1997, 1998); Hubbard, Skinner, and Zeldes (1995); Laibson, Repetto and Tobacman (1998); Ludvigson (1999).

[2] See Attanasio (1998) for an excellent recent survey of the literature on consumption.

[3] Recent empirical research on household portfolios documents increased stock market participation in Europe and in the United States and the importance of precautionary motives for portfolio choice. See Guiso, Jappelli and Terlizzese (1996), Attanasio, Banks and Tanner (1998), Alessie et al. (2000), Banks and Tanner (2000), Bertaut and McCluer (2000), Boersch-Supan and Eymann (2000), and Guiso and Jappelli (2000).

[4] Gakidis (1998) examines the interaction between borrowing constraints and undiversifiable labor income risk over the life cycle while Cocco, Gomes and Maenhout (1999) extend the life-cycle model of Bertaut and Haliassos (1997) to many periods and introduce constraints preventing borrowing through either risky or riskless assets. Haliassos and Hassapis (1998) retain the small-scale aspect but extend Bertaut and Haliassos (1997) to allow for income- and collateral-based borrowing constraints of various degrees of tightness. They show that borrowing constraints can have a major influence on the portfolio effects of risk aversion and of earnings risk, and that the presence of constrained households tends to bias empirical estimates of precautionary effects downwards. Constantinides, Donaldson and Mehra (1998) consider a general-equlibrium model of households that live for three periods and argue that liquidity constraints faced by younger cohorts

behavior, an infinite-horizon model with such constraints yields a counterfactual portfolio specialization result, as demonstrated by Heaton and Lucas (1997, 2000). In the absence of correlation between stock returns and earnings shocks, such a model implies that households should invest all of their wealth in stocks. This implication is robust to habit persistence, transactions costs, risk aversion, and to an equity premium as low as two percent.[5] Heaton and Lucas (2000) find that positive correlation between stock returns and shocks to labor income (or income from business ownership) tends to discourage portfolio specialization. In view of the surprising robustness of the portfolio specialization result to other modifications they have considered, they offer such positive correlation as an explanation of why small savers do not hold only stocks in their portfolios.[6]

Although the result regarding positive correlation between earnings and stock returns is a useful first step towards understanding the nature of the portfolio specialization puzzle, it is unlikely to provide a full resolution of the puzzle. Emerging empirical evidence on the correlation between earnings and stock returns for different population groups is hard to reconcile with observed stockholding behavior. In one of the first studies attempting to quantify this correlation, Davis and Willen (1999) obtain estimates ranging between .1 and .3 over most of the working life for college educated males and around $-.25$ at all ages for male high school dropouts.[7] Heaton and Lucas (1999) find that entrepreneurial risk is positively

who expect higher earnings in the future can be crucial in accounting for the equity premium. Storeslettern, Telmer and Yaron (1998) show how a general equilibrium life cycle model with short sales and borrowing constraints and persistent idiosyncratic shocks can explain part of the observed equity premium puzzle.

[5] Michaelides (2000) shows that co-existence of stocks and bonds in a portfolio can arise in some instances in the presence of stock market predictability, undiversifiable labor income risk, borrowing and short sales constraints. Nevertheless, this prediction arises for only some realizations of the factor predicting future returns; in most cases the household is either fully invested in the stock market or holds no stocks. Moreover, the model (counterfactually) predicts that median stockholding should be one.

[6] Koo (1995) analyzes a similar infinite-horizon model, but only for the case of zero correlation between earnings shocks and stock returns. Campbell, Cocco, Gomes, Maenhout and Viceira (1998) solve an infinite horizon model of optimal portfolio allocation when stock market returns exhibit mean reversion, but they assume that individual labor income is riskless.

[7] They use the Annual Demographic Files of the March Current Population Survey (CPS) to construct panel data on mean annual earnings between 1963 and 1994.

correlated with stock returns and reaches levels around .2. In contrast to positive correlation, negative correlation between earnings and stock returns implies increased willingness to invest in the stock market as a hedge against earnings risk. Thus, based on the empirical estimates of such correlations, low education households should be more heavily invested in the stock market while college graduates and entrepreneurs should tend to abstain from stock holding. Portfolio data are at variance with this implication, and they exhibit a strong positive correlation between education level and stock holding (see Mankiw and Zeldes, 1991, and Haliassos and Bertaut, 1995).[8]

In this paper, we revisit the portfolio specialization puzzle. We first subject the infinite-horizon model with liquidity constraints to closer scrutiny to shed light on its properties and explore the sources of the puzzle. As part of this exercise, we consider a labor income process that allows us to decompose the consumption and portfolio effects of permanent and transitory shocks to labor income and show their interaction with liquidity constraints and their relative importance in producing precautionary effects and the portfolio specialization result. We then offer an alternative explanation for observed stock holding patterns. Specifically, we find that the interactions explored in the first part of the paper imply that relatively small fixed stock market entry costs are sufficient to deter households from participating in the stock market. Such entry costs could arise, for example, from informational considerations, sign-up fees, and investor inertia.

Why do small fixed entry costs deter portfolio specialization in stocks? In models that allow for precautionary motives, there is a tension between the desire to accumulate precautionary wealth to buffer consumption effects of earnings shocks (prudence) and the desire to limit exposure to stockholding risk. In models of impatient households, there is a further conflict between prudence and impatience: the former encourages asset holding, while the latter encourages current consumption at the expense of future well-being. We find that when liquidity constraints are present, and for a plausible range of parameter values, infinite-horizon households find themselves constrained with zero holdings of stocks and bonds around thirty percent of the time. Moreover, very small mean saving is consistent with the consumption

[8] Mankiw and Zeldes use data from the Panel Study of Income Dynamics, while Haliassos and Bertaut employ the Survey of Consumer Finances.

smoothing objectives. Both factors make the gain from entering the stock market very small; thirty percent of the time the equity premium is given up due to the liquidity constraint, while for the rest of the times the gain is small due to the limited amount of savings. As a result, a small initial entry cost can deter a rational individual from entering the stock market, suggesting that entry costs could generate the observed reluctance of households to undertake stockholding. The finding that small costs can deter stockholding is also consistent with the observation that the recent emergence of an "equity culture" among households came in response to the proliferation and aggressive advertizing of mutual funds as well as to systematic education of workers regarding retirement accounts.

On the purely technical side, we show how to generalize to portfolio models the numerical solution technique proposed by Deaton (1991) for single-asset models, and how to use the invariant wealth distribution to compute time- and population averages in portfolio models. We also demonstrate a methodological point that is broadly applicable to dynamic programming optimization models. Specifically, policy function shifts induced by a certain type of shocks (e.g., transitory earnings shocks) may be considerably smaller than those induced by another type (e.g. permanent earnings shocks), while their influence on the distribution of outcomes (e.g. mean asset holdings) may be larger. This suggests that inferences on the relative importance of shocks for economic behavior should not be based solely on policy functions but should also be verified through stochastic simulation or computation of invariant distributions.

The paper is organized as follows. Section 2 describes the economic environment in the portfolio and saving models. Section 3 discusses the numerical solution method for the portfolio model that generalizes the Deaton (1991) method for solving the saving model. Section 4 discusses policy functions and time series moments of consumption, stock and bond holdings, and the portfolio share of risky assets. It examines effects of risk aversion, and of precautionary motives arising from transitory and permanent shocks to labor incomes. Section 5 analyzes the effects of correlation between stock market returns and both types of labor income shocks. Section 6 derives threshold entry costs sufficient to keep households out of the stock market under alternative parameter configurations, while Section 7 concludes.

2 The Model

We consider the problem of an infinitely-lived household that maximizes expected intertemporal utility faced with a menu of a risky and a riskless asset. The household solves

$$MAX_{\{B_t,S_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t), \tag{1}$$

subject to

$$C_t + B_t + S_t \leq X_t \tag{2}$$

$$X_{t+1} = S_t \widetilde{R}_{t+1} + B_t R_f + Y_{t+1} \tag{3}$$

$$C_t \geq 0 \tag{4}$$

$$B_t \ \geq 0 \tag{5}$$

$$S_t \geq 0 \tag{6}$$

All variables are in real terms. B_t and S_t are real amounts of the riskless asset (bonds) and of the risky asset (stocks), respectively, that are held between the beginning of period t and the beginning of period $t+1$. E_t denotes the mathematical expectation operator based on information available up to the beginning of period t, while β is the discount factor that satisfies $0 < \beta < 1$. $U(C_t)$ is the felicity derived from consumption in period t, X_t is cash on hand at the beginning of period t, $\widetilde{R}_{t+1}$ is the risky gross return on stocks held between the beginning of period t and that of period $t+1$, R_f is the gross riskless rate which is assumed time-invariant, and Y_t is labor income received at the beginning of period t.

The budget constraint (2) will hold with equality, given the assumption of non-satiation. We assume that the period-by-period felicity function is of the constant relative risk aversion (CRRA) form

$$U(C_t) = \frac{C_t^{1-\rho} - 1}{1-\rho}, \quad \rho \neq 1, \quad \rho > 0 \tag{7}$$

$$U(C_t) = \ln C_t, \quad \text{when } \rho = 1. \tag{8}$$

Constraint (4) is never binding under CRRA utility, since $\lim_{C_t \to 0} U'(C_t) = \infty$.

Constraints (5) and (6) are a direct generalization of the liquidity constraint imposed by Deaton in a single-asset model. The Deaton constraint precludes borrowing via short sales of the single asset, while (5) and(6) preclude short sales of either available asset, namely borrowing at the riskless or the risky rate. We refer to the benchmark model with no portfolio choice as the "saving model", since households can only choose B_t and have no access to the stock market.

2.1 Labor Income

Labor income risk is nondiversifiable because of moral hazard and adverse selection considerations, and it cannot be ignored by households concerned about their consumption paths. We assume that labor income of household i follows:

$$Y_{it} = P_{it} U_{it}, \tag{9}$$

where

$$P_{it} = G P_{it-1} N_{it} \tag{10}$$

This process, first used in a nearly identical form by Carroll (1992), is decomposed into a "permanent" component, P_{it}, and a transitory component, U_{it}. We assume that $\ln U_{it}$ and $\ln N_{it}$ are each independent and identically (normally) distributed with means $\{-.5 * \sigma_u^2$, $-.5 * \sigma_n^2\}$, and variances σ_u^2 and σ_n^2, respectively. The lognormality of U_{it} and the assumption about the mean of its logarithm imply that

$$EU_{it} = \exp(-.5 * \sigma_u^2 + .5 * \sigma_u^2) = 1 \tag{11}$$

and similarly for EN_{it}. Thus, precautionary wealth and portfolio effects can be computed despite the introduction of lognormally distributed multiplicative shocks. Computation of precautionary effects involves comparison of models in which household i is guaranteed in period t a certain level of income $\overline{Y}_{it}$ versus models in which the same household faces income risk but still has expected income equal to $\overline{Y}_{it}$.

The log of P_{it}, evolves as a random walk with a deterministic drift, $\mu_g = \ln G$, assumed to be common to all individuals. Given these assumptions, the growth in individual labor income follows

$$\Delta \ln Y_{it} = \ln G + \ln N_{it} + \ln U_{it} - \ln U_{it-1}, \tag{12}$$

where the unconditional mean growth for individual earnings is $\mu_g - .5 * \sigma_n^2$, and the unconditional variance equals $(\sigma_n^2 + 2\sigma_u^2)$. Individual income growth in (12) has a single Wold representation that is equivalent to the MA(1) process for individual earnings growth estimated using household level data (MaCurdy [1982], Abowd and Card [1989], and Pischke [1995]).[9]

2.1.1 Calibration of Parameters

We set the rate of time preference, δ, equal to 0.1, and the constant real interest rate, r, equal to 0.02. Carroll (1992) estimates the variances of the idiosyncratic shocks using data from the *Panel Study of Income Dynamics*, and our benchmark simulations use values close to those: 0.1 percent per year for σ_u and 0.08 percent per year for σ_n. We set μ_g equal to 0.03 and the benchmark coefficient of relative risk aversion equal to 8, while we also experiment with coefficients equal to $\{2, 4, 6\}$. The mean equity premium equals 4.2 percent in the benchmark case, while we also consider a value equal to 6 percent. Its standard deviation is 18 percent. Numerical quadrature is used to take expectations, in the spirit of Tauchen (1986).

3 Solution Method

Analytical first order conditions for bonds and for stocks respectively can be written as follows:

$$U'(C_t) = \frac{1+r}{1+\delta} E_t U'(C_{t+1}) + \lambda_B \tag{13}$$

and

$$U'(C_t) = \frac{1}{1+\delta} E_t \left[U'(C_{t+1}) \, \widetilde{R}_{t+1} \right] + \lambda_S \tag{14}$$

[9] Although these studies generally suggest that individual income changes follow an MA(2), the MA(1) is found to be a close approximation.

where λ_B and λ_S refer to the Lagrange multipliers for the no short sales constraints on bonds and on stocks. Recalling that the budget constraint in period t is

$$C_t = X_t - B_t - S_t \tag{15}$$

where X_t is cash on hand, a binding short sales constraint on bonds, implies that $C_t = X_t - S_t$ since bond holdings are at a corner of zero. Similarly, when the constraint preventing short sales of stock is binding, (15) implies that $C_t = X_t - B_t$. We generalize the Deaton (1991) solution to allow for portfolio choice by writing the two Euler equations in the following way:

$$U'(C_t) = MAX\left[U'(X_t - S_t), \frac{1+r}{1+\delta} E_t U'(C_{t+1})\right] \tag{16}$$

and

$$U'(C_t) = MAX\left[U'(X_t - B_t), \frac{1}{1+\delta} E_t \widetilde{R}_{t+1} U'(C_{t+1})\right]. \tag{17}$$

Given the nonstationary process followed by labor income, we normalize asset holdings and cash on hand by the permanent component of earnings P_{it}, denoting the normalized variables by lower case letters (Carroll, 1992). Defining $Z_{t+1} = \frac{P_{t+1}}{P_t}$ and taking advantage of the homogeneity of degree $(-\rho)$ of marginal utility implied by CRRA preferences, we have

$$U'(x_t - s_t - b_t) = MAX\left[U'(x_t - s_t), \frac{1+r}{1+\delta} E_t U'(c_{t+1}) Z_{t+1}^{-\rho}\right] \tag{18}$$

and

$$U'(x_t - s_t - b_t) = MAX\left[U'(x_t - b_t), \frac{1}{1+\delta} E_t \widetilde{R}_{t+1} U'(c_{t+1}) Z_{t+1}^{-\rho}\right]. \tag{19}$$

The normalized state variable x evolves according to

$$x_{t+1} = (s_t \widetilde{R}_{t+1} + b_t R_f) Z_{t+1}^{-1} + U_{it+1} \tag{20}$$

We use the identity $c_{t+1} = x_{t+1} - b_{t+1} - s_{t+1}$ where both b_{t+1} and s_{t+1} will be functions of x_{t+1} to substitute out c_{t+1} on the right hand sides of (18) and (19) (see appendix for the proposed algorithm).

In order for the algorithm to work, we must make sure that the two functional equations of interest define a contraction mapping. Two sufficient conditions for the individual Euler equations (18) and (19) to define a contraction mapping for $\{b(x), s(x)\}$ respectively are the conditions in Theorem 1 of Deaton and Laroque (1992):

$$\frac{1+r}{1+\delta} E_t Z_{t+1}^{-\rho} < 1 \tag{21}$$

and

$$\frac{1}{1+\delta} E_t \widetilde{R}_{t+1} Z_{t+1}^{-\rho} < 1 \tag{22}$$

If these conditions hold simultaneously, there will exist a unique set of optimum policies satisfying the two Euler equations. We next simplify these conditions to gain an intuitive understanding of the economics of the problem. Given that $Z_{t+1} = GN_{t+1}$, with $\{N\}$ being log normally distributed, we have $E_t(GN_{t+1})^{-\rho} = \exp(-\rho\mu_g) * \exp(-\rho\mu_n + \frac{\rho^2\sigma_n^2}{2})$. Assume for now that stock returns are uncorrelated with Z. Then

$$\begin{aligned} E_t \widetilde{R}_{t+1} Z_{t+1}^{-\rho} &= E_t \widetilde{R}_{t+1} E_t Z_{t+1}^{-\rho} \\ &= (1+\mu_r) * \exp(-\rho\mu_g) * \exp(-\rho\mu_n + \frac{\rho^2\sigma_n^2}{2}) \end{aligned} \tag{23}$$

Taking logs of the two conditions and using the approximation $\log(1+x) \approx x$ for small x, (21) becomes

$$\frac{r-\delta}{\rho} + \frac{\rho}{2}\sigma_n^2 < \mu_g + \mu_n \tag{24}$$

which is the condition derived by Deaton (1991) with $\mu_n = 0$. (22) becomes

$$\frac{\mu_r-\delta}{\rho} + \frac{\rho}{2}\sigma_n^2 < \mu_g + \mu_n \tag{25}$$

Note that the two conditions collapse into one when the stock market investment opportunity has the same return characteristics as the risk free rate.

With a positive equity premium ($\mu_r > r$), satisfaction of (25) guarantees (24). Impatience must now be even higher than in the saving model to prevent the accumulation of infinite stocks, since the condition involving $\mu_r - \delta$ must be satisfied. Two other distinct cases can also guarantee the existence of a solution. First, a high expected earnings growth profile (as measured by μ_g) guarantees that the individual will not want to accumulate an infinite amount of stocks or bonds but would rather borrow now, expecting earnings to increase in the future. Second, if the rate of time preference exceeds the expected stock return, more risk averse (higher ρ) individuals will not satisfy the convergence conditions.

4 Labor Income Uncorrelated to Stock Returns

4.1 Portfolio Specialization and Effects of Risk Aversion

In this Section, we solve the portfolio model for different degrees of (constant) relative risk aversion. Our findings confirm the puzzling result of complete portfolio specialization in stocks derived by Heaton and Lucas (1997) and show why it is robust to changes in risk aversion. Figures 1 and 2 show respectively consumption and stock holdings, each normalized by the permanent component of income, as functions of similarly normalized cash on hand. We consider risk aversion coefficients of 2,4,6 and 8.

Fig. 1 shows that the household does not save at levels of normalized cash on hand below a cutoff x^* (typically around 97% of the permanent component of labor income), as it is bound by both short sales constraints. It would like to borrow at the riskless rate, expecting higher future realizations of cash on hand. Unable to do so, it is even willing to engage in short sales of stock so as to boost consumption, and the short-sales constraint on stocks binds (Fig. 2).

The mechanism by which short-sales constraints on stocks and bonds justify zero stockholding in this range of normalized cash on hand can be seen as follows. In the absence of such constraints, an expected utility maximizer exhibits second-order risk aversion, in the sense that the premium it is willing to pay to avoid risk is proportional to the variance of the risk and goes to zero faster than the standard deviation of the risk (Segal and Spivak, 1990). Viewed from a different angle, households with no stocks will always choose to invest at least ε in stocks, since stocks offer the equity premium and have (locally) zero covariance with the marginal utility of consumption. As Haliassos and Bertaut (1995) have shown, imposition of a nonnegativity constraint on wealth, requiring

$$A_t = S_t + B_t \geq 0, \tag{26}$$

cannot alter this result, because it treats bonds and stocks symmetrically. However, the presence of two separate short sales constraints for bonds and stocks with (potentially) different shadow values breaks this symmetry.

The policy function for normalized wealth is the difference between the 45-degree line and the policy function for consumption. Figs. 1 and 2 show that households with normalized cash on hand above x^* start saving, but they put all their savings in stocks. This confirms the portfolio specialization result of Heaton and Lucas (1997), for a different earnings process. The source of this result, and of its robustness to degrees of risk aversion, size of equity premia, and earnings processes, can also be seen with reference to the different shadow values of the two short-sales constraints. Combining (13) and (14) yields

$$\frac{1}{1+\delta} E_t \left[U'(C_{t+1}) \left(\widetilde{R}_{t+1} - R_f \right) \right] = \lambda_B - \lambda_S. \tag{27}$$

Under no stockholding and no correlation between earnings and stock returns, the covariance between the equity premium and the marginal utility of consumption is zero. Thus, equation (27) can be rewritten as

$$\frac{1}{1+\delta} E_t \left[U'(C_{t+1}) \right] \; E_t \left[\widetilde{R}_{t+1} - R_f \right] = \lambda_B - \lambda_S \tag{28}$$

Given nonsatiation and an equity premium, the left hand side of (28) is positive, i.e. $\lambda_B > \lambda_S$. This difference in shadow values of relaxing constraints reflects the superior attributes of the riskless asset as a borrowing vehicle compared to the risky and costlier (in expected terms) alternative of short sales of stock. Since $\lambda_B > \lambda_S$ at zero stockholding, households in the neighborhood of x^* would like to borrow risklessly not only to consume but also to invest in stocks that offer an equity premium and have zero covariance with consumption. Households are prevented from borrowing and devote all saving to stocks.

Changes in the degree of risk aversion cannot reverse this result, since they do not affect the sign of marginal utility. Although we have assumed CRRA preferences throughout, the same holds for habit persistence. This is because habits influence first-order conditions for bonds and stocks in equivalent ways, failing to reverse the sign of the difference in shadow values. As long as there is an equity premium, its size does not matter, either. This explains the robustness of the portfolio specialization result to the experiments in Heaton and Lucas (1997) and to the different earnings process that we consider here. As long as earnings processes are uncorrelated with stock returns, the nature of these processes does not influence this result, and $\lambda_B > \lambda_S$ continues to hold.

Fig. 2 also shows that normalized stock holdings are increasing in risk aversion at levels of normalized cash on hand that justify saving. This surprising result is due to a conflict between risk aversion and "prudence" in the presence of binding short sales constraints. In an expected-utility framework, the degree of risk aversion is tied to the elasticity of intertemporal substitution and it is inversely related to it. Prudence is the tendency of an expected utility maximizer to accumulate additional wealth to buffer consumption from shocks to labor income (see Kimball, 1990), and it is positively related to risk aversion. Thus, higher risk aversion implies lower elasticity of substitution and higher prudence. Both make households want to increase their net wealth beyond x^* (Fig. 1), but none of this increase comes from changes in realized borrowing, which is still at zero because of the binding short sales constraint. Their desire to increase wealth dominates their motive to reduce exposure to stockholding risk, leading to increased stockholding for higher degrees of risk aversion.

Table 1 uses the invariant distribution of normalized cash on hand (see Appendix B) to show that mean and median bondholding are zero. Intuitively, and viewed in the context of an infinite-horizon household, this distribution indicates the proportion of time that the household receives normalized cash on hand realizations in each specified region.[10] Consistent with policy functions, mean and median normalized stock holdings are not only positive, but also increasing in risk aversion. Such portfolio behavior by the more risk averse is justified, since it results in smaller standard deviation of normalized consumption, as well as in higher mean normalized consumption.

[10] Viewed in the context of a continuum of households facing *ex ante* the same earnings process, it indicates the proportion of the population, at a point in time, with normalized cash on hand in each specified region.

4.2 Precautionary Effects

4.2.1 Policy Functions

Let us now focus on the role of labor income risk in this type of portfolio behavior. Precautionary effects on asset accumulation are derived as differences from a model in which households are guaranteed the expected value of labor incomes. In the absence of short sales constraints, an expected utility maximizer will accumulate precautionary wealth to buffer consumption from shocks to labor income if the utility function exhibits prudence, i.e. has positive third derivative (Kimball, 1990). Kimball (1993) used an atemporal model to derive conditions under which uninsurable labor income risk discourages investment in a risky asset ("temperance").[11] Our CRRA utility function exhibits both properties. In the current setup, the answer to how wealth and portfolios are altered in response to uninsurable labor income risk involves a comparison between models in which short sales constraints are present.

Figures 3 to 5 depict the effects on policy functions from varying the standard deviation of permanent and of transitory shocks to labor income, for unchanged mean earnings. The policy function for bonds is not shown, since constraints are binding throughout the relevant range of normalized cash on hand, forcing bondholding to be zero. The benchmark standard deviations of transitory and permanent shocks are set to 0.1 and .08 respectively. Comparison to a model with no labor income risk shows the combined precautionary effects of both types of shocks. Comparison of the benchmark model to a setup without permanent shocks but with standard deviation of transitory shocks at the benchmark identifies the role of permanent shocks.[12] Finally, comparison of this setup to the model with no earnings risk shows the role of transitory shocks.

When cash on hand is below a threshold x^*, total labor income risk from both sources has no effect on the policy functions for consumption and for asset holdings. Below this cash on hand threshold, both short sales constraints are binding regardless of whether labor income

[11] See also Pratt and Zeckhauser (1987).

[12] Due to numerical problems with convergence, the case of "no" labor income risk involves standard deviations of transitory and of permanent shocks equal to 0.02 and 0.02, respectively.

is risky. Thus, binding constraints eliminate precautionary effects on desired consumption, wealth, stockholding, and bondholding. In a second region of normalized cash on hand, earnings risk encourages wealth accumulation as expected. Yet liquidity constraints force bond holdings to be at the zero floor regardless of earnings risk. Prudence dictates a precautionary increase in wealth, and all of this increase is achieved through higher stockholding. As a result, the policy function for stockholding under labor income risk lies above that under income certainty (Fig. 5).[13] Because borrowing constraints are binding both under risky and under riskless labor income, the portfolio share of stocks is unity in both cases (Fig. 4).[14] Thus, there are no precautionary effects on the risky portfolio share, unlike what happens in models without liquidity constraints. Had one looked only at the portfolio share of risky assets, one would have missed the precautionary effects on the level of stockholding.

In Figures 3-5, we also remove permanent shocks, maintaining transitory shocks. Figure 3 shows that removal of permanent labor income shocks encourages a substantial increase in normalized consumption (reduction in wealth) in the range above x^*, consistent with prudence. Fig. 5 demonstrates the major influence of permanent shocks on desired portfolio composition. Since the short sales constraint on bonds is binding, removal of the permanent shock induces a decrease in stockholding. In this region, stocks act as precautionary buffers, and households take advantage of the equity premium to generate wealth in order to respond to the long term risk created by permanent shocks to labor income. Comparison of the intermediate setup without permanent earnings shocks to the model without earnings risk shows that the effects that transitory shocks have on policy functions are similar to those obtained for permanent shocks, but quantitatively much smaller.

[13] Haliassos and Hassapis (1998) obtain positive effects of earnings risk on stockholding for a variety of income-based and collateral borrowing constraints in a small-scale model with finite horizons.

[14] The vertical lines in fig. 4 do not represent jumps in the portfolio shares but are drawn to emphasize the different levels of normalized cash on hand that induce stock market entry in the three different cases. At levels of normalized cash on hand below these thresholds, saving is zero and portfolio shares are not defined. When positive saving takes place, portfolio shares equal unity.

4.2.2 Time Series Moments

Based on policy functions, we would expect that total labor income risk and permanent income shocks alone would increase mean and median stockholding substantially, while transitory shocks would have a smaller effect. Table 2 confirms our expectations about permanent earnings shocks and total labor income risk. Contrary to results on policy functions, however, transitory shocks increase mean and median stockholding more than permanent shocks do. This extends to a model of portfolio choice a result obtained in the single-asset saving literature, namely that permanent shocks have a wealth effect and thus do not generate a big savings response, while transitory shocks result in more savings.

The first column reports the case of no labor income risk. Starting from zero initial wealth, the household receives certain labor income that grows at 3% per period, and consumes it, since it cannot borrow. Thus, short sales constraints imply zero asset holding under no income risk. The second column introduces transitory shocks to labor income. Since asset holding can only be positive or zero, and given the portfolio specialization in stocks at likely levels of normalized cash on hand, mean and median normalized stockholding become positive but mean bondholding remains at zero. Thus, transitory shocks increase stockholding. Mean normalized consumption and consumption smoothing are enhanced through the use of stocks as a saving vehicle. So far, time series results are qualitatively similar with those obtained from policy functions.

Column 3 introduces permanent shocks to labor income, but removes transitory shocks. Mean and median normalized stocks are below those under transitory shocks. Indeed, we find the biggest discrepancies in mean (and median) stockholding between the benchmark (col. 4) and column 3 in which transitory shocks are eliminated. Transitory shocks, which were seen to have very small effects on policy functions at any given level of normalized cash on hand compared to permanent shocks, are now shown to be more important than permanent shocks for mean and median stockholding in time series simulation. Time-series mean and median asset holdings are the joint product of policy functions and of the relative frequencies with which different levels of normalized cash on hand occur. While effects of transitory shocks at a *given* level of normalized cash on hand are smaller than the effects of permanent shocks at that same level, the two types of shocks induce different (long-run, invariant) distributions of normalized cash on hand, as shown on Fig. 6. With transitory shocks (but no permanent shocks), households spend more time at higher levels of normalized cash on hand that warrant sizeable saving and stockholding compared to the case of permanent shocks only. This reversal illustrates a more general principle applicable to a wide set of stochastic optimization models, namely that policy functions should be taken together with time series simulations in assessing the relative importance of different types of shocks for economic behavior.

Table 2 also shows the extent of consumption smoothing accomplished under the two different types of shocks. In the benchmark case, normalized consumption is half as volatile as normalized earnings (the standard deviation of the former is .05 versus 0.1 for the latter). When transitory shocks are removed, households are unable to smooth any of the volatility in earnings (the two standard deviations are equal). By contrast, when households face only transitory shocks, they are able to smooth 40 percent of earnings variability (see column 2).

The results on consumption smoothing and on asset holdings are not unrelated but are both jointly endogenously determined in response to exogenous earnings shocks. For any given mean level of asset holding, consumption smoothing is more difficult in the face of permanent rather than transitory earnings shocks. The effect of this consideration on consumption smoothing and on asset holding is unclear a priori. The household might decide to hold more assets in the face of permanent earnings shocks to achieve a higher degree of consumption smoothing. On the other hand, the household might decide to accept a lower degree of consumption smoothing and hold fewer assets on average. Our findings in this section show that the household chooses the latter option.[15]

[15] This extends to models of portfolio choice a result obtained by Deaton (1991) in a single-riskless-asset model.

5 Correlation between stock market returns and labor income risk

Our findings suggest that labor income shocks provide a major impetus for stockholding, thus contributing to the puzzling portfolio specialization result in the infinite-horizon model with uncorrelated stock returns and labor incomes. Positive correlation between labor incomes and stock market returns raises the covariance between the marginal utility of consumption and stock returns at any given level of stockholding. In the absence of short sales constraints, this should make stocks less attractive.[16] Heaton and Lucas (2000) found that moderate positive correlation between earnings shocks and stock returns could mitigate the portfolio specialization result under short sales constraints. In this section, we refine their findings by contrasting correlation with transitory and permanent earnings shocks. The method used to induce positive correlation is described in Appendix A.

In unreported experiments, we found that positive correlation between stock returns and transitory earnings shocks is unlikely to be important in reversing the portfolio specialization result. Correlation equal to 0.2 yields small effects on policy functions. Correlation of unity induces households to move first into bonds, but portfolio specialization in stocks continues to occur for most of the relevant range of normalized cash on hand. At any rate, there is no empirical support for assuming that such extreme levels of correlation characterize an important subset of the population.

Figures 7 to 10 illustrate the effects of positive correlation between stock returns and permanent shocks to labor income equal to 0.1, 0.3, and 0.5. The increased correlation between stock returns and permanent income shocks makes stocks significantly less attractive and induces households to start investing in bonds at lower levels of normalized cash on hand (Fig. 10). For correlation of 0.3, the household still enters the stock market first, but the range of cash on hand for which only stocks are used is already severely limited (Fig. 9). At correlation of 0.5, we find sizeable portfolio shifts away from stocks, a reversal in the order in which the household enters the stock and the bond market that is more in line with empirical

[16] This intuitively appealing conjecture is corroborated by the findings of Viceira (2001) in a long-horizon model without liquidity constraints.

observation, and a justification for zero stockholding in a likely range of normalized cash on hand. In Table 4, a positive correlation of 0.5 drives mean and median stock holdings to zero, while mean and median bond holdings are positive. This improvement in portfolio predictions comes with minor effects on the policy function for consumption (wealth).

If positive correlation between earnings shocks and stock returns is the key to resolving the stockholding puzzle, then we should observe high levels of positive correlation in those segments of the population where the proportion of stockholders is particularly small. In one of the first studies attempting to quantify this correlation, Davis and Willen (1999) obtain estimates ranging between .1 and .3 over most of the working life for college educated males and around $-.25$ at all ages for male high school dropouts.[17] Heaton and Lucas (1999) argue that entrepreneurial risk is positively correlated with stock returns and reaches levels around .2. These numbers are not only fairly small, but also of the opposite sign than what would be needed to account for the incidence of zero stockholding in the relevant population categories. They imply that zero stockholding should be prevalent for college graduates or entrepreneurs who in fact tend to hold stocks, and they predict that low education households should actually be holding stocks as a hedging instrument when in fact they tend not to do so.

6 Zero Stockholding and Entry Costs

In this section we explore an alternative route to accounting for zero stockholding. Suppose that access to stockholding opportunities entails some cost. Such costs arise naturally, given the informational requirements for investing in the stock market and commissions charged by brokers and fund managers. They are augmented if one includes the opportunity cost of the household's time spent, as well as possible misperceptions about the level of costs and effort required to participate in the stock market that generate inertia (see Haliassos and Bertaut, 1995). We can then compute the normalized entry cost to the stock market that would make agents indifferent between entering the stock market or not participating and

[17] They use the Annual Demographic Files of the March Current Population Survey (CPS) to construct panel data on mean annual earnings between 1963 and 1994.

using the riskless asset market to generate the wealth buffer.

To compute this threshold entry cost, we solve for the associated value functions. Details of its computation are found in Appendix C. Not surprisingly, the value function of the portfolio model exceeds that of the saving model at any level of normalized cash on hand, since households are no worse off when they have the option to invest in stocks. Positive correlation between stock returns and permanent shocks to labor income lowers the value function for the portfolio model since it makes stocks less useful for buffering labor income risk (but still remains above the value function for the saving model).

If we denote the value function associated with participating in the stock market by V_s and the value function when using the bond market by V_B, the normalized threshold entry cost as a function of normalized cash on hand is $k(x)$, such that

$$V_S(x - k(x)) = V_B(x) \tag{29}$$

Given the monotonicity in cash on hand of the value function, we can use a numerical interpolation procedure to invert the value functions and derive the entry cost as

$$k(x) = x - V_S^{-1}(V_B(x)) \tag{30}$$

Since $k(x)$ varies with the realized cash on hand, we can now make use of the time-invariant distribution of normalized cash on hand[18] to find the maximum level of x that the household will experience. We compute this from the invariant distributions depicted in Figs. 13-14 as the level $\widehat{x}$, such that $\Pr(x \leq \widehat{x}) = 1$. Our threshold entry cost is then computed as $k(\widehat{x})$.[19]

This threshold entry cost, or equivalently the minimum compensation that any household in the model would accept in order to stay out of the stock market, is an overestimate of the entry cost needed to generate observed population splits between stockholders and non-stockholders in at least three respects. First, it is computed using a model which implies that, if the household gains access to the stock market, it can make use of stockholding opportunities over an infinite horizon. Second, we have assumed that the cost of accessing

[18] See Appendix B for the computation of the time invariant distribution.

[19] We use the invariant distribution associated with the saving model to compute $\widehat{x}$ since we are assuming that the household is contemplating entry in the stock market for the first time.

the stock market is a ticket fee that is paid only once. Third, we use the value $k(\widehat{x})$, which is sufficient to keep everybody out of the stock market forever. As seen from Figs. 11 and 12, $k(x)$ is monotonically increasing in x, implying that wealthier individuals require larger compensation to stay out of the stock market.

Despite these considerations, the computed threshold entry costs tend to be relatively small. Table 5 reports the values of these threshold costs for different combinations of risk aversion and impatience. We consider two alternative values for the equity premium (4.2 and 6.0 percent) and for the correlation between permanent earnings shocks and stock returns (zero and 0.3). For an impatient household ($\delta = .1$) with risk aversion of 2 whose labor income is uncorrelated with stock returns, the threshold, one-time, entry cost is 4 percent of mean annual labor income. For other parameter configurations, impatient households ($\delta = .1$) will abstain from the stock market for entry costs ranging between 3 and 24 percent of mean annual labor income.

Interestingly, when the coefficient of relative risk aversion rises from 2 to 8, the threshold entry cost rises from 4 to 16 percent (see Table 5, Panel I). The reason for the higher entry cost arises from the conflict between prudence and risk aversion discussed above. When risk aversion rises, prudence also rises to the point that prudence dominates risk aversion and dictates that more wealth be accumulated in the form of stocks. Since the importance of stocks is enhanced, the compensation for abstaining from the stock market has to rise, and therefore a higher entry cost must exist to rationalize stock market non-participation.

When the correlation between the permanent earnings shock and the stock return innovation is increased to 0.3 (see Panel III), the threshold cost drops somewhat to 3% from 4% when the correlation is zero. Positive correlation reduces the attractiveness of stocks and thus smaller costs are sufficient to keep households out of the stock market. When the equity premium is increased to six percent, the effects of risk aversion and correlation between stock returns and permanent shocks to earnings remain qualitatively the same (see Panels II and IV). Threshold costs now range from 5 to 24 percent (compared to a corresponding range of 3 to 16 percent when the equity premium equals 4.2 percent)[20]. When impatience is halved

[20] The case of $\{\rho = 8, \delta = .05, \text{Corr}=0\}$ which yielded the maximum threshold cost (34 percent) with an equity premium of 4.2 percent cannot be solved for with an equity premium equal to six percent because the

from 0.1 to 0.05, threshold entry costs rise, approximately doubling in most cases (see table 5).

One may wonder why entry costs tend to be low, given that the household gains access to stocks over an infinite horizon. Two factors are at work. First, access to stocks does not necessarily imply stockholding in every period. The invariant distributions for portfolio models in figures 6, 13 and 14, combined with the corresponding policy functions for stockholding show that households are likely to spend a substantial fraction of their time at levels of normalized cash on hand that do not justify any stockholding. Specifically, when $\rho = 2$ and stock returns are uncorrelated with labor income, the household does not save anything ($c < x^*$) around 38% of the time. When the coefficient of relative risk aversion rises to 8, on the other hand, the liquidity constraint is binding only 10% of the time, enhancing the value of entering the stock market and justifying the higher cost needed to generate stock market non-participation (cost rises from 4% to 16% of mean labor income). Finally, with positive correlation between stock returns and labor income and $\rho = 2$, the household does not save anything approximately 47% of the time. Having zero saving for such substantial periods of time detracts from the appeal of having access to stocks and tends to lower the threshold entry costs.

The finding that households with previous stockholding experience may decide not to hold stocks over certain periods of time is consistent with the empirical findings in Bertaut (1998), who used the panel sample of the *Survey of Consumer Finances*. Bertaut showed that zero stockholding in a given period is not confined to households that have never entered the stock market, but may also apply to households that previously held stocks.

A second reason for the low threshold costs arises from the small total amount of saving after access to the stock market is obtained. As tables 1-4 have illustrated, households build a small buffer of assets to smooth consumption fluctuations. When stock returns and labor income are uncorrelated, mean normalized stock holdings equal .14 when $\rho = 8$ and .032 when $\rho = 2$. With correlation between stock returns and labor income equal to .3, mean stock holdings equal .11 when $\rho = 8$ and .029 when $\rho = 2$. The small asset

contraction mapping convergence condition is not satisfied.

accumulation implied by the model is a direct consequence of impatience and higher future expected earnings growth against which no borrowing is allowed. Consequently, the benefit from entering the stock market is correspondingly small. The finding that relatively low entry costs can generate stock market non-participation by most households is consistent with recent empirical findings by Vissing-Jorgensen (1999) and Paiella (2000).

If entry costs are crucial in determining stock market participation, threshold entry costs should not only be low for those who tend to be out of the stock market, but also significantly higher for the wealthy who tend to participate in the stock market. There are good a priori reasons for high threshold costs among wealthy households. Such households tend to hold larger amounts of wealth in stocks and to run into binding short sales constraints less frequently. Since their average stockholding tends to be larger due to both reasons, wealthy households stand to gain more through stock market participation. A way to confirm and quantify this intuition in the context of the present model is to consider households who tend to accumulate more wealth because they are more patient than our benchmark household. Table 6 considers households with risk aversion of 2 and rates of time preference ranging from .03 to 0.1. A reduction in the rate of time preference raises threshold entry costs. When the rate of time preference is halved (from 0.1 to 0.05), threshold costs approximately double in most cases (see table 6). As we further reduce the degree of impatience, threshold entry costs increase at faster rates. Less impatient households tend to accumulate more wealth. At $\delta = .04$, mean normalized wealth (stockholding) equals .26, whereas at $\delta = .1$ it falls to .04.[21] Consequently, they tend to run into binding borrowing and short sales constraints less frequently. At $\delta = .04$ households save nothing 5 percent of the time, whereas at $\delta = .1$ no saving takes place 35 percent of the time. The increase in average stockholding implies that much higher entry costs are needed to discourage those households from participating in the stock market.

Although the infinite-horizon model is a good benchmark for computing the threshold entry costs that we described, it does not resolve all aspects of stockholding behavior. The model can account for zero stock holding and either zero or positive holdings of riskless

[21] The equity premium is six percent.

assets by households that have never entered the stock market. On the other hand, it cannot account for the co-existence of positive portfolio holdings of stocks and riskless assets. Once the entry fee is paid, positive asset holding implies complete portfolio specialization in stocks for empirically plausible degrees of correlation between earnings and stock returns. Such co-existence is observed in the data for a subset of stockholders (King and Leape, 1984; Mankiw and Zeldes, 1991; Bertaut and Haliassos, 1997).

7 Concluding Remarks

This paper has extended Deaton's approach to solving single-asset models of saving, in order to incorporate portfolio choice subject to short sales constraints. We first subjected the infinite-horizon model with liquidity constraints to closer scrutiny so as to shed light on its properties and explore the sources of the portfolio specialization puzzle. We provided an explanation why the puzzle has proved robust to a number of model variations attempted in the literature. We then decomposed the consumption and portfolio effects of permanent and transitory shocks to labor income and showed their interaction with liquidity constraints and their relative importance in producing precautionary effects. This illustrated the possibility that policy functions and time series simulations might yield conflicting implications on the relative importance of the different types of shocks. Even though positive correlation between shocks to earnings and stock market innovations could discourage portfolio specialization in stocks, we argued that existing empirical evidence seems to be at variance with the pattern of correlations required to explain stock holding by different segments of the population.

We therefore explored the potential of fixed entry costs to explain stock holding behavior. Specifically, we derived an upper bound to the entry cost required to keep households out of the stock market under different degrees of risk aversion, rates of time preference, equity premia, and correlation between stock returns and labor income shocks. This threshold entry cost tends to be small, suggesting that entry costs arising from informational considerations, sign-up fees, and investor inertia could generate the observed reluctance of households to undertake stockholding even when they hold liquid assets. The costs might also explain the delay in the spreading of an "equity culture" among households. A remaining puzzle, however, is the observed portfolio co-existence of riskless assets and stocks for a subset of the population. Part of this puzzle is due to the assumed positive labor income floor which acts essentially as a riskless asset crowding out bond holdings. The alternative of assigning positive probability to a zero labor income state could also generate zero stock holding; whether it could also provide a plausible justification for the co-existence of bonds and stocks in the portfolio is an interesting topic for further research.

A Appendix A: Numerical Dynamic Programming

The pair of Euler equations are given by

$$\begin{aligned} U'(x_t - s(x_t) - b(x_t)) &= MAX[U'(x_t - s(x_t)), \beta E_t R_f (G_{t+1} N_{t+1})^{-\rho} * \\ &\quad U'(x_t - s(x_{t+1}) - b(x_{t+1}))] \end{aligned} \tag{31}$$

and

$$\begin{aligned} U'(x_t - s(x_t) - b(x_t)) &= MAX[U'(x_t - b(x_t)), \beta E_t \widetilde{R}_{t+1} (G_{t+1} N_{t+1})^{-\rho} * \\ &\quad U'(x_{t+1} - s(x_{t+1}) - b(x_{t+1}))] \end{aligned}$$

where $x_{t+1} = (s_t \widetilde{R}_{t+1} + b_t R_f) Z_{t+1}^{-1} + U_{it+1}$. The single state variable (cash on hand, x_t) is discretized into 100 equidistant grid points between (.3 and 5). Given that the two conditions that guarantee that the above system defines a contraction mapping are satisfied, we can solve simultaneously for $\{s(x), b(x)\}$. Starting with any initial guess (say $s(x) = .1 * x$ and $b(x) = .1 * x$), we use the right hand side of the first Euler equation to get an update for b and continue doing so until b converges to its time invariant solution b_1^* (see Deaton (1991)). We then use the second Euler equation with b_1^* taken as given, to find the solution for the time invariant optimal s, call it s_1^*. We know have two updated functions $\{s_1^*, b_1^*\}$; the process can be repeated until these functions converge to their time invariant solutions (this in practice depends on the parameters of the problem but is much faster than using a grid search method to pick the values of $\{b, s\}$ that would maximize the value function).

A.0.3 Contemporaneous Correlation

To find the probabilities associated with different state realizations in the presence of contemporaneous correlation, we discretize the joint probability distribution of a bivariate standard normal in the following way. The univariate standard normal distribution is divided into ten equiprobable intervals using eleven points; $\{\pm 10,\ \pm 1.28155156,\ \pm 0.84162123,\ \pm 0.52440051,\ \pm 0.25334710,\ 0\}$. A discrete approximation of the formula

$$F(y_1 \leq Y \leq y_2, z_1 \leq Z \leq z_2) = F(y_2, z_2) - F(y_2, z_1) - F(y_1, z_2) + F(y_1, z_1)$$

where F is the bivariate standard normal of the two random variables (Y, Z) is then derived using the $CDFBVN$ command in GAUSS.

B Appendix B: Computing the Time- Invariant Distribution

Normalized cash on hand follows a renewal process[22] and therefore has an associated invariant distribution. To find the time invariant distribution of cash on hand, we first compute the bond and stock policy functions; $b(x)$ and $s(x)$ respectively. Note that the normalized cash on hand evolution equation is

$$\begin{aligned} x_{t+1} &= [b(x_t)R_f + s(x_t)\widetilde{R}_{t+1}]\frac{P_t}{P_{t+1}} + U_{t+1} \\ &= w(x_t|\widetilde{R}_{t+1}, \frac{P_t}{P_{t+1}}) + U_{t+1} \end{aligned} \tag{32}$$

where $w(x)$ is defined by the last equality and is conditional on $\{\widetilde{R}_{t+1}, \frac{P_t}{P_{t+1}}\}$. Denote the transition matrix of moving from x_j to x_k,[23] as T_{kj}. Let Δ denote the distance between the equally spaced discrete points of cash on hand on the grid. The risky asset return $\widetilde{R}$ and

[22] The proof for a mathematically equivalent model of commodity prices with non-negative inventories is given by Deaton and Laroque (1992, theorem 2).

[23] The normalized grid is discretized between $(x\min, x\max)$ where $x\min$ denotes the minimum point on the equally spaced grid and $x\max$ the maximum point.

$\frac{P_t}{P_{t+1}}$ are discretized using 10 grid points respectively: $R = \{R_l\}_{l=1}^{l=10}$ and $\frac{P_t}{P_{t+1}} = \{GN_m\}_{m=1}^{m=10}$. $T_{kj} = \Pr(x_{t+1=k}|x_{t=j})$ is found using

$$\sum_{l=1}^{l=10}\sum_{m=1}^{m=10} \Pr(x_{t+1}|x_t, \widetilde{R}_{t+1} = R_l, \frac{P_t}{P_{t+1}} = N_m) * \Pr(\widetilde{R}_{t+1} = R_l) * \Pr(\frac{P_t}{P_{t+1}} = N_m) \tag{33}$$

where both the independence of $(\widetilde{R}_{t+1}, \frac{P_t}{P_{t+1}})$ from x_t and the independence of $\frac{P_t}{P_{t+1}}$ from $\widetilde{R}_{t+1}$ were used. Numerically, this probability is calculated using

$$T_{kjlm} = \Pr(x_k + \frac{\Delta}{2} \geqslant x_{t+1} \geqslant x_k - \frac{\Delta}{2}|x_t = x_j, \frac{P_{it}}{P_{it+1}} = N_m, R_{t+1} = R_l)$$

Making use the approximation that for small values of σ_u^2, $U \sim N(\exp(\mu_u + .5 * \sigma_u^2), (\exp(2 * \mu_u + (\sigma_u^2)) * (\exp(\sigma_u^2) - 1)))$, and denoting the mean of U by $\overline{U}$ and its standard deviation by σ, the transition probability conditional on N_m and R_l then equals

$$\begin{aligned} T_{kjlm} &= \Phi(\frac{x_k + \frac{\Delta}{2} - w(x_t|N_m, R_l) - \overline{U}}{\sigma} \geqslant x_{t+1} \geqslant \frac{x_k - \frac{\Delta}{2} - w(x_t|N_m, R_l) - \overline{U}}{\sigma} \\ |x_t &= x_j, \frac{P_{it}}{P_{it+1}} = N_m, R_{t+1} = R_l) \end{aligned}$$

The unconditional probability from x_j to x_k is then given by

$$T_{kj} = \sum_{l=1}^{l=10}\sum_{m=1}^{m=10} T_{kjlm} \Pr(N_m) \Pr(R_l) \tag{34}$$

Given the matrix T, the probabilities of each of the states are updated by

$$\pi_{kt+1} = \sum_j T_{kj} * \pi_{jt} \tag{35}$$

so that the invariant distribution can be found by repeatedly multiplying the transition matrix by itself until all its columns stop changing. The invariant distribution π is instead calculated (faster) as the normalized eigenvector of T corresponding to the unit eigenvalue by solving the linear equations

$$\begin{pmatrix} T - I & e \\ e' & 0 \end{pmatrix}\begin{pmatrix} \pi \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{36}$$

where e is an M-vector of ones.

Once the limiting distribution of cash on hand is derived, average cash on hand can be computed using

$$\sum_j \pi_j * x_j \tag{37}$$

Similar formulae can be used to compute the mean, median and standard deviations of the variables of interest, as reported in the tables.

C Appendix C: Value Function Computation

An induction argument is sufficient to show that the value function inherits the properties of the utility function; in particular, the value function is homogeneous of degree $(1-\rho)$ when the utility function in of the CRRA form. As a result, the equation that determines the value function

$$V(X_t, P_t) = MAX_{B_t, S_t} U(C_t) + \beta E_t V(X_{t+1}, P_{t+1}) \tag{38}$$

can be rewritten as

$$V(x_t) = MAX_{b(x_t), s(x_t)} U(c_t) + \beta E_t \{\frac{P_t}{P_{t+1}}\}^{1-\rho} V(x_{t+1}) \tag{39}$$

Starting from any initial guess of the value function (say $V(x) = \frac{x^{1-\rho}}{1-\rho}$) and substituting this along with the optimal consumption, bond and stock policy functions on the right hand side of (39), we obtain an update of $V(x)$; this procedure can be repeated until the value function converges at all grid points.

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Table 1: Effects on consumption, bond and stock holdings from varying coefficient of relative risk aversion

	$\rho = 6$	$\rho = 7$	$\rho = 8$
Mean Normalized Bond Holdings	0.00	0.00	0.00
Mean Normalized Stock Holdings	0.09	0.11	0.14
Mean Normalized Consumption	1.004	1.005	1.006
Median Normalized Bond Holdings	0.00	0.00	0.00
Median Normalized Stock Holdings	0.07	0.09	0.11
Median Share of Wealth in Stocks	1.00	1.00	1.00
Median Normalized Consumption	1.009	1.009	1.009
σ(Normalized Bond Holdings)	0.00	0.00	0.00
σ(Normalized Stock Holdings)	0.08	0.10	0.12
σ(Normalized Consumption)	0.06	0.05	0.05
σ(Normalized Earnings)	0.10	0.10	0.10

Notes to Table 1: Normalized variables are with respect to the permanent component of labor income (P_{it} in the text). The reported numbers are generated using the time invariant distributions associated with each model, as described in the text. Other parameters are set to $\delta = .1$, mean equity premium is 4.2 percent, standard deviation of excess returns is 18 percent, $\sigma_u = .1, \sigma_n = .08$.

Table 2: Effects on consumption, bond and stock holdings from transitory and permanent labor income uncertainty

	$\sigma_u = .02$ $\sigma_n = .02$	$\sigma_u = .10$ $\sigma_n = .02$	$\sigma_u = .02$ $\sigma_n = .08$	$\sigma_u = .10$ $\sigma_n = .08$
Mean Norm Bonds	0.00	0.00	0.00	0.00
Mean Norm Stocks	0.00	0.05	0.01	0.14
Mean Norm Consumption	1.000	1.003	1.000	1.006
Median Norm Bonds	0.00	0.00	0.00	0.00
Median Norm Stocks	0.00	0.03	0.003	0.11
Median Share of Wealth in Stocks	1.00	1.00	1.00	1.00
Median Norm Consumption	1.00	1.005	1.004	1.009
σ(Norm Bonds)	0.00	0.00	0.00	0.00
σ(Norm Stocks)	0.00	0.06	0.01	0.12
σ(Norm Cons)	0.02	0.06	0.02	0.05
σ(Norm Earnings)	0.02	0.10	0.02	0.10

Notes to Table 2: See Table 1.

Table 3: Effects on consumption, bond and stock holdings from varying the correlation between transitory labor income uncertainty and stock market risk

	Benchmark	$Corr = .2$	$Corr = 1$
Mean Normalized Bond Holdings	0.00	0.00	0.03
Mean Normalized Stock Holdings	0.14	0.15	0.03
Mean Normalized Consumption	1.006	1.006	.97
Median Normalized Bond Holdings	0.00	0.00	0.03
Median Normalized Stock Holdings	0.12	0.12	0.00
σ(Normalized Bond Holdings)	0.00	0.00	0.03
σ(Normalized Stock Holdings)	0.12	0.13	0.09
σ(Normalized Consumption)	0.05	0.05	0.06

Notes to Table 3: See Table 1. Corr is the contemporaneous correlation between transitory labor income shocks and stock market returns.

Table 4: Effects on consumption, bond and stock holdings from varying the correlation between permanent labor income uncertainty and stock market risk

	Benchmark	$Corr = .1$	$Corr = .3$	$Corr = .5$
Mean Norm Bonds	0.00	0.00	0.01	0.11
Mean Norm Stocks	0.14	0.14	0.11	0.00
Mean Norm Consumption	1.009	1.005	1.004	1.000
Median Norm Bonds	0.00	0.00	0.00	0.09
Median Norm Stocks	0.11	0.11	0.09	0.00
Median Share of Wealth in Stocks	1.00	1.00	1.00	.00
Median Norm Consumption	1.009	1.009	1.007	1.007
σ(Norm Bonds)	0.00	0.00	0.02	0.09
σ(Norm Stocks)	0.12	0.11	0.08	0.00
σ(Norm Con)	0.05	0.05	0.05	0.05

Notes to Table 4: See Table 1. Corr is the contemporaneous correlation between permanent labor income shocks and stock market returns.

Table 5: Fixed Costs Generating Stock Market Non-Participation

	I. Equity Premium = 4.2%, Corr=0		II. Equity Premium = 6%, Corr=0	
	$\delta = .05$	$\delta = .1$	$\delta = .05$	$\delta = .1$
$\rho = 2$	8	4	14	6
$\rho = 5$	14	8	22	13
$\rho = 8$	34	16	*	24
	III. Equity Premium = 4.2%, Corr=0.3		IV. Equity Premium = 6%, Corr=0.3	
$\rho = 2$	6	3	12	5
$\rho = 5$	7	4	14	8
$\rho = 8$	7	6	23	11

Notes to Table 5: The table reports the fixed costs necessary to generate stock market non-participation as a percentage of mean labor income (at an annual horizon). Corr refers to the correlation between the permanent labor income shocks and the stock market return innovations. ρ is the CRRA coefficient and δ is the discount rate. Mean growth rate equals 3 percent, the standard deviation of permanent shocks (σ_n) equals .08 and the standard deviation of transitory shocks (σ_u) equals .1. * denotes a parameter configuration for which the contraction mapping convergence condition is violated.

Table 6: Fixed Costs Generating Stock Market Non-Participation

Varying the impatience parameter

	I. Equity Premium = 4.2% or 6%, Corr=0			
	$\delta = .03$	$\delta = .04$	$\delta = .05$	$\delta = .1$
$\rho = 2$	15 (32)	12 (20)	8 (14)	4 (6)
	II. Equity Premium = 4.2% or 6%, Corr=0.3			
$\rho = 2$	12 (22)	9 (16)	6 (12)	3 (5)

Notes to Table 6: See Table 5. The first entry in each cell is for an equity premium equal to 4.2 percent. Entries in parentheses are for an equity premium equal to 6.0 percent.

Fig.1 : Normalized Consumption (varying rho)

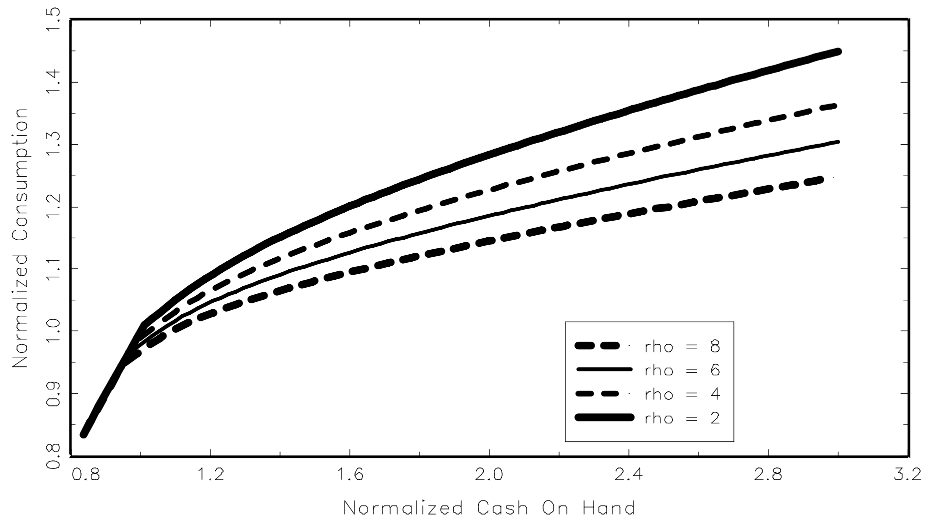


Fig.2 : Normalized Stock Holdings (varying rho)

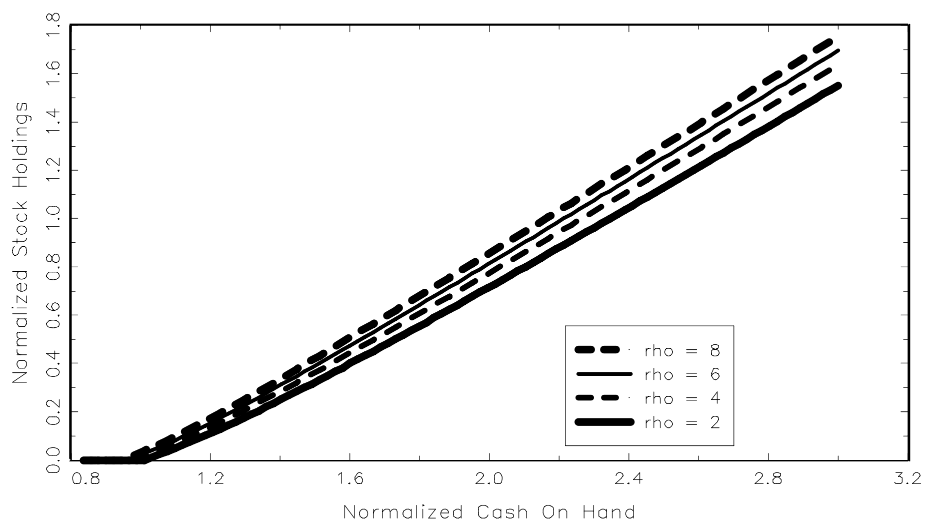


Fig.3 : Normalized Consumption varying permanent uncertainty

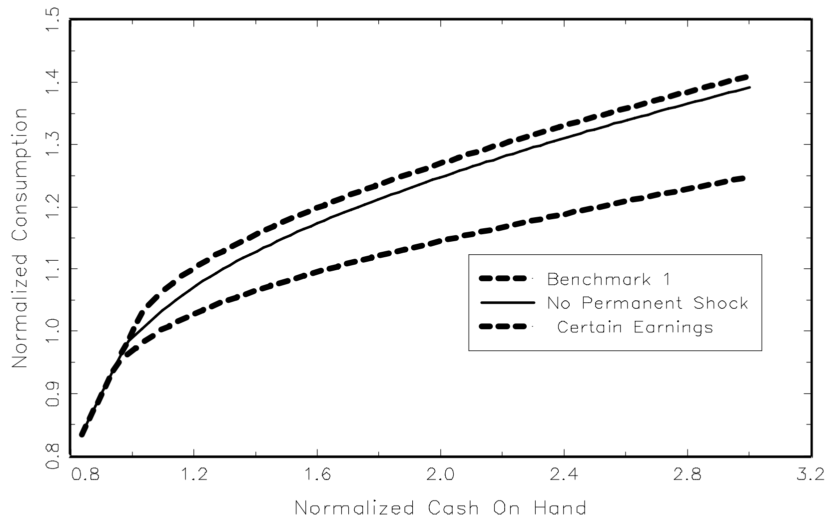


Fig.4 : Share of Wealth in Stocks varying permanent uncertainty

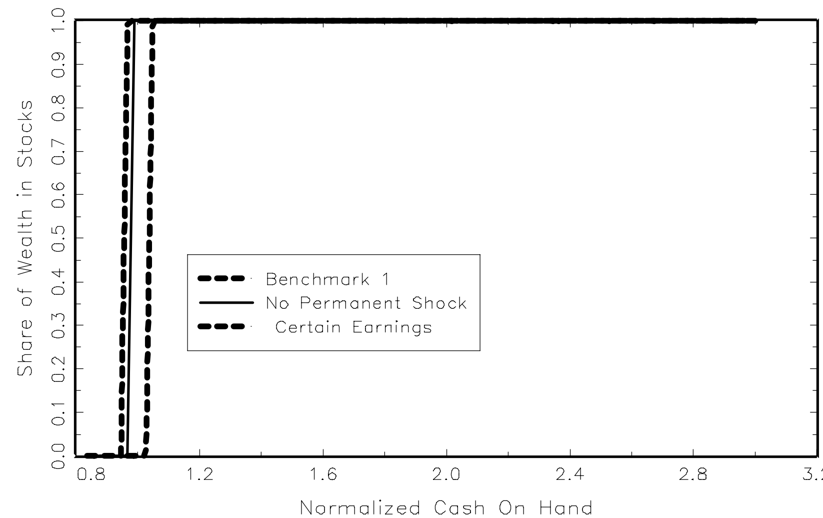


Fig.5 : Normalized Stock Holdings varying earnings uncertainty

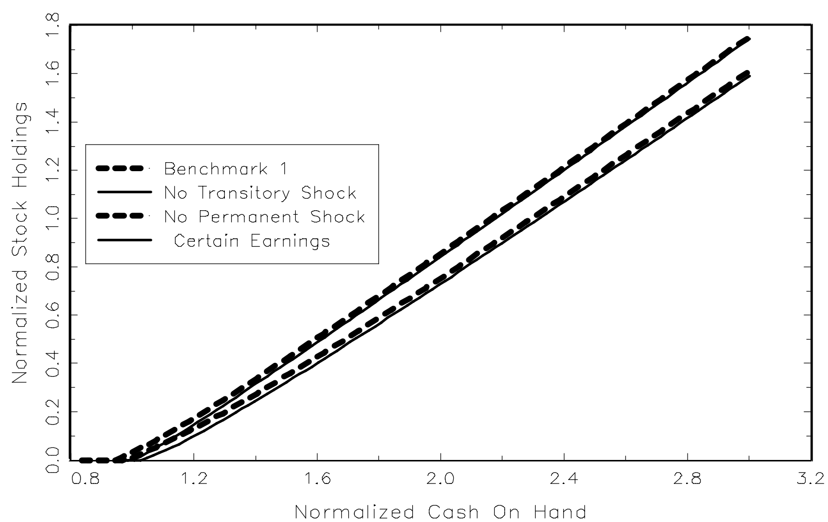
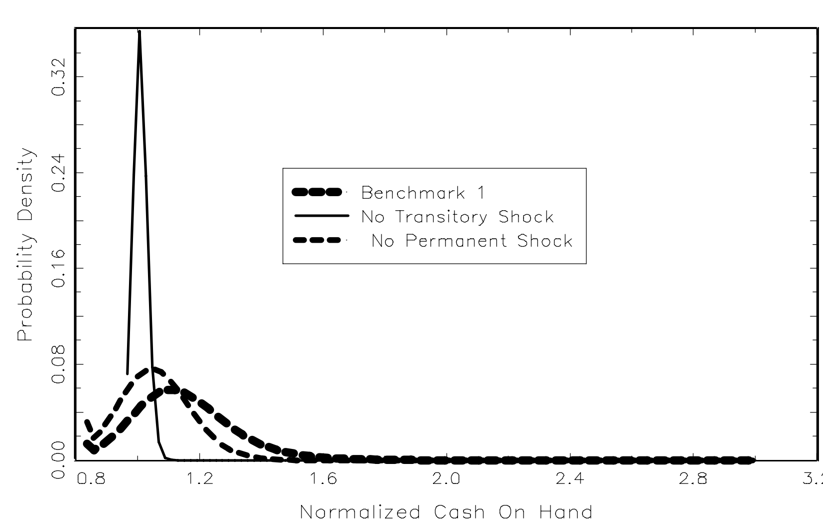


Fig.6 : Cash on Hand Invariant Distribution



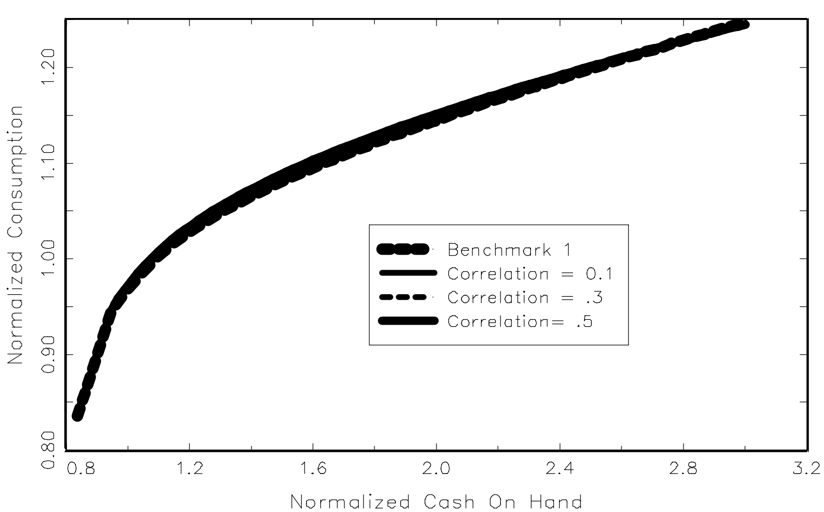


Fig.7 : Normalized Consumption

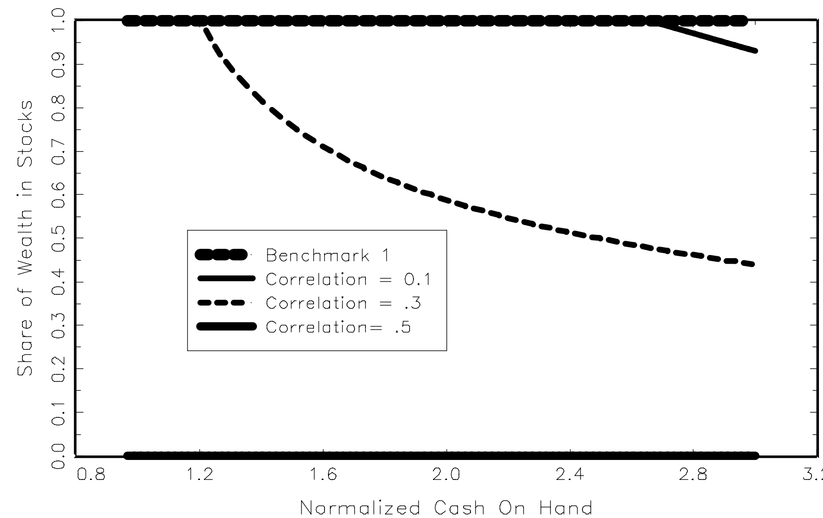


Fig.8 : Share of Wealth in Stocks

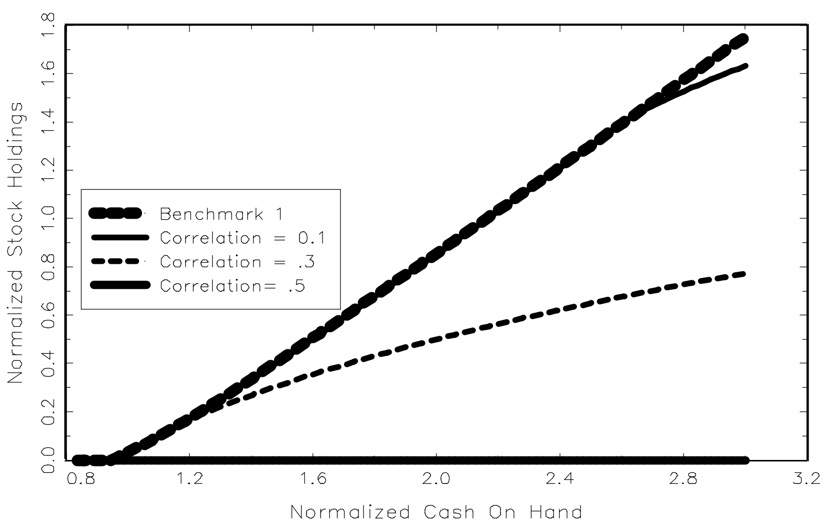


Fig.9 : Normalized Stock Holdings varying permanent uncertainty

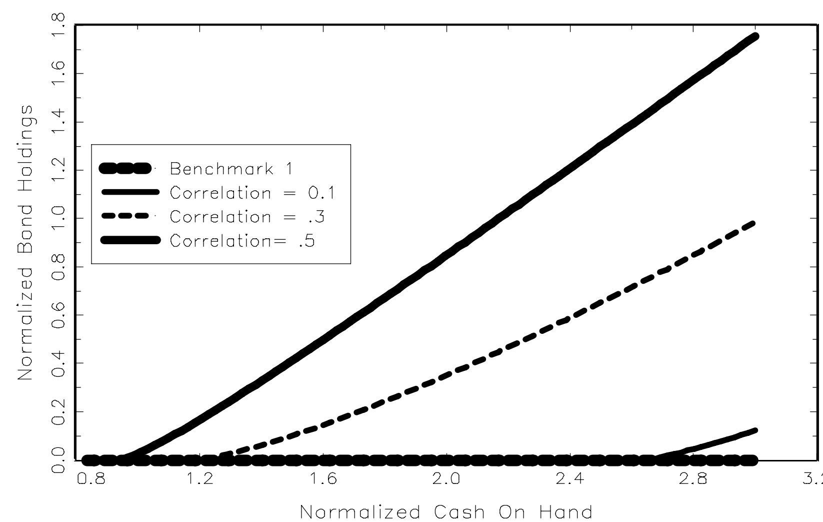


Fig.10 : Normalized Bond Holdings varying permanent uncertainty

Fig.11 : Certainty Equivalent to stay out of Stock Market

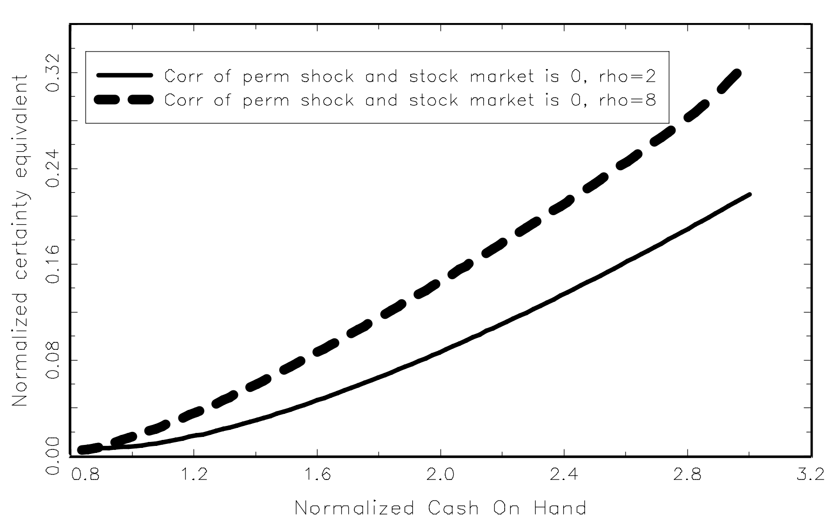


Fig.12 : Certainty Equivalent to stay out of Stock Market

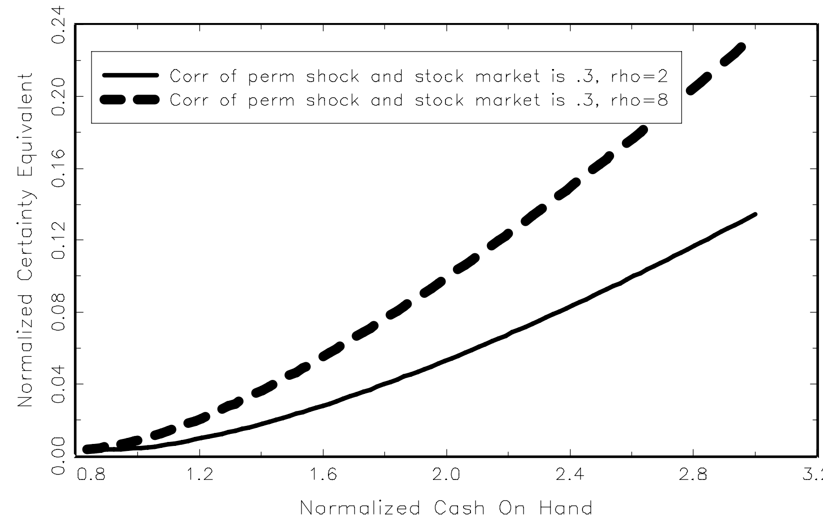


Fig.13 : Invariant Distributions, rho=2 vs rho=8

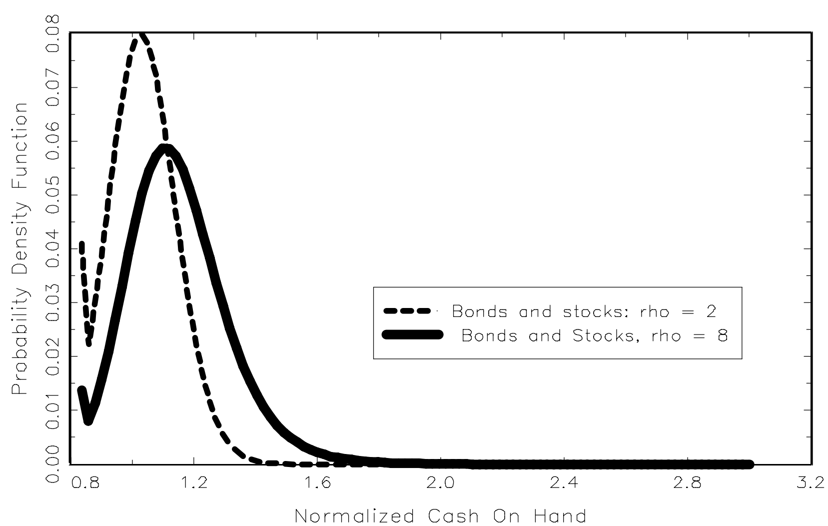


Fig.14 : Invariant Distributions

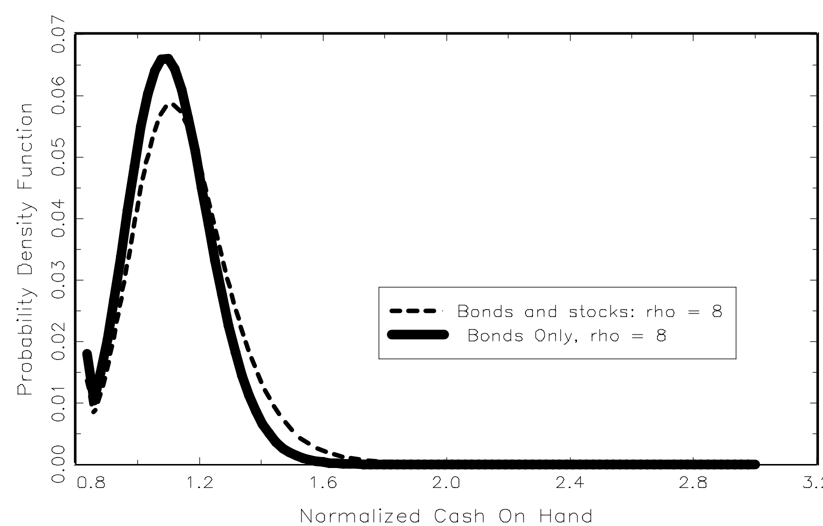


Fig.15: Cert Equiv changing Discount Rates, Corr=0, Eqpr=4.2%

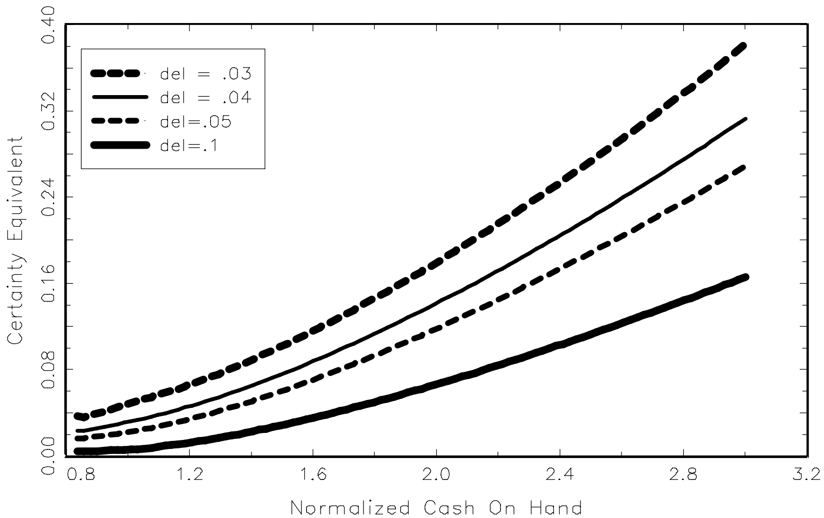


Fig.16: Cert Equiv changing Discount Rates, Corr=0.3, Eqpr=4.2%

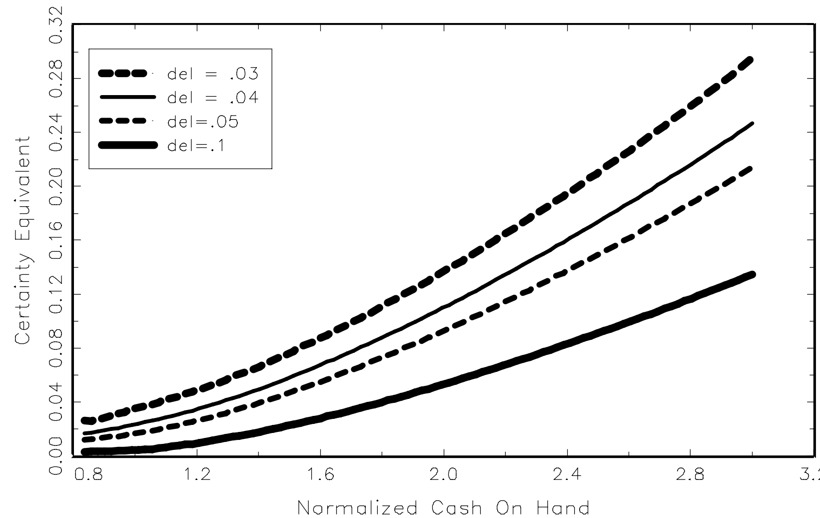


Fig.17: Cert Equiv changing Discount Rates, Corr=0, Eqpr=6%

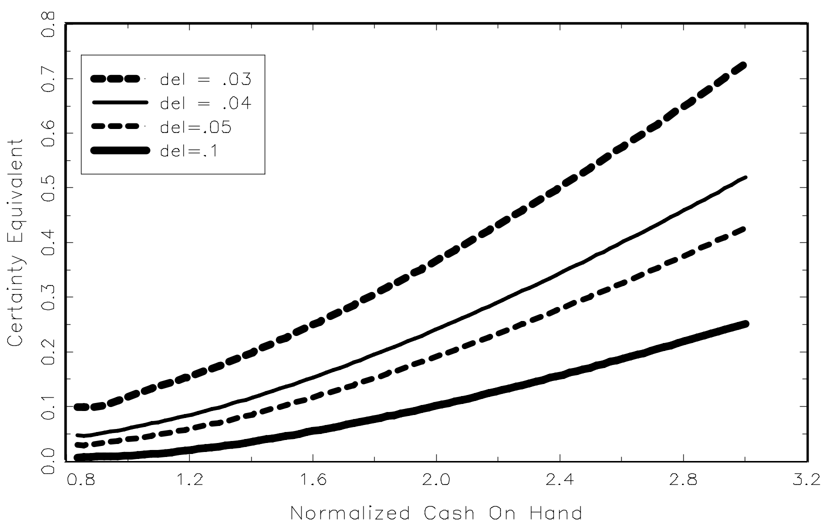


Fig.18: Cert Equiv changing Discount Rates, Corr=0.3, Eqpr=6%