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No. 2807

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*INTERNATIONAL MACROECONOMICS*



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Discussion Paper No. 2807  
May 2001

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May 2001

## ABSTRACT

### Monetary Policy Rules and the Exchange Rate\*

A positive and normative evaluation of alternative monetary policy regimes is addressed in a two-country general equilibrium model. The behaviour of the exchange rate, as well as of the other macroeconomic variables, depends crucially on the monetary regime chosen, though not necessarily on monetary shocks. The centralized welfare criterion presents a trade-off between stabilizing the economy around the flexible-price allocation and reducing the volatility of the nominal interest rates. In this framework, some form of control of the exchange rate is welfare improving.

JEL Classification: E52, F41

Keywords: exchange rate regimes, monetary policy rules, welfare criterion

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\*We are grateful to Maurice Obstfeld and Michael Woodford for useful comments and discussions. We also thank Peter Andrews, Kosuke Aoki, Giancarlo Corsetti, Paolo Pesenti, Kenneth Rogoff, Lars Svensson and Martin Uribe. All errors are our own. Pierpaolo Benigno gratefully acknowledges financial support from the Alfred P Sloan Doctoral Dissertation Fellowship. The views expressed herein are those of the authors and should not be interpreted as those of the Bank of England.

Submitted 03 April 2001

## NON-TECHNICAL SUMMARY

This Paper develops a theory of exchange rate determination under interest rate rules. The framework is a two-country optimizing model with sticky prices, incorporating elements from both the recent closed economy literature on the effects of monetary policy, in the spirit of the Neo-Wicksellian framework (Woodford, 2000), and the recent open economy literature on exchange rate determination. We design monetary policy regimes by the interaction of interest rate rules followed by the monetary policy-makers of both countries.

We consider a world economy composed of two countries, Home and Foreign respectively. A generic agent is both producer and consumer: a producer of a single differentiated product and a consumer of all the goods produced in both countries.

Each agent derives utility from consuming an index of consumption goods, a composite index of the Home and Foreign goods, and from the liquidity services of money holding, which derives disutility from producing the differentiated product. The whole economy is subject to two country-specific sources of fluctuations: demand and supply shocks. Households maximize the expected discounted value of the utility flow. We assume that markets are complete within and across countries by allowing agents to trade in a set of bonds that span all the states of nature.

Money matters because agents derive utility from its liquidity services. If real money balances and consumption are separable in utility and prices are flexible, money is neutral. In order to give a role to monetary policy for the short-run fluctuations of the economy, we introduce both nominal price rigidity and a market structure characterized by monopolistic competition. The latter assumption rationalizes the existence of price stickiness without violating the participation constraints of the producers. Nominal rigidities are introduced using a Calvo model, thus allowing fluctuations around the equilibrium for a longer period of time. We allow the degrees of rigidity to vary across countries.

The instrument of monetary policy is the domestic short-term nominal interest rate. In terms of our equilibrium conditions, this means that the money market equilibrium condition can be neglected, provided we are not interested in characterizing the path of real money balances or that of money supply in the whole area. Monetary policy is endogenous, meaning that the interest rate is set to react to other macroeconomic variables. Through the plethora of rules that different reactions can generate, we analyse the positive and normative consequences of alternative monetary regimes.

We then characterize a floating, fixed and managed exchange rate regime.

As a first step, the design of monetary policy rules is crucial for the determinacy of the equilibrium. In an open-economy framework the richness of the set of relevant variables complicates the analysis. A policy rule followed by a country, if considered aside from the rules followed by other countries, can no longer be sufficient for the determinacy of the equilibrium. The interdependence among the rules followed by different countries, even in a non-coordinated way, is crucial in determining the characteristics of various monetary regimes, within and across countries, and affects the response of the economies to domestic and foreign shocks. We show that simple interest rate feedback rules can determine a unique and stable equilibrium. The important result is that there is no need to include an explicit reaction toward the nominal exchange rate. In a floating regime, the Taylor principle that monetary policy rules should be aggressive toward domestic inflation holds provided that the principle is applied to both rules. We further show that appropriately designed fixed exchange rates are not destabilizing.

Once the equilibrium is determinate, we examine how the dynamic behaviour of the terms of trade and of the nominal exchange rate depends on the exchange rate regime adopted. Under a floating regime, the exchange rate is, in general, non-stationary. An important feature of our findings is that this excess volatility comes directly from real shocks, which are instead drawn from a stationary distribution. On the other hand, these real shocks generate a stationary pattern for the real macroeconomic variables of the model. The way monetary rules are designed is crucial for determining the non-stationary property of the nominal exchange rate: rules that target the level of the nominal exchange can stabilize its long-run value.

Finally, we evaluate the different regimes by using a centralized micro-founded welfare function that displays a trade-off between stabilizing the economy around the flexible price allocation and reducing the distortions implied by the excess volatility of nominal interest rates. Given the 'calibrated' economy, in the first best the spread in nominal interest rates across countries is close to zero, but unexpected movements in the exchange rate are allowed. An interest rate-smoothing Taylor rule in which monetary policy-makers react explicitly to changes in the exchange rate can approximate the first best.

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<sup>1</sup>An effort in this direction has been made by Ball (1998), Ghironi (1998), McCallum and Nelson (1999), Monacelli (1998), Svensson (2000) and Weeparana (1998) who have analyzed the implications of Taylor rules in small open-economy models.

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The structure of the paper is the following: in the first section we present the model and we briefly discuss its most novel aspects. We start our analysis from the log-linear approximation to the equilibrium conditions. We then specify the monetary policy rules that we are considering. The analysis of equilibrium determinacy is addressed in Section 2. In Section 3 we explore the positive consequences of different rules for the dynamic behavior of the terms of trade and the nominal exchange rate. Section 4 analyzes the properties of the regimes in terms of the implied volatility of the nominal interest rates. In Section 5 we evaluate the different rules according to social welfare criteria. Section 6 concludes.

## 1 The Model

We consider a world economy composed by two countries,  $H$  and  $F$ , Home and Foreign respectively. The economy is populated by a continuum of agents on the interval  $[0, 1]$ . The population on the segment  $[0, n)$  belongs to country  $H$ , while the segment  $[n, 1]$  belongs to  $F$ . A generic agent is both producer and consumer: a producer of a single differentiated product and a consumer of all the goods produced in both countries.

Each agent derives utility from consuming an index of consumption goods, a composite index of the Home and Foreign goods, and from the liquidity

services of money holding, while derives disutility from producing the differentiated product. The whole economy is subjected to two country-specific sources of fluctuations: demand, supply shocks. Households maximize the expected discounted value of the utility flow.

We assume that markets are complete within and across countries by allowing agents to trade in a set of bonds that span all the states of nature.

Money matters because agents derive utility from its liquidity services. If real money balances and consumption are separable in utility and prices are flexible, money is neutral. In order to give a role to monetary policy for the short-run fluctuations of the economy, we introduce both nominal price rigidity and a market structure characterized by monopolistic competition. The latter assumption rationalizes the existence of price stickiness without violating the participation constraints of the producers. Nominal rigidities are introduced using a model *a la* Calvo (1983), thus allowing fluctuations around the equilibrium for a longer period of time.<sup>2</sup> In each period a seller faces a fixed probability  $1 - \alpha$  of adjusting its price, irrespective on how long it has been since the seller had changed its price. In this event the price is chosen to maximize the expected discounted profits under the circumstance that the decision on the price is still maintained. We have that  $1/(1 - \alpha)$  represents the average duration of contracts within a country. We allow the degrees of rigidity to vary across countries.

The instrument of monetary policy is the domestic short-term nominal interest rate. In terms of our equilibrium conditions, this means that the money market equilibrium condition can be neglected, provided we are not interested in characterizing the path of real money balances or that of money supply in the whole area. Monetary policy is endogenous, meaning that the interest rate is set to react to other macroeconomic variables. Through the plethora of rules that different reactions can generate, we analyze the positive and normative consequences of alternative monetary regimes.

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<sup>2</sup>Yun (1996), in a closed-economy model, and Kollmann (1996), in a open-economy model, introduce Calvo's type of price-setting into dynamic general equilibrium monetary models.



In the next section we present the log-linear approximation of the structural equations of the model.

## 1.1 AD Block

The aggregate demand block is derived from the log-linear approximation to the first-order conditions of the representative consumers in the Home and Foreign countries.<sup>3</sup>

The Euler equation describes the intertemporal link between current and one-period ahead expected consumption and relates it to the risk-free real return in units of the consumption index.

The assumption of complete international markets, combined with the law of one price and the fact that the consumption index is common across countries, implies that there is perfect risk sharing of consumption across countries. It follows that the allocation of the consumption bundle can be described by only one Euler equation. In a log-linear form

$$E_t \widehat{C}_{t+1} = \widehat{C}_t + \rho^{-1} n (\widehat{i}_t^H - E_t \pi_{t+1}^H) + \rho^{-1} (1 - n) (\widehat{i}_t^F - E_t \pi_{t+1}^F), \quad (1)$$

where  $\widehat{C}$  is the consumption index,  $\widehat{i}^H$  and  $\widehat{i}^F$  are the nominal interest rates in the Home and Foreign countries,  $\pi^H$  and  $\pi^F$  are the respective producer inflation rates (where  $\pi_t^H = \ln P_{H,t} / P_{H,t-1}$  and  $\pi_t^F = \ln P_{F,t} / P_{F,t-1}$ ) and  $\rho$  is the inverse of the intertemporal elasticity of substitution in consumption.<sup>4</sup> Expected consumption growth depends on the real return in units of the

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<sup>3</sup>In what follows, given a variable  $X^H$  for country  $H$  and the respective variable  $X^F$  for country  $F$ , we define

$$\begin{aligned} X^W &= nX^H + (1 - n)X^F, \\ X^R &= X^F - X^H. \end{aligned}$$

Instead  $\widetilde{X}$  denotes the natural rate of  $X$ , i.e. the value that would be achieved in the case prices were perfectly flexible, while  $\widehat{X}$  denotes its sticky-price equilibrium. All variables should be interpreted as deviations from a steady state level.

<sup>4</sup>We have denoted with  $P_H$  the price of the home produced goods in the home currency and with  $P_F$  the price of the foreign produced goods in the foreign currency.

same index, which can be written as a weighted average of the real returns in units of the Home and Foreign consumption goods indices. In fact,  $n$  and  $(1 - n)$  represent respectively the shares of Home and Foreign goods in the total consumption index. In this model,  $n$  and  $1 - n$  coincide with the population size of the Home and Foreign countries.

Output in each country is determined according to

$$\widehat{Y}_t^H = (1 - n)\widehat{T}_t + \widehat{C}_t + g_t^H, \quad \widehat{Y}_t^F = -n\widehat{T}_t + \widehat{C}_t + g_t^F, \quad (2)$$

where  $\widehat{Y}^H$  and  $\widehat{Y}^F$  are output produced in countries  $H$  and  $F$ ,  $g$  is a country-specific demand shock. We denote with  $S$  the nominal exchange rate (the price of foreign currency in terms of home currency) while  $T$ , the terms of trade, is defined as  $T \equiv SP_F/P_H$ . Each country's output is affected by a common aggregate consumption component,  $C$ . However, the country-specific demand shocks and the terms of trade can create dispersion of output across countries. Using (2), world output can be written as the sum of world consumption and demand shock

$$\widehat{Y}_t^W = \widehat{C}_t + g_t^W.$$

The world output gap is the difference between its sticky-price equilibrium and the natural rate that arises under flexible prices

$$y_t^W = \widehat{Y}_t^W - \widetilde{Y}_t^W,$$

We can then rewrite the Euler equation (1) in terms of world output gaps as

$$E_t y_{t+1}^W = y_t^W + \rho^{-1} n (\widehat{i}_t^H - E_t \pi_{t+1}^H - \widetilde{R}_t^W) + \rho^{-1} (1 - n) (\widehat{i}_t^F - E_t \pi_{t+1}^F - \widetilde{R}_t^W), \quad (3)$$

where  $\widetilde{R}^W$  represents the perturbations to the world natural real interest rate, the Wicksellian rate. It is the rate that would arise in the case prices were perfectly flexible and each country's inflation rate were zero. As it is shown in Appendix A, it is a combination of world supply and demand shocks. Equation (3) can be interpreted as a microfounded 'open-economy' IS curve.<sup>5</sup>

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<sup>5</sup>For a closed-economy counterpart, see Kerr and King (1996) and Woodford (1996).

The terms of trade are state variables that depend on their past value, the producer inflation differential as well as on the exchange rate depreciation. In fact, in a log-linear approximation, we obtain

$$\widehat{T}_t = \widehat{T}_{t-1} + \Delta S_t + \pi_t^F - \pi_t^H. \quad (4)$$

The expected depreciation of the exchange rate is linked to the nominal interest rate differential through the uncovered interest parity

$$E_t \Delta S_{t+1} = \widehat{i}_t^H - \widehat{i}_t^F. \quad (5)$$

## 1.2 AS Block

The supply block of the model contains the two aggregate supply equations

$$\begin{aligned} \pi_t^H &= \lambda^H \widehat{m}c_t^H + \beta E_t \pi_{t+1}^H, \\ \pi_t^F &= \lambda^F \widehat{m}c_t^F + \beta E_t \pi_{t+1}^F, \end{aligned}$$

where  $\widehat{m}c$  represents the deviation of the real marginal costs from the steady state;  $\lambda^H$  and  $\lambda^F$  are a combinations of the structural parameters of the model while  $\beta$  is the intertemporal discount factor in the consumer preferences.<sup>6</sup>

As in the closed-economy case, under the Calvo-style price-setting model, producer inflation rates exhibit a forward-looking behavior. They depend on the current and expected deviations of the real marginal costs from the steady state. The short-run response of inflation to real marginal costs is related to the probability that in each period sellers adjust their prices. When the two countries have the same degree of nominal rigidity, the producer inflation rates react similarly to movements in the real marginal costs. Using the labor supply decision of the households, it is possible to write the real marginal costs as a function of the marginal rate of substitution between consumption

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<sup>6</sup>We have defined  $\lambda^i \equiv [(1-\alpha^i\beta)(1-\alpha^i)/\alpha^i] \cdot [1/(1+\sigma\eta)]$  for  $i = H$  or  $F$ . We have that  $\sigma$  is the elasticity of substitution in consumption between the differentiated goods produced within a country while  $\eta$  is the elasticity of the disutility of producing the differentiated goods.

and the production of the domestic goods. We can then write the aggregate supply equations as

$$\begin{aligned}\pi_t^H &= \lambda^H[(1-n)(1+\eta)(\widehat{T}_t - \widetilde{T}_t) + (\rho + \eta)(y_t^W)] + \beta E_t \pi_{t+1}^H \\ &= (1-n)k_T^H(\widehat{T}_t - \widetilde{T}_t) + k_C^H(y_t^W) + \beta E_t \pi_{t+1}^H,\end{aligned}\quad (6)$$

$$\begin{aligned}\pi_t^F &= \lambda^F[-n(1+\eta)(\widehat{T}_t - \widetilde{T}_t) + (\rho + \eta)(y_t^W)] + \beta E_t \pi_{t+1}^F \\ &= -nk_T^F(\widehat{T}_t - \widetilde{T}_t) + k_C^F(y_t^W) + \beta E_t \pi_{t+1}^F,\end{aligned}\quad (7)$$

where  $\eta$  is the elasticity of the disutility of producing the goods.  $\widetilde{T}_t$  is the flexible-price terms of trade which, as we show in Appendix A, is a combination of relative demand and supply shocks.<sup>7</sup>

There are some interesting novelties that these AS equations imply in an open economy.

First, the real marginal costs are not proportional to the output gap, as a consequence of the interdependence induced by the terms of trade. The smaller and more open is a country, the more relative prices influence the real marginal costs and thus the inflation rates. Focusing on the AS equation in country  $H$ , an increase in the terms of trade shifts the AS equation and increases inflation of country  $H$  through two channels. The first is the expenditure-switching effect: an increase in the price of goods produced in country  $F$  relative to goods produced in  $H$  boosts the demand of goods produced in country  $H$ , pushing up inflation in this country. The second is the reduction in the marginal utility of nominal income: the optimal response is to increase prices in order to offset the fall in revenues.

Second, the relation between real marginal costs and the terms of trade may create an intrinsic inertia in the real marginal costs. In fact, as equation (4) shows, terms of trade are state variables that depend on past values of the inflation rate differential and the exchange rate. If, conditional on the monetary policy regime chosen, terms of trade are sluggish then the adjustment of real marginal costs is slow.

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<sup>7</sup>Where we have defined  $k_C^i \equiv \lambda^i(\rho + \eta)$  and  $k_T^i \equiv k_C^i \left(\frac{1+\eta}{\rho+\eta}\right)$ .

### 1.3 Interest Rate Rules

The model is closed by specifying the monetary policy rules followed by the Central Banks. Here we assume that the monetary authority controls the short-term nominal interest rate. In the classical Taylor rule, the instrument is set to react to domestic inflation and output gap. However, in an open-economy model, the specification of the rules is more controversial, because the set of variables toward which monetary policy can react is larger.<sup>8</sup>

Our strategy is to explore the analytical property of our model by first specifying simple, and reasonable, rules that lead to equilibria that can be solved analytically, in order to infer informations on shocks' transmission mechanism in open economies. We analyze three regimes: a fixed exchange rate, a floating exchange rate and a managed exchange rate.

In the first regime, we design rules that imply a determined and fixed nominal exchange rate. It will be shown that, in principle, many fixed exchange rate regimes exist depending on the specification of the underlying rules.

Then we consider a floating regime, which is defined as a regime in which the interest rates in both countries do not react explicitly to the exchange rate. A class of policies with this characteristic is

$$\begin{aligned}\widehat{i}_t^H &= \gamma \widehat{i}_{t-1}^H + \phi \pi_t^H + \psi y_t^H, \\ \widehat{i}_t^F &= \gamma^* \widehat{i}_{t-1}^F + \phi^* \pi_t^F + \psi^* y_t^F,\end{aligned}$$

with  $\phi, \phi^*, \psi, \psi^*, \gamma$  and  $\gamma^*$  positive. Rules of this kind have been extensively used in the closed-economy literature.<sup>9</sup> Each policymaker reacts to past movements in the interest rate, the current domestic producer inflation rate and output gap. When the coefficients  $\gamma$  and  $\gamma^*$  are zero, this class boils down to classical Taylor rules.

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<sup>8</sup>Only recently the works of Ball (1998), Ghironi (1998), McCallum and Nelson (1998), Monacelli (1998), Svensson (2000) and Weeparana (1998) have started this analysis using open-economy models.

<sup>9</sup>An exhaustive analysis is presented in Woodford (1999a,b, 2000).

As a third regime, we consider a managed exchange rate, a ‘dirty’ floating. We consider the cases in which one country reacts either to changes in the nominal exchange rate or to the deviations of the level of the exchange rate from a defined target. A managed exchange rate (I) is defined as a couple of rules of the form

$$\begin{aligned}\widehat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \widehat{i}_t^F &= \phi^*\pi_t^F + \psi^*y_t^F - \lambda\widehat{S}_t,\end{aligned}$$

while a managed exchange rate (II) is defined as

$$\begin{aligned}\widehat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \widehat{i}_t^F &= \phi^*\pi_t^F + \psi^*y_t^F - \mu\Delta S_t,\end{aligned}$$

with  $\phi, \phi^*, \psi, \psi^*, \lambda, \mu$  positive and  $\widehat{S}_t \equiv \ln(S_t/S^*)$  where  $S^*$  is the exchange rate target.<sup>10</sup>

## 1.4 Summing Up

Equations (3), (4), (5), (6) and (7) combined with the log-linear version of the interest rate rule characterize completely our log-linear equilibrium dynamics.

In this model we can identify three roles for the exchange rate.

First, the exchange rate affects relative prices and by this channel the demand of the produced goods. This is the expenditure-switching effect: an exchange rate depreciation shifts demand from goods produced in country  $F$  to goods produced in  $H$ . As an indirect impact, it then increases producer price inflation in country  $H$ .

Second, the exchange rate is like a price of an asset that is linked to the nominal-interest-rate differential through the uncovered interest parity.

Third, if the instrument of monetary policy responds directly or implicitly to the exchange rate, there exists another channel of transmission through

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<sup>10</sup>This class of rules can be further enlarged by incorporating a response to past values of the interest rate.

which the exchange rate has a direct impact on the real return and consumption growth.

## 2 Equilibrium Determinacy

A critical issue in an open-economy framework is the determinacy of the nominal exchange rate. Indeed, purely ‘sunspot-driven’ movements in the nominal exchange rate can induce indeterminacy in the ‘real’ side of the economy through the relative price channel. The specification of the monetary policy rules can be relevant for eliminating any dependence of the equilibrium allocation of the economy from fluctuations unrelated to the fundamentals.

In this section, we discuss how monetary policy rules should be designed in order to determine a unique and stable rational expectations equilibrium for the log-linear approximation to the equilibrium conditions of our model.<sup>11</sup> Here we focus on the cases in which both countries have the same degree of nominal rigidity and the same coefficients on the targets in the rules.

In a closed-economy model, Woodford (1999a, 2000) derives the conditions under which interest rate feedback rules imply a determinate equilibrium. In particular, Taylor rules in which the interest rate reacts only to the current inflation rate lead to a determinate equilibrium when the reaction toward inflation is aggressive, i.e. if the weight on the inflation deviations from the target is bigger than one. This is known as the Taylor’s principle.

In an open-economy framework the richness of the set of relevant variables complicates the analysis. Moreover, a policy rule followed by a country, if considered aside from the rules followed by other countries, can be no longer sufficient for the determinacy of the equilibrium. The interdependence among the rules followed by different countries, even in a non coordinated way, is crucial in determining the characteristics of various monetary regimes, within

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<sup>11</sup>Since the analysis of Sargent and Wallace (1975), interest rate targeting has often been associated with the problem of indeterminacy of the equilibrium. Woodford (1999a) shows that their result applies to the special kind of interest rate rules that specify a path for a short term nominal interest rate instrument as a function of exogenous variables.

and across countries, and affects the response of the economies to domestic and foreign shocks.

We first analyze a class of interest rate rules that can implement a fixed exchange rate regime. If the Foreign nominal interest rate is tied to the Home nominal interest rate, i.e.  $\widehat{i}_t^F = \widehat{i}_t^H$  the uncovered interest parity, equation (5) implies that the exchange rate expectations are always zero. However, this is not sufficient to determine the nominal exchange rate. In fact, consider a simple bounded process  $\{\zeta_t\}_{t=0}^{+\infty}$  such that  $E_{t-1}\zeta_t = 0$ . Then, consistently with the rule followed, there exists an equilibrium in which the exchange rate depreciation follows the path  $\Delta S_t = \zeta_t$  and in which the exchange rate can be moved by exogenous disturbances that are not related to the fundamentals. One way to obtain determinacy is when the nominal interest rate in the Foreign country follows the Home nominal interest rate and reacts with a feedback to deviations of the exchange rate from a desired target:

$$\widehat{i}_t^F = \widehat{i}_t^H - \lambda \widehat{S}_t, \quad (8)$$

with  $\lambda > 0$  where  $\widehat{S}_t \equiv \ln(S_t/S^*)$  and  $S^*$  is the exchange rate target. On a path in which the exchange rate is above the target, the foreign interest rate is lowered, viceversa if the exchange rate is below.

Substituting (8) into equation (5) and noting that  $\Delta S_{t+1} = \widehat{S}_{t+1} - \widehat{S}_t$ , we obtain

$$E_t \widehat{S}_{t+1} = (1 + \lambda) \widehat{S}_t.$$

In order to have a unique and bounded rational expectation equilibrium for the nominal exchange rate, is sufficient to have  $\lambda$  greater than 0. In this equilibrium  $S_t = S^*$  at all dates  $t$ .<sup>12</sup>

To close the determination of all the variables of the model, we have to specify a reaction function for the “leader”.<sup>13</sup> There are many possible spec-

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<sup>12</sup>It is worth stressing that the explicit feedback toward the exchange rate is not operative in the equilibrium, but private sector should believe that the monetary policymaker is committed to this reaction function in a credible way.

<sup>13</sup>In what follows we will use the terminology leader and follower only as a reference but without any implications for the timing of monetary policy decisions.



ifications of the interest rate rules for the Home country all compatible with the definition of a fixed exchange rate regime but with different implications for macroeconomic variability. Here we restrict the instrument rule of the leader to

$$\widehat{i}_t^H = \phi\pi_t^H + \psi y_t^H.$$

We have the following proposition.

**Proposition 1** *Under a fixed exchange rate regime defined by a couple of rules of the following form*

$$\begin{aligned}\widehat{i}_t^F &= \widehat{i}_t^H - \lambda\widehat{S}_t, \\ \widehat{i}_t^H &= \phi\pi_t^H + \psi y_t^H,\end{aligned}$$

*with  $\phi, \psi, \lambda$  non negative, if the degrees of rigidity are equal across countries, there is equilibrium determinacy if condition*

$$(\phi - 1)k_C + \psi(1 - \beta) > 0. \tag{9}$$

*holds and  $\lambda$  is greater than zero.*

This result has some interesting features.

First, it shows that in general fixed exchange rate regimes are not destabilizing. The restriction to be satisfied are fairly broad and reasonable: we have assumed that the interest rate rule of the “follower” reacts to deviations of the exchange rate from the target, while the instrument rule of the “leader” follows a simple Taylor rule.

Second, once the exchange rate is determined, the restrictions required for the determinacy of the equilibrium are the same as in the closed economy case under a Taylor-rule regime. In the case  $\psi = 0$ , the Taylor’s principle holds.

We now characterize the determinacy of the equilibrium under the floating exchange rate regime. Considering the rules

$$\begin{aligned}\widehat{i}_t^H &= \gamma\widehat{i}_{t-1}^H + \phi\pi_t^H + \psi y_t^H, \\ \widehat{i}_t^F &= \gamma\widehat{i}_{t-1}^F + \phi\pi_t^F + \psi y_t^F,\end{aligned}$$

we obtain the following proposition.

**Proposition 2** *Under a floating exchange rate regime with  $\phi$ ,  $\psi$  and  $\gamma$  non negative, if the degrees of rigidity are equal across countries, there is equilibrium determinacy if the following conditions hold*

$$k_T(\gamma + \phi - 1) + \psi(1 - \beta) > 0, \quad (10)$$

$$k_C(\gamma + \phi - 1) + \psi(1 - \beta) > 0. \quad (11)$$

When  $\rho < 1$  then  $k_T > k_C$ , and (11) implies (10). Viceversa when  $\rho > 1$ . The conditions are the same as in the closed economy counterpart. The only, but important, qualification is that both reaction function should be simultaneously “aggressive”. Indeed, if only one country were following a policy of interest rate pegging, we would observe indeterminacy of equilibrium. The higher the smoothing parameter, the less aggressive monetary policy need to be with respect to inflation and output gap. When the weight on the output gap is zero, then determinacy simply requires that  $\gamma + \phi > 1$ .

Finally we analyze the determinacy under the third regime, the managed exchange rate regime (I),

$$\begin{aligned} \widehat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \widehat{i}_t^F &= \phi\pi_t^F + \psi y_t^F - \lambda\widehat{S}_t, \end{aligned}$$

and the managed exchange rate regime (II)

$$\begin{aligned} \widehat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \widehat{i}_t^F &= \phi\pi_t^F + \psi y_t^F - \mu\Delta S_t. \end{aligned}$$

**Proposition 3** *Under the managed exchange rate regime (I) with  $\phi$ ,  $\psi$ ,  $\lambda$  non negative, if the degrees of rigidity are equal, there is equilibrium determinacy if condition (9) holds along with*

$$(\phi - 1)k_T + \psi(1 - \beta) > 0,$$

and  $\lambda$  is greater than zero. Under the managed exchange rate regime (II) with  $\phi, \psi, \mu$  non negative, if the degrees of rigidity are equal across countries, there is equilibrium determinacy if condition (9) holds along with

$$k_T(\phi + \mu - 1) + \psi(1 - \beta) > 0.$$

### 3 Terms of Trade and Exchange Rate Dynamics Under Different Regimes

In this section, we focus on the sticky-price equilibrium and continue to restrict the analysis to the case in which the two countries have the same degree of nominal rigidities. We analyze how the path of the terms of trade and the exchange rate is affected by different monetary policy regimes.

A natural benchmark of comparison is a particular flexible-price allocation in which the producer inflation rates are zero in both countries. As shown in the Appendix A, in this allocation  $T_t = \tilde{T}_t$  and  $S_t = \tilde{T}_t$ . All the movements that occur in the terms of trade are undertaken by the exchange rate while prices do not move. As we will show in the normative section, this equilibrium is the efficient equilibrium, from a central planner point of view, when we abstract from the liquidity preferences of holding money.

In the sticky-price equilibrium, a non efficient path of the terms of trade determines a dispersion of output and inflation across countries. In fact, considering the definition of Home and Foreign output gap, we can write the output gap differential as

$$y_t^F - y_t^H = (\tilde{T}_t - \hat{T}_t),$$

from which deviations of the terms of trade from the efficient equilibrium cause a spread in the output gap across countries.

The output-gap differential has also an immediate consequence on the path of the inflation rate differential. In fact, from the aggregate supply

equation we obtain

$$\pi_t^F - \pi_t^H = k_T E_t \left\{ \sum_{j=0}^{+\infty} \beta^j [y_{t+j}^F - y_{t+j}^H] \right\},$$

where the inflation differential depends on the expected path of the output gap differential. It is then implicit that a non-efficient path of the terms of trade creates a spread in the inflation differential across the economies. Instead, efficiency would require to absorb any asymmetric transitory shock with no variation in the output gap and inflation differential.

In the next sub-section, we investigate the path of the exchange rate and of the terms of trade under a fixed exchange rate system.

### 3.1 Fixed Exchange Rate Regime

Combining equations (6) and (7) and imposing the equilibrium condition of a fixed exchange rate, we obtain that the terms of trade behave according to a second-order stochastic difference equation which has always a unique and stable rational expectation solution of the form

$$\widehat{T}_t = \lambda_1 \widehat{T}_{t-1} + \lambda_1 k_T E_t \sum_{j=0}^{+\infty} (\beta \lambda_1)^j \widetilde{T}_{t+j}, \quad (12)$$

where  $\lambda_1$  is the stable eigenvalue, with  $0 < \lambda_1 < 1$ . Furthermore if the process followed by  $\widetilde{T}_t$  is Markovian of the form

$$\widetilde{T}_t = \rho_1 \widetilde{T}_{t-1} + \varepsilon_t, \quad (13)$$

with  $0 < \rho_1 < 1$ , then (12) can be simplified to

$$\widehat{T}_t = \lambda_1 \widehat{T}_{t-1} + \nu \widetilde{T}_t,$$

where we have  $0 < \nu < 1$ .<sup>14</sup> It is important to note that the dynamical properties of the terms of trade – in the case the degrees of rigidity are equal

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<sup>14</sup> $\nu$  is defined as  $\nu \equiv \frac{\lambda_1 k_T}{1 - \beta \lambda_1 \rho_1}$ .

across countries – do not depend on the underlying rules that characterize the fixed exchange rate and are only affected by real asymmetric shocks.

Moreover the terms of trade are inertial and their short-run response to natural terms of trade shocks is less than proportional.  $\lambda_1$  is a monotone increasing function of the overall degree of nominal rigidity. As the rigidity decreases, the inertia in the terms of trade decreases. On the other hand,  $\nu$  is a monotone decreasing function of the degree of nominal rigidities, and at the limit, with perfect flexibility,  $\nu$  approaches one. As the degree of rigidity decreases, the short run response approaches the flexible-price response. By dampening the variability of the exchange rate, the variability of the terms of trade is reduced:

$$\text{var}(\widehat{T}_t^F) = \frac{1 + \rho_1 \lambda_1}{1 - \rho_1 \lambda_1} \frac{\nu^2}{1 - \lambda_1^2} \text{var}(\widetilde{T}_t) < \text{var}(\widetilde{T}_t)$$

where again the variability of the terms of trade increases monotonically as the degree of rigidity decreases and at the limit approaches the variability that efficiency would require.

### 3.2 Floating Exchange Rate Regime

First, we restrict the analysis to the classical Taylor rules

$$\begin{aligned} \widehat{i}_t^H &= \phi \pi_t^H + \psi y_t^H, \\ \widehat{i}_t^F &= \phi \pi_t^F + \psi y_t^F. \end{aligned}$$

As we show in the Appendix B, the equilibrium paths of the terms of trade, the interest rate differential and the exchange rate depreciation is given by

$$\begin{aligned} \widehat{T}_t &= c_1 \widetilde{T}_t, \\ \pi_t^F - \pi_t^H &= c_2 \widetilde{T}_t \\ \Delta S_t &= -\widehat{T}_{t-1} + c_3 \widetilde{T}_t. \end{aligned}$$

An interesting implication for the dynamical properties of the terms of trade is that the inertia is completely eliminated. Instead changes in the exchange

rate react negatively to past movements in the terms of trade (with a unitary coefficient). Moreover,  $c_1 > c_2$  as well as  $c_1 > c_3$ . Instead  $c_3$  becomes bigger than  $c_2$  the more aggressive monetary policy is with respect to inflation and output (i.e. as  $\phi$  and  $\psi$  increase). When the reaction toward inflation becomes infinite – i.e. as  $\phi \rightarrow +\infty$  –, the parameters converge in the following way  $c_1 \rightarrow 1$ ,  $c_2 \rightarrow 0$  and  $c_3 \rightarrow 1$ .

Differently from the fixed exchange rate system, the weights that monetary policy put on inflation and output stabilization affect the dynamical properties of the terms of trade. In fact, under the floating regime, the volatility of the terms of trade is

$$\text{var}(\widehat{T}_t) = c_1^2 \text{var}(\widetilde{T}_t) < \text{var}(\widetilde{T}_t),$$

and approaches the efficient volatility as the parameters  $\phi$  or  $\psi$  increase.

At first pass this analysis allows us to address some empirical regularities.

For example, Obstfeld (1997) shows that in shifting from a fixed exchange rate system to a floating regime the volatility of the terms of trade increases. In our model, this result is ambiguous and depends on the coefficients of the interest-rate rules.<sup>15</sup> When monetary policy is aggressive with respect to inflation (high values of  $\phi$ ) then the terms of trade volatility is higher under the floating regime. For higher values of  $\phi$ , the volatility approaches the efficient volatility, while the volatility under fixed exchange rate system is independent of the parameters of the rule. However, for lower values of  $\phi$ , the volatility of the terms of trade under the floating regime decreases and can be lower than the volatility under the fixed exchange rate regime.

Another empirical regularity documented by Obstfeld (1997) is that, in a floating regime, changes in the terms of trade display more variability than

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<sup>15</sup>Our result differs from Monacelli (1998). He addresses this issue in a dynamic small-open-economy model with an endogenous monetary policy by specifying a reaction function in which there is a feedback toward deviations of the nominal exchange rate from a target. He labels as a floating exchange rate regime a regime in which there is no feedback. He finds that the terms of trade variability is a decreasing function of the strengthen of the feedback.

changes in the nominal exchange rate. Considering the terms of trade identity (4), we can write

$$\text{var}(\Delta\widehat{T}) = \text{var}(\Delta\widehat{S}) + \text{var}(\pi^R) + 2\text{cov}(\Delta\widehat{S}, \pi^R).$$

The variability of the changes in the terms of trade is higher than that of changes in the nominal exchange rate if

$$\text{var}(\pi^R) > -2\text{cov}(\Delta\widehat{S}, \pi^R),$$

which in our model is satisfied if and only if

$$c_2 < 2c_1(1 - \rho_1).$$

The latter condition holds for reasonable calibrations of the parameters, and in general is always satisfied provided  $\rho_1$  is not high, e.g.  $\rho_1 < 0.5$ .

More interestingly in this simple example we can analyze the equilibrium path of the exchange rate. By exploiting the initial conditions  $S_{t-1} = \widetilde{T}_{t-1} = \widehat{T}_{t-1} = 0$ , we have that at a generic time  $\tau \geq t$  the exchange rate is equal to the sum of current and past perturbations to the natural terms of trade

$$S_\tau = c_3\widetilde{T}_\tau - \sum_{j=t}^{\tau-1} c_2\widetilde{T}_j.$$

It is important to note that the exchange rate is a non-stationary variable. In the long run, temporary shocks do not die out and have a permanent effect. Here we denote with  $\Delta E_t S_\tau$  the innovation in the time  $t$  rational forecast of the time  $\tau$  nominal exchange rate following an innovation in the shock process at time  $t$ , i.e.<sup>16</sup>

$$\Delta E_t S_\tau = E_t S_\tau - E_{t-1} S_\tau.$$

Assuming a process of the form (13) and a temporary perturbation  $\varepsilon_t$  at time  $t$ , we can write the innovation in the time  $t$  rational forecast of the time  $\tau$  exchange rate as

$$\Delta E_t S_\tau = c_3 \rho_1^{\tau-t} \varepsilon_t - c_2 \frac{1 - \rho_1^{\tau-t}}{1 - \rho_1} \varepsilon_t,$$

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<sup>16</sup>The difference operator is applied to the conditional expectations operator.

from which we obtain its long-run innovation

$$\Delta E_t S_\infty = -c_2 \frac{1}{1 - \rho_1} \varepsilon_t. \quad (14)$$

The nominal exchange rate displays a non-stationary behavior.

In fact,  $\Delta E_t S_\infty$  is the innovation in the rational forecast of the long-run nominal exchange rate, i.e. the innovation of the stochastic trend in the definition of Beveridge and Nelson (1981), and it measures the magnitude of the non-stationary component.

As well, we can compute the short-run unexpected response, which is given by  $\Delta E_t S_t = c_3 \varepsilon_t$ . The long run response of the nominal exchange rate has the opposite sign of the short-run's one.

Following a positive perturbation to the natural terms of trade (i.e. an increase in  $\tilde{T}$ ), the Home exchange rate suddenly depreciates – but less than proportionally. Instead, in the long run, it experiences an appreciation. The short-run reaction follows the efficient path, though the magnitude is smaller.

Consider for example a positive productivity shock that affects the Home country. The efficient equilibrium would require an appreciation of the Home terms of trade and a depreciation of the Home exchange rate ( $T$  and  $S$  should increase). In fact, there is a need to shift the demand to the Home produced goods, in order to equilibrate the disutility of working across countries.

However, under the floating regime, the Home terms of trade depreciate less while the exchange rate first depreciates and then appreciates. The time at which the appreciation is reached depends on the nature of the process  $\tilde{T}_t$ . If it follows a white-noise process, the appreciation occurs in the period immediately after the shock, while as  $\rho_1$  increases the appreciation is delayed. The long-run appreciation is a function both of the parameters  $\rho_1$  and  $c_2$ . As the monetary policy rules becomes more aggressive, the long-run appreciation is reduced while the short-run response is amplified.

The magnitude of the non-stationary behavior is then a function of the values of the parameters of the policy rule chosen. Moreover, in the short run, the correlation between expected changes in the terms of trade and the



exchange rate is positive, while becomes negative in the long-run.<sup>17</sup>

The intuition for the non-stationary behavior of the exchange rate is directly related to the state equation

$$\widehat{T}_t = \widehat{T}_{t-1} + \Delta S_t + \pi_t^F - \pi_t^H.$$

Although the terms of trade is stationary and is expected to revert to the initial value, there is nothing, under the floating regime specified, that restrict the exchange rate to revert to the pre-shock value, unless the producer inflation rates are always stabilized to zero. The long-run value of the terms of trade differs from the initial value for a different decomposition between the exchange rate and the domestic and foreign price levels. Taylor rules in which the reaction toward inflation is infinite can produce long-run stationarity of the exchange rate. But, we will show later that these rules are undesirable if we take into account the costs implied by the excess volatility of the nominal interest rate.

An important implication of this analysis is that, even if financial and monetary shocks can further exacerbate the excess volatility of the nominal exchange rate, real shocks do have an important role and can be source of the excess volatility of the nominal exchange rate. In particular, they can generate persistent effects. And we do not need to rely on a non-stationary distribution of such shocks. Nominal exchange rates are non stationary following stationary real shocks, while the same perturbations generate a stationary distribution for the real macroeconomic variables.

Here we investigate if and how the nonstationary behavior changes when the floating regime is defined by smoothing rules of the form

$$\begin{aligned}\widehat{i}_t^H &= \gamma \widehat{i}_{t-1}^H + \phi \pi_t^H + \psi y_t^H, \\ \widehat{i}_t^F &= \gamma \widehat{i}_{t-1}^F + \phi \pi_t^F + \psi y_t^F.\end{aligned}$$

As it is shown in Appendix B, the equilibrium path of the terms of trade and

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<sup>17</sup>This might also explain the empirical finding that the correlation between changes in the terms of trade and the exchange rate is not so strong, as shown in Obstfeld (1997).

of the exchange rate is of the form

$$\begin{aligned} A^s(L)\widehat{T}_t &= R^s(L)\widetilde{T}_t, \\ A^s(L)\Delta S_t &= U^s(L)\widetilde{T}_t, \end{aligned}$$

where  $A^s(L)$ ,  $R^s(L)$  are first-order polynomials while  $U^s(L)$  is of second order. Note that when the inertia coefficient  $\gamma$  is zero, there is no autoregressive component in  $A^s(L)$ .

We investigate if the inertia originated from the interest-rate-smoothing element can pin down a stationary behavior for the nominal exchange rate. In figure 1 we plot the innovation in the rational forecast of the long-run exchange rate following a perturbation to the natural terms of trade, under different rules in this class. We set  $\phi$  equal to 1.5, while  $\psi$  assumes values in the interval between 0 and 1. We allow the coefficient  $\gamma$  to vary between 0 and 10. For low values of the smoothing coefficient, in general for values below one, the exchange rate experiences a permanent appreciation. The opposite happens when the smoothing rules are super inertial. Again, only for particular parameters, the exchange rate becomes stationary. For example, this is the case if  $\gamma$  is equal to 1, when  $\psi$  is equal to 0.

### 3.3 Managed Exchange Rate Regimes

We now analyze the case in which monetary policy rules react to the nominal exchange rate.

The main result of this section is that the existence of a small feedback on the level of the exchange rate induces a stationary nominal exchange rate while a feedback on exchange rate changes is in general unsuccessful.

As shown in Appendix B, under a managed exchange rate (I), where the feedback is on the level, it is possible to obtain a solution for  $\widehat{T}_t$  and  $\widehat{S}_t$  of the form

$$\begin{aligned} A(L)\widehat{T}_t &= R(L)\widetilde{T}_t, \\ A(L)\widehat{S}_t &= U(L)\widetilde{T}_t, \end{aligned}$$

where  $A(L)$  is a second-order polynomial, while  $R(L)$  and  $U(L)$  are first-order polynomials.

The exchange rate is then a stationary variable that converges to the target  $S^*$  and behaves according to an ARMA process in which the AR component is of the second order. It is the case that such form of managed exchange rate succeeds in stabilizing the long-run fluctuations of the exchange rate.<sup>18</sup>

Instead, under the managed exchange rate (II), where there is a feedback to the exchange rate depreciation, we obtain a solution for  $\widehat{T}_t$  and  $\Delta S_t$  of the form

$$\begin{aligned} A^m(L)\widehat{T}_t &= R^m\widetilde{T}_t, \\ A^m(L)\Delta S_t &= U^m(L)\widetilde{T}_t, \end{aligned}$$

where  $A^m(L)$  and  $U^m(L)$  are first-order polynomials, while  $R^m$  is a constant. So the source of inertia in this case arises from the explicit target on exchange rate changes. However, from the solution of  $\Delta S$ , changes in the nominal exchange rate are a stationary variable, but nothing assures that the level is stationary. So this class of managed exchange rates can pin down a determined equilibrium for the exchange rate, but it does not necessarily imply a stationary value, unless for particular parameters. Figure 2 plot the innovation in the rational forecast of the long-run nominal exchange rate under different rules belonging to this class. We set  $\phi$  equal to 1.5, while  $\psi$  assumes values in the interval between 0 and 1. We allow the coefficient  $\mu$  to vary between 0 and 10. When  $\psi$  is equal to zero, the exchange rate experiences always a long-run appreciation as a consequence of a positive perturbation to the natural terms of trade, unless the feedback on the exchange rate depreciation becomes infinite. However, when the rules react explicitly also to the output gap, a strong response to the exchange rate implies a long-run permanent depreciation. The sign of the long-run behavior of the exchange

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<sup>18</sup>We can further show that the terms of trade variability is not monotone decreasing in the parameter  $\lambda$ . For low values of  $\lambda$  the variability of the terms of trade under this regime is lower than under the fixed exchange rate.

rate depends on the parameters of the rule. Only incidentally the exchange rate reverts to its initial steady state.

### 3.4 Some Comparisons

In this section, we analyze in a calibrated economy the impulse response function of the exchange rate and the terms of trade following a perturbation to the natural terms of trade. We calibrate the model according to the parametrization used in Rotemberg and Woodford (1998) while we continue to assume that the degrees of rigidity are equal, i.e.  $\alpha^H = \alpha^F = \alpha$ . The parameters  $\alpha$ ,  $\beta$ ,  $\eta$ ,  $\rho$ ,  $\sigma$  assume the values 0.66, 0.99, 0.47, 0.16, 7.88 respectively. We consider the following regimes in comparisons to the efficient equilibrium:

1. A fixed exchange rate system;
2. A symmetric Taylor-rule regime with parameters  $\phi = 1.5$ ,  $\psi = 0.5$ ;
3. A smoothing Taylor-rule regime with parameters  $\phi = 0.3$ ,  $\psi = 0.1$  and  $\gamma = 0.8$ ;
4. A smoothing Taylor-rule regime with parameters  $\phi = 0.1$ ,  $\psi = 0.025$  and  $\gamma = 1.2$ .
5. A managed exchange rate regime (I) with  $\phi = 1.5$ ,  $\psi = 0.5$  and  $\lambda = 0.5$ ;
6. A managed exchange rate regime (II) with parameters  $\phi = 1.5$ ,  $\psi = 0.5$  and  $\mu = 0.5$ ;

Note that in the smoothing rule 3), the parameters that indicate the short-run reaction to inflation and the output gap have been adjusted to imply the same long-run response as under the Taylor rule 2). In rule 4) these parameters have been further lowered because of the super-inertial component implied by the smoothing coefficient. Rules 2), 3) and 4) belong

to the class of floating regimes. Instead, rules 5) and 6) belong to the class of managed exchange rate regimes.

First, we analyze the case of no persistence in the natural terms of trade process, i.e.  $\rho_1 = 0$ . In Figure 3, we plot the impulse response of the terms of trade to a temporary positive shock to the natural terms of trade, for each of the rules considered. In the efficient equilibrium and in the Taylor-rule regime there is no endogenous persistence in the economy independently of the contract length; moreover, we note that the short-run response of the terms of trade under the Taylor rule is smaller than under efficiency. An interest-rate smoothing argument implies a weaker short-run response than the Taylor-rule regime while determines an inertial behavior in the terms of trade. In the fixed exchange rate regime and in the managed exchange rate regimes (I) and (II) the response is dampened.

Figure 4 shows the impulse response of the nominal exchange rate.

Under the Taylor-rule regime, the exchange rate depreciates and suddenly appreciates without reverting to the initial equilibrium. By introducing interest rate smoothing, the short-run response is dampened, and the shock is propagated over time. The long-run value depends on the smoothing coefficient: when the coefficient  $\gamma$  is equal to 0.8, the nominal exchange rate is close, in the long run, to the initial value, while with a coefficient 1.2 it persistently depreciates. Under this regime, the exchange rate depreciates instantaneously and remains around the depreciated value for all periods. Once the shock occurred, the expected exchange rate depreciation or appreciation is approximately zero. The managed exchange rate regime (I) corrects for the non-stationary path and implies an inertial adjustment toward the equilibrium value.<sup>19</sup> In the case of managed exchange rate (II), in the long run, the exchange rate persistently appreciates, though less than under the Taylor-rule regime.

In Figure 5 and 6 we repeat the same experiment, but assuming autocorrelation in the natural terms of trade,  $\rho_1 = 0.4$ . Consistently with the theo-

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<sup>19</sup>We are assuming that the target coincides with the initial value.

retical findings, the inertial behavior of the terms of trade, under the efficient path and the Taylor-rule regime, is a consequence of the exogenous inertia in the natural terms of trade. The response of the terms of trade under the fixed exchange rate system presents a hump-shaped pattern and in a short period of time overshoots the efficient path. The managed exchange rate systems (I) and (II) present a dampened short-run response but an higher persistence than the Taylor-rule regime. Similarly for the smoothing regimes, where persistency is amplified. These regimes present also overshooting of the terms of trade above the natural terms of trade. A similar path, but less evident, arises under the managed exchange rate regime (I).

Looking at the long-run response of the exchange rate, a high smoothing coefficient can induce a persistent exchange rate depreciation, while when  $\gamma$  is equal to 0.8 the nominal exchange rate is stationary. Under the Taylor rules and the managed exchange rate regime (I), the exchange rate first depreciates and then appreciates, but in the managed exchange rate regime (I) the exchange rate reverts to the target. Instead, under the managed exchange rate (II) there a long-run appreciation less than under the Taylor-rule regime. Consistently with the previous analysis, note that the property of stationary exchange rate obtained in the particular smoothing regime with  $\gamma = 0.8$  is not robust. With different smoothing parameters, the exchange rate would be non stationary, as it is the case with  $\gamma = 1.2$ .

## 4 Efficient Frontiers

In this section, under the maintained hypothesis of equal degrees of rigidity across countries, we explore the properties of the regimes in terms of the implied volatility of the nominal interest rate. We also continue to assume that the process for  $\tilde{R}^W$  and  $\tilde{T}$  are Markovian of the kind

$$\begin{aligned}\tilde{T}_t &= \rho_1 \tilde{T}_{t-1} + \varepsilon_t, \\ \tilde{R}_t^W &= \rho_2 \tilde{R}_{t-1}^W + u_t,\end{aligned}$$

where  $\varepsilon$  and  $u$  are white noise process. We decompose the nominal interest rate in its world and relative components. As it shown in Appendix A, the world and relative nominal interest rates in the efficient equilibrium follow

$$\begin{aligned}\widehat{i}_t^W &= \widetilde{R}_t^W, \\ \widehat{i}_t^R &= (1 - \rho_1)\widetilde{T}_t,\end{aligned}$$

while the Home and Foreign nominal interest rates can be written as a combination of world and relative variables.

As explained in Woodford (1999b,c), there are several reasons why the central bank should avoid a high volatility of the nominal interest rates.

First, if the economy experiences a deflationary shock, monetary policy is constrained by the lower bound on the nominal interest rate and then cannot reduce the interest rates as efficiency requires.

Second, the level of the nominal interest rate determines the distortions on the household behavior induced by the effort to economize on cash balance: a high volatility of the nominal interest rates would increase these distortions.

In our context, the decomposition of the nominal interest rate in the world and relative components shows that in implementing the efficient equilibrium, even if the world natural rate is not that volatile, an excess volatility of the natural terms of trade can conduct at most one nominal interest rate in proximity or below the zero floor. An excess volatility of the relative nominal interest rate implies also an excess volatility of the expected exchange rate depreciation. This can produce an unnecessary effort for producers or consumers to economize in the transaction services denominated in the foreign currency.

To illustrate the trade-off between stabilizing the economy toward the efficient outcome and maintaining a low volatility of the nominal interest rate, we construct the locus of all the efficient combinations between inflation and nominal interest rate volatility. In particular, we consider as a separated problem the frontiers associated respectively with world and relative variables

$$\ell_1 \equiv \text{var}(\pi^W) + \zeta \text{var}(\widehat{i}^W) \qquad \ell_2 \equiv \text{var}(\pi^R) + \zeta \text{var}(\widehat{i}^R),$$

in which the coefficient  $\zeta$  is varied. When the degrees of rigidity are equal, there is a rationale in decomposing the problem in two parts, given that the structural equations that determine world and relative variables are completely separated.

As shown in Figure 7, there is a trade-off, between stabilizing world inflation rates, and keeping the component of world nominal interest rate less volatile. As well there is a trade-off between stabilizing the variability of relative inflation and keeping low the variability of relative nominal interest rates, as shown in Figure 8.<sup>20</sup> In both figures, we indicate also the values that can be achieved under the policies we have considered in the numerical comparisons of the previous section.

Concerning the trade-off between the world variables, all the policies considered, except for the smoothing rule with  $\gamma = 1.2$ , manage to reduce the variability of world inflation, but at the cost of high volatility of the nominal interest rates. The policy that exploits in a better way the trade-off is the interest-rate-smoothing rule with  $\gamma = 0.8$ . Fixed and managed exchange rate regimes perform worse, because they introduce unnecessary dependence on relative prices shocks. The smoothing rule with  $\gamma = 1.2$  reduces the volatility of the nominal interest rates at a cost of higher volatility of the world inflation.

Instead, looking at the trade-off between relative variables, the Taylor rule is the most inefficient policy, while the smoothing regimes and the managed exchange rate regimes are characterized by lower volatility of the relative nominal interest rate. However the smoothing regime with  $\gamma = 0.8$  exploits better the trade-off, while the regime with  $\gamma = 1.2$  induces high volatility of the inflation rate differential above that implied by the fixed exchange rate regime.

In the case in which the degrees of rigidity are equal across countries, there is no trade-off between stabilizing the deviations of the terms of trade

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<sup>20</sup>We have assumed  $\rho_1 = 0.4$ , while  $\rho_2 = 0.35$ .



from their natural rate and stabilizing relative inflation. In fact

$$\pi_t^F - \pi_t^H = -k_T(\widehat{T}_t - \widetilde{T}_t) + \beta E_t(\pi_{t+1}^F - \pi_{t+1}^H).$$

In Figure 9, we plot the efficient frontier obtained by the combinations of relative interest rate variability and terms of trade gap variability that minimize the following function

$$\ell_3 \equiv \text{var}(\widehat{T} - \widetilde{T}) + \zeta \text{var}(\widehat{i}^R),$$

when the parameter  $\zeta$  is varied. There is again a trade-off between reducing the volatility of relative nominal interest rates and stabilizing the volatility of the terms of trade deviations from their natural rates.

## 5 General Case and Welfare Comparisons

In this section, we extend the analysis to the case in which the degrees of rigidity are different across countries. It is no longer possible to obtain a closed-form solution for the equilibrium implied by the log-linear structural equations. However, some of the conclusions and intuitions of the previous sections can be extended to this general case.<sup>21</sup>

### 5.1 Calibration

We calibrate a US-EMU model in which country  $H$  is the US, while country  $F$  is the EMU. Given that both areas contribute to world production in the same proportion, we consider their size to be equal. We set  $n = 0.5$ . The other parameters are calibrated according to Rotemberg and Woodford (1997), except for the degrees of nominal rigidity. We then set  $\beta, \eta, \rho, \sigma$  to assume the values 0.99, 0.47, 0.16, 7.88. Instead for the degrees of nominal rigidity we use the estimated values of Galí, Gertler and Lopes-Salides (2000). In their work, by assuming a mark-up of prices over marginal costs equal to the

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<sup>21</sup>It can be shown that even in this general case, the equilibrium implied by a Taylor-rule regime does not exhibit inertia.

10%, they estimate the degree of nominal rigidity in the US –the parameter  $\alpha^H$  in our model– in the interval between 0.407 and 0.660, while for the EMU they estimate a value in the interval between 0.670 and 0.771. We choose the middle points of these intervals, setting then  $\alpha^H = 0.487$  and  $\alpha^F = 0.720$ . These values imply an average duration of contracts of approximately 2.3 quarters in the US and of 3.65 quarters in the EMU area. The parameters  $k_C^H$  and  $k_C^F$  are respectively 0.0738 and 0.0151, while  $k_T^H$  and  $k_T^F$  assume the values 0.1705 and 0.0349. We assume that  $\rho_1 = 0.4$  and  $\rho_2 = 0.35$ . The standard deviation of the natural terms of trade is calibrated to 3.3, while the standard deviation of the world natural rate is 3.58.<sup>22</sup> Table 1 summarizes the parameters' values used in this section.

We compare the performance of rules that belongs to the following general class

$$\begin{aligned}\widehat{i}_t^H &= \gamma \widehat{i}_{t-1}^H + \phi \pi_t^H + \psi y_t^H + \lambda \widehat{S}_t + \mu \Delta S_t, \\ \widehat{i}_t^F &= \gamma^* \widehat{i}_{t-1}^F + \phi^* \pi_t^F + \psi^* y_t^F - \lambda^* \widehat{S}_t - \mu^* \Delta S_t,\end{aligned}$$

in which all the parameters are assumed to be positive. All the rules we have analyzed so far are nested in this general class. Table 2 and 3 gives a

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<sup>22</sup>To obtain a measure of the standard deviation of the natural terms of trade, we construct the deviations of the logarithm of the US terms of trade relative to their mean. In the last ten years their standard deviation has been of the order of the 2 per cent. However, in order to retrieve the standard deviation of the natural terms of trade, we need an hypothesis on the monetary policy regime occurred in the sample analyzed . As a rough method, we just suppose that both countries have followed a Taylor-rule regime. In the particular case in which the degrees of rigidity are equal across countries, the standard deviation of the natural terms of trade is  $1/c_1$  times the standard deviation of the terms of trade. Our approximate value is equal to 3.3. In calibrating the standard deviation of the world natural rate, we exploit the value estimated by Rotemberg and Woodford (1997) for the US economy. They find a standard deviation equal to 3.72 per cent. Using the decomposition of country H's natural real rate in

$$\widetilde{i}_t^H = \widetilde{R}_t^W + (1 - n)E_t\{\widetilde{T}_{t+1} - \widetilde{T}_t\},$$

and assuming that the changes in the terms of trade and the world natural rate are uncorrelated, we fix the standard deviation of the world real natural rate to 3.58.

summary of the rules that we will consider in this section.

The rules labeled with the letter  $T$  belongs to the class of Taylor rules. In  $T_1$  the coefficient on inflation is equal to 1.5 for both countries, while in  $T_2$  and  $T_3$  is alternatively equal to 1 or 2. The coefficient on the domestic output gap is set to 0.5.

The rules labeled with  $S$  belong to the class of instrument rules with interest rate smoothing. We assume that the parameters of smoothing are alternatively 0.6 or 0.9.<sup>23</sup> Rules  $S$  and  $T$  belong to the class of floating regime.

The rules labeled with  $M$  indicate managed exchange rate regimes.  $M_1$  and  $M_2$  are direct extensions of the rule  $T_1$  in which we allow also for a reaction respectively to the level and to the first difference of the exchange rate. In the same way,  $M_3$  and  $M_4$  extend rule  $S_4$ . Rule  $M_5$  allows for a contemporaneous reaction toward the level and the difference of the exchange rate.

Finally  $O_1$  and  $O_2$  are optimal rules according to the criterion of optimality that will be explained in the following paragraph. In particular in  $O_1$ , we constrain the rule of the Home country to assume values that approximately describes the behavior of the monetary authority in the U.S. (see the contributions by Taylor (1999) and Clarida, Galí and Gertler (1999)).

## 5.2 Evaluating Welfare Effects of Different Monetary Policy Rules

The purpose of this section is to evaluate, from a normative perspective, the behavior of different monetary policy rules. We abstract completely from any strategic interaction and we compare the rules using a centralized welfare criterion, which corresponds to the sum of the utility of all the consumers

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<sup>23</sup>The short-run responses to the inflation rates are adjusted in a way that relatively long-run responses assume alternatively the values 1, 1.5 or 2.

belonging to the world economy.<sup>24</sup>

For the moment, we abstract also from the liquidity services of holding real money balances. When the monopolistic distortions are completely eliminated by an appropriate taxation subsidy, a second-order approximation of the welfare function assumes the form

$$W = E[E_0(1 - \beta) \sum_{t=0}^{+\infty} \beta^t L_t],$$

where

$$L_t \equiv \Lambda_y [y_t^W]^2 + \Lambda_T [\widehat{T}_t - \widetilde{T}_t]^2 + \Lambda_{\pi^H} (\pi_t^H)^2 + \Lambda_{\pi^F} (\pi_t^F)^2,$$

and all the parameters  $\Lambda_y$ ,  $\Lambda_T$ ,  $\Lambda_{\pi^H}$  and  $\Lambda_{\pi^F}$  are positive with  $\Lambda_{\pi^H} + \Lambda_{\pi^F} = 1$ . The expectation  $E$  is taken over the possible initial states of the economy  $\widetilde{T}_0$  and  $\widetilde{R}_0^W$  at the time the commitments to the policy rules are taken, using the distribution of the processes for  $\widetilde{T}$  and  $\widetilde{R}^W$ .

Under the welfare loss  $W$ , the first-best coincides indeed with the particular flexible-price equilibrium in which producer inflations are zero.<sup>25</sup> Thus the loss function  $W$  measures the departure of the world economy from this particular flexible-price equilibrium. The coefficients  $\Lambda$  are function of the structural parameters of the model and, given our calibration, they assume the values  $\Lambda_y = 0.0512$ ,  $\Lambda_T = 0.0288$ ,  $\Lambda_{\pi^H} = 0.17$ ,  $\Lambda_{\pi^F} = 0.83$ .

The criterion  $W$  does not take into account variations in the nominal interest rates. By imposing the constraint on the lower bound on the nominal

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<sup>24</sup>The construction of an appropriate open-economy microfounded country-specific welfare criterion is still an open issue. See Kim and Kim (1999) for a discussion of the problems associated with quadratic approximation of the welfare based on the utility of the consumers. Given that we rely on log-linear approximation to the equilibrium condition, a second-order expansion of the welfare of the consumers represents a correct rank of alternative regime if in its evaluation it is only needed a first-order approximation to the equilibrium conditions.

<sup>25</sup>Some recent contributions of Devereux and Engel (2000) and Obstfeld and Rogoff (2000) have shown that in a stochastic model, with prices fixed one-period in advance and with pre-commitment, the flexible-price equilibrium is the first best, in the case of producer currency pricing, from a decentralized and centralized perspective. This is not the case, as shown by Devereux and Engel (2000), with local currency pricing.

interest rate or by assuming positive liquidity services of holding real money balances, as in Woodford (1999b,c), we can rationalize a welfare function  $W^*$  that includes the cost associated with nominal interest rate volatility. The appropriate  $L^*$  is then

$$L_t^* = L_t + n\Lambda_i(\widehat{i}_t^H)^2 + (1 - n)\Lambda_i(\widehat{i}_t^F)^2$$

where  $\Lambda_i$  has been restricted to be equal across the Home and Foreign consumers. We adopt Woodford's calibration that set  $\Lambda_i$  equal to 0.277. Along with costs in terms of the volatility of the nominal interest rate, the criterion  $W^*$  implicitly consider a cost in terms of volatility of the expected changes in the nominal exchange rate. Indeed, we can write

$$n\Lambda_i(\widehat{i}_t^H)^2 + (1 - n)\Lambda_i(\widehat{i}_t^F)^2 = \Lambda_i(\widehat{i}_t^W)^2 + n(1 - n)\Lambda_i(E_t\Delta S_{t+1})^2.$$

We evaluate the policy rules through the criteria  $W$  and  $W^*$ . The policy denoted  $O_1$  minimized the loss  $W^*$  under the constraint that the US policymaker follows an interest rate smoothing rule with coefficients  $\phi = 0.18$ ,  $\psi = 0.05$ ,  $\gamma = 0.9$ . The policy labeled with  $O_2$  minimizes without constraints the loss  $W^*$ , in the class of rules identified in this section. While the policy labeled with  $FB$  represents the first best for the economy, it minimizes the criterion  $W^*$  taking into account the dynamics implied by the structural equation of the model.

We compare the policy rules also in terms of the volatility of the relevant macroeconomic variables, where consistently with the measure of welfare, the volatility is defined using the operator  $v(\cdot)$  applied to the generic variable  $x$

$$v(x) = E[E_0(1 - \beta) \sum_{t=0}^{+\infty} \beta^t x_t^2].$$

Another criterium of comparison is the evaluation of the costs implied by the volatility of the exchange rate.<sup>26</sup>

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<sup>26</sup> Among others, there are two reasons why we argue that the volatility of the nominal exchange rate may not be desirable for the world economy. First, an excess volatility of

Considering a world in which agents can freely take position in the market of forward exchange rate, it is reasonable to argue that the costs induced by the excess volatility are not only associated with the volatility *per se* but also, with the unexpected component of this volatility. A natural measure is in the deviations of the forward exchange rate from the realized exchange rate.

In our model the forward rate is the best predictor of the exchange rate. Deviations from the uncovered interest parity can arise only ex-post. However these deviations are function of the monetary policy regime. We denote with  $f_t$  the forward exchange rate, contracted at a generic time  $t - 1$  for period  $t$ . Then, we construct the statistic  $v()$  applied to the difference between the realized value  $S_t$  and the forward rate  $f_t$ . This is a measure of the short-run unexpected volatility of the nominal exchange rate. Note that  $S_t - f_t = \Delta E_t S_t$ .

Equivalently if we want to evaluate the long-run capital losses, we use the statistic  $v()$  applied to  $\Delta E_t S_\infty$ , the volatility of the innovation in the stochastic trend of the exchange rate, if any exists. With  $\Delta E_t S_\infty | \tilde{T}$  we denote the innovation in the long-run nominal exchange rate conditional on a innovation to the natural terms of trade.

Table 4 and 5 summarize the comparisons among the various rules.<sup>27</sup>

Taylor rules in which the policymakers react only to contemporaneous inflation rates and output gaps perform better in terms of reducing the distance from the efficient equilibrium, measured by  $W$ . However, they imply a high cost in terms of volatility of the nominal interest rates. Rules with smoothing drastically reduce such volatility. Rules in which the smoothing parameter is 0.9 perform better than rules in which the parameter is 0.6 ac-  
the nominal exchange rate creates a cost for agents that are constrained to carry on foreign transactions by holding foreign currency. This cost is then due to the cost opportunity of holding money and is rationalized by the volatility of the nominal interest rates. Second, an excess volatility of the nominal exchange rate affects negatively trade between nations, as in Rose (2000).

<sup>27</sup>Interest rates, changes in the exchange rate and inflation rates have been annualized.

according to the criterion  $W^*$ , because they can further reduce the volatility of the nominal interest rates. The best rule among the ‘smoothing class’ is the one in which the country with higher degree of rigidity reacts more to its domestic inflation relative to the reaction of the other policymaker.

Regimes  $M_1$  and  $M_2$  perform better than regime  $T_1$  from which they are constructed, if the criterion is  $W^*$ . However, they induce more inefficient variability in the relevant macroeconomic variables, according to the criterion  $W$ . The volatility of the exchange rate changes is reduced by half, while the volatility of the deviations of the terms of trade from their natural rates is increased. Instead, rules  $M_3$  performs better than  $S_4$ , according to both criteria  $W$  and  $W^*$  suggesting that a control on the level of the nominal exchange rate might be desirable. Similarly a comparisons between  $M_4$  and  $S_4$  implies that reacting toward changes in the nominal exchange rate might improve the performance of the rule.

The desirability of targeting changes in the exchange rate is also present in the constrained optimal policy  $O_1$ . Given the assumed reaction function of the Home policymaker, the Foreign policymaker should react aggressively to the inflation rate with a high smoothing parameter. A reaction toward changes in the exchange rate is desirable, while the weight on the exchange rate level is zero. The unconstrained optimal policy,  $O_2$ , is more aggressive toward the inflation rates, especially toward the inflation rate in the Foreign country. It presents a interest rate smoothing coefficient above unity and a control toward the exchange rate, in particular toward the exchange rate depreciation. The control toward the level is marginal and slightly improves the welfare.<sup>28</sup> This kind of rule performs well in terms of the volatility of the macroeconomic variables compared to other interest rate smoothing rules, without inducing too much variability of the nominal interest rates. Moreover, it can well approximate the first-best.

There is a striking characteristic of the first best,  $FB$ , as well as of the rule

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<sup>28</sup>When the reaction to the level is removed, the welfare passes from 0.997266 to 0.997268. This control of the exchange rate level can be justified as a device to increase the inertia of the policy as to approximate the first best.

$O_2$ . They reduce the volatility of the expected changes in the exchange rate to zero. This is in fact the cost of exchange rate fluctuations that arises in the welfare criterion. They act as if they were pure interest rate pegging rules but they lead to determinacy of the equilibrium. However, even if they imply a path close to zero for the nominal interest rate differential, they are different from a pure fixed exchange rate system, because they allow for unexpected short-run deviations of the exchange rate. This unexpected component is large compared to other rules. Alternatively these rules produces changes in the exchange rate that are only a function of current shocks with no dependence on the past history of the perturbations that the economy has experienced. Moreover, under the first best, there is no control of the long-run behavior of the exchange rate following a perturbation to the world natural real rate, while perturbations to the natural terms of trade are not allowed to produce high volatility of the innovation in the long-run rational forecast of the exchange rate.

In this framework, there is then scope for managing the exchange rate even from a centralized welfare criterion.

## 6 Conclusions

This paper provides a simple framework for addressing empirical and policy issues in an open economy framework. We have emphasized the crucial role of policy rules in determining a different pattern for the nominal exchange rate and the terms of trade. A managed exchange rate is desirable because it can reduce the losses given by the departure of the equilibrium allocation from the flexible-price equilibrium and by the volatility of the nominal interest rates.

Here we want to underline how the simplicity and the flexibility of this model make it suitable for analyzing many other important issues that we have neglected at first pass. The extension to the analysis of monetary policy rules to an open economy framework provide new insights on the desirability



of alternative rules, and raises a number of issues of great interest, among which the choice of exchange rate regime, the potential benefit from monetary policy coordination, the optimal response to shock originating from abroad and the choice of consumer price indexes versus domestic inflation targeting.

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## Appendix A

In any flexible price equilibrium, only real shocks affect real variables.<sup>29</sup> In fact the equilibrium path of real variables is described by

$$\begin{aligned}\tilde{C}_t &= \frac{\eta}{\eta + \rho}(a_t^W - g_t^W), \\ \tilde{T}_t &= \frac{\eta}{1 + \eta}(g_t^R - a_t^R), \\ \tilde{Y}_t^H &= (1 - n)\tilde{T}_t + \tilde{C}_t^W + g_t^H, \\ \tilde{Y}_t^F &= -n\tilde{T}_t + \tilde{C}_t^W + g_t^F,\end{aligned}$$

where  $a$  and  $g$  are respectively supply (originated from productivity disturbances) and demand shocks. Only world shocks affect the natural rate of world consumption, while only relative shocks perturb the natural rate of the terms of trade. An increase in world productivity increases consumption while an increase in world demand has an opposite effect. The terms of trade are affected only by relative disturbances. In fact its crucial role is that of balancing the burden of exerting output across countries. Risk sharing in consumption implies that the marginal disutilities of labor supply are equated between the two countries. Whenever there are asymmetric disturbances that induce the households in a country to work more, changes in the terms of trade optimally shift part of the burden to the household in the other country. A larger demand shock in country  $H$  than in country  $F$  appreciates its terms of trade, while a larger supply shock leads to a depreciation.

Consider now the equilibrium condition

$$\tilde{T}_t = \tilde{T}_{t-1} + \Delta\tilde{S}_t + \tilde{\pi}_t^F - \tilde{\pi}_t^H.$$

There are infinitely many equilibria which differ because of the different decomposition of the changes in the terms of trade into exchange rate depreciation and inflation rate differential. The monetary policy regime is crucial in determining this split. Here we analyze a particular equilibrium in which the producer inflation rate is zero in each country. As explained in Benigno P. (1999), in the presence of

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<sup>29</sup>This is a consequence of the assumption of additive separability in the consumer's utility.

nominal rigidities, a non-zero producer inflation rate induces an inefficient dispersion of demand across goods produced within a country using the same technology. As we will emphasize in the section on the welfare, this particular flexible price equilibrium represents the efficient outcome that a central planner would induce in the case prices are sticky and the monopolistic distortions are small in size. Under the assumption of zero producer inflation rates in both countries, the nominal exchange rate follows directly the path of the natural terms of trade. In fact, equation (4) implies that

$$\Delta \tilde{S}_t = \tilde{T}_t - \tilde{T}_{t-1},$$

and, given the initial conditions  $\tilde{S}_{-1} = \tilde{T}_{-1} = 0$ , that

$$\tilde{S}_t = \tilde{T}_t.$$

An increase in the productivity of the Home country relative to that of the Foreign country reduces the disutility of labor in the Home economy. An appreciation of the Home terms of trade (an increase in  $T$ ) allows Home goods to be more competitive restoring the labor supply-consumption trade-off. All the adjustment is obtained by depreciation of the nominal exchange rate (i.e.  $S$  increases). Instead, an exogenous increase in the demand of Home goods relative to Foreign goods can be equilibrated by a depreciation in the Home terms of trade that shift part of the increased demand to the Foreign country.

By using the uncovered interest rate parity, it is possible to express the natural nominal (and real) interest rates differential as

$$\tilde{i}_t^H - \tilde{i}_t^F = \mathbb{E}_t\{\tilde{T}_{t+1} - \tilde{T}_t\} \equiv \tilde{R}_t^R, \quad (\text{A.1})$$

which is driven only by the expected changes in the natural terms of trade; the world natural nominal interest rate instead is purely driven by world demand and supply shocks, in fact

$$n\tilde{i}_t^H + (1-n)\tilde{i}_t^F = \rho\mathbb{E}_t\{\tilde{C}_{t+1} - \tilde{C}_t\} \equiv \tilde{R}_t^W. \quad (\text{A.2})$$

Finally, each country's natural nominal interest rate is obtained by combining (A.1) and (A.2) in

$$\begin{aligned}\tilde{i}_t^H &= \tilde{R}_t^W + (1-n)\tilde{R}_t^R, \\ \tilde{i}_t^F &= \tilde{R}_t^W - n\tilde{R}_t^R.\end{aligned}$$

## Appendix B

### Derivation of the equilibrium path under the floating regime.

Taylor rules.

In this appendix, we assume that both countries have the same degree of nominal rigidities. Under the Taylor-rules regime specified by the rules

$$\begin{aligned}\hat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi\pi_t^F + \psi y_t^F,\end{aligned}$$

there is complete separation between the determination of world variables and relative variables. It is the case that the equilibrium paths of the terms of trade, the exchange rate and the inflation rate differential can be obtained from the following equilibrium conditions

$$\pi_t^F - \pi_t^H = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t(\pi_{t+1}^F - \pi_{t+1}^H), \quad (\text{A.1})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta S_t, \quad (\text{A.2})$$

$$E_t \Delta S_{t+1} = \phi(\pi_t^H - \pi_t^F) + \psi(\hat{T}_t - \tilde{T}_t), \quad (\text{A.3})$$

where condition (A.1) is obtained by subtracting (6) from (7); equation (A.2) is equation (4) in the text while the third equation is (5) in which the interest rate rules have been substituted. Given the Markovian nature of the process  $\tilde{T}_t$ , a rational expectations equilibrium assumes the following form

$$\begin{aligned}\hat{T}_t &= b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t, \\ \pi_t^F - \pi_t^H &= b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t, \\ \Delta S_t &= b_3 \hat{T}_{t-1} + c_3 \tilde{T}_t.\end{aligned}$$

where the coefficients satisfy the following restrictions

$$\begin{aligned}
b_1 &= 1 + b_2 + b_3 \\
c_1 &= c_2 + c_3, \\
b_1 b_3 &= -\phi b_2 + \psi b_1, \\
c_1 b_3 + \rho_1 c_3 &= -\phi c_2 + \psi c_1 - \psi, \\
c_1(\beta b_2 - k_T) + \beta c_2 \rho_1 &= c_2 - k_T, \\
b_1(\beta b_2 - k_T) &= b_2.
\end{aligned}$$

In a unique and stable rational expectations equilibrium (stability requires that  $|b_1| < 1$ ), it is always the case that  $b_1 = b_2 = 0$  while  $b_3 = -1$ .

The coefficients on the natural terms of trade are given by

$$\begin{aligned}
c_1 &= \frac{(\phi - \rho_1)k_T + \psi(1 - \beta\rho_1)}{(\phi - \rho_1)k_T + (1 - \rho_1)(1 - \beta\rho_1) + \psi(1 - \beta\rho_1)}, \\
c_2 &= \frac{(1 - \rho_1)k_T}{(\phi - \rho_1)k_T + (1 - \rho_1)(1 - \beta\rho_1) + \psi(1 - \beta\rho_1)}, \\
c_3 &= \frac{(\phi - 1)k_T + \psi(1 - \beta\rho_1)}{(\phi - \rho_1)k_T + (1 - \rho_1)(1 - \beta\rho_1) + \psi(1 - \beta\rho_1)}.
\end{aligned}$$

### Taylor rules with interest-rate smoothing.

Under the class of interest-rate smoothing rules

$$\begin{aligned}
\widehat{i}_t^H &= \gamma \widehat{i}_{t-1}^H + \phi \pi_t^H + \psi y_t^H, \\
\widehat{i}_t^F &= \gamma \widehat{i}_{t-1}^F + \phi \pi_t^F + \psi y_t^F,
\end{aligned}$$

the relevant equilibrium conditions for the determination of the terms of trade and the exchange rate are

$$\pi_t^F - \pi_t^H = -k_T(\widehat{T}_t - \widetilde{T}_t) + \beta E_t(\pi_{t+1}^F - \pi_{t+1}^H), \quad (\text{A.4})$$

$$\widehat{T}_t = \widehat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta S_t, \quad (\text{A.5})$$

$$E_t \Delta S_{t+1} = \phi(\pi_t^H - \pi_t^F) + \gamma(\widehat{i}_{t-1}^H - \widehat{i}_{t-1}^F) + \psi(\widehat{T}_t - \widetilde{T}_t), \quad (\text{A.6})$$

$$\widehat{i}_t^H - \widehat{i}_t^F = \gamma(\widehat{i}_{t-1}^H - \widehat{i}_{t-1}^F) + \phi(\pi_t^H - \pi_t^F) + \psi(\widehat{T}_t - \widetilde{T}_t), \quad (\text{A.7})$$



where condition (A.4) is obtained by subtracting (6) from (7); equation (A.5) is equation (4) in the text while (A.6) is (5) in which the interest rate rules have been substituted. Equation (A.7) is the difference between the two interest rate rules. The above set of equations can be compacted in a system of the form

$$E_t \begin{bmatrix} y_{t+1}^s \\ z_t^s \end{bmatrix} = \begin{bmatrix} M_1^s & M_2^s \\ M_3^s & M_4^s \end{bmatrix} \begin{bmatrix} y_t^s \\ z_{t-1}^s \end{bmatrix} + \begin{bmatrix} m_1^s \\ m_2^s \end{bmatrix} \tilde{T}_t,$$

where  $y_t^{s'} = [\pi_t^R \ \Delta S_t]$ ,  $z_{t-1}^{s'} = [\hat{i}_{t-1}^R \ \hat{T}_{t-1}]$ ,  $M_j^s$  are  $2 \times 2$  matrices, while  $m_j^s$  are  $2 \times 1$  vectors. Under the conditions for determinacy, there are two eigenvalues outside the unit circle. Let denote these eigenvalues as  $\omega_1$  and  $\omega_2$  collected in the diagonal matrix  $\Omega$ . Let  $V$  a  $2 \times 4$  matrix of the left eigenvectors associated with the unstable eigenvalues, with the property that  $VM^s = \Omega V$ . Furthermore we decompose  $V$  in two  $2 \times 2$  matrices with  $V = [V_1 \ V_2]$ . Now, if  $\tilde{T}_t$  follows a Markovian process of the form (13), we have that the solution for  $y_t$  is of the form

$$y_t^s = \Psi_1^s z_{t-1}^s + \Psi_2^s \tilde{T}_t,$$

where  $\Psi_1^s \equiv -V_1^{-1}V_2$  and  $\Psi_2^s \equiv V_1^{-1}(I\rho - \Omega)^{-1}Vm$ . We then obtain

$$\begin{aligned} z_t^s &= M_3^s y_t^s + M_4^s z_{t-1}^s + m_2^s \tilde{T}_t, \\ &= (M_3^s \Psi_1^s + M_4^s) z_{t-1}^s + (M_3^s \Psi_2^s + m_2^s) \tilde{T}_t, \\ &= Z_1^s z_{t-1}^s + Z_2^s \tilde{T}_t, \end{aligned}$$

where  $Z_1^s$  and  $Z_2^s$  have been appropriately defined. We have seen that in a pure floating regime the terms of trade do not introduce any intrinsic inertia. The inclusion of a smoothing argument into the Taylor rules does not alter this property. In fact the second column of  $Z_1^s$  is of zeros. It follows that the only source of inertia in the system is coming only from the interest rate smoothing component. Reminding that  $z_t^{s'} = [\hat{i}_t^R \ \hat{T}_t]$ , it can be possible to obtain a solution for  $\hat{T}_t$  and  $\hat{i}_t^R$  of the form

$$\begin{aligned} A^s(L)\hat{T}_t &= R^s(L)\tilde{T}_t, \\ A^s(L)\hat{i}_t^R &= Q^s(L)\tilde{T}_t, \end{aligned}$$

where  $A^s(L)$ ,  $Q^s(L)$ ,  $R^s(L)$  are first-order polynomials. While for  $\Delta S_t$  we obtain

$$A^s(L)\Delta S_t = U^s(L)\tilde{T}_t,$$

where  $U^s(L)$  is a second-order polynomials.

### Managed Exchange Rate (I)

Under the class of managed exchange rate (I)

$$\begin{aligned}\widehat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \widehat{i}_t^F &= \phi\pi_t^F + \psi y_t^F - \lambda\widehat{S}_t,\end{aligned}$$

the relevant equilibrium conditions for the determination of the terms of trade and the exchange rate are

$$\pi_t^F - \pi_t^H = -k_T(\widehat{T}_t - \tilde{T}_t) + \beta E_t(\pi_{t+1}^F - \pi_{t+1}^H), \quad (\text{A.8})$$

$$\widehat{T}_t = \widehat{T}_{t-1} + \pi_t^F - \pi_t^H + \widehat{S}_t - \widehat{S}_{t-1}, \quad (\text{A.9})$$

$$E_t\widehat{S}_{t+1} = \phi(\pi_t^H - \pi_t^F) + \psi(\widehat{T}_t - \tilde{T}_t) + (1 + \lambda)\widehat{S}_t, \quad (\text{A.10})$$

where condition (A.8) is obtained by subtracting (6) from (7); equation (A.9) is equation (4) in the text while the third equation is (5) in which the interest rate rules have been substituted. This set of equations can be compacted in a system of the form

$$E_t \begin{bmatrix} y_{t+1} \\ z_t \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \begin{bmatrix} y_t \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \tilde{T}_t,$$

where  $y'_t = [\pi_t^R \ \widehat{S}_t]$ ,  $z'_{t-1} = [\widehat{T}_{t-1} \ \widehat{S}_{t-1}]$ ,  $M_j$  are  $2 \times 2$  matrices, while  $m_j$  is a  $2 \times 1$  vector. Under the conditions for determinacy, there are two eigenvalues outside the unit circle. Let denote these eigenvalues as  $\omega_1$  and  $\omega_2$  collected in the diagonal matrix  $\Omega$ . Let  $V$  a  $2 \times 4$  matrix of the left eigenvectors associated with the unstable eigenvalues, with the property that  $VM = \Omega V$ . Furthermore we decompose  $V$  in two  $2 \times 2$  matrices with  $V = [V_1 \ V_2]$ . Now, if  $\tilde{T}_t$  follows a Markovian process of the form (13), we have that the solution for  $y_t$  is of the form

$$y_t = \Psi_1 z_{t-1} + \Psi_2 \tilde{T}_t,$$

where  $\Psi_1 \equiv -V_1^{-1}V_2$  and  $\Psi_2 \equiv V_1^{-1}(I\rho - \Omega)^{-1}Vm$ . Furthermore from the system (A.14), we have

$$\begin{aligned} z_t &= M_3y_t + M_4z_{t-1} + m_2\tilde{T}_t, \\ &= (M_3\Psi_1 + M_4)z_{t-1} + (M_3\Psi_2 + m_2)\tilde{T}_t, \\ &= Z_1z_{t-1} + Z_2\tilde{T}_t, \end{aligned}$$

where  $Z_1$  and  $Z_2$  have been appropriately defined. Reminding that  $z' = [\hat{T}_t \ \hat{S}_t]$ , it can be possible to obtain a solution for  $\hat{T}_t$  and  $\hat{S}_t$  of the form

$$\begin{aligned} A(L)\hat{T}_t &= R(L)\tilde{T}_t, \\ A(L)\hat{S}_t &= U(L)\tilde{T}_t, \end{aligned}$$

where  $A(L)$  is a second-order polynomial with  $A(L) = \det[I - LZ_1]$  and  $R(L)$  and  $U(L)$  are first-order polynomials.

### Managed exchange rate (II)

Under the class of managed exchange rate (I)

$$\begin{aligned} \hat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi\pi_t^F + \psi y_t^F - \mu\Delta S_t, \end{aligned}$$

the relevant equilibrium conditions for the determination of the terms of trade and the exchange rate are

$$\pi_t^F - \pi_t^H = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t(\pi_{t+1}^F - \pi_{t+1}^H), \quad (\text{A.11})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta S_t, \quad (\text{A.12})$$

$$E_t\Delta S_{t+1} = \phi(\pi_t^H - \pi_t^F) + \psi(\hat{T}_t - \tilde{T}_t) + \mu\Delta S_t, \quad (\text{A.13})$$

where condition (A.11) is obtained by subtracting (6) from (7); equation (A.12) is equation (4) in the text while the third equation is (5) in which the interest rate rules have been substituted. The above set of equations can be compacted in a system of the form

$$E_t \begin{bmatrix} y_{t+1}^m \\ \hat{T}_t \end{bmatrix} = \begin{bmatrix} M_1^m & M_2^m \\ 1' & 1 \end{bmatrix} \begin{bmatrix} y_t^m \\ \hat{T}_{t-1} \end{bmatrix} + \begin{bmatrix} m_1^m \\ 0 \end{bmatrix} \tilde{T}_t, \quad (\text{A.14})$$

where  $y'_t = [\pi_t^R \ \Delta S_t]$ ,  $M_1^m$  is a  $2 \times 2$  matrix,  $1'$  and  $M_2^m$  are  $2 \times 1$  vectors, while  $m_1^m$  is a  $2 \times 1$  vector. Under the conditions for determinacy, there are two eigenvalues outside the unit circle. Because there is one predetermined endogenous variable ( $\widehat{T}_{t-1}$ ), the system has a unique bounded solution if and only if exactly two eigenvalues of the  $3 \times 3$  matrix lie outside the unit circle. Let denote these eigenvalues as  $\omega_1$  and  $\omega_2$  collected in the diagonal matrix  $\Omega$ . Let  $V$  a  $2 \times 3$  matrix of the left eigenvectors associated with the unstable eigenvalues, with the property that  $VM^m = \Omega V$ . Following the same steps as in the previous case, we obtain

$$y_t^m = \Psi_1^m \widehat{T}_{t-1} + \Psi_2^m \widetilde{T}_t,$$

where  $\Psi_1^m \equiv -V_1^{-1}V_2$  and  $\Psi_2^m \equiv V_1^{-1}(I\rho - \Omega)^{-1}Vm$ . Furthermore from the system (A.14), we have

$$\begin{aligned} \widehat{T}_t &= 1' y_t + \widehat{T}_{t-1} \\ &= (1' \Psi_1 + 1) \widehat{T}_{t-1} + (1' \Psi_2) \widetilde{T}_t, \\ &= Z_1^m \widehat{T}_{t-1} + Z_2^m \widetilde{T}_t, \end{aligned}$$

where  $Z_1^m$  and  $Z_2^m$  have been appropriately defined. So that we obtain that the solution for  $\widehat{T}_t$  is of the form

$$A^m(L) \widehat{T}_t = R^m \widetilde{T}_t$$

where

$$A^m(L) = 1 - LZ_1^m$$

and  $R^m = Z_2^m$ . Note that  $|Z_1^m| < 1$  and  $Z_1^m$  would be zero if there is no weight on exchange rate depreciation. We obtain then that

$$A^m(L) \Delta S_t = U^m(L) \widetilde{T}_t$$

where  $U^m(L)$  is a second-order polynomial.

Table 1: Calibrated Parameters

$\alpha^H = 0.48,$	$\alpha^F = 0.72,$
$\beta = 0.99,$	$\eta = 0.47,$
$\rho = 0.16,$	$\sigma = 7.88,$
$\text{s.d.}(\widetilde{R}_t^W) = 3.58,$	$\text{s.d.}(\widetilde{T}_t) = 3.3,$
$\rho_1 = 0.4,$	$\rho_2 = 0.35,$
$k_C^H = 0.0738,$	$k_C^F = 0.0151,$
$k_T^H = 0.1705,$	$k_T^F = 0.0349,$
$\Lambda_y = 0.0512,$	$\Lambda_T = 0.0288,$
$\Lambda_{\pi^H} = 0.17,$	$\Lambda_{\pi^F} = 0.83,$
$\Lambda_i = 0.277,$	$n = 0.5.$

Table 2: Rules

	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>	S <sub>6</sub>
$\phi$	1.5	2	1	0.8	0.4	0.2	0.1	0.15	0.6
$\phi^*$	1.5	1	2	0.4	0.8	0.1	0.2	0.6	0.15
$\psi$	0.5	0.5	0.5	0.2	0.2	0.05	0.05	0.05	0.2
$\psi^*$	0.5	0.5	0.5	0.2	0.2	0.05	0.05	0.2	0.05
$\gamma$	0	0	0	0.6	0.6	0.9	0.9	0.9	0.6
$\gamma^*$	0	0	0	0.6	0.6	0.9	0.9	0.6	0.9
$\lambda$	0	0	0	0	0	0	0	0	0
$\lambda^*$	0	0	0	0	0	0	0	0	0
$\mu$	0	0	0	0	0	0	0	0	0
$\mu^*$	0	0	0	0	0	0	0	0	0

Table 3: Rules

	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	O <sub>1</sub>	O <sub>2</sub>
$\phi$	1.5	1.5	0.1	0.1	0.1	0.18	0.2156
$\phi^*$	1.5	1.5	0.2	0.2	0.2	5.9407	1.6693
$\psi$	0.5	0.5	0.05	0.05	0.05	0.05	0.0767
$\psi^*$	0.5	0.5	0.05	0.05	0.05	0	0.0603
$\gamma$	0	0	0.9	0.9	0.9	0.90	1.1792
$\gamma^*$	0	0	0.9	0.9	0.9	3.7386	1.2634
$\lambda$	0	0	0	0	0	0	0.0001
$\lambda^*$	0.5	0	0.05	0	0.05	0	0
$\mu$	0	0	0	0	0	0	0.0345
$\mu^*$	0	0.5	0	0.05	0.05	0.6446	0.0763

Table 4: Comparisons of Rules

	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>	S <sub>6</sub>
$v(y^W)$	1.73	1.66	1.82	4.37	4.55	12.19	12.05	6.794	7.38
$v(y^H)$	1.62	1.47	1.81	4.18	4.48	11.80	11.54	7.70	6.15
$v(y^F)$	1.92	1.92	1.92	4.66	4.73	12.74	12.73	6.36	8.87
$v(\pi^W)$	0.12	0.10	0.14	0.09	0.10	0.27	0.29	0.19	0.12
$v(\pi^H)$	0.33	0.28	0.40	0.26	0.29	0.70	0.75	0.62	0.35
$v(\pi^F)$	0.02	0.02	0.02	0.01	0.01	0.05	0.05	0.01	0.04
$v(\widehat{T} - \widetilde{T})$	0.16	0.17	0.16	0.20	0.20	0.32	0.32	0.33	0.54
$v(\Delta S)$	3.36	3.76	3.09	1.75	1.52	0.66	0.75	4.96	6.02
$v(E\Delta S)$	1.47	1.64	1.31	0.49	0.39	0.08	0.06	2.82	3.74
$v(\widehat{S} - f)$	0.12	0.13	0.11	0.08	0.07	0.03	0.04	0.13	0.14
$v(\Delta ES^\infty)$	0.030	0.025	0.038	0.004	0.004	0.004	0.028	0.07	0.06
$v(\Delta ES^\infty   \widetilde{T})$	0.009	0.008	0.010	0.002	0.002	0.001	0.002	0.000	0.000
$v(\widehat{i}^H)$	11.52	12.03	10.95	5.46	4.86	1.08	0.91	0.68	7.30
$v(\widehat{i}^F)$	8.86	8.51	9.25	4.39	4.65	1.05	1.06	5.92	0.80
W	0.1660	0.1548	0.1812	0.2864	0.3003	0.7985	0.7940	0.4843	0.4936
W*	2.9903	3.0006	2.9808	1.6525	1.6171	1.0942	1.0666	1.3997	1.6179

Table 5: Comparisons of Rules

	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	O <sub>1</sub>	O <sub>2</sub>	FB
$v(y^W)$	1.77	1.78	12.06	12.00	12.05	11.20	8.7818	8.8603
$v(y^H)$	1.60	1.61	11.63	11.52	11.62	11.12	8.2209	8.2944
$v(y^F)$	2.06	2.05	12.69	12.65	12.69	11.43	9.5117	9.5950
$v(\pi^W)$	0.12	0.11	0.29	0.28	0.29	0.26	0.3430	0.3359
$v(\pi^H)$	0.28	0.31	0.73	0.75	0.73	0.69	0.9036	0.8915
$v(\pi^F)$	0.04	0.02	0.07	0.05	0.07	0.05	0.0558	0.0528
$v(\widehat{T} - \widetilde{T})$	0.25	0.19	0.38	0.34	0.39	0.30	0.3378	0.3377
$v(\Delta S)$	1.67	2.12	0.30	0.58	0.26	0.91	1.6719	1.6738
$v(E\Delta S)$	0.65	0.68	0.01	0.05	0.04	0.13	0.0584	0.0595
$v(\widehat{S} - f)$	0.06	0.09	0.01	0.03	0.01	0.05	0.1008	0.1005
$v(\Delta ES^\infty)$	0	0.029	0	0.022	0	0.006	0	0.1075
$v(\Delta ES^\infty   \widetilde{T})$	0	0.004	0	0.002	0	0.001	0	0.0048
$v(\widehat{i}^H)$	10.93	11.27	0.95	0.91	0.94	0.98	1.1377	1.1360
$v(\widehat{i}^F)$	9.29	8.97	0.98	1.06	0.99	1.49	1.3168	1.3155
W	0.1790	0.1679	0.8121	0.7932	0.8119	0.7377	0.6573	0.6568
W*	2.9863	2.9729	1.0809	1.0661	1.0804	1.0811	0.9973	0.9964

Figure 1: Stochastic Trend in the Exchange Rate following different Interest-Rate-Smoothing Regimes

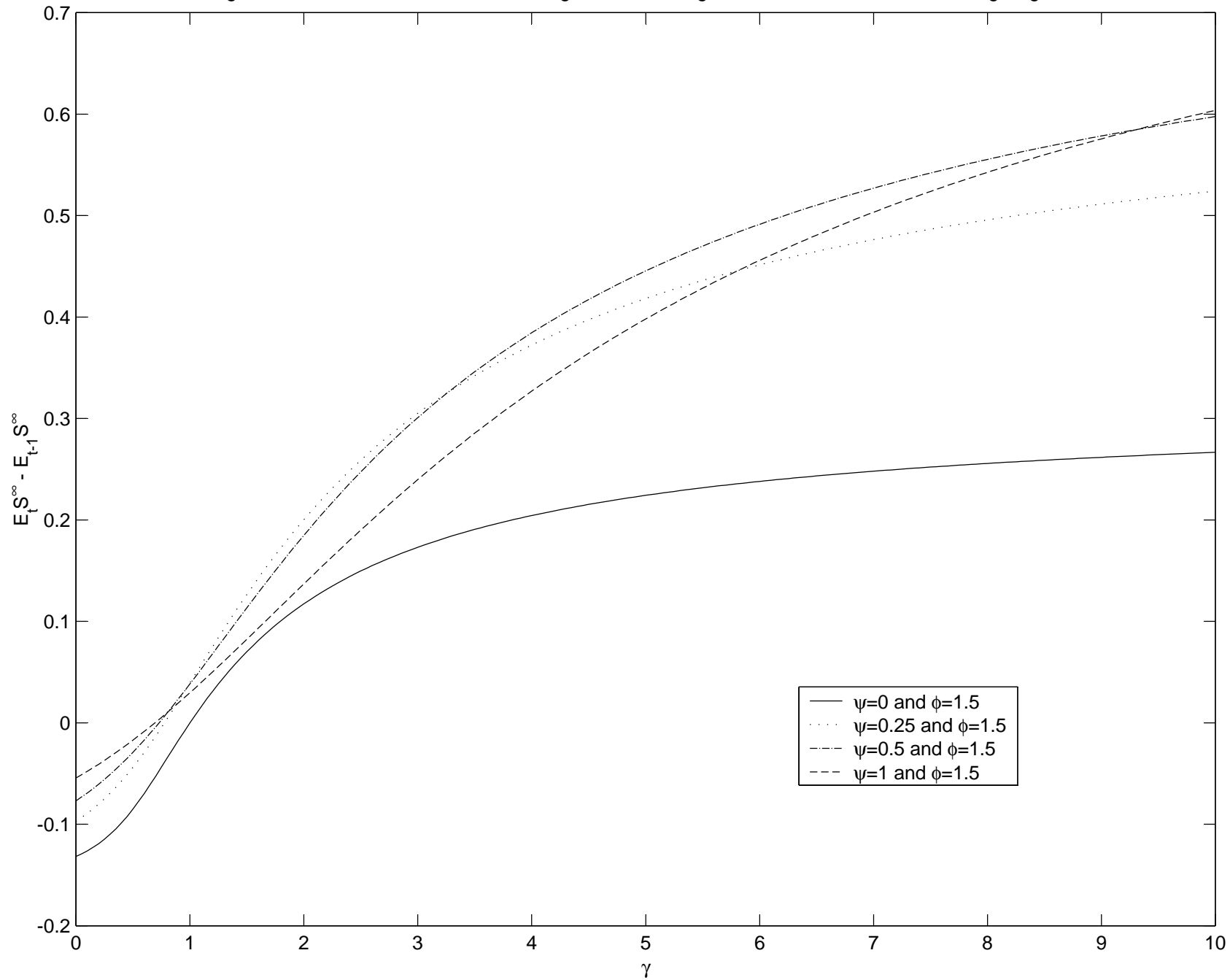


Figure 2: Stochastic Trend in the Exchange Rate following Different Managed-Exchange-Rate Regimes (II)(Target to the Difference)

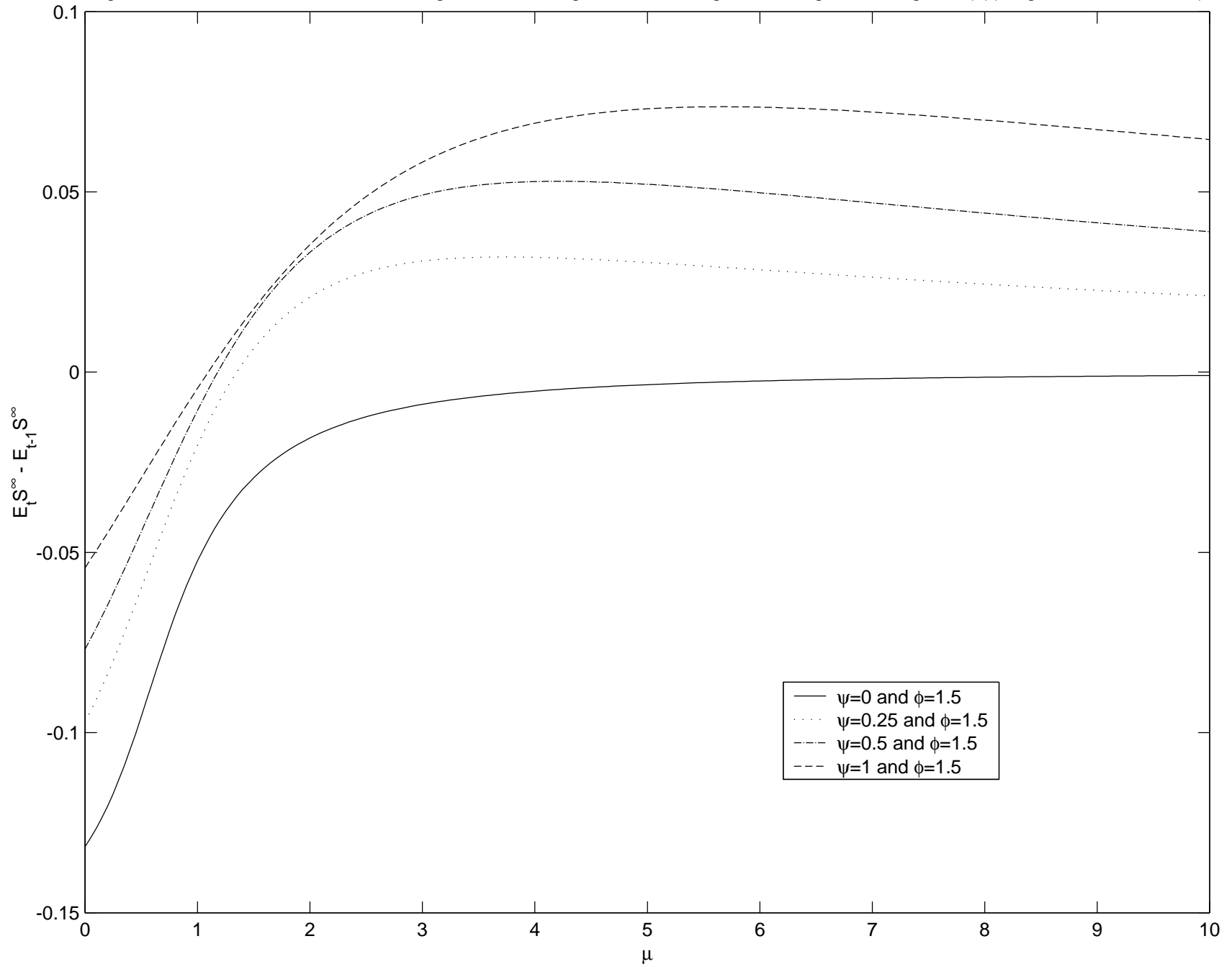




Figure 3: Impulse Response of the Terms of Trade to a Shock to the Natural Rate of the Terms of Trade,  $\rho_1=0$

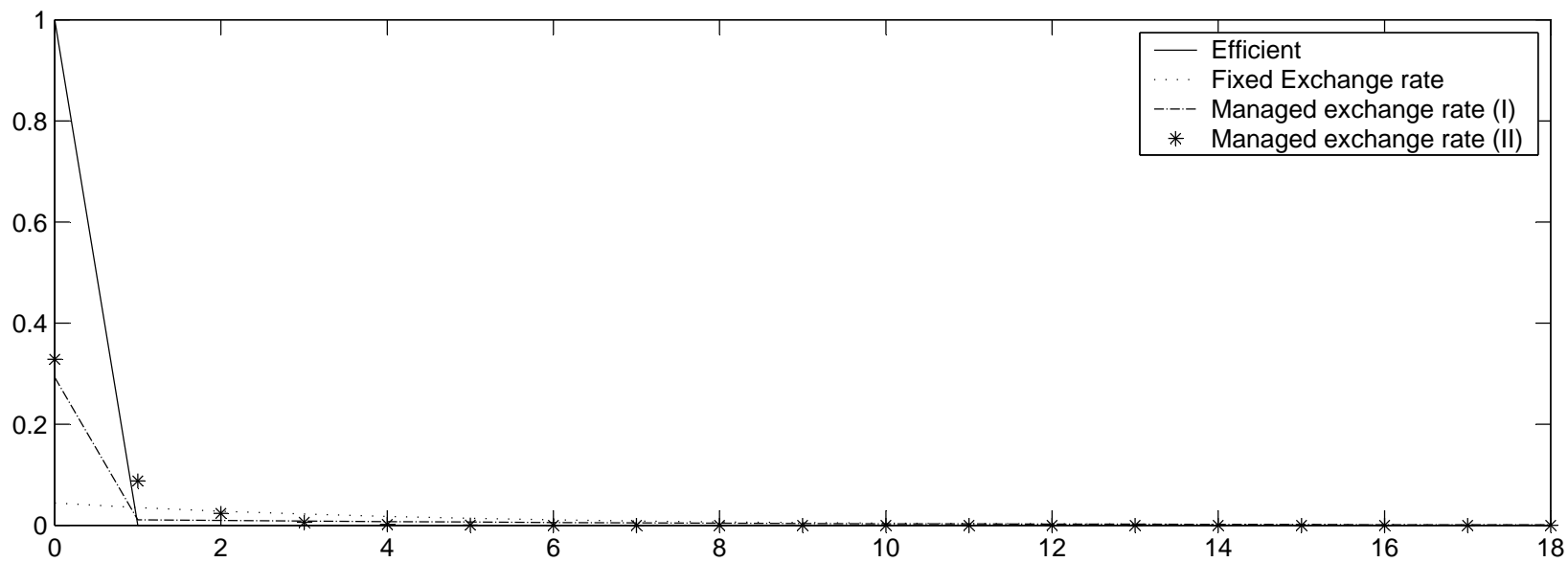
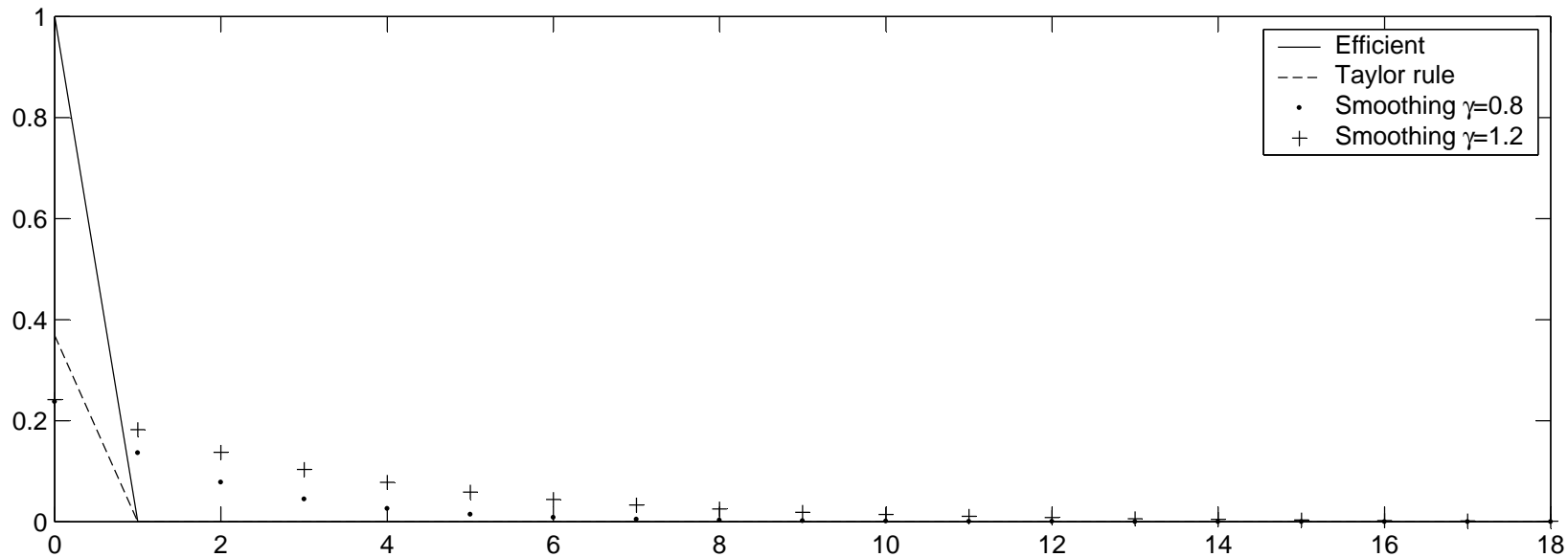


Figure 4: Impulse Response of the Exchange Rate to a Shock to the Natural Rate Terms of Trade,  $\rho_1=0$

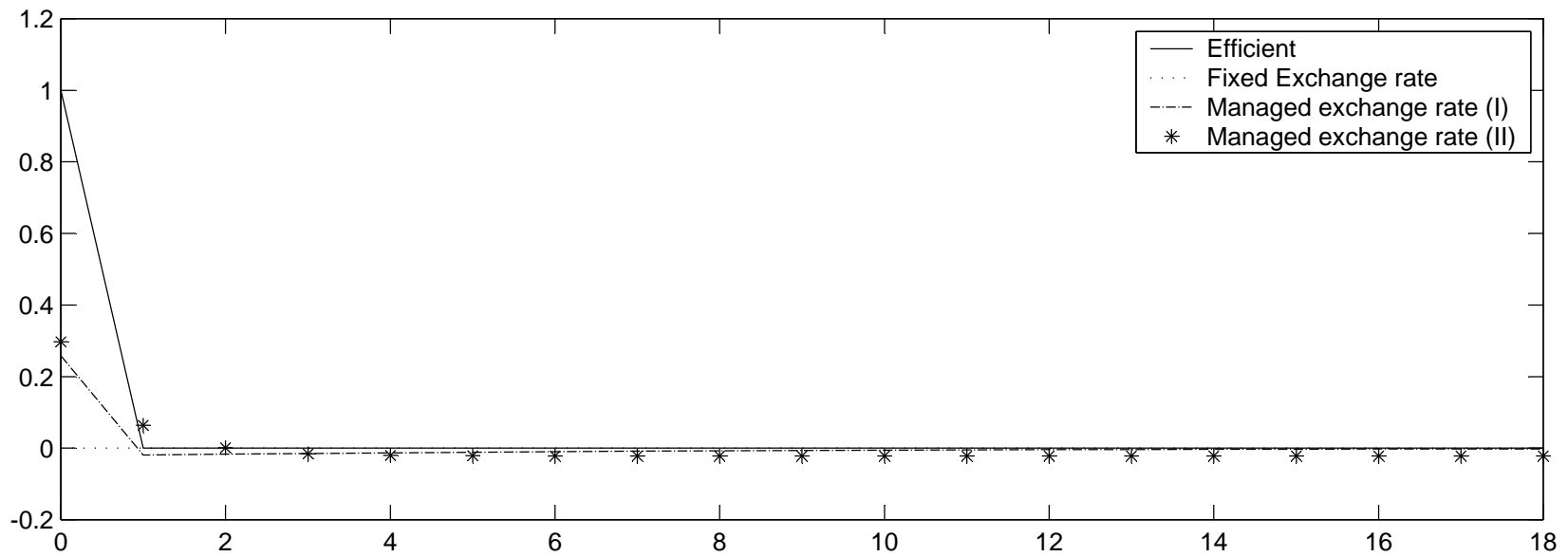
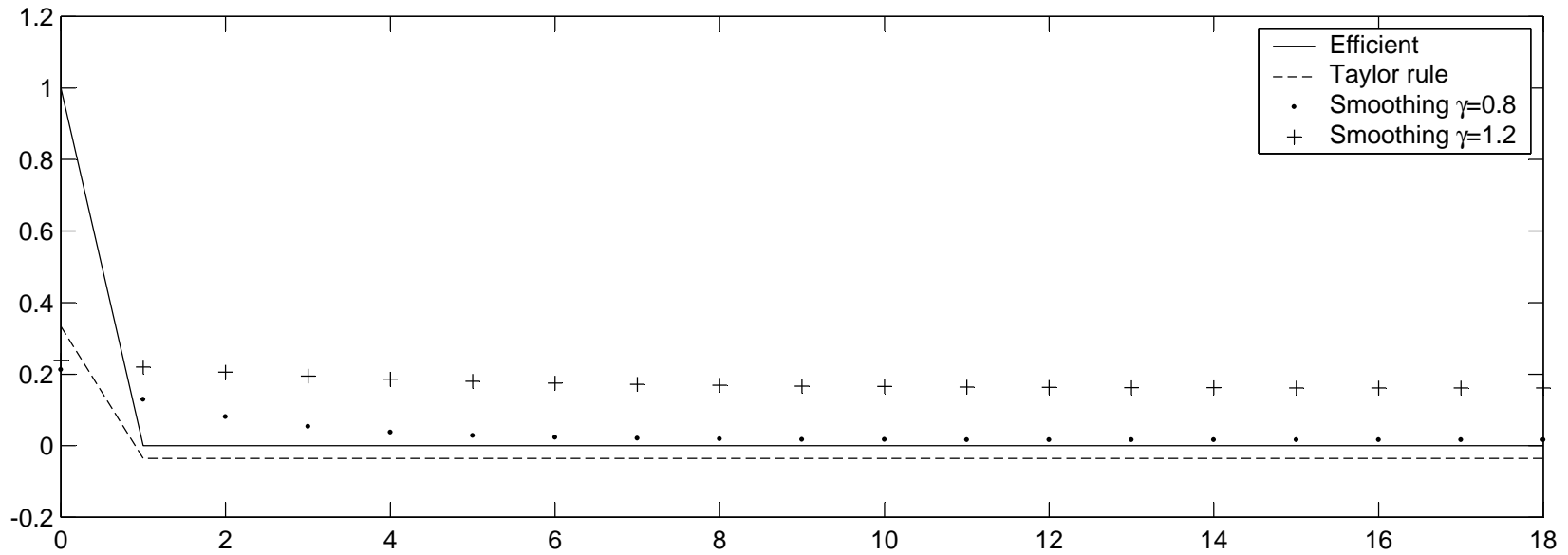


Figure 5: Impulse Response of the Terms of Trade to a Shock to the Natural Rate of the Terms of Trade,  $\rho_1=0.4$

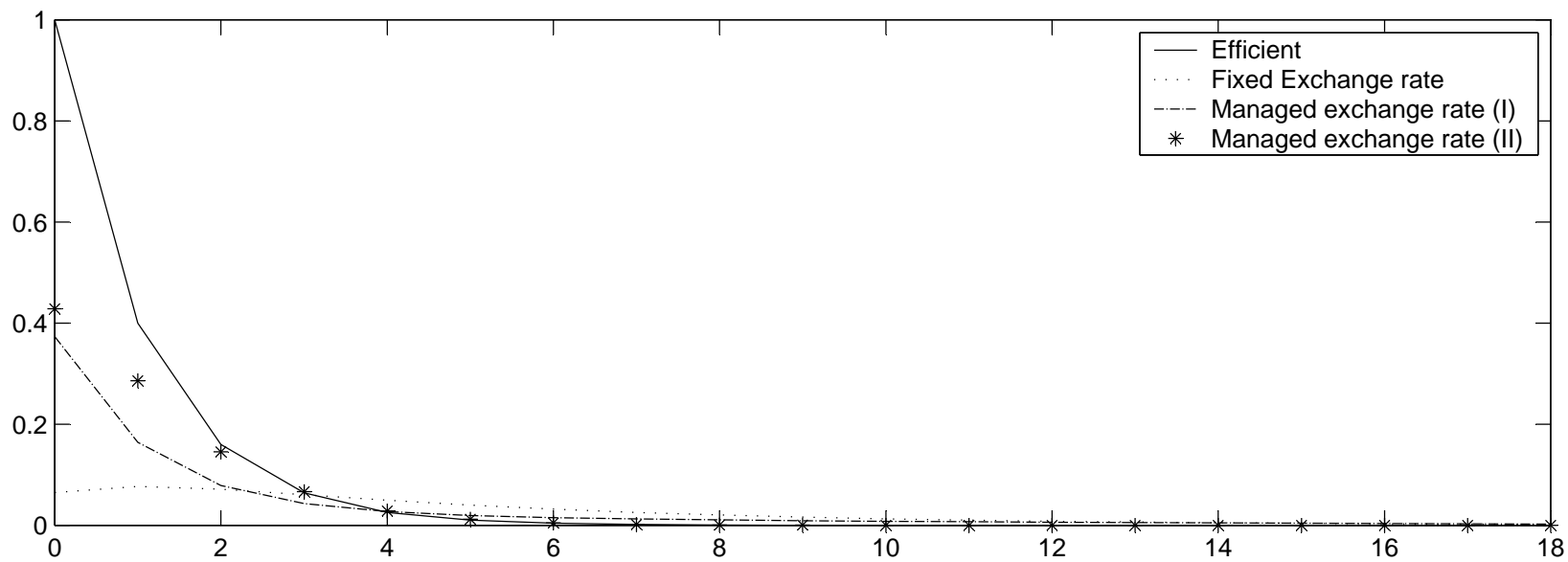
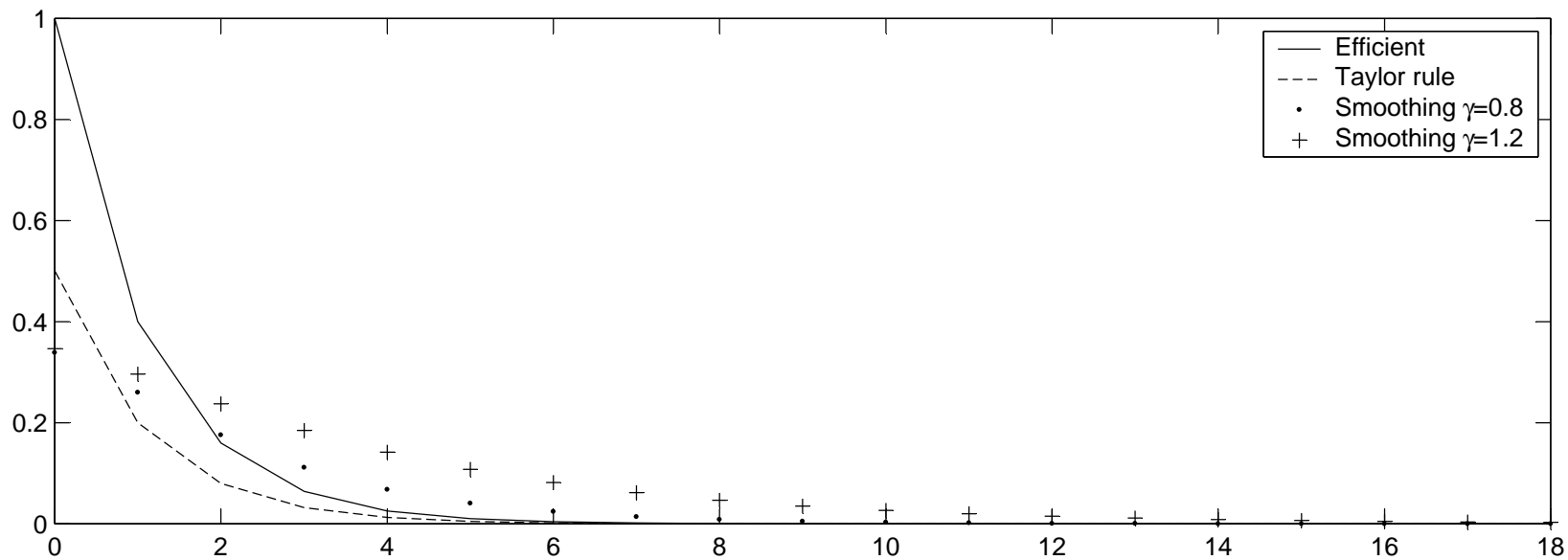


Figure 6: Impulse Response of the Exchange Rate to a Shock to the Natural Rate Terms of Trade,  $\rho_1=0.4$

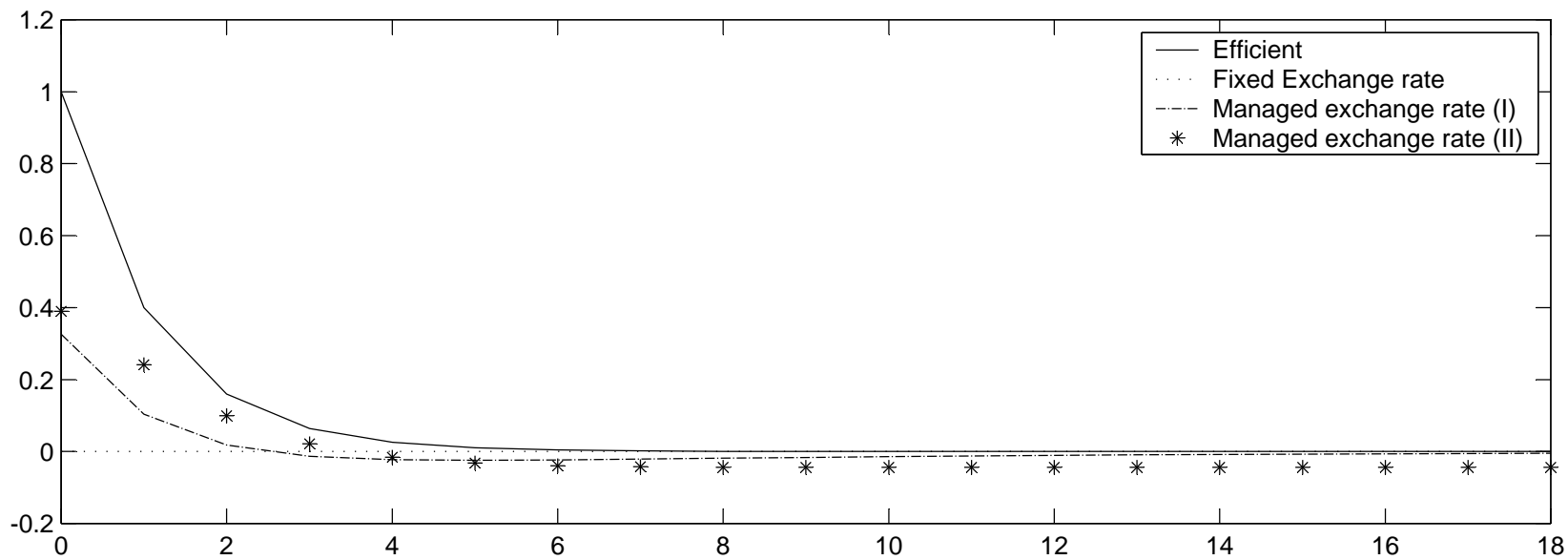
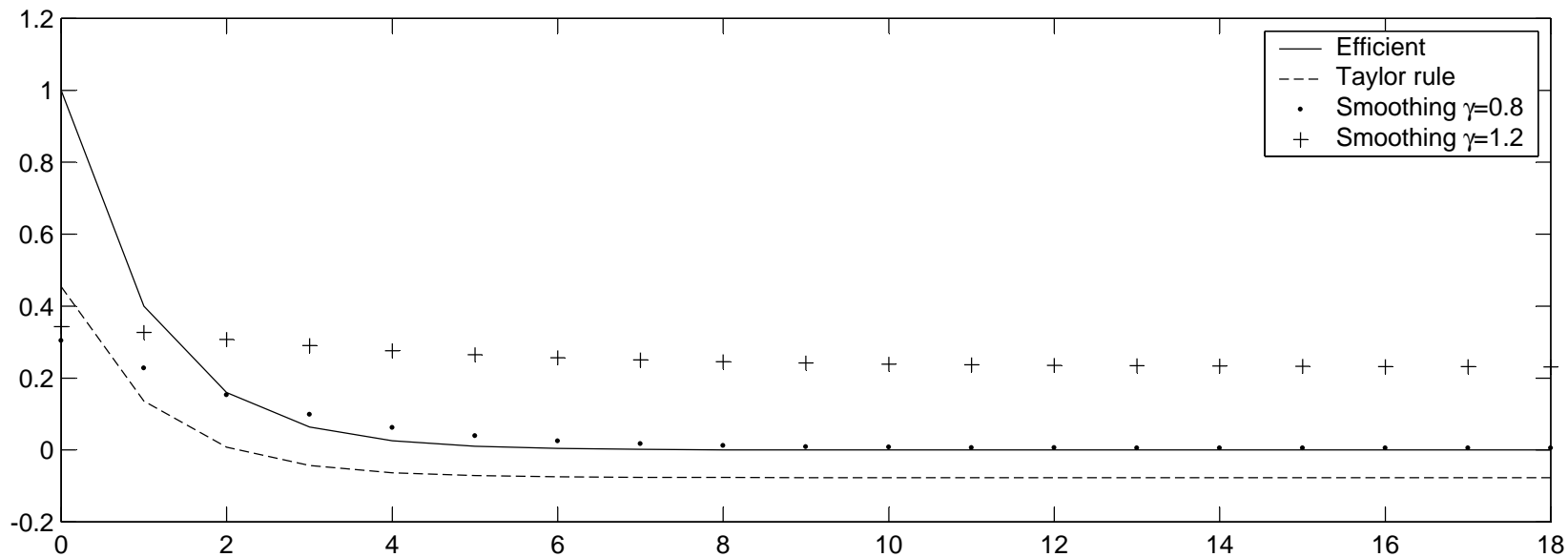


Figure 7: Efficient Frontier between World Inflation and World Nominal Interest Rate Variability

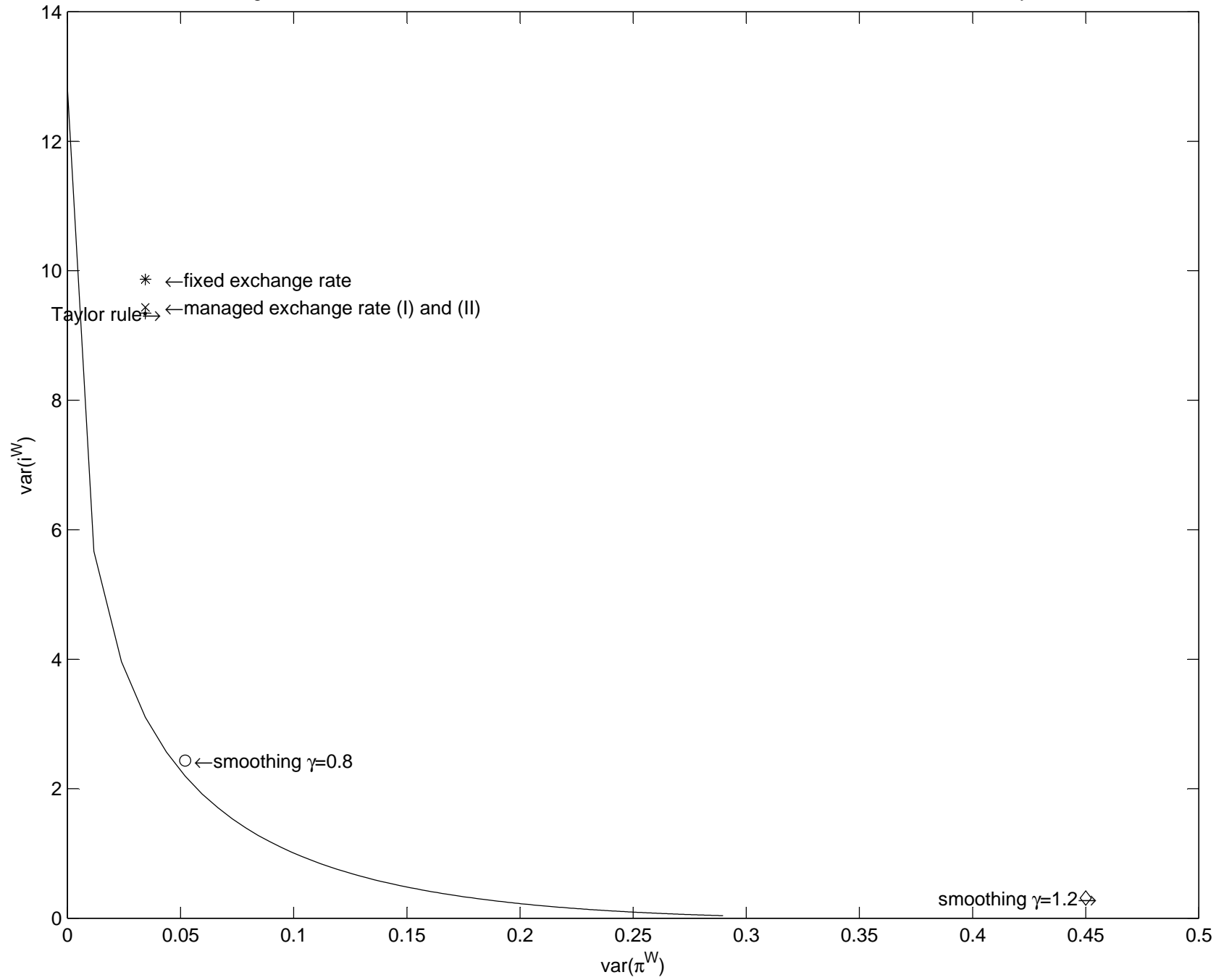


Figure 8: Efficient Frontier between Relative Inflation and Relative Nominal Interest Rate Variability

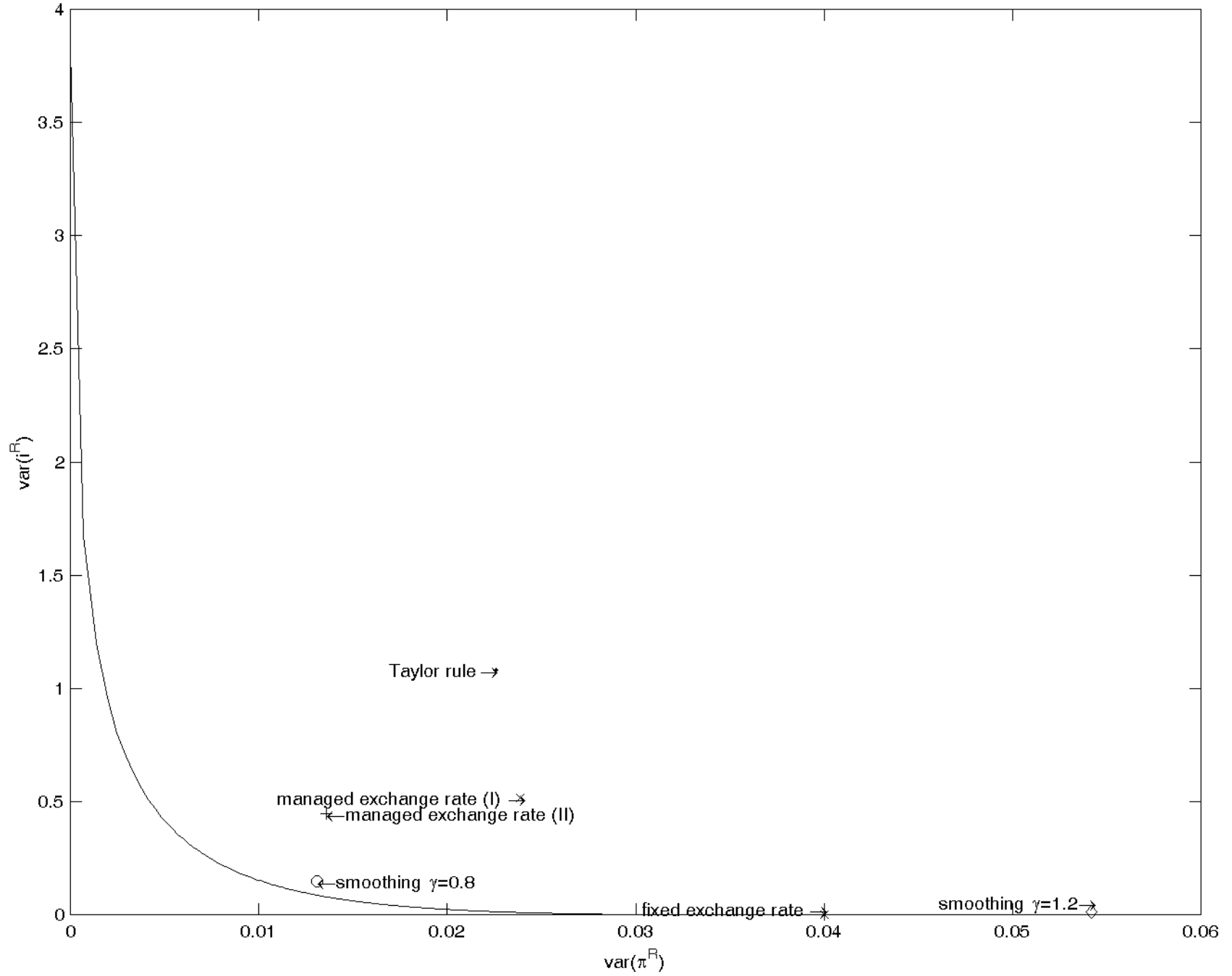


Figure 9: Efficient Frontier between Relative Inflation and Relative Nominal Interest Rate Variability

