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## ABSTRACT

# Job Matching, Social Network and Word-of-Mouth Communication*

In our model, workers are embedded within a network of social relationships and can communicate by word of mouth. They can find a job either through formal agencies or through informal networks of contacts (word-of-mouth communication). From this micro scenario, we derive an aggregate matching function that has the standard properties but fails to be homogenous of degree one. The latter is due to negative externalities generated by indirect acquaintances (contacts of contacts) that slow down word-of-mouth information transmission, especially in dense networks. We then show that there exists a unique labour market equilibrium and that, because of these negative externalities, the equilibrium unemployment rate increases with the network size in dense networks.

JEL Classification: D83, J64
Keywords: job search, microfoundation of the matching function, personal communication, social network

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# NON-TECHNICAL SUMMARY

It is well documented that resorting to word of mouth and newspaper advertisements are two major job search methods that are used by workers. Sociologists and labour economists have both produced a broad empirical literature on labour market networks. In fact, the pervasiveness of social networks and their relative effectiveness varies with the social group considered. For instance, Holzer (1988) shows that among 16–23 year old workers who reported job acceptance, 66% used informal search channels (30% direct application without referral and 36% friends/relatives), while only 11% use state agencies and 10% newspapers. More recently, Topa (2001) argues that the observed spatial distribution of unemployment in Chicago is consistent with a model of local interactions and information spillovers, and may thus be generated by agent's reliance on informal methods of job search such as networks of personal contacts.

In our framework, the bulk of information about jobs can be obtained through employed friends and employment agencies (or newspapers). In other words, there are two ways of learning about jobs: either employed workers hear about the job in the workplace and transmit this information to all their unemployed friends, or the directly unemployed read about job opportunities in the newspapers or in the local employment agency.

In this context, the aim of this Paper is twofold. First, we provide an explicit micro scenario in which finding a job depends both on formal and informal methods. According to this scenario, a social network links workers to each other, the members of this network can communicate through word of mouth and agents rely partly on friends to gather information about employment opportunities. Second, we establish a relationship between the network of personal contacts, the information transmission protocol and the aggregate job matching process. Our analysis of a labour market based on search and an explicit social network structure sheds light on the social dimension of job matching.

In our model, individuals are not isolated one way or another. Rather, they are embedded within a network of social relationships. We represent this social network with an undirected graph where nodes stand for the agents and a link between two nodes means that the corresponding agents can communicate directly. For most of the analysis, we focus on symmetric social networks where all agents have the same number of direct acquaintances. We refer to this number as the network size. Given a network of contacts, information about employment opportunities can be transmitted between any two direct neighbours through word-of-mouth communication. More precisely, when a job is available in the economy, workers can match with such a vacancy using either formal or informal methods. When an unemployed worker hears directly from a vacancy, we assume that they take the job, and this is considered as a formal method (since the social network plays no role). If on the contrary the worker hearing directly from a vacancy is currently employed, we assume that they transmit this information to their direct unemployed neighbours. Unemployed workers getting a job with the help of their local social network – as described above – rely on informal methods of job search.

We first show that the relationship between network structure (namely size) and job finding is not as straightforward as is commonly viewed. Indeed, in the standard social network literature (especially in sociology), more contacts are thought to be an advantage since they are more network members who can potentially broker job vacancies and job seekers. We show that this result depends crucially on the size of the network. Indeed, in our model direct neighbours are beneficial whereas indirect neighbours are detrimental. More direct contacts provide job seekers with a higher probability of receiving information about job openings and the unemployed prefer a large set of direct acquaintances to broaden their potential employment channels. But the better a worker is connected, the higher the number of unemployed direct neighbours that can potentially benefit from the information the worker holds about available jobs. As a result, the unemployed prefer a small set of indirect acquaintances to release the constraints of information sharing with a potentially bigger set of information recipients. In other words, indirect neighbours generate a negative externality over their direct set of acquaintances. We show that increasing the network size has a positive impact on the individual's probability of finding a job through friends in sparse social networks. On the contrary, increasing the network size in dense networks slows down word-of-mouth information transmission

We then obtain a well defined aggregate matching function that gives the number of job matches per unit of time. This endogenous matching function is derived from an explicit micro scenario where the structure of personal contacts and the job information transmission process is spelled out in detail. Our framework can be seen as an extension of the standard urn–ball model, where firms play the role of urns, workers play the role of balls, and balls (workers) are randomly placed in urns (firms). This random placing is because of a coordination failure; not all pairs are matched exactly. Rather, this uncoordinated process yields to overcrowding in some jobs and no applications to others. Such coordination failures are the sources of frictions captured by the matching function. In our context, the network of personal contacts allows for a (partial) replacement of redundant jobs thus reducing coordination failures and alleviating matching frictions, whose intensity is now related explicitly to network size. The link between the matching function and the network size is precisely the key element of our model.

This matching function is increasing and strictly concave in both the unemployment and the vacancy rates but fails to exhibit constant returns to scale. The latter means that, if social networks and word-of-mouth communications are integrated in the job search process, then the matching function is more likely **not** to be homogeneous of degree one. There is a huge body of empirical work to assess whether this property of the matching function is encountered in real life labour markets. Even if the results lean towards constant returns to scale, they are very much controversial and most of these empirical studies do not include informal methods in finding a job. By taking into account these methods, it would be interesting to see if the results would be altered in such a way that the matching function would fail to exhibit constant returns to scale.

With this matching function, we can fully characterize the labour market equilibrium whose existence and uniqueness is established. We show that the resulting equilibrium unemployment rate decreases with the network size in sparse networks while it increases when the pattern of links is dense. In other words, social networks increase frictions in the labour market for dense networks whereas they reduce them for sparse networks.

# 1 Introduction

Individuals seeking for jobs read newspapers, go to employment agencies, browse in the web and mobilize their local networks of friends and relatives. Although underestimated by the bulk of the search and matching literature, personal contacts often play a prominent role in matching job-seekers with vacancies. Empirical evidence suggests indeed that about half of all jobs are filled through contacts.[1] Networks of personal contacts mediate employment opportunities which flow through word-of-mouth and, in many cases, constitute a valid alternative source of employment information to more formal methods.

The aim of this paper is twofold. First, we provide an explicit micro scenario in which finding a job depends both on formal and informal methods. According to this scenario, workers are linked to each other by a social network, the members of this network can communicate through word-of-mouth and agents partly rely on friends to gather information about employment opportunities. Second, we establish a relationship between the network of personal contacts, the information transmission protocol and the aggregate job matching process. Our analysis of a labor market based on search and an explicit social network structure sheds light on the social dimension of job matching.

In our model, individuals are not isolated one with respect to the other. Rather, they are embedded within a network of social relationships. We represent this social network by an undirected graph where nodes stand for the agents and a link between two nodes means that the corresponding agents can communicate directly. For most of the analysis, we focus on symmetric social networks where all agents have the same number of direct acquaintances. We refer to this number as the network size. Given a network of contacts, information about employment opportunities can be transmitted

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between any two direct neighbors through word-of-mouth communication. More precisely, when a job is available in the economy, workers can match with such a vacancy using either formal or informal methods. When an unemployed worker hears directly from a vacancy, we assume that s/he takes the job, and this is considered as a formal method (since the social network plays no role). If on the contrary the worker hearing directly from a vacancy is currently employed, we assume that s/he transmits this information to her/his direct unemployed neighbors. Unemployed workers getting a job with the help of their local social network −as described above− rely on informal methods of job search.

We first show that the relationship between network structure (namely size) and job-finding is not as straightforward as it is commonly viewed. Indeed, in the standard social network literature (especially in sociology), more contacts are thought to be an advantage since they are more network members who can potentially broker job vacancies and job seekers. We show that this result crucially depends on the size of the network. Indeed, in a symmetric social network, each individual worker can receive information from her/his *direct* neighbors. However, each of her/his neighbors also has a direct set of acquaintances −*indirect* neighbors from the viewpoint of the first worker− that may benefit from this information. In our model direct neighbors are beneficial whereas indirect neighbors are detrimental. More direct contacts provide job seekers with a higher probability of receiving information about job openings and the unemployed prefer a large set of direct acquaintances to broaden their potential employment channels. But the better a worker is connected, the higher the number of unemployed direct neighbors that can potentially benefit from the information s/he holds about available jobs. As a result, the unemployed prefer a small set of indirect acquaintances to release the constraints of information sharing with a potentially bigger set of information recipients. In other words, *indirect* neighbors generate a negative externality over their direct set of acquaintances. We show that rising the network size has a positive impact on the individual probability to find a job through friends in sparse social networks. On the contrary, increasing the network size in dense networks slows down word-of-mouth information transmission

We then obtain a well-defined aggregate matching function which gives the number of job matches per unit of time. This endogenous matching function is derived from an explicit micro scenario where the structure of personal contacts and the job information transmission process is spelled out in detail. The corresponding reduced form is expressed in terms of the unemployed worker and vacant firm pools, and the social network underlying players talks. Contrarily to previous contributions also providing for micro foundations for matching functions, the expression obtained here is neither an exponential nor a min one. This matching function is increasing and strictly concave in both the unemployment and the vacancy rates. Moreover, the (extension of the standard) matching function we provide clearly relates job matching to individual social embeddedness and captures complex spillovers within social networks of interrelated personal contacts. In particular, we find a non-monotonic relationship between network size and the rate at which matches occur. With this matching function in hand, we can fully characterize the labor market equilibrium whose existence and uniqueness is established. We show that the resulting equilibrium unemployment rate decreases with the network size in sparse networks while it increases when the pattern of links is dense.

There have been several attempts to find a micro foundation of the standard macroeconomic matching function. The most popular reduced form is the exponential matching function that was first employed by Butters (1977) to model contacts between buyers and sellers in commodity markets.[2] More recently, Lagos (2000) has proposed an alternative micro approach by deriving an aggregate matching function which takes the form of a min function. Our micro foundation of the matching function based on word-of-mouth communication gives insights on the relationship between job search, job matching and social network. In fact, there have been few theoretical attempts to model this link. Notable exceptions include Diamond (1981), Montgomery (1991, 1992), Mortensen and Vishwanath (1994) and Kugler (2000) that contribute to the theoretical literature on equilibrium wage determination in search markets. However, in all these approaches, the modelling of the social network is quite shallow. To our knowledge, the first paper to explic-

[2] This matching function owes its origin to the well-known and extensively analysed urn-ball model in probability theory. According to this model, the labor market is visualized as 'urns' (vacancies) to be filled by 'balls' (workers). Because of a coordination failure inherent to any random placing of the balls in the urns, matching is not perfect and one can interpret the resulting mismatches in terms of labor market frictions. In most cases, the system steady state can be approximated by an exponential-type matching function as the population becomes large. See for instance Hall (1979), Pissarides (1979), Peters (1991), Blanchard and Diamond (1994), Burdett, Shi and Wright (2001), Smith and Zenou (2001).

itly model the structure of social contacts by an undirected network in a labor market context is Boorman (1975).[3] Following this early contribution, Calvó-Armengol (2001) develops a model specifying at the individual level both the decision to establish or to maintain social ties with other agents, and the process by which information about jobs is obtained and transmitted. The analysis focuses on the impact that an endogenous determination of job contact networks has on the effectiveness of information transmission and on the aggregate unemployment level. On the contrary, the present paper builds an aggregate matching function stemming from an explicit network structure, and determines the impact a partial reliance on social networks as a method of job search has on labor market outcomes.

The remaining of the paper is as follows. The next section describes the social network, the labor market and the information transmission protocol within this network. Section 3 derives the aggregate matching function and examines its main properties. The characterization, the existence and the uniqueness of the labor market equilibrium is established in section 4. Section 5 concludes and all the proofs are presented in Appendix.

# 2 Social Network and Word-of-Mouth Communication

Social networks are links and associations between people of a common ilk. These can be friends, acquaintances and colleagues. Networks are evident between family members, but are also established between friends and neighborhood residents. In this section, we model the social network between people by means of graph theory.

## 2.1 The social network

We consider a finite population of workers $N = \{1, ..., n\}$. In our model, individuals are not isolated one with respect to the other. Rather, they are embedded within a network of social relationships. More precisely, each

[3] A recent and growing literature stresses the role of networks in explaining a wide range of economic phenomema among which labor markets are just an example. See for instance Jackson and Wolinsky (1996), Bala and Goyal (2000) and the references therein. For a previous model of word-of-mouth communication see for instance Ellison and Fudenberg (1995).

worker $i$ is in *direct* contact with a group of workers (her/his set of friends or relatives) and we assume that each pair of directly connected workers can communicate with each other through word-of-mouth. A direct link between two individuals $i$ and $j$ is denoted by $ij$. The collection of all existing links constitutes the prevailing social network of personal relationships denoted by $g$. Such a social network is modelled as an undirected graph in which binary relationships are symmetric that is, whenever $i$ is connected to $j$ according to $g$ ($ij \in g$), then $j$ is also connected to $i$ according to $g$ ($ji \in g$).

Given a social network $g$, we denote by $N_i(g)$ the set of all direct neighbors of worker $i$. Formally, $N_i(g) = \{j \in N \backslash \{i\} : ij \in g\}$. We also denote by $n_i(g)$ the cardinal of the set $N_i(g)$ that is, the number of direct neighbors of $i$ with whom s/he can directly communicate. For example, Figure 1a corresponds to a star-shaped graph in which worker 1 can communicate with every other individual in the economy whereas workers 2 to $n = 6$ can directly communicate only with worker 1. Figure 1b illustrates the case of the complete graph where every worker can directly communicate with everybody.

An interesting case to be considered is when all workers have the same number of direct neighbors that is, $n_i(g) = s$ for all $i \in N$. Such a graph is called a symmetric graph and $s$ is the size of the corresponding social network. The complete graph described in Figure 1b is a particular case of a symmetric network where $s = n - 1 = 5$.

[*Insert Figures* 1*a and* 1*b here*]

## 2.2 The labor market

The labor market environment is as follows. Time is discrete and continues forever. At any point in time, each of the $n$ workers is either employed or unemployed. At period $t$, the unemployment pool is denoted by $U_t$ and the corresponding unemployment rate by $u_t = U_t/n$. There are also $V_t$ vacancies to be filled and each worker directly hears of a vacancy with probability $v_t = V_t/n$. We refer to $v_t$ as the job arrival rate or the vacancy rate. Each employer posts a vacancy by advertising this job both in employment agencies (and/or national newspapers) and to her/his current workers.

At each period, currently employed workers lose their jobs with some probability $\delta$. This process is taken to depend only on the general state of the economy and hence is treated as exogenous to the labor market. The timing of the model is as follows. At the end of period $t$, the unemployment and employment rates are respectively equal to $u_t$ and $1-u_t$. At the beginning of period $t+1$, there is a technological shock and employed workers lose their jobs with the breakdown probability $\delta$. The resulting employment rate is $(1-\delta)(1-u_t)$. Then, $V_{t+1}$ vacancies are posted and jobs are filled according to the procedure described below. At the end of period $t+1$, the unemployment and employment rates are respectively equal to $u_{t+1}$ and $1-u_{t+1}$. And so on. From now on, and for notational simplicity, we omit the subscript $t$ when no confusion is possible.

## 2.3 Word-of-mouth information transmission

At each period, and once the technological shock has occurred, any worker (employed or unemployed) *directly* hears of a vacant job with probability $v = V/n$. Recall that jobs are systematically posted both through employment agencies (or newspapers) and within firms. Hence, the probability that a worker directly hears of a job (i.e. through the employment agency for the unemployed or from the employer her/himself for the employed) is always equal to $v$ irrespective of the current employment status. There are now two cases to be considered. First, the directly informed worker is unemployed. Then, s/he takes this job immediately. This means that this worker has found the job through an employment agency (or an ad in the newspapers) and, consequently, does not rely on her/his social network to be reemployed. Second, the directly informed worker is employed, meaning that s/he has been directly informed by her/his current employer. Obviously, this worker does not need this job and transmits this information to one of her/his direct unemployed neighbors, if any. We assume that unemployed workers are treated on an equal footing, which means that all unemployed direct neighbors have the same probability to be informed.

Observe that, according to this information transmission protocol, job information can only flow through word-of-mouth from an employed to an unemployed worker that is, between workers with different employment status. Indeed, vacancies are assumed to be posted for one period which coincides with the time required to transmit information to direct neighbors. Therefore, if the informed worker is both employed and does not have any unemployed worker in her/his direct vicinity, the job slot is lost. Similarly, if an unemployed worker hears of two (or more) vacancies through word-of-mouth from two (or more) direct employed neighbors, we assume that s/he selects one job randomly, the other job(s) being lost. Finally, one (or more) job(s) is (are) also lost when an unemployed worker hears of jobs both directly and through friends.

Assuming that job information cannot be relayed further away than the direct neighborhood of the initially informed employed worker is not completely at odds with empirical findings. Indeed, Granovetter (1995) shows that information transmission with no relay (as assumed here) accounts for 39.1% of the jobs found through contacts (p. 57). To keep things tractable, we maintain this simplifying assumption throughout and, in section 2.5, we discuss how our results are robust to generalizations of this information transmission protocol.

## 2.4 Finding a job through contacts

In our model, workers partly rely on friends to gather information about potential jobs. Denote by $\theta \equiv (1-\delta)(1-u)$ the individual probability of remaining employed after the technological shock and before vacancies are posted for the current period. Conditional on being unemployed and not hearing directly of a vacancy, the individual probability of finding a job through contacts for worker $i$ depends on the prevailing social network $g$ and is given by:

$$P_i(g,u,v) = 1 - \prod_{j \in N_i(g)} \left[1 - v\theta \frac{1-\theta^{n_j(g)}}{(1-\theta)\, n_j(g)}\right] \tag{1}$$

The explanation for this result is the following. Fix a worker $j \in N_i(g)$ in the direct neighborhood of player $i$. Then, $v\theta$ is the probability of this particular neighbor $j$ knowing of a job opportunity (probability $v$) and not needing it (probability $\theta$). This employed and informed neighbor $j$ transmits this available job information to her/his direct neighbor $i$ with probability $\frac{1-\theta^{n_j(g)}}{(1-\theta)n_j(g)}$. Indeed, the probability of $i$ being the unemployed worker selected among all the unemployed neighbors of $j$ to be told about the existing vacancy can be decomposed as follows:

$$\frac{1-\theta^{n_j(g)}}{(1-\theta)\, n_j(g)} = \theta^{n_j(g)-1} + \sum_{k=1}^{n_j(g)-1} \binom{n_j(g)-1}{k} \frac{1}{k+1} \theta^{n_j(g)-k-1} (1-\theta)^k$$

According to this expression, worker $i$ is the recipient of the job information held by her/his employed neighbor $j$ if either s/he the only unemployed neighbor of $j$ (probability $\theta^{n_j(g)-1}$) or s/he is the one selected among the $k+1$ unemployed friends of $j$ (probability $\frac{1}{k+1}\theta^{n_j(g)-k-1}(1-\theta)^k$). Therefore, $v\theta\frac{1-\theta^{n_j(g)}}{(1-\theta)n_j(g)}$ is the probability of player $i$ finding a job thanks to his direct neighbor $j \in N_i(g)$, whereas with complementary probability $1-v\theta\frac{1-\theta^{n_j(g)}}{(1-\theta)n_j(g)}$ the employed direct neighbor $j$ of player $i$ does not prove useful to find a job. Finally, $\prod_{j\in N_i(g)}\left[1-v\theta\frac{1-\theta^{n_j(g)}}{(1-\theta)n_j(g)}\right]$ denotes the individual probability of worker $i$ not hearing of a vacancy through word-of-mouth communication from any of her/his direct acquaintances.

In Figures 1a and 1b, we have calculated this probability $P_i(g,u,v)$ for a star-shaped graph and a complete graph. From Figure 1a, it is clear that individual 1 has the highest probability to find a job through word-of-mouth since s/he is connected to everybody whereas all the others have the same probability since they are only connected to individual 1 ($P_1 > P_2 = P_3 = P_4 = P_5 = P_6$). In Figure 1b, all individuals have the same number of direct neighbors (symmetric graph), which implies that they all have the same probability to find a job through contacts ($P_1 = P_2 = P_3 = P_4 = P_5 = P_6$). Observe however that, in both cases, all individuals have the same probability $v$ to find a job through formal methods since this job-finding process does not depend on the social network.

From now on, we focus on symmetric social networks with uniform mix in which all workers have both the same number of neighbors equal to $s$ (symmetry) and the same number of employed and unemployed direct contacts equal respectively to $(1-u)\,s$ and $us$ (uniform mix). We refer to $s$ as the network size. In a symmetric network of size $s$, the individual probability of hearing of a job through word-of-mouth is then:

$$P(s,u,v) = 1 - \left[1 - v\theta\frac{1-\theta^s}{(1-\theta)\,s}\right]^s \tag{2}$$

As stated above, Figure 1b depicts a particular example of a symmetric social network when $s = n-1 = 5$.

**Proposition 1** *The properties of $P(s,u,v)$ are the following:*

*(i) $P(\cdot,u,v)$ is increasing between 0 and $\overline{s}$ and decreasing between $\overline{s}$ and $n-1$, where $\overline{s}$ is the unique global maximum of $P(\cdot,u,v)$. Also, $P(\cdot,u,v)$ is strictly concave on $[0,K)$ for some $K > \overline{s}$;*

*(ii)* $P(s,\cdot,v)$ *is decreasing in* $u$*. Moreover, there exists some* $\widetilde{\delta}\in[0,1)$ *such that* $P(s,\cdot,v)$ *is strictly convex in* $u$ *when* $\delta\geq\widetilde{\delta}$*;*

*(iii)* $P(s,u,\cdot)$ *is increasing and strictly concave in* $v$*.*

The following comments are in order. First, fix $u$ and $v$. The individual probability $P(\cdot,u,v)$ to find a job through word-of-mouth within the network of social contacts exhibits diminishing marginal returns to network size $s$.[4] In other words, the marginal impact of adding a new connection to everybody decreases with the total number of pairwise links in the society. Moreover, $P(\cdot,u,v)$ increases with $s$ in sparse networks ($s<\overline{s}$) while it decreases with $s$ in densely connected labor market networks ($s>\overline{s}$). To understand this result, observe that increasing the network size has both a (positive) direct and (negative) indirect effect. On one hand, rising the network size expands the available *direct* connections to every worker. Workers become better connected and, consequently, the potential job information they can benefit from increases. Indeed, the probability that at least one direct contact is informed about a job opening is $1-(1-v)^s\nearrow 1$ as $s\to+\infty$. On the other hand, rising the network size also increases the potential number of unemployed workers directly connected to an employed and informed worker. The information held by every employed worker is now shared by a larger group of unemployed workers. The individual probability of being randomly selected by an employed direct friend as the information recipient is $\frac{1-\theta^s}{(1-\theta)s}\searrow 0$ as $s\to+\infty$. Therefore, every unemployed worker suffers from the information sharing constraints exerted by the unemployed *indirectly* connected to her/him. Stated differently, expanding one's neighborhood has a negative impact on the current direct friends as it reduces their (individual) probability to gather job opportunities through social contacts. Workers relative locations thus create a *negative network externality* for their direct vicinity. This indirect negative effect prevails in networks of large size as $[1-(1-v)^s]\left[\frac{1-\theta^s}{(1-\theta)s}\right]\to 0$ when $s\to+\infty$. Increasing the network size of dense networks ($s>\overline{s}$) slows down word-of-mouth information transmission.[5] This result contradicts the common view that more contacts always yield positive effects since more network members can potentially reduce one's chance to obtain a job.

[4]In fact, this is true only on a restricted domain $[0,K)$ including the unique global maximum $\overline{s}$. However, observe that concavity holds on the whole domain where $s$ is allowed to vary whenever $K\geq n-1$.

[5]The threshold value $\overline{s}$ is uniquely determined by $\frac{\partial P(s,u,v)}{\partial s}=0$.

Second, when the unemployment rate $u$ increases, two effects are in order: $(i)$ the likelihood that a worker, who is directly informed of a vacancy through formal channels (arrival rate $v$), is unemployed increases, and also $(ii)$ the number of unemployed directly connected to every informed and employed worker rises. This implies that $u$ and $P(s, \cdot, v)$ are negatively correlated. To understand the positive impact of the vacancy rate $v$ on the individual probability of finding a job through friends $P(s, u, \cdot)$ a similar intuition applies.

## 2.5 Generalizing the communication protocol

So far, we have assumed that information about job opportunities can only flow from employed workers to unemployed direct acquaintances. In particular, the informed worker cannot transmit any information to any other employed friend that may then relay it to some unemployed direct contact, if any. Hence, the rate at which employed workers hear of a job opportunity is completely determined by the vacancy rate $v$ and does not depend on the network of social contacts $g$. As a consequence, one's indirect neighbors do not constitute a potential source of job information. Rather, they are perceived as potential competitive information recipients. The resulting information sharing constraints they exert on indirect neighbors generate the negative externality arising in information transmission.

Suppose now that we relax this assumption and we allow for information to be relayed through word-of-mouth from employed worker to employed worker, with no restrictions whatsoever on the length of transmission. Assume, though, that relayed information is correctly transmitted with some probability strictly less than one, to account, for instance, for forgetfulness.[6] Now, the rate $a_j$ at which some employed worker $j$ acquires job information depends both on the vacancy rate $v$ and on the network of contacts $g$ and thus can be written as $a_j(v, g)$. The individual probability of finding a job through contacts then becomes

$$P_i(g, u, v) = 1 - \prod_{j \in N_i(g)} \left[ 1 - a_j(v, g)\, \theta \frac{1 - \theta^{n_j(g)}}{(1 - \theta)\, n_j(g)} \right]$$

Indeed, indirect connections do not only induce information sharing constraints but may now also constitute a valuable source of job information.

[6] Equivalently, we could assume that there is no forgetfulness at all but that the length of job information transmission is finite and arbitrarily fixed.

Still, one can show that in dense enough networks, the negative effect of information sharing constraints outweighs the positive impact of possibly acceding a broader range of information channels. In other words, allowing for information to flow on the network of contacts along any path connecting two workers does not alter the qualitative relationship between job matching and social embeddedness stressed in this paper.[7]

# 3 The matching function

As stated above, unemployed workers find jobs from two different channels. Either they find their job directly through *formal* methods −such as advertisement or employment agencies− with probability $v$, or they gather information about jobs through *informal* methods −in our case, the network of social contacts− with probability $P(s,u,v)$. In this context, the job acquisition rate or individual hiring probability of an unemployed worker is:

$$h(s,u,v) = v + (1-v)P(s,u,v) \tag{3}$$

At each period of time, there are $n\,u = U$ unemployed workers that find a job with probability $h(s,u,v)$. Since this probability is independent across different individuals, the number of job matches taking place per unit of time is just $n\,u\,h(s,u,v)$. Therefore, the matching function for our labor market where workers partly rely on personal contacts to find a job is given by:[8,9]

$$m(s,u,v) = u\,[v + (1-v)P(s,u,v)] \tag{4}$$

[7]For more details on this issue, see Calvó-Armengol (2001). Note, however, that allowing for information to flow through word-of-mouth with no restrictions on the length of transmission complicates sharply the analysis. Indeed, when relays are permitted, the local topology of the network may play a role and has to be taken explicitly into account.

[8]To be more precise this matching function corresponds to the rate at which job matches occur per unit of time. It suffices therefore to multiply $m(s,u,v)$ by $n$ to get the number of matches per unit of time.

[9]It is easy to verify that the matching function for a general social network $g$, not necessarily symmetric, is equal to:

$$m(g,u,v) = n\,u\left[v + (1-v)\frac{1}{n}\sum_{i\in N} P_i(g,u,v)\right]$$

where $P_i(g)$ is given by (1).

We can thus express the aggregate rate at which job matches occur as a function of the unemployed worker and vacant firm pools, and the social network underlying players talks. This endogenous matching function is derived from an explicit micro scenario where the structure of personal contacts and the job information transmission process is spelled out in detail. Contrary to previous contributions also providing micro foundations for matching functions, the well-defined reduced function obtained here is neither an exponential nor a min one. Moreover, the central role of the network of contacts in matching job-seekers with vacancies is made explicit, and the link between $m(s,u,v)$ and the network size $s$ is precisely the key element of our model.

**Proposition 2** *The properties of the matching function $m(s,u,v)$ are the following:*

- *(i)* $m(\cdot,u,v)$ *is increasing between* $0$ *and* $\overline{s}$ *and decreasing between* $\overline{s}$ *and* $n-1$*, where* $\overline{s}$ *is the unique global maximum of* $P(\cdot,u,v)$*. Also,* $m(\cdot,u,v)$ *is strictly concave on* $[0,K)$ *for some* $K>\overline{s}$*;*
- *(ii)* $m(s,\cdot,v)$ *is increasing and strictly concave in* $u$ *on* $[0,\overline{u}]$ *for some* $0<\overline{u}\leq 1$*;*
- *(iii)* $m(s,u,\cdot)$ *is increasing and strictly concave in* $v$*.*

We have the following comments. First, even though our matching function is quite different to the ones found in the literature, it has the same natural properties: it is increasing and strictly concave in both $u$ and $v$.[10]

Second, it is easily verified that $P(s,u,v)$ is not homogeneous of degree one, implying in turn that the matching function $m(s,u,v)$ also fails to exhibit constant returns to scale (with respect to $u$ and $v$). The intuition for this result is as follows. Suppose first that the network size $s$ is fixed. Increasing the vacancy rate from $v$ to $\lambda v$ (where $\lambda>1$) has a positive direct impact on all workers in the population. By contrast, increasing the unemployment rate by the same amount (from $u$ to $\lambda u$) has both a direct and an indirect negative effect. Indeed, the number of unemployed direct acquaintances increases, thus reducing the value of such personal contacts as job providers (direct negative effect). Moreover, the number of unemployed indirect acquaintances also increases, thus imposing a stronger information

[10] For $u$, this is true only on a restricted domain, i.e. on $[0,\overline{u}]$, where $\overline{u}$ is quite large.

sharing constraint (indirect negative effect). These two combined negative effects outweigh the positive direct effect of additional vacancies. In order to see that, let us write (2) as[11]

$$P(s,u,v) = 1 - \left[1 - \frac{1}{s}v\theta\left(1+\theta+\cdots+\theta^{s-1}\right)\right]^{s}$$

where $\theta = (1-\delta)(1-u)$. Therefore, increasing $v$ has a positive linear impact on $\frac{1}{s}v\theta\left(1+\theta+\cdots+\theta^{s-1}\right)$ whereas increasing $u$ has both a negative linear impact through $\theta$ of the same order and a magnifying negative impact through the polynomial form $\left(1+\theta+\cdots+\theta^{s-1}\right)$. This result is at odds with the standard hypothesis of a constant-return-to-scale aggregate matching function made in the theoretical literature on job matching (Mortensen and Pissarides, 1999 and Pissarides, 2000). It says that, if social networks and word-of-mouth communications are integrated in the job-search process, then the matching function is more likely *not* to be homogeneous of degree one. Besides, there is a huge body of empirical work to assess whether this property of the matching function is encountered in real-life labor markets. Even if the results lean towards constant returns to scale, they are very much controversial[12] and most of these empirical studies do not include informal methods in finding a job. By taking into account these methods, it would be interesting to see if the results would be altered in such a way that the matching function would fail to exhibit constant returns to scale.

Third, there is a non-monotonic relationship between the job matching rate and the network size. In fact, because the word-of-mouth communication plays a crucial role in our model, the workers and their direct set of acquaintances impose an important externality to each other. This externality between workers and personal contacts implies that network size is relevant in determining the rate at which unemployed find jobs. The non-monotonic relationship is just a direct consequence of the ambiguous impact network size has on the individual probability $P(s,u,v)$ to find a job through friends. Recall that network size has a positive impact on $P(s,u,v)$ in sparse networks, whereas is has a negative impact on $P(s,u,v)$ dense ones.

Finally, we can deduce from (4) the following simple expression for the

[11] Simply note that $(1-\theta^{s})/(1-\theta) = 1+\theta+\cdots+\theta^{s-1}$.

[12] See for instance Coles and Smith (1996), Petrongolo and Pissarides (2001) and the references therein.

individual probability $f(s,u,v)$ for firms to fill a vacancy:

$$f(s,u,v) = \frac{m(s,u,v)}{v} = u\left[1 - \left(1 - \frac{1}{v}\right) P(s,u,v)\right] \tag{5}$$

Clearly, the properties of both the job-hiring rate $h(s,u,v)$ and the job-filling rate $f(s,u,v)$ as functions of the network size $s$ are immediately deduced from that of $P(s,u,v)$ namely, strictly concave in $s$, increasing between 0 and $\overline{s}$ and decreasing between $\overline{s}$ and $n-1$. Moreover, the job-hiring rate $h(s,u,v)$ is decreasing in $u$ and increasing in $v$ whereas the job-filling rate $f(s,u,v)$ is increasing in $u$ and decreasing in $v$.[13] In other words, given a vacancy rate $v$ (and a network size $s$), when the number of unemployed increases, it is more difficult to find a job but easier to fill a vacancy. Similarly, given an unemployment rate $u$ (and a network size $s$), it becomes easier to find a job but more difficult to fill a vacancy as the number of vacancies increases.[14]

# 4 The labor market equilibrium

## 4.1 Characterization of the equilibrium

Firms and workers are all identical. A firm is a unit of production that can either be filled by a worker whose production is $y$ units of output or be unfilled and thus unproductive. We denote by $\gamma$ the search cost for the firm per unit of time, by $w$ the wage paid by the firms when a match is realized and by $r$ the discount factor. We assume that the wage is exogenous. This is because our focus is not on wage determination but rather on the communication mechanisms through which job information is gathered and transmitted, the network of personal contacts underlying such communication processes, and their impact on labor market outcomes. In particular, one of the salient features of our framework is to derive an explicit matching function from a model of communication and networks (see Proposition 2).[15] In section 4.3,

[13] See Lemmata 1 and 2 in the appendix.

[14] See Pissarides (2000) for a thorough account and description of such trading externalities. Note also that $1/h$ and $1/f$ can be interpreted as the mean duration respectively of unemployment and of vacancies.

[15] There are papers that have explored the wage premium associated with the use of personal contacts in finding a job. See for instance Montgomery (1991) and Kugler (2000) for

we will however discuss how our model can take into account endogenous wages.

At every period, matches between workers and firms depend upon the current network of social contacts of size $s$ and the current state of the economy given by the unemployment rate $u$ and the vacancy rate $v$. We focus on the steady state equilibrium.

**Definition 1** *Given a network size $s$ and the associated matching technology $m(s,\cdot,\cdot)$, a (steady-state) labor market equilibrium $(u^*(s), v^*(s))$ is determined by a free-entry condition for firms and a steady-state condition on unemployment flows.*

At the steady state labor market equilibrium, every worker has $s$ direct acquaintances consisting of $su^*(s)$ unemployed and $s(1-u^*(s))$ employed contacts. We now characterize such a steady state equilibrium. We first establish the free-entry condition and the resulting labor demand. At period $t$, the intertemporal profit of a filled job and of a vacancy are denoted respectively by $I_{F,t}$ and $I_{V,t}$. Recall that the job-filling rate $f$ is defined by (5). Since time is discrete, we have the following standard Bellman equations:

$$\begin{aligned} I_{F,t} &= y - w + \frac{1}{1+r}\left[(1-\delta)I_{F,t+1} + \delta\, I_{V,t+1}\right] \\ I_{V,t} &= -\gamma + \frac{1}{1+r}\left[(1-f)I_{V,t+1} + f\, I_{F,t+1}\right] \end{aligned}$$

In steady state, both $I_{F,t} = I_{F,t+1} = I_F$ and $I_{V,t} = I_{V,t+1} = I_V$. Following Pissarides (2000), we assume that firms post vacancies up to a point where $I_V = 0$. We deduce from this free entry condition the following relation between $u$ and $v$:

$$\frac{m(s,u,v)}{v} = \gamma\,\frac{r+\delta}{y-w} \tag{6}$$

In other words, the value of a job is equal to the expected search cost, i.e. the cost per unit of time multiplied by the average duration of search for the firm. This equation can be mapped in the plane $(u,v)$ and is referred

analyses of this issue in an adverse selection setting, Mortensen and Vishwanath (1994) for an equilibrium search models with wage posting and on-the-job search, and Montgomery (1992) for a model with weak and strong ties.

to as the labor demand curve. We then close the model by the following steady-state condition on flows:

$$m(s, u, v) = \delta(1 - u) \tag{7}$$

As above, this equation can be mapped in the plane $(u, v)$ and is referred to as the Beveridge curve. The two equations (6) and (7) with two unknowns $u$ and $v$ fully characterize the labor market equilibrium $(u^*(s), v^*(s))$ as a function of the network size $s$.

**Proposition 3** *Suppose that $\gamma(r + \delta)/(y - w) > \delta/(1 + \delta)$. Then, for all network size $s$, there exists a labor market equilibrium $(u^*(s), v^*(s))$. If $\gamma(r+\delta)/(y - w)$ is small enough, this equilibrium is unique.*

Observe that the condition on the parameters $\gamma(r+\delta)/(y-w) > \delta/(1+\delta)$ that guarantees the existence of the equilibrium is very likely to be satisfied. Indeed, we deduce from (6) that $\gamma(r + \delta)/(y - w)$ is equal to the job-filling rate $f(s, u, v)$. A sufficient condition for $f(s, u, v) > \delta/(1 + \delta)$ to hold is $f(s, u, v) > \delta$ that is, the job-filling rate be higher than the job-destruction rate, which is obviously true in most labor markets.

## 4.2 Social network and unemployment

We now investigate the different properties of the labor market equilibrium and focus on the relationship between the equilibrium unemployment rate $u^*(s)$ and the size of the social network $s$. We assume from now on that the conditions for uniqueness are met.

**Proposition 4** *The equilibrium unemployment rate $u^*(s)$ decreases with $s$ when $s < \overline{s}$, while it increases when $s \geq \overline{s}$.*

Our matching function depends explicitly on the structure of personal contacts and the labor market equilibrium captures the influence of the frictions due to workers social embeddedness on market outcomes. In particular, we know from propositions 1 and 2 that in a sparse network $(s < \overline{s})$ , both the individual probability $P(\cdot, u, v)$ to find a job through word-of-mouth and the matching function increase with the network size $s$. We deduce from the free entry condition (6) that, holding the arrival rate $v$ fixed, unemployment decreases. The Beveridge curve (7) then implies that unemployment must also decrease to equalize flows out with flows in. Since the two effects have the same sign, $u^*(s)$ decreases with $s$. When the social network of contacts is dense ($s \geq \overline{s}$), the opposite result holds since negative network externalities prevail in networks of large size and both $P(\cdot, u, v)$ and $m(\cdot, u, v)$ decrease with $s$.

[*Insert Figures* 2*a and* 2*b here*]

The impact of the network size $s$ on the equilibrium vacancy rate $v^*(s)$ is ambiguous both when the network is sparse ($s < \overline{s}$) or dense ($s \geq \overline{s}$). Indeed, two opposite effects are now in place. On one hand, increasing the size of a sparse network improves the transmission of information through word-of-mouth communication. As a result, matches are more frequent and we deduce from the free entry condition (6) that more vacancies are posted. In other words, $v^*(s)$ and $s$ are positively correlated. On the other hand, rising the size of a sparse network by creating additional direct connections increases the number of matches between workers and firms. We then deduce from the Beveridge curve (7), that vacancies decrease in order to guarantee that the flows out of unemployment are still equal to the flows into unemployment. Therefore, $v^*(s)$ and $s$ are negatively correlated. When the network is dense, this ambiguity remains and is sustained by the opposite intuition: $v^*(s)$ and $s$ are both negatively and positively correlated due to (6) and (7) respectively.

## 4.3 Endogenous wages

So far, we have assumed that wages were exogenous so that employed workers systematically transmit information about job opportunities to their unemployed friends. One may argue that, if wages were endogenous and negotiated between workers and firms, the employed could exploit a job offer to increase their bargaining power and thus their wages. In this case, it would not always be optimal for employed workers to communicate job offers to their unemployed neighbors. In fact, it is easy to see that currently employed workers who had never been offered an outside job would always use any available outside opportunity to increase their wages. It should be clear that after some finite iterations of such negotiations, these workers would obtain the highest possible wage.[16] This implies that all employed workers who have

[16] For instance, if there is Bertrand competition between two employers (the current and the outside ones), the employed worker who has the two offers obtains all the surplus and therefore gets straightaway the highest possible wage.

been working for a fixed number of periods (greater or equal than two) in the same firm and have exploited all possible wage negotiations, always transmit additional job information to her/his unemployed friends. Formally, the individual probability of finding a job through contacts for any unemployed worker within a symmetric network of size $s$ can now be written as:

$$P(s,u,kv) = 1 - \left[1 - kv\theta\frac{1-\theta^s}{(1-\theta)\,s}\right]^s$$

where $k < 1$, and the corresponding matching function is given by:

$$m(s,u,v) = u\left[v + (1-v)P(s,u,kv)\right]$$

In words, compared to the case of exogenous wages, the unemployed workers have less chances to hear from a vacancy from their employed direct friends ($kv < v$) because the latter can now use job offers to increase their wages. Observe that $k$ is endogenous and determined by the labor market equilibrium, and represents the reduction in available job information sources.

In this context, a wage distribution endogenously emerges in equilibrium. Indeed, apart of the unemployment benefit received by the unemployed, employed workers earn different wages depending on their work history (in terms of outside offers and thus negotiations). The lowest wage is received when they leave unemployment and start working in a firm whereas in the highest wage they obtain all the surplus because they have exhausted all possible negotiations. Even if this extension enriches the working of the labor market, it leads to a much more complicated analysis without altering the qualitative features of our framework. Indeed, the closed-form expression of our microfounded matching function remains similar. More importantly, the frictions induced by the social network explicitly characterized in terms of information sharing constraints still hold.

# 5 Conclusion

In recent years, a growing literature consisting both of empirical work and theoretical contributions has stressed the prominence of social networks in explaining a wide range of economic phenomena. In particular, the prevalent social contacts strongly determine, or at least influence, economic success of individuals in a labor market context.

In this paper, we have analyzed the matching between unemployed workers and vacant jobs in a social network context. More precisely, each individual, who is embedded within a network of social relationships, can find a job either through formal methods (employment agencies or advertisements) or through informal networks (word-of-mouth communication). From this micro scenario, we first derive an aggregate matching function that has the standard properties but fails to be homogenous of degree one. This is because there is a non-monotonic relationship between the size of the social network and the probability to find a job: increasing the size of sparse networks is beneficial to workers whereas it is detrimental in dense networks. Indeed, increasing the network size of dense networks slows down word-of-mouth information transmission and creates negative network externalities. We then close the model by introducing the behavior of firms and show that there exists a unique labor market equilibrium under mild conditions on the parameters of the economy. Finally, and because of the previous result, we show that the equilibrium unemployment rate decreases with the network size in sparse networks while it increases in dense networks.

The results obtained in this paper are robust to generalizations of the communication mechanism in the network of personal contacts. Also, allowing for wages to be endogenously determined would not affect our main conclusions.

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# A Appendix

**Proof of Proposition 1.**

Let $q(s,\theta) \equiv \frac{\theta(1-\theta^s)}{s(1-\theta)}$. Then, $P(s,u,v) = 1 - Q(s,u,v)$ where $Q(s,u,v) = [1 - vq(s,\theta)]^s$. The properties of $P(\cdot)$ can thus be deduced from that of $Q(\cdot)$ established below:

$(a)$ $Q(s,u,\cdot)$ is decreasing and strictly convex with respect to $v$. Indeed, differentiating once with respect to $v$ gives: $\frac{\partial Q}{\partial v} = -sQ\frac{q}{1-vq} < 0$. Differentiating twice we get $\frac{\partial^2 Q}{\partial v^2} = -s\frac{\partial Q}{\partial v}\frac{q}{1-vq} - sQ\frac{q^2}{(1-vq)^2}$. Replacing $\frac{\partial Q}{\partial v}$ by its expression above gives $\frac{\partial^2 Q}{\partial v^2} = s(s-1)Q\frac{q^2}{(1-vq)^2} > 0$.

$(b)$ $Q(s,\cdot,v)$ is increasing with respect to $u$. Moreover, there exists $\widetilde{\delta} \in [0,1)$ such that $Q(s,\cdot,v)$ is strictly concave with respect to $u$ as long as $\delta \geq \widetilde{\delta}$. Indeed, simplifying by $(1-\theta)$ gives $q(s,\theta) = \frac{1}{s}(\theta + \cdots + \theta^s)$. Hence, $q(s,\cdot)$ is increasing with respect to $\theta$, implying that $Q(s,u,v) = [1 - vq(s,1-u)]^s$ is increasing with respect to $u$. From $\theta = (1-\delta)(1-u)$ we deduce that $\frac{\partial^2 Q}{\partial u^2} = (1-\delta)^2 \times \frac{\partial^2}{\partial \theta^2}[1 - vq(s,\theta)]^s$. Differentiating twice gives

$$\frac{\partial^2}{\partial \theta^2}[1-vq]^s = -vs[1-vq]^{s-2}\left[(1-vq)\frac{\partial^2 q}{\partial \theta^2} - v(s-1)\left(\frac{\partial q}{\partial \theta}\right)^2\right]$$

Hence, $\frac{\partial^2 Q}{\partial u^2} < 0$ is equivalent to $(1-vq)\frac{\partial^2 q}{\partial \theta^2} - v(s-1)\left(\frac{\partial q}{\partial \theta}\right)^2 > 0$ where

$$\begin{cases} q(s,\theta) = \frac{1}{s}(\theta + \cdots + \theta^s) \\ \frac{\partial q}{\partial \theta} = \frac{1}{s}\left(1 + \cdots + s\theta^{s-1}\right) \\ \frac{\partial^2 q}{\partial \theta^2} = \frac{1}{s}\left(2 + \cdots + s(s-1)\theta^{s-2}\right) \end{cases}$$

At $\theta = 0$ we have: $q(s,\theta) = 0$, $\frac{\partial q}{\partial \theta}|_{\theta=0} = \frac{1}{s}$ and $\frac{\partial^2 q}{\partial \theta^2}|_{\theta=0} = \frac{2}{s}$. Therefore, $\frac{\partial^2 Q}{\partial u^2}|_{\theta=0} < 0$ is equivalent to $2s > v(s-1)$ which is true. Denote by $\widetilde{\theta}$ the smallest positive root of the polynomial $R$ in $\theta$ of degree $2(s-1)$ given by: $R(\theta) \equiv (1-vq)\frac{\partial^2 q}{\partial \theta^2} - v(s-1)\left(\frac{\partial q}{\partial \theta}\right)^2$. If $R(\theta) > 0$ for all $\theta > 0$ we set $\widetilde{\theta} = +\infty$ by definition. From $R(0) > 0$ and by continuity, we deduce that $R(\theta) > 0$ on $\left[0, \widetilde{\theta}\right)$. Let $\widetilde{\delta} = 1 - \min\left\{\widetilde{\theta}, 1\right\}$. Then, $R(\theta) > 0$ on $\left[0, 1-\widetilde{\delta}\right)$ implying that $\frac{\partial^2 Q}{\partial u^2} < 0$ for all $u \in [0,1]$ that is, $Q(s,\cdot,v)$ strictly concave with respect to $u$, as long as $\delta \geq \widetilde{\delta}$.

($c$) $Q(\cdot, u, v)$ is decreasing in $[0, \overline{s}]$ and increasing on $[\overline{s}, +\infty)$. Moreover, is strictly convex on $[0, K)$ for some $K > \overline{s}$. We prove this result in four steps.[17] Fix $u$ and $v$ and let $\phi(s) = 1 - vq(s, \theta)$. Then, $Q(s, u, v) = [\phi(s)]^s$ from which we deduce that $\frac{\partial Q}{\partial s} = \Phi(s) \times Q$ where $\Phi(s) = \ln \phi(s) + s\frac{\phi'(s)}{\phi(s)}$.

**Step 1.** We show that $\left.\frac{\partial Q}{\partial s}\right|_{s=1} < 0$ which is equivalent to proving that $\Phi(1) < 0$. With some algebra, $\phi'(s) = \frac{v\theta}{1-\theta}\left[\frac{1-\theta^s}{s^2} + \ln\theta\frac{\theta^s}{s}\right]$, implying that $\frac{1}{1-v\theta}\Phi(1) = v\theta\left(1 + \frac{\theta}{1-\theta}\ln\theta\right) + (1 - v\theta)\ln(1 - v\theta)$. Establishing that $\Phi(1) < 0$ is thus equivalent to showing that for all $\theta \in (0, 1)$, $\rho_\theta(v) < 0$ on $(0, 1)$, where $\rho_\theta(v) = v\theta\left(1 + \frac{\theta}{1-\theta}\ln\theta\right) + (1 - v\theta)\ln(1 - v\theta)$. Fix $\theta$. Differentiating twice gives $\rho'_\theta(v) = \frac{\theta^2}{1-\theta}\ln\theta - \theta\ln(1 - v\theta)$ and $\rho''_\theta(v) = \frac{\theta^2}{1-v\theta} > 0$. Therefore, $\rho_\theta$ is strictly convex, implying that $\rho'_\theta$ increases on $(0, 1)$ with supremum $\rho'_\theta(1) = \frac{\theta}{1-\theta}[\theta\ln\theta - (1-\theta)\ln(1-\theta)]$. It is straightforward to see that $x \mapsto x\ln x - (1-x)\ln(1-x)$ is worth 0 at $x = 0, \frac{1}{2}$ and 1, takes negative values on $\left(0, \frac{1}{2}\right)$ and positive values on $\left(\frac{1}{2}, 1\right)$. Therefore, $\rho_\theta$ decreases on $\left(0, \frac{1}{2}\right)$ and increases on $\left(\frac{1}{2}, 1\right)$ with supremum given by $\max\{\rho_\theta(0), \rho_\theta(1)\}$. We have $\rho_\theta(0) = 0$ and $\rho_\theta(1) = \theta\left(1 + \frac{\theta}{1-\theta}\ln\theta\right) + (1-\theta)\ln(1-\theta)$. If $\theta < \frac{1}{2}$, $\theta\ln\theta < (1-\theta)\ln(1-\theta)$ implying that $\rho_\theta(1) < 0$. If $\theta > \frac{1}{2}$, $\theta\ln\theta > (1-\theta)\ln(1-\theta)$, therefore $\rho_\theta(1) < \frac{\theta}{1-\theta}(1 - \theta + \ln\theta)$. It is easy to check that $x \mapsto 1 - x + \ln x$ is negative on $(0, 1)$. Hence, $\rho_\theta(1) < 0$. In both cases, $\sup_{v\in(0,1)} \rho_\theta = \rho_\theta(0) = 0$. *Q.E.D.*

**Step 2.** We show that $Q(\cdot, u, v)$ increases towards its asymptotic limit for high values of $s$. It is easy to check that $\Phi(s) \sim \left[\frac{v\theta}{(1-\theta)s}\right]^2$ when $s \to +\infty$, implying that $\frac{\partial Q}{\partial s} > 0$ for high values of $s$. Therefore, $Q(\cdot, u, v)$ increases towards its limit $\exp\left(-\frac{v\theta}{1-\theta}\right)$ when $s \to +\infty$. *Q.E.D.*

**Step 3.** We show that $\frac{\partial Q}{\partial s} \leq 0$ implies that $\frac{\partial^2 Q}{\partial s^2} > 0$. We have $\frac{\partial Q}{\partial s} = \Phi(s)Q$. Therefore, $\frac{\partial^2 Q}{\partial s^2} = \Phi'(s)Q + \Phi(s)\frac{\partial Q}{\partial s} =$

[17] This proof follows closely that of Lemma 2 in Calvó-Armengol (2001).

$\left(\Phi'(s)+[\Phi(s)]^2\right)Q$. Therefore, $\Phi'>0$ implies that $\frac{\partial^2 Q}{\partial s^2}$. Suppose on the contrary that $\Phi'\leq 0$. We have $\phi(s)=1-v\theta\frac{1-\theta^s}{(1-\theta)s}\to 1$ and $s\phi'(s)=\frac{v\theta}{1-\theta}\left(\frac{1-\theta^s}{s}+\ln\theta\times\theta^s\right)\to 0$ when $s\to+\infty$. Therefore, $\lim\limits_{s\to+\infty}\Phi(s)=0$. Hence, $\Phi'\leq 0$ implies that $\Phi>0$. Reciprocally, $\Phi\leq 0$ implies that $\Phi'>0$, which in turn implies that $\frac{\partial^2 Q}{\partial s^2}>0$. But $\Phi\leq 0$ is equivalent to $\frac{\partial Q}{\partial s}\leq 0$. Hence, $\frac{\partial Q}{\partial s}\leq 0$ implies that $\frac{\partial^2 Q}{\partial s^2}>0$. *Q.E.D.*

**Step 4.** We deduce from steps 1 and 2 that $\frac{\partial Q}{\partial s}=0$ for some $\overline{s}\in[1,+\infty)$. Therefore, from step 3, $\left.\frac{\partial^2 Q}{\partial s^2}\right|_{s=\overline{s}}>0$. Therefore, $Q(\cdot,u,v)$ does not have any local maxima and there exists a unique such point $\overline{s}$, and $Q(\cdot,u,v)$ reaches its global minimum at $\overline{s}$. Moreover, by continuity of $\frac{\partial^2 Q}{\partial s^2}$, there exists some $K>\overline{s}$ such that $Q(\cdot,u,v)$ is strictly convex on $[1,K)$. *Q.E.D.*

■

**Proof of Proposition 2.**

Recall that $m(s,u,v)=u[v+(1-v)P(s,u,v)]$. Therefore,

(*a*) the properties of the matching function $m(\cdot,u,v)$ with respect to $s$ are deduced from that of $P(\cdot,u,v)$ given in Proposition 1(*ii*).

(*b*) With some algebra and using Proposition 1 we get:

$$\begin{cases}\frac{\partial m(s,u,v)}{\partial v}=u[1-P(s,u,v)]+u(1-v)\frac{\partial P(s,u,v)}{\partial v}>0\\ \frac{\partial^2 m(s,u,v)}{\partial v^2}=-2u\frac{\partial P(s,u,v)}{\partial v}+u(1-v)\frac{\partial^2 P(s,u,v)}{\partial v^2}<0\end{cases}$$

proving that $m(s,u,\cdot)$ is increasing and concave with respect to $v$.

(*c*) With some algebra we get:

$$\begin{cases}\frac{\partial m(s,u,v)}{\partial u}=v+(1-v)\frac{\partial}{\partial u}[uP(s,u,v)]\\ \frac{\partial^2 m(s,u,v)}{\partial u^2}=(1-v)\frac{\partial^2}{\partial u^2}[uP(s,u,v)]\end{cases}$$

Simplifying by $(1-\theta)$, we deduce from (2) that $P(s,u,v)=1-\left[1-\frac{v}{s}\left(\theta+\theta^2+\cdots+\theta^s\right)\right]^s$, where $\theta=(1-\delta)(1-u)$. Fix $v$ and $s$ and let $R(u)\equiv uP(s,u,v)$. Clearly, $R(u)$ is a polynomial in $u$ of degree $2s+1$, with roots 0 and 1 (that is, $R(0)=R(1)=0$) and strictly positive on $(0,1)$ (that is, $R(u)>0$, $\forall 0<u<1$). Therefore, $R'(u)=u\frac{\partial P(s,u,v)}{\partial u}+P(s,u,v)$ is a polynomial of degree $2s$ that has a unique root $\widetilde{u}\in(0,1)$ corresponding to the global maximum of $R$ on $[0,1]$. From $R'(u)$ continuous and $R'(0)=P(s,0,v)>0$ we deduce that $R'(u)>0$ on $(0,\widetilde{u})$ and that $R''(u)$ is negative locally around $\widetilde{u}$ that is, $R''(u)<0$ on $(\widetilde{u}-\varepsilon,\widetilde{u}+\varepsilon)$ for some $\varepsilon>0$. We also deduce from $R''(u)=u\frac{\partial^2 P(s,u,v)}{\partial u^2}+2\frac{\partial P(s,u,v)}{\partial u}$ and Proposition 1$(ii)$ that $R''(0)=2\frac{\partial P(s,u,v)}{\partial u}\mid_{u=0}<0$. If $R''(u)$ were to change sign on $[0,\widetilde{u}]$, by continuity of $R''$ and because both $R''(0)<0$ and $R''(\overline{u})<0$, it would imply that $R''(u)$ had two distinct roots on $(0,\widetilde{u})$, which is impossible because successive derivatives of polynomials have nested roots, and $R'(u)$ has only one root on $[0,1]$. Therefore, $R''(u)<0$ on $[0,\widetilde{u}]$. Let $\overline{u}=\arg\max\{u\in[0,1]\mid R'>0 \text{ and } R''<0 \text{ on } [0,u]\}$. Clearly, $0<\widetilde{u}\leq\overline{u}\leq 1$. ■

**Lemma 1** *The hiring probability $h(s,u,v)=\frac{m(s,u,v)}{u}$ is decreasing and convex in $u$ and increasing and concave in $v$. The properties of $h(\cdot,u,v)$ with respect to $s$ are the same than that of $P(\cdot,u,v)$.*

**Proof.** Recall that $h(s,u,v)=v+(1-v)P(s,u,v)$. With some algebra and using Proposition 1 we get:

$$\begin{cases} \frac{\partial h(s,u,v)}{\partial u}=(1-v)\frac{\partial P(s,u,v)}{\partial u}<0 \\ \frac{\partial^2 h(s,u,v)}{\partial u^2}=(1-v)\frac{\partial^2 P(s,u,v)}{\partial u^2}>0 \\ \frac{\partial h(s,u,v)}{\partial v}=1-P(s,u,v)+(1-v)\frac{\partial P(s,u,v)}{\partial v}>0 \\ \frac{\partial^2 h(s,u,v)}{\partial v^2}=-2\frac{\partial P(s,u,v)}{\partial v}+(1-v)\frac{\partial^2 P(s,u,v)}{\partial v^2}<0 \end{cases}$$

which completes the proof. ■

**Lemma 2** *The filling probability $f(s,u,v)=\frac{m(s,u,v)}{v}$ is increasing in $u$ and decreasing in $v$. The properties of $h(\cdot,u,v)$ with respect to $s$ are the same than that of $P(\cdot,u,v)$.*

**Proof.** Recall that $f(s,u,v)=u\left[1-\left(1-\frac{1}{v}\right)P(s,u,v)\right]$. With some algebra and using Proposition 1 we get:

$$\begin{cases} \frac{\partial f(s,u,v)}{\partial u}=\frac{f(s,u,v)}{u}-u\left(1-\frac{1}{v}\right)\frac{\partial P(s,u,v)}{\partial u}>0 \\ \frac{\partial f(s,u,v)}{\partial v}=-\frac{u}{v^2}P(s,u,v)-u\left(1-\frac{1}{v}\right)\frac{\partial P(s,u,v)}{\partial v}<0 \end{cases}$$

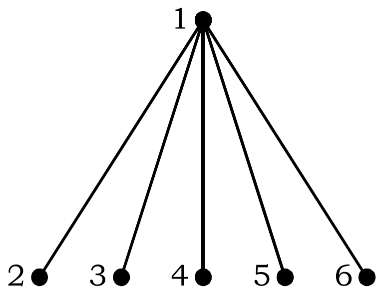
which completes the proof. ■

**Proof of Proposition 3.**

Fix the network size $s$. We first prove that along the Beveridge curve, $u$ is decreasing in $v$. Indeed, let $(u,v)$ and $(u',v')$ both satisfying (7) with $v'>v$. By definition, $m(s,u,v)=\delta(1-u)$ and $m(s,u',v')=\delta(1-u')$. Suppose that $u'\geq u$. Then, $m(s,u',v')\leq m(s,u,v)$. But we deduce from Proposition 2 that $m(s,u,v)<m(s,u,v')\leq m(s,u',v')$ which yields to a contradiction. Therefore, $u'<u$. We now prove that along the curve in the plane $(u,v)$ obtained from the free entry condition (6), $u$ is increasing in $v$. Indeed, from the implicit function theorem we get: $\frac{dv}{du}=-\frac{\frac{\partial(m/v)}{\partial u}}{\frac{\partial(m/v)}{\partial v}}>0$ according to Lemma 2. If a labor market equilibrium exists on $[0,\overline{u}]\times[0,1]\subseteq[0,1]^2$, it is thus unique. We now prove existence. At $v=1$, $m(s,u,1)=u$. We deduce from (7) that $\left(\frac{\delta}{1+\delta},1\right)$ belongs to the Beveridge Curve and from (6) that $\left(\gamma\frac{r+\delta}{y-w},1\right)$ satisfies the free entry condition (which requires that $\gamma\frac{r+\delta}{y-w}\leq 1$). A necessary and sufficient condition for an equilibrium to exist is thus $\gamma\frac{r+\delta}{y-w}>\frac{\delta}{1+\delta}$. Clearly, when $\gamma\frac{r+\delta}{y-w}\leq\overline{u}$, the equilibrium is unique. ■

**Proof of Proposition 4.**

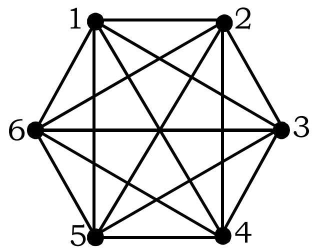
Suppose first that $s<\overline{s}$. Let $(u,v)$ on the Beveridge Curve, thus satisfying (7), and let $s'$ such that $s<s'<\overline{s}$. We know from Proposition 2 that $m(s,u,\cdot)$ increases with $v$ and that $m(s',u,v)>m(s,u,v)$. Therefore, if we keep $u$ constant while increasing the network size from $s$ to $s'$, the vacancy rate adjusts by decreasing. As a result, the Beveridge Curve (that decreases on the plane $(u,v)$) shifts downwards. Let now $(u,v)$ satisfy (6). We know from Lemma 2 that $f(s,u,v)=\frac{m(s,u,v)}{v}$ is an decreasing function of $v$ and that $f(s',u,v)>f(s,u,v)$. Therefore, the vacancy rate adjusts by increasing and the curve associated to the free entry condition shifts upwards on the plane $(u,v)$. One can check geometrically that $u^*(s')<u^*(s)$. Suppose now that $s\geq\overline{s}$ and let $s'>s$. Following a similar reasoning it is straightforward to see that the Beveridge Curve now shifts upwards while the free entre condition curve shifts downwards, implying that $u^*(s')>u^*(s)$. ■




$P_1 = 1 - [1 - \iota\theta]^5$

$P_2 = \ldots = P_6 = \iota\theta(1 - \theta^5)/5(1 - \theta)$

**Figure 1a** Star centered on 1 (n = 6).




$P_1 = \ldots = P_6 = 1 - [1 - \iota\theta(1 - \theta^5)/5(1 - \theta)]^5$

**Figure 1b.** Complete graph (n = 6).

Figure 1: Two Examples of Networks

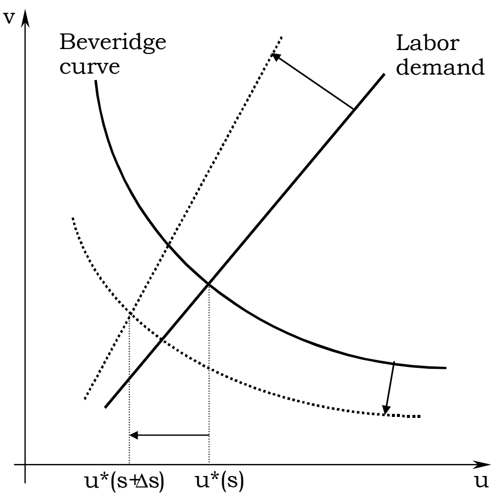



**Figure 2a**. Sparse networks ($s < \underline{s}$).

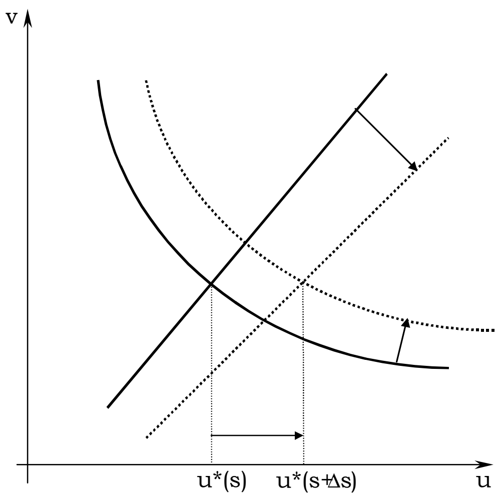



**Figure 2b.** Dense networks ($s > \underline{s}$).

Figure 2: Equilibrium Unemployment Level