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ABSTRACT

Universal Service and Entry: the Role of Uniform Pricing and Coverage Constraints *

Universal service objectives are pervasive in telecommunications, and have gained new relevance after liberalization and the introduction of competition in many markets. Despite their policy relevance, little work has been done allowing for a thorough discussion of instruments designed to achieve universal service objectives under competition. We intend to fill this gap, and consider various policy instruments, such as constraints on pricing and coverage. It is shown that these are not competitively neutral and may have far-reaching strategic effects. Equilibrium coverage of both incumbent and entrant may be lower than without regulation, and firms may even (non-cooperatively) leave each others' markets to lessen competitive pressure in their remaining markets. These effects depend on which measures are imposed at the same time, thus no single measure can be evaluated in isolation. We also point out that different groups of consumers are affected in different ways, making welfare comparisons difficult.

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NON-TECHNICAL SUMMARY

Consumer access to telecommunications services has been an important issue for a long time, and in the 'information society' being able to communicate and access information is more decisive than ever. Ideally, everyone should have telephone access: This goal has been termed *universal service*. There are several reasons why governments and regulators may want to pursue the goal of universal service – reasons of equity, of economic development, and even of economic efficiency (if there are sizeable network externalities).

For a long time, in order to attain the objective of universal service, restrictions were imposed on incumbent carriers, referred to as *universal service obligations* (USOs). 'Universal service' as it is now understood refers to achieving at least a 'minimum quality level' of a 'basic package' of telecommunications (or other) services to all consumers and at 'affordable prices'. We will not offer a new definition, but rather study the actual impact of USOs after the introduction of competition.

Recent changes in technology and regulatory philosophies have been the engines of huge transformations in the telecommunications sector as well as in other public utilities industries. Cost characteristics have made feasible greater reliance on *competition* instead of direct regulation in many market segments. These policies coexist with the concern that services must also be supplied to less profitable segments of the market, which had been subject to cross-subsidization prior to the advent of competition. Since competition and universal service requirements based on cross-subsidies are at odds with each other, universal service policies gain new dimensions. First, they must be redefined to pursue the previous goals of guaranteeing a basic service in the new environment. Second, their design must explicitly take account of their impact on competition. One stated aim is to devise policies that are 'competitively neutral', i.e. do not influence competition and let the market determine the efficient allocation of services. This may neglect the facts that telecommunications markets are subject to the existence of access bottlenecks, and characterized by a small number of networks.

We argue that existing theoretical analyses do not address the issue of universal service objectives and instruments in an entirely satisfactory way. The traditional way of thinking of USOs typically concentrates on net avoided costs (NAC), i.e. the total cost savings that the incumbent could get by withdrawing from loss-making areas. This was a valid approach with the stable (monopoly) market structure of the past. As long as the incumbent remains the sole supplier in a rural area and technology is the same, the calculation of NAC is invariant to the market structure in other areas. An approach that focuses only on NAC would miss the important linkages between the prices set by the incumbent when facing entrants in *other* areas.

On the contrary, all policy instruments, be they (pricing or coverage) constraints imposed by regulators, or financing mechanisms created to alleviate additional costs caused by universal service obligations, have strategic effects, and therefore do affect competition. We take a first step in this direction by analysing the effects of pricing and coverage constraints, which are already quite rich and sometimes unexpected.

Constraints on prices may take the form of *uniform pricing* (UP), which forces the firms to offer their service at a geographically uniform price to all their consumers, or a *price cap*, which establishes a maximum average price of a firm's services. Firms (most likely the incumbent) may also be obliged to cover at least a given area, i.e. to have a *coverage constraint* (CC) imposed upon them. These constraints have direct effects in terms of lowering firms' profits. More importantly, they may profoundly change the nature of competition because the strategic interaction between firms changes. On the one hand, regulation may make firms more defensive or more aggressive (or both, as we will explain below), and on the other hand, may create strategic links between hitherto unrelated markets. These various policies interact and may lead to quite different results depending on their combination. Therefore they cannot be evaluated in isolation but must be analysed when imposed together. We perform a positive analysis, indicating the type of interactions and the winners or losers on the sides of firms and groups of consumers.

In our analysis, coverage choices play a central role. In particular we show that price competition is affected critically by *relative* coverage, i.e. the ratio between the entrant's coverage and the incumbent's. This aspect is neglected in the existing literature and represents our main contribution to the problem of USOs.

Our results are as follows: the uniform pricing constraint creates strategic links across markets; price will be a compromise between a low price where the incumbent competes and a high price where the incumbent is a monopolist. Therefore the price of the incumbent will in general be higher than the one of the entrant (if their costs are the same), i.e. the incumbent will be less aggressive. We also show that if both incumbent and entrant can choose their coverage, under the UP constraint, coverage of both incumbent and entrant can be *smaller* than without this constraint. If in addition to the uniform pricing constraint the incumbent is subject to a coverage constraint, the entrant's chosen coverage increases in the mandated coverage of the incumbent. While this is good in principle since more customers will be able to select services from alternative providers, we demonstrate that prices *increase* as a consequence, hence welfare of previously served consumers falls. This is a typical example of a potential unintended consequence of poorly designed (or poorly understood) USOs.

The fundamental result emerging from these considerations is that there are clear trade-offs between larger coverage and higher welfare of served customers, and between the welfare of customers in markets with competition or monopoly. Uniform pricing distributes the benefits of competition in the form of lower prices to customers who will not be served by the entrant. On the other hand it increases the prices of duopoly customers and can lower the coverage of both firms. Higher coverage imposed by the regulator naturally raises the number of customers, but previous customers lose welfare due to higher prices. This last effect arises *not* because the incumbent wants to recoup the additional fixed cost (these are sunk), but because the larger coverage makes the incumbent more accommodating, and therefore competition will be less effective in bringing prices down.

1 Introduction

Consumer access to telecommunications services has been an important issue for a long time, and in the "information society" being able to communicate and access information is more decisive than ever. Ideally, everyone should have phone access, therefore this goal has been termed *universal service*. There are several reasons why governments and regulators may want to pursue the goal of universal service, reasons of equity, of economic development, and even of economic efficiency (if there are sizeable network externalities).[1]

For a long time, in order to attain the objective of universal service, restrictions were imposed on incumbent carriers, referred to as *universal service obligations* (USOs), which e.g. could be restrictions on prices, or the obligation to offer certain services in certain geographic areas. Losses caused by these restrictions were commonly financed by enormous cross-subsidies from long-distance and business services.[2]

Recent changes in technology and regulatory philosophies have been the engines of huge transformations in the telecommunications sector as well as in other public utilities industries. Cost characteristics have made feasible greater reliance on competition instead of direct regulation in many market segments. These policies coexist with the concern that services must also be supplied to less profitable segments of the market. Since competition and universal service requirements based on cross-subsidies are at odds with each other, in this new scenario universal service policies gain new dimensions: First, they must be redefined to pursue the previous goals of guaranteeing a basic service in the new environment; second, their design must explicitly take account of their impact on competition. One stated aim is to devise policies that are "competitively neutral", i.e. do not influence competition and let the market determine the efficient allocation of services. This may neglect the fact that telecommunications markets will be subject to the ex-

[1] For a discussion of these and other arguments see Cremer *et al.* (1998).

[2] The exact definition of "universal service" is less than clear. In the beginning, the term referred to linking up the independent local networks which had developed at the turn of the century in the US. Later it changed its meaning as the US market was monopolized by AT&T (Mueller, 1997). "Universal service" as it is understood now refers to achieving at least a "minimum quality level" of a "basic package" of telecommunications (or other) services to all consumers and at "affordable prices" (as defined by the Federal Communications Commission in its CC Docket 96-45 or the European Commission in its communication COM(96) 73). In the implementation of this definition, "affordable prices" have two dimensions: Either the price *level* may be regulated, or the *spread* in prices for different consumers or different locations may be controlled through "averaged" tariffs, or both. We will not offer a new definition, but rather study the actual impact of USOs after the introduction of competition.

istence of access bottlenecks (as discussed in great length in Laffont and Tirole, 2000). Telecommunications markets are also generally characterized by a small number of networks, so that the resulting competition will be oligopolistic. In our work we will concentrate on the second issue and do not discuss questions related to access.[3]

We argue that existing theoretical analyses do not address the issue of universal service objectives and instruments in an entirely satisfactory way. The traditional way of thinking of USOs typically concentrates on net avoided costs (NAC), i.e. the total cost savings that the incumbent could get by withdrawing from loss-making areas. This was a valid approach with the stable (monopoly) market structure of the past. As long as the incumbent remains the sole supplier in a rural area and technology is the same, the calculation of NAC is invariant to the market structure in other areas. An approach that focuses only on NAC, would miss the important linkages between the prices set by the incumbent when facing entrants in *other* areas. On the contrary, all policy instruments, be it (pricing or coverage) constraints imposed by regulators, or financing mechanisms created to alleviate additional costs caused by universal service obligations, have strategic effects, and therefore do affect competition. We do a first step in this direction by analyzing the effects of pricing and coverage constraints, which are already quite rich and sometimes unexpected.

In this work we analyze universal service obligations as a form of regulation that puts constraints on a firm's strategy space, in particular on prices and coverage. Constraints on prices may take the form of *uniform pricing* (UP), which forces the firms to offer its service at geographically uniform price to all its consumers, or a *price cap*, which establishes a maximum average price of a firm's services. Firms (most likely the incumbent) may also be obliged to cover at least a given area, i.e. be imposed a *coverage constraint* (CC). These constraints have direct effects in terms of lowering firms' profits. More importantly, they may profoundly change the nature of competition because the strategic interaction between firms changes. On the one hand, regulation may make firms more defensive or more aggressive (or both, as we will explain below), and on the other hand, may create strategic links

[3] The interaction between entry, access and universal service is analyzed by Armstrong (2001). He argues that a retail instrument (such as a universal service fund) should be used to address retail-level distortions caused by USOs (such as geographically uniform retail tariffs), while wholesale instruments should be used to address productive inefficiencies (which means that access charges should be cost based). In the absence of bypass opportunities, this boils down to the Efficient Component Pricing Rule, split into two parts. In his analysis, strategic interaction between players is absent since retail prices are always determined exogenously.

between hitherto unrelated markets. These various policies interact and may lead to quite different results depending on their combination. Therefore they cannot be evaluated in isolation but must be analyzed when imposed together. We perform a positive analysis, indicating the type of interactions and the winners or losers on the sides of firms and groups of consumers.

A uniform pricing constraint on firms has the appealing property that the regulator needs no information whatsoever to impose it, while a price cap involves information about technology and its evolution, and demand. The uniform pricing constraint creates strategic links across markets: price will be a compromise between a low price where the incumbent competes, and a high price where the incumbent is a monopolist. Therefore the price of the incumbent will in general be higher than the one of the entrant (if their costs are the same), i.e. the incumbent will be less aggressive. We also show that if both incumbent and entrant can choose their coverage, under the UP constraint, coverages of both incumbent and entrant can be *smaller* than without this constraint.

If in addition to the uniform pricing constraint the incumbent is subject to a coverage constraint, the entrant's coverage increases in the mandated coverage of the incumbent. While this is good in principle since more customers will be able to select services from alternative providers, we demonstrate that prices *increase* as a consequence, hence welfare of previously served consumers falls. This is a typical example of a potential unintended consequence of poorly designed (or poorly understood) USOs.[4]

The fundamental result emerging from these considerations is that there are clear trade-offs between larger coverage and higher welfare of served customers, and between the welfare of customers in markets with competition or monopoly. Uniform pricing distributes the benefits of competition in the form of lower prices to customers who will not be served by the entrant. On the other hand it increases the prices of duopoly customers and can lower the coverage of both firms. Higher coverage imposed by the regulator naturally raises the number of customers, but previous customers lose welfare due to higher prices. This last effect arises *not* because the incumbent wants to

[4]We can cite other examples of discrepancies between the intentions of the regulator and their practical implementation with respect to USOs. Crandall and Waverman (2000) argue that there is no real economic need for the extensive distortions built into the rate structures of most developed countries, particularly the US. In a similar vein, Rosston and Wimmer (2000), using data for the US, challenge the myth of affordability by showing that the elimination of subsidies would have a only mild impact on the size of the network. They also show that universal service programs fail with respect to horizontal equity considerations, since net recipients of subsidies include rich households living in suburban areas, while losers include a disproportionate percentage of poor, Black and Hispanic households living in urban areas.

recoup the additional fixed cost (these are sunk), but because the larger coverage makes the incumbent more accommodating, and therefore competition will be less effective in bringing prices down.

The idea of this paper is closely related to the more general issue of multimarket oligopoly (Bulow *et al.,* 1985). Since coverage choices are made before (possibly uniform) prices are set, this analysis is also related to the role of commitment to future actions as a means to influencing rival's behavior (see Bagwell and Wolinsky, 2000 for a recent survey). There are two papers that are close to the present analysis. Choné *et al.* (2000) and Anton *et al.* (1998) also consider USOs and their implications on a firm's strategy space. They obtain results in a spirit similar to ours on the strategic links created through pricing restrictions, but their emphasis is more on the funding mechanism (Choné *et al.*, 2000) and on the strategic effects of subsidies, possibly attributed in an auction (Anton *et al.*, 1998). Both these papers have a simpler setting in term of the geography of a country (there are only 2 areas, a rural and an urban one), and are not suited to analyze coverage constraints which, on the contrary, are treated in this work.[5] In our analysis, coverage choices play a central role. In particular we show that price competition is critically affected by *relative* coverage, i.e. the ratio between the entrant's coverage and the incumbent's. This aspect is neglected in the existing literature and represents our main contribution to the problem of USOs. We would like to stress that while the telecommunications industry represents the main motivation for this paper, the ideas presented here are also relevant for other industries, including electricity, natural gas and postal services.

The rest of the paper continues as follows: In section 2 we introduce the model, and in section 3 we shortly discuss the pre-entry monopoly with and without regulation. Section 4 introduces a competitive benchmark, and analyses the equilibrium prices and coverages under a UP constraint. Price caps are discussed shortly in section 5. Section 6 deals with coverage constraints, with or without UP, and discusses their welfare properties. Since some equilibrium and welfare properties of the various combinations of USOs are ambiguous, in section 7 we analyze the equilibrium under a linear demand specification, in order to resolve such ambiguities at least in a relevant case. Section 8 concludes.

[5] Since the UP constraint that is typical of USOs effectively implies that firms are not allowed to price discriminate, Armstrong and Vickers (1993) is also related to the present paper. They analyse the effects of price discrimination when an incumbent firm faces a (non-strategic) entrant that may enter only in some parts of the market. They show how the responses to entry change according to whether the incumbent is allowed to charge discriminatory prices across areas.

2 The Model

There are two firms $i = 1, 2$, each one offering one type of telecommunications services (phone calls) which are imperfect substitutes, and a continuum $[0, \bar{x}] \subset \mathbb{R}_+$ of different locations ("local markets", towns, villages) to be served, where $\bar{x}$ is the size of the country. Each market has the same number of customers, but markets are ordered by fixed cost.[6] If firm i decides to enter a certain location x it has to pay the fixed cost associated to that location denoted as $c(x)$, where $c(0) = 0$ and $c'(x) > 0$. We assume that this fixed cost is the same for both firms.

At each location there is a mass 1 of identical (representative) consumers. If a consumer lives in an area which is supplied only by firm i at a price p_i, then his indirect utility is $v_m(p_i)$, where the subscript m stands for "monopoly". If on the other hand he lives in an area supplied by both firms, his indirect utility is $v_d(p_1, p_2)$, where d stands for "duopoly". Therefore the demand functions in a monopoly area and in a duopoly area are, respectively, $q_m(p_i) = -\partial v_m/\partial p_i$, and $q_i(p_1, p_2) = -\partial v_d/\partial p_i$. Demand functions are well-behaved and have compact support. In the duopoly areas demands are symmetric and goods are demand substitutes, while in the monopoly areas there exists a unique monopoly price.

Firms play a two-stage game where they first decide how many locations to serve, and pay the associated fixed costs, and then in the second stage they both set prices, having zero marginal production cost.[7] This captures the situation where firms first bring their network to each location, e.g. by installing fibre optics cables and switches, and then set call prices where the marginal transmission cost is negligeable.

We make the important assumption that each firm starts building its own network from the cheapest location and leaves no gaps between served locations. Therefore if firm i serves an area $[0, x_i]$, its total fixed investment cost will be $C(x_i) = \int_0^{x_i} c(x)\,dx$ which is convex since $C'' = c' > 0$. This assumption implies that only one type of asymmetric equilibrium can arise, where one firm is bigger than the other. We denote by 1 or "incumbent" the bigger firm, and by 2 or "entrant" the smaller firm. This assumption tries to capture the natural scenario where high-capacity networks are typically built starting from business districts and are then rolled out to more peripheral

[6] Alternatively, one may assume that fixed cost per location is constant while consumer density or consumers' willingness-to-pay decreases. The important assumption is that locations are ordered by decreasing profitability.

[7] The same analysis can be made for quantity competition, leading to qualitatively similar results to price competition. Still, restrictions on pricing seem less consistent with competition in quantites.

areas. It is also related to typical entry patterns where entrants tend to follow cream-skimming strategies, starting from the most profitable locations (where profitability can be defined either in terms of customer groups or by geographic characteristics).[8] As a matter of notation, we denote by $\pi_i(x)$ the profit obtained at a generic location x, while overall gross profit is $\Pi_i(x_i) = \int_0^{x_i} \pi_i(x)\,dx$.

3 Benchmarks

Useful benchmarks are given by the allocations that would be chosen by a monopolist or by a benevolent social planner. These are obtained from the following simple program:

$$\max_{p,x} v_m(p)\,x + (1+\lambda)\left[pq_m(p)\,x - C(x)\right]$$

The monopoly solution corresponds to the case $\lambda \to \infty$, while the first-best is obtained when $\lambda = 0$. In the latter situation, the firm would make losses. If no fixed transfers are available, then the social planner maximizes the unweighted sum of consumer surplus and firm's profits, subject to the firm's balanced budget constraint $pq_m(p)\,x \geq C(x)$. Hence the second-best solution is obtained from the same program, where λ is the endogenous Lagrange multiplier of the budget constraint. Given this remark on the interpretation of λ, the solution to the program for the three cases (monopoly, first- and second-best) is the following:

$$p = \frac{\lambda}{1+\lambda}\frac{q_m(p)}{-q_m'(p)}, \tag{1}$$

$$c(x) = \frac{v(p)}{1+\lambda} + pq_m(p). \tag{2}$$

In particular, we obtain for the monopoly allocation $p = p_m$, yielding profits π_m at each location. The monopolist covers the area $[0, x_m]$, where $c(x_m) = \pi_m$. At the marginal location fixed costs of serving this location are equal to gross profits, while gross profits exceed fixed costs at all other locations served. All locations where fixed cost exceed gross profits are not served.

[8] On the other hand, if we did allow for disjoint intervals, the analysis would be considerably more complicated because of the possibility of multiple equilibria. For instance, in the unregulated equilibrium (section 3) our assumption implies that the incumbent always covers the natural monopoly areas, while without it the identity of the firm at any single monopoly location is undetermined. We consider this as an interesting but separate question, see e.g. Hoernig (2001) on homogeneous goods and sequential entry.

In the first-best, the optimal price is $p^* = 0$ and coverage x^* given by $c(x^*) = v_m(0)$. Since $v_m(0) > \pi_m$ it follows immediately that $x^* > x_m$. However in the first-best the firm would be making losses equal to $C(x^*)$. Therefore in the second-best situation with binding budget constraint the Lagrange multiplier λ is strictly positive. It can be shown that the right-hand sides of equations (1) and (2) are, respectively, increasing and decreasing in λ, and the second-best solution involves "bracketing", $p^* < p^{**} < p_m$ and $x_m < x^{**} < x^*$.

In an unregulated monopoly the optimal price is the same at all locations, hence a uniform pricing constraint imposed by the government would not be binding. This feature of the model of course would be different if the marginal cost of production was not constant.[9] With constant marginal costs, the imposition of the UP constraint becomes relevant if the incumbent would otherwise offer lower prices at locations where he competes with the entrant, i.e. for strategic reasons.

4 Duopoly

4.1 Benchmark equilibrium

Let us consider first an unregulated duopoly. At each location the outcome is that of a local duopoly or monopoly. We make the following regularity assumptions on the profits at each location that are also needed for later use:

1. Per firm equilibrium duopoly profits are unique and lower than monopoly profits: $0 < \pi_d < \pi_m$.

2. Prices are strategic complements: $\partial^2 \pi_d(p_1, p_2) / \partial p_1 \partial p_2 > 0$.

3. Marginal revenue in a monopoly area exceeds marginal revenue in a duopoly area[10]:

$$A = \frac{\partial \pi_d(p_1, p_2)}{\partial p_1} < B = \frac{\partial \pi_m(p_1)}{\partial p_1}. \tag{3}$$

[9] Similarly, the imposition of a larger coverage through a coverage constraint would imply losses to the monopolist in the additional locations, but would not lead to higher prices. The reason is that the additional fixed costs are sunk and that the monopolist is already maximizing profits at each location.

[10] As a remark, one can interpret the demand function in a monopoly area as equivalent to the demand in a duopoly area when a rival firm charges at least the price $\bar{p}_j(p_i)$ where his demand falls to zero given p_i, $q_m(p_i) = q_i(p_i, \bar{p}_j(p_i))$. Then condition 3 is implied by condition 2.

At each duopoly location, firm i maximizes $(j \neq i)$

$$\pi_d(p_i, p_j) = p_i q_i(p_i, p_j), \tag{4}$$

resulting in equilibrium prices $p_1 = p_2 = p_d$ and profits π_d. In the first stage the entrant will serve all consumers in $\left[0, x_2^b\right]$ where $c\left(x_2^b\right) = \pi_d$. The incumbent makes profits π_d in duopoly and π_m in monopoly areas, hence his locations served are $[0, x_m]$ exactly as under monopoly.

In the absence of cross-market linkages competition and technology simply lead to some natural monopoly areas (served by the incumbent) and to some duopoly areas. The latter are characterized by low investment costs, and are made of a greater number of locations the more differentiated the products are, or equivalently, the less tough competition is.

4.2 Second-stage equilibrium under UP

We now describe the second-stage equilibria of the game, where given their coverages both the entrant and the incumbent set their prices. It is assumed that this decision is subject to a uniform pricing (UP) constraint imposed by the regulator, i.e. firms are not allowed to discriminate between consumers at different locations. This restriction is binding for the incumbent, since he would prefer to offer lower prices where he competes with the entrant, and set the monopoly price where he is alone.[11]

Both firms compete at locations x with $0 \leq x \leq x_2$, and the incumbent serves alone locations $x_2 < x \leq x_1$, while locations $x > x_1$ are not served.

In the second stage the incumbent and the entrant solve

$$\begin{aligned} \max_{p_1} \Pi_1 &= x_2 \pi_d(p_1, p_2) + (x_1 - x_2)\, \pi_m(p_1), \\ \max_{p_2} \Pi_2 &= x_2 \pi_d(p_2, p_1), \end{aligned}$$

respectively. At each duopoly location, firms have the same profit function π_d since demand *functions* are the same, but actual demands differ because they charge different prices. We have $\pi_m(p_1) > \pi_d(p_1, p_2)$ for all p_2, and under reasonable assumptions $\pi_m(p_1) > \pi_d(p_1, p_2) + \pi_d(p_2, p_1)$ holds as well.

[11] It should be noticed that the assumption of UP on the entrant is irrelevant in the present context because the entrant would always be active in duopoly areas with identical customers, so he will always adopt the same pricing policies everywhere. On the other hand, if for some reason the entrant can become the only supplier over some areas, then this assumption would play an important role (see section 7 and Hoernig 2001).

Necessary first-order conditions for an interior Nash equilibrium are

$$\begin{aligned} x_2\left(q_1+p_1\frac{\partial q_1}{\partial p_1}\right)+(x_1-x_2)\left(q_m+p_1\frac{\partial q_m}{\partial p_1}\right) &= x_2A+(x_1-x_2)\,B=0, \\ x_2\left(q_2+p_2\frac{\partial q_2}{\partial p_2}\right) &= 0. \end{aligned}$$

The following lemma characterizes the pricing equilibria in the second-stage game (all the proofs are in the Appendix):[12]

Lemma 1 *Equilibrium prices and profits in the second stage of the game depend only on relative coverage $k = x_2/x_1 \leq 1$. Under uniform pricing (UP) there is price bracketing $p_d < p_i^{UP} < p_m$, $i = 1, 2$. Prices are decreasing in relative coverage k. This is also true at any given location for the profits of firm 2, and for the profits of firm 1 in the monopoly areas. On the other hand, if k is small or if goods are sufficiently differentiated, the profit of firm 1 in the duopoly area may increase with k.*

Under UP the incumbent has to find a common price over different markets. Competition in the duopoly areas is relaxed relative to the unconstrained situation. This is easily understood by noting that if the incumbent did set the unconstrained duopoly price everywhere, this would impose a huge penalty on its own monopoly locations. When relative coverage increases, differences between the firms decrease. With a higher relative coverage the incumbent depends relatively more on the duopoly areas and is therefore willing to compete for market share by setting a lower price. The incumbent is made "tougher" and this results in an inward shift of its reaction function. This direct effect has an indirect strategic impact on the rival's price due to his upward sloping best reply function.

4.3 Equilibrium coverage under UP

We now analyze the coverage choices of both the incumbent and the entrant in the first stage. Firms simultaneously solve

$$\max_{x_i} \Pi_i\left(x_2/x_1\right) - C\left(x_i\right) \tag{5}$$

to determine their optimal coverage.[13]

[12] Existence of equilibria is assured by strategic complementarity and compactness. We do not deal here with the additional issue of unicity, and simply assume it. This would follow from standard additional assumptions on the slope of reaction functions.

[13] Alternatively, one could assume that entry is sequential. For the case of differentiated goods the results are qualitatively the same. This is not so in the case of homogeneous goods.

From the previous section we know that at each location local profits depend only on relative coverage, $\pi_1(k)$ and $\pi_2(k)$ in the duopoly areas, and $\pi_m(k)$ in the monopoly areas. In order to characterize the best responses of the firms in terms of coverage we need to introduce additional assumptions on local profits. In particular we make the following technical assumptions:

1. The elasticity of the local marginal revenue of the entrant $\pi_2'(k)$ with respect to relative coverage is either positive or not "too" negative: $\frac{k\pi_2''}{\pi_2'} > -2$.

2. The elasticity of the difference between local marginal revenues of the incumbent $f(k) = \pi_1' - \pi_m'$ is either positive or not "too" negative: $\frac{\partial f}{\partial k}\frac{k}{f} > -2$.

Assumption 1 is better understood by noting that it implies that revenue $x_2\pi_2(x_2/x_1)$ is concave in x_2. Hence it rules out pathological cases where the entrant would always match the incumbent's coverage. Imagine a situation where entry costs are very low everywhere. Under Assumption 1, the entrant would still prefer to have a coverage lower than the incumbent in order not to depress prices too low. On the other hand, he would match the incumbent if the products were independent. Assumption 1 can then be understood as ruling out cases where products are "too" *differentiated*. If products were extremely differentiated, there would be very little strategic interaction and the whole exercise would be meaningless.

Assumption 2 effectively means that products are not too *homogeneous*. As relative coverage increases, price differences between the two firms decrease. However, so long as products are sufficiently heterogenous the marginal revenue in the duopoly areas does not fall too sharply. Therefore it is optimal for the incumbent to lower price to fight for market share (instead of resigning himself to a small market share at a high price).

The following proposition characterizes the best responses in terms of coverage:[14]

Proposition 1 *For the incumbent coverages are strategic substitutes, while for the entrant they are strategic complements,*

$$\frac{dx_1^{inc}}{dx_2} < 0,\ 0 < \frac{dx_2^{ent}}{dx_1} < k. \tag{6}$$

The incumbent covers less than x_m, while coverage of the entrant may be higher or lower than the benchmark x_2^b.

That is, if the entrant increases its coverage, the incumbent serves less high-cost areas. If x_2 increases then the incumbent's profits are reduced because some monopoly areas are substituted for less profitable duopoly areas.

[14] In the proof of the proposition we also show that an equilibrium exists.

However, by itself this should cause no change on the incentive to supply marginal locations that belong to the high-cost monopoly areas. What matters is that an increase in x_2 reduces relative coverage and pushes down prices everywhere. For this reason, the incumbent prefers to serve a lower number of high-cost locations. This is the main effect, although the response of the incumbent is dampened by the fact that a reduction in own coverage would push relative coverage k even further up, damaging profits on all infra-marginal locations.

On the other hand, the entrant optimally chooses a higher coverage when the incumbent's coverage increases. A large incumbent sets prices less aggressively because he depends more on monopoly areas. The associated increase in prices makes more locations become profitable in duopoly. Hence the entrant raises its coverage, even though this is counterbalanced by an effect analogous to the one outlined before (higher k).

Therefore in equilibrium under the uniform pricing constraint the coverage of the incumbent is always lower than the monopoly coverage without such a constraint, while the entrant's coverage may be higher or lower than the benchmark duopoly coverage.

5 Price caps

In this section we consider a different form of price regulation, namely the imposition of an upper limit P (price cap) on the incumbent. In general, price caps are understood as a limit on some weighted average of the prices of a basket made of different services. In the present context, consumers are identical and just differ in their locations, hence a weighted average boils down to a simple price ceiling. If we had different consumers at each location, e.g. business and residential users, then the two notions would not coincide. Armstrong and Vickers (1993) show that in this case the incumbent may set a very low price (even below marginal cost) in the duopoly area in order to charge higher prices in the monopoly area.

Relatively to the unregulated benchmark, it is easy to see that the imposition of a price cap alone has no effect on the price and size of the duopoly region, while lowering the coverage of the incumbent if $p_d < P < p_m$. If the price cap is particularly tight, $P < p_d$, then both firms choose a lower coverage.[15]

[15] This result holds whether or not the price cap is imposed on the entrant, too. If it is imposed as well, equilibrium profits and duopoly coverage decrease even further. This remark applies also to the analysis that follows.

Imagine now a price cap is imposed together with a UP constraint. The strategic links between the duopoly and monopoly markets that we have highlighted in the previous sections are now cut. In fact, if the price cap is binding for the incumbent, then the incumbent's best reply is to set $p_1 = P$ everywhere. The entrant's best reply does not depend on coverage, but only on the incumbent's price (which is equal to the price cap). So the entrant's choice is either to set the best reply to the price cap, or adopt the cap if it is imposed on himself and is binding.

Let us turn to the determination of coverage. Compared to the unregulated benchmark, a joint price cap and UP lead to the same incumbent's coverage as a price cap alone. The entrant's coverage depends only on whether the price cap is binding for him or not. If $P > p_d$ then it is not binding since his best reply p_2 is such that $p_d < p_2 < P$, and therefore profits and coverage are higher than in the unregulated case. If on the other hand $P < p_d$, profits and coverage are lower, even more so if the price cap is also imposed on the entrant.

As compared to the equilibrium under UP alone, the imposition of an additional price cap produces to opposite effects. Imagine a price cap is imposed on the incumbent starting at the (privately chosen) equilibrium price level under UP alone. The incumbent now will serve less locations. There is a direct effect of lower profits due to the binding cap that makes the incumbent serve less locations. This reduction is reinforced by the fact that he cannot benefit from the strategic incentive to increase x_1 in order to reduce relative coverage and increase prices everywhere. In the same vein, the entrant will now serve a larger area, because his strategic incentive to be small has disappeared: If the entrant increases x_2, he will not punish himself through lower prices. What we have just described is valid for a small departure from the UP equilibrium. If the price cap is very tight both coverages will go down.

6 Coverage constraints and welfare considerations

In this section we turn our discussion to the imposition by the regulator of a coverage constraint (CC) on the incumbent. This kind of constraint specifies a minimum area that he is obliged to serve. First of all we note that if CC is imposed alone, the only effect is that of the corresponding increase in the monopoly coverage (with the incumbent making losses in the new locations), while prices and the entrant's coverage do not change with respect to the unregulated benchmark. This scenario does not change qualitatively if a price cap is imposed. Competition remains localized since the coverage constraint by itself does not create links between the local markets.

The more interesting case that may arise is when CC and UP are imposed together. The cross-market linkages created by the UP constraint imply that equilibrium variables are affected by relative coverage. By interfering with the coverage choices via a CC, there will be additional strategic effects that have an impact on all consumers through prices.

Recall from section 4.3 that the entrant's coverage best response function is increasing. Hence if a CC is imposed on the incumbent, this leads to a higher coverage by the entrant as well. This is in principle beneficial for consumers, since now more consumers are served overall, and more consumers have the choice between the incumbent and the entrant in duopoly locations. However, there is also an important strategic effect. The coverage of the entrant increases less than proportionately than the incumbent's, and relative coverage actually decreases with an increase in the incumbent's coverage ($dx_2/dx_1 < k$). This decrease in relative coverage causes prices to increase everywhere. Hence existing customers may actually loose from the adoption of CC on top of UP.

In order to conduct a welfare analysis, we draw a distinction between local and aggregate consumer surplus in the monopoly and duopoly areas as follows: Let $v_d(k) = v_d(p_1(k), p_2(k))$ and $v_m(k) = v_m(p_1(k))$ denote consumer surplus in duopoly and monopoly areas, respectively, while aggregate surplus in the groups of areas is denoted as $V_d(x_1, x_2) = x_2 v_d(k)$ and $V_m(x_1, x_2) = (x_1 - x_2) v_m(k)$. The welfare impacts of a binding CC are described in the following proposition:

Proposition 2 *Imagine the incumbent has to satisfy both a UP and a CC constraint. Compared to the situation where only UP is imposed, prices in all markets and coverages are higher. Consumers that do not change supplier are negatively affected. However, in aggregate consumers in monopoly and duopoly areas benefit if fixed costs increase slowly. Total profits of the incumbent decrease, while the entrant's profits increase.*

We see that there is a complex trade-off between achieving high coverage and higher value for served customers. The unintended outcome of imposing a minimum coverage on the incumbent at the same time as uniform pricing constraints is less intense competition in the marketplace.[16]

[16] A binding price cap would avoid this because the incumbent would not be able to raise its price as a response to a mandated higher coverage. Obviously, the more instruments the regulator has at its disposal, the higher social welfare could be. The downside of having

7 The Linear case

In this work we have shown how the notion of a USO, interpreted as a constraint on the pricing policies of an incumbent operator, introduces cross-market linkages and has a strong impact on the strategic interaction among firms in all markets. We have been able to obtain general results, under assumptions that we claimed to be realistic.

In this section, we briefly sketch the equilibrium for a particular case: Linear demand functions and linear marginal costs. We do this for two reasons. First of all, we assure that the assumptions we adopted in the previous sections hold true for a relevant case that is used widely in the literature. Secondly, we are able to characterize explicitly the equilibrium thus resolving, at least for this relevant case, the direction of some welfare effects that could not be signed unambiguously under more general circumstances.

The utility function that we adopt is quasi-linear, and takes the following functional form:[17]

$$U(q_1, q_2) = q_1 + q_2 - \frac{1}{2}q_1^2 - \frac{1}{2}q_2^2 - bq_1q_2,$$

where $b \in [0, 1]$ is the coefficient of product differentiation. For $b = 1$ goods are homogeneous, while for $b = 0$ they are completely independent. This gives rise to linear demand functions $q_i(p_1, p_2) = \frac{1-b-p_i+bp_j}{1-b^2}$ in duopoly areas. In monopoly areas demand becomes $q_m(p_i) = 1 - p_i$. On the cost side, we assume that $C(x_i) = \frac{1}{2}x_i^2$, or equivalently, $c(x_i) = x_i$.

Given these specifications, we are able to prove that all the assumptions we made before are indeed satisfied.[18] Moreover, we can prove that if goods are sufficiently differentiated ($b \leq \sqrt{3} - 1$), a subgame-perfect equilibrium in pure strategies under UP exists. An example of the reaction functions in coverages in the first stage is reported below in figure 1.

a high number of instruments is that it creates a very intrusive regulatory environment. This is why we deem particularly useful to analysise a type of intervention that is both realistic and enforceable, such as UP and CC constraints that we analyze in this paper.

[17] See Singh and Vives (1984).

[18] The calculations are available from the authors on request.

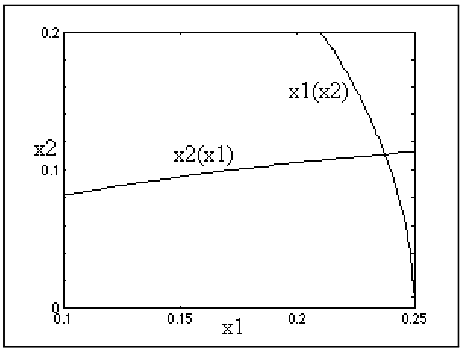

Figure 1: Coverage best reactions under UP ($b = 0.65$).

In Proposition 1 we showed that under UP the incumbent would always set a coverage below the level in the unregulated benchmark. This is confirmed under the linear specification, where it also turns out that the entrant sets a lower coverage, too. This is shown in figure 2, where the dotted lines refer to the unregulated case.

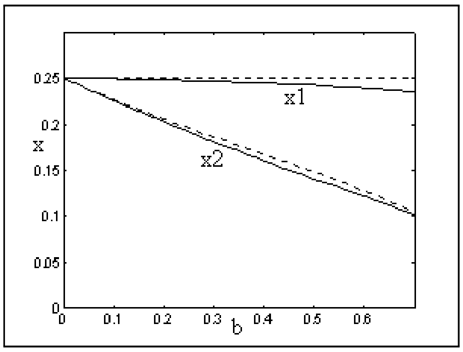

Figure 2: Equilibrium and unregulated benchmark coverages under UP.

Table 1 summarizes most of the results that we have obtained in this paper. The comparisons are between UP and the competitive unregulated benchmark (third column), and UP alone with the situation where CC is imposed on top (fourth column). For example, the entrant's coverage under UP alone is lower than in the unregulated benchmark and also lower than under UP+CC. The results that we could only prove in the linear case are in italics. It can be seen from the table how all the general results do translate to the linear case. An additional important effect that is obtained for the linear case is that the imposition of a UP constraint would decrease the coverage of both firms. Also notice that in the linear case, the imposition of CC on top of UP would benefit consumers in aggregate both in monopoly and duopoly areas. On the other hand, a policy that introduces UP alone makes duopoly customers worse off in aggregate relative to the unregulated benchmark: the entrant serves less customers and prices increase. The impact of UP on aggregate consumers surplus in monopoly areas depends on two opposite tendencies: There are relatively fewer monopoly areas but customers pay less than the unregulated price. The net effect is positive.

What	Where / Who	UP compared to: Unregulated benchmark	UP compared to: UP + CC
Coverage	Entrant Incumbent	*lower* lower	lower lower
Relative cov.		*lower if b small*	higher
Prices	Duopoly Monopoly	higher lower	lower lower
Consumer surplus	Duopoly Monopoly	lower higher	higher higher
Aggregate cons. surplus	Duopoly Monopoly	*lower* *higher*	*lower* *lower*
Profits per local market	Duopoly Monopoly	higher lower	lower lower
Total profits	Entrant Incumbent	higher lower	lower higher

As a side remark, note that when goods are sufficiently homogeneous ($b > \sqrt{3} - 1$) there would be equilibria possibly involving mixed strategies in the pricing game. These mixed equilibria arise because quantities cannot be negative, and arise due to the finite choke-off price of the linear demand specification. If goods are sufficiently homogeneous the incumbent may not want to sell at all in the duopoly areas, which would be left to the entrant alone. In this way the incumbent avoids competition in the duopoly areas and concentrates on his monopoly areas. This result is rather extreme in that the incumbent either is present in all duopoly areas or in none.

If we adopt a different timing of coverage decisions, then we could allow the possibility of the incumbent withdrawing from some duopoly area after having installed capacity. One may think of this as having installed some "dark" fibre optics, but not "lightening" it. The incumbent may choose to do this in order to alleviate the strategic burden associated to serving duopoly locations. This could also be done by not using the existing facilities, not advertising, or offering bad quality.[19] This strategic effect operates through two dimensions: First, the incumbent now depends relatively less on his

[19] On the regulation of minimum quality standards see e.g. Ronnen (1991) and Valletti (2000).

duopoly areas, and therefore is willing to post higher prices. Second, if also the entrant is subject to a uniform pricing constraint, then he prices less aggressively since he has become a monopolist over some areas. Both these effects lead to higher prices in equilibrium. This is a purely strategic ("competitive") phenomenon, serving to alleviate competitive pressure on the other markets, and even though the outcome looks as if firms were dividing markets between them there is no collusion involved. A proof of this claim is available from the authors on request.

8 Conclusions

Different policy measures aimed at fulfilling universal service objectives have quite intricate consequences even in the simple setup of our model, where we were able to single out the possible strategic effects of these measures. The result of each additional measure, be it uniform pricing constraints, price caps or coverage constraints, can be quite different depending on which other policies are imposed at the same time. Our results show that a welfare evaluation of single policies is impossible without specifying the context of other policies, and that "baskets" of measures must be analyzed together. In particular, uniform pricing constraints create strategic links between otherwise unrelated geographical areas, which can lead to lower coverage chosen by both incumbent and entrant. The imposition of a minimum coverage for the incumbent in the presence of a uniform pricing constraint raises market prices, not because firms try to recoup installation costs (these are sunk), but because *ex post* the incumbent relies relatively less on the revenue generated under competition.

In relation to consumer welfare, we argued that there are different groups of consumers, those that have access to two, one or no provider before and after the imposition of the policy measures. Welfare of these different groups will in general move into different directions. We pointed out how different groups would fare under various policy programs, but leave the question of *optimal* policies for further research. To be realistic, we believe that the best one can hope for is to find a policy mix that minimizes distortions while fulfilling universal service objectives.

An important question which should also receive renovated attention in the research agenda is the strategic role of financing universal service through subsidies or universal service funds. It is easy to see that, for example, a direct effect of a commitment of loss-covering state subsidies for local markets is to make the incumbent price more aggressively. The entrant then will enter less markets, reversing part of the previous effect, and the overall effect is ambiguous. On the other hand, lump-sum subsidies do not have this effect but are more difficult to set to the correct levels *ex ante* or may be subject to renegotiation. Therefore the precise way of financing universal service may strongly affect market outcomes.

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9 Appendix

Proof of Lemma 1:

We can write the first-order conditions as

$$\begin{aligned} kA + (1-k)\,B &= 0, \\ q_2 + p_2 \tfrac{\partial q_2}{\partial p_2} &= 0. \end{aligned}$$

Since the monopoly solution corresponds to $B = 0$, and the duopoly solution to $A = 0$, this shows that for firm 1 $p_d < p_1^{UP} < p_m$. Since the reaction function of firm 2 is upward sloping, then also $p_d < p_2^{UP} < p_m$. Doing comparative statics at equilibrium gives

$$\begin{aligned} \tfrac{\partial p_1}{\partial k} &= -\tfrac{\partial^2 \Pi_2}{\partial p_2^2} \frac{(A-B)}{D} < 0, \\ \tfrac{\partial p_2}{\partial k} &= \tfrac{\partial^2 \Pi_2}{\partial p_1 \partial p_2} \frac{(A-B)}{D} < 0, \end{aligned}$$

where $D = \frac{\partial^2 \Pi_1}{\partial p_1^2} \frac{\partial^2 \Pi_2}{\partial p_2^2} - \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2} > 0$ in order for the equilibrium to be locally stable, $\frac{\partial^2 \Pi_2}{\partial p_2^2} < 0$ from the second-order conditions, and $\frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2} > 0$ due to strategic complementarity. Since prices are decreasing in k and goods are demand substitutes, an increase in relative coverage k also reduces local profits for firm 2. In fact, it is sufficient to apply the envelope theorem to see that

$$\frac{d\pi_2}{dk} = \frac{\partial \pi_2}{\partial p_1} \frac{\partial p_1}{\partial k} = p_2 \frac{\partial q_2}{\partial p_1} \frac{\partial p_1}{\partial k} < 0.$$

This is also true in monopoly areas since an increase in k causes p_1^{UP} to fall,

$$\frac{d\pi^m}{dk} = \frac{\partial \pi^m}{\partial p_1} \frac{\partial p_1}{\partial k} < 0.$$

On the other hand, the effect of an increase in k on firm 1's duopoly profits π_1 cannot be signed,

$$\frac{d\pi_1}{dk} = \underbrace{A \frac{\partial p_1}{\partial k}}_{>0} + \underbrace{\frac{\partial \pi_1}{\partial p_2} \frac{\partial p_2}{\partial k}}_{<0}.$$

The second term is the same negative effect as above. We cannot apply the envelope theorem to the first term, and it can be seen from the first-order condition that marginal revenue A is negative in equilibrium. When k tends to 1, A tends to zero, $d\pi_1/dk$ becomes negative, while it tends to be positive for small k because p_1^{UP} tends to p^m, where $A << 0$. It is also positive if goods are sufficiently differentiated, since $\partial\pi_1/\partial p_2 = p_1 \partial q_1/\partial p_2$ is small.

Proof of Proposition 1:

First note that

$$\frac{\partial k}{\partial x_2} = \frac{1}{x_1}, \ \frac{\partial k}{\partial x_1} = -\frac{x_2}{x_1^2}.$$

Firm 2 maximizes, given x_1,

$$x_2\pi_2(k) - C(x_2),$$

leading to the following first- and second-order conditions:

$$\begin{aligned} \pi_2 + k\pi_2' &= c(x_2), \\ (2\pi_2' + k\pi_2'')\frac{\partial k}{\partial x_2} - c'(x_2) &\leq 0. \end{aligned}$$

Let us consider the term $(2\pi_2' + k\pi_2'')$. Clearly, if π_2 is concave in k, then this term is negative. In general, it suffices that π_2'' is not too big. This is ensured by our assumption on the elasticity of π_2' that ensures the concavity of $k\pi_2(k)$ in k: $\frac{d^2 k\pi_2}{dk^2} = 2\pi_2' + k\pi_2'' = \frac{d^2 x_2\pi_2}{dx_2^2} < 0$.

The derivative of optimal coverage of firm 2 with respect to the coverage of firm 1 is

$$\begin{aligned} \frac{dx_2}{dx_1} &= -\frac{2\pi_2'\frac{\partial k}{\partial x_1} + k\pi_2''\frac{\partial k}{\partial x_1}}{2\pi_2'\frac{\partial k}{\partial x_2} + k\pi_2''\frac{\partial k}{\partial x_2} - c(x_2)} \\ &= k\frac{2\pi_2' + k\pi_2''}{2\pi_2' + k\pi_2'' - x_1 c(x_2)}. \end{aligned}$$

The denominator is negative by the second-order condition, and our assumption on $k\pi_2$ assures that the denominator is negative as well, thus $dx_2/dx_1 \in (0,k)$. From the first-order condition we can also observe that if firm 1 is setting a very high coverage then the best reply for firm 2 is to set $x_2 < x_m$ because it cannot exceed the monopoly coverage in a best response. Therefore $k \to 0$ and the remaining term of the left-hand side of the first-order condition is π_2, calculated at $p_1 \to p_m$ and p_2 at the best response to this p_1 which is greater than p_d, which means that $\pi_2 > \pi_d$ and firm 2 would set $x_2 > x_2^b$. Conversely, if firm 1 sets $x_1 = x_2^b$, then the best reply of firm 2 is to set a lower coverage: If $x_2 = x_1$ then $k = 1$ and $\pi_2 = \pi_d$ and the first-order condition becomes $\pi_d + \pi_2' - c(x_2) = 0$, which is negative at

$x_2 = x_2^b$. Therefore in equilibrium the coverage of firm 2 may be either higher or lower than x_2^b.

Firm 1 maximizes

$$x_2 \pi_1 (k) + (x_1 - x_2) \pi_m (k) - C (x_1),$$

with first- and second-order conditions:

$$\begin{aligned} -k \left(k\pi_1' + (1-k) \pi_m'\right) + \pi_m &= c (x_1), \\ -k[2\pi_1' + k\pi_1'' - 2\pi_m' + (1-k) \pi_m''] \frac{\partial k}{\partial x_1} - c' (x_1) &\leq 0. \end{aligned}$$

Let us consider the expression in the square brackets. It can be rewritten as:

$$2\pi_1' + k\pi_1'' - 2\pi_m' - k\pi_m'' + \pi_m'' = \frac{1}{k} \frac{d}{dk} \left(k^2(\pi_1' - \pi_m')\right) + \pi_m''.$$

First, note that $\pi_m'' > 0$. This is implied by the fact that $x_1 \pi_m (k)$ is increasing in x_1 (from Lemma 1) and concave in x_1 for sufficiently differentiated products ($\frac{d(x_1 \pi_m(k))}{dx_1}$ is bounded above by π_m as x_1 increases and $k \to 0$): $\frac{d^2 x_1 \pi_m(k)}{dx_1^2} = -k\pi_m'' \frac{\partial k}{\partial x_1} < 0$. Second, $\frac{d}{dk} \left(k^2(\pi_1' - \pi_m')\right)$ is also positive since we have assumed that the elasticity of $f(k) = \pi_1' - \pi_m' = -(B - A)\frac{\partial p_1}{\partial k} + \frac{\partial \pi_1}{\partial p_2}\frac{\partial p_2}{\partial k}$ is bounded below by -2. This assumption is a sufficient but not necessary condition for our results.

In order for the second-order condition to be satisfied, the cost function needs to be sufficiently convex. These results also imply that the slope of the reaction function of firm 1 is negative:

$$\frac{dx_1}{dx_2} = -\frac{-k \left(2\pi_1' + k\pi_1'' - 2\pi_m' + (1-k) \pi_m''\right) \frac{\partial k}{\partial x_2}}{-k \left(2\pi_1' + k\pi_1'' - 2\pi_m' + (1-k) \pi_m''\right) \frac{\partial k}{\partial x_1} - c' (x_1)} < 0.$$

Since at $x_2 = 0$ the optimal x_1 is clearly equal to x_m, at any other $x_2 > 0$ the coverage chosen by firm 1 is below x_m.

Under our assumptions, payoffs are concave and best replies are continuous. Since firm 1 and firm 2' best replies are decreasing (starting from $x_1 (0) = x_m$) and increasing (while remaining less than x_1), respectively, they must cross at some point (x_1, x_2) with $x_2 < x_1 < x_m$, which is the (unique) equilibrium. ∎

Proof of Proposition 2:

Note that now

$$\frac{dk}{dx_1} = \frac{d}{dx_1} \left(\frac{x_2}{x_1}\right) = \frac{x_1 dx_2/dx_1 - x_2}{x_1^2} = \frac{1}{x_1} \left(\frac{dx_2}{dx_1} - k\right) < 0.$$

Since k is decreasing, both prices are higher (Proposition 1), therefore v_d and v_m both decrease. As for aggregate monopoly consumer surplus, consider:

$$\frac{d}{dx_1}V_m\left(x_1,x_2\right)=\left(1-\frac{dx_2}{dx_1}\right)v_m\left(k\right)+\left(1-k\right)v_m'\left(k\right)\left(\frac{dx_2}{dx_1}-k\right)$$

This expression is positive when $dx_2/dx_1 \approx k$: Price does not change and there are more monopoly consumers in aggregate since $k < 1$. However, if fixed costs increase rapidly ($c' >> 0$), then the reaction function of firm 2 is flat, $dx_2/dx_1 \approx 0$. Then there are 2 effects: The highest possible positive effect from having more monopoly customers in aggregate, but the price increases and all former monopoly customers pay a bit more. The overall effect may be negative. The same is true for aggregate consumer surplus in duopoly:

$$\frac{d}{dx_1}V_d\left(x_1,x_2\right)=\frac{dx_2}{dx_1}v_d\left(k\right)+kv_d'\left(k\right)\left(\frac{dx_2}{dx_1}-k\right).$$

It is not possible to sign unambiguously this expression. When the reaction function of firm 2 is flat ($dx_2/dx_1 \approx 0$) then the above expression is negative. Firm 2 does not expand its coverage, and the only effect is to push prices up. If on the other hand $dx_2/dx_1 \approx k$, prices are almost unaffected, and there is an unambiguous positive impact by attracting additional duopoly customers.

As far as total profits of firm 1 are concerned,

$$\frac{d}{dx_1}\left(\Pi_1-C\left(x_1\right)\right)=\left(\frac{\partial\Pi_1}{\partial x_1}-c\left(x_1\right)\right)+\frac{\partial\Pi_1}{\partial x_2}\frac{dx_2}{dx_1}<0,$$

where both terms on the right-hand side are negative, and the expression in the bracket would be zero by the envelope theorem if CC is imposed only slightly above the coverage chosen under UP alone. The effect on total profits of firm 2 is

$$\frac{d}{dx_1}\left(\Pi_2-C\left(x_2\right)\right)=x_2\frac{\partial\pi_2}{\partial k}\frac{\partial k}{\partial x_1}=-k^2\frac{\partial\pi_2}{\partial k}>0. \blacksquare$$