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ABSTRACT

New Extreme-Value Dependence Measures and Finance Applications*

In the finance literature, cross-sectional dependence in extreme returns of risky assets is often modelled implicitly assuming an asymptotically dependent structure. If the true dependence structure is asymptotically independent then existing finance models will lead to over-estimation of the risk of simultaneous extreme events. We provide simple techniques for deciding between these dependence classes and for quantifying the degree of dependence in each class. Examples based on daily stock market returns show that there is strong evidence in favour of asymptotically independent models for dependence in extremal stock market returns, and that most of the extremal dependence is due to heteroskedasticity in stock returns processes.

JEL Classification: C13, C22, G11, G15
Keywords: asymptotic independence, extreme value theory, Hill's estimator, tail index

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NON-TECHNICAL SUMMARY

Estimating dependence between risky asset returns is the cornerstone of portfolio theory and many other finance applications such as hedging, credit spread analysis, valuation of exotic option written on more than one asset, and risk management in general. Unfortunately, the conventional dependence measure, Pearson correlation, is constructed as an average of deviations from the mean. As a consequence, the weight given to extreme realizations is the same as for all of the other observations in the sample. If the dependence characteristics for these extreme realizations differs from all others in the sample, the conclusions drawn from such measures could result in a financial institution risking bankruptcy. This suggests that correlation is not a good measure of dependency in the case where extreme realizations are important.

As an alternative to the traditional approach, it is possible to draw on statistical developments in extreme value theory (evt). Even though earlier applications of these techniques are widespread in the engineering literature, only more recently has evt been brought to finance. While the number of univariate contributions to finance increases steadily, multivariate finance applications are also beginning to appear either to explore the dependency of stock market returns or to derive directly the Value at Risk of a position.

In this Paper we draw attention to a pitfall that arises in the estimation of cross-sectional dependence among extreme returns and provide a remedy for this. The pitfall comes from the fact that there are two classes of extreme value dependence, asymptotic dependence and asymptotic independence, for which the characteristics of events behave quite differently as the events become more extreme. Both forms of extremal dependence permit dependence between moderately large values of each variable, but the very largest values from each variable can occur together only when the variables exhibit asymptotic dependence. This type of behaviour may be detected simply by considering a scatterplot of joint realizations of stock market returns. For instance, for the US-UK pair, one notices that the number of times of jointly exceeding a high threshold decreases quickly whereas it does not for the German-French pair. This suggests that the US-UK pair is asymptotically independent whereas the German-French pair is asymptotically dependent. It should be emphasized that even though the US-UK pair is asymptotically independent, its Pearson correlation is significantly different from zero. As a consequence, these returns are not independent in the usual sense.

The conventional multivariate extreme value theory has emphasized the asymptotically dependent class, resulting in its wide use in all current finance applications. If the series are truly asymptotically independent, such an approach will result in the over-estimation of extreme value dependence, and consequently of the financial risk. The degree of this over-estimation depends on the degree of asymptotic independence. Despite this potential for bias, the case for asymptotically independent models has so far been omitted in the finance literature.

In this Paper, we provide techniques for distinguishing between asymptotically dependent and asymptotically independent variables and for quantifying the degree of dependence for the appropriate dependence class. We introduce two extremal dependence measures, which require no knowledge of the distribution of stock returns to measure the degree of asymptotic dependence and asymptotic independence respectively. We also develop a test strategy, whereby one first investigates if given pairs of series are asymptotically independent. If the series are asymptotically independent then the test stops. If the first test rejects asymptotic independence then we measure the degree of asymptotic dependency with a second measure.

In an empirical study, using daily returns on five stock indices (S&P, FTSE, DAX, CAC and Nikkei), over a 31.5-year period from 26th December 1968 to 31st May 2000, we find left-tail dependence to be usually stronger than right-tail dependence. In addition, we demonstrate that most of these stock index returns do not exhibit asymptotic dependence, suggesting that much of the extreme value dependence reported in previous studies is likely to be over-estimated. Using an asymmetric version of a GARCH(1,1) filter, we find that much, but not all, of the extreme value dependence is caused by changing stock market volatility.

New Extreme-Value Dependence Measures and Finance Applications

1 Introduction

Estimating dependence between risky asset returns is the cornerstone of portfolio theory and many other finance applications such as hedging, credit spread analysis, valuation of exotic option written on more than one asset, and risk management in general. Unfortunately, the conventional dependence measure, Pearson correlation, is constructed as an average of deviations from the mean. As a consequence, the weight given to extreme realizations is the same as for all of the other observations in the sample. If the dependence characteristics for these extreme realizations differs from all others in the sample the conclusions drawn from such measures could result in a financial institution risking bankruptcy. This suggests that correlation is not a good measure of dependency in the case where extreme realizations are important.

As an alternative to the traditional approach, it is possible to draw on statistical developments in extreme value theory (evt). Even though earlier applications of this field are widespread in the engineering literature, only more recently, has evt been brought to finance. Most of the applications are univariate. Jansen and de Vries (1991) showed that the crash of 19th October 1987 may not be an isolated event. Loretan and Phillips (1994) used evt to study the existence of moments of financial returns and Longin (1996) who showed that the tails of stock market returns belong to the Fréchet class. Embrechts, Klüppelberg, and Mikosch (1997) provide a summary of general evt results and comprehensive references. Diebold, Schuerman and Stroughair (1998) sketch a number of pitfalls associated with the application of evt techniques to financial data. They emphasize the role of small samples and the dependency of financial data, especially considering volatility.[1] While the number of univariate contributions increases steadily, multivariate finance applications are also beginning to appear. Longin and Solnik (2000) explore the use of multivariate extreme value methods for stock market returns, which Longin (2000) uses to demonstrate how VaR of a position can be derived. Stariça (2000) finds a high level of dependence between the extreme movements of most of the currencies in the EU. Marsh and Wagner (2000) find extremal dependence between stock returns and trading volume among equity markets. Hartmann, Straetmans and de Vries (2000) find co-crashes between stock and bond markets and some evidence of extreme cross-border linkages.

We draw attention to a pitfall that arises in the estimation of cross-sectional dependence among extreme returns and provide a remedy for this. The problems arise from the fact that there are two classes of extreme value dependence, *asymptotic dependence* and *asymptotic independence*, for which the characteristics of events behave quite differently as the events become more extreme. Both forms of extremal dependence permit dependence between moderately large values of each variable, but the very largest values from each variable can occur together only when the variables exhibit asymptotic dependence. To illustrate this, Figure 1 presents scatter-plots of the most recent 1,000 daily stock market returns in the US against those in the UK, and those of Germany against France. The dependence for the German-French stock market returns is persistent for both positive and negative extremes, which is indicative of the variables being asymptotically dependent. In contrast, the extremal dependence between US and UK stock market returns is much weaker although the largest values in each tail for one variable coincide with moderately large values of the same sign for the other variable, suggesting the variables are asymptotically independent but not exactly independent.

The conventional multivariate extreme value theory has emphasized the asymptotically dependent class resulting in its wide use in all the finance applications listed above. If the series are truly asymptotically independent, such an approach will result in the over-

[1]There exist some contributions, such as Harvey and Siddique (1999) or Rockinger and Jondeau (2001) where asset prices get modeled within a GARCH framework but where a conditional distribution is chosen to accomodate conditional skewness and kurtosis. This approach may be viewed as an alternative to evt.

estimation of extreme value dependence, and consequently of the financial risk. The degree of this over-estimation depends on the degree of asymptotic independence. Despite this potential for bias, the case for asymptotically independent models has so far been omitted in the finance literature.

In this paper, we provide techniques for distinguishing between asymptotically dependent and asymptotically independent variables and for quantifying the degree of dependence for the appropriate dependence class. We introduce two extremal dependence measures, χ and $\overline{\chi}$, that require no knowledge of the distribution of stock returns to measure the degree of asymptotic dependence and asymptotic independence respectively. Only when $\overline{\chi}$ suggests the variables are asymptotically dependent is the measure χ a correct measure of asymptotic dependence. For the data in Figure 1, we show how these two measures quantify the empirical conclusions above.

More generally, using daily returns on five stock indices (viz. S&P, FTSE, DAX, CAC and Nikkei), over a 31.5-year period from 26th December 1968 to 31st May 2000, we find left-tail dependence to be usually stronger than right-tail dependence. This result corresponds to recent findings in Longin and Solnik (2000). In addition, we demonstrate that most of these stock index returns do not exhibit asymptotic dependence, suggesting that much of the extreme value dependence reported in previous studies is likely to be over-estimated. Using an asymmetric version of a GARCH(1,1) filter, we find that much, but not all, of the extreme value dependence is caused by changing stock market volatility.

The remaining sections are organized as follows: Section 2 briefly describes univariate extreme value theory and recent developments in the measurement of dependence in multivariate extreme values. Section 3 describes the empirical analyses, which include a description of the data sources and a report of empirical findings. Section 4 provides a brief discussion on how the concepts underpinning the measures for extreme value dependence can be extended into useful portfolio management tools. Section 5 concludes.

2 Extreme Value Theory and Extremal Dependence

By their nature, data on extreme values are relatively scarce, and from this rather uncertain basis we often need to extrapolate to rarer events than those observed. Extreme value theory is a rigorous and broad mathematical limit theory which provides an asymptotic justification for modeling and extrapolating extreme values. For assessing the financial risk of a portfolio, the complete joint distribution of the various assets during periods of great turmoil needs to be estimated. This involves estimating the marginal distributions and the dependence structure. We will focus on the dependence estimation in the bivariate context, though the ideas and techniques extend naturally to higher dimensions. First we describe briefly univariate extreme value methods as they are used both to determine the marginal distributions and as they provide the inference techniques for the dependence measures.

2.1 Univariate Methods

There is a long history, and a large associated literature, on probability characterizations and statistical models for univariate extremes. The numerous approaches by which extreme values may be statistically modelled separate into two forms: methods for maxima over fixed intervals and methods for exceedances over high thresholds. We outline the fundamental aspects of each method. Further details can be found in Embrechts, Klüppelberg and Mikosch (1997) and Reiss and Thomas (1997).

The limit theory for the maximum of a sample of n independent and identically distributed random variables is based on a location-scale normalization of the maximum so that its distribution is non-degenerate as $n \to \infty$. Provided a non-degenerate limit can be achieved, it follows that, whatever the distribution of the original variables, the limiting distribution of the maximum has a small set of possible distributions, namely the Gumbel, the Fréchet or the negative Weibull distributions. The generalized extreme value distribution (GEV) is a unifying model that encompasses these three types of extreme value distributions. The GEV has three parameters, μ, σ and ξ, denoting the location, scale and shape parameters respectively. The shape parameter, ξ, also called the tail index, determines the three extreme value types.[2] For example, if the original variables follow a normal distribution then a Gumbel distribution will result for the maximum. Similarly, the Fréchet (negative Weibull) distributions arise as the distribution of the maximum for variables with heavier (lighter) tails than the normal distribution. There is now a growing consensus that many financial series have heavy tails (see for example, Loretan and Phillips, 1994), so the Fréchet distribution is to be expected.

When observations on all exceedances of a high threshold are available then using simply the fixed interval maximum values is inefficient as it may exclude large observations from the analysis. The appropriate limit theory in this context is one based on a point process result of Pickands (1971), which has been advocated for statistical modeling by Smith (1989). The limit result suggests modeling exceedances of a high threshold by a non-homogeneous Poisson process. A consequence of this model is that the excess values over the threshold follow the generalized Pareto distribution (GPD) and that maximum values are modelled by the generalized extreme value distribution, both distributions having a common shape parameter ξ. The GPD model, advocated by Pickands (1975) and Davison and Smith (1990), provides a flexible family of tail behaviors, with $\xi = 0$ corresponding to the exponential distribution. A key modeling aspect with threshold methods is the selection of the threshold. A number of diagnostic techniques exist for threshold selection, including a bootstrap method which produces an optimal value under certain criteria (Danielsson and de Vries, 1997). The critical aspect of threshold selection is that inference conclusions should be insensitive to increases in threshold above a suitable level.

For the subsequent dependence measures we introduce a special example of threshold modeling linked to the generalized Pareto distribution for the case where $\xi > 0$, i.e. Fréchet tailed. In this case the tail of the variable Z above a high threshold u can be approximated as

$$\Pr(Z > z) = \frac{\mathcal{L}(z)}{z^{1/\xi}} \text{ for } z > u, \tag{1}$$

[2]Specifically, when ξ takes negative values, positive values or the value 0, the GEV distribution becomes the negative Weibull, the Fréchet and the Gumbel distributions respectively.

where $\mathcal{L}(z)$ is a slowly varying function of z (see Embrechts (1997), page 325). Treating the slowly varying function as a constant for all $z > u$, i.e. $\mathcal{L}(z) = c$, and under the assumption of independent observations the maximum likelihood estimators for ξ, known as Hill's estimator (Hill, 1975), and c are

$$\hat{\xi} = \frac{1}{n_u} \sum_{j=1}^{n_u} \log \left(\frac{z_{(j)}}{u} \right), \tag{2}$$

$$\hat{c} = \frac{n_u}{n} u^{1/\hat{\xi}}, \tag{3}$$

where $z_{(1)}, \ldots, z_{(n_u)}$ are the n_u observations of variable Z that exceed u.

The above discussion applies to independent variables. When the variables are dependent, the statistical approaches for analyzing maxima are unchanged as the limit distribution of the maximum is also a generalized extreme value distribution. Unlike the maximum over interval method, temporal dependence due to the use of threshold method adds some complications. One approach is to ignore the dependence and apply the methods as if the data were independent. This approach leads to unbiased estimators but with standard errors that are too small.[3] But many finance applications have directly assumed temporal independence and omitted the well documented volatility dependence.

2.2 Measuring Extreme Value Dependence

Much effort has been made to extend univariate extreme value theory for applications in a multivariate context. In almost all multivariate studies, it is helpful to remove the influence of marginal aspects first by transforming the original variables to a common marginal distribution. After such a transformation, differences in distributions are purely due to dependence aspects. Hence, our dependence measures, unlike the correlation, are no longer influenced by the form of the marginal distribution.[4] In this spirit, we transform the bivariate returns (X, Y) to unit Fréchet marginals (S, T) using the transformation

$$S = -1/\log F_X(X) \text{ and } T = -1/\log F_Y(Y), \tag{4}$$

[3] Two approaches are used to overcome this problem: declustering of the exceedances of the threshold to produce approximately independent data (see Davison and Smith, 1990); or using robust methods for standard error evaluation based on estimating equations (see Coles and Walshaw, 1994).

[4] For a further discussion of this issue, see Embrechts, McNeil and Strautman (1999).

where F_X and F_Y are the respective marginal distribution functions for X and Y. Consequently, $\Pr(S > s) = \Pr(T > s) \sim s^{-1}$ as $s \to \infty$, and (S, T) possess the same dependence structure as (X, Y). In practice, the values of F_X and F_Y that are used in the transformation (4) are obtained using the empirical distribution functions of the separate variables.

2.2.1 The conventional approach

To understand extremal dependence, one must first appreciate that the form and degree of such dependence determine the chance of obtaining large values of both variables. As variables S and T are on a common scale then events of the form $\{S > s\}$ and $\{T > s\}$, for large values of s, correspond to equally extreme events for each variable. As all such probabilities will tend to zero as $s \to \infty$ it is natural to consider conditional probabilities of one variable given that the other is extreme. Specifically, consider the behaviour of $\Pr(T > s \mid S > s)$ for large s. If (S, T) are perfectly dependent then $\Pr(T > s \mid S > s) = 1$. In contrast, if (S, T) are *exactly independent* then $\Pr(T > s \mid S > s) = \Pr(T > s)$, which tends to 0 as $s \to \infty$. Defining

$$\chi = \lim_{s \to \infty} \Pr(T > s \mid S > s), \tag{5}$$

where $0 \leq \chi \leq 1$, we have that variables are termed *asymptotically dependent* if $\chi > 0$ and *asymptotically independent* if $\chi = 0$. Clearly χ measures the degree of dependence that is persistent into the limit. An example of a non-trivial asymptotically dependent joint distribution is the logistic model in the bivariate extreme value family, see Tawn (1988) and Longin and Solnik (2000), which for unit Fréchet margins has

$$\Pr(S \leq s, T \leq t) = \exp\{-(s^{-1/\alpha} + t^{-1/\alpha})^{\alpha}\} \tag{6}$$

with $0 < \alpha \leq 1$. When $\alpha = 1$ then the variables are exactly independent and $\chi = 0$. When $\alpha < 1$ then $\chi = 2 - 2^{\alpha}$, so the variables are asymptotically dependent to a degree depending on α. Generally, when $\chi = 0$ the two random variables are not necessarily exactly independent. For example, if the dependence structure is that of a bivariate normal random variable with any value for the correlation coefficient less than one, then $\chi = 0$ (Sibuya, 1960).

When exact independence is rejected, the traditional multivariate extreme value methods,[5] assume $\Pr(T > s \mid S > s) = \chi > 0$ for all large s. If the true distribution of the variables is asymptotically independent, the use of the traditional multivariate extreme value methods will over-estimate $\Pr(S > s, T > s)$ and all other probabilities of joint extreme events since $\Pr(T > s \mid S > s) \to 0$ as $s \to \infty$. The degree of bias will depend on the difference between the estimated χ and the true value of $\Pr(T > s \mid S > s)$, which is determined by the value of s and the rate at which $\Pr(T > s \mid S > s) \to 0$ as $s \to \infty$.

2.2.2 An alternative measure of dependence

More recently, Ledford and Tawn (1996, 1997), Bruun and Tawn (1998), Bortot and Tawn (1998) have provided a range of extremal dependence models, derived from a different form of multivariate limit theory, that describe dependence but have $\chi = 0$. Although the random variables are asymptotically independent in this case, different degrees of dependence are attainable at finite levels of s. Two simple extremal dependence measures were developed by Coles *et al.* (1999) for identifying the form and the associated degree of dependence for the two types of extremal dependence. One of these measures is χ, which provides the degree of asymptotic dependence if the variables are asymptotically dependent. For all asymptotically independent variables $\chi = 0$, so χ cannot provide a measure of the degree of asymptotic independence. Coles *et al.* (1999) suggest that $\overline{\chi}$, defined by

$$\overline{\chi} = \lim_{s \to \infty} \frac{2 \log \Pr(S > s)}{\log \Pr(S > s, T > s)} - 1, \tag{7}$$

where $-1 < \overline{\chi} \leq 1$, is an appropriate measure of asymptotic independence as it gives the rate that $\Pr(T > s \mid S > s) \to 0$. Values of $\overline{\chi} > 0$, $\overline{\chi} = 0$ and $\overline{\chi} < 0$ loosely correspond respectively to when (S, T) are positively associated in the extremes, exactly independent, and negatively associated. For the bivariate normal dependence structure $\overline{\chi}$ is equal to the correlation coefficient, which aids in interpreting the value of $\overline{\chi}$. For other examples see Heffernan (2000).

The pair of dependence measures $(\chi, \overline{\chi})$ together provide all the necessary information to characterise the form and degree of extremal dependence. For asymptotically depen-

[5] See for example, de Haan (1985), de Haan and de Ronde (1998) and Coles and Tawn (1991, 1994).

dent variables $\overline{\chi} = 1$ with the degree of dependence given by $\chi > 0$. For asymptotically independent variables $\chi = 0$ with the degree of dependence given by $\overline{\chi}$. It is important to test if $\overline{\chi} = 1$ first before drawing conclusions about asymptotic dependence based on estimates of χ.

2.3 $\overline{\chi}$ and χ : Estimation and Statistical Inference

To estimate $\overline{\chi}$ and χ, we use results in Ledford and Tawn (1996, 1997, 1998), where it was established that under weak conditions

$$\Pr(S > s, T > s) = \mathcal{L}(s)s^{-1/\eta} \qquad \text{as } s \to \infty, \tag{8}$$

where $0 < \eta \leq 1$ is a constant and $\mathcal{L}(s)$ is a slowly varying function. From this representation it follows that

$$\overline{\chi} = 2\eta - 1 \tag{9}$$

and that if $\overline{\chi} = 1$, corresponding to $\eta = 1$, then $\chi = \lim_{s\to\infty} \mathcal{L}(s)$. Thus estimating η and $\lim_{s\to\infty} \mathcal{L}(s)$ provide the basis for estimating χ and $\overline{\chi}$.[6]

Inference follows using univariate extreme value techniques by identifying that if $Z = \min(S, T)$ then

$$\begin{aligned} \Pr(Z > z) &= \Pr\{\min(S,T) > z\} \\ &= \Pr(S > z, T > z) \\ &= \mathcal{L}(z)z^{-1/\eta} \\ &= dz^{-1/\eta} \text{ for } z > u, \end{aligned} \tag{10}$$

for some high threshold u. From this representation and the univariate tail form (1), it can be seen that η is the tail index of the univariate variable Z, and so can be easily estimated

[6]There is the possibility that $\eta = 1$ and $\mathcal{L}(s) \to 0$ as $s \to \infty$ leading to asymptotic independence. This boundary case cannot be identified from data as the slowly varying function cannot be identified other than as a constant, and mis-specification of the dependence structure in this situation is unlikely to be important. Thus, we focus on inference for η and $\lim_{s\to\infty} \mathcal{L}(s)$, treating the slowly varying function as constant over some threshold u, i.e. $\mathcal{L}(s) = d$ for $s > u$.

using the Hill estimator from equation (2), **truncated to the interval (0,1]**, and that d is the associated scale parameter which can be estimated by equation (3). The following development is based on the assumption of independent observations on Z. From this formulation we obtain our estimator for $\overline{\chi}$ to be

$$\begin{aligned}\hat{\overline{\chi}} &= \frac{2}{n_u}\left(\sum_{j=1}^{n_u} \log\left(\frac{z_{(j)}}{u}\right)\right) - 1 \\ \mathrm{Var}(\hat{\overline{\chi}}) &= (\hat{\overline{\chi}}+1)^2/n_u,\end{aligned}$$

with the notation as in equations (2) and (3). If $\hat{\overline{\chi}}$ is significantly less than 1 (i.e. if $\hat{\overline{\chi}} + 1.96\sqrt{\mathrm{Var}(\hat{\overline{\chi}})} < 1$) then we infer the variables to be asymptotically independent and take $\chi = 0$. Only if there is no significant evidence to reject $\overline{\chi} = 1$ do we estimate χ, which we do under the assumption that $\overline{\chi} = \eta = 1$. Using the maximum likelihood estimator given by (3), under the constraint $\hat{\overline{\chi}} = 1$, our estimator of χ is

$$\begin{aligned}\hat{\chi} &= \frac{u n_u}{n}, \\ \mathrm{Var}(\hat{\chi}) &= \frac{u^2 n_u (n - n_u)}{n^3}.\end{aligned}$$

Furthermore, we can assess whether the variables have a joint tail which decays with the same form as for exact independence by testing if $\hat{\overline{\chi}}$ is significantly different from 0.

3 Empirical Analyses

Our data consists of closing stock index levels of S&P 500 from the US, FTSE 100 from the UK, DAX 30 from Germany, CAC 40 from France, and Nikkei 225 from Japan. Our sample period spans from 26th December 1968 to 31st May 2000 giving rise to 8,200 daily return observations for each series. Three of the indices (viz. S&P, FTSE and CAC) were created by grafting two returns series from the same country. For example, the UK returns are represented by the FT All Shares returns before 1st January 1980 and FTSE returns after that date. Daily index returns are generated by taking first differences of the logarithmic indices. Although some of the returns series do not include the dividend distribution, as dividends do not generate extreme movements this is not a problem for our analysis.

It has been widely documented elsewhere that the US market has, by far, the greatest influence on all the other stock markets (see, for example, Martens and Poon (2001)). The US market is also the latest to close on any particular day among the five stock markets in our sample. This means that any extreme movements in the US stock market are likely to impact on the other stock markets' on the following day. Hence, in the following analyses, we use previous day US returns whenever the returns pair involves S&P returns.

3.1 Descriptive statistics

Table 1 presents some summary statistics for the five stock index returns. Panel A presents the mean, variance, skewness and excess kurtosis, evaluated using generalized method of moments to ensure robustness to heteroskedasticity.[7] The average mean returns for all five returns is 0.033% (or 8.5% per annum) excluding dividends. All series have a significant negative skewness which implies that extreme negative returns are a dominant feature for all five indices. Kurtosis is significantly greater than three for all series, except the case of S&P returns in the US, which suggests Fréchet type tails. An Engle test (not reported) for heteroskedasticity shows significant first lag autocorrelation among the squared returns.

To disentangle variations in volatility, known to generate excess kurtosis, from extreme returns, we consider also a filtered version of stock returns. The filter we used is an asymmetric version of the GARCH(1,1) model, see Zakoian (1994), which is based on the model that the return R_t at time t follows a normal distribution with mean ω and standard deviation $\sqrt{h_t}$, where

$$\begin{aligned} R_t &= \omega + \sqrt{h_t} Z_t \\ h_t &= \alpha_0 + \alpha^+ Z_{t-1}^2 h_{t-1} D_{Z_{t-1} \geq 0} + \alpha^- Z_{t-1}^2 h_{t-1} D_{Z_{t-1} < 0} + \beta h_{t-1}, \end{aligned}$$

where α_0, α^+, α^- and β are parameters, and D_E is the indicator function that event E occurs. Using estimated values of these parameters the filtered series of Z_t values is derived.

The second panel of Table 1 presents the Hill index for both left and right tails of the unfiltered and filtered univariate data series. There are approximately 2% of the

[7] We achieve this using the methodology developed by Richardson and Smith (1993).

8,200 returns observations falling into the tail region for each variable when the method of Danielsson and de Vries (1997) is used for threshold selection. All the tail indices reported in Table 1 are significantly greater than zero. The GARCH filter reduces the tail index by 18% on average for the left tail, and by 25% on average for the right tail, indicating that heteroskedasticity is a contributing factor to extreme price movements.

3.2 Results on Extremal Dependence

Throughout this section we report our analyses based on the crude assumption of temporal independence of the data. In footnote 3, approaches which account for temporal dependence were discussed. But, given that previous extremal dependence studies in finance (see Introduction) have exclusively ignored this aspect, and that our analysis is largely exploratory, we do not see this assumption as restrictive. Moreover, we argue that although the assumption of *iid* (identical and independent distribution) is questionable for the unfiltered returns, there are reasonable grounds to believe that temporal independence cannot be rejected for filtered returns series. This is because the filtered returns have revealed no autocorrelations in the first and second moments and we did not detect any pattern of clustering in time of extremes in each separate filtered returns series. Nevertheless, in drawing conclusions we are cautious that the standard errors we present are likely to be too small, particularly for the unfiltered series.

Estimates of $\overline{\chi}$, for selected pairs of stock index returns, are reported in Table 2 for three non-overlapping sub-periods. Each sub-period is over ten years with the world market crash taking place in sub-period 2 and the integration of the European Union evolving throughout sub-period 3. Note that $-1 < \overline{\chi} \leq 1$ from equation (7), but our estimator based on the Hill's estimator (2) is not constrained to ensure the upper bound is satisfied. Here, we present unconstrained estimates of $\overline{\chi}$.

The table presents estimates corresponding to different extremal dependence aspects of the series, enabling identification of: (i) differences between left-tail dependence and right-tail dependence, (ii) the influence of heteroskedasticity, and (iii) the difference between dependence among European countries and between Europe and the US or Japan.

Table 2 also presents the correlation coefficients for selected returns pairs. All the correlation coefficients are significantly positive and appeared to have increased through time, especially among the European countries. The correlation for the filtered series is slightly weaker, but is still statistically significant. The higher correlation for the unfiltered returns in contrast to that for the filtered returns reflects a widespread dependency among stock market volatilities.

In most cases the estimates in Table 2 of $\overline{\chi}$ are both significantly greater than 0 and significantly less than 1. From the interpretation of $\overline{\chi}$ in Section 2.2, this implies that there is significant dependence between large values of the paired series but that the very largest values do not occur concurrently. Thus, most of the pairs are asymptotically independent, which means that they are not well described by existing methods in finance.

A number of features of the form of the extremal dependence become clear by studying the estimates in Table 2. For all pairs, whether filtered or not, estimated values of $\overline{\chi}$ are larger for left tails than for right tails, which implies that inter-series dependence for extreme returns is stronger among the left tails than that among the right tails[8]. Similarly, for all series, the $\overline{\chi}$ estimates are larger for unfiltered than filtered series, indicating that volatility is a major contributing factor to the between-series extremal dependence.

There is evidence of complex non-stationarity in the extremal dependence with a general increase over time in the $\overline{\chi}$ estimates for the left tail in the raw series but stability over time period for the corresponding estimates for the filtered data. The same pattern is found for the right tail amongst the European markets. This suggests that the increased dependence is due to an increase in volatility linkage instead of contemporaneous extreme returns realisations. This is an important finding indicating that, to successfully model the dependence of the extreme values in these data, it is necessary to incorporate the changing pattern of volatility.

Estimates of χ for pairs where asymptotic dependence cannot be rejected are given in Table 3. This corresponds to only 15 of the 84 pairs, indicating that the assumption of asymptotic dependence is inappropriate in most cases. In Section 4 we assess how important

[8]Similar findings were reported in Ang and Chen (2000), Longin and Solnik (2000) and Martens and Poon (2001).

this error is by obtaining an approximation to the bias in a portfolio risk assessment that is incurred by falsely assuming asymptotic dependence. Of the cases where asymptotic dependence is not inappropriate, almost all correspond to the unfiltered series, with the most consistent pattern of estimates being for the left tails within European markets in the last decade. This provides a considerable strengthening to the understanding provided by correlation coefficients.

The thresholds used for the above analyses were each determined using the method of Danielsson and de Vries (1997). The percentage of observations that exceed the threshold varies a great deal between returns pairs, but remains very stable for each returns pair and across the three sub-periods. For example, the percentages of observations falling into the left tail of German-French unfiltered returns distribution in sub-periods 1, 2 and 3 are 6%, 6% and 5% respectively. The corresponding figures for US-UK filtered returns are 21%, 22% and 22% for the right tail.

4 Implications of asymptotic independence for portfolio risk assessment

So far we have shown how to characterise extremal dependence and measure its associated degree in stock returns. For both forms of extremal dependence, the measures introduced are based on evaluation of the properties of the joint distribution function at equal probability marginal quantiles. To use these measures to characterise extremes of a portfolio, i.e., of a linear combination of assets, the full joint distribution function needs to be estimated in the tail region. For the case where returns are asymptotically dependent or exactly independent such methods exist, see Coles and Tawn (1994) and de Haan and de Ronde (1998), but for asymptotically independent returns equivalent methods are still being developed.[9]

In this paper, we focus on estimating portfolio risk. Again, the emphasis here is that if the portfolio risk is estimated based on the assumption of asymptotic dependence when the

[9] Different approaches are considered in Ledford and Tawn (1997), Bruun and Tawn (1998), Bortot *et al.* (2000), Coles *et al.* (1999), and Heffernan and Tawn (2000).

returns are asymptotically independent variables, then portfolio risk will be over-estimated. Here, we follow the Ledford and Tawn (1997) approach for handling asymptotic independence to develop a method for providing bounds on the portfolio risk, which are relatively tight if the variables are asymptotically dependent, but for asymptotically independent variables the bounds differ, with the lower bound being the more likely to provide a better approximation in many cases.

To illustrate the estimation of bounds on portfolio risk we focus on the bivariate case. Define (X, Y) to be two returns. In a portfolio risk management context, one would seek a convex combination of these returns, i.e., $aX + (1-a)Y$ for $0 < a < 1$, such that the probability of the combination exceeding a high threshold, k, is minimized over a. The key stage of this process is the evaluation of the probability for a given a. This is the aspect we focus on, estimating

$$\Pr\{aX + (1-a)Y > k\}$$

for fixed a and k, with k large. For any (x, y), where $y = (k - ax)/(1-a)$, we obtain

$$\Pr(X > x, Y > y) \leq \Pr(aX + (1-a)Y > k) \leq 1 - \Pr(X < x, Y < y). \qquad (11)$$

Generally, these bounds will be uninformative, however the point (x, y) can be selected to minimize the errors for each bound simultaneously. The appropriate (x, y) point is the one with the largest joint density, which generally occurs when x and y are at equal marginal quantile values, i.e., we seek (x, y) with $y = (k - ax)/(1-a)$ and

$$\Pr(X > x) = \Pr(Y > y). \qquad (12)$$

Define this point by (x_0, y_0). To evaluate (x_0, y_0) we need a model for the marginal distribution tail form. From Section 2.1 the univariate tail model (1) is appropriate for finance data, with different parameters c_X, ξ_X and c_Y, ξ_Y for each margin. Thus, equation (12) corresponds to

$$y_0 = (c_Y/c_X)^{\xi_Y} x_0^{\xi_Y/\xi_X}.$$

It follows that

$$\begin{aligned} \Pr(X > x_0, Y > y_0) &= \Pr(S > s_0, T > s_0) \\ 1 - \Pr(X < x, Y < y) &= 2\Pr(X > x_0) - \Pr(S > s_0, T > s_0), \end{aligned}$$

where (S, T) have unit Fréchet marginal distributions and $s_0 \approx 1/\Pr(X > x_0) = x_0^{\xi_X}/c_x = y_0^{\xi_Y}/c_Y$. Then the bounds (11) become

$$\begin{aligned} \Pr(X > x_0)\Pr(T > s_0 \,|\, S > s_0) &\leq \Pr(aX + (1-a)Y > k) \\ &\leq \Pr(X > x_0)\{2 - \Pr(T > s_0 \,|\, S > s_0)\}, \end{aligned}$$

so from expressions (8) and (10)

$$ds_0^{-(1-\overline{\chi})/(1+\overline{\chi})} \leq \Pr(aX + (1-a)Y > k)/\Pr(X > x_0) \leq 2 - ds_0^{-(1-\overline{\chi})/(1+\overline{\chi})}. \quad (13)$$

Using estimators of the marginal tail parameters and estimators of the extremal dependence structure characteristics, $\overline{\chi}$ and d, these bounds can be evaluated. Together with the estimate of $\Pr(X > x_0)$ these bounds provide the required bounds on $\Pr(aX+(1-a)Y > k)$. If the variables are asymptotically dependent, so $\overline{\chi} = 1$ and $d = \chi$, then as $k \to \infty$ the bounds on $\Pr(aX + (1-a)Y > k)/\Pr(X > x_0)$ converge to χ and $2 - \chi$ so are very tight. For asymptotically independent variables these limits for the bounds are 0 and 2 respectively. However, the rate of approach to these limits depends on the degree of asymptotic independence, $\overline{\chi}$. Typically, for finite k, the lower bound in (13) provides a much closer approximation to the truth than the upper bound, this is as the probability excluded in the calculation of the bound relates to a more extreme region than that included in the upper bound.

Figure 2 presents the upper and lower bounds of the portfolio risk, $\Pr(aX + (1-a)Y > k)$, calculated based on equation (13) for two country pairs, viz. US vs. UK and Germany vs. France, using left tail parameter estimates for the most recent subperiod unfiltered returns. We assume in each portfolio that, the weights for the first and second assets are 0.25 and 0.75 respectively. The daily percentage loss, k, is selected such that it is always greater than the individual univariate extreme value thresholds obtained through bootstrap estimation as mentioned in Section 2.1.

As one would expect when the daily percentage loss, k, increases, the logarithmic expected waiting time between such losses in Figure 2 increases, and the band between the upper and lower bounds widens. We observed previously from Table 2 that asymptotic dependence cannot be rejected for both US-UK and German-French returns pairs. Table

3 shows that the degree of dependence between German and French returns ($\chi = 0.476$) is about twice as strong as that between US and UK returns ($\chi = 0.275$). This corresponds to the wider band between upper and lower bounds of the US-UK portfolio compared with that of the German-French portfolio. If we translate the probability of portfolio risk into waiting time of threshold exceedance, the probability of the US-UK portfolio having a one-day loss equal to or exceeding 3.25% ranges between 5 months and 4 years. The estimated waiting time for the equivalent loss for the German-French portfolio is much tighter and ranges between 2 and 7 months. This reinforces the observation made earlier that for portfolio extreme risk diversification, the stock combination, US and UK, is more effective than the German-French stock combination.

5 Conclusion

In this paper, we introduce two measures for extreme value dependency to applications in finance and demonstrate their use in portfolio risk management. The introduction of a set of dependence measures that characterises all classes of extreme value dependency is an important development. These new tools have allowed us to document, for the first time, the widespread asymptotic independence among stock market returns; a phenomenon that has so far been overlooked in the finance literature. The omission of asymptotic independence models has led to over-estimation of portfolio risk. We have shown here that such an over-statement can be quite substantial.

Other empirical findings include a confirmation that extreme value dependence is much stronger in bear markets than in bull markets, and that much (but not all) of the extreme value dependency is due to correlated conditional volatility. In general, the correlation between volatilities has increased over time to produce asymptotically dependent stock markets within Europe and strong, but still asymptotically independent stock markets between Europe (UK, Germany and France), North America (US) and Asia (Japan). Future research could explore the extreme value dependencies across a larger set of stock markets and that across different financial markets and asset classes.

An accurate measurement of asset-returns behaviour during extreme period is useful in many important finance applications. With the new dependency measures, the dynamic of conditional correlation can be better understood. Further work could investigate the hedging efficiency gain and option pricing improvement with a model for conditional correlation that captures the characteristics observed here. The modeling of portfolio joint tail distribution would require detailed calibration of individual tail distributions and asset extreme value dependency. Ongoing research in this area should provide better tools for portfolio management and risk diversification. Future research to identify exogenous and leading relationships and economic conditions will also help us narrow the confidence intervals when we make statements about the probability of extreme value occurrence.

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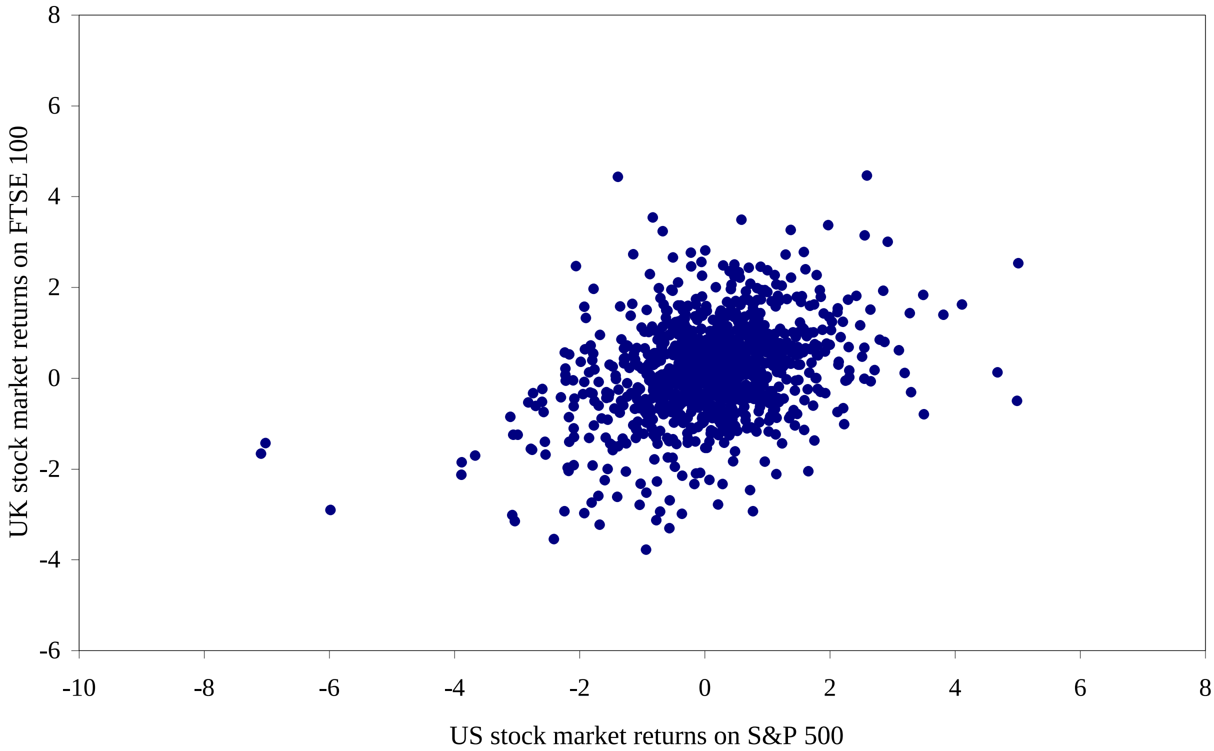
Figure 1
Scatter plot of 1,000 pairs of daily returns on selected stock market indices
for the period 19 December 1995 to 31 May 2000



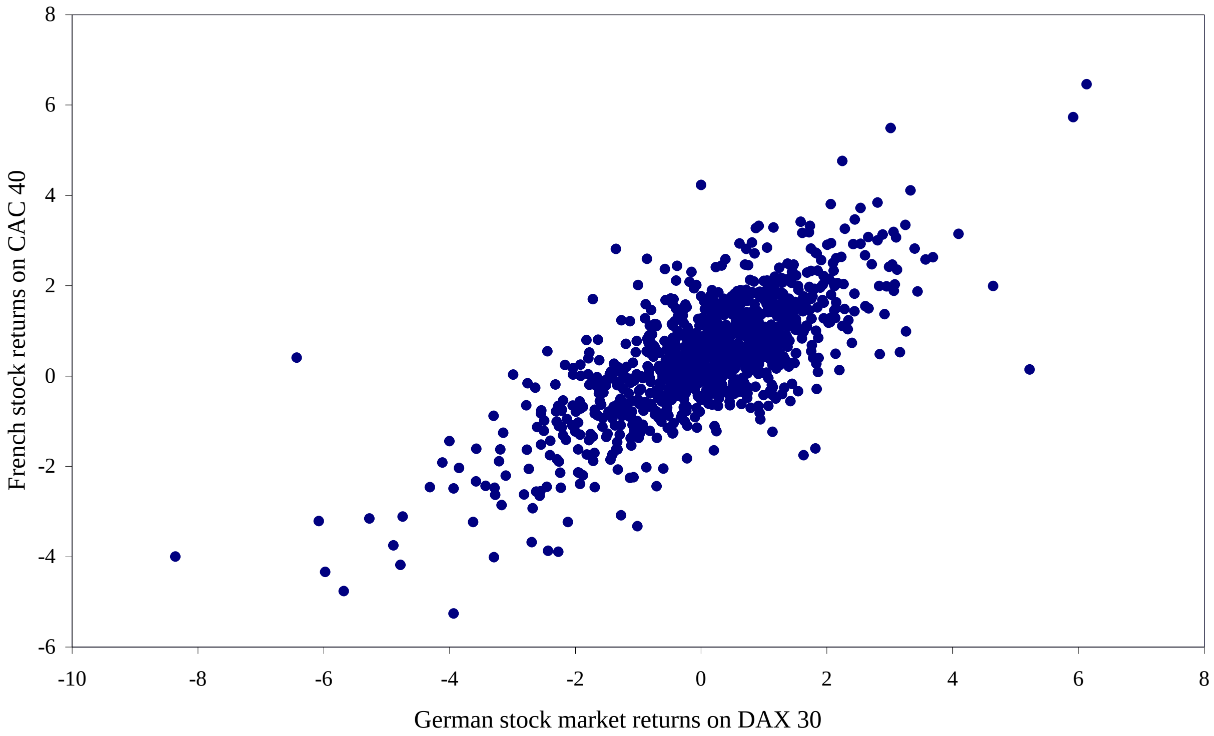
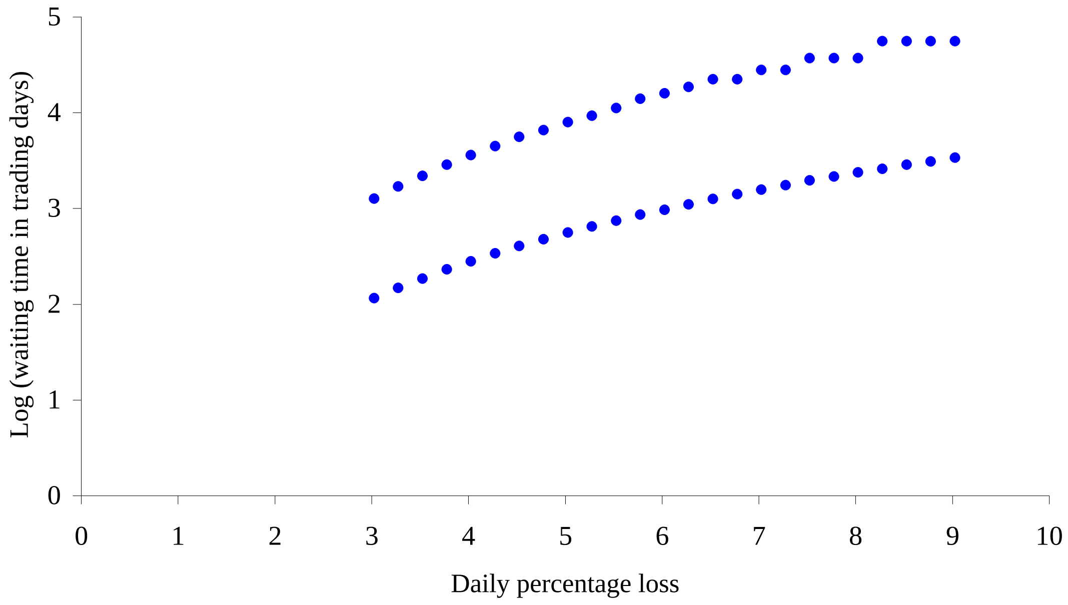
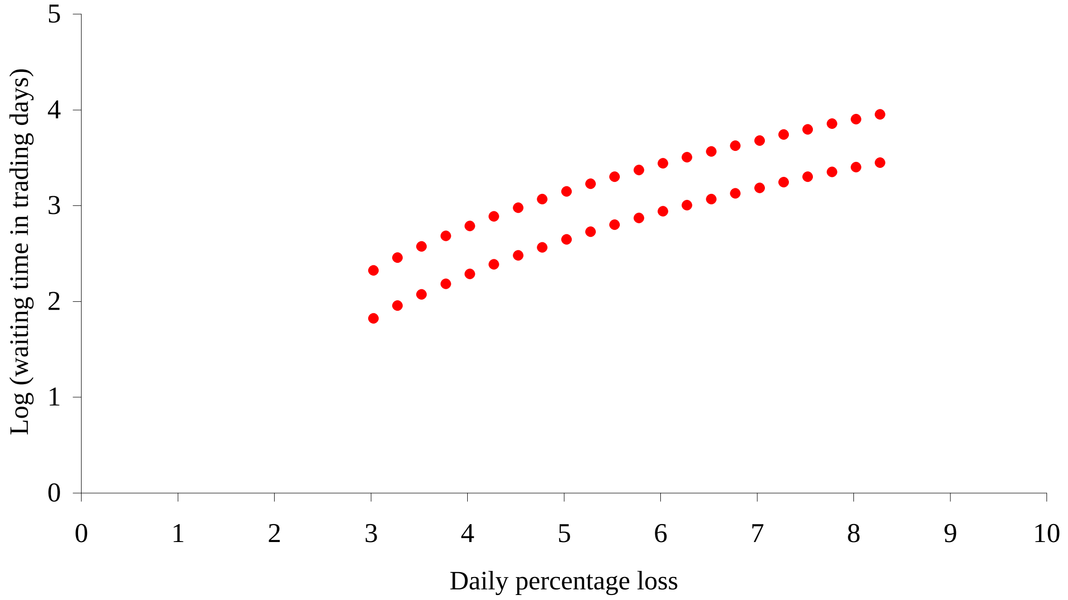


Figure 2
Bounds on Portfolio Risk Estimated for Subperiod 3
from 11 December 1989 to 31 May 2000

US and UK Stock Markets



German and French Stock Markets



Note:
Portfolio weights for first and second assets are 0.25 and 0.75 respectively.

Table 1.
Summary statistics and tail indices for stock market returns
over the period 27 December 1968 to 31 May 2000

	US	UK	Germany	France	Japan
Mean	0.032	0.035	0.031	0.039	0.028
Std Deviation	0.953	1.042	1.082	1.055	1.092
Skewness	-1.79	-0.31	-0.55	-0.61	-0.16
Excess Kurtosis	45.11	9.41	9.67	10.17	15.17
Tail index for daily stock index returns					
Left Tail	0.30 (0.021)	0.30 (0.025)	0.32 (0.025)	0.31 (0.025)	0.32 (0.026)
Right Tail	0.26 (0.021)	0.32 (0.021)	0.30 (0.025)	0.27 (0.020)	0.36 (0.029)
Tail index for daily return residuals from the asymmetric GARCH(1,1) model					
Left Tail	0.25 (0.018)	0.23 (0.017)	0.25 (0.017)	0.27 (0.018)	0.28 (0.020)
Right Tail	0.19 (0.016)	0.20 (0.015)	0.22 (0.015)	0.22 (0.017)	0.31 (0.020)

Notes:

1. The stock market indices are S&P 500 (for the US), FTSE 100 (for the UK), DAX 30 (for Germany), CAC 40 (for France) and Nikkei 225 (for Japan). There are 8,200 observations in each returns series, and the returns are defined as log differences of stock index.
2. Mean, standard deviation, skewness and kurtosis are computed based on the generalised method of moments using a weighting matrix correcting for possible first order autocorrelation. Hence, they are robust against heteroskedasticity and serial correlation.
3. Standard errors are in parentheses.

Table 2.
Measures of extreme tail independency, $\bar{c}$
for selected daily stock market return pairs over three sub-periods

Subperiod 1: 27 December 1968 – 19 June 1979 (2,733 observations)

	Unfiltered Raw Data					Filtered Residuals				
		Left tail		Right tail			Left tail		Right tail	
	r	$\bar{c}$	s.e.	$\bar{c}$	s.e.	r	$\bar{c}$	s.e.	$\bar{c}$	s.e.
US-UK	0.220	0.472	0.118	0.601	0.170	0.213	0.444	0.082	0.362	0.056
US-GER	0.211	0.820	0.185	0.355	0.098	0.186	0.514	0.060	0.281	0.051
US-FRA	0.261	0.381	0.109	0.683	0.238	0.227	0.447	0.074	0.266	0.064
US-JAP	0.116	0.333	0.104	0.368	0.095	0.114	0.418	0.060	0.314	0.104
UK-GER	0.102	0.332	0.097	0.299	0.089	0.080	0.746	0.153	0.105	0.101
UK-FRA	0.141	0.502	0.125	0.344	0.092	0.126	0.598	0.068	0.393	0.055
GER-FRA	0.163	0.438	0.111	0.183	0.127	0.167	0.616	0.072	0.272	0.089

Subperiod 2: 20 June 1979 – 8 December 1989 (2,733 observations)

	Unfiltered Raw Data					Filtered Residuals				
		Left tail		Right tail			Left tail		Right tail	
	r	$\bar{c}$	s.e.	$\bar{c}$	s.e.	r	$\bar{c}$	s.e.	$\bar{c}$	s.e.
US-UK	0.347	0.800	0.243	0.765	0.156	0.302	0.451	0.081	0.361	0.056
US-GER	0.319	0.688	0.116	0.442	0.106	0.366	0.513	0.059	0.256	0.057
US-FRA	0.257	0.568	0.107	0.272	0.081	0.341	0.469	0.066	0.271	0.064
US-JAP	0.407	0.626	0.111	0.764	0.193	0.290	0.411	0.061	0.324	0.103
UK-GER	0.345	0.658	0.114	0.591	0.173	0.273	0.766	0.159	0.089	0.103
UK-FRA	0.287	0.845	0.234	0.449	0.102	0.249	0.605	0.069	0.398	0.056
GER-FRA	0.358	0.809	0.140	0.518	0.104	0.259	0.616	0.072	0.271	0.086

Subperiod 3: 11 December 1989 – 31 May 2000 (2,733 observations)

	Unfiltered Raw Data					Filtered Residuals				
		Left tail		Right tail			Left tail		Right tail	
	r	$\bar{c}$	s.e.	$\bar{c}$	s.e.	r	$\bar{c}$	s.e.	$\bar{c}$	s.e.
US-UK	0.311	0.724	0.177	0.462	0.119	0.292	0.426	0.070	0.354	0.055
US-GER	0.361	0.593	0.110	0.452	0.099	0.353	0.513	0.059	0.254	0.058
US-FRA	0.275	0.575	0.109	0.345	0.123	0.266	0.473	0.066	0.279	0.065
US-JAP	0.264	0.482	0.118	0.493	0.114	0.260	0.411	0.061	0.312	0.101
UK-GER	0.570	1.043	0.166	0.850	0.142	0.510	0.770	0.160	0.089	0.103
UK-FRA	0.670	0.824	0.167	0.711	0.136	0.648	0.601	0.069	0.390	0.056
GER-FRA	0.664	1.023	0.177	0.913	0.156	0.611	0.616	0.072	0.270	0.087

Notes:

1. The stock market indices are S&P 500 (for the US), FTSE 100 (for the UK), DAX 30 (for Germany), CAC 40 (for France) and Nikkei 225 (for Japan).
2. r is the correlation coefficient calculated using all observations in the subperiod.
3. $\bar{c}$ is computed based on tail index estimation on Frechet transformed margins of daily co-exceedances of stock market returns pair.
4. The filter used is an asymmetric version of univariate GARCH,

$$R_t = w + \sqrt{h_t} Z_t , \quad h_t = a_0 + a^+ Z_{t-1}^2 h_{t-1} D_{Z_{t-1} \geq 0} + a^- Z_{t-1}^2 h_{t-1} D_{Z_{t-1} < 0} + b\, h_{t-1} ,$$

where R_t is the stock return at time *t*, a_0, a^+, a^- and b are parameters, and D_E is the indicator function that event *E* occurs.

Table 3.
Measures of extreme tail dependency, c, based on tail index estimation
on Frechet transformed margins of co-exceedances
constructed from daily stock market return pairs over three sub-periods

	Unfiltered Raw Data				Filtered Residuals			
	Left tail		Right tail		Left tail		Right tail	
	c	s.e.	c	s.e	c	s.e	c	s.e
Subperiod 1: 27 December 1968 – 19 June 1979 (2,733 observations)								
US-GER	0.239	0.024						
US-FRA			0.173	0.025				
UK-GER					0.206	0.018		
Subperiod 2: 20 June 1979 – 8 December 1989 (2,733 observations)								
US-UK	0.195	0.026	0.257	0.022				
US-JAP			0.238	0.026				
UK-GER					0.203	0.018		
UK-FRA	0.222	0.028						
GER-FRA	0.309	0.023						
Subperiod 3: 11 December 1989 – 31 May 2000 (2,733 observations)								
US-UK	0.275	0.028						
UK-GER	0.421	0.033	0.361	0.027	0.203	0.018		
GER-FRA	0.476	0.041	0.413	0.033				

Notes:
1. The stock market indices are S&P 500 (for the US), FTSE 100 (for the UK), DAX 30 (for Germany), CAC 40 (for France) and Nikkei 225 (for Japan).
2. c is computed based on tail index estimation on Frechet transformed margins of daily co-exceedances of stock market returns pair, and the assumption that $\bar{c}$ estimated for Table 2 is equal to 1.
3. The filter used is an asymmetric version of univariate GARCH,

$$R_t = w + \sqrt{h_t} Z_t, \quad h_t = a_0 + a^+ Z_{t-1}^2 h_{t-1} D_{Z_{t-1} \geq 0} + a^- Z_{t-1}^2 h_{t-1} D_{Z_{t-1} < 0} + b\, h_{t-1},$$

where R_t is the stock return at time *t*, a_0, a^+, a^- and b are parameters, and D_E is the indicator function that event *E* occurs.