# DISCUSSION PAPER SERIES

No. 2757

## PRICE STABILITY AS A NASH EQUILIBRIUM IN MONETARY OPEN ECONOMY MODELS

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***INTERNATIONAL MACROECONOMICS***

Centre for Economic Policy Research

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ISSN 0265-8003

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Discussion Paper No. 2757
April 2001

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CEPR Discussion Paper No. 2757

April 2001

# ABSTRACT

## Price Stability as a Nash Equilibrium in Monetary Open-Economy Models*

A two-country dynamic general equilibrium model with imperfect competition and price stickiness is considered. This work shows the conditions under which price stability can implement the flexible-price allocation as a Nash equilibrium. This is possible if and only if both countries maintain a certain positive degree of monopolistic competition. In such equilibrium, the monetary policy-makers have no incentive to surprise price setters *ex post*.

JEL Classification: E52, F41
Keywords: Nash equilibrium, open economy, optimal monetary policy, price stability

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*We are indebted to Michael Woodford for helpful and detailed comments. We thank also Jinill Kim and Paolo Pesenti. All errors are our own. The views expressed herein are those of the authors and should not be interpreted as those of the Bank of England.

Submitted 15 February 2001

# NON-TECHNICAL SUMMARY

There is a large consensus among policy-makers and students of monetary policy that price stability should be the main objective of a central bank. This is a desirable goal of monetary policy insofar as it can induce an efficient allocation of resources across different uses and times. An increasing literature on monetary policy evaluation has started to address the issue of optimal monetary policy in stochastic general equilibrium models with imperfect competition and price stickiness. As shown in closed economy models by Woodford (1999), the case for price stability is quite robust once necessary qualifications are considered.

In this Paper, we analyse a two-country general equilibrium model with imperfect competition and price stickiness. The open economy environment enriches the spectrum of analysis to non-cooperative *versus* cooperative solutions. A further arising issue is whether monetary policy should also control for movements in the exchange rate or can only focus on the stabilization of a domestic price index. In fact, inefficient fluctuations of the exchange rate can create a misallocation of resources across countries.

We find necessary and sufficient conditions for price stability – defined as a stable level of the GDP price index – to be a Nash equilibrium. This allocation is supported by positive degrees of monopolistic distortions. Moreover, the fluctuations of the economy will reproduce the same fluctuations that would arise if the case prices were perfectly flexible. Thus there is no further need to stabilize the exchange rate once each monetary policy-maker controls for its domestic price level.

There is a natural intuition on why the stable-price allocation should be supported by a positive degree of monopolistic competition.

In fact, were this allocation supported by a competitive level – in which real wages are equated to the marginal rate of substitution between consumption and labour – each policy-maker would have an incentive to deflate. The contraction in consumption would reduce both the marginal utility of consumption and the disutility of production, but the latter would be further reduced by the appreciation of the terms of trade.

We show our main propositions both under a special case, with isoelastic utility functions and a quadratic approximation of the welfare criteria, as well as under a general case, with general utility functions and without any approximation. In the former case, we are able to provide a correct evaluation of the second-order approximation of the welfare by using only a log-linear approximation to the equilibrium conditions.

Moreover, we consider both the cases in which prices are set according to a Calvo-style price-setting behaviour and in which all prices are fixed one period in advance. In order to unify both frameworks, we introduce the artificial definition of 'notional' price, as the price that would be set by a flexible-price agent that takes as given the evolution of the equilibrium implied by the price mechanism considered. The strategy space is then described in terms of notional or actual inflation. It is worth noting that there are other strategies that can support the flexible-price allocation in a Nash equilibrium. Indeed, with all prices fixed in advance, Devereux and Engel (2000) and Obstfeld and Rogoff (2000) show that specific money rules can support such allocation in a pre-commitment framework, in which any monetary authority's incentive to surprise *ex post* price setters is ruled out. Here, instead, we focus on an equilibrium which is time-consistent given that each policy-maker maintains the domestic price stable.

The possibility that a non-cooperative allocation can support the flexible price outcome is certainly desirable. In our context, this feature is more appealing given that the strategy of each policy-maker is rather simple and can be described by the complete stabilization of the domestic price level. Also, in a cooperative framework, a central planner would choose to reproduce the same flexible-price fluctuations while maintaining fixed domestic prices. A complete elimination of the monopolistic distortions is, however, needed. The enforcement of such equilibrium is an open issue.

# 1 Introduction

There is a large consensus among policymakers and students of monetary policy that price stability should be the main objective of a Central Bank. This is a desirable goal of monetary policy insofar it can induce an efficient allocation of resources across different uses and times. An increasing literature on monetary policy evaluation has started to address the issue of optimal monetary policy in stochastic general-equilibrium models with imperfect competition and price stickiness. As shown in closed-economy models by Woodford (1999), the case for price stability is quite robust once necessary qualifications are considered.

In this paper, we analyze a two-country general equilibrium model with imperfect competition and price stickiness. The open-economy environment enriches the spectrum of analysis to non-cooperative versus cooperative solutions. A further arising issue is whether monetary policy should also control for movements in the exchange rate or can only focus on the stabilization of a domestic price index. In fact inefficient fluctuations of the exchange rate can create a misallocation of resources across countries.

We find necessary and sufficient conditions for price stability – defined as a stable level of the GDP price index – to be a Nash equilibrium. This allocation is supported by positive degrees of monopolistic distortions. Moreover, the fluctuations of the economy will reproduce the same fluctuations that would arise in the case prices were perfectly flexible. Thus there is no further need to stabilize the exchange rate once each monetary policymaker controls for its domestic price level.

There is a natural intuition on why the stable-price allocation should be supported by a positive degree of monopolistic competition.[1] In fact, were

[1]Corsetti and Pesenti (2000), Tille (2000), Benigno (1999a) provide an intuition for this result in a perfect foresight model with unanticipated money supply shocks.

this allocation supported by a competitive level – in which real wages are equated to the marginal rate of substitution between consumption and labor – each policymaker would have an incentive to deflate. The contraction in consumption would reduce both the marginal utility of consumption and the disutility of production, but the latter would be further reduced by the appreciation of the terms of trade.

We show our main propositions both under a special case, with isoelastic utility functions and a quadratic approximation of the welfare criteria, as well as under a general case, with general utility functions and without any approximation.[2] In the former case, we are able to provide a correct evaluation of the second-order approximation of the welfare by using only a log-linear approximation to the equilibrium conditions.[3]

Moreover, we consider both the cases in which prices are set according to a Calvo-style price-setting behavior and in which all prices are fixed one-period in advance. In order to unify both frameworks, we introduce the artificial definition of 'notional' price, as the price that would be set by a flexible-price agent that takes as given the evolution of the equilibrium implied by the price mechanism considered. The strategy space is then described in terms of notional or actual inflation. It is worth noting that there are other strategies that can support the flexible-price allocation in a Nash equilibrium. Indeed, with all prices fixed in advance, Devereux and Engel (2000) and Obstfeld and Rogoff (2000) show that specific money rules can support such allocation in a pre-commitment framework, in which any monetary authorities' incentive to surprise ex-post price setters is ruled out. Here, instead, we focus on an equilibrium which is time-consistent given that each policymaker maintains the domestic price stable.

The possibility that a non-cooperative allocation can support the flexible price outcome is certainly desirable. In our context, this feature is more appealing given that the strategy of each policymaker is rather simple and

[2] In the former case, we follow Rotemberg and Woodford (1997) and Woodford (1999).

[3] Problems with quadratic approximation of the welfare are discussed in Woodford (1999) and Kim J. and S.H. Kim (1999).

can be described by the complete stabilization of the domestic price level. As well, in a cooperative framework, a central planner would choose to reproduce the same flexible-price fluctuations while maintaining domestic prices fixed. However, a complete elimination of the monopolistic distortions is needed. The enforcement of such equilibrium is an open issue.

The structure of this work is the following. Section 2 discusses the key elements of the model. Section 3 presents the welfare criterion. Section 4 introduces the hypotheses on the price mechanisms. Section 5 describes the strategic game. Section 6 presents the main results under the restricted hypothesis of isoelastic utility function. Section 7 generalizes. Section 8 presents some additional remarks. Finally, section 9 concludes.

# 2 Key Elements

The model belongs to a recent class of stochastic general equilibrium models with imperfect competition and price stickiness that have been used for normative analysis.[4] The detailed analysis is presented in Benigno (1999b). In this section we emphasize the main structure of the model and the crucial assumptions.

**Condition 1** *The world economy is populated by a continuum of agents on the interval* $[0,1]$*. The population on the segment* $[0,n)$ *belongs to the country* $H$*, while the segment* $[n,1]$ *belongs to* $F$*. A generic agent* $j$ *belonging to the world economy is both producer and consumer: a producer of a single differentiated product and a consumer of all the goods produced in both countries* $H$ *and* $F$*. Thus all goods produced are traded between countries. Preferences of the generic household* $j$ *are given by*

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ U(C_s^j) - V(y_s^j, z_s^i) \right],$$

[4] Closed economy models are described in Woodford (1996, 1999). Devereux and Engel (2000), Obstfeld and Rogoff (2000a,2000b) consider open-economy models, in which all prices are fixed one-period in advance.

*where the upper index $j$ denotes a variable that is specific to agent $j$, while the upper index $i$ denotes a variable that is specific to country $i$.*[5] *$U$ and $V$ are generic utility function, while $C^j$ is an index of consumption bundles, $y^j$ is the production of the differentiated good produced by agent $j$, while $z^i$ is a country-specific shock. We have that $i = H$ if $j \in [0, n)$, while $i = F$ if $j \in [n, 1]$. $E_t$ denotes the expectation conditional on the information set at date $t$, while $\beta$ is the intertemporal discount factor, with $0 < \beta < 1$.*

**Condition 2** *The index $C^j$ is defined as*

$$C^j \equiv \frac{(C_H^j)^n (C_F^j)^{1-n}}{n^n (1-n)^{1-n}} \tag{1}$$

*and $C_H^j$ and $C_F^j$ are indexes of consumption across the continuum of differentiated goods produced respectively in country $H$ and $F$. Specifically,*

$$C_H^j \equiv \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_o^n c^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^j \equiv \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}. \tag{2}$$

*Here $\sigma$, which is assumed greater than one, is the elasticity of substitution across goods produced within a country, while the elasticity of substitution between the bundles $C_H$ and $C_F$ is 1.*

Given the indexes of consumption (1) and (2), the appropriate consumption-based price indexes are defined as

$$P^i \equiv (P_H^i)^n (P_F^i)^{1-n},$$

$$P_H^i \equiv \left[ \left( \frac{1}{n} \right) \int_o^n p^i(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \qquad P_F^i \equiv \left[ \left( \frac{1}{1-n} \right) \int_n^1 p^i(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}},$$

where $p^i(h)$ is the price of good $h$ produced in country $H$ sold in the market of country $i$ and $p^i(f)$ is the price of good $f$ produced in country $F$ sold in the market of country $i$.

[5] Here we consider that households derive liquidity services from holding real money balances, which, however, are small in size and close to zero.

**Condition 3** *We assume that there is producer-currency-pricing and that there are no transaction costs in transporting goods across countries. The law of one price holds with $p^H(h) = S \cdot p^F(h)$ and $p^H(f) = S \cdot p^F(f)$, where $S$ is the nominal exchange rate. Given these assumptions and given the structure of the preferences, it is also the case that purchasing power parity holds, i.e. $P^H = SP^F$ and that $P^H_H = SP^F_H$ and $P^H_F = SP^F_F$.*

**Condition 4** *We assume that the sellers of the differentiated product act in a market characterized by monopolistic competition.*

**Condition 5** *At a country level, the asset markets are complete; while at an international level are either complete or incomplete. In the latter case, we assume a zero initial condition on the holdings of the internationally traded bond.*

In what follows, we define the terms of trade $T$ of country $F$ as the ratio of the price of the bundle of goods produced in region $F$ relative to the price of the bundle imported from region $H$. We have then $T \equiv P^F_F/P^F_H = P^H_F/P^H_H$. In a flexible-price equilibrium, by using conditions 1-5, we obtain that consumption and terms of trade are determined by

$$(1-\Phi^H)U_C(C_t) = T_t^{1-n} V_y\left(T_t^{1-n} C_t, z_t^H\right), \tag{3}$$

$$(1-\Phi^F)U_C(C_t) = T_t^{-n} V_y\left(T_t^{-n} C_t, z_t^F\right), \tag{4}$$

at "all dates" $t$ while their steady-state values are determined by[6]

$$(1-\Phi^H)U_C(\overline{C}) = \overline{T}^{1-n} V_y\left(\overline{T}^{1-n}\overline{C}, \overline{z}^H\right), \tag{5}$$

$$(1-\Phi^F)U_C(\overline{C}) = \overline{T}^{-n} V_y\left(\overline{T}^{-n}\overline{C}, \overline{z}^F\right). \tag{6}$$

where we have defined the overall degrees of monopolistic distortions, $\Phi^H$ and $\Phi^F$, as the degrees of monopolistic competition corrected for the distorting

[6] With the expression "at all dates t" we mean "at all times $t$ and across all states of nature at time $t$".

taxation

$$
\begin{aligned}
(1-\Phi^H) &\equiv \frac{\sigma-1}{\sigma}(1-\tau^H), \\
(1-\Phi^F) &\equiv \frac{\sigma-1}{\sigma}(1-\tau^F).
\end{aligned}
$$

In order to get some intuition on conditions (3) and (4), we define the Home and Foreign real marginal costs

$$
\begin{aligned}
mc_t^H &= \frac{T_t^{1-n} V_y\left(T_t^{1-n} C_t, z_t^H\right)}{U_C(C_t)}, \\
mc_t^F &= \frac{T_t^{-n} V_y\left(T_t^{-n} C_t, z_t^F\right)}{U_C(C_t)}.
\end{aligned}
$$

In the flexible-price allocation real marginal costs are constant and equal to the factor $(1-\Phi)$. Or, with an alternative interpretation, real wages are above the marginal rate of substitution between consumption and labor by a constant. As $\Phi$ approaches 0, the economy reaches a competitive allocation in the factor market, in which real wages are equated to the marginal rate of substitution, between consumption and leisure, and real marginal costs are unitary.

# 3 Welfare Criterion

As it is common in the recent micro-founded literature on monetary policy evaluation, we assume that the policymakers are benevolent and maximize the discounted sum of the utility flows of the consumers. The welfare criteria of the Home and Foreign Central Bank are defined as

$$
\begin{aligned}
W^H &\equiv \mathrm{E}_0 \left\{ \sum_{j=0}^{\infty} \beta^j w_j^H \right\}, \\
W^F &\equiv \mathrm{E}_0 \left\{ \sum_{j=0}^{\infty} \beta^j w_j^F \right\},
\end{aligned}
$$

where the average utility flow among all the households belonging to country $H$ is defined as

$$
w_t^H \equiv U(C_t) - \frac{\int_0^n v(y_t(h), z_t^H) dh}{n},
$$

while that of country $F$ is

$$w_t^F \equiv U(C_t) - \frac{\int_n^1 v(y_t(f), z_t^F)df}{1-n}.$$

In the analysis that follows we will also make use of a second-order expansion of the utility flows. The details are presented in the appendix. We obtain

$$\begin{aligned} w_t^H &= U_C\overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1-\Phi^H)\cdot\widehat{Y}_{H,t} - \frac{(1-\Phi^H)}{2}\cdot[\widehat{Y}_{H,t}]^2 + \\ &\quad -(1-\Phi^H)\frac{\eta^H}{2}\cdot[\widehat{Y}_{H,t}]^2 - \frac{(1-\Phi^H)}{2}(\sigma^{-1}+\eta^H)\cdot \mathrm{var}_h\widehat{y}_t(h) + \\ &\quad (1-\Phi^H)\eta^H\cdot\widehat{Y}_{H,t}\overline{Y}_t^H] + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \tag{7}$$

for country $H$ while for country $F$ we get

$$\begin{aligned} w_t^F &= U_C\overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1-\Phi^F)\cdot\widehat{Y}_{F,t} - \frac{(1-\Phi^F)}{2}\cdot[\widehat{Y}_{F,t}]^2 + \\ &\quad -(1-\Phi^F)\frac{\eta^F}{2}\cdot[\widehat{Y}_{F,t}]^2 - \frac{(1-\Phi^F)}{2}(\sigma^{-1}+\eta^F)\cdot \mathrm{var}_f\widehat{y}_t(f) + \\ &\quad +(1-\Phi^F)\eta^F\cdot\widehat{Y}_{F,t}\overline{Y}_t^F] + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \tag{8}$$

where $\rho$, $\eta^i$, with $i = H, F$ are the inverse of the intertemporal elasticity of substitution of consumption and of the elasticity of labor supply. It is worth noting that in deriving equations (7) and (8), we have not imposed any assumption on the price-setting mechanism.

# 4 Sticky Prices

Finally, in this section, we assume that prices are sticky. We analyze both a static context in which all sellers fix their prices one-period in advance and a dynamic context in which sellers set their prices according to a Calvo-style price-setting model. In the former case, the prices that are set at time $t-1$ for time $t$ exploit all the information available at time $t-1$, according to the following conditions.

**Condition 6** *With prices fixed one-period in advance, the optimal pricing decisions, respectively for country $H$ and $F$, imply that*

$$E_{t-1}\{[(1-\Phi^H)U_C(C_t)T_t^{n-1} - V_y(T_t^{1-n}C_t, z_t^H)]T_t^{1-n}C_t\} = 0, \qquad (9)$$
$$E_{t-1}\{[(1-\Phi^F)U_C(C_t)T_t^{n} - V_y(T_t^{-n}C_t, z_t^F)]T_t^{-n}C_t\} = 0. \qquad (10)$$

Sellers maximize their expected profits. The flexible-price conditions (3) and (4) hold in expected value, with appropriate weights for each contingency. Instead, in a Calvo's model, each firm faces a fixed probability $(1-\alpha^i)$, with $i = H, F$, of adjusting its price at a certain date independently of the time has elapsed since the last adjustment. We obtain the following conditions.

**Condition 7** *In a Calvo-style price-setting model, the optimal pricing decisions for sellers that are changing their prices – $\widetilde{p}_t^H(h)$ and $\widetilde{p}_t^F(f)$ respectively for the Home and Foreign countries– at a generic time $t$ are*

$$E_t\sum_{k=0}^{\infty}(\alpha^H\beta)^k\left\{\left[(1-\Phi^H)U_C(C_{t+k})\frac{\widetilde{p}_t^H(h)}{P_{H,t+k}^H}T_{t+k}^{n-1} - V_y(\widetilde{y}_{t,t+k}^d(h), z_{t+k}^H)\right]\widetilde{y}_{t,t+k}^d(h)\right\} = 0, \qquad (11)$$

$$E_t\sum_{k=0}^{\infty}(\alpha^F\beta)^k\left\{\left[(1-\Phi^F)U_C(C_{t+k})\frac{\widetilde{p}_t^F(f)}{P_{F,t+k}^F}T_{t+k}^{n} - V_y(\widetilde{y}_{t,t+k}^d(f), z_{t+k}^F)\right]\widetilde{y}_{t,t+k}^d(f)\right\} = 0, \qquad (12)$$

*where*

$$\widetilde{y}_{t,t+k}^d(h) = \left(\frac{\widetilde{p}_t^H(h)}{P_{H,t+k}^H}\right)^{-\sigma} T_{t+k}^{1-n}C_{t+k}.$$

$$\widetilde{y}_{t,t+k}^d(f) = \left(\frac{\widetilde{p}_t^F(f)}{P_{F,t+k}^F}\right)^{-\sigma} T_{t+k}^{-n}C_{t+k},$$

*and condition (11) and (12) hold at all dates $t$.*

In the analysis that follows, we are primarily interested in describing a strategic game in which each policymaker takes an action expressed in terms of the inflation rate. This is possible in the Calvo-style price-setting model. In fact, the inflation rate can assume values different from zero, given that a fraction of sellers can change their prices at a certain date. However, in the model where all prices are fixed one-period in advance, the domestic inflation rate is zero, by definition. We introduce the fictitious role of notional price. The notional price is the price that would be set by a seller that can choose its price freely in each period and in each contingency, taking as given the evolution of the other macroeconomic variables –that are relevant for its pricing decision– as they were following the equilibrium path implied by the price-mechanism considered. In other words, the notional seller acts as a flexible-price seller which has no influence on the equilibrium path of the variables.

**Condition 8** *The notional prices, $p^N(h)$ and $p^N(f)$ respectively for the Home and Foreign countries, are*

$$\frac{p_t^N(h)}{P_{H,t}^H} = \frac{1}{1-\Phi^H} \frac{T_t^{1-n} V_y\left(\left(\frac{p_t^N(h)}{P_{H,t}^H}\right)^{-\sigma} T_t^{1-n} C_t, z_t^H\right)}{U_C(C_t)} \tag{13}$$

$$\frac{p_t^N(f)}{P_{F,t}^F} = \frac{1}{1-\Phi^F} \frac{T_t^{-n} V_y\left(\left(\frac{p_t^N(f)}{P_{F,t}^F}\right)^{-\sigma} T_t^{-n} C_t, z_t^F\right)}{U_C(C_t)} \tag{14}$$

*at each date $t$ where the sequences $\{P_{H,t}^H, P_{F,t}^F, T_t, C_t\}$ follow the equilibrium path implied by the price mechanism considered. Notional inflation rates are defined as*

$$\begin{aligned}
\pi_{H,t}^N &= \frac{p_t^N(h)}{P_{H,t-1}^H} - 1, \\
\pi_{F,t}^N &= \frac{p_t^N(f)}{P_{F,t-1}^F} - 1.
\end{aligned}$$

# 5 Strategic Game

In this section we characterize the strategic game played by the two policymakers. Each policymaker chooses a strategy $s^i$ that belongs to its proper set of strategies $S^i$, with $i = H, F$. We characterize the property of the strategy set.

**Definition 9** *A rule is an assignment that relates a set of endogenous variables at a certain date $t$ to other variables –endogenous or exogenous, even of different dates– or to numerical values. A generic strategy $s^i \in S^i$ is a commitment to a sequence of rules each for each date $t$.*

As an example, imposing $\pi_t^H = 0$ at date $t$ is a rule, while $\pi_t^H = 0$ at *all* dates $t$ is a strategy. We further specify some desirable characteristics of the set of strategies.

**Definition 10** *A couple of strategies $(s^H, s^F)$ is feasible if they imply a unique rational expectations equilibrium for the set of equilibrium conditions derived from the household-firm's optimizing behavior.*

We consider a game in which each policymaker can commit to its strategy in order to maximize its own welfare function, respectively $W^H$ and $W^F$ for the Home and Foreign policymakers. In the solution of the strategic game, we use the notion of Nash equilibrium.

**Definition 11** *A couple of feasible strategies $(s_*^H, s_*^F)$ is a Nash equilibrium if $s_*^H \in \arg\max W^H(s^H, s_*^F)$ and $s_*^F \in \arg\max W^F(s_*^H, s^F)$.*

In the analysis that follows, the following lemmas will be useful.

**Lemma 12** *With prices fixed one-period in advance, a strategy of zero Home notional inflation implies condition (3) at all dates $t$ and a strategy of zero Foreign notional inflation implies condition (4) at all dates $t$.*

**Proof.** We prove this lemma only for the case of the Home country, the other follows in a specular way. From the definition of notional price we obtain

$$\frac{p_t^N(h)}{P_{H,t-1}^H} = \frac{1}{1-\Phi^H} \frac{T_t^{1-n} V_y \left( \left( \frac{p_t^N(h)}{P_{H,t-1}^H} \right)^{-\sigma} T_t^{1-n} C_t, z_t^H \right)}{U_C(C_t)},$$

where we have imposed $P_{H,t}^H = P_{H,t-1}^H$, given that all prices are fixed one-period in advance. It follows directly that by setting $\frac{p_t^N(h)}{P_{H,t-1}^H} = 1$ condition (3) holds. ∎

Moreover, in the Calvo-style price-setting model, we obtain the following results.

**Lemma 13** *With a Calvo-style price-setting model, the strategy of zero Home producer inflation implies condition (3) at all dates t and a strategy of zero Foreign producer inflation implies condition (4) at all dates t.*

**Proof.** We prove this lemma only for the case of the Home country, the other follows in a specular way. If the Home producer inflation is zero at all dates $t$ we have that $\widetilde{p}_t^H(h)/P_{H,t+k}^H$ is 1 for all $k$ and at all dates $t$. We can then write (11) as

$$\begin{aligned}
0 &= [(1-\Phi^H)U_C(C_t)T_t^{n-1} - V_y(T_t^{1-n}C_t, z_t^H)]T_t^{1-n}C_t + \\
&\quad +(\alpha^H\beta)\mathrm{E}_t\{\sum_{k=0}^{\infty}(\alpha^H\beta)^k\left\{\left[\begin{array}{c}(1-\Phi^H)U_C(C_{t+1+k})T_{t+1+k}^{n-1}+\\ -V_y(T_{t+1+k}^{1-n}C_{t+1+k}, z_{t+1+k}^H)\end{array}\right]T_{t+1+k}^{1-n}C_{t+1+k}\right\},
\end{aligned}$$

where the term in expectations is zero because (11) holds at all dates $t$ and in particular it holds under an equilibrium path of zero inflation. Then it follows that

$$(1-\Phi^H)U_C(C_t)T_t^{n-1} = V_y(T_t^{1-n}C_t, z_t^H)$$

at date $t$. The argument can be extended at all dates $t$. ■

**Lemma 14** *In a Calvo-style price-setting model the strategy of zero producer inflation coincides with the strategy of zero notional inflation.*

# 6 Results: CES Utility Functions

In this section, we restrict our analysis to particular utility functions and we consider only small deviations of the state variables from their steady state. We consider a log-linear approximation to the equilibrium conditions and a second-order approximation of the welfare criteria as in equations (7) and (8). Even though this analysis is not general, it is a good starting point to get some insights on the more general case. Most importantly, it shows an interesting case in which a second-order approximation of the welfare criteria can be correctly evaluated only by relying on a first-order approximation to the equilibrium conditions. In a closed economy, Woodford (1999) has shown that a log-linear approximation can be sufficient to evaluate a second-order approximation of the welfare criterion, if in the latter approximation the magnitude of the first-order terms is kept small by appropriate taxation subsidies. In an open economy, things become more complicated because production is not simply equated to domestic absorption but depends also on relative prices. In this context, we are able to provide a correct evaluation because the first-order terms in the welfare approximation are kept small by the combination of the strategy of the other policymaker with an appropriate level of the overall monopolistic distortions.[7]

**Condition 15** *We assume that $U()$ and $V()$ have the following specifications*

$$\begin{aligned} U(C_s^j) &\equiv \frac{(C_s^j)^{1-\rho}}{1-\rho}, \\ V(y_s^j, z_s^i) &\equiv \frac{z_s^i (y_s^j)^{\upsilon^i}}{\upsilon^i}, \end{aligned}$$

*where $\rho$ is the intertemporal elasticity of substitution in consumption while $\eta^i \equiv \upsilon^i - 1$, with $\upsilon^i \geq 1$, is the country-specific elasticity of labor supply.*

Note that this restriction nests the specifications of Devereux and Engel (2000) and Obstfeld and Rogoff (2000). Using the assumption (16), we can write the conditions that determine the flexible-price allocation for consumption and the terms of trade as

$$(1 - \Phi^H) C_t^{-\rho} = T_t^{1-n} (T_t^{1-n} C_t)^{\eta^H} z_t^H, \tag{15}$$

$$(1 - \Phi^F) C_t^{-\rho} = T_t^{-n} (T_t^{-n} C_t)^{\eta^F} z_t^F. \tag{16}$$

[7] See J. Kim and S.H. Kim (1999) for a discussion of the problems in approximating welfare in an open economy.

We can then obtain a closed-form solution for the terms of trade and consumption both in the steady-state and in the flexible price equilibrium.[8] Furthermore, in the flexible-price equilibrium, the deviations of the log of consumption and the terms of trade from their steady state can be written in an exact form as

$$\begin{aligned}
\widetilde{C}_t &\equiv \frac{\eta^H(1+\eta^F)n\overline{Y}_t^H + \eta^F(1+\eta^H)(1-n)\overline{Y}_t^F}{n(\rho+\eta^H)(1+\eta^F)+(1-n)(\rho+\eta^F)(1+\eta^H)}, \\
\widetilde{T}_t &\equiv \frac{\eta^F(\rho+\eta^H)\overline{Y}_t^F - \eta^H(\rho+\eta^F)\overline{Y}_t^H}{n(\rho+\eta^H)(1+\eta^F)+(1-n)(\rho+\eta^F)(1+\eta^H)}.
\end{aligned}$$

Equipped with these results, we start analyzing the strategic game. In the next proposition we show that a strategy of zero producer inflation can be a Nash equilibrium.

**Proposition 16** *With Calvo-style price-setting behavior, given lemma 13 and 14 and conditions 1-5, 7 and 15, if the overall degrees of monopolistic competition assume the values*

$$\begin{aligned}
\Phi^{*H} &= \frac{D^H}{1+D^H} > 0, \qquad \textit{with} \qquad D^H \equiv \frac{1-n}{n}\frac{\rho+\eta^F}{1+\eta^F} \\
\Phi^{*F} &= \frac{D^F}{1+D^F} > 0, \qquad \textit{with} \qquad D^F \equiv \frac{n}{1-n}\frac{\rho+\eta^H}{1+\eta^H}
\end{aligned}$$

*the strategy of zero producer inflation (or zero notional inflation) is a Nash equilibrium.*

**Proof.** First, we remind that the strategy of zero notional inflation and zero producer inflation coincide in the Calvo's model. We show that given that one country is following a strategy of zero producer inflation, then the

[8] Note how the conditions stated are crucial for the closed-form solution. The Cobb-Douglas assumption allows to write the terms of trade in a closed-form. The assumption on the structure of the asset markets allows to neglect the dynamics of the current account. The only assumption that could be relaxed is that of producer-currency-pricing. In fact, given the structure of the preferences producer-currency-pricing and local-currency-pricing deliver the same allocation under flexible prices.

strategy of zero producer inflation is also optimal for the other policymaker and viceversa. If policymaker F follows the strategy of zero producer inflation, then from lemma 13 and condition 15 it follows that

$$(1-\Phi^F)C_t^{-\rho} = T_t^{-n}(T_t^{-n}C_t)^{\eta^F} z_t^F,$$

at each date $t$, which in a log-linear *exact* form implies that

$$\widehat{T}_t = \frac{(\rho+\eta^F)}{n(1+\eta^F)}\widehat{C}_t - \frac{\eta^F}{n(1+\eta^F)}\overline{Y}_t^F. \tag{17}$$

Condition (17) in (7), combined with the value of $\Phi^{*H}$, implies that the linear term $\widehat{C}_t - (1-\Phi^{*H})\cdot \widehat{Y}_{H,t}^d$ disappears.

Furthermore we can write

$$\begin{aligned}
(\widehat{Y}_{H,t})^2 &= (1+D^H)^2\cdot \widehat{C}_t^2 - 2(1+D^H)\cdot \frac{1-n}{n}\frac{\eta^F}{1+\eta^F}\cdot \widehat{C}_t\overline{Y}_t^F + \text{t.i.p},\\
\widehat{Y}_{H,t}\overline{Y}_t^H &= (1+D^H)\cdot \widehat{C}_t\overline{Y}_t^H + \text{t.i.p.}
\end{aligned}$$

From which we can simplify $w_t^H$ to

$$\begin{aligned}
w_t^H = {} & U_C\overline{C}[\frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1+\eta^H)\frac{(1-\Phi^{*H})}{2}\cdot(1+D^H)^2\cdot\widehat{C}_t^2 + \\
& +(1-\Phi^{*H})\frac{(1-n)}{n}\frac{\eta^F(1+\eta^H)}{1+\eta^F}\cdot(1+D^H)\cdot\widehat{C}_t\overline{Y}_t^F + \\
& +(1-\Phi^{*H})\eta^H\cdot(1+D^H)\cdot\widehat{C}_t\overline{Y}_t^H + \\
& -\frac{(1-\Phi^{*H})}{2}(\sigma^{-1}+\eta^H)\cdot \text{var}_h\widehat{y}_t(h)] + \text{t.i.p.} + o(\|\xi\|^3),
\end{aligned}$$

Noting that $(1-\Phi^{*H})\cdot(1+D^H) = 1$, we can further simplify to

$$\begin{aligned}
w_t^H = {} & U_C\overline{C}[-\frac{1}{2n(1+\eta^F)}[n(\rho+\eta^H)(1+\eta^F)+(1-n)(\rho+\eta^F)(1+\eta^H)]\widehat{C}_t^2 + \\
& +\frac{1}{n(1+\eta^F)}[\eta^H(1+\eta^F)n\overline{Y}_t^H + \eta^F(1+\eta^H)(1-n)\overline{Y}_t^F]\cdot\widehat{C}_t \\
& -\frac{(1-\Phi^H)}{2}(\sigma^{-1}+\eta^H)\cdot \text{var}_h\widehat{y}_t(h)] + \text{t.i.p.} + o(\|\xi\|^3),
\end{aligned}$$

and to

$$\begin{aligned}
w_t^H = {} & U_C\overline{C}[-\frac{1}{2n(1+\eta^F)}[n(\rho+\eta^H)(1+\eta^F)+(1-n)(\rho+\eta^F)(1+\eta^H)](\widehat{C}_t-\widetilde{C}_t)^2 + \\
& -\frac{(1-\Phi^H)}{2}(\sigma^{-1}+\eta^H)\cdot \text{var}_h\widehat{y}_t(h)] + \text{t.i.p.} + o(\|\xi\|^3),
\end{aligned}$$

using the definition of $\widetilde{C}$.

Following Woodford (1999) and using the assumption of Calvo-style price-setting behavior, we can write the welfare criterion $W^H$ as

$$W^H = -\Omega \mathrm{E}_0 \sum_{j=0}^{\infty} \beta^j [\Lambda(\widehat{C}_j - \widetilde{C}_j)^2 + (\pi_j^H)^2] + \text{t.i.p.} + o(\|\xi\|^3),$$

where $\Omega$ and $\Lambda$ are functions of the structural parameters of the model. Given the zero producer inflation strategy of the Foreign policymaker, the optimal policy for the Home policymaker is to stabilize its producer price inflation at all dates $t$. The other side of the construction of the Nash equilibrium follows in a specular way. ■

This proposition needs some further comments. We have shown that, by appropriately choosing the overall degrees of monopolistic competition, the strategy of zero producer inflation is a Nash equilibrium. Furthermore, this equilibrium implements the flexible-price allocation.

There is an intuition on why such equilibrium can only be supported at a positive level of monopolistic competition. In an open-economy non-cooperative game, the complete elimination of the monopolistic distortions is in contrast with the strategic use of the terms of trade. In fact each policymaker can use the terms of trade as a tool that shifts the "burden" of production to the other country. Were the equilibrium coincident with the competitive level, in which all the monopolistic distortions are eliminated, each country would gain by contracting inflation. The reduction in the utility of consumption would be more than offset by the reduction in the disutility of producing the goods, because of the appreciation of the terms of trade. It follows that a time consistent zero-inflation equilibrium can be sustained only at a positive overall level of monopolistic competition.

It is worth stressing that there might exist other strategies that implement the flexible-price allocation in a Nash equilibrium. Not necessarily these strategies require the same level of monopolistic competition, $\Phi^{*H}$ and $\Phi^{*F}$, nor they imply that conditions (3) and (4) can be taken as given in the strategic game. In our context, it is sufficient that each policymaker commits to stabilize its domestic inflation taking as given the strategy of the other policymaker in order to reproduce, in a strategic game, the fluctuations of the economy that would arise under flexible prices. Finally, this allocation is time consistent, in the sense that monetary policymakers have no incentive to surprise in an unexpected way price setters, i.e. to change their strategies at future dates $t$, once the commitment is taken.[9]

Here we move to the case in which prices are fixed one-period in advance. It is sensible to assume that the welfare functions are

$$\begin{aligned} W^H &= \mathrm{E}_{t-1} w_t^H, \\ W^F &= \mathrm{E}_{t-1} w_t^F, \end{aligned}$$

and that the strategic game lasts one period.

**Proposition 17** *With prices fixed one-period in advance, given lemma 12 and conditions 1-6 and 15, if the degrees of monopolistic competition assume the values $\Phi^{*H}$ and $\Phi^{*F}$, the strategy of zero notional inflation is a Nash equilibrium.*

The proof follows proposition 16 with the appropriate qualifications. Here we show, instead, that if both policymakers are committed to rules that avoid any anticipated variations in the average level of variables, i.e. $\mathrm{E}_{t-1}\widehat{C}_t = 0$, the zero notional inflation strategy is a Nash equilibrium independently of the overall degrees of monopolistic competition, under the further assumption that $\rho = 1$. Just recall equation (7)

$$\begin{aligned} w_t^H &= U_C \overline{C}[\widehat{C}_t - (1-\Phi^H) \cdot \widehat{Y}_{H,t} + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - \frac{(1-\Phi^H)}{2} \cdot [\widehat{Y}_{H,t}]^2 + \\ &\quad -(1-\Phi^H)\frac{\eta^H}{2} \cdot [\widehat{Y}_{H,t}]^2 - \frac{(1-\Phi^H)}{2}(\sigma^{-1} + \eta^H) \cdot \mathrm{var}_h \widehat{y}_t(h) + \\ &\quad (1-\Phi^H)\eta^H \widehat{Y}_{H,t}\overline{Y}_t^H] + \text{t.i.p.} + o(\|\xi\|^3). \end{aligned}$$

[9] It is worth noting that the conditions on $\Phi^{*H}$ and $\Phi^{*F}$ are similar to the conditions that characterize the absence of the incentive to strategically use the terms of trade in the perfect foresight model of Corsetti and Pesenti (2000), Tille (2000) and Benigno P. (1999a). However, in these frameworks the set of strategies comprises only unexpected and exogenously movements in the money supply.

Under the assumption of $\rho = 1$, we can write[10]

$$\begin{aligned} w_t^H &= U_C\overline{C}[\widehat{C}_t - (1-\Phi^H)\cdot\widehat{Y}_{H,t} - \frac{(1-\Phi^H)}{2}\cdot[\widehat{Y}_{H,t}]^2 + \\ &\quad -(1-\Phi^H)\frac{\eta^H}{2}\cdot[\widehat{Y}_{H,t}]^2 + (1-\Phi^H)\eta^H\widehat{Y}_{H,t}\overline{Y}_t^H] + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned}$$

which can be simplified (assuming that the other country is following the zero notional inflation strategy) to

$$w_t^H = U_C\overline{C}[(\Phi^H + (1-\Phi^H)D^H)\widehat{C}_t - \frac{(1-\Phi^H)}{2}\Psi(\widehat{C}_t - \widetilde{C}_t)^2 + \text{t.i.p.} + o(\|\xi\|^3).$$

where $\Psi$ is a function of the structural parameters of the model. We obtain then

$$\begin{aligned} \mathrm{E}_{t-1}w_t^H &= U_C\overline{C}[(\Phi^H + (1-\Phi^H)D^H)\mathrm{E}_{t-1}\widehat{C}_t - \frac{(1-\Phi^H)}{2}\Psi\mathrm{E}_{t-1}(\widehat{C}_t - \widetilde{C}_t)^2 \\ &\quad + \text{t.i.p.} + o(\|\xi\|^3). \end{aligned}$$

It follows that given that one country is pursuing the zero notional inflation strategy, this strategy is also optimal for the other country independently of the size of the monopolistic distortions. Furthermore the implied allocation satisfies conditions (9) and (10). However, the condition $\mathrm{E}_{t-1}\widehat{C}_t = 0$ is not necessarily incentive compatible unless the monopolistic distortions assume the values $\Phi^{*H} = 1-n$ and $\Phi^{*F} = n$. In this case the linear term would automatically disappear and the monetary policymaker would avoid to surprise systematically price setters. As it is the case of the previous proposition, there are as well other strategies that can support the flexible-price allocation in a Nash equilibrium. Indeed, in the same framework of this proposition, Devereux and Engel (2000) and Obstfeld and Rogoff (2000) find specific money rules that can support such equilibrium, in a pre-commitment framework.

[10] Note that with prices fixed one-period in advance $\mathrm{var}_h\widehat{y}_t(h) = 0$.

# 7 Results: General Case

In the previous section we have restricted the analysis to isoelastic utility functions and to an equilibrium path of the state variables that is close to the initial steady state. Here we relax these assumptions. We consider general utility functions and an exact solution to the equilibrium path of the variables. We further introduce demand shocks in the form of country-specific public expenditure shocks as

$$\begin{aligned} y(h) &= \left(\frac{p(h)}{P_H}\right)^{-\sigma} [(T)^{1-n} C^W + G^H], \\ y(f) &= \left(\frac{p(f)}{P_F}\right)^{-\sigma} [(T)^{-n} C^W + G^F], \end{aligned}$$

where $G^H$ and $G^F$ are the Home and Foreign public-expenditure shocks. As an important difference with respect to the propositions of the previous paragraph, we offer a necessary and sufficient condition for the strategies to be a Nash equilibrium.

**Proposition 18** *With Calvo-style price-setting behavior, given conditions 1-5, 7, lemma 13 and 14, the strategy of zero producer inflation rate (or of zero notional inflation rate) is a Nash equilibrium if and only if the degrees of monopolistic competition assume the values*

$$\begin{aligned} \Phi_t^{*H} &= \frac{D_t^H}{1+D_t^H} > 0, \qquad \textit{with} \qquad D_t^H \equiv \frac{1-n}{n}\frac{\widetilde{\rho}_t + \widetilde{\eta}_t^F}{1+\widetilde{\eta}_t^F}, \\ \Phi_t^{*F} &= \frac{D_t^F}{1+D_t^F} > 0, \qquad \textit{with} \qquad D_t^F \equiv \frac{n}{1-n}\frac{\widetilde{\rho}_t + \widetilde{\eta}_t^H}{1+\widetilde{\eta}_t^H}, \end{aligned}$$

*at all dates t, with*[11]

$$\widetilde{\rho}_t = -\frac{U_{CC}(\widetilde{C}_t)\widetilde{C}_t}{U_C(\widetilde{C}_t)}, \quad \widetilde{\eta}_t^H = \frac{V_{yy}(\widetilde{Y}_t^H, z_t^H)\widetilde{Y}_t^H}{V_y(\widetilde{Y}_t^H, z_t^H)} \textit{ and } \widetilde{\eta}_t^F = \frac{V_{yy}(\widetilde{Y}_t^F, z_t^F)\widetilde{Y}_t^F}{V_y(\widetilde{Y}_t^F, z_t^F)}.$$

[11] Proposition 18 and 19 can be further generalized by assuming an intratemporal elasticity of substitution different from one, while maintaining the complete intenational market assumption.

**Proof.** If country F is following the strategy of zero producer inflation, it follows that

$$(1-\Phi_t^F)U_C(C_t) = T_t^{-n}V_y\left(T_t^{-n}C_t + G_t^F, z_t^F\right), \tag{18}$$

in all states of nature at date $t$. The Home policymaker is maximizing the welfare criterion

$$W^H \equiv \mathrm{E}_0\left\{\sum_{j=0}^{\infty}\beta^j w_j^H\right\}, \tag{19}$$

where the average utility flow among all the households belonging to country $H$ is

$$w_t^H = U(C_t) - \frac{\int_0^n v(\left(\frac{p_t^H(h)}{P_{H,t}^H}\right)^{-\sigma}[(T_t)^{1-n}C_t^W + G_t^H], z_t^H)dh}{n}, \tag{20}$$

while

$$(P_{H,t}^H)^{1-\sigma} = \alpha^H(P_{H,t-1}^H)^{1-\sigma} + (1-\alpha^H)p_t^H(h)^{1-\sigma}. \tag{21}$$

We restrict our attention to strategies in which the Home policymakers have to choose the sequence $\{\Pi_t^H\}_{t=0}^{\infty}$ with $\Pi_t^H = P_{H,t}^H/P_{H,t-1}^H$in order to maximize (19) under the constraints given by (11), (18) and (21). We identify this maximization problem as problem (A). First, we consider the general problem, problem (B), in which we disregard the constraints (11) and in which the Home policymaker can freely choose in the strategy set that controls the sequences $\{\Pi_t^H, C_t, T_t\}_{t=0}^{\infty}$. By enlarging the set of controls to all the variables involved in problem (B), it is possible to obtain its first-best. Moreover, the maximum value of the welfare $W^H$ attainable in problem (B) is always at least as good as the maximum value in problem (A), because the latter is nested in the former. Given the convexity of the disutility function in supplying labor and the fact that $\sigma > 1$, for any path of $C$ and $T$, a necessary condition for a plan in the problem (B) to be optimal is to avoid dispersion of prices across the goods produced in the same country. It follows that, in problem (B), it is optimal to stabilize the producer price level. Instead, the sequences of consumption and terms of trade are chosen to satisfy the following first-order conditions

$$
\begin{aligned}
U_C(C_t) - T_t^{1-n} V_y(., z_t^H) &= \lambda_t[(1-\Phi_t^F) U_{CC}(C_t) - T_t^{-2n} V_{yy}\left(.z_t^F\right)], \\
-\frac{(1-n)}{n} T_t^{1-n} V_y(., z_t^H) &= \lambda_t[T_t^{-n} C_t^{-1} V_y\left(., z_t^F\right) + T_t^{-2n} V_{yy}\left(., z_t^F\right)],
\end{aligned}
$$

in all states of nature at date $t$ where $\lambda_t$ is the state-dependent Lagrangian multiplier. We can simplify to

$$
U_C(C_t) - T_t^{1-n} V_y(., z_t^H) = \frac{(1-n)}{n} T_t^{1-n} V_y(., z_t^H) \frac{[T_t^{-2n} V_{yy}\left(., z_t^F\right) - (1-\Phi_t^F) U_{CC}(C_t)]}{[T_t^{-n} C_t^{-1} V_y\left(., z_t^F\right) + T_t^{-2n} V_{yy}\left(., z_t^F\right)]},
$$

which can be rewritten as

$$
U_C(C_t) - T_t^{1-n} V_y(C_t T_t^{1-n} + G_t^H, z_t^H) = T_t^{1-n} V_y(C_t T_t^{1-n} + G_t^H, z_t^H) \frac{1-n}{n} \frac{\rho_t + \eta_t^F}{1+\eta_t^F}. \tag{22}
$$

Equation (22) is satisfied by the flexible-price allocation for the Home country

$$
(1-\Phi_t^H) U_C(C_t^*) - T_t^{*1-n} V_y(C_t^* T_t^{*1-n} + G_t^H, z_t^H) = 0
$$

if and only if $\Phi_t^H = \Phi_t^{*H}$. We have then characterized the optimal path $\{\Pi_t^{H*}, C_t^*, T_t^*\}_{t=0}^{\infty}$ in problem (B). Looking back at the problem (A), the strategy of zero producer inflation, by using lemma 14, can replicate the optimal path of problem (B). It further satisfies the constraints (11) at all dates $t$. It is then the optimal strategy in problem (A). The strategy of zero inflation rate is then a Nash equilibrium. Invoking lemma 14, we obtain that the same result holds also for the strategy of zero notional inflation rate. ■

We have shown that the strategy of zero producer inflation can be enforced as a Nash equilibrium if and only if the overall degrees of monopolistic competition in the two countries are positive. They can be made state dependent through the use of state-dependent taxations, $\tau^H$ and $\tau^F$. This equilibrium coincides with the flexible-price allocation. Even in this case, it is worth stressing that there might exist other strategies that can implement the flexible-price allocation in a Nash equilibrium.

An equivalent proposition holds for the case in which all the prices are fixed one-period in advance.

**Proposition 19** *With prices fixed one-period in advance, given conditions 1-6 and lemma 12, the strategy of zero notional inflation is a Nash equilibrium if and only if the degrees of monopolistic competition assume the values*

$$\Phi_t^{*H} = \frac{D_t^H}{1+D_t^H} > 0, \qquad \textit{with} \qquad D_t^H \equiv \frac{1-n}{n}\frac{\widetilde{\rho}_t+\widetilde{\eta}_t^F}{1+\widetilde{\eta}_t^F},$$
$$\Phi_t^{*F} = \frac{D_t^F}{1+D_t^F} > 0, \qquad \textit{with} \qquad D_t^F \equiv \frac{n}{1-n}\frac{\widetilde{\rho}_t+\widetilde{\eta}_t^H}{1+\widetilde{\eta}_t^H},$$

*at all dates $t$, with*

$$\widetilde{\rho}_t = -\frac{U_{CC}(\widetilde{C}_t)\widetilde{C}_t}{U_C(\widetilde{C}_t)}, \quad \widetilde{\eta}_t^H = \frac{V_{yy}(\widetilde{Y}_t^H, z_t^H)\widetilde{Y}_t^H}{V_y(\widetilde{Y}_t^H, z_t^H)} \ \textit{and} \ \widetilde{\eta}_t^F = \frac{V_{yy}(\widetilde{Y}_t^F, z_t^F)\widetilde{Y}_t^F}{V_y(\widetilde{Y}_t^F, z_t^F)}.$$

**Proof.** Following lemma 12, if the Foreign policymakers is following the zero notional inflation strategy, then condition (18) holds at all dates $t$. The Home policymakers is maximizing its welfare function

$$W^H = \mathrm{E}_{t-1}\left[U(C_t) - V(C_t T_t^{1-n} + G_t^H, z_t^H)\right], \tag{23}$$

under the constraints given by (18) and

$$\mathrm{E}_{t-1}\{[(1-\Phi^H)U_C(C_t)T_t^{n-1} - V_y(T_t^{1-n}C_t, z_t^H)]T_t^{1-n}C_t\} = 0. \tag{24}$$

In a discretionary equilibrium constraint (24) is taken as given, but has to be satisifed in equilibrium.[12] First, we consider the general problem, in which we

[12]In an equilibrium with commitment, (24) constitutes an incentive compatibility constraint that the monetary policymaker has to take in consideration. It can be shown, with isoelastic utility functions, that if each country has a strategy that implies its respective flexible-price condition (3) or (4) independently of the strategy of the other country, then the flexible price allocation is a Nash equilibrium no matter the level of the monopolistic distortions. However, only one of such equilibria, with appropriate monopolistic distortions, is rational from the point of view of the monetary policymakers. In fact, when the monopolistic distortions are not at the appropriate level, each policymaker can gain by surprising price-setters, even from an ex-ante perspective. There is only one of such equilibria in which this incentive is completely eliminated. This coincides with the case in which the discretionary equilibrium can be implemented, as we have emphasized here. Goodfriend and King (2000) and Obstfeld and Rogoff (2000), respectively in closed and open economy models, focus instead on equilibria with commitment.

disregard the constraint (24) and in which the policymaker is free to choose $C_t$ and $T_t$. The optimal policy for this general problem implies the flexible allocation for the Home country

$$(1-\Phi_t^H)U_C(C_t) - T_t^{1-n}V_y(C_tT_t^{1-n} + G_t^H, z_t^H) = 0$$

if and only if $\Phi_t^H = \Phi_t^{*H}$. In the original problem, this allocation satisfies also the constraint (24) and can be implemented by the strategy of zero notional inflation. The zero notional inflation strategy is then the optimal strategy in the original problem. Thus it is a Nash equilibrium. ■

# 8 Further Remarks

## 8.1 Implementation

We have shown that the strategy of zero notional inflation is a Nash equilibrium, provided the monopolistic distortions assume some specific values. In the model with staggered prices, this strategy coincides with a strategy of zero actual producer inflation. Monetary policymakers need to control only a domestic variable, without any reference to the variables or shocks of the other country. A relevant issue to study is whether the policymaker has actually a control on its strategy. For this purpose, in the model with all prices fixed in advance, we study if and how the Home policymaker can use its instrument of monetary policy, the interest rate, to control its domestic notional inflation rate taking as given the strategy of the other policymaker as it happens in the Nash equilibrium.

We rewrite conditions (13) and (14) in an implicit form

$$\begin{aligned} F(C_t, T_t, \Pi_t^{N,H}, z_t^H) &= 1 - \Phi^H, & (25)\\ F^*(C_t, T_t, \Pi_t^{N,F}, z_t^F) &= 1 - \Phi^F. & (26) \end{aligned}$$

where $F$ is a function increasing in $C$ and $T$ and decreasing in the Home gross notional inflation $\Pi^{N,H}$, while $F^*$ is decreasing in $T$ and $\Pi^{N,F}$ and increasing in $C$. By applying the global theorem[13] on the implicit function to (26), we obtain that for any given strategy $\Pi^{N,F}$

$$T_t = Q(C_t, \Pi_t^{N,F}, z_t^F) \tag{27}$$

where $Q$ is increasing in $C$ and decreasing in $\Pi^{N,F}$. After plugging condition (27) into condition (25) and using the global theorem on the implicit function, we obtain

$$C_t = L(\Pi_t^{N,H}, \Pi_t^{N,F}, z_t^H, z_t^F),$$

where $C$ is an increasing function of $\Pi_t^{N,H}$ and $\Pi_t^{N,F}$. Finally we can write (27) as

$$T_t = L^*(\Pi_t^{N,H}, \Pi_t^{N,F}, z_t^H, z_t^F),$$

where $T$ is increasing in $\Pi^{N,H}$ and decreasing in $\Pi^{N,F}$. Using the pricing-decision rule for country H, equation (9) we can then write

$$\mathrm{E}_{t-1}\{[(1-\Phi^H)U_C(L(.))L^*(.)^{n-1} - V_y(L^*(.)^{n-1}L(.), z_t^H)]L^*(.)^{n-1}L(.)\} = 0.$$

From which, considering that $\Pi_t^{N,H} \equiv P_{H,t}^N / P_{H,t-1}^H$, it can be seen that $P_{H,t-1}^H$ is only a function of the joint distribution of $\{P_{H,t}^N, z_t^H, z_t^F\}$ based on the information at time $t-1$, once the notional inflation strategy of policymaker $F$ is taken as given. Moreover $P_{H,t-1}^H$ is homogenous of degree one in the distribution $P_{H,t}^N$ as at $t-1$. It is also the case that notional inflation $\Pi_t^{N,H}$ depends on the realized choice of the notional price $P_H^N$ at time $t$ relative to the distribution of notional price as expected at time $t-1$. Given this results, it can be further shown, using the Euler equation, that there exists a monotone relation between the notional price and the nominal interest rate. The monetary policymaker can thus control the notional price by moving its instrument of monetary policy.

[13]The global theorem states that: given $f(x,y) = c$ with $x = (x_1, x_2 ... x_n)$, $f$ real function on $\Re_+^{n+1}$, if $\forall\, x \in \Re_+^n$, i) f is continuous in $\Re_+^{n+1}$; ii) $f(x,y)$ is strictly increasing (or decreasing) in $y$; iii) $\underset{y \longrightarrow 0^+}{lim} f(x,y) < c$ and $\underset{y \longrightarrow \infty}{lim} f(x,y) > c$ ; then it exists one and only one continuous $y = g(x)$ defined $\Re_+^n$ and with values in $\Re_+$ such that $f(x, g(x)) = c$ $\forall\, x \in \Re_+^n$.

A further indicator that can show if the policymaker is close to the zero notional inflation target can be constructed by using the real marginal cost function. In fact, from equation (13), a departure of notional inflation from the zero target is equivalent to a departure of real marginal costs from the value implied by the monopolistic distortions in the flexible price allocation. It follows that if the observable domestic real marginal cost is constant and equal to the value implied by the overall degree of monopolistic distortions $1 - \Phi^{*H}$ then the notional inflation is on the right track of zero

## 8.2 International monetary policy coordination

A further remark is on the issue of international monetary policy coordination. We have shown that the Nash equilibrium strategy does not imply any sort of coordination between the two countries. In fact, it is sufficient that each country controls its own notional or actual inflation. This equilibrium replicates the same fluctuations as the flexible price equilibrium. However, as already discussed, the incentive to strategically use the terms of trade implies that the only possibility to support such allocation is with a positive overall degree of monopolistic distortions.

Considering a central planner that maximize the global welfare function $W$ defined as

$$W = nW^H + (1 - n)W^F,$$

where each country has a weight equal to its population size, it is possible to show that the optimal plan coincides with the flexible-price allocation with zero domestic notional or actual inflation rates and with zero monopolistic distortions. Moreover, this equilibrium is time consistent. Cooperative and non-cooperative equilibria are similar for what concerns the fluctuations of the economy, but they imply different levels of monopolistic distortions. In fact, consumption and production are higher under the cooperative solution because the strategic role of the terms of trade is completely internalized. There is a potential gain for cooperation, but the enforcement of such equilibrium is an open issue.

# 9 Conclusion

We have characterized strategic games in which price stability can be implemented as a Nash equilibrium. Other direction of research should include models with pricing-to-market and with non-tradeables goods, as well as a context with incomplete markets and intratemporal elasticity of substitution different from one.

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# 10 Appendix

Following Woodford (1999), we take a Taylor expansion of each term of the utility function

$$w_t^H \equiv U(C_t) - \frac{\int_0^n v(y_t(h), z_t^H)dh}{n}. \tag{28}$$

Taking a second-order linear expansion of $U(C_t)$ around the steady state value $\overline{C}$ defined by equation (5) and (6), we obtain

$$U(C_t) = U(\overline{C}) + U_C(C_t - \overline{C}) + \frac{1}{2}U_{CC}(C_t - \overline{C})^2 + o(\|\xi\|^3), \tag{29}$$

where in $o(\|\xi\|^3)$ we group all the terms that are of third or higher order in the deviations of the various variables from their steady-state values. Furthermore expanding $C_t$ with a second-order Taylor approximation we obtain

$$C_t = \overline{C}(1 + \widehat{C}_t + \frac{1}{2}\widehat{C}_t^2) + o(\|\xi\|^3), \tag{30}$$

where $\widehat{C}_t = \ln(C_t/\overline{C})$. Substituting (30) into (29) we obtain

$$U(C_t) = U_C\overline{C}\widehat{C}_t + \frac{1}{2}(U_C\overline{C} + U_{CC}\overline{C}^2)\widehat{C}_t^2 + \text{t.i.p.} + o(\|\xi\|^3), \qquad (31)$$

which can be written as

$$U(C_t) = U_C\overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2] + \text{t.i.p.} + o(\|\xi\|^3),$$

where we have defined the intertemporal elasticity of substitution in consumption as $\rho \equiv -U_{CC}\overline{C}/U_C$ and where in t.i.p. we include all the terms that are independent of monetary policy. Similarly we take a second-order Taylor expansion of $v(y_t(h), z_t^H)$ around a steady state where $y_t(h) = \overline{Y}^H = \overline{CT}^{1-n}$ for each $h$, and at each date $t$, and where $z_t^H = \overline{z}^H$ at each date $t$. We obtain

$$\begin{aligned} v(y_t(h), z_t^H) &= v(\overline{Y}^H, 0) + v_y(y_t(h) - \overline{Y}^H) + v_z(z_t^H - \overline{z}^H) + \frac{1}{2}v_{yy}(y_t(h) - \overline{Y}^H)^2 \\ &\quad + v_{yz}(y_t(h) - \overline{Y}^H)(z_t^H - \overline{z}^H) + \frac{1}{2}v_{zz}(z_t^H - \overline{z}^H)^2 + o(\|\xi\|^3). \quad (32) \end{aligned}$$

Here we take a second-order Taylor expansion of $y_t(h)$ obtaining

$$y_t(h) = \overline{Y}^H \cdot (1 + \widehat{y}_t(h) + \frac{1}{2} \cdot [\widehat{y}_t(h)]^2) + o(\|\xi\|^3),$$

where $\widehat{y}_t(h) = \ln(y_t(h)/\overline{Y}^H)$. We can simplify (32) to

$$\begin{aligned} v(y_t(h), z_t^H) &= v_y\overline{Y}^H \cdot [\widehat{y}_t(h) + \frac{1}{2} \cdot \widehat{y}_t(h)^2 + \frac{\eta^H}{2} \cdot \widehat{y}_t(h)^2 \\ &\quad -\eta^H \cdot \widehat{y}_t(h)\overline{Y}_t^H] + \text{ t.i.p.} + o(\|\xi\|^3), \qquad (33) \end{aligned}$$

where $\overline{Y}_t^H$ has been defined by the relation $v_{yz}(z_t^H - \overline{z}) \equiv -v_{yy}\overline{Y}^H\overline{Y}_t^H$ and where the elasticity of labor supply is $\eta^H \equiv V_{yy}(\overline{Y}^H, \overline{z}^H)\overline{Y}^H/V_y(\overline{Y}^H, \overline{z}^H)$. By using (5), we can write (33) as

$$\begin{aligned} v(y_t(h), z_t^H) &= U_C\overline{C}(1-\Phi^H) \cdot [\widehat{y}_t(h) + \frac{1}{2} \cdot \widehat{y}_t(h)^2 + \frac{\eta^H}{2} \cdot \widehat{y}_t(h)^2 \\ &\quad -\eta^H \cdot \widehat{y}_t(h)\overline{Y}_t^H] + \text{ t.i.p.} + o(\|\xi\|^3). \qquad (34) \end{aligned}$$

We integrate (34) across the households belonging to region $H$, obtaining

$$\begin{aligned} \frac{\int_0^n v(y_t(h), z_t^H)dh}{n} &= U_C\overline{C}(1-\Phi^H) \cdot \{E_h\widehat{y}_t(h) + \frac{1}{2} \cdot [\text{var}_h\widehat{y}_t(h) + [E_h\widehat{y}_t(h)]^2] \\ &\quad + \frac{\eta^H}{2} \cdot [\text{var}_h\widehat{y}_t(h) + [E_h\widehat{y}_t(h)]^2] - \eta^H E_h\widehat{y}_t(h)\overline{Y}_t^H\} \\ &\quad + \text{t.i.p.} + o(\|\xi\|^3). \qquad (35) \end{aligned}$$

We recall that

$$y(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} (T)^{1-n} C^W,$$

from which, using the aggregator

$$Y_H \equiv \left[\left(\frac{1}{n}\right) \int_o^n y^d(h)^{\frac{\sigma-1}{\sigma}} dh\right]^{\frac{\sigma}{\sigma-1}},$$

we obtain

$$Y_{H,t} = T_t^{1-n} C_t.$$

We take a second-order approximation of the aggregator obtaining

$$\widehat{Y}_{H,t} = E_h \widehat{y}_t(h) + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right) \mathrm{var}_h \widehat{y}_t(h) + o(\|\xi\|^3). \quad (36)$$

Finally substituting (36) into (35) we obtain

$$\begin{aligned} \frac{\int_0^n v(y_t(h), z_t)}{n} &= U_C \overline{C}(1-\Phi^H) \cdot [\widehat{Y}_{H,t} + \frac{1}{2} \cdot [\widehat{Y}_{H,t}]^2 + \frac{\eta^H}{2} \cdot [\widehat{Y}_{H,t}]^2 \\ &\quad + \frac{1}{2}(\sigma^{-1} + \eta^H) \cdot \mathrm{var}_h \widehat{y}_t(h) - \eta^H \widehat{Y}_{H,t}^d \overline{Y}_t^H] \\ &\quad + \text{t.i.p.} + o(\|\xi\|^3). \end{aligned} \quad (37)$$

Combining (31) and (37) into (28), we obtain

$$\begin{aligned} w_t^H &= U_C \overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1-\Phi^H) \cdot \widehat{Y}_{H,t} - \frac{(1-\Phi^H)}{2} \cdot [\widehat{Y}_{H,t}]^2 + \\ &\quad -(1-\Phi^H)\frac{\eta^H}{2} \cdot [\widehat{Y}_{H,t}]^2 - \frac{(1-\Phi^H)}{2}(\sigma^{-1} + \eta^H) \cdot \mathrm{var}_h \widehat{y}_t(h) + \\ &\quad (1-\Phi^H)\eta^H \cdot \widehat{Y}_{H,t} \overline{Y}_t^H] + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \quad (38)$$

while for country $F$ we have

$$\begin{aligned} w_t^F &= U_C \overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1-\Phi^F) \cdot \widehat{Y}_{F,t} - \frac{(1-\Phi^F)}{2} \cdot [\widehat{Y}_{F,t}]^2 + \\ &\quad -(1-\Phi^F)\frac{\eta^F}{2} \cdot [\widehat{Y}_{F,t}]^2 - \frac{(1-\Phi^F)}{2}(\sigma^{-1} + \eta^F) \cdot \mathrm{var}_f \widehat{y}_t(f) + \\ &\quad +(1-\Phi^F)\eta^F \cdot \widehat{Y}_{F,t} \overline{Y}_t^F] + \text{t.i.p.} + o(\|\xi\|^3). \end{aligned} \quad (39)$$