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ABSTRACT

Equilibrium Exchange Rate Policies: Complicit Renegotiation-Proof Outcomes*

Countries can repeatedly and opportunistically renegotiate the terms of agreements to which they can only complicitly assent. Therefore, when attempting to coordinate exchange rate policies, they continuously play partnership games. We develop a reduced form model of exchange rate management where, as a starting point, (a) sequences of discrete realignments and (b) shared intervention are desirable. We show that the implementation of the *ex ante* optimal policy suffers from severe time inconsistencies. We analyse the Stackelberg equilibria of the stochastic differential game played by partner countries. We find that equilibrium complicit renegotiation-proof policies are supported by net cross-country wealth transfers from the weaker to the stronger bargaining power country. Our theoretical results provide a game-theoretic interpretation of the evolution of monetary arrangements in Europe and the emergence of EMU.

JEL Classification: C73, F31, F33
Keywords: bargaining power, cross-country wealth transfers, exchange rate management, renegotiation

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NON-TECHNICAL SUMMARY

Most policy-makers would recognize that the implementation of desirable common exchange rate policies is greatly complicated by free-riding and moral hazard issues: while countries may agree *ex ante* upon the foreign exchange policy to follow, *ex post* they might non-cooperatively prefer to repudiate the agreement. Such problems cannot be easily solved because it is difficult for sovereign countries to write contracts credibly. As a consequence, repeated non-cooperation threats and opportunistic renegotiation cannot be prevented: a country with strong bargaining power might strategically refuse to carry out the level of intervention that it promised and demand that its weaker 'partner' country carry out a larger share than previously agreed.

When agreements can only be complicit and no simple precommitment devices are available, allowing for *ex post* renegotiation is essential as bargaining power considerations become predominant. The policy that ultimately prevails is the equilibrium outcome of the games which countries continuously play. Indeed, by nature, sovereign countries can only complicitly assent to agreements and can always renegotiate them. Thus, in equilibrium, partner countries internalize the fact that they can continuously and repeatedly revise, in a possibly opportunistic fashion, the terms of an agreement.

Within a continuous-time model of bilateral exchange rate management, we have analysed this bargaining process. Thus, we assume that it is cooperatively optimal (a) to endorse a policy consisting of combinations of exchange rate pegs and repeated realignments, and (b) to share the burden of intervention. Then, we show that the implementation of the *ex ante* jointly optimal realignment policy suffers from severe time inconsistencies: *ex post*, as soon as countries have different basic characteristics, one country will wish to trigger realignments *too frequently*, and the other one *too rarely*. The *ex ante* optimal realignment dates will not be chosen *ex post*.

We then analyse strategic bargaining among the two partner countries allowing for infinitely repeated renegotiation: countries optimize dynamically over intervention policies in a non-cooperative fashion and are allowed to bargain over their realization, at any time, and as many times as they wish. Restricting our attention to Stackelberg equilibria, we see that the solution of the resulting dynamic bargaining game produces an equilibrium outcome which is time consistent and renegotiation-proof, in that no country would benefit from either repudiating or renegotiating it *ex post*.

When nothing prevents countries from internalizing the renegotiation surplus, we find that the *ex ante* optimal realignment policy is reached. The striking feature of this efficient equilibrium outcome is that it must be supported by net cross-country wealth transfers from the weak to the strong bargaining power

country: in equilibrium, the leader's incentive to strategically renegotiate is only removed if *the follower continuously transfers wealth to the leader*.

When wealth payments among sovereign countries are precluded, the bargaining process leads to an inefficient outcome, as the resulting policy is furthest away from the first best. Comparing these two equilibria we find that the absence of cross-country wealth transfers yields high inefficiencies. Thus we conclude that although cross-country wealth transfers might appear controversial they should simply be viewed as a very efficient repeated commitment device.

We finally recast the evolution of monetary arrangements in Europe in the past 25 years within our theoretical framework. We claim that: 1) earlier arrangements, such as the Snake and EMS, were inefficient as the system was too rigid and wealth transferability was insufficient; 2) monetary integration, and hence an efficient outcome, could be reached only when the leader country, Germany, could be sufficiently compensated through a large enough concession, such as reunification.

1 Introduction

Most policy makers would recognize that the implementation of desirable common exchange rate policies is greatly complicated by free-riding and moral hazard issues: While countries may agree ex-ante upon the foreign exchange policy to follow, ex-post they might non-cooperatively prefer to repudiate the agreement. Such problems cannot be easily solved because it is difficult for sovereign countries to credibly write contracts.[1] As a consequence, repeated non-cooperation threats and opportunistic renegotiation cannot be prevented: A country with strong bargaining power might strategically refuse to carry out the level of intervention it promised to, and demand its weaker "partner" country carry out a larger share than previously agreed.

A strand of the exchange rate literature has examined coordination problems between two countries. Hamada (1974) introduced strategic behavior and pointed out that, because of externalities, Nash equilibrium strategies do not always implement the efficient policy. Two mechanisms which mitigate such inefficiencies, by inducing a country not to repudiate an agreement, were then put forward: (1) In Canzoneri and Henderson (1991), countries play reputation games, where if one of the two players deviates from the cooperative strategy, its partner will punish it playing non-cooperative strategies forever in the future.[2] (2) In Persson and Tabellini (1995), the implementation of some sort of institutional reform, involving a degree of sovereignty devolution, generates the first-best policy.[3]

However, policy makers change over time and therefore have little need to build up reputation. Furthermore, its seems clear that sovereign countries find it hard, or politically difficult, to design institutional reforms which efficiently tie themselves. Hence, it is questionable whether the equilibria proposed in the literature, which are based on (1) reputation mechanisms and/or (2) the complete ability to contract upon exchange rate policies, are the relevant ones.

When agreements can only be complicit and no simple precommitment devices are avail-

[1]A clear example of this is given by the current failure of the European Union to comply with the WTO ruling in a series of trading disputes with the United States.

[2]In their setting the policy makers of two countries need to adjust their monetary policies in response to a common negative technological shock. In a simple one-shot game, the resulting Nash equilibrium does not internalize all positive externalities cooperation provides, as it entails an excessive monetary contraction on the part of the two countries. If the same game is repeated over time, the efficient outcomes are still supported by trigger mechanisms. However, these trigger mechanisms are not enough if the characteristics of this game evolve over time. See also Currie and Levine (1993), and Ghosh and Masson (1994).

[3]It would then possible to write special contracts between policy makers and central banks to which monetary authority is delegated, in such a way that in playing non-cooperative Nash strategies the two central banks select efficient policies.

able, allowing for ex-post renegotiation is essential as bargaining power considerations become predominant. The policy that ultimately prevails is the equilibrium outcome of the games which countries continuously play.

The purpose of this paper is to take into account that, by nature, sovereign countries can only complicitly assent to agreements, and can always renegotiate them. It establishes how, in equilibrium, partner countries internalize the fact that they can continuously and repeatedly revise, in a possibly opportunistic fashion, the terms of an agreement.

Within a continuous-time framework, we construct a highly stylized model of bilateral exchange rate management. We do not develop a fully blown macroeconomic model. Instead we starts assuming that it is cooperatively optimal (a) to endorse a policy consisting of combinations of exchange rate pegs and repeated realignments, and (b) to share the burden of intervention. Indeed most policy makers consider that realignments are worthwhile whenever the intervention required to maintain the current exchange rate peg becomes too onerous relative to abandoning it.[4]

We first show that the implementation of the ex-ante jointly optimal realignment policy suffers from severe time inconsistencies: Ex-post, as soon as countries have different basic characteristics, one country will wish to trigger realignments *too frequently*, and the other one *too rarely*. The ex-ante optimal realignment dates will not be chosen ex-post, and no simple pre-commitment devices are available to guarantee the efficient outcome.

We then analyze strategic bargaining among the two partner countries allowing for infinitely repeated renegotiation: Countries optimize dynamically over intervention policies in a non cooperative fashion and are allowed to bargain over their realization, at any time, and as many times as they wish. Now, incorporating game theoretic arguments in a continuous time model is complicated by the fact that most stochastic differential games cannot be solved for analytically. As a first step, we restrict our attention to Stackelberg equilibria, where a leader country policy maker is in position to make take-it-or-leave-it offers to a follower. The solution of the resulting dynamic bargaining game produces an equilibrium outcome which is time consistent and renegotiation-proof, in that no country would benefit from either repudiating or renegotiating it ex-post.

We begin considering the unconstrained equilibrium where nothing prevents countries from internalizing the renegotiation surplus. The striking feature of this efficient equilibrium outcome is that it must be supported by net cross-country wealth transfers from the weak to

[4]In this way, our model prescribes fairly flexible foreign exchange policies that can mimic the experience of several countries in the past fifty years. Among various regimes, we can include Bretton Woods and the European Monetary System.

the strong bargaining power country: In equilibrium, the leader's incentive to strategically renegotiate is only removed if *the follower continuously transfers wealth to the leader.*

Wealth payments among sovereign countries are certainly feasible and indeed we can think of several mechanisms through which wealth is transferred in practice. However, the ability of a weak country to continuously transfer wealth is often not perfect. We therefore also determine the constrained equilibrium that prevails in the extreme case when cross-country wealth transfers are absolutely impossible. This equilibrium outcome leads to the worse result, as the resulting policy is most further away from the first best.

We then compare these two equilibria, to quantify the inefficiency a limited wealth transferability can generate. Numerical simulations highlight that the absence of cross-country wealth transfers indeed yields high inefficiencies. That is, although cross-country wealth transfers might appear controversial they should simply be viewed as a very efficient repeated commitment device.[5]

We finally recast the evolution of monetary arrangements in Europe in the past 25 years within our theoretical framework. The road towards European monetary union represents one prominent case of prolonged negotiation between sovereign countries, where the advantages from monetary cooperation were clear, but partner countries struggled to design adequate compensations for the party in the stronger bargaining position. It can be seen as a process of continuous bargaining between a leader Germany and follower France, over exchange rate mechanisms and intervention shares.

Our theory's reading of events is as follows: First, earlier arrangements, notably the Snake and EMS, were inefficient outcomes which prevailed because the system was too rigid (1979-1986) and wealth transferability was insufficient (1986-1993). Second, monetary integration could only be implemented if the leader Germany could be sufficiently compensated. This was only possible with a large enough concession such as reunification.

Our research also contributes to the literature which has analyzed exchange rates management in continuous time within specific currency regimes. Our basic modeling is close to these studies in that the focus of their analysis lies with the determination of the dynamics of exchange rates. However, an important limit of this literature is that it only considers exogenously given target zones.[6] Taking into account counter parties behavior enables us to rationally determine regime switches endogenously and highlight incentive incompatibilities which were silenced. Allowing for renegotiation, we expand the strategy space open to

[5] In an analysis of tax harmonization, Chari and Kehoe (1990), and Tabellini and Persson (1994) also find that the implementation of the first-best policy is possible only if institutional arrangements do not preclude such cross-country wealth transfers.

[6] Krugman (1991), Svensson (1992), and Bertola (1994) provide good surveys.

partner countries and bring game-theoretic arguments to this strand of research.

The structure of the paper is as follows. Section 2 sets out the basic structure of the model we use to analyze bilateral management of exchange rates. It also describes the characteristics of realignment policies and determines under which conditions they are optimal. Section 3 outlines the moral hazard problem, showing the difference between ex-ante and ex-post optimal policies. Section 4 introduces renegotiation between the two partner countries and the implementation of Stackelberg equilibria. Section 5 provides a full characterization of restricted and unrestricted renegotiation proof equilibria. Section 6 discusses the role that cross-country wealth transfers have in the implementation of efficient equilibria. Section 7 interprets the recent evolution of monetary arrangements in Europe in light of our analytical framework. Section 8 concludes, while an Appendix gives details of the mathematics.

2 A Reduced Form Model

2.1 Countries, Uncertain Fundamental and Intervention Policies

We consider two countries that we indicate with A and B. The fundamental value of the spot exchange rate between these two countries depends on several macroeconomic factors, such as income levels, industrial productions, price levels, and so on. Within our simplified framework we abstract from all the details that a fully blown macroeconomic model requires and summarize the dynamics of this fundamental value by a single uncertain state variable, v_t, which reflects all macroeconomic shocks continuously perturbing it.[7] For simplicity and tractability, we take v_t to follow a driftless arithmetic Brownian motion,

$$dv_t \;=\; \sigma\, dW_t \;, \tag{1}$$

where W_t is the Wiener process, while $\sigma \in \mathbf{R}^+$.

Countries A and B can endorse an *intervention policy*, that we indicate with $\mathcal{I}$.[8] For a given intervention policy, $\mathcal{I}$, the *spot rate*, which we will denote $s_t \equiv s(v_t \mid \mathcal{I})$, is therefore a function of the forcing process, v_t.[9] Intervention policies result in spot rate behavior ranging

[7]Although we are not specific about the nature v_t, it could be derived using an asset market approach model of exchange rate determination, as in Froot and Obstfeld (1992), and Weller (1992).

[8]For intervention we mean *any* action on the part of the two countries policy makers aimed at controlling exchange rates. Whilst, active exchange rate management is mostly conducted through money market operations, it can also involve operations in foreign exchange and other financial markets.

[9]Under the usual convention, s_t indicates the log of the exchange rate between A and B, hence s_t is the log of the number of units of country B currency required to purchase one unit of country A currency.

from the following two extremes: In a free-float regime, s_t simply equals v_t at all times t, as no intervention operations in the money market is aimed at influencing the spot rate's evolution.[10] Conversely, in a permanent peg regime, s_t is kept forever, and at whatever cost, equal to a constant.

Such extremely lax or strict monetary policies are however considered suboptimal: On the one hand constant exchange rates are beneficial to each one of the partner economies as they reduce both exchange rate risk and transaction costs, while providing a nominal anchor for their monetary policies. On the other hand, in fixing the exchange rate countries lose monetary policy independence. Essentially, having a volatile exchange rate entails instability costs associated with the unpredictability of movements of the spot rate, but curbing this volatility demands intervention operations which also generate losses.[11]

2.2 Infrequent Realignments and Shared Intervention

(a) Policy makers have historically followed an intermediate form of intervention policies characterized by sequences of discrete realignments which can be defined as follows:[12]

Definition 1 *An infrequent realignment policy, $\mathcal{I}(\{T_i\})$, is an intervention policy which involves (i) realigning the exchange rate at a series of infrequent random dates $\{T_i\}$, where $i \in \mathbf{N}$, and (ii) endorsing a peg so that the exchange rate remains equal to the level to which it was last realigned, at all other times. The resulting managed spot exchange rate being*

$$s(v_\tau \mid \mathcal{I}) \;=\; v_{T_i} \;, \qquad \textit{for all} \;\; \tau \in [T_i, T_{i+1}) \;, \quad \textit{and for all} \;\; i \in \mathbf{N} \;. \tag{2}$$

(b) Another feature of exchange rate management is that cooperation between partner countries yields substantial positive externalities: When a unilateral peg regime is under speculative attack, its defense might require high short-term interest rates, limits to capital flows, and a massive use of foreign reserves. In a bilateral peg regime, the monetary authorities of the partner countries can move short term interest rates in opposite directions and access

[10] The "hands-off" foreign exchange policy officially endorsed by the United States for many years, consistently with the free floating equilibrium of a standard monetary model (see for instance Krugman (1992)), results in a spot rate floating freely according to $s_t = v_t$.

[11] See Frankel (1999) for a summary of arguments sustaining that the appropriate exchange rate regime is country and state dependent.

[12] Obstfeld and Rogoff (1995) report that only six major economies with open capital markets, in addition to a number of very small economies had maintained a fixed exchange rate for five years or more, as of 1995. Klein and Marion (1997) report that the mean duration of pegs among Western Hemisphere countries is about 10 months.

credit lines for their respective national currencies, making the defense of the peg a less onerous exercise.[13] It is therefore in the joint interest of two countries to share the burden of the intervention required to implement a policy, $\mathcal{I}$.

In the context of sovereign countries, cooperation consists of a complicit agreement by countries A and B to implement a policy, $\mathcal{I}$, applying a certain intervention *sharing rule*: A sharing rule is a function $\phi(v_t)$, where $\phi(v_t) \in [0,1]$ for all possible state v_t, representing the share of the necessary instantaneous amount of intervention, $s(v_t \mid \mathcal{I}) - v_t$, that country A agrees to undertake in the joint implementation of policy $\mathcal{I}$.[14] The extreme cases where country A or B intervene unilaterally to implement single-handedly a policy $\mathcal{I}$, correspond to $\phi(v_t)$ equals 1 or 0, for all v_t.

2.3 Realignment Losses and Intervention Cost

Our intention is to capture these two essential features of bilateral exchange rate management. Our reduced-form model therefore assumes, from the outset, that desirable intervention policies involve (a) infrequent realignments and (b) shared intervention:

We summarize all pressures in favor and against intervention actions in a single overall trade-off. We refer to all the negative impacts of an action on a country's economy in the following unit period of time, as the country's *unit period loss* associated with this action. Policy makers are assumed to act in the best interests of the country they represents, and use a common constant discount factor, ρ. Their actions are intended to minimize the discounted sum of unit period losses expected to be incurred by their country over an infinite horizon.

Clearly, policy makers only wish to follow realignment policies if the trade-off they face induces them to prefer such policies. Now, the distinctive characteristic of an infrequent realignment policy is that it involves a series of *rare* dates, $\{T_i\}$, at which the exchange rate is altered, whereas at *all other* dates it is kept unchanged. In this context, infrequent action is the solution to repeated stochastic optimization problems of the following type:

[13] The recent experience of the European Monetary System gives a good example of this, as attacks to the system's central parities were more successful when their defenses were not coordinated properly (see Buiter *el al.* (1998)).

[14] Within a basic monetarist model of exchange rate determination to peg the spot rate at a given parity $\bar{s}$ requires fixing the relative monetary supply in the two countries. If we apply the usual interpretation that movements in the forcing process v_t reflect shocks to the velocity of circulation of money, we know that the monetary base will need to adjust to absorb such shocks and that the required level of relative monetary base will be $\bar{s} - v_t$. Then, within our reduced form model, we can use the difference $s(v_t \mid \mathcal{I}) - v_t$ as the simplest measure of the required intervention.

1. Altering or keeping unchanged the exchange rate must be chosen each unit of time.
2. At any date t and for all states of the world, the unit period loss that results from altering the exchange rate is higher than that of keeping it unchanged.
3. The discounted stream of expected future losses is
   (a) higher if the exchange rate is altered than if it is kept unchanged, in the states of the world which follow a realignment;
   (b) lower if the exchange rate is altered than if it is kept unchanged, in states of the world where large intervention is required to keep the exchange rate unchanged.

We contrast the action of altering the exchange rate to that of keeping it unchanged, considering the influence of two vectors of parameters, (Λ_A, Λ_B) and (κ_A, κ_B):

On the one hand, we denote Λ_A and Λ_B the immediate losses altering the exchange rate imposes on countries A and B, respectively. These *realignment losses* are meant to capture *all* the costs encountered by sovereign countries when exchange rates suddenly shift. These realignment losses can be interpreted as follows: (i) Large changes in prices and (ii) the deterioration of the policymakers' credibility cause significant destruction to national wealth. In particular, abrupt swings in exchange rates disturb international trade, while appreciations (depreciations) of national currencies reduce (increase) the value of foreign assets (liabilities). Moreover, when countries actively defend a particular exchange rate level, if the exchange rate is then altered, the credibility of the pegging countries commitment to stabilization is largely annihilated and hence reintroducing a new peg for the spot rate becomes very onerous.[15]

On the other hand, we will capture the immediate losses keeping the exchange rate unchanged imposes on countries A and B with country specific parameters κ_A and κ_B, as follows: To keep the exchange rate unaltered for one unit period, country A and B incur instantaneous intervention losses equal to

$$\kappa_A \,.\, \phi(v_t)^2 \,.\, [s(v_t \mid \mathcal{I}) - v_t]^2 \qquad \text{and} \qquad \kappa_B \,.\, [1 - \phi(v_t)]^2 \,.\, [s(v_t \mid \mathcal{I}) - v_t]^2, \tag{3}$$

respectively, where κ_A and κ_B are two positive constants.

This is the simplest functional form assumption which captures all the above features (1., 2., 3.(a) and (b)) we intend to. Maintaining the exchange rate unchanged results in a unit-period loss for country A (and B) which is a product of three separable functions:

[15] Indeed, in several countries, over-valued domestic currencies have been employed as nominal anchors in order to fight high inflation; after their devaluations inflation resumed, interest rates hiked and the monetary authorities struggled to convince investors and consumers of their commitment to low inflation rates.

- A *country specific* function, κ_A (and κ_B). We just take *constants.*

- A *cooperation gain* function, $\phi(v_t)^2$ (and $[1-\phi(v_t)]^2$). For a given state, this is *quadratic* in the share of intervention, $\phi(v_t)$ (and $[1-\phi(v_t)]$), carried out by a country: Essentially, countries collectively stand to gain from coordinating their intervention operations when each country's unit-period loss function is increasing and *convex* in the share of intervention it carries out. That is, the sharing rule, hence cooperation, would be irrelevant if we took the cooperation gain function to be linear, $\phi(v_t)$, instead of $\phi(v_t)^2$. The important ingredient is convexity. We therefore take the simplest convex function.

- A *state-dependent* function, $[s(v_t \mid \mathcal{I}) - v_t]^2$. This is *quadratic* in the overall amount of intervention required: maintaining the exchange rate unchanged must involve a unit-period loss which is symmetric and *increasing* in the amount of required intervention. We could have taken the absolute value, $|s(v_t \mid \mathcal{I}) - v_t|$, as state-dependent function, instead of $[s(v_t \mid \mathcal{I}) - v_t]^2$. This however generates difficulties associated with non-differentiability. We therefore take a quadratic function because is the simplest twice-differentiable function which is increasing and symmetric in the amount of required intervention.

The quadratic form for cooperation gains has also a very helpful property: Under this assumption, the *ex-ante optimal sharing rule*, which consists of the country A share of intervention minimizing (cooperatively) the collective loss due to intervention, $\phi_{A\cup B} \equiv \arg\min_{\phi(.)} \{ \, (\kappa_A \, \phi(v_t)^2 + \kappa_B \, [1-\phi(v_t)]^2 \,) \, [s(v_t \mid \mathcal{I}) - v_t]^2 \, \}$ is state-independent:

$$\phi_{A\cup B} \;=\; \frac{\kappa_B}{\kappa_A + \kappa_B} \;, \quad \text{for all } v_t \; . \tag{4}$$

This reduced form modeling is close to Bertola and Caballero (1992), Bertola and Svensson (1993), and Lewis (1995) who develop stochastic models of discrete, repeated realignments within specific currency regimes: Essentially, instead of taking the realignment trigger levels as given, we take as given the parameters of the trade-off which leads policy makers to select this form of infrequent action. Our approach is however less reduced form in that we then consider the policy makers' objective functions to determine the realignment trigger levels they select endogenously.

Let us for clarity detail the timing of events, when an infrequent realignment policy $\mathcal{I}(\{T_i\})$ and a sharing rule $\phi(v_t)$ is implemented: Take the current date t to be in the interval (T_i, T_{i+1}), where T_i and T_{i+1} are realignment dates. At a previous date, T_i, countries A and B realigned the spot rate to the level v_{T_i}, incurring realignment losses Λ_A and Λ_B, respectively. After T_i and until now, countries A and B have actively pegged the exchange rate to v_{T_i}.

This was done through continuous intervention generating losses $\kappa_A \; \phi(v_\tau)^2 \; [s(v_\tau \mid \mathcal{I}) - v_\tau]^2$ and $\kappa_B \; [1 - \phi(v_\tau)]^2 \; [s(v_\tau \mid \mathcal{I}) - v_\tau]^2$, where $\tau \in (T_i, t]$, to countries A and B, respectively, every unit of time. Countries A and B will continue to do so until the next realignment date, T_{i+1}, which seen from today's date t is a random time. The state of the world will be such that they will then again realign the spot rate, but now to the level $v_{T_{i+1}}$, incurring realignment losses Λ_A and Λ_B, respectively. The sequence is then repeated in time.

2.4 Symmetric Realignment Policies of Fixed Amplitude

Notice that the model we have constructed is (i) *symmetric* with respect to current state and (ii) *time-homogeneous*. Intervention policies that policy makers find optimal to choose are therefore clearly within a subset of the set of infrequent realignment policies:

This subset encompasses *symmetric* infrequent realignment policies where the realigmnent is triggered whenever the *amplitude* of the intervention, $|s(v_t \mid \mathcal{I}(\{T_i\}) \; - \; v_t|$, reaches a time-independent threshold size to be determined. Such policies that can therefore be characterized by a single constant, δ, such that for all date $t \in [T_i, T_{i+1})$, the next realignment occurs at the random time T_{i+1}, defined as:

$$T_{i+1} \;=\; \inf \; \{ \tau \geq T_i : |s(v_\tau \mid \mathcal{I}(\{T_i\})) \; - \; v_\tau| \;=\; \delta \} \; . \tag{5}$$

The dynamics of the state variable and the exchange rate for a given symmetric realignment policy of fixed amplitude are represented in Figure 1.

A good way to capture what a realignment policy amplitude δ effectively means, consists of describing it in terms of the expected time-interval between realignments it yields: When a realign occurs whenever the amplitude of the intervention reaches a level δ, then it is easy to show (see Karlin-Taylor (1975), p.360) that, at a realignment date T_i, the expected time to the following realigmnent date, T_{i+1}, equals

$$E[\Delta T(\delta)] \;\; \equiv \;\; E_{T_i} \, [T_{i+1} - T_i] \;\; = \;\; \frac{\delta^2}{\sigma^2} \; , \qquad \text{for all} \;\; i \in \mathbf{N} \; . \tag{6}$$

To considerably simplify the algebra later, we now introduce our central aggregate loss operator: It only applies to (a) symmetric infrequent realignment policies, and (b) state-independent sharing rules, i.e. $\phi(v_t)$ independent of v_t. Consider a policy maker facing (i) unit-period intervention loss function $\kappa \; (v_t - s_t)^2$, where κ is a constant, and (ii) a lump-sum realignment loss Λ at each realignment. Let $C(v_t - s_t, \delta \mid \kappa, \Lambda)$ denote the discounted sum across time of losses expected to be incurred if a symmetric realignment policy of trigger amplitude δ is followed. If the spot rate is currently pegged at v_{T_i}, we therefore have

$$C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda) \;\equiv\; E_t \left[\int_t^{\infty} \kappa \, (v_\tau - s_\tau)^2 \, e^{-\rho(\tau - t)} \, d\tau \right] + \sum_{j=i+1}^{\infty} \Lambda \, \mathbf{1}_{\{T_j\}} \, e^{-\rho(T_j - t)} \, . \quad (7)$$

Here $s_\tau = v_{T_j}$ for all $\tau \in [T_j, T_{j+1})$, where $T_{j+1} = \inf\{\tau \geq T_j : |v_{T_j} - v_\tau| = \delta\}$. $\mathbf{1}_{\{T\}} \equiv 1$ at date T and zero otherwise. Denoting $\gamma \equiv \sqrt{2\rho}/\sigma$, we obtain the following expressions:[16]

– For a realignment date $t = T_i$, $s_{T_i} = v_{T_i}$ and

$$C(v_{T_i} - s_{T_i}, \delta \mid \kappa, \Lambda) \;=\; K(0) - \Big(K(\delta) \;-\; K(0) \;-\; \Lambda \Big) \frac{1}{\cosh[\gamma\delta] - 1} \;+\; \Lambda \, . \quad (8)$$

– For a non-realignment date $t \in (T_i, T_{i+1})$, $s_t = v_{T_i}$ and

$$C(v_t - s_t, \delta \mid \kappa, \Lambda) \;=\; K(v_t - v_{T_i}) - \Big(K(\delta) \;-\; K(0) \;-\; \Lambda \Big) \frac{\cosh[\gamma(v_t - v_{T_i})]}{\cosh[\gamma\delta] - 1} \, . \quad (9)$$

$K(v_t - v_{T_i}) \equiv \lim_{\delta \to \infty} C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)$ corresponds to the expected aggregate intervention losses *if* the exchange rate was *forever* pegged at v_{T_i}, and is equal to[17]

$$K(v_t - v_{T_i}) \;=\; \frac{\kappa}{\rho} \left(\frac{\sigma^2}{\rho} \;+\; (v_t - v_{T_i})^2 \right) \, . \quad (10)$$

The dynamics of $C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)$ with respect to the realignment trigger amplitude, δ, are exhibited in Figure 2.[18] On the one hand, extremely frequent realignments involve incurring extremely frequently a lump-sum realignment loss.[19] On the other hand, extremely infrequent realignments are almost equivalent to pegging forever, but extremely infrequent realignments are nevertheless preferable to no realignments at all.[20]

Overall, there is a unique intermediate realignment frequency that minimizes $C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)$. We find that the realignment trigger amplitude, $\tilde{\delta} \equiv \tilde{\delta}(\kappa, \Lambda)$, minimizing the aggregate loss function $C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)$, i.e. $\tilde{\delta} \equiv \arg\min_\delta \, C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)$, solves the first order optimality condition[21]

$$\frac{\tilde{\delta}}{\rho} \left(\tilde{\delta} \;-\; \frac{2\,(\cosh[\gamma\tilde{\delta}] - 1)}{\gamma \, \sinh[\gamma\tilde{\delta}]} \right) \;=\; \frac{\Lambda}{\kappa} \, . \quad (11)$$

[16]See Appendix B.III.

[17]See Appendix B.I.

[18]It is also easy to verify from equations (8) and (9) that values will be continuous through time.

[19]$\lim_{\delta \to 0} \; C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda) = K(v_t - v_{T_i}) \;+\; \Lambda \, \cosh[\gamma(v_t - v_{T_i})] \; \lim_{\delta \to 0} (\cosh[\gamma\delta] - 1)^{-1}$. Given that $\lim_{\delta \to 0} \cosh[\gamma\delta] = 1^+$, this implies $\lim_{\delta \to 0} \; C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda) = \infty$.

[20]$\lim_{\delta \to \infty} [C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda) - K(v_t - v_{T_i})] \;=\; -\kappa/\rho \, \cosh[\gamma(v_t - v_{T_i})] \; \lim_{\delta \to \infty} (2\,\delta^2)/\exp[\gamma\delta])$. Given that $\lim_{\delta \to \infty} 2\delta^2/\exp[\gamma\delta] = 0^+$, $C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)$ tends to $K(v_t - v_{T_i})$ from below.

[21]We write $\partial\, C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)/\partial\delta = 0$, using expression (9).

3 Contrasting Ex-ante and Ex-post Behavior

We now show that implementation of a desirable policy which involves (a) infrequent realignments and (b) shared intervention, is complicated by free-riding and moral hazard problems:

The intervention policy that would be chosen by policy makers of countries A and B, if they behave *cooperatively*, minimizing the sum of *both* countries aggregate losses, is clearly a symmetric realignment policy of fixed trigger amplitude (by state-space symmetry and time-homogeneity of the problem). Furthermore, the sharing rule it is ex-ante cooperatively optimal for countries A and B to initially agree upon, is the state-independent sharing rule $\phi_{A\cup B} = \kappa_B/(\kappa_A + \kappa_B)$ (see equation (4)). Therefore, the ex-ante optimal intervention policy is characterized by the ex-ante optimal realignment trigger amplitude

$$\begin{aligned} \delta_{A\cup B} \quad &\equiv \quad \arg\min_{\delta}\ C(v_t - v_{T_i}, \delta \mid \kappa_A\, \phi^2_{A\cup B} \,+\, \kappa_B\, (1-\phi_{A\cup B})^2\,,\, \Lambda_A \,+\, \Lambda_B)\ , \\ &= \quad \tilde{\delta}(\kappa_A\, \phi^2_{A\cup B} \,+\, \kappa_B\, (1-\phi_{A\cup B})^2\,,\, \Lambda_A \,+\, \Lambda_B)\ . \end{aligned} \tag{12}$$

Here $C(v_t, \delta \mid \kappa_A\, \phi^2_{A\cup B} + \kappa_B\, (1-\phi_{A\cup B})^2\,,\, \Lambda_A + \Lambda_B)$ measures the collective expected burden of a symmetric realignment policy of trigger amplitude δ, when intervention is shared optimally.

We now contrast this first-best policy with the ex-post behavior of countries A and B attempting to implement the cooperatively optimal sharing rule, $\phi_{A\cup B}$, without renegotiation. The difference lies in the fact that an individual country's preferred trigger amplitude only minimizes the expected sum across time of the discounted losses incurred by that country. Now, country A and country B's ex-post non-cooperatively optimal intervention policies are also within the set of symmetric realignment policies of fixed trigger amplitude (again, by state-space symmetry and time-homogeneity of the problem). Country A and B's ex-post optimal realignment trigger amplitudes solve respectively

$$\delta_A(\phi_{A\cup B}) \;\equiv\; \arg\min_{\delta}\ C(v_t - v_{T_i}, \delta \mid \kappa_A\, \phi^2_{A\cup B}, \Lambda_A) \;=\; \tilde{\delta}(\kappa_A\, \phi^2_{A\cup B}, \Lambda_A)\ , \qquad \text{and} \tag{13}$$

$$\delta_B(\phi_{A\cup B}) \;\equiv\; \arg\min_{\delta}\ C(v_t - v_{T_i}, \delta \mid \kappa_B\, (1-\phi_{A\cup B})^2, \Lambda_B) \;=\; \tilde{\delta}(\kappa_B\, (1-\phi_{A\cup B})^2, \Lambda_B) \tag{14}$$

In the vast majority of situations, $\{\sigma; \rho; \kappa_A; \kappa_B; \Lambda_A; \Lambda_B\}$, the ex-post non-cooperatively optimal intervention policy of country A (and that of country B) differs from the ex-ante cooperatively optimal one. The origins of these differences are best understood examining the first order optimality condition satisfied by optimal realignment trigger amplitudes:

Equation (11) reveals that differences in the ratio Λ/κ, on the RHS, result in differences in optimal realignment trigger amplitude, $\tilde{\delta} \equiv \tilde{\delta}(\kappa, \Lambda)$.[22] This ratio Λ/κ reflects the strength

[22] In this comparison, environment parameters $\{\sigma; \rho\}$ are kept unchanged.

of Λ, the force inducing less frequent realignments, relative to κ, the force inducing more frequent realignments. That is the magnitude of Λ/κ is inversely related to the frequency of realignments it induces. Very intuitively, when altering the exchange rate is expensive (large Λ) and keeping it unchanged is cheap (small κ), hence the ratio Λ/κ is large, less frequent realignments will be preferred.

Individual countries' ex-post choice of realignment dates will only correspond to the ex-ante optimal one, if $\delta_A(\phi_{A\cup B})$ and $\delta_B(\phi_{A\cup B})$ equal $\delta_{A\cup B}$. This will however only be the case if the relative strength of forces influencing countries' individual choice of realignment timing is exactly equal to the relative strength of the sum of these forces.

Clearly, by symmetry, this condition is satisfied if the two countries have *identical* characteristics, $(\Lambda_A, \kappa_A) = (\Lambda_B, \kappa_B)$. However, if this is not the case, the ex-post non-cooperatively optimal intervention policy of country A (and that of country B) will most often differ from the ex-ante cooperatively optimal one. Overall, we more precisely have that,[23] if

$$\frac{\Lambda_A}{\kappa_A\,\phi^2_{A\cup B}} \ \begin{matrix} > \\ < \end{matrix}\ \frac{\Lambda_B}{\kappa_B\,(1-\phi_{A\cup B})^2}\,, \quad \text{then} \quad \delta_A(\phi_{A\cup B}) \ \begin{matrix} > \\ < \end{matrix}\ \delta_{A\cup B} \ \begin{matrix} > \\ < \end{matrix}\ \delta_B(\phi_{A\cup B})\,. \qquad (15)$$

Lemma 1 *Ex-post, one country will always wish to realign earlier and the other one later than ex-ante optimal. There is only consensus if the LHS of (15) holds as an equality.*

Essentially, countries are different, hence in the vast majority of cases, the implementation of an agreement to just share the burden of intervention according to the first-best sharing rule will suffer from a time consistency problem. One country wishing to trigger realignments too frequently, and the other one too rarely. Such an agreement is therefore simply not credible. As the ex-ante optimal realignment dates would not be chosen ex-post, the first best policy would not be implemented. Importantly, sovereign countries do not have simple commitment devices available to them in order to guarantee the efficient outcome.

Notice that Lemma 1 only highlights the existence of moral hazard problem associated with the choice of realignement dates. However, this is just one element of a much wider problem. The implementation of the first-best policy is not only plagued by free-riding issued at the infrequent dates of realignment, but also at other dates (the vast majority of the time), when intervention is required to peg the exchange rate.

The reason is that if "partner" countries attempt to just share the burden of intervention according to the first-best sharing rule, a renegotiation surplus exists and remains to be

[23] Our specific assumption of quadratic unit-period intervention costs yields $\phi_{A\cup B} = \kappa_B/(\kappa_A + \kappa_B)$ for all v_t (equation 4). Then, $\Lambda_A/(\kappa_A\,\phi^2_{A\cup B}) \ > \ \Lambda_B/\kappa_B\,(1-\phi_{A\cup B})^2$ is actually equivalent to a simpler but less intuitive condition $\kappa_A\,\Lambda_A \ > \ \kappa_B\,\Lambda_B$.

internalized. Then, such an agreement is not state-by state renegotiation-proof, as ex-post, countries will attempt to renegotiate the agreed sharing rule, point-by-point, in order to internalize this renegotiation surplus. The extent to which individual countries succeed in doing so depends on their relative bargaining power.

4 Allowing for Renegotiation

4.1 Stackelberg Equilibria

It is therefore essential to take into account that partner countries can continuously and repeatedly revise, in a possibly opportunistic fashion, the terms of an agreement they can only complicitly assent to:

A country can strategically refuse to carry out the level intervention it promised to and demand the partner country carry out a larger share than previously agreed. Furthermore, stronger countries are in a position to demand their partners to carry out larger supplementary intervention and will therefore be able to carry out less themselves.

That is, nothing prohibits countries to renegotiate, at any time, and as many time as they wish, the terms of an initial agreement. No international court can prevent them from doing so. Furthermore, the extend to which a country will attempt to behave opportunistically depends on its relative bargaining power.

We will see that ex-post renegotiation possibilities and countries' relative bargaining power are crucial determinants of exchange rate policies that prevail in equilibrium. Allowing for dynamic renegotiation in our symmetric information set-up generates repeated partnerships games with perfect monitoring and discounting: the problem involves the indefinite repetition of a fixed strategic situation where information is perfect and instantaneous.[24]

Differential games, such as the one played here by both countries policy makers, are difficult to solve for. In order to introduce in a tractable fashion such game-theoretic elements into this continuous-time model, we restrict our attentions to the limiting cases: We will only consider the equilibria that result if (i) country A's policy maker is in a position to make take-it-or-leave-it offers to country B's policy maker, and (ii) vice-versa.

Such situations result in a hierarchical Stackelberg equilibria.[25] The leader policy maker,

[24] In discrete-time environments, such games with perfect and imperfect monitoring are treated in Rubinstein (1979), Fudenberg and Maskin (1986), Radner, Myerson and Maskin (1986), and Abreu (1988).

[25] See Basar and Olsder (1982) for an extensive discussion of stochastic differential games.

L, who has all the bargaining power, commits to a particular strategy. The follower policy maker, F, then reacts optimally, taking the leader's strategy as given. Here, $(L, F) \in \{(A, B)\,;\,(B, A)\}$ denotes to the two limiting cases above. In these simple games, the follower player is nevertheless entitled to reject the take-it-or-leave-it offer made by the leader. Non-coerced agreements can be reached as resulting gains to one side do not imply losses to the other. The strategies are here Markov, open loop (state dependent), and perfect state (perfect information).

To determine the equilibrium complicit renegotiation-proof common policy, we examine the requests the leader, L, could find optimal to make ex-post. Then, the complicit renegotiation-proof agreement consists of the set of actions which generate the equilibrium behavior, in the sense that by backward induction, no deviations from this equilibrium outcome can benefit a player. In other words, for the given relative bargaining power situation, it is the agreement such that although it is possible, no renegotiation will occur ex-post.

In the following, subscripts L and F will replace subscripts A and B (or B and A) when the leader and follower countries are A and B (or B and A), respectively.

4.2 Follower Country's Reservation Strategy

The optimal ex-post attitude of the leader, L, depends in turn on the outside option, or reservation strategy of the follower, F, which we here determine first.

Given his bargaining power, the Stackelberg leader of this game can commit to a strategy in which, if his offer was rejected, he would not cooperate with the follower in the future. However, in case of no cooperation, the leader cannot impede the follower to manage the exchange rate single-handedly. That is, if the follower country, F, rejected the leader's offer, it could still decide to manage the exchange rate, but would be left alone to do so.

If the follower country, F, was to unilaterally manage the exchange rate, it would select the policy that minimizes its expected aggregate losses, taking into account that it would have to bear *all* the burden of intervention, i.e. $\phi_F = 1$. The losses incurred would therefore include (i) unit-period intervention losses $\kappa_F\ (v_t - s_t)^2$, and (ii) a realignment loss Λ_F at each realignment. Again, the optimal policy is a symmetric realignment policy of fixed trigger amplitude (by state-space symmetry and time-homogeneity). The follower's optimal unilateral management strategy is then characterized by the realignment trigger amplitude

$$\underline{\delta}_F \quad \equiv \quad \arg\min_{\delta}\ C(v_t - s_t, \delta \mid \kappa_F\,,\, \Lambda_F) \quad = \quad \tilde{\delta}(\kappa_F\,,\, \Lambda_F)\ . \tag{16}$$

Interestingly, we find that the realignment trigger value of the optimal follower reservation strategy, $\underline{\delta}_F$, is always smaller than the first-best one, $\delta_{A\cup B}$, for both $F \in \{A, B\}$ and for all possible situations $\{\sigma; \kappa_A; \kappa_B; \Lambda_A; \Lambda_B\}$:[26] $\underline{\delta}_F \neq \delta_{A\cup B}$ reflects the fact that (even optimally) defending unilaterally a fixed exchange rate is more onerous than optimally defending it jointly (unilateral intervention is not first-best). $\underline{\delta}_F < \delta_{A\cup B}$ tells us more precisely that, if let alone, it is optimal for a country to endorse an intervention policy that entails *more frequent* realignments than the cooperatively optimal one.

The fact that $\underline{\delta}_F < \delta_{A\cup B}$ has important repercussions on the follower's reservation value. If country F rejected the leader country's offer, at a date $t \in [T_i, T_{i+1})$, while the exchange rate is at v_{T_i}, it's best available strategy would be:

1. For v_t *inside* the interval $(v_{T_i} - \underline{\delta}_F, v_{T_i} + \underline{\delta}_F)$, country F would defend the *prevailing* peg and implement its optimal unilateral management strategy, characterized by (16).
2. For v_t *outside* the interval $(v_{T_i} - \underline{\delta}_F, v_{T_i} + \underline{\delta}_F)$, it would be in country F's best interest to begin realigning *immediately*, setting a new peg at the current level v_t. Then, it would implement its optimal unilateral management policy, characterized by (16).

Overall, the follower's reservation expected aggregate loss function, $\underline{F}(v_t)$, is therefore

$$\underline{F}(v_t) = \begin{cases} C(v_t - v_{T_i}, \underline{\delta}_F \mid \kappa_F, \Lambda_F) & \text{for } v_t \in (v_{T_i} - \underline{\delta}_F,\ v_{T_i} + \underline{\delta}_F), \\ \underline{F}^{(o)} \equiv \Lambda_F + C(0, \underline{\delta}_F \mid \kappa_F, \Lambda_F) & \text{for } v_t \in (-\infty, v_{T_i} - \underline{\delta}_F] \cup [v_{T_i} + \underline{\delta}_F, \infty). \end{cases} \tag{17}$$

Figure 3 gives a graphical representation of $\underline{F}(v_t)$. Notice that it is a continuous function. Furthermore, in the outside intervals, this function is independent of v_t:

$$\underline{F}^{(o)} \;=\; \Lambda_F \;+\; \frac{\kappa_F}{\rho}\left(\frac{\sigma^2}{\rho} \;-\; \frac{2\,\underline{\delta}_F}{\gamma\ \sinh[\gamma\underline{\delta}_F])}\right)\;. \tag{18}$$

We can actually derive the leader's expected aggregate loss function that results if the follower pursues his reservation strategy. For dates $t \in [T_i, T_{i+1})$, denoting it $\underline{L}(v_t)$, we have

$$\underline{L}(v_t - v_{T_i}) \;=\; \sum_{j=i+1}^{\infty} \Lambda_L\, \mathbf{1}_{\{T_j\}}\, e^{-\rho(T_j - t)} \;=\; C(v_t - v_{T_i}, \underline{\delta}_F \mid 0, \Lambda_L)\;. \tag{19}$$

[26]See Appendix C.

5 Equilibrium Renegotiation-Proof Outcome

5.1 Unconstrained Equilibrium

The time consistent renegotiation-proof outcome that prevails in equilibrium is such that no country would benefit from either repudiating or renegotiating it ex-post. We will infer this equilibrium complicit agreement, examining the renegotiation strategies the leader country could otherwise find optimal to pursue ex-post.

We begin considering the case where there are no restrictions on ex-post renegotiation. Here, since the follower would accept an offer that makes him marginally better off with respect to his outside option, the equilibrium complicit renegotiation-proof agreement is such that the leader extracts *all* the rent generated by the difference between the first best policy and the reservation value of the follower.

That is, on the one hand, given that renegotiation is unconstrained, the leader's offers internalize all surplus to be obtained from renegotiation hence the first-best intervention policy is ultimately implemented. On the other hand, the leader's optimal offers leave the follower country marginally better off than in its reservation strategy. We will see that combining these two considerations is sufficient to show that the equilibrium renegotiation proof outcome must be supported by net inter-country wealth transfers:

On the one hand, countries A and B (i) implement the ex-ante optimal symmetric realignment policy of fixed amplitude, $\delta_{A\cup B}$ (equation (12)), and (ii) share the intervention according to the ex-ante optimal sharing rule, $\phi_{A\cup B}$ (equation (4)). Therefore, the sum of L and F's expected aggregate losses equal the cooperatively optimal one. On the other hand, the leader L's requests push the follower F's expected aggregate losses to a level equal to his reservation value, $\underline{F}(v_t)$, in every state: this is, point by point, the limit above which the leader cannot go, because otherwise the follower would reject his demands.

Consider a date $t \in [T_i, T_{i+1})$, the spot rate being pegged at v_{T_i}. By backward induction, the renegotiation-proof agreement which prevails in the unconstrained equilibrium is therefore such that the leader and follower's expected aggregate losses are, respectively,

$$L(v_t - v_{T_i}) = C(v_t - v_{T_i}, \delta_{A\cup B} \mid \kappa_A \, \phi_{A\cup B}^2 + (1-\phi_{A\cup B})^2 \, , \Lambda_A + \Lambda_B) - \underline{F}(v_t) \; , \quad (20)$$

$$F(v_t - v_{T_i}) = \underline{F}(v_t) \; . \quad (21)$$

For all date $t \in [T_j, T_{j+1})$ where $j \in \{i, i+1, ...\}$, the spot rate is pegged at v_{T_j} until the next realignment occurs at the random time $T_{j+1} = \inf \left\{ \tau \geq T_j : |v_\tau - v_{T_j})| = \delta_{A\cup B} \right\}$. Both functions $L(v_t - v_{T_i})$ and $F(v_t - v_{T_i})$ are therefore defined for $v_t - v_{T_i} \in [-\underline{\delta}_{A\cup B}, \underline{\delta}_{A\cup B}]$.

We can then calculate the unit period loss functions, which we denote $l(v_t - s_t)$ and $f(v_t - s_t)$, respectively, generating the above expected aggregate cost functions:

$$E_t \left(\int_t^{\infty} e^{-\rho\,(\tau - t)}\; l(v_\tau - s_\tau)\, d\tau \right) \quad \equiv \quad L(v_t - v_{T_i})\ , \tag{22}$$

$$E_t \left(\int_t^{\infty} e^{-\rho\,(\tau - t)}\; f(v_\tau - s_\tau)\, d\tau \right) \quad \equiv \quad F(v_t - v_{T_i})\ . \tag{23}$$

To solve for $f(v_t - v_{T_i})$ and $l(v_t - v_{T_i})$ we apply Îto's lemma to equations (22) and (23).[27] The follower's reservation expected aggregate losses $\underline{F}(v_t)$ being a non-differentiable function at both v_t equal $v_{T_i} - \underline{\delta}_F$ and $v_{T_i} + \underline{\delta}_F$, the above functions $L(v_t - v_{T_i})$ and $F(v_t - v_{T_i})$ are only twice continuously differentiable functions within the intervals $v_t - v_{T_i} \in [-\underline{\delta}_{A\cup B}, -\underline{\delta}_F]$, $(-\underline{\delta}_F,\, \underline{\delta}_F)$, and $[\underline{\delta}_F, \underline{\delta}_{A\cup B}]$. Îto's lemma can therefore only be applied within each of these three intervals. We obtain

$$l(v_t - v_{T_i}) = \begin{cases} (\,\kappa_A\, \phi_{A\cup B}^2 \;+\; \kappa_B\,(1 - \phi_{A\cup B})^2 \;-\; \kappa_F\,)\;(v_t - v_{T_i})^2 \\ \qquad\qquad\qquad \text{for} \quad v_t - v_{T_i} \;\in\; (-\underline{\delta}_F,\, \underline{\delta}_F)\ , \\ (\,\kappa_A\, \phi_{A\cup B}^2 \;+\; \kappa_B\,(1 - \phi_{A\cup B})^2\,)\;(v_t - v_{T_i})^2 \;-\; \rho\, \underline{F}^{(o)} \\ \qquad\qquad\qquad \text{for} \quad v_t - v_{T_i} \;\in\; [-\underline{\delta}_{A\cup B}, -\underline{\delta}_F] \cup [\underline{\delta}_F, \underline{\delta}_{A\cup B}]\ , \end{cases} \tag{24}$$

and

$$f(v_t - v_{T_i}) = \begin{cases} \kappa_F\;(v_t - v_{T_i})^2 & \text{for} \quad v_t - v_{T_i} \;\in\; (-\underline{\delta}_F, \underline{\delta}_F)\ , \\ \rho\, \underline{F}^{(o)} & \text{for} \quad v_t - v_{T_i} \;\in\; [-\underline{\delta}_{A\cup B}, -\underline{\delta}_F] \cup [\underline{\delta}_F, \underline{\delta}_{A\cup B}]\ . \end{cases} \tag{25}$$

We now gather our results: On the one hand, the unit period losses incurred *purely* implementing the first-best intervention policy are $\kappa_A \phi_{A\cup B}^2 (v_t - v_{T_i})^2$ for country A, and $\kappa_B (1 - \phi_{A\cup B})^2) (v_t - v_{T_i})^2$ for country B. On the other hand, given the leader's bargaining power, the complicit agreement is only an equilibrium outcome if the leader L's unit period loss function, $l(v_t - v_{T_i})$, reaches (24).

Consequently, in equilibrium, the follower, F, is pushed to *transfer* every unit of time an amount of wealth to the leader, L, equal to the *difference* between (a) the leader's pure policy implementation loss and (b) the leader's equilibrium loss, $l(v_t - v_{T_i})$. Let $\theta(v_t - v_{T_i})$ denote this unit period net inter-country wealth transfer. We have

$$\theta(v_t - v_{T_i}) = \begin{cases} \kappa_F\,(1 - \phi_F^2)\;(v_t - v_{T_i})^2 & \text{for } v_t - v_{T_i} \in\; (-\underline{\delta}_F,\, \underline{\delta}_F), \\ \rho\, \underline{F}^{(o)} -\; \kappa_F\, \phi_F^2\,(v_t - v_{T_i})^2 & \text{for } v_t - v_{T_i} \in [-\underline{\delta}_{A\cup B}, -\underline{\delta}_F] \cup [\underline{\delta}_F, \underline{\delta}_{A\cup B}]\,. \end{cases} \tag{26}$$

Here, if (L, F) is (A, B) or (B, A), (ϕ_L^2, ϕ_F^2) denotes $(\phi_{A\cup B}^2, (1 - \phi_{A\cup B})^2)$ or $((1 - \phi_{A\cup B})^2, \phi_{A\cup B}^2)$.

[27]See Appendix D.

Figure 4 breaks-down the countries' unit-period losses by nature: First, $\kappa_L \phi_L^2 (v_t - v_{T_i})^2$ and $\kappa_F \phi_F^2 (v_t - v_{T_i})^2$ represent the leader and follower unit period loss which is solely related to the *implementation* of the first-best intervention policy. Second, $\theta(v_t - v_{T_i})$ represents, respectively, the equilibrium unit period transfer from the follower to the leader, whose existence is only related to *bargaining power*. Adding-up, the resulting leader's unit period loss function, $l(v_t - v_{T_i})$, equals $\kappa_L \phi_L^2 (v_t - v_{T_i})^2$ minus $\theta(v_t - v_{T_i})$. Similarly, the resulting follower's unit period loss function, $f(v_t - v_{T_i})$, equals $\kappa_F \phi_L^2 (v_t - v_{T_i})^2$ plus $\theta(v_t - v_{T_i})$.

Clearly, when $(L, F) = (A, B)$ or $(L, F) = (B, A)$, the unconstrained equilibrium transfer flow from follower to leader, $\theta(v_t - v_{T_i})$, are not mirror images. For example, the situation, $\{\sigma; \rho; \kappa_A; \kappa_B; \Lambda_A; \Lambda_B\}$, depicted in Figure 5 is such that the unilateral optimal realignment policy of country A entails a trigger amplitude $\underline{\delta}_A$ larger than $\underline{\delta}_B$, the corresponding one for country B. Then, if country A is the leader, the internal regime is narrower and the external regime is wider than if country B is the leader.

We have established that, when renegotiation is considered and there are no restrictions to it, partner countries will converge to an equilibrium in which (i) the best policy they could cooperatively select is sustained and (ii) bargaining power related wealth transfers are made from the weaker to the stronger country.

Persson and Tabellini (1994) actually argued that side payments among partner countries are one way to implement efficient policies. Their analysis was however based on the convenient but unsatisfactory assumption that partner countries can write and endorse contracts. We have here taken a different angle: taking into account the fact that it is virtually impossible for sovereign countries to credibly write such contracts, and that countries have different bargaining power, we have shown that the efficient outcome prevails in equilibrium if there are direct transfers from the weak to the strong country.

5.2 Limited Wealth Transferability

In general, there are many ways in which net cross-country wealth transfers can be organized. They can take different forms, from trade arrangements to military cooperation, from aid donations to joint investment projects.[28] Essentially, when a country makes a concession on such items to another country, the losses to the former and benefits to the latter amount to a cross country wealth transfer.

[28] As examples of such transfers we can cite the trade concessions (in the form of the Most Favored Nation status) made in the past to Communist China by the United States or the recent increase in the Saudi Arabia quota of oil production aimed at reducing oil prices and the United States energy bill.

In practice however, it is usually difficult to identify sets of concession-items which yield the unconstrained equilibrium cross-country wealth transfers. Such items may not be physically available or simply do not exist. The country transferring out wealth has weak bargaining power, hence is often also struggling economically. It may have little to transfer, and therefore little to be requested from his dominant "partner".

In such circumstances, both player-countries of the partnership game are aware that wealth transferability is limited, when they are to strike a complicit exchange rate policy agreement. Intuitively, the impact of such limits on the equilibrium outcome is as follows:

Our determination of the unrestricted equilibrium outcome was based on two considerations: On the one hand, because renegotiation is unconstrained, the leader's offers internalize all surplus to be obtained from renegotiation and the first-best intervention policy is ultimately implemented. On the other hand, the leader's optimal offers leave the follower country marginally better off than in its reservation strategy.

If cross country wealth transferability is limited, this puts a restriction on potential ex-post renegotiations. Now, the leader's optimal offers would still leave the follower country marginally better off than in its outside option. That is, the most opportunistic requests the leader can make still push the follower expected aggregate losses up to a level everywhere equal to his reservation value, $\underline{F}(v_t)$. This remains, point by point, the most the leader obtain from the follower, without the follower rejecting his demands.

However, given that renegotiation is constrained, the leader would be unable to internalize all the renegotiation surplus. The more wealth transferability is limited, the less the leader will gain in equilibrium from his dominant position in would be renegotiations. When transfers are limited, the intervention policy which is implemented is only second-best, and interestingly, the leader country is the only one loosing out.

Essentially, the leader cannot organize to his own benefit the exchange of more personal intervention for direct wealth receipts from his partner. The non-availability of direct wealth transfer removes the flexible tool by which the leader could otherwise manage to internalize not just part, but the entire renegotiation surplus.

The requested concession-items in the constrained equilibrium will therefore be such that resulting direct wealth transfers mimic as much as possible the unrestricted equilibrium transfer function, $\theta(v_t - v_{T_i})$, or more coarsely, have its main characteristics:

1. The unrestricted equilibrium inter-country transfer, $\theta(v_t - v_{T_i})$, is a bargaining power compensation tool in the uninterrupted partnership game countries play continuously. Hence, an important feature of these direct transfers is that wealth is repeatedly trans-

ferred – in our model, every unit of time.

2. This transfer flow, $\theta(v_t - v_{T_i})$, is an increasing and decreasing function of the amplitude of intervention, $|v_t - v_{T_i}|$, within the internal and the external regimes, respectively.

   In the external regime, the follower, F, has a decreasing incentive to stick to an agreement: Endorsing the agreement implies an expected aggregate loss which is increasing in $|v_t - v_{T_i}|$. Conversely, the expected aggregate loss of his reservation strategy becomes insensitive to state, because the follower left alone would immediately realign (see Section 4.2). As a consequence, when the fundamental deviates much from the prevailing peg, the opportunistic leader, L, cannot extract as much rent from the follower.

3. For very large values of the ex-ante optimal realignment trigger amplitude, $\delta_{A\cup B}$, this transfer becomes negative in the periods that precede realignments dates.[29]

4. A discontinuity emerges in $\theta(v_t - v_{T_i})$ at both $v_t - v_{T_i}$ equal $-\underline{\delta}_F$ and $\underline{\delta}_F$.

   The change in follower reservation strategy at these points (as established in Section 5.2.) drives this dynamic. Essentially, a change in convexity in an aggregate loss function around a regime switch point results in a discrete jump in its supporting instantaneous loss function. Here the change in second derivative with respect to v_t in the follower's reservation value, $\underline{F}(v_t)$, at both v_t equal $v_{T_i} - \underline{\delta}_F$ and $v_{T_i} + \underline{\delta}_F$ yields discontinuities in $f(v_t - v_{T_i})$, hence in $\theta(v_t - v_{T_i})$ and $l(v_t - v_{T_i})$.[30]

5.3 Equilibrium Outcome without Wealth Transfers

We now determine the equilibrium that prevails in the extreme case where cross-country wealth transfers are not feasible at all. Here, the internalized surplus from renegotiation is minimum. Comparison with the unconstrained equilibrium will then enable us to quantify the inefficiency a limited wealth transferability can generate, hence to assess whether cross-country wealth transfers are at all important.

Even without wealth transfers, the most opportunistic requests the leader can make would still leave the follower country marginally better off than in its outside option. In equilibrium, this pushes the follower expected aggregate losses up to a level everywhere equal

[29] $\theta(v_t - v_{T_i})$ is non-negative for $v_t - v_{T_i} \in (-\underline{\delta}_F, \underline{\delta}_F)$. It is increasing for $v_t - v_{T_i} \in (\delta_{A\cup B}, \delta_F]$ and decreasing for $v_t - v_{T_i} \in [\delta_F, \delta_{A\cup B})$. Therefore, it is non-negative throughout the interval $(-\delta_{A\cup B}, \delta_{A\cup B})$ if and only if $\theta(\delta_{A\cup B})$ is non-negative. Now, $\theta(\delta_{A\cup B}) = \rho \underline{F}^{(o)} - \kappa_F \, \phi_F^2 \, \delta_{A\cup B}^2$. Hence, if $\delta_{A\cup B} > \sqrt{\rho \underline{F}^{(o)} / (\kappa_F \phi_F^2)}$, the transfer flow $\theta(v_t - v_{T_i})$ becomes negative for $|v_t - v_{T_i}|$ close to $\delta_{A\cup B}$.

[30] Algebraically, given that $\underline{F}(v_t)$ is countinous in v_t, we have $F(\underline{\delta}_F^-) = \underline{F}^{(o)}$. Therefore the difference $f(\underline{\delta}_F) - f(\underline{\delta}_F^-) = (\sigma^2/2) \; d^2 \, C(v_t - v_{T_i}, \delta \mid \kappa_F, \Lambda_F) / d\, v_t^2 \big|_{\underline{\delta}_F^-} \neq 0$.

to his reservation value, $\underline{F}(v_t)$, exactly as in equation (21). However, given that he cannot obtain any direct transfers, the leader is unable to internalize more of the renegotiation surplus than what can be extracted requesting more follower intervention.

At a date $t \in [T_i, T_{i+1})$, a follower's expected aggregate loss function, which we denote $F^*(v_t - v_{T_i})$, everywhere equal his reservation value,

$$F^*(v_t - v_{T_i}) \;=\; \underline{F}(v_t) \qquad \text{for} \quad v_t - v_{T_i} \;\in (-\delta^*, \delta^*) \;, \tag{27}$$

is the discounted sum over time of a follower unit-period intervention loss,

$$f^*(v_t - v_{T_i}) = \begin{cases} \kappa_F \; (v_t - v_{T_i})^2 & \text{for} \quad v_t - v_{T_i} \;\in (-\underline{\delta}_F, \underline{\delta}_F) \;, \\ \rho \, \underline{F}^{(o)} & \text{for} \quad v_t - v_{T_i} \;\in [-\delta^*, -\underline{\delta}_F] \cup [\underline{\delta}_F, \delta^*] \;. \end{cases} \tag{28}$$

Here δ^* denotes the leader's optimal realignement trigger amplitude when transfers are impossible. This is clearly controlled by the Stackelberg leader. We therefore begin considering δ^* as given, assuming it to be larger than $\underline{\delta}_F$, the realignement trigger amplitude of the follower's optimal reservation strategy. We will then determine, in equation (40), the first order optimality condition δ^* actually satisfies and verify it is larger than $\underline{\delta}_F$.

The share of intervention, $\phi^*_F(v_t)$, generating the unit-period intervention loss function in equation (28), is the solution to $f^*(v_t - v_{T_i}) = \kappa_F \; (\phi^*_F(v_t))^2 \; (v_t - v_{T_i})^2$. That is, the leader can at most request that the follower carries a share of intervention

$$\phi^*_F(v_t) = \begin{cases} 1 & \text{for} \quad v_t - v_{T_i} \;\in (-\underline{\delta}_F, \underline{\delta}_F) \;, \\ \sqrt{\rho \, \underline{F}^{(o)} / (\kappa_F (v_t - v_{T_i})^2)} & \text{for} \quad v_t - v_{T_i} \;\in [-\delta^*, -\underline{\delta}_F] \cup [\underline{\delta}_F, \delta^*] \;. \end{cases} \tag{29}$$

Given that he cannot obtain direct transfers, this is the follower intervention the Stackelberg leader will request in equilibrium. The leader's share of intervention, ϕ^*_L, just complements that of the follower, $\phi^*_L = 1 - \phi^*_F$. The equilibrium leader L unit period costs is therefore

$$l^*(v_t - v_{T_i}) = \begin{cases} 0 & \text{for} \quad v_t - v_{T_i} \;\in (-\underline{\delta}_F, \underline{\delta}_F) \;, \\ \kappa_L \left[|v_t - v_{T_i}| \; - \; \sqrt{\rho \, \underline{F}^{(o)} / \kappa_F} \right]^2 & \text{for} \quad v_t - v_{T_i} \;\in [-\delta^*, -\underline{\delta}_F] \cup [\underline{\delta}_F, \delta^*] \;. \end{cases} \tag{30}$$

Let $L^*(v_t - v_{T_i})$ denote the discounted stream of the leader's unit period losses, $l^*(v_t - v_{T_i})$, when a realignment is triggered each time $|v_t - s_t|$ reaches the amplitude δ^*. If the spot rate is currently pegged at v_{T_i}, it is

$$L^*(v_t - v_{T_i}) \;\equiv\; E_t \left[\int_t^\infty l^*(v_\tau - s_\tau) \, e^{-\rho(\tau - t)} \, d\tau \right] + \sum_{j=i+1}^{\infty} \Lambda_L \, \mathbf{1}_{\{T_j\}} \, e^{-\rho(T_j - t)} \;. \tag{31}$$

Here $s_\tau = v_{T_{j-1}}$ for all $\tau \in [T_{j-1}, T_j)$, where $T_j = \inf\{\tau \geq T_{j-1} : |v_{T_{j-1}} - v_\tau| = \delta^*\}$. $\mathbf{1}_{\{T\}} \equiv 1$ at date T and zero otherwise. Denoting $\gamma \equiv \sqrt{2\rho}/\sigma$, we obtain the following expressions:[31]

– For a realignment date $t = T_i$, $s_{T_i} = v_{T_i}$ and

$$L^*(v_{T_i} - s_{T_i}) \;=\; K^*(0) \;+\; \Lambda_L \;-\; \left(K^*(\delta^*) \;-\; K^*(0) \;-\; \Lambda_L\right) \frac{1}{\cosh[\gamma\delta^*] - 1} \;. \tag{32}$$

– For a non-realignment date $t \in (T_i, T_{i+1})$, $s_t = v_{T_i}$ and

$$L^*(v_t - s_t) \;=\; K^*(v_t - v_{T_i}) \;-\; \left(K^*(\delta^*) \;-\; K^*(0) \;-\; \Lambda_L\right) \frac{\cosh[\gamma(v_t - v_{T_i})]}{\cosh[\gamma\delta^*] - 1} \;. \tag{33}$$

$K^*(v_t - v_{T_i}) \equiv \lim_{\delta^* \to \infty} \; L^*(v_t - v_{T_i})$ corresponds to the discounted stream of leader's unit period losses, $l^*(v_t - v_{T_i})$, if the exchange rate was *forever* pegged at v_{T_i}, and[32]

$$K^*(v_t - v_{T_i}) \;=\; \frac{\kappa_L}{\rho}\left[\omega_t^2\,\Omega_0 \;+\; \omega_t\,\frac{\sigma}{\sqrt{2\rho}}\,\Omega_1 \;+\; \frac{\sigma^2}{\rho}\,\Omega_2\right] \;, \tag{34}$$

$$\text{where} \quad \omega_t \;\equiv\; \left[(v_t - v_{T_i}) \;-\; \mathrm{sg}[v_t - v_{T_i}]\,\sqrt{\rho\,\underline{F}^{(o)}/\kappa_F}\right] \;, \tag{35}$$

$$\Omega_0 \;\equiv\; 1 \;+\; \frac{\mathrm{sg}[\xi_2]}{2}\left(1 \;-\; e^{-|\xi_2|}\right) \;-\; \frac{\mathrm{sg}[\xi_1]}{2}\left(1 \;-\; e^{-|\xi_1|}\right) \;, \tag{36}$$

$$\Omega_1 \;\equiv\; (1 + |\xi_1|)\,e^{-|\xi_1|} \;-\; (1 + |\xi_2|)\,e^{-|\xi_2|} \;, \tag{37}$$

$$\Omega_2 \;\equiv\; \frac{\xi_1}{4}\,(2 + |\xi_1|)\,e^{-|\xi_1|} \;-\; \frac{\xi_2}{4}\,(2 + |\xi_2|)\,e^{-|\xi_2|} \;+\; \Omega_0 \;, \tag{38}$$

$$\text{with} \quad \xi_1 \;\equiv\; \frac{\sqrt{2\rho}}{\sigma}\,(s_t + \underline{\delta}_F - v_t) \;, \quad \text{and} \quad \xi_2 \;\equiv\; \frac{\sqrt{2\rho}}{\sigma}\,(s_t - \underline{\delta}_F - v_t) \;. \tag{39}$$

The first order optimality condition satisfied by the realignment trigger amplitude δ^* which minimizes the leader's expected costs, i.e. $\delta^* \equiv \arg\min_{\delta^*} \; L^*(v_t - v_{T_i})$, is then[33]

$$\frac{\partial\, K^*(\delta^*)}{\partial \delta^*}\;\frac{(\cosh[\gamma\delta^*] - 1)}{\gamma\,\sinh[\gamma\delta^*]} \;=\; \left(K^*(0) \;+\; \Lambda_L \;-\; K^*(\delta^*)\right) \;. \tag{40}$$

---

[31] See Appendix E.II.

[32] See Appendix E.I.

[33] The derivation is similar to that of equation (11). It can then be shown that $\delta^* > \underline{\delta}_F$ as initially assumed, following the same steps as Appendix C.

6 Measuring the Importance of Wealth Transfers

We now consider a numerical example, to assess best the impact of renegotiation and inter-country wealth transferability on realignment trigger amplitude and countries' aggregate losses, as well as appreciate the differences these issues can generate graphically.

Figure 6 represents the aggregated expected losses of the leader and follower countries as functions of the amount of intervention, $s_t - v_t$, under three scenarios: Panel (a) refers to the scenario without renegotation, where the follower unilaterally undertakes its reservation intervention policy and the leader simply free-rides on the activity of the former, as in Section 4.2 equations (19) and (17). Panel (b) considers the restricted equilibrium when wealth transfers between the two partner countries are absolutely impossible, as in Section 5.3 equations (32), (33) and (27). Finally, Panel (c) refers to the unrestricted equilibrium, where wealth transfers are perfectly possible, as in Section 5.1 equations (20) and (21). In the three panels, input parameters are those of two countries with identical intervention costs, but where the follower suffers a smaller realignment loss when a peg is abandoned, i.e. $\kappa_L = \kappa_F$ and $\Lambda_L > \Lambda_F$.

In terms of the implemented intervention policy, these input parameters yield devaluation trigger values of (a) $\underline{\delta}_F = 0.395$ in the follower unilateral reservation policy, (b) $\delta^* = 0.769$ in the restricted renegotiation-proof equilibrium, and (c) $\delta_{A\cup B} = 0.724$ in the unrestricted renegotiation-proof equilibrium:

1. The equilibrium devaluation trigger values that prevail in the renegotiation-proof equilibria, $\delta_{A\cup B}$ and δ^*, *largely* exceed the devaluation trigger amplitude, $\underline{\delta}_F$, the follower chooses if he unilaterally endorses a realignment policy.[34] The expected time between realignments, δ^2/σ^2, in the renegotiation-proof equilibria being roughly four times larger than in the unilateral policy. These differences remain of that magnitude when we use other input values.

2. The difference between $\delta_{A\cup B}$ and δ^* is however minimal and its sign can be reversed with different input parameters. That is, no precise ordering exists between the restricted and unrestricted equilibrium devaluation trigger amplitudes, and importantly, limited wealth transferability does not affect much the timing of realignments in equilibrium.

In terms of aggregate losses, Figure 6 shows that the leader benefits from very significant renegotiation opportunities, as its aggregate expected losses are much greater when renegotiation is not considered (Panel (a)). The leader's gain with renegotiation is then larger

[34]Notice that we have analytically established $\delta_{A\cup B} > \underline{\delta}_F$ and $\delta^* > \underline{\delta}_F$.

when wealth is transferable (Panel (c) compared with Panel (b)), as he is capable of appropriating a substantial renegotiation surplus: Comparing the constrained and un-constrained equilibria we see that the relative gain is state-dependent and in percentage terms varies significantly from values above 100% in the internal regime to values of around 20% at the edges of the external regime. In the internal section of the state space, limited wealth transferability makes that the leader has to renounce receiving largest wealth transfers, where the follower would be otherwise most willing to compensate him for his cooperation.

Interestingly, limited wealth transferability does not affect much the equilibrium frequency of realignments, which outsiders can observe, but changes drastically the aggregate losses of the leader relative to the follower, which only insiders can observe.

7 Bargaining, Monetary Cooperation and EMU

Sovereign countries continuously negotiate over international policy issues. These may vary from monetary policy coordination to international treaties, from the design of super-national institutions, such as the World Trade Organization or the European central bank, to national security. Since economic and political conditions evolve agreements can be repudiated or revised, sovereign countries engage themselves in a process of continuous revision of proposals and solutions that eventually leads to some equilibrium agreement, where parties make concessions to their partners according to their relative bargaining strength.

In this Section, we recast the evolution of monetary arrangements between Germany and France and the road to monetary union in Europe (EMU) within our theoretical framework. The design of EMU, with the creation of a common currency and a super-national central bank, represents one of the most striking examples of wrangling and twisting among partner countries (Szász (1999)). With Germany being in a Stackelberg leader position relative to France, the process of negotiation between them lasted for more than twenty years, but eventually produced an agreement to lock in the monetary policies of the two countries. We can distinguish three phases which our theory essentially associates to the three scenarios compared in the previous Section 6 and depicted in Figure 6:[35]

• *First Phase (1979-1986):* The process began with the creation of the Snake and then continued with the development of the European Monetary System (EMS). Given the rather narrow margins of the currency bands for the exchange rates of the member countries,

[35]Because of their superior reputation, German monetary authorities would sustain larger credibility losses than the French ones in correspondence of realignments of the central parity. This asymmetry justifies the particular choice of the parameters made in Figure 6, where $\Lambda_L > \Lambda_F$.

the snake and EMS have always been seen as a system of adjustable pegs with infrequent realignements.

The formulation of these two similar exchange rate mechanisms was the result of the stronger bargaining position of the German authorities. Both parties agreed that they should tight together their monetary policies, as they would benefit from a stable exchange rate. However, the French authorities had to accept an exchange rate mechanism where it was responsibility of the central bank issuing the *weak* currency, to maintain the central parity when the system was under speculative pressure. This was only seemingly an even exchange rate mechanism, in that the French franc was clearly the weak currency. In practice, France would therefore bear *all* the burden of intervention defending the system imposed.

Limited support from the German monetary authorities was provided through the Very Short Term Financing (VSFT) facility. Importantly however, while the Bank of France could borrow un-limited amounts of Deutsche marks during periods of balance of payments difficulties, automatic access was allowed *only* when the central parity was under attack and VSFT liabilities were to be settled within a very short period of time. The system was therefore not capturing the value of renegotiation because it was rather inflexible. Essentially, the leader Germany was just free-riding on the activity of the follower France which had nothing better than undertaking its reservation unilateral intervention policy.

In terms of our analysis, this situation yields country-specific aggregate losses developed in Section 4.2 equations (19) and (17) and depicted in Panel (a) in Figure 6. The shares of intervention were $\phi_F = 1$ and $\phi_L = 0$, where $(L, F) = (Germany, France)$, and the devaluation trigger amplitude implemented by the French authorities was $\delta = \underline{\delta}_F$. Notice that in line with our analysis, the French franc was actually devalued seven times between 1979 and 1986, which is much more than afterwards.

• *Second Phase (1986-1993):* Not surprisingly, a significant step towards an efficient outcome was taken with the Basle-Nyborg Agreement in 1987, when participants to the EMS relaxed the crucially inefficient conditions for access to the VSFT facility: The maturity of debt contracted with other central banks was extended to 75 days and the limits of automatic renewal were doubled. More importantly, the access to the facility to finance infra-marginal intervention, i.e. to finance intervention which takes place when the exchange rate is well within its band, was no longer subject to the authorization of the lending central bank.

In practice, these apparently minor amendments changed substantially the way the two partner countries shared intervention. By providing extended borrowing facilities to their counter-party, the German authorities agreed to sustain part of the intervention required to defend the exchange rate mechanism. Notice, this does not mean that Germany was loosing out in the process, in that countries, by bringing more flexibility to the system are internalizing parts of a renegotiation surplus.

Our theory's reading of events is that, in that period (as opposed to the following third Phase), the concessions France could possibly make, could not generate wealth transfers to Germany of a magnitude close to those of the unrestricted equilibrium transfer function, $\theta(v_t - v_{T_i})$. Germany could not capture all the positive externalities that monetary cooperation could provide because the prevailing equilibrium was restricted by France's limited wealth transferability.

The situation was closer to the restricted equilibrium developed in Section 5.3, with country-specific aggregate losses given by equations (32), (33) and (27) and depicted in Panel (b) in Figure 6. Here Germany internalizes parts of the renegotiation surplus and France none of it, so that these changes to the EMS were mostly to Germany's benefit. The restricted equilibrium devaluation trigger amplitude, δ^*, leading to a large reduction in the frequency of realignment with respect to the first Phase. As a matter of fact, after the last realignment of December 1986, the central parity between the French franc and the Deutsche mark remained stable until the EMS collapsed in 1993.

• *Third Phase (1993 onwards):* In the end, policy makers considered that establishing a common currency and an independent European central bank was desirable. However, the German authorities would agree upon renouncing their monetary independence only in exchange of important concessions in the area of Germany's highest political interest, east-west relations and national security.

Our theory's reading of events is that, given their strong bargaining power, Germany would not agree until concessions of a magnitude comparable to the unrestricted equilibrium transfer function, $\theta(v_t - v_{T_i})$, were made. This because with the prospect of German reunification such concessions became available at that time.

No wonder that the negotiations which led to the drafting of the Delors Report and to the Maastricht Treaty were parallel to the process of German reunification. Essentially, German authorities did not oppose the creation of a common currency and an independent European central bank in exchange of the French support to the reunification of West and East Germany.

Our model has little to say about EMU since then. However it is suggestive of the magnitude of the concession it took to get Germany's acceptance of EMU: This is the difference in aggregate expected losses between the restricted and the unrestricted equilibrium outcome, $L^*(v_t - s_t)$ (equations (32), (33) and Figure 6 Panel (b)) minus $L(v_t - s_t)$ (equation (20) and Figure 6 Panel (c)). This appears as very substantial and probably only the continuous benefit German people receive from their homeland reunification could generate such values.

8 Concluding Remarks and Extensions

Sovereign countries are often engaged in prolonged negotiations over international policy issues. Commonly proposed solutions to the associated moral hazard problems require either the ability of writing complete contracts or designing precommitment devices which efficiently tie the actions of partner countries. However, these solutions, when they exist, generally impose some measure of sovereign devolution, cast into international treaties or super-national institutions, that are politically hard to accept and/or realize.

When such devices are not available, the actions that partner countries eventually undertake are the result of a bargaining process characterized by continuous renegotiation, where the terms of any complicit agreement can be revised at any time. One of the prominent context of such negotiations is in the area of coordination of monetary and exchange rate policies, as shown in the recent history of monetary arrangements in Europe.

We have offered a formal study of this bargaining process in exchange rate policy coordination, analyzing the strategic games policy makers play and deriving the complicit renegotiation-proof common policies that prevail in equilibrium. These consist of the actions which generate the equilibrium behavior, in the sense that backward induction implies that no deviations from this equilibrium outcome can benefit one of the partners. That is, for the given relative bargaining power situation and regulation, it is the agreement such that although it is possible, no renegotiation will occur ex-post.

When renegotiation is not limited in any way, the ex-ante optimal policy is clearly implemented. However, we find that efficiency is only reached in equilibrium, when substantial net cross-country wealth transfers from the weak to the strong bargaining power country occur. Cross-country wealth transfers are possibly disturbing and politically difficult to reveal. However, when such transfers are only partially feasible or impossible altogether, the equilibrium outcome which then prevails implements a second-best intervention policy.

We have established our results with a highly stylized reduced-form model of exchange rate management. We have clearly skipped important issues which certainly deserve attention in future research:

1. It would be desirable to link our study of foreign exchange policies with welfare analysis, developing a fully blown macroeconomic model, where production and consumption decisions affect the determination of the exchange rate.

2. One would like to analyze more realistic intermediate distributions of bargaining power than the limiting cases we have examined. Current treatment of stochastic differential games does not allow to fully describe them. Notice however that there will also be wealth transfers in equilibrium with intermediate cases, given the complex state dependency of our solutions in the limiting ones.

3. Another interesting extension would be to consider multiple agent problems created by political divisions within a country, as power is most often shared between different institutions with different authority and conflicting agendas.

4. One could reflect the fact that some currencies tend to appreciate (depreciate) with a generating data process for the fundamental value, v_t, which allows for a drift term. It would then be possible to relate the sign of that drift with the identity of the leader country.

5. Finally, we could capture the fact that stronger currencies improve the policy maker's reputation introducing different realignment losses for devaluations and appreciations.

APPENDIX

A. Calculation of the Operators $H_k(v_t, s_t \mid a)$, for $k \in \{0,1,2\}$ and $a \in \mathbf{R}^+$:

$$
\begin{aligned}
H_k(v_t, s_t \mid a) &\equiv \int_t^\infty J_k(v_t, s_t \mid a)\, e^{-\rho(\tau-t)}\, d\tau \;, \\
\text{where} \quad J_k(v_t, s_t \mid a) &\equiv \int_{s_t-a}^{s_t+a} \frac{(v_\tau - v_t)^k}{\sigma\sqrt{2\pi\,(\tau-t)}} \exp\left[-\frac{1}{2}\frac{(v_\tau-v_t)^2}{\sigma^2\,(\tau-t)}\right] dv_\tau \;.
\end{aligned}
$$

Let $\qquad \xi_1 \equiv \dfrac{\sqrt{2\rho}}{\sigma}\,(s_t + a - v_t) \qquad$ and $\qquad \xi_2 \equiv \dfrac{\sqrt{2\rho}}{\sigma}\,(s_t - a - v_t)$.

(a) To begin with, we calculate $J_0(v_t, s_t \mid a)$, $J_1(v_t, s_t \mid a)$ and $J_2(v_t, s_t \mid a)$:

$$
J_0(v_t, s_t \mid a) \;=\; \Phi\left[\frac{\xi_1/\sqrt{2\rho}}{\sqrt{\tau-t}}\right] \;-\; \Phi\left[\frac{\xi_2/\sqrt{2\rho}}{\sigma\sqrt{\tau-t}}\right] \;. \tag{41}
$$

Using the change of variable $x_\tau \equiv (v_\tau - v_t)/[\sigma\sqrt{(\tau-t)}]$, for which $dv_\tau = dx_\tau\,\sigma\sqrt{(\tau-t)}$, we have

$$
J_k(v_t, s_t \mid a) \;=\; \frac{\left(\sigma\sqrt{\tau-t}\right)^k}{\sqrt{2\pi}} \left(\left[\, -(x_\tau)^{k-1}\, e^{-x_\tau^2/2} \,\right]_{\frac{\xi_2/\sqrt{2\rho}}{\sqrt{\tau-t}}}^{\frac{\xi_1/\sqrt{2\rho}}{\sqrt{\tau-t}}} + \int_{\frac{\xi_2/\sqrt{2\rho}}{\sqrt{\tau-t}}}^{\frac{\xi_1/\sqrt{2\rho}}{\sqrt{\tau-t}}} (k-1)(x_\tau)^{k-2}\, e^{-x_\tau^2/2}\, dx_\tau \right) \;.
$$

Consequently,

$$
J_1(v_t, s_t \mid a) \;=\; \frac{\sigma\sqrt{\tau-t}}{\sqrt{2\pi}} \left(\exp\left[-\frac{(\xi_2/\sqrt{2\rho})^2}{2(\tau-t)}\right] \;-\; \exp\left[-\frac{(\xi_1/\sqrt{2\rho})^2}{2(\tau-t)}\right] \right) \;. \tag{42}
$$

$$
\begin{aligned}
J_2(v_t, s_t \mid a) \;=\; & \frac{\sigma^2\sqrt{\tau-t}}{\sqrt{2\pi}} \left(\frac{\xi_2}{\sqrt{2\rho}} \exp\left[-\frac{(\xi_2/\sqrt{2\rho})^2}{2(\tau-t)}\right] \;-\; \frac{\xi_1}{\sqrt{2\rho}} \exp\left[-\frac{(\xi_1/\sqrt{2\rho})^2}{2(\tau-t)}\right] \right) \\
& + \sigma^2\,(\tau-t) \left(\Phi\left[\frac{\xi_1/\sqrt{2\rho}}{\sqrt{\tau-t}}\right] \;-\; \Phi\left[\frac{\xi_2/\sqrt{2\rho}}{\sigma\sqrt{\tau-t}}\right] \right) \;.
\end{aligned} \tag{43}
$$

In particular, $J_0(v_t, s_t \mid \infty) = 1$, $J_1(v_t, s_t \mid \infty) = 0$ and $J_2(v_t, s_t \mid \infty) = \sigma^2\,(\tau - t)$.

(b) We now calculate $H_0(v_t, s_t \mid a)$, $H_1(v_t, s_t \mid a)$ and $H_2(v_t, s_t \mid a)$. We use results concerning Laplace transforms in Erdélyi (1954) page 146, and Zwillinger (1996) pages 559 and 563:

$$
H_0(v_t, s_t \mid a) \;=\; \int_0^\infty \Phi\left[\frac{\xi_1/\sqrt{2\rho}}{\sqrt{y}}\right] e^{-\rho y}\, dy \;-\; \int_0^\infty \Phi\left[\frac{\xi_2/\sqrt{2\rho}}{\sqrt{y}}\right] e^{-\rho y}\, dy \;.
$$

Using the fact that for $x > 0$ and $c > 0$,

$$
\operatorname{erf}[x] \equiv \int_0^x \frac{2}{\sqrt{\pi}}\, e^{-u^2}\, du \quad \text{and} \quad \int_0^\infty \operatorname{erf}\left[\frac{c}{2\sqrt{y}}\right] e^{-\rho y}\, dy \;=\; \frac{1}{\rho}\, \left[1 - e^{-c\sqrt{\rho}}\right] \;,
$$

$$
\begin{aligned}
\int_0^\infty \Phi\left[\frac{\xi/\sqrt{2\rho}}{\sqrt{y}}\right] e^{-\rho y}\, dy \;&=\; \int_0^\infty \left[\frac{1}{2} + \operatorname{sg}[\xi] \int_0^{|\xi|/\sqrt{2\rho y}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{u^2}{2}\right] du\right] e^{-\rho y}\, dy \\
&=\; \frac{1}{2\rho} + \frac{\operatorname{sg}[\xi]}{2} \int_0^\infty \operatorname{erf}\left[\frac{|\xi|}{2\sqrt{\rho y}}\right] e^{-\rho y}\, dy \;=\; \frac{1}{2\rho} + \frac{\operatorname{sg}[\xi]}{2\rho}\left(1 - e^{-|\xi|}\right) \;.
\end{aligned}
$$

Consequently,

$$H_0(v_t, s_t \mid a) \;=\; \frac{\mathrm{sg}[\xi_1]}{2\rho}\left(1 \,-\, e^{-|\xi_1|}\right) \;-\; \frac{\mathrm{sg}[\xi_2]}{2\rho}\left(1 \,-\, e^{-|\xi_2|}\right) \,. \tag{44}$$

In particular, $H_0(v_t, s_t \mid \infty) = 1/\rho$, and $H_0(v_t, v_t \mid a) = [1 - e^{-\gamma a}]/\rho$.

$$\begin{aligned} H_1(v_t, s_t \mid a) &= \frac{\sigma}{\sqrt{2\pi}}\left\{\int_0^\infty \sqrt{y}\exp\left[\frac{-(\xi_2/\sqrt{2\rho})^2}{2y}\right] e^{-\rho y} dy - \int_0^\infty \sqrt{y}\exp\left[\frac{-(\xi_1/\sqrt{2\rho})^2}{2y}\right] e^{-\rho y} dy\right\} \\ &= \frac{\sigma}{(2\rho)^{3/2}}\left[(1+|\xi_2|)\, e^{-|\xi_2|} \;-\; (1+|\xi_1|)\, e^{-|\xi_1|}\right] \qquad (45) \\ &\left(\text{as} \int_0^\infty \sqrt{y}\; \exp\left[\frac{-(\xi/\sqrt{2\rho})^2}{2y}\right] e^{-\rho y}\, dy \;=\; \frac{\sqrt{\pi}}{2\rho^{3/2}}\,(1+|\xi|)\, e^{-|\xi|} \right) . \end{aligned}$$

In particular, $H_1(v_t, s_t \mid \infty) = 0$, and $H_1(v_t, v_t \mid a) = 0$.

$$\begin{aligned} H_2(v_t, s_t \mid a) &= \frac{\sigma^2}{\sqrt{2\pi}}\frac{\xi_2}{\sqrt{2\rho}}\int_0^\infty \sqrt{y}\exp\left[\frac{-(\xi_2/\sqrt{2\rho})^2}{2y}\right] e^{-\rho y}\, dy \\ &- \frac{\sigma^2}{\sqrt{2\pi}}\frac{\xi_1}{\sqrt{2\rho}}\int_0^\infty \sqrt{y}\exp\left[\frac{-(\xi_1/\sqrt{2\rho})^2}{2y}\right] e^{-\rho y}\, dy \,+\, \sigma^2\int_0^\infty y\; J_0(v_t, s_t \mid a)\; e^{-\rho y}\, dy \;, \\ &= \frac{\sigma}{2\rho\gamma}\left[\frac{\xi_2}{\sqrt{2\rho}}(1+|\xi_2|)\, e^{-|\xi_2|} \;-\; \frac{\xi_1}{\sqrt{2\rho}}(1+|\xi_1|)\, e^{-|\xi_1|}\right] \\ &\quad + \frac{2}{\gamma^2}\, H_0(v_t, s_t \mid a) \,+\, \frac{\sigma}{2\rho\gamma}\left[\frac{\xi_2}{\sqrt{2\rho}}\, e^{-|\xi_2|} \;-\; \frac{\xi_1}{\sqrt{2\rho}}\, e^{-|\xi_1|}\right] \\ &\quad\left(\text{as } \sigma^2\int_0^\infty y\; J_0(v_t, s_t \mid a)\; e^{-\rho y}\, dy \;=\; \sigma^2\; \frac{-\partial\, H_0(v_t, s_t \mid a)}{\partial\rho} \right), \\ H_2(v_t, s_t \mid a) &= \frac{\sigma^2}{\rho^2}\left[\frac{\xi_2}{4}(2+|\xi_2|)\, e^{-|\xi_2|} \;-\; \frac{\xi_1}{4}(2+|\xi_1|)\, e^{-|\xi_1|}\right] \\ &\quad + \frac{\sigma^2}{\rho}\left[\frac{\mathrm{sg}[\xi_1]}{2\rho}\left(1 \,-\, e^{-|\xi_1|}\right) \;-\; \frac{\mathrm{sg}[\xi_2]}{2\rho}\left(1 \,-\, e^{-|\xi_2|}\right)\right] . \qquad (46) \end{aligned}$$

In particular, $H_2(v_t, s_t \mid \infty) = \sigma^2/\rho^2$, and $H_2(v_t, v_t \mid a) = [1 \,-\, (1+\gamma a + (\gamma a)^2/2)e^{-\gamma a}]\sigma^2/(2\rho^2)$.

B.I. Calculation of the Instability Loss Operator, $K(v_t - v_{T_i})$:

$$K(v_t - v_{T_i}) \;=\; \int_t^\infty \left(\int_{-\infty}^\infty \frac{\kappa\,(v_\tau - v_{T_i})^2}{\sigma\,\sqrt{2\pi\,(\tau-t)}}\,\exp\left[-\frac{1}{2}\,\frac{(v_\tau - v_t)^2}{\sigma^2\,(\tau - t)}\right] dv_\tau\right)\, e^{-\rho(\tau-t)}\, d\tau \;.$$

Breaking down $(v_\tau - v_{T_i})^2$ in $[(v_\tau - v_t) + (v_t - v_{T_i})]^2$ and expanding, we decompose $K(v_t - v_{T_i})$:

$$K(v_t - v_{T_i}) \;=\; \kappa\,(v_t - v_{T_i})^2\, H_0(v_t, v_{T_i} \mid \infty) \,+\, 2\kappa\,(v_t - v_{T_i})\, H_1(v_t, v_{T_i} \mid \infty) \,+\, \kappa\, H_2(v_t, v_{T_i} \mid \infty) \;,$$

where the operators $H_k(v_t, s_t \mid a)$, for $k \in \{0, 1, 2\}$ and $a \in \mathbf{R}^+$, are defined and calculated in Appendix A. Using expressions (44), (45) and (46) we obtain:

$$K(v_t - v_{T_i}) \;=\; \frac{\kappa}{\rho}\left(\frac{\sigma^2}{\rho} \,+\, (v_t - v_{T_i})^2\right). \;\; \Box$$

B.II. Laplace Transforms of Conditional Probabilities of First Hitting Times.

For $t \in (T_i, T_{i+1}]$, the *next* realignment date T_{i+1}, can be characterized by a couple (v^d_{i+1}, v^u_{i+1}) where $v^d_{i+1} \le v_t \le v^u_{i+1}$, such that realignment occurs the first time v_t reaches either the "upper" threshold level, v^u_{i+1}, or the "lower" threshold level, v^d_{i+1}. Let $t^u_{i+1}(v^d_{i+1}, v^d_{i+1}) \equiv \inf\{\, \tau > T_i \mid v_\tau = v^u_{i+1}$ and $\inf_{T_i \le s \le \tau}\{v_s\} > v^d_{i+1}\}$ be the first time at which v_t hits the "upper" threshold level, v^u_{i+1}, without having hit a "lower" level, v^d_{i+1}. Similarly, let $t^u_{i+1}(v^d_{i+1}, v^d_{i+1}) \equiv \inf\{\, \tau \mid v_\tau = v^d$ and $\sup_{T_i \le s \le \tau}\{v_s\} < v^u\}$. These two random variables are well-defined since the sample paths of v_t are continuous almost surely. Overall, we therefore have $T_{i+1} = t^u_{i+1} \wedge t^d_{i+1}$.

Taking Laplace transforms in the backward Kolmogorov equation subject to appropriate boundary conditions (See Cox and Miller (1965)), and denoting $\gamma \equiv \sqrt{2\rho}/\sigma$, we can derive the Laplace transforms of $\varphi_t(t^u_{i+1})$ and $\varphi_t(t^d_{i+1})$, the probability densities of $t^u_{i+1}(v^d_{i+1}, v^d_{i+1})$ and $t^d_{i+1}(v^d_{i+1}, v^d_{i+1})$, respectively, conditional on information at date t,

$$\int_t^\infty e^{-\rho(t^u_{i+1}-t)}\ \varphi_t(t^u_{i+1})\ d\,t^u_{i+1} \;=\; \frac{e^{-\gamma(v_t - v^d_{i+1})} - e^{\gamma(v_t - v^d_{i+1})}}{e^{-\gamma(v^u_{i+1} - v^d_{i+1})} - e^{\gamma(v^u_{i+1} - v^d_{i+1})}}\,,$$

$$\int_t^\infty e^{-\rho(t^d_{i+1}-t)}\ \varphi_t(t^d_{i+1})\ d\,t^d_{i+1} \;=\; \frac{e^{-\gamma(v^u_{i+1} - v_t)} - e^{\gamma(v^u_{i+1} - v_t)}}{e^{-\gamma(v^u_{i+1} - v^d_{i+1})} - e^{\gamma(v^u_{i+1} - v^d_{i+1})}}\,.$$

B.III. Calculation of the Aggregate Loss Operator, $C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)$:

At time $t \in [T_i, T_{i+1})$, we can decompose the aggregate cost operator of a symmetric realignment policy of trigger amplitude δ as follows:

$$\begin{aligned} C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda) \;\equiv\;& E_t\left[\int_t^{T_{i+1}} \kappa\,(v_\tau - v_{T_i})^2\, e^{-\rho(\tau - t)}\, d\tau\right] \\ &\quad +\; C(v_{T_{i+1}} - s_{T_{i+1}}, \delta \mid \kappa, \Lambda)\ E_t\left[e^{-\rho(T_{i+1}-t)}\right] \qquad (47)\\ =\;& E_t\left[\int_t^\infty \kappa\,(v_\tau - v_{T_i})^2\, e^{-\rho(\tau - t)}\, d\tau\right] \\ +\;& \left(C(0, \delta \mid \kappa, \Lambda) \;-\; E_t\left[\int_{T_{i+1}}^\infty \kappa\,(v_\tau - v_{T_i})^2\, e^{-\rho(\tau - T_{i+1})}\, d\tau\right]\right) E_t\left[e^{-\rho(T_{i+1}-t)}\right] \\ =\;& K(v_t - v_{T_i}) + \Big(C(0, \delta \mid \kappa, \Lambda) - K(\delta)\Big)\, E_t\left[e^{-\rho(T_{i+1}-t)}\right]\,. \qquad (48) \end{aligned}$$

(a) The aggregate cost of a realignment policy, at a future realignment date, T_j, can be written in the following recursive fashion, as a function of that at the following realignment date, T_{j+1}:

$$\begin{aligned} C(v_{T_j} - s_{T_j}, \delta \mid \kappa, \Lambda) \;=\;& \Lambda \,+\, E_{T_j}\left[\int_{T_j}^{T_{j+1}} \kappa\,(v_\tau - v_{T_j})^2\, e^{-\rho(\tau - T_j)}\, d\tau\right] \\ &+\, E_{T_j}\left[C(v_{T_{j+1}} - s_{T_{j+1}}, \delta \mid \kappa, \Lambda)\ e^{-\rho(T_{j+1}-T_j)}\right]\,. \qquad (49) \end{aligned}$$

Breaking down the random time $T_{j+1} \equiv t^u_{j+1} \wedge t^d_{j+1}$ as in Appendix A.II., we have that at time T_j, the discount factor for the next realignment (at the random time T_{j+1}) is:

$$E_{T_j}\left[e^{-\rho(T_{j+1}-T_j)}\right] \;=\; \int_{T_{j+1}}^\infty e^{-\rho(t^u_{j+1} - T_{j+1})} \varphi_{T_{j+1}}(t^u_{j+1})\ d\,t^u_{j+1} \;+\; \int_{T_{j+1}}^\infty e^{-\rho(t^d_{j+1} - T_{j+1})} \varphi_{T_{j+1}}(t^d_{j+1})\ d\,t^d_{j+1}$$

where $t^u_{j+1} \equiv t^u_{j+1}(v_{T_j} - \delta, v_{T_j} + \delta)$ and $t^d_{j+1} \equiv t^d_{j+1}(v_{T_j} - \delta, v_{T_j} + \delta)$. Now, by symmetry,

$$\int_{T_j}^{\infty} e^{-\rho(t^u_{j+1}-T_j)}\, \varphi_{T_j}(t^u_{j+1})\, d\,t^u_{j+1} \;=\; \int_{T_j}^{\infty} e^{-\rho(t^d_{j+1}-T_j)}\, \varphi_{T_j}(t^d_{j+1})\, d\,t^d_{j+1} \;=\; \frac{1}{e^{-\gamma\delta} + e^{\gamma\delta}} \;=\; \frac{1}{2\cosh[\gamma\delta]}\,.$$

Hence, $E_{T_j}[e^{-\rho(T_{j+1}-T_j)}] = 1/\cosh[\gamma\delta]$, and we can then write equation (49) as

$$C(0,\delta \mid \kappa,\Lambda) \;=\; \Lambda + K(0) + \Big(C\,(0,\delta \mid \kappa,\Lambda) \;-\; K(\delta)\Big)\, \frac{1}{\cosh[\gamma\delta]}\,.$$

$$\text{So,}\quad C(0,\delta \mid \kappa,\Lambda) \;=\; \Lambda \;+\; K(0) + \Big(\Lambda \;+\; K(0) \;-\; K(\delta)\Big)\, \frac{1}{\cosh[\gamma\delta] - 1}\,. \tag{50}$$

(b) Breaking down the random time $T_{i+1} \equiv t^u_{i+1} \wedge t^d_{i+1}$ as in Appendix B.II., we have that at time $t \in [T_i, T_{i+1})$, the discount factor for the next realignment (at the random date T_{i+1}) is:

$$E_t\left[e^{-\rho(T_{i+1}-t)}\right] \;=\; \int_{T_{i+1}}^{\infty} e^{-\rho(t^u_{i+1}-T_{i+1})} \varphi_{T_{i+1}}(t^u_{i+1})\; d\,t^u_{i+1} \;+\; \int_{T_{i+1}}^{\infty} e^{-\rho(t^d_{i+1}-T_{i+1})} \varphi_{T_{i+1}}(t^d_{i+1})\; d\,t^d_{i+1}$$

where $t^u_{i+1} \equiv t^u_{i+1}(v_{T_i} - \delta, v_{T_i} + \delta)$ and $t^d_{i+1} \equiv t^d_{i+1}(v_{T_i} - \delta, v_{T_i} + \delta)$. Hence,

$$E_t\left[e^{-\rho(T_{i+1}-t)}\right] \;=\; \frac{\Big(e^{-\gamma(v_t - v_{T_i})} \;+\; e^{\gamma(v_t - T_i)}\Big).\big(e^{-\gamma\delta} \;-\; e^{\gamma\delta}\big)}{e^{-2\gamma\delta} \;-\; e^{2\gamma\delta}} \;=\; \frac{\cosh[\gamma(v_t - v_{T_i})]}{\cosh[\gamma\delta]}\,. \tag{51}$$

(c) Replacing (50) and (51) in (48) yields:

$$C(v_t,\delta \mid \kappa,\Lambda) \;=\; K(v_t - v_{T_i}) \;-\; \Big(K(\delta) \;-\; K(0) \;-\; \Lambda\Big)\, \frac{\cosh[\gamma(v_t - v_{T_i})]}{\cosh[\gamma\delta] - 1}\,. \;\square$$

C. Proof that the Trigger Amplitudes $\underline{\delta}_A$ and $\underline{\delta}_B$ are Smaller than $\delta_{A\cup B}$.

Defining $\nu \equiv \Lambda/\kappa$, the first order optimality condition (11) can be written as $\nu = g(\tilde{\delta}).h(\tilde{\delta})$, where

$$g(\tilde{\delta}) \;\equiv\; \frac{\tilde{\delta}}{\rho}\,, \quad \text{and} \quad h(\tilde{\delta}) \;\equiv\; \tilde{\delta} \;-\; \frac{2\,(\cosh[\gamma\tilde{\delta}] - 1)}{\gamma\,\sinh[\gamma\tilde{\delta}]}\,.$$

Now, simple derivation yields $dh(\tilde{\delta})/d\tilde{\delta} = (\,(\cosh[\gamma\tilde{\delta}] \;-\; 1)/\sinh[\gamma\tilde{\delta}]\,)^2 > 0$. Therefore, given that $g(\tilde{\delta}) > 0$, $h(\tilde{\delta}) > 0$ and $dg(\tilde{\delta})/d\tilde{\delta} > 0$, it follows that $d[g(\tilde{\delta}).h(\tilde{\delta})]/d\tilde{\delta} > 0$, hence $d\tilde{\delta}/d\nu > 0$. Let

$$\nu_{A\cup B} \;\equiv\; \frac{\Lambda_A \;+\; \Lambda_B}{\phi^2_{A\cup B}\kappa_A \;+\; (1-\phi_{A\cup B})^2\kappa_B}\,, \qquad \nu_A \;\equiv\; \frac{\Lambda_A}{\kappa_A}\,, \qquad \text{and}\;\; \nu_B \;\equiv\; \frac{\Lambda_B}{\kappa_B}\,.$$

Replacing we immediately obtain $\nu_{A\cup B} > \nu_A$ and $\nu_{A\cup B} > \nu_B$. Given that $\nu_{A\cup B} = g(\delta_{A\cup B}).h(\delta_{A\cup B})$, $\nu_A = g(\underline{\delta}_A).h(\underline{\delta}_A)$ and $\nu_B = g(\underline{\delta}_B).h(\underline{\delta}_B)$, using $d\tilde{\delta}/d\nu > 0$, we have $\delta_{A\cup B} > \underline{\delta}_A$, and $\delta_{A\cup B} > \underline{\delta}_B$. $\square$

D. Derivation of the leader and follower's unit-period loss functions, $l(v_t - v_{T_i})$ and $f(v_t - v_{T_i})$, when renegotiation is unconstrained.

Applying Îto's lemma within each of the three intervals $v_t - v_{T_i} \in [-\underline{\delta}_{A\cup B}, -\underline{\delta}_F]$, $(-\underline{\delta}_F, \underline{\delta}_F)$, and $[\underline{\delta}_F, \underline{\delta}_{A\cup B}]$, to equations (22) and (23), we obtain that that within each of these three intervals, the leader and follower unit-period cost functions satisfy the differential equations:

$$l(v_t - v_{T_i}) \quad = \quad \rho\, L(v_t - v_{T_i}) \; - \; \frac{\sigma^2}{2} \frac{d^2}{d\, v_t^2} L(v_t - v_{T_i}) \; , \tag{52}$$

$$f(v_t - v_{T_i}) \quad = \quad \rho\, F(v_t - v_{T_i}) \; - \; \frac{\sigma^2}{2} \frac{d^2}{d\, v_t^2} F(v_t - v_{T_i}) \; . \tag{53}$$

Furthermore, taking derivatives of equations (9) and (10), we see that for $t \in [T_i, T_{i+1})$,

$$\rho \; C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda) \; - \; \frac{\sigma^2}{2} \; \frac{\partial^2 \, C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)}{\partial^2 \; v_t} \quad = \quad \kappa \; (v_t - v_{T_i})^2 \; . \tag{54}$$

Replace within each interval (17) in (20) and (21) to express the RHS of (52) and (53) in terms of the operator $C(v_t - v_{T_i}, \delta \mid \kappa, \Lambda)$. Simplifying using (54), we obtain the expressions of $l(v_t - v_{T_i})$ and $f(v_t - v_{T_i})$ given in equations (24) and (25). □

E.I. Calculation of the Instability Loss Operator, $K^*(v_t - v_{T_i})$:

$$\begin{aligned} K^*(v_t - v_{T_i}) \quad &= \quad \int_t^\infty \left(\int_{-\infty}^{v_{T_i} - \underline{\delta}_F} \frac{g(v_\tau - v_{T_i})}{\sigma \, \sqrt{2\pi \, (\tau - t)}} \exp\left[-\frac{1}{2} \frac{(v_\tau - v_t)^2}{\sigma^2 \, (\tau - t)} \right] dv_\tau \right) \, e^{-\rho(\tau - t)} \, d\tau \\ &+ \quad \int_t^\infty \left(\int_{v_{T_i} + \underline{\delta}_F}^\infty \frac{g(v_\tau - v_{T_i})^k}{\sigma \, \sqrt{2\pi \, (\tau - t)}} \exp\left[-\frac{1}{2} \frac{(v_\tau - v_t)^2}{\sigma^2 \, (\tau - t)} \right] dv_\tau \right) \, e^{-\rho(\tau - t)} \, d\tau \; , \end{aligned}$$

where $g(v_\tau - v_{T_i}) \equiv (v_\tau - v_{T_i})^2 \alpha_2 - 2|v_\tau - v_{T_i}|\alpha_2 \alpha_1 + \alpha_2(\alpha_1)^2$, with $\alpha_1 \equiv \sqrt{\rho\, \underline{F}^{(o)}/\kappa_F}$ and $\alpha_2 \equiv \kappa_L$. Writing $|v_t - v_{T_i}|$ as $\mathrm{sg}[v_t - v_{T_i}]\,(v_t - v_{T_i})$, then breaking down $(v_\tau - v_{T_i})^k$ in $[(v_\tau - v_t) + (v_t - v_{T_i})]^k$, for $k = 1, 2$, and expanding, the integral $K^*(v_t - v_{T_i})$ can be decomposed in three parts:

$$K^*(v_t - v_{T_i}) \; = \; \sum_{k=0}^{k=2} \beta_k \; \Big[\, H_k(v_t, v_{T_i} \mid \infty) \; - \; H_k(v_t, v_{T_i} \mid \delta) \Big] \; ,$$

$$\text{where} \quad \beta_0 \equiv \alpha_2 \; (\omega_t)^2 \; , \; \beta_1 \equiv 2\alpha_2 \, \omega_t \; , \; \beta_2 \equiv \alpha_2 \; , \quad \text{and} \quad \omega_t \equiv [(v_t - v_{T_i}) \; - \; \mathrm{sg}[v_t - v_{T_i}]\, \alpha_1] \; .$$

The operators $H_k(v_t, s_t \mid a)$, for $k \in \{0, 1, 2\}$ and $a \in \mathbf{R}^+$, are defined and calculated in Appendix A. Denoting Ω_0, Ω_1 and Ω_2 the following, and using (44), (45) and (46) yields the expression,

$$\Omega_0 \;\; \equiv \;\; \rho \, [H_0(v_t, s_t \mid \infty) \; - \; H_0(v_t, s_t \mid \underline{\delta}_F)] \;\; , \qquad \Omega_1 \;\; \equiv \;\; \frac{(2\rho)^{3/2}}{\sigma} \; [H_1(v_t, s_t \mid \infty) \; - \; H_1(v_t, s_t \mid \underline{\delta}_F)] \;\; ,$$

$$\Omega_2 \;\; \equiv \;\; \frac{\rho^2}{\sigma^2} \; [H_2(v_t, s_t \mid \infty) \; - \; H_2(v_t, s_t \mid \underline{\delta}_F)] \;\; . \;\; □$$

E.II. Calculation of the Aggregate Loss, $L^*(v_t - v_{T_i})$:

At time $t \in [T_i, T_{i+1})$, we can decompose $L^*(v_t - v_{T_i})$ as we did in Appendix A.II.:

$$\begin{aligned} L^*(v_t - v_{T_i}) \;&\equiv\; \int_t^{T_{i+1}} \left(\int_{-\infty}^{v_{T_i} - \underline{\delta}_F} \frac{g(v_\tau - v_{T_i})}{\sigma\,\sqrt{2\pi\,(\tau - t)}} \exp\left[-\frac{1}{2}\,\frac{(v_\tau - v_t)^2}{\sigma^2\,(\tau - t)} \right] dv_\tau \right) e^{-\rho(\tau - t)}\, d\tau \\ &+\; \int_t^{T_{i+1}} \left(\int_{v_{T_i} + \underline{\delta}_F}^{\infty} \frac{g(v_\tau - v_{T_i})}{\sigma\,\sqrt{2\pi\,(\tau - t)}} \exp\left[-\frac{1}{2}\,\frac{(v_\tau - v_t)^2}{\sigma^2\,(\tau - t)} \right] dv_\tau \right) e^{-\rho(\tau - t)}\, d\tau \\ &+\; L^*(v_{T_{i+1}} - s_{T_{i+1}})\; E_t\left[e^{-\rho(T_{i+1} - t)} \right] \\ &=\; K^*(v_t - v_{T_i}) + \Big(L^*(0) - K^*(\delta^*) \Big) \frac{\cosh[\gamma(v_t - v_{T_i})]}{\cosh[\gamma\delta^*]}\;, \end{aligned}$$

where $g(v_\tau - v_{T_i}) \equiv (v_\tau - v_{T_i})^2\alpha_2 - 2|v_\tau - v_{T_i}|\alpha_2\alpha_1 + \alpha_2(\alpha_1)^2$, with $\alpha_1 \equiv \sqrt{\rho\,\underline{F}^{(o)}/\kappa_F}$ and $\alpha_2 \equiv \kappa_L$. Similarly, writing in recursive fashion the aggregate cost of a realignment policy, at a future realignment date, T_j, as a function of that at the following realignment date, T_{j+1}, yields

$$L^*(0) \;=\; \Lambda \,+\, K^*(0) + \Big(\Lambda \,+\, K^*(0) \,-\, K^*(\delta^*) \Big)\, \frac{1}{\cosh[\gamma\delta^*] - 1}\;.$$

Replacing, we obtain the expression in the paper. □

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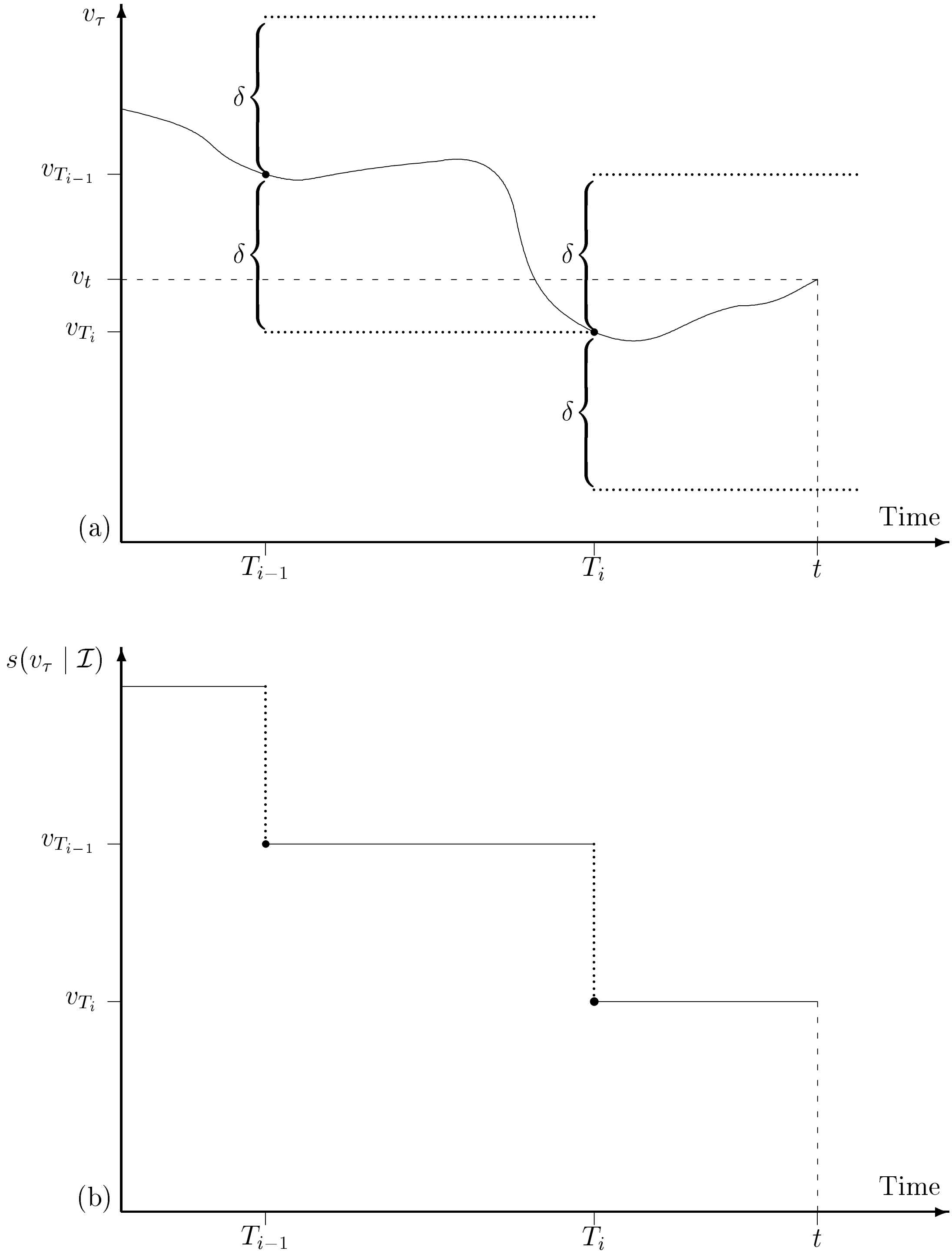


Figure 1: Random time line under a symmetric realignment policy of fixed amplitude δ. Figure (a) exhibits a possible path followed by the fundamental value, v_t, until a date $t \in [T_i, T_{i+1})$. Figure (b) exhibits the resulting evolution of the spot rate, $s(v_t \mid \mathcal{I})$.

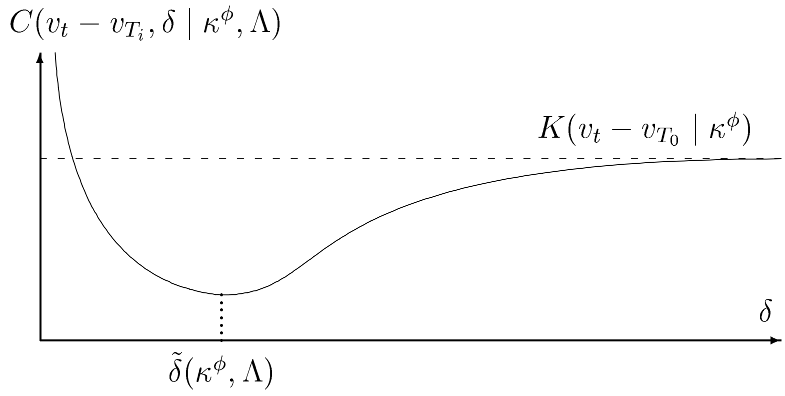


Figure 2: Realignment trigger amplitude, $\tilde{\delta}(\kappa, \Lambda)$, characterizing the intervention policy which minimizes the aggregate cost operator, $C(v_t, \delta \mid \kappa, \Lambda)$.

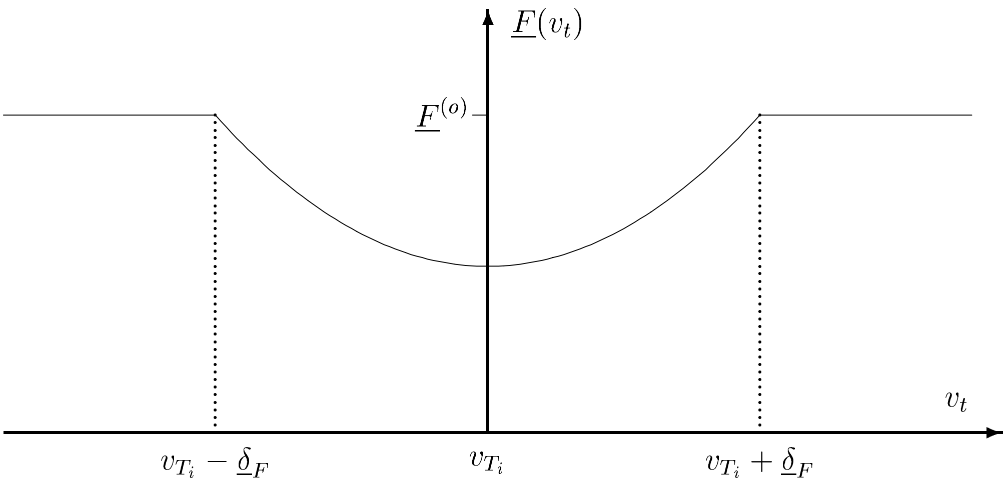


Figure 3: Follower's reservation value, at a date $t \in [T_i, T_{i+1})$.

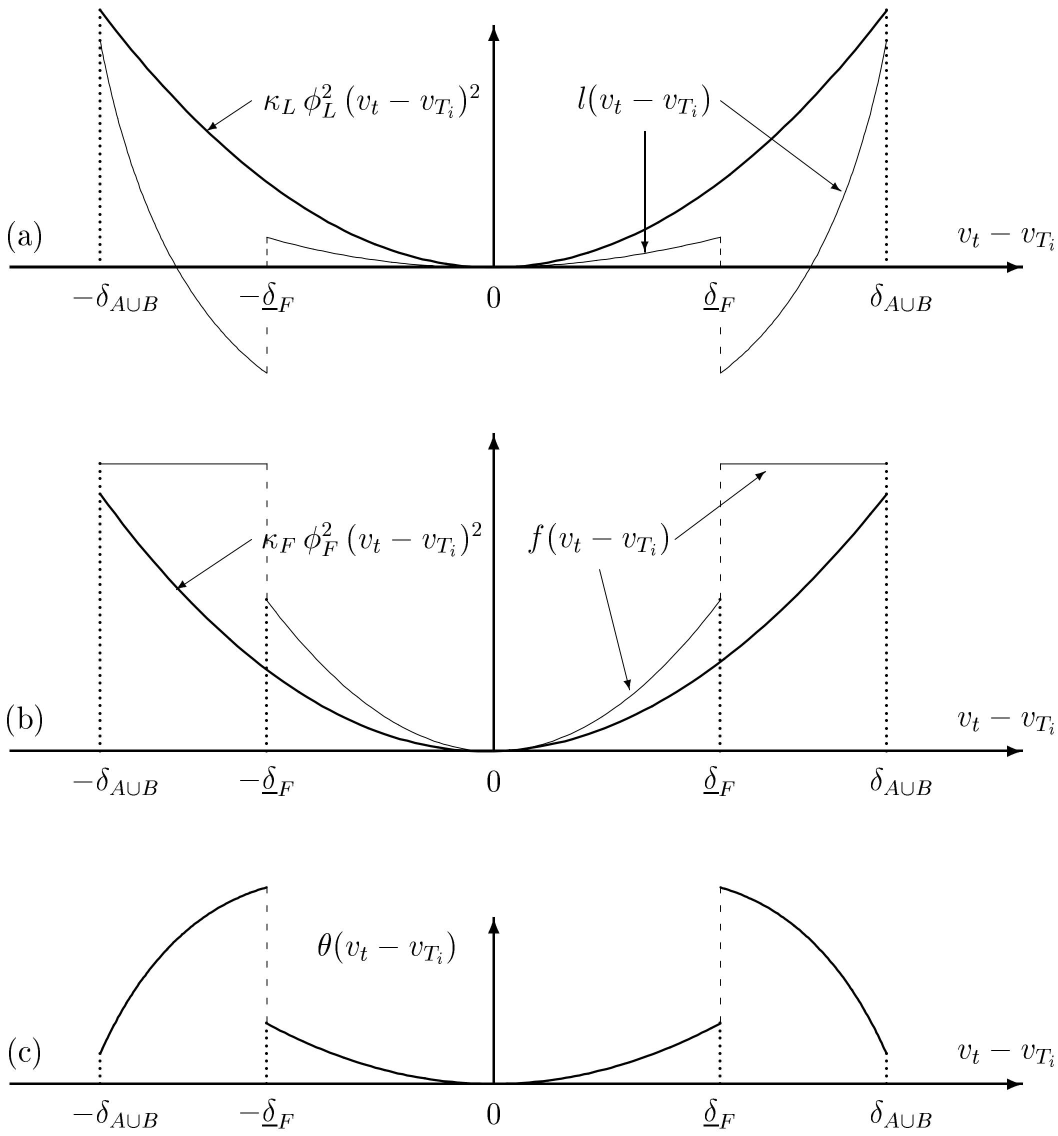


Figure 4: Equilibrium complicit renegotiation-proof agreement when renegotiation is unconstrained.

Figures (a) and (b) exhibit the leader and follower instantaneous costs, that result purely from the implementation of the first-best policy, $\kappa_L\phi_L^2(v_t - v_{T_i})^2$ and $\kappa_F\phi_F^2(v_t - v_{T_i})^2$, respectively, at a date $t \in [T_i, T_{i+1})$. Figure (c) exhibits the follower to leader instantaneous transfer $\theta(v_t - v_{T_i})$ supporting the equilibrium renegotiation proof outcome. Figures (a) and (b) exhibit the resulting overall leader and follower instantaneous costs, $l(v_t - v_{T_i})$ and $f(v_t - v_{T_i})$.

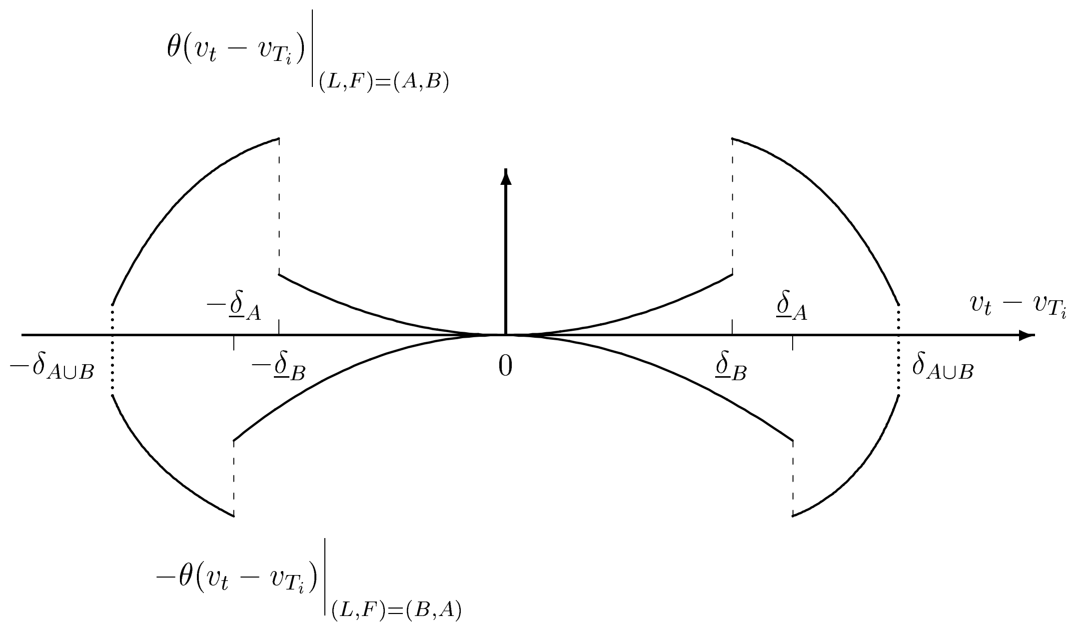
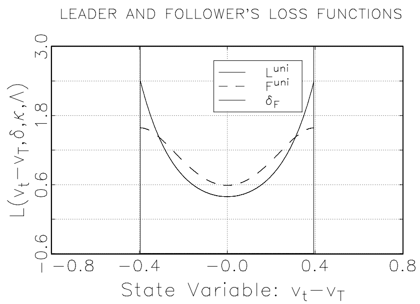


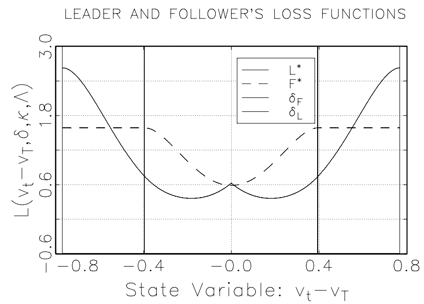
Figure 5: Influence of the Countries Relative Bargaining Power on the Equilibrium Instantaneous Transfer from Country A to country B, when renegotiation is unconstrained.

The top curve gives the country A to B transfer $\theta(v_t - v_{T_i})$ with $(L, F) = (A, B)$, corresponding to the first polar case where country A is the leader and country B is the follower. Conversely the bottom curves gives the country A to B transfer $-\theta(v_t - v_{T_i})$ with $(L, F) = (B, A)$, corresponding to the alternative polar case where country B is the leader and country A is the follower. In this figure $\Lambda_A/\kappa_A > \Lambda_B/\kappa_B$, which generates $\underline{\delta}_A > \underline{\delta}_B$.

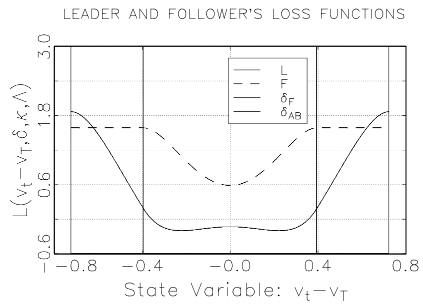
Figure 6: Country-specific aggregate loss functions



(a) Unilateral equilibrium



(b) Restricted equilibrium



(c) Unrestricted equilibrium

Notes: The three panels exhibit the aggregate expected losses of the leader –solid line– and follower –dashed line– as functions of the amount of intervention, $v_t - s_t$, under three scenarios: Panel (a) refers to the scenario without renegotiation, where the follower unilaterally undertakes its reservation intervention policy, as in Section 4.2. Panel (b) considers the restricted equilibrium in Section 5.3, and Panel (c) refers to the unrestricted equilibrium in Section 5.1. Input parameters are: $\sigma = 0.05$, $\rho = 0.05$, $\kappa_F = 1, \kappa_L = 1$, $\Lambda_F = 1, \Lambda_F = 2$. The inner vertical lines in the three Panels indicate the reservation strategy realignment trigger points of the follower, $v_{T_i} - \delta_F$ and $v_{T_i} + \delta_F$, where $\delta_F = 0.395$. The outer vertical lines in the middle and right Panels indicate the prevailing realignment trigger points in the restricted –middle Panel– ($v_{T_i} - \delta^*$, $v_{T_i} + \delta^*$, where $\delta^* = 0.769$) and unrestricted equilibria –right Panel– ($v_{T_i} - \delta_{A\cup B}$ and $v_{T_i} + \delta_{A\cup B}$, where $\delta_{A\cup B} = 0.724$).