DISCUSSION PAPER SERIES

No. 2707

WAGES AND PRODUCTIVITY GROWTH IN A DYNAMIC MONOPOLY

Helmut Bester and Emmanuel Petrakis

INDUSTRIAL ORGANIZATION

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP2707.asp

ISSN 0265-8003

WAGES AND PRODUCTIVITY GROWTH IN A DYNAMIC MONOPOLY

Helmut Bester, Freie Universität Berlin and CEPR
Emmanuel Petrakis, University of Crete

Discussion Paper No. 2707
February 2001

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 2707

February 2001

ABSTRACT

Wages and Productivity Growth in a Dynamic Monopoly*

This Paper studies the intertemporal problem of a monopolistic firm that engages in productivity-enhancing innovations to reduce its labour costs. If the level of wages is sufficiently low, the firm's rate of productivity growth approaches the rate of wage growth and eventually the firm reaches a steady state where its unit labour cost remains constant over time. Otherwise, it will gradually reduce its innovation effort over time and ultimately terminate production. Productivity-dependent wage differentials do not affect productivity growth in the steady state; they increase, however, the firm's long-run equilibrium cost level.

JEL Classification: D24, D42, D92, J30, J51
Keywords: dynamic programming, innovation, labour productivity, monopoly, wage differentials, wages

Helmut Bester
Freie Universität Berlin
Department of Economics
Boltzmannstr. 20
D-14195 Berlin
GERMANY
Tel: (49 30) 838 55257
Fax: (49 30) 838 54142
Email: hbester@wiwiss.fu-berlin.de

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=109515

Emmanuel Petrakis
Department of Economics
University of Crete
University Campus at Gallos
Rethymnon 74100
GREECE
Tel: (30 831) 77409
Fax: (30 831) 77406
Email: petrakis@econ.soc.uoc.gr

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=122022

* We wish to thank Rabah Amir and Andreas Irmen for helpful comments. Helmut Bester is grateful for financial support from the German Science Foundation (DFG) under the programme Industrieökonomik und Inputmärkte.

Submitted 07 December 2000

NON-TECHNICAL SUMMARY

This Paper addresses the interaction between labour market conditions and the innovative performance of firms. It studies the intertemporal problem of a firm that engages in productivity-enhancing innovations to reduce its labour costs. While the firm acts as a monopoly in the output market, it takes the current competitive wage in the labour market as given. At each point in time, the wage rate and the firm's productivity determine its unit labour cost. Through investments in process innovations it can raise labour productivity and thus reduce its cost at subsequent dates. The firm thus faces a dynamic optimization problem because innovation affects future labour costs and the incentives for innovation depend on the evolution of these costs.

In our model it is the evolution of wages that stimulates productivity enhancing innovations. Higher total labour costs create stronger incentives for process innovations that raise the productivity of labour. The innovative performance of the firm thus depends on the growth of wages as it seeks to reduce its labour cost by increasing labour productivity.

By formulating the firm's innovation strategy as an infinite horizon optimization programme we can study the long-run evolution of productivity growth when the competitive wage increases at an exogenous rate. As long as the firm's initial labour cost lies below some critical level, its rate of productivity growth eventually approaches the rate of wage growth under the optimal innovation policy. The firm then reaches a steady state where these two rates coincide so that its unit labour cost remains constant over time. Interestingly, this steady state is independent of the level of wages; it only depends on their growth rate.

The findings in the present Paper complement our study in Bester and Petrakis (CEPR Discussion Paper No. 2031, 1998) on the relation between wages and productivity growth in a competitive industry with free entry and exit. The properties of the steady state in the present monopoly model are essentially equivalent to the competitive case. There is, however, an important difference on the adjustment path towards the steady state: in the competitive model the current rate of innovation always depends positively on the industry's current unit labour cost. In contrast, in the monopoly model the optimal innovation policy does not have this characteristic. Instead, the highest rate of innovation occurs at some intermediate level of the firm's unit labour cost. The reason is that low costs create little incentive for cost cutting. The same happens also when costs are rather high, because then the firm's output is low and so the overall gains from innovation are small.

Our analysis also addresses the role of productivity-dependent wage differentials. Such differentials have been investigated in a number of

empirical studies and may be explained, for example, by bargaining power on the part of the firm's workforce. A well known theoretical argument in this context is that a positive relation between wages and productivity reduces the firm's incentive for innovations increasing productivity. The idea is that the firm cannot realize the full gain from its investment in innovation when it anticipates that its labour force will capture some share of the rents from technological improvements.

Our dynamic analysis reveals that rent sharing has some quite different long-run effects. Indeed, the firm's steady state rate of innovation is independent of the degree of rent sharing. This is so because in the steady state the rate of productivity growth equals the rate of wage growth and the ratio of the firm's wage to the competitive wage remains constant over time. The higher the degree of rent sharing, however, the higher the firm's long-run equilibrium cost level. Also, rent sharing decreases the critical value of the initial cost, above which the firm cannot survive in the market in the long run.

One might extend our model by considering dynamic imperfectly competitive markets. In an intertemporal model where at each date firms compete on prices or quantities it will be interesting to analyse how strategic interactions between the firms affect productivity growth in the short run and in the long run. Stimulated by the work of Schumpeter, a large part of the literature on R&D relates the pace of innovative activity to market structure. An imperfect competition version of our model could combine this approach with our cost–push argument.

1 Introduction

This paper studies the intertemporal problem of a firm that engages in productivity enhancing innovations to reduce its labor costs. While the firm acts as a monopoly in the output market, it takes the current competitive wage in the labor market as given. At each point in time, the wage rate and the firm's productivity determine its unit labor cost. Through investments in process innovations it can raise productivity and thus reduce its cost at subsequent dates. This generates a dynamic optimization problem because innovation affects future labor costs and the incentives for innovation depend on the evolution of these costs.

We formulate this problem as an infinite horizon optimization programme. In this way we can study the long–run evolution of productivity growth when the competitive wage increases at an exogenous rate. It turns out that the monopoly's long–run behavior depends critically on the initial level of its labor cost. If this level is too high, it will gradually reduce its innovation effort over time. Ultimately its cost becomes too high to operate profitably and so it will terminate production. The more interesting case prevails when the initial cost lies below the critical level. In this case the rate of productivity growth approaches the rate of wage growth under the optimal innovation policy. Eventually, the firm reaches a steady state where these two rates coincide so that its unit labor cost remains constant over time. Interestingly, this steady state is independent of the level of wages; it only depends on their growth rate. The level of wages determines only whether the optimal policy tends towards the steady state or whether the firm will go extinct in the long–run.

In our model it is the evolution of wages that stimulates productivity enhancing innovations at the firm level. Our cost push argument addresses the interaction between labor market conditions and the innovative performance of industries and countries. This interaction is the subject of a number of empirical studies both at the macroeconomic (see e.g. Gordon (1987)) and the microeconomic level (see e.g. Doms, Dunne and Troske (1997), Chennells and Van Reenen (1997), Mohnen *et. al.* (1986) and Van Reenen (1996)). An important theoretical insight in this context is Grout's (1984) analysis, who argues that the firm's labor force will capture some share of the rents from technological improvements. This in turn will reduce the firm's incentives to innovate.[1]

We address the issue of rent sharing in Section 5, where we endogenize the firm's wage rate by introducing productivity dependent wage differentials. These may reflect the employees' bargaining power within the firm. In contrast with Grout's (1984) static model, our dynamic analysis reveals that rent sharing has some quite different long–run effects. Indeed, the firm's steady state rate of innovation is independent of the degree of rent sharing. This is so because in the steady state the rate of productivity growth equals the rate of wage growth and the ratio of the firm's wage to the competitive wage remains constant over time. The higher the degree of rent sharing, however, the higher is the firm's long–run equilibrium cost level. Also, rent sharing decreases the critical value of the initial cost, above which the firm will eventually quit operating at some point in the future.

This paper complements our study in Bester and Petrakis (1998) of the relation between wages and productivity growth in a competitive industry with free entry and exit where the last period's best technology is freely available to any firm.[2] Also in the competitive framework it is the growth rate of wages that determines the industry's long–run behavior. There are, however, some important differences between the competitive case and the present model. First, under perfect competition the individual firm does not face a truly dynamic optimization problem to determine its optimal innovation policy. This is so because free entry and exit, and free availability of last period's best technology imply that a firm's future profits are zero, independently of its innovation decision. Second, and more importantly, under perfect competition the individual firm does not have to consider the impact of its production cost on the output price. Therefore, there is a posi-

[1]Ulph and Ulph (1994) argue, however, that union bargaining power and innovative investments may be positively related.

[2]An extension of our study towards a general equilibrium model is presented in Hellwig and Irmen (1999).

tive relation between the incentive for innovation and the current cost level. This is no longer the case in the present monopoly model or, more generally, under imperfect competition. Instead, under the optimal policy the relation between $R\&D$ investment and current cost has an inverted U-shape: When the monopoly's cost are relatively low, there is little incentive to reduce these costs even further. Also relatively high costs, however, reduce the gains from cost reduction because of the low output associated with a high monopoly price. As a result, the monopoly's highest innovation effort occurs for some intermediate level of labor costs and its adjustment path towards the steady state may not be monotone.

This paper is organized as follows. Section 2 describes the firm's infinite horizon optimization problem. The steady state solutions of this problem are derived in Section 3. Section 4 analyses the adjustment dynamics of the optimal innovation policy. The impact of productivity dependent wage differentials is studied in Section 5. Section 6 presents conclusions. All proofs are relegated to the Appendix in Section 7.

2 The Model

Our model depicts the evolution of labor productivity in a dynamic monopoly. Time is discrete and at each date $t = 0, 1, 2...$, the monopolist employs labor in combination with other inputs to produce a single good. Also, he engages in process innovation to increase labor productivity. His innovation behavior at date t determines his technology at the subsequent date. The incentive for reducing labor costs through productivity enhancing innovations depends on the intertemporal path of wages. We first analyse the evolution of productivity under the assumption that the wage rate is exogenous and that the monopolist takes the going rate as given. Later, in Section 5 we consider the possibility of productivity dependent wage differentials, which may reflect the employees' bargaining power within the firm.

More formally, the model is specified as follows. At each date the monopolist faces the inverse demand function $P(x)$ and so his revenue is $R(x) \equiv P(x)x$. To produce x units of output at date t, he has to invest the amount

$C(x)$ in inputs other than labor. The required labor input is x/a_t, where a_t is the firm's labor productivity at date t.

The firm can engage in labor productivity enhancing process innovation. We assume that it can increase current productivity by the factor $(1+q)$ by spending the amount $K(q)$. Thus, if a_t describes the technology available at date t, labor productivity at $t+1$ becomes

$$a_{t+1} = (1+q_t)a_t \tag{1}$$

if the amount $K(q_t)$ is spent on innovations. Let $K(0) = 0, K'(0) = 0, K'(\infty) = \infty$ and $K'(q) > 0$, $K''(q) > 0$ for all $q > 0$. The initial level of productivity $a_0 = \bar{a}_0$ is exogenous.

The exogenous wage rate w_t grows at the rate $\gamma > 0$ so that $w_{t+1} = (1+\gamma)w_t$, with $w_0 = \bar{w}_0 > 0$. One possible interpretation is that γ represents the average growth rate of labor productivity in the entire economy. Therefore, also wages grow at the rate γ in the equilibrium of the economy - wide labor market. Since the firm under consideration constitutes only a small part of the whole economy, its impact on the equilibrium wage rate is negligible.

Let $c_t \equiv w_t/a_t$. Then, after investing $K(q)$ at date t, the firm's labor cost per unit of output at the next date is

$$c_{t+1} = \frac{1+\gamma}{1+q}c_t. \tag{2}$$

The firm's sales profit Π depends on its unit labor cost according to

$$\Pi(c) \equiv \max_x \; R(x) - C(x) - c\,x. \tag{3}$$

We assume that $R(x) - C(x)$ is strictly concave and $R(0) \geq C(0), R'(0) > C'(0)$ and $R'(\infty) < C'(\infty)$. Therefore, the profit maximizing output $x^*(c)$ is uniquely determined and continuous in c. Moreover, $x^*(c)$ positive and strictly decreasing in c as long as c is not too large. Notice that, by the envelope theorem, we have $\Pi'(c) = -x^*(c)$.

For our analysis the shape of the function $-\Pi'(c)\,c = x^*(c)\,c$ is crucial. To guarantee that this function is well–behaved we make the following assumption:

Assumption 1 $[R'(x) - C'(x)]x$ is strictly quasi–concave whenever $P(x) > 0$. Moreover, $[R'(x) - C'(x)]x \to 0$ as $x \to 0$.

This assumption is satisfied for standard cost and demand functions: For instance, $R'(x)\,x$ is strictly concave if $P(x)$ is linear or if it is iso–elastic with a price elasticity of demand larger than unity. In addition, $-C'(x)\,x$ is concave if $C(x) = k\,x^\omega$ with $k > 0$ and $\omega \geq 1$. Furthermore, also the second part of Assumption 1 is satisfied by these demand and cost functions.

Lemma 1 *There is a $c^m > 0$ such that $-\Pi'(c)c$ is increasing in c if $c < c^m$. If $c > c^m$ and $\Pi(c) > 0$, then $-\Pi'(c)c$ is decreasing in c. Moreover, $-\Pi'(0) < \infty$ and $-\Pi'(c)c \to 0$ for $c \to \infty$.*

The monopolist discounts future payoffs by the factor $0 < \delta < 1$. We assume that

$$-\delta\,\Pi'\left(c^m(1+\gamma)\right)c^m > K'(\gamma). \tag{4}$$

For a given discount factor δ, this condition is satisfied if the growth rate γ of wages is sufficiently small. As we show in the proof of Proposition 1 below, this assumption ensures that for values of c in the neighborhood of c^m the monopolist chooses an innovation rate $q > \gamma$.

Example In what follows we illustrate our analysis by the following specification of our model. Let

$$P(x) = A - x,\ C(x) = x^2,\ K(q) = q^2. \tag{5}$$

For $A > c$, we then have

$$\Pi(c) = \frac{(A-c)^2}{8},\ \Pi'(c) = -\frac{A-c}{4}. \tag{6}$$

Thus condition (4) is satisfied if

$$\delta > \frac{32\,\gamma}{A^2(1-\gamma)}. \tag{7}$$

because $-\Pi'(c)c$ attains its maximum at $c^m = A/2$. $\diamondsuit$

We can now state the monopolist's intertemporal problem of choosing an optimal innovation strategy. He solves

$$\max_{c_t, q_t} \sum_{t=0}^{\infty} \delta^t [\Pi(c_t) - K(q_t)] \tag{8}$$

subject to

$$c_{t+1} = \frac{1+\gamma}{1+q_t} c_t, \;\; c_0 = \bar{c}_0. \tag{9}$$

The solution of this problem determines an optimal trajectory $\{c_t^*, q_t^*\}_{t=0}^{\infty}$. We can view (8) as an optimal control problem. This has a single state variable c, a single control q, an infinite horizon and is autonomous. Therefore, the optimal innovation policy may also be described by a function $q^*(c)$ with the interpretation that the monopolist optimally chooses $q_t^* = q^*(c_t)$ when his unit labor cost at t equals c_t.

3 Steady States

To derive the optimal innovation policy $q^*(\cdot)$ we employ the technique of dynamic programming. Let $V(c)$ denote the value function associated with problem (8). Thus, at date t the monopolist's present value of profits under the optimal policy is $V(c_t)$ when his current unit cost is c_t. The Bellman equation for problem (8) is

$$\begin{aligned} V(c) &= \Pi(c) - K(q^*(c)) + \delta V\left(c \frac{1+\gamma}{1+q^*(c)}\right) \\ &= \max_q \Pi(c) - K(q) + \delta V\left(c \frac{1+\gamma}{1+q}\right). \end{aligned} \tag{10}$$

We assume that $V(\cdot)$ is continuously differentiable. The optimal innovation policy $q^*(\cdot)$ then necessarily satisfies the first order condition

$$-\delta\, V'\left(c \frac{1+\gamma}{1+q^*(c)}\right) \frac{(1+\gamma)c}{1+q^*(c)} = (1+q^*(c))K'(q^*(c)). \tag{11}$$

By the envelope theorem we obtain from (10) that

$$V'(c) = \Pi'(c) + \delta\, V'\left(c \frac{1+\gamma}{1+q^*(c)}\right) \frac{1+\gamma}{1+q^*(c)}. \tag{12}$$

In what follows we characterize the optimal policy $q^*(\cdot)$ by examining the implications of conditions (11) and (12).

The optimal innovation policy determines the dynamic path of the monopolist's wage cost according to

$$c_{t+1}^* = \frac{1+\gamma}{1+q^*(c_t^*)} c_t^*. \tag{13}$$

We first look at the possible steady state values of c_t^*. These values are the candidates for the long–run level of the firm's wage–productivity ratio. In a steady state, c_t^* is stationary over time and so $c_t^* = \hat{c}$ for all t. Obviously, (13) implies that $\hat{c}_I = 0$ always constitutes a steady state for the evolution of c_t^*. In this steady state the firm requires no labor input and so its optimal policy trivially satisfies $q^*(\hat{c}_I) = 0$. More relevant are the steady states in which $\hat{c}$ is positive. By (13) this requires $q^*(\hat{c}) = \gamma$. In such a steady state, the firm matches the growth rate of wages by the rate of productivity growth so that its labor cost per unit of output remains constant.

For $q^*(\hat{c}) = \gamma$, condition (12) becomes $V'(\hat{c}) = \Pi'(\hat{c})/(1-\delta)$ so that by (11)

$$-\frac{\delta}{1-\delta}\Pi'(\hat{c})\hat{c} = K'(\gamma)(1+\gamma). \tag{14}$$

By Lemma 1 and (4) this equation has exactly two solutions, $\hat{c}_{II}$ and $\hat{c}_{III}$. Moreover, $0 < \hat{c}_{II} < c^m < \hat{c}_{III}$.

It is easy to see how the values $\hat{c}_{II}$ and $\hat{c}_{III}$ change with the parameters of the model. Since $\Pi(c)$ is independent of δ and γ and $K'' > 0$, by Lemma 1 and (14), we observe that $\widehat{c}_{II}$ increases with growth rate of wages and decreases with the monopolist's discount factor, while $\widehat{c}_{III}$ decreases with γ and increases with δ.

Example For the specification in (5) we have $c^m = A/2$ and (14) has the two solutions

$$\begin{aligned} \hat{c}_{II} &= \frac{A}{2} - \frac{\sqrt{\delta A^2 - 32\gamma(1+\gamma)(1-\delta)}}{2\sqrt{\delta}}, \\ \hat{c}_{III} &= \frac{A}{2} + \frac{\sqrt{\delta A^2 - 32\gamma(1+\gamma)(1-\delta)}}{2\sqrt{\delta}}, \end{aligned} \tag{15}$$

provided δ satisfies (7). We can interpret A as a measure of market size. Thus $\widehat{c}_{II}$ increases with growth rate of wages and decreases with the market size and the discount factor, while $\widehat{c}_{III}$ decreases with γ and increases with A and δ. $\diamondsuit$

We now prove that $c \in \{\hat{c}_{II}, \hat{c}_{III}\}$ implies $q^*(c) = \gamma$ to establish the following result:

Proposition 1 *The optimal policy satisfies $q^*(c) = \gamma$ if and only if $c \in \{\hat{c}_{II}, \hat{c}_{III}\}$.*

Interestingly, there are two steady state values for the firm's wage–productivity ratio where the monopolist has the same incentive to spend on labor productivity enhancing innovation. The intuition is as follows. Obviously, the firm has a stronger *direct* incentive to invest in productivity enhancing activities at the steady state $\widehat{c}_{III}$ where the unit labor costs are higher. Note, however, that at $\widehat{c}_{III}$ the monopolist produces a lower output than at the low wage–productivity steady state $\widehat{c}_{II}$, i.e. $x^*(\widehat{c}_{II}) > x^*(\widehat{c}_{III})$. Hence, any given unit labor cost savings apply to a larger output at the low wage–productivity steady state. Therefore, the monopolist has a stronger *indirect* incentive to invest in productivity enhancing activities at the steady state $\widehat{c}_{II}$. This latter *output effect* counterbalances the *direct effect*, and the monopolist has the same incentive for innovation in the two steady states $\widehat{c}_{II}$ and $\widehat{c}_{III}$.

4 Innovation Dynamics

In the foregoing section we have shown that the trajectory $\{c_t^*, q_t^*\}_{t=0}^{\infty}$ has three steady states, in which c_t^* and q_t^* remain constant over time. Typically, however, the initial value $\bar{c}_0$ of the monopolist's wage–productivity ratio will only accidentally coincide with one of the three steady state values, $\hat{c}_I, \hat{c}_{II}$ and $\hat{c}_{III}$. In general c_t^* will evolve over time until it possibly reaches one of the steady states. From the theory of dynamic optimization it is well-known that the optimal path of c_t^* is monotone, i.e. the sign of $c_{t+1}^* - c_t^*$ is the same for all t (see e.g. Kamien and Schwartz (1991, p. 179)).

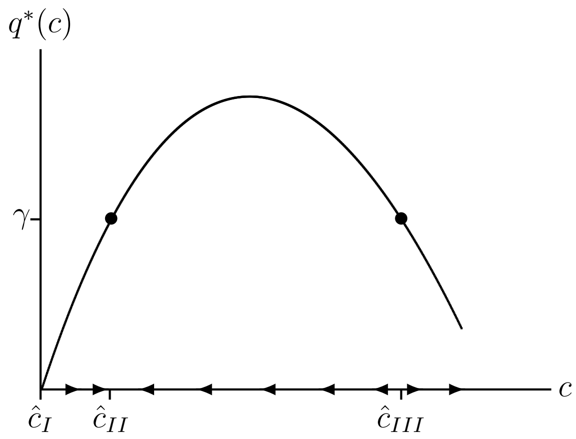

Figure 1: *The Optimal Innovation Policy*

We first consider the optimal policy $q^*(c)$ for values of c in the interval $(\hat{c}_{II},\ \hat{c}_{III})$. It turns out that for this intermediate range the monopolist has a strong incentive for cost reductions:

Proposition 2 *The optimal policy satisfies $q^*(c) > \gamma$ for all $c \in (\hat{c}_{II}, \hat{c}_{III})$.*

Within the range $c \in (\hat{c}_{II}, \hat{c}_{III})$, the monopoly chooses an innovation rate that exceeds the growth rate of wages. This means that c_t^* decreases monotonically over time when the initial value $\bar{c}_0$ happens to lie between $\hat{c}_{II}$ and $\hat{c}_{III}$. For intermediate values of wage–productivity ratio, the monopolist has a strong incentive to invest in labor productivity enhancing activities. Not only its unit labor cost is not too low so that the *direct* innovation incentive is strong. But also the firm's output is high enough to induce a strong *indirect* incentive to invest in cost reduction. In summary, both the direct effect and the output effect are rather strong for intermediate values of c.

For values of c for which Proposition 2 does not apply, wages will grow faster than productivity under the optimal policy:

Proposition 3 *The optimal policy satisfies $q^*(c) < \gamma$ for all $c \notin [\hat{c}_{II}, \hat{c}_{III}]$.*

Figure 1 summarizes our results. For any starting point $\bar{c}_0$ in the range $(0, \hat{c}_{II})$, the firm's wage–productivity ratio c_t^* ultimately approaches $\hat{c}_{II}$. On the optimal path c_t^* increases monotonically over time because for each t one has $q^*(c_t^*) < \gamma$. Also when $\bar{c}_0$ is in the interval $(\hat{c}_{II}, \hat{c}_{III})$, c_t^* approaches $\hat{c}_{II}$ in the long–run. On such a path, however, c_t^* decreases over time because $q^*(c_t^*) > \gamma$. If $\bar{c}_0$ exceeds the critical value $\hat{c}_{III}$, the firm will become eventually extinct under the optimal policy, which satisfies $q^*(c_t^*) < \gamma$. On this path, the monopoly's labor cost increases over time and so its output decreases until it ultimately equals zero.

Unless $\bar{c}_0 > \hat{c}_{III}$, the rate of productivity growth converges to the rate of wage growth. The long–run behavior of the monopoly's innovation strategy is thus independent of its initial productivity $\bar{a}_0$ and the level of the wage rate $\bar{w}_0$. It is not the level of wages but the growth rate of wages that eventually determines the monopoly's innovation behavior and its wage–productivity ratio in the steady state $\hat{c}_{II}$. In this steady state, also the firm's output $x^*(\hat{c}_{II})$ and its price $P(x^*(\hat{c}_{II}))$ are constant and do not depend on the wage level. This level, however, has a profound effect on the long–run level of employment: As $\hat{c}_{II}$ is constant, a one percent increase in $\bar{w}_0$ raises the long–run level of labor productivity by one percent. Since the steady state output $x^*(\hat{c}_{II})$ is not affected, employment falls by one percent. In other words, the monopoly's long–run elasticity of labor demand with respect to the wage level equals minus unity.

Recall that the higher the exogenous growth rate of wages γ is, the higher is the steady state value of wage–productivity ratio $\widehat{c}_{II}$ and the lower is the steady state value $\widehat{c}_{III}$. Therefore, as γ increases, the range of parameters for which the firm will become eventually extinct becomes larger. Moreover, for the rest of the parameter values, the higher the growth rate of wages γ is, the higher is the monopolist's long–run wage–productivity ratio $\widehat{c}_{II}$. The monopolist will thus face a higher unit cost of labor in the long–run in economies where wages grow faster. The monopolist's output will then be lower and his price higher in the long–run. Further, as labor productivity will increase faster in the steady state, the monopolist's demand for labor will decrease faster in the long–run. Similarly, the firm's adjustment behavior depends on its discount factor δ and the size of the market: If demand is relatively weak or the discount factor relatively high, it is more likely that the firm eventually becomes extinct. Also, it faces a higher wage–productivity ratio in the long–run if it is optimal to stay in the market forever.

5 Productivity Dependent Wages

Our analysis has so far considered a monopoly that pays the competitive wage rate to its labor force. It is often argued, however, that there is some sharing of the rents from innovation between workers and firms. For instance, if wage determination is the result of bargaining between a firm and a union, the wage will be positively related to the firm's productivity.[3]

To investigate the impact of productivity dependent wages on the monopoly's optimal innovation strategy, we now consider the case where the firm specific wage w_t depends on the competitive wage $\tilde{w}_t$ and current labor productivity a_t according to

$$w_t = \alpha\, \tilde{w}_t + \beta\, a_t. \tag{16}$$

We assume $\alpha > 0$, $\beta > 0$ and $\beta/(1-\alpha) \geq \tilde{w}_t/a_t$ so that $w_t \geq \tilde{w}_t$. The factor β reflects the extent to which the employees benefit from the firm's productivity. In a wage bargaining model, it is positively related to the union's bargaining power.

Example In the right–to–manage model of wage bargaining the firm chooses the output $x^*(c_t)$ after the wage rate w_t has been determined. Therefore its employment equals $x^*(w_t/a_t)/a_t$. The union's payoff at date t may be specified as $U(w_t) = (w_t - \tilde{w}_t)x^*(w_t/a_t)/a_t$. Suppose that at each date the firm and the union bargain about the current wage so that w_t is determined by the generalized Nash bargaining solution. Then w_t maximizes

$$\Pi\left(w_t/a_t\right)^{1-r}\, U\left(w_t\right)^r, \tag{17}$$

[3]See e.g. Grout (1984). For a survey of wage bargaining models with trade unions see e.g. Booth (1995), Ulph and Ulph (1990), Layard *et al.* (1991). An empirical analysis can be found in Van Reenen (1996).

where $r \in (0, 1]$ indicates the union's bargaining power. For the specification in (5) one gets

$$w_t = \frac{2-r}{2}\,\tilde{w}_t + \frac{A\,r}{2}\,a_t, \tag{18}$$

which is of the same form as equation (16). $\diamondsuit$

The monopoly's wage–productivity ratio at date t equals

$$c_t = \alpha\,\frac{\tilde{w}_t}{a_t} + \beta. \tag{19}$$

As before, we assume that the competitive wage grows at the rate γ so that $\tilde{w}_{t+1} = (1+\gamma)\tilde{w}_t$. If at date t the firm increases future labor productivity by the factor q_t, its labor cost per unit of output at the subsequent date is

$$c_{t+1} = \alpha\,\frac{\tilde{w}_t(1+\gamma)}{a_t(1+q_t)} + \beta, \tag{20}$$

By combining (19) and (20) it becomes apparent that the factor β affects the evolution of c_t according to

$$c_{t+1} = [c_t - \beta]\,\frac{1+\gamma}{1+q_t} + \beta. \tag{21}$$

Ceteris paribus, c_{t+1} increases with β if $q_t > \gamma$. This is so because the firm's workforce captures some share of the gains from cost reduction. Conversely, the available rents decrease if $q_t < \gamma$ because then $c_{t+1} > c_t$. In this case, c_{t+1} and β are negatively related.

When wages depend on the firm's productivity, the monopoly's innovation behavior takes into account the effect of productivity changes on its wage cost. Thus equation (21) replaces constraint (9) in the monopoly's problem (8). As in the previous analysis of this problem, the critical steady states of the state variable c are those where $q^*(\hat{c}) = \gamma$ so that $c^*_{t+1} = c^*_t$ if $c^*_t = \hat{c}$. Analogously to equations (11) and (12), it is straightforward to show that such a $\hat{c}$ must satisfy

$$-\delta\,V'(\hat{c})[\hat{c} - \beta] = (1+\gamma)K'(\gamma), \;\; V'(\hat{c}) = \Pi'(\hat{c}) + \delta\,V'(\hat{c}). \tag{22}$$

Combining these conditions yields

$$-\frac{\delta}{1-\delta}\Pi'(\hat{c})[\hat{c}-\beta] \quad = K'(\gamma)(1+\gamma). \tag{23}$$

Similarly to (14), for β not too large and γ sufficiently small this equation has exactly two solutions, $\hat{c}_{II}(\beta)$ and $\hat{c}_{III}(\beta)$. Clearly, for $\beta = 0$ these solutions coincide with the state states $\hat{c}_{II}$ and $\hat{c}_{III}$ in the foregoing analysis.

Example For the specification in (5) we have

$$\begin{aligned} \hat{c}_{II}(\beta) &= \frac{A+\beta}{2} - \frac{\sqrt{\delta\,(A-\beta)^2 - 32\gamma(1+\gamma)(1-\delta)}}{2\sqrt{\delta}}, \\ \hat{c}_{III}(\beta) &= \frac{A+\beta}{2} + \frac{\sqrt{\delta\,(A-\beta)^2 - 32\gamma(1+\gamma)(1-\delta)}}{2\sqrt{\delta}}, \end{aligned} \tag{24}$$

are the two solutions of equation (23). ◇

By Lemma 1 we get immediately the following result:

Proposition 4 *The steady state value $\hat{c}_{II}(\beta)$ is increasing in β, and the steady state value $\hat{c}_{III}(\beta)$ is decreasing in β.*

Proposition 4 describes the impact of β on the firm's long–run behavior. The properties of the optimal policy $q^*(c)$ are analogous to the results in Propositions 2 and 3: In the interval $(\hat{c}_{II}(\beta), \hat{c}_{III}(\beta))$ it is the case that $q^*(c) > \gamma$ while $q^*(c) < \gamma$ for all values of $c \notin [\hat{c}_{II}(\beta), \hat{c}_{III}(\beta)]$. On the adjustment path, the dependence of wages on productivity tends to reduce the firm's investment in cost reduction: Since the length of the interval $(\hat{c}_{II}(\beta), \hat{c}_{II}(\beta))$ decreases with β, this factor is negatively related to the range of c–values where $q^*(c) > \gamma$. The intuition is simply that sharing the productivity gains with its employees reduces the firm's innovation incentive. By the same argument, $\hat{c}_{III}(\beta) < \hat{c}_{III}$ so that productivity dependent wages reduce the range of initial values $\bar{c}_0$ that allow the firm to survive in the long–run.

If, however, $\bar{c}_0 < \hat{c}_{III}(\beta)$, the optimal trajectory $\{c_t^*, q_t^*\}_{t=0}^{\infty}$ converges towards $(\hat{c}_{II}(\beta),\, \gamma)$. In this case, rent sharing between the monopoly and its labor force does not affect the long–run rate of productivity growth and the long–run innovation effort $K(\gamma)$. On the optimal path, the firm specific wage w eventually grows at the same rate as the competitive wage $\tilde{w}$ and so the relative wage differential $w/\tilde{w}$ remains constant over time. Because of this differential, the monopoly's long–run labor cost $\hat{c}_{II}(\beta)$ is positively related to the factor β. Since $\Pi'(c) < 0$, its profit in the steady state $\hat{c}_{II}(\beta)$ depends negatively on β.

Consider, for instance, a unionized firm where bargaining between the monopolist and the union determines the wage rate. Our analysis then predicts the following: First, long–run productivity growth does not depend on the union's bargaining power; it is simply determined by the growth rate of the competitive wage. Second, as the union's power increases, the critical value of the initial wage for which the monopolist is able to survive in the market is lowered. Finally, as the union's claims over productivity rents increase, the monopolist's long–run unit labor cost increases, and output and profits decrease. Hence, due to the presence of the union, the firm is more likely to become unprofitable over time and to leave the market. Moreover, even when it remains profitable, its long–run unit labor costs are higher and its output is lower than in a non–unionized firm.

6 Conclusions

Modern industrialized countries are characterized by rapid technical progress accompanied with substantial increases in real wages. We have shown that, in a dynamic monopolistic industry, the firm optimally invests each period in productivity enhancing innovations to counterbalance increasing wages. Our analysis presents a dynamic cost–push argument of productivity growth. Higher current labor costs create stronger incentives for process innovations that raise the productivity of labor in future periods. If the level of wages is too high, the monopolist is unable to survive in the market in the long–run. It will gradually reduce its innovation effort over time and ultimately will exit the market. If, however, the wage level is sufficiently low, the monopolist's rate of productivity growth monotonically approaches the growth rate of wages and eventually the firm reaches a steady state where its unit cost of labor remains constant over time. Interestingly, long–run productivity growth only depends on the growth rate of wages and is thus independent of the initial level of wages. The latter only determines whether the monopolist will be able to survive, or not, in the market. While the monopolist's unit labor cost as well as its output in the long–run depend only on the growth rate of wages, the long–run levels of labor productivity and employment are determined by the initial level of wages.

We have also analyzed the case where the rents stemming from productivity enhancing innovations are shared between the monopolist and its workers. Surprisingly, also in this case the long–run productivity growth only depends on the growth rate of wages and is independent of the share of profits over which the workers have claims (measured e.g. by the union's power). Hence, unionization does not influence long–run productivity growth, despite the fact that it depresses the short–run incentives for innovation. The union's bargaining power, however, determines the monopolist's long–run unit cost of labor as well as its likelihood of survival in the market. When union power is high, the monopolist can survive in the market only if the initial level of wages is low enough. If, however, the monopolist's optimal policy is to stay in the market forever, then its long–run unit cost of labor is the higher, the higher the union's power is.

Our model could be extended to consider dynamic imperfectly competitive markets. In an intertemporal model where at each date firms compete in prices or quantities it will be interesting to analyze how strategic interactions between the firms affect productivity growth in the short–run and in the long–run. Stimulated by the work of Schumpeter (1947), a large part of the literature on $R\&D$ relates the pace of innovative activity to market structure. An imperfect competition version of our model could combine this approach with our cost-push argument.

7 Appendix

This appendix contains the proof of Lemma 1 and Propositions 1 – 3.

Proof of Lemma 1: When the optimal output $x^*(c)$ is positive, the first order condition is $R'(x^*) - C'(x^*) = c$. Therefore

$$-\Pi'(c)c = x^*(c)\, c = [R'(x^*(c)) - C'(x^*(c))]x^*(c). \tag{25}$$

Since $x^*(c) < \infty$, one has $-\Pi'(0) = x^*(0) < \infty$. For $c \to \infty$ one has $x^*(c) \to 0$. Therefore, the second part of Assumption 1 and (25) imply that $-\Pi'(c)c \to 0$ for $c \to \infty$.

For c sufficiently small $x^*(c) > 0$ and so $-\Pi'(c)c > 0$. Since $-\Pi'(c)c \to 0$ for $c \to 0$ and for $c \to \infty$, the function $-\Pi'(c)c$ attains a maximum for some $0 < c^m < \infty$. Since $x^*(c)$ is strictly decreasing, Assumption 1 and (25) imply that c^m is unique and that $-\Pi'(c)c$ is increasing in c for $c < c^m$ and decreasing in c for $c > c^m$ as long as $\Pi(c) > 0$. Q.E.D.

Proof of Proposition 1: Since we have already shown that $q^*(c) = \gamma$ implies $c \in \{\hat{c}_{II}, \hat{c}_{III}\}$, it remains to show that the equation $q^*(c) = \gamma$ has two solutions $\hat{c}_{II}$ and $\hat{c}_{III}$. Notice that combining (11) and (12) yields

$$V'(c)c = \Pi'(c)c - K'(q^*(c))(1 + q^*(c)) \tag{26}$$

for all c. Since $K'' > 0$, this equation has a unique solution $q^*(c)$ for all c. Moreover, by continuity of $V'(\cdot)$ and $\Pi'(\cdot)$ the optimal policy $q^*(\cdot)$ is continuous.

We first show that $q^*(c^m) > \gamma$. Note that by (26) we have $-V'(c)c \geq -\Pi'(c)c$ for all c. Suppose $q^*(c^m) \leq \gamma$. Then this together with (11) implies

$$\begin{aligned}
-\delta\, \Pi'\left(c^m \frac{1+\gamma}{1+q^*(c^m)}\right) \frac{(1+\gamma)c^m}{1+q^*(c^m)} &\leq \\
-\delta\, V'\left(c^m \frac{1+\gamma}{1+q^*(c^m)}\right) \frac{(1+\gamma)c^m}{1+q^*(c^m)} &= \\
(1+q^*(c^m))K'(q^*(c^m)) &\leq (1+\gamma)K'(\gamma).
\end{aligned} \tag{27}$$

As $q^*(c^m)$ increases from 0 to γ, the term $[(1+\gamma)c^m]/[1+q^*(c^m)]$ decreases from $c^m(1+\gamma)$ to c^m. By Lemma 1, $-\Pi'(c)c$ is decreasing in c over the interval

$[c^m, c^m(1+\gamma)]$. Accordingly, the first term (27) is increasing in $q^*(c^m)$ over the interval $[0,\gamma]$ and so it follows from $q^*(c^m) \leq \gamma$ that

$$-\Pi'\left(c^m \frac{1+\gamma}{1+q^*(c^m)}\right)\frac{(1+\gamma)c^m}{1+q^*(c^m)} \geq -\Pi'\left(c^m(1+\gamma)\right)(1+\gamma)\,c^m. \tag{28}$$

This in combination with (27) yields

$$-\delta\,\Pi'\left(c^m(1+\gamma)\right)(1+\gamma)\,c^m \leq (1+\gamma)K'(\gamma), \tag{29}$$

a contradiction to assumption 4. This proves that $q^*(c^m) > \gamma$.

Next we show that $q^*(\bar{c}) < \gamma$ for some $\bar{c} > c^m$. Suppose the contrary. Then $q^*(c) \geq \gamma$ and so by (10)

$$V(c) \leq \Pi(c) - K(\gamma) + \delta V\left(c\frac{1+\gamma}{1+q^*(c)}\right) \tag{30}$$

for all c sufficiently large. Note that $\Pi(c) \to 0$ as $c \to \infty$. Moreover, we have $q^*(c) < \infty$ for all c, because $K(\infty) = \infty$. Therefore, (30) implies

$$\lim_{c\to\infty} V(c) \leq -\frac{K(\gamma)}{1-\delta} < 0, \tag{31}$$

a contradiction. This proves that $q^*(\bar{c}) < \gamma$ for some $\bar{c} > c^m$.

Since $q^*(0) = 0$, $q^*(c^m) > \gamma$ and $q^*(\bar{c}) < \gamma$ for some $\bar{c} > c^m$, continuity of $q^*(\cdot)$ implies that there is a $\hat{c}_{II} \in (0, c^m)$ such that $q^*(\hat{c}_{II}) = \gamma$ and a $\hat{c}_{III} > c^m$ such that $q^*(\hat{c}_{III}) = \gamma$. Q.E.D.

Proof of Proposition 2: Note that the sign of $q^*(c) - \gamma$ cannot change over some interval $[c', c'']$ if this interval does not contain a steady state. Otherwise, one would obtain a contradiction to the fact that the optimal trajectory c_t^* is monotone. By Proposition 1, therefore, the sign of $q^*(c) - \gamma$ is constant over the interval $c \in (\hat{c}_{II}, \hat{c}_{III})$. The proof of Proposition 1 shows that $q^*(c^m) > \gamma$. Since $c^m \in (\hat{c}_{II}, \hat{c}_{III})$, this proves that $q^*(c) > \gamma$ for all $c \in (\hat{c}_{II}, \hat{c}_{III})$. Q.E.D.

Proof of Proposition 3: By the same argument as in the proof of Proposition 2, the sign of $q^*(c) - \gamma$ cannot change over the interval $c \in [0, \hat{c}_{II})$ or the interval $(\hat{c}_{III}, \infty)$. Therefore it is sufficient to show that each interval contains a c' such that $q^*(c') < \gamma$. Clearly, this is the case for the interval $[0, \hat{c}_{II})$ because $q^*(0) = 0$.

Now consider the interval $(\hat{c}_{III}, \infty)$. The proof of Proposition 1 shows that $q^*(\bar{c}) < \gamma$ for some $\bar{c} > c^m$. Since Proposition 2 shows that $q^*(c) > \gamma$ for all $c \in (\hat{c}_{II}, \hat{c}_{III})$, it must be the case that $\bar{c} > \hat{c}_{III}$. Therefore also the interval $(\hat{c}_{III}, \infty)$ contains a c' such that $q^*(c') < \gamma$. Q.E.D.

8 References

Bester, Helmut and Emmanuel Petrakis, 1998, Wages and Productivity Growth in a Competitive Industry, CEPR-Discussionpaper No. 2031.

Booth, Alison L., 1995, *The Economics of the Trade Union*, University Press, Cambridge.

Chennells, Lucy and Van Reenen, John, 1997, Technical Change and Earnings in British Establishments, *Economica* 64, 587-604.

Doms, Mark, Dunne, Timothy, and Troske, Kenneth R., 1997, Workers, Wages and Technology, *Quarterly Journal of Economics* 112, 253-290.

Gordon, Robert J., 1987, Productivity, Wages, and Prices Inside and Outside of Manufacturing in the US, Japan, and Europe, *European Economic Review* 31, S. 685 - 733.

Grout, Peter A., 1984, Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Approach, *Econometrica* 52, 499-460.

Hellwig, Martin and Irmen, Andreas, 1999, Endogenous Technical Change in a Competitive Economy, University of Mannheim, mimeo.

Kamien, Morton I. and Schwartz, Nancy L., 1991, *Dynamic Optimization,* North Holland: Amsterdam.

Layard, Richard, Nickell, Stephen and Jackman, Richard, 1991, *Unemployment: Macroeconomic Performance and the Labour Market*, Oxford University Press.

Mohnen, Pierre A., Nadiri, M. Ishaq and Prucha, Ingmar R., 1986, R&D, Production Structure and Rates of Return in the US, Japanese and German Manufacturing Sectors, *European Economic Review* 30, 749-771.

Ulph, Alistair M. and Ulph, David T., 1990, Union Bargaining: A Survey of Recent Work, in D. Sapsford and Z. Tzannatos (eds.), *Current Issues in Labour Economics*, London, Macmillan.

Ulph, Alistair M. and Ulph, David T., 1994, Labour Markets and Innovation: Ex-Post Bargaining, *European Economic Review* 38, 195-210.

Van Reenen, John, 1996, The Creation and Capture of Rents: Wages and Innovation in a Panel of U. K. Companies, *Quarterly Journal of Economics* 111, 195-226.