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ABSTRACT

Gaps and Triangles*

We derive principles of optimal short run monetary policy in a real business cycles model, with money and with monopolistic firms that set prices one period in advance. The only distortionary policy intruments are the nominal interest rates and the money supplies. In this environment it is feasible to undo both the cash in advance and the price setting restrictions. We show that the optimal allocation is achieved under the Friedman rule. We also show that, in general, it is not optimal to undo the restriction that prices are set one period in advance. Sticky prices provide the planner with tools to improve upon a distorted flexible prices allocation.

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Keywords: Friedman rule, optimal cyclical monetary policy, prices set in advance

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NON-TECHNICAL SUMMARY

In this Paper we derive general principles on how to conduct short-run monetary policy by analysing a real business cycles model without capital, to which three main restrictions are added: monopolistic firms, a cash-in-advance restriction on household transactions, and a restriction on firms, stating that prices must be set one period in advance. In this sticky prices environment, monetary policy can affect allocations both through the path of nominal interest rates and money supplies. These are the only distortionary policy instruments that the government can use.

For a given interest rate path it is possible in this environment to conduct money supply policy in order to undo the restriction of price stickiness, so that the allocations under sticky and flexible prices coincide. By setting the nominal interest rates to zero, the effects of the cash in advance restriction can also be eliminated. With the available policy instruments, because of the zero bound on nominal interest rates, the mark up distortion cannot be undone.

The first major result of the Paper is that the Friedman rule of setting the nominal interest rates equal to zero must be followed in order to achieve the optimal allocation. This is a general result and thus extends the result in Ireland (1996), which shows that it is optimal to follow the Friedman rule when the utility function is separable, linear in leisure and logarithmic in consumption. In addition, we obtain that in general it is not optimal to completely undo the effects of price stickiness. Ramsey rules in this environment require the sticky prices allocation to deviate from the flexible prices one. These deviations between the flexible prices and the sticky prices allocations for a given interest rate path are our definition of gaps. Sticky prices provide the planner with policy tools to improve upon a distorted flexible prices allocation.

Under flexible prices only interest rates matter. The path of money supply affects the price levels but not the real allocation. Because there are lump sum taxes, the optimal monetary policy is to set nominal interest rates to zero. That way the distortion is minimized in each date and state, and expected utility is maximized. The optimal allocation will be distorted by a constant mark up. Instead, the optimal allocation under sticky prices will be characterized by variable mark-ups around an average mark-up induced by monopolistic competition. These wedges cannot be attained under flexible prices because nominal interest rates cannot be negative. Under sticky prices the planner is able to sidestep the zero bound restriction on nominal interest rates and achieve higher utility.

There are problems with following the Friedman rule under sticky prices. Under the Friedman rule, since the cash in advance constraint is not binding, the money supply cannot be used to determine the allocation. As Carlstrom

and Fuerst (1998b) show in a comment to Ireland (1996), the allocation is indeterminate and there are equilibria with sunspot fluctuations. This indeterminacy is no longer there for positive but arbitrarily small interest rates. It seems to us that all that is economically relevant is the behaviour of these economies as the interest rates converge to zero. Along this sequence the optimal allocations can be achieved by using money supply.

1. Introduction

In this paper we derive general principles on how to conduct short run monetary policy, by analyzing a real business cycles model without capital, to which three main restrictions are added: monopolistic firms, a cash-in-advance restriction on the households transactions, and a restriction on firms that prices must be set one period in advance. In this sticky prices environment, monetary policy can affect allocations both through the path of nominal interest rates and money supplies. These are the only distortionary policy instruments that the government can use.

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The literature on optimal monetary policy in an imperfectly competitive and sticky prices world is relatively recent. Papers related to the current analysis include Ireland (1996), Rotemberg and Woodford (1997, 1999), Goodfriend and King (1997), Carlstrom and Fuerst (1998a, 1998b), King and Wolman (1998), Gali and Monacelli (1999) and Erceg et al. (2000).

Our analysis is orthogonal to the analysis in Rotemberg and Woodford (1997, 1999), Gali and Monacelli (1999) and Erceg et al (2000). They analyze an environment with our basic structure but with three main differences. They allow for fiscal instruments that undo the monopolistic competition distortion; one way or another those are cash-less economies where money is only a unit of account; and prices are set in a staggered fashion. Erceg et al (2000) also consider sticky wages. The nominal rigidities are the only distortions so that the flexible prices allocation, if feasible, is optimal. Since by assumption there is no money demand in these models, money supply does not play a role. The nominal interest rate is the sole instrument of monetary policy. When the flexible prices allocation is feasible, the optimal nominal interest rate policy is to target the real interest rate under flexible prices.

King and Wolman (1998) study a model with staggered price setting and keep the monopolistic competition distortion but get rid of the nominal interest rate distortion by allowing interest to be paid on currency. They show that the flexible prices solution is optimal in the deterministic case. Kahn, King and Wolman (2000) allow for the money demand distortion and solve the optimal policy problem numerically. They show that the optimal allocation is quantitatively close to the allocation under flexible prices.

Carlstrom and Fuerst (1998a) study a similar environment to the one we address here. They do not analyze optimal policy. Rather they investigate the implications of following restrictive policies such as an interest rate target, with endogenous money supply, or an inflation target, with varying nominal interest rates. Under both policies there are sunspot fluctuations.

The remainder of the paper is organized as follows. In section 2 we describe the model and define the equilibria under flexible and sticky prices. Under flexible prices the allocation is a function of monetary policy only through the nominal interest rate path. Instead, under sticky prices, the allocation depends on the money supply policy for a given nominal interest rate policy. In Section 3 we first define the Ramsey problem in the economy with sticky prices and determine the optimal interest rate policy (Section 3.2). Subsequently we take the process for the nominal interest rates as exogenous and determine the optimal money supply policy consistent with it (Section 3.3). We identify conditions on preferences, technology and shocks for which the Ramsey solution and the allocation under flexible prices coincide. As we believe it is unlikely that these conditions are satisfied, in general the optimal policy under sticky prices achieves higher utility than under flexible prices. Section 4 contains concluding remarks.

2. The economy

Our model economy has a simple structure as in Ireland (1996) or Carlstrom and Fuerst (1998a). The economy consists of a representative household, a continuum of firms indexed by $i \in [0, 1]$, and a government or central bank. We consider a vector of shocks that follow a Markov process, $s_t = [\varkappa_t, z_t, v_t, G_t]$, to preferences, $\varkappa_t$, technology, z_t, velocity, v_t, and government expenditures, G_t. The history of these shocks up to period t, $(s_0, s_1, ..., s_t)$, is denoted by s^t. In order to simplify the exposition we assume that the histories of shocks have a discrete joint distribution.

Each firm produces a distinct, perishable consumption good, indexed by i. The production uses labor, according to a concave technology.

We impose a cash in advance constraint to the households transactions with the timing as in Lucas and Stokey (1983). Each period is divided into two subperiods, with the assets market in the first subperiod and the goods market in the second.

2.1. The households

The households have preferences over composite consumption C_t, and leisure $1 - N_t$, described by the expected utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_t, 1 - N_t, \varkappa_t\right) \right\} \tag{2.1}$$

where β is a discount factor, $\varkappa_t$ is the referred preference shock, and C_t is

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1.$$

where θ is the elasticity of substitution between any two goods.

The households start period t with outstanding money balances, M_t, and decide to buy B_{t+1} units of money in nominal bonds that pay $R_t B_{t+1}$ units of money one period later. They also decide to spend $E_t Q_{t,t+1} A_{t+1}$ in state contingent nominal securities. Each security pays one unit of money at the beginning of period $t+1$ in a particular state. $Q_{t,t+1}$ is the price, normalized by the probability of the occurrence of the state, at the beginning of period t. The households also receive monetary transfers X_t. A fraction $\frac{1}{v_t}$ of the purchases of consumption, $\int_0^1 P_t(i) c_t(i) di$, where $P_t(i)$ is the price of good i, in units of money, must be made with money so that the following cash in advance constraint must be satisfied,

$$\int_0^1 P_t(i) c_t(i) di \leq (M_t - B_{t+1} + R_{t-1} B_t - E_t Q_{t,t+1} A_{t+1} + A_t + X_t) v_t \tag{2.2}$$

$v_t \geq 1$ represents the velocity of money. As v_t becomes arbitrarily large money ceases to play a role in this economy. At the end of the period, the households receive the labor income, $W_t N_t$ where W_t is the nominal wage rate. They also receive the dividends from the final goods firms $\int_0^1 \Pi_t(i) di$ and they pay lump sum taxes, T_t. The households face the budget constraints

$$\begin{aligned} M_{t+1} \leq & \left[M_t - B_{t+1} + R_{t-1} B_t - E_t Q_{t,t+1} A_{t+1} + A_t + X_t - \int_0^1 P_t(i) c_t(i) di \right] + \\ & W_t N_t + \int_0^1 \Pi_t(i) di - T_t \end{aligned} \tag{2.3}$$

The Bellman equation that describes the households' problem is

$$V(M_t, B_t, A_t, s_t) = \text{Max } \{ u(C_t, 1 - N_t, \varkappa_t) + \beta E_t V(M_{t+1}, B_{t+1}, A_{t+1}, s_{t+1}) \},$$

where the maximization is subject to (2.2) and (2.3), together with no-Ponzi games conditions on the holdings of assets.

Let $P_t = \left[\int P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$. The following are first order conditions of this problem, for all t and s^t,

$$\frac{c_t(i)}{C_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \tag{2.4}$$

$$\frac{u_{1-N}(t)}{u_C(t)} = \frac{W_t}{P_t}\frac{1}{R'_t}, \text{ where } R'_t = \frac{R_t - 1 + v_t}{v_t} \tag{2.5}$$

$$\frac{u_C(t)}{P_t} = R'_t E_t\left[\frac{\beta u_C(t+1)}{P_{t+1}}\frac{R_{t+1}}{R'_{t+1}}\right] \tag{2.6}$$

and, for all t, $t+1$, s^t, s^{t+1},

$$Q_{t,t+1} = \beta\frac{\left(\frac{u_C(t+1)}{P_{t+1}}v_{t+1} - \frac{u_{1-N}(t+1)}{W_{t+1}}(v_{t+1}-1)\right)}{\left(\frac{u_C(t)}{P_t}v_t - \frac{u_{1-N}(t)}{W_t}(v_t-1)\right)} \tag{2.7}$$

From (2.6) and (2.7), we have

$$E_t\left[Q_{t,t+1}\right] = \frac{1}{R_t}, \text{ for all } t \text{ and } s^t. \tag{2.8}$$

Condition (2.4) defines the demand for each of the intermediate goods i and condition (2.5) sets the intratemporal marginal rate of substitution between leisure and consumption equal to the real wage corrected for the cost of using money. Condition (2.6) is a requirement for the optimal choice of risk free nominal bonds. Condition (2.7) determines the price of one unit of money at time $t+1$, for each state of nature s^{t+1}, normalized by the conditional probability of occurrence of state s^{t+1}, in units of money at time t. Condition (2.8) says that at t the money value of an unit of money at $t+1$, $\frac{1}{R_t}$, is equal to the expenditure at t necessary to get one unit of each contingent nominal security.

When $v_t = 1$ then $R'_t = R_t$. In this case our model coincides with the standard cash in advance model and first order conditions of the household (2.5),(2.6) and (2.7) can be written as

$$\frac{u_{1-N}(t)}{u_C(t)} = \frac{W_t}{P_t}\frac{1}{R_t}, \text{ for all } t \text{ and } s^t, \tag{2.9}$$

$$\frac{u_C(t)}{P_t} = R_t E_t\left[\frac{\beta u_C(t+1)}{P_{t+1}}\right], \text{ for all } t \text{ and } s^t, \tag{2.10}$$

$$Q_{t,t+1} = \beta \frac{\left(\frac{u_C(t+1)}{P_{t+1}}\right)}{\left(\frac{u_C(t)}{P_t}\right)}, \text{ for all } t,\ t+1,\ s^t,\ s^{t+1}. \tag{2.11}$$

As $v_t \to \infty$ the cash constraint becomes irrelevant and the model reduces to a real model. Note that in that case $R_t' = 1$.

2.2. Government

The government has to finance an exogenous stream of government purchases, G_t. These purchases result from the aggregation of the expenditures on each good produced in the economy,

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \theta > 0.$$

Given the prices on each good, $P_t(i)$, the government chooses the quantities, $g_t(i)$, in order to minimize total spending and achieve a certain exogenous value of G_t. Thus, the purchases of each good i must satisfy

$$\frac{g_t(i)}{G_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \tag{2.12}$$

The government finances these government expenditures, G_t, with lump sum taxes $T_t = P_t G_t$. These taxes are collected at the goods market. In addition the government makes a lump-sum monetary transfer X_t at the assets market to the representative household.

As government debt is irrelevant in this environment we choose to write the government budget constraint as a balanced budget constraint. Therefore

$$M_t^s = M_{t-1}^s + P_t G_t + X_t - T_t.$$

The money supply evolves according to $M_t^s = M_{t-1}^s + X_t$.

2.3. Firms

Each firm i has the production technology

$$y_t(i) \leq z_t F\left(n_t(i)\right) \tag{2.13}$$

where $y_t(i)$ is the production of good i and z_t is an aggregate technology shock. $y_t(i)$ can be used for private and public consumption, $y_t(i) = c_t(i) + g_t(i)$.

The problem of the firm is to choose the price in order to maximize profits that can be used for consumption in period $t+1$ taking the demand function,

$$\frac{y_t(i)}{Y_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \tag{2.14}$$

obtained from (2.4) and from (2.12), as given, and satisfying the technology constraint.

2.3.1. Under flexible prices

Under flexible prices, the value of period t profits in units of money is $E_t Q_{t,t+1} \Pi_t(i)$. The maximization of this expression is equivalent to the maximization of $\Pi_t(i)$, which is given by

$$\Pi_t(i) = P_t(i) y_t(i) - W_t n_t(i)$$

where $P_t(i)$ satisfies the demand function (2.14). The first order condition of this problem is

$$P_t(i)\left[1 + \frac{d \ln P_t(i)}{d \ln y_t(i)}\right] - \frac{W_t}{z_t F'(t)} = 0$$

where $\frac{d \ln P_t(i)}{d \ln y_t(i)} = -\frac{1}{\theta}$, since θ is the demand elasticity. Therefore, it must be that

$$P_t = P_t(i) = \frac{\theta}{\theta - 1} \frac{W_t}{z_t F'(t)}. \tag{2.15}$$

As there is a complete symmetry across firms they all set an identical price, which is a constant mark-up over marginal cost. As the elasticity of demand θ gets larger, the mark-up converges down to 1.

2.3.2. When prices are set in advance

We consider now an environment where firms set the prices one period in advance and sell the output on demand in period t at the previously chosen price.[1]

[1] This only makes sense if the size of the shocks is sufficiently small so that the firms still prefer to satisfy demand instead of shutting down.

When firm i sets prices one period in advance, it solves the problem of choosing at $t-1$, the price $P_t(i)$ that maximizes the value of profits, i.e.

$$E_{t-1}\left[Q_{t-1,t}Q_{t,t+1}\left(P_t(i)y_t(i)-W_t n_t(i)\right)\right]$$

The objective function for each firm can be written as

$$E_{t-1}\left[\left(\frac{u_C\,(t+1)}{P_{t+1}}v_{t+1}-\frac{u_{1-N}(t+1)}{W_{t+1}}\left(v_{t+1}-1\right)\right)\left(P_t(i)y_t(i)-W_t n_t(i)\right)\right]$$

or equivalently, using (2.5),

$$E_{t-1}\left[\left(\frac{u_C\,(t+1)}{P_{t+1}}\frac{R_{t+1}}{R'_{t+1}}\right)\left(P_t(i)y_t(i)-W_t n_t(i)\right)\right]. \qquad (2.16)$$

The firm's problem is to maximize (2.16) subject to (2.13) and (2.14). The price chosen by the firm is

$$P_t(i)=P_t=\frac{\theta}{(\theta-1)}E_{t-1}\left[\eta_t\frac{W_t}{z_t F'(t)}\right] \qquad (2.17)$$

where

$$\eta_t=\frac{\left(\frac{u_C(t+1)}{P_{t+1}}\frac{R_{t+1}}{R'_{t+1}}\right)Y_t}{E_{t-1}\left[\left(\frac{u_C(t+1)}{P_{t+1}}\frac{R_{t+1}}{R'_{t+1}}\right)Y_t\right]} \qquad (2.18)$$

2.4. Market clearing:

In each period there are markets for goods, labor, money, risk free nominal bonds and state contingent nominal securities. Market clearing conditions are given by the following equations, for each date and state

$$c_t(i)+g_t(i)=y_t(i),$$

so that the demand for each of the goods is equal to the supply;

$$N_t=\int_0^1 n_t(i)\;di$$

so that the total demand of labor is equal to the supply;

$$B_{t+1}=0$$

and

$$A_{t+1} = 0$$

since nominal bonds and the contingent securities are in zero net supply.

Since M_{t+1} denotes the money carried by the household into period $t+1$, market clearing requires that $M_t^s = M_{t+1}$, for each date and state.

From the cash in advance constraints (2.2) of the households and the market clearing conditions we have,

$$\frac{P_t C_t}{v_t} = M_{t-1}^s + X_t \equiv M_t^s. \tag{2.19}$$

2.5. Imperfectly competitive equilibrium:

An equilibrium are prices $\{(P_t(s^t), P_t(i)(s^t), W_t(s^t), Q_{t,t+1}(s^t, s_{t+1}), R_t(s^t))_{s^t \in S^{t+1}, s_{t+1} \in S}\}_{t=0}^{\infty}$, and allocations $\{(C_t(s^t), C_t(i)(s^t), N_t(s^t), M_{t+1}(s^t), B_{t+1}(s^t), A_{t+1}(s^t, s_{t+1}), n_t(i)(s^t), y_t(i)(s^t))_{s^t \in S^{t+1}, s_{t+1} \in S}\}_{t=0}^{\infty}$, given initial s^0, M_0, B_0,and A_0, shocks $\{s^t \in S^t\}_{t=0}^{\infty}$, and policy variables $\{X_t(s^t), T_t(s^t)\}_{t=0}^{\infty}$ such that:

(i) given the prices $\left\{(P_t(s^t), P_t(i)(s^t), W_t(s^t), Q_{t,t+1}(s^t, s_{t+1}), R_t(s^t))_{s^t \in S^{t+1}, s_{t+1} \in S}\right\}_{t=0}^{\infty}$ the sequences

$\{(C_t(s^t), C_t(i)(s^t), N_t(s^t), M_{t+1}(s^t), B_{t+1}(s^t), A_{t+1}(s^t, s_{t+1}))_{s^t \in S^{t+1}, s_{t+1} \in S}\}_{t=0}^{\infty}$ solve the problem of the representative household;

(ii) either,

a) given prices $\left\{(P_t(s^t), W_t(s^t))_{s^t \in S^t}\right\}_{t=0}^{\infty}$, the sequence $\left\{P_t(i)(s^t)_{s^t \in S^t}\right\}_{t=0}^{\infty}$ solves the problem of the firms as stated in subsection (2.2.1) if prices are set contemporaneously, or,

b) given prices $\{(P_t(s^{t-1}), W_t(s^t), Q_{t-1,t}(s^{t-1}, s_t), Q_{t,t+1}((s^{t-1}, s_t), s_{t+1}), R_t(s^t))_{s^{t-1} \in S^{t+1}, s_t \in S, s_{t+1} \in S}\}_{t=0}^{\infty}$, the sequence $\left\{(P_t(i)(s^{t-1}))_{s^{t-1} \in S^{t-1}}\right\}_{t=0}^{\infty}$ solves the problem of the firms as stated in subsection (2.2.2) if prices are set in advance;

(iii) all markets clear;

2.6. Allocations under flexible and under sticky prices

It is useful to notice that the two first order conditions of the households and firms, under flexible prices, (2.5) and (2.15), can be written as

$$\frac{u_{1-N}(C_t, 1-N_t, \varkappa_t)}{u_C(C_t, 1-N_t, \varkappa_t)} = \frac{W_t}{P_t R_t'} = \frac{(\theta - 1)}{\theta R_t'} z_t F'(N_t), \tag{2.20}$$

where

$$R_t' = \frac{R_t - 1 + v_t}{v_t}.$$

The feasibility constraints are

$$C_t + G_t = z_t F(N_t). \tag{2.21}$$

Conditions (2.20) and (2.21) determine the flexible prices allocation for consumption, C_t, and labor, N_t, as a function of the nominal interest rate, R_t, and the shocks.

The level of the interest rate and the mark up of the monopolistic competition affect the equilibrium on the same margin: both introduce a wedge between the marginal rate of transformation and the marginal rate of substitution. Under flexible prices that wedge is reduced to its minimum by setting the nominal interest rate equal to zero, i.e. when the Friedman rule is followed. Given the interest rate path, the money supply, compatible with that interest rate, determines the price level.

In the environment with sticky prices the money supply has a direct effect on the real wage, since prices are set in advance. The real wage satisfies the following condition,

$$E_{t-1}\left[\eta_t \frac{\frac{W_t}{P_t}}{\frac{(\theta-1)z_t F'(t)}{\theta}}\right] = 1. \tag{2.22}$$

Given a path for the nominal interest rates, there is a continuum of money supply policies that are consistent with that path, and are associated with different real allocations. This is the sense in which Carlstrom and Fuerst (1998a) call attention to the real indeterminacy associated with an interest rate rule under sticky prices. Our approach is very different, since we allow the planner to decide on the money supply and, therefore, to use the degrees of freedom implied by (2.22) to achieve the optimal allocation.

3. Optimal allocations under sticky prices

In this section we determine allocations under commitment that maximize the representative agent's expected utility subject to the feasibility constraints and the first order conditions of the households' problem and of the firms' problem. Under sticky prices the first order condition of the households' problem (2.5) can be combined with the first order condition of the firms' problem (2.17), to obtain an implementability condition,

$$E_{t-1}\left[\frac{E_t\left(\frac{u_C(t+1)}{P_{t+1}}\frac{R_{t+1}}{R'_{t+1}}\right)Y_t}{E_{t-1}\left[E_t\left(\frac{u_C(t+1)}{P_{t+1}}\frac{R_{t+1}}{R'_{t+1}}\right)Y_t\right]}\frac{1}{\frac{(\theta-1)}{\theta R'_t}z_tF'(t)}\frac{u_{1-N}(t)}{u_C(t)}\right]=1. \tag{3.1}$$

Using the intertemporal condition (2.6) in (3.1), we have

$$E_{t-1}\left[\frac{\frac{u_C(t)}{\beta P_t R'_t}Y_t}{\frac{(\theta-1)}{\theta R'_t}z_tF'(t)}\frac{u_{1-N}(t)}{u_C(t)}\right]=E_{t-1}\left[\frac{u_C(t)}{\beta P_t R'_t}Y_t\right]$$

and therefore

$$E_{t-1}\left[\frac{u_C(t)}{\frac{\theta R'_t}{(\theta-1)}}Y_t-\frac{F(t)}{F'(t)}u_{1-N}(t)\right]=0 \tag{3.2}$$

It is easy to verify that for a given interest rate path there is a large set of allocations satisfying the feasibility constraint and this implementability condition. In the appendix we show that, as long as interest rates are strictly positive, $R_t > 1$, any allocation that satisfies the feasibility constraint and this implementability condition can be decentralized uniquely by an appropriate money supply. This is in accordance with what we observed previously, that there is a large set of money supply processes compatible with a given interest rate path. The existence of a large set of money supply processes compatible with a given interest rate path implies the existence of a large set of allocations compatible with that same interest rate path.

This way of formalizing the government's choices illustrates that in an environment with nominal rigidities the monetary policy cannot be defined only by the interest rate path. The monetary policy needs to specify the trajectory of the money supply also. On the other hand, in the flexible prices environment the monetary policy is completely characterized by the interest rate path. Thus, when we compare the sticky prices and the flexible prices allocation we do it for policies that imply identical interest rates in both environments. The deviation of the allocation under flexible prices from the allocation under sticky prices, for any monetary policy that implies the same interest rate path is our definition of gaps.

When $R_t = 1$, because the cash in advance restriction is no longer binding, it is not possible to use the money supply to determine one particular allocation.

There is a large set of allocations that solve the feasibility an implementability constraints. The flexible prices allocation is one of them.

Proposition 3.1. *(Carlstrom and Fuerst, 1998a and 1998b)*[2] *A flexible prices equilibrium allocation with an interest rate path $\{R_t\}$, such that $R_t > 1$, is feasible and implementable under sticky prices. If $R_t = 1$, then the flexible prices allocation is one in a large set of allocations that satisfy the feasibility and implementability restrictions, (2.21) and (3.2).*

Proof: This is straightforward since for an interest rate path $\{R_t\}$, the implementability condition under sticky prices, (3.2), is exactly the expected value of the constraints (2.20) of the flexible prices allocation. The feasibility constraints are the same in both problems.

If $R_t > 1$, the cash in advance constraints bind and money supply can be used to determine the flexible prices allocation.■

The optimal allocations, under sticky and flexible prices, are compared in terms of welfare in the following corollary:

Corollary 3.2. *The optimal allocation under sticky prices will make the households at least as well off as under flexible prices.*

In the next section we compute the optimal allocation under sticky prices, and compare this to the optimal allocation under flexible prices. If the two allocations differ for the same optimally chosen interest rate path, that means that sticky prices allow the planner to choose an allocation that is strictly better than the optimal allocation under flexible prices.

3.1. Ramsey problem with sticky prices

The Ramsey problem with sticky prices is the choice of sequences of consumption, C_t, labor, N_t, and interest rates R_t, that maximize (2.1) subject to the feasibility constraint (2.21), the implementability condition (3.2) and the condition that gross nominal interest rates are greater than or equal to one

$$R_t \geq 1$$

[2] This has been shown in Carlstrom and Fuerst (1998a), even though the emphasis is on policies where this possibility of targeting both the nominal interest rates and the price level is excluded. See also Adao, Correia and Teles (1999).

The Lagrangian for the Ramsey problem under sticky prices can be written as

$$\begin{aligned}\mathfrak{L} &= \sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t \Pr(s^t)u\left(C_t,1-N_t,\varkappa_t\right)\\ &+\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\lambda_t(s^t)\Pr(s^t)(z_tF(N_t)-G_t-C_t)\\ &+\sum_{t=0}^{\infty}\sum_{s^{t-1}\in S^{t-1}}\beta^t\varphi_t(s^{t-1})\sum_{s^t\in S^t}\Pr(s^t)\left(\frac{u_C(t)}{\frac{\theta R_t'}{(\theta-1)}}\left(C_t+G_t\right)-\frac{F(t)}{F'(t)}u_{1-N}(t)\right)\end{aligned}$$

where $\beta^t\lambda_t(s^t)\Pr(s^t)$, $\beta^t\varphi_t(s^{t-1})\Pr(s^{t-1})$ are the multipliers of the resources constraint and the implementability condition, respectively, and $\Pr(s^t)$ are the probabilities of the shocks.

The formalization of the Ramsey problem in terms of the choice of the allocations and the nominal interest rates allows for a very simple representation of the set of possibilities and allows for the determination of the optimal policy in two stages. In one stage we determine the optimal allocation for a given path of the nominal interest rates. In the other stage, the indirect utility function, which is a function of the path for the nominal interest rate, is maximized. We start by analyzing the choice of the interest rate policy.

3.2. The choice of the optimal interest rate policy

In this environment, it is necessary to follow the Friedman rule in order to achieve the optimal allocation under sticky prices, as stated in the following proposition:

Proposition 3.3. *The interest rate, R_t, that allows to achieve the optimal allocation when prices are set one period in advance, is equal to one.*

Proof:

A marginal increase in the nominal interest rate has a negative impact on utility, which is given by

$$-\beta^t\varphi_t(s^{t-1})\Pr(s^t)\frac{u_C\left(t\right)}{\frac{\theta R_t'^2}{(\theta-1)}}\frac{1}{v_t}\left(C_t+G_t\right). \tag{3.3}$$

Given the non-negativity constraint on the net nominal interest rate and the fact that in this second best environment φ_t is always strictly positive, it is optimal to set $R_t'=R_t=1$.■

Therefore, the optimal interest rate path under flexible prices coincides with the optimal interest rate path under the sticky prices environment. As we restricted the fiscal instruments of the governments to lump sum taxes and transfers, we do not need to rely on distortionary taxes to finance government expenditures. Also, the distortion that results from monopolistic competition cannot be eliminated by fiscal instruments and the nominal interest rate affects directly the level of the mark up. Because of that, $R = 1$ is optimal in both the flexible and the sticky prices environments, as zero is the lower bound on the interest rate level.

When the nominal interest rate is zero, the level of real balances is indeterminate as the cash in advance constraint is nonbinding. Under flexible prices there is a unique equilibrium allocation, as the desire of agents to spend cash is matched with fluctuations in the current price level. However, in a sticky prices environment sunspot equilibria have real consequences. The fact that output is demand determined and agents in the economy can vary the amount of cash they spend in response to some sunspot event leads to real indeterminacies (see Carlstrom and Fuerst, 1998b). This indeterminacy is no longer there for positive but arbitrarily small interest rates. It seems to us that all that is economically relevant is the behavior of these economies as the interest rates converge to zero.[3] Along this sequence the optimal allocations can be achieved by using money supply.

3.3. Optimal policy for a given interest rate path

We will pursue the analysis of the optimal policy assuming that the interest rates $R_t \geq 1$ are exogenous. Given this interest rate path we choose the allocation that maximizes the utility subject to the feasibility constraint and the implementability condition. The first order conditions of the Ramsey problem, for a given interest rate path, are

$$u_C(t) - \lambda_t^s + \varphi_t \left\{ \frac{u_{C,C}(t)}{\frac{\theta R_t'}{(\theta-1)}} Y_t + \frac{u_C(t)}{\frac{\theta R_t'}{(\theta-1)}} - \frac{F(t)}{F'(t)} u_{1-N,C}(t) \right\} = 0$$

$$-u_{1-N}(t) + \lambda_t^s z_t F'(t) + \varphi_t \left\{ -\frac{u_{C,1-N}(t)}{\frac{\theta R_t'}{(\theta-1)}} Y_t - \frac{F'(t)^2 - F''(t)F(t)}{F'(t)^2} u_{1-N}(t) + \frac{F(t)}{F'(t)} u_{1-N,1-N}(t) \right\} = 0$$

[3] We thank Robert Lucas and V.V. Chari for discussions on this issue. The approach we follow in this paper is normative. We do not model the game played by the government and the private agents. If we were to do that the Friedman rule would be the policy in the unique subgame perfect equilibrium.

Manipulating these first order conditions, we obtain

$$\varphi_t = \frac{\frac{z_t F'(t) u_C(t)}{u_{1-N}(t)} - 1}{\frac{z_t F'(t) u_C(t)}{u_{1-N}(t)} \frac{(\theta-1)}{\theta R'_t} \left(-\frac{\frac{Y_t}{C_t} u_{C,C}(t) C_t}{u_C(t)} - 1 + \frac{u_{C,1-N}(t)\varpi(t)}{u_C(t)} \right) + \varpi'(t) - \frac{\varpi(t) u_{1-N,1-N}(t)}{u_{1-N}(t)} + \frac{\frac{Y_t}{C_t} u_{1-N,C}(t) C_t}{u_{1-N}(t)}} \tag{3.4}$$

where

$$\varpi(t) = \frac{F(t)}{F'(t)} \text{ and } \varpi'(t) = \frac{F'(t)^2 - F''(t)F(t)}{F'(t)^2}$$

The set of equations described by (3.4) says that the marginal utility of the distortion has to be be the same across states and equal to φ_t.

As we want to characterize the optimal allocations by analyzing under which conditions the optimal solution under sticky prices coincides with the allocation under flexible prices, we will determine under which conditions the value of the right hand side of (3.4) for the allocation of flexible prices is the same across states.

Lemma 3.4. *If R'_t is constant across states, the value of the right hand side of (3.4) is constant across states for the flexible prices allocation if and only if the expression*

$$D_t \equiv -\frac{u_{C,C}(t) C_t}{u_C(t)} \frac{Y_t}{C_t} - 1 + \frac{u_{C,1-N}(t)}{u_C(t)} \varpi(t) + \varpi'(t) - \frac{u_{1-N,1-N}(t)}{u_{1-N}(t)} \varpi(t) + \frac{u_{1-N,C}(t) C_t}{u_{1-N}(t)} \frac{Y_t}{C_t} \tag{3.5}$$

does not depend on s_t.

Proof: Under flexible prices, $\frac{u_C(t) z_t F'(t)}{u_{1-N}(t)} = \frac{\theta}{\theta-1} R'_t$, so that the right hand side of (3.4) becomes

$$\frac{\frac{\theta}{\theta-1} R'_t - 1}{-\frac{u_{C,C}(t) C_t}{u_C(t)} \frac{Y_t}{C_t} - 1 + \frac{u_{C,1-N}(t)}{u_C(t)} \varpi(t) + \varpi'(t) - \frac{u_{1-N,1-N}(t)}{u_{1-N}(t)} \varpi(t) + \frac{u_{1-N,C}(t) C_t}{u_{1-N}(t)} \frac{Y_t}{C_t}}. \tag{3.6}$$

If R'_t is not state dependent then as long as D_t is not state dependent, the expression of the right hand side of (3.4) is not state dependent for the flexible prices allocation.■

Therefore, we need to determine the conditions under which D_t is constant across states for the flexible prices allocation.

Proposition 3.5. *If* $v_t = v$, $G_t = 0$, *and* $\varkappa_t = \varkappa$, *the flexible prices allocation is the optimal allocation for a constant interest rate path when*

i) Preferences are described by monotonic transformations of

$$u = \frac{C_t^{1-\sigma}}{1-\sigma}\mathcal{F}(1-N_t), \mathcal{F}' > 0, \sigma \geq 0,\ \sigma \neq 1.$$

These preferences are such that labor is constant across states in the flexible prices allocation; and when

ii) Preferences are described by monotonic transformations of

$$u = \frac{C_t^{1-\sigma}}{1-\sigma} - \alpha N_t^{\psi},\ \sigma \geq 0,\ \sigma \neq 1,\ \psi \geq 1,$$

and the technology is

$$F(t) = \gamma N_t^{\varrho},\ \gamma > 0,\ \varrho \leq 1.$$

Proof: If $v_t = v$ then R_t' is constant for a constant interest rate path. If $G_t = 0$, then $\frac{Y_t}{C_t} = 1$.

i) For these preferences the flexible prices allocation, which satisfies $\frac{u_C(t)}{u_{1-N}(t)} = \frac{\theta}{(\theta-1)}\frac{R_t'}{z_t F'(t)}$, can be written as

$$\frac{\mathcal{H}(1-N_t)}{C_t\mathcal{H}'(1-N_t)} = \frac{\mathcal{H}(1-N_t)}{z_t F(N_t)\mathcal{H}'(1-N_t)} = \frac{\theta}{(\theta-1)}\frac{R_t'}{z_t F'(N_t)}$$

where $\mathcal{H}(1-N_t) = (\mathcal{F}(1-N_t))^{\frac{1}{1-\sigma}}$.

Then

$$\frac{\mathcal{H}(1-N_t)}{F(N_t)\mathcal{H}'(1-N_t)} = \frac{\theta}{(\theta-1)}\frac{R_t'}{F'(N_t)}$$

and labor allocations under flexible prices are state independent. As D_t is invariant to monotonic transformations of the momentary utility (see Appendix) it is easy to check that D_t of $u = \log C_t + \log(\mathcal{H}(1-N_t))$ is given by

$$D_t \equiv \varpi'(t) - \frac{\varpi(t)\, u_{1-N,1-N}(t)}{u_{1-N}(t)} = \varpi'(t) - \varpi(t)\left(\frac{\mathcal{H}''(t)}{\mathcal{H}'(t)} - \frac{\mathcal{H}'(t)}{\mathcal{H}(t)}\right).$$

Since D_t is a function of N only, it is state independent for the flexible prices allocation. Lemma 3.4 implies that the optimal solution is the flexible prices allocation.

ii) When preferences are given by $u = \frac{C_t^{1-\sigma}}{1-\sigma} - \alpha N_t^{\psi}$, the flexible prices allocation is given by

$$\frac{(z_t F(N_t))^{-\sigma}}{\alpha \psi N_t^{\psi-1}} = \frac{\theta}{(\theta-1)} \frac{R_t'}{z_t F'(N_t)}$$

Since $\sigma \neq 1$, labor in the flexible prices allocation depends on the technological shock. The expression for D_t is now given by

$$D_t = \sigma - 1 + \varpi'(t) + \frac{(\psi-1)}{N_t} \varpi(t) = \sigma - 1 + \frac{\psi}{\varrho}$$

and is therefore state independent. The flexible prices allocation also satisfies in this case the first order condition of the Ramsey problem. ■

When the cost of holding money, R', is constant across states, the constant mark up of the flexible prices allocation implies that the proportionate wedge between the marginal rate of substitution and the marginal rate of transformation is constant across states. Proposition 3.5 identifies conditions under which the smoothing of distortions across states corresponds to the smoothing of proportionate wedges across states.

The classes of preferences i) and ii) in Proposition 3.5 include preferences commonly used in macroeconomics. Class i) includes preferences that are aggregable and consistent with balanced growth,

$$u = \frac{(C_t(1-N_t)^{\psi})^{1-\sigma} - 1}{1-\sigma}, \ \sigma > 0,$$

and

$$u = \log C_t + \alpha(1 - N_t),$$

which are the preferences assumed in Ireland (1996).

Class ii) includes

$$u = \frac{C_t^{1-\sigma}}{1-\sigma} - \alpha N_t, \ \sigma \geq 0, \ \sigma \neq 1,$$

and

$$u_t = \frac{\left(C_t - \zeta N_t^{\psi}\right)^{1-\sigma} - 1}{1-\sigma}, \ \sigma > 0, \ \psi > 1, \ \zeta > 0$$

which are the preferences assumed in Greenwood, Hercowitz and Huffman (1988).

There are preferences (for $v_t = v$, $G_t = 0$, and $\varkappa_t = \varkappa$) that do not satisfy the constancy of D_t at the flexible prices allocation. One simple example is $u = \alpha C_t + (1 - N_t)^{\frac{1}{2}}$. For a linear technology, $D_t = \frac{1}{2(1-N_t)}$. Since in the flexible prices allocation N_t depends on the state, the right hand side of (3.4) (for the flexible prices allocation) is state dependent and therefore the sticky prices optimal allocation achieves higher utility than the flexible prices one. For the parametrization of two equally probable i.i.d. shocks, $S = \{1, 2\}$, with $\alpha = 2$, $\theta = 2$, $F' = 1$ and $R_t = 1$, the flexible prices allocation is given by $C_t = 1.875$ in the good state and $C_t = .75$ in the bad state. The pair of consumptions, 1.845 in the good state and .834 in the bad state, satisfies the conditions of the Ramsey problem and gives a higher expected utility to the representative agent. In this particular case the social planner prefers a smoother consumption to the flexible prices consumption allocation.

If there are velocity shocks the result of optimality of the flexible prices allocation can still be obtained, as stated by the following proposition:

Proposition 3.6. *Let $v_t \neq v$, $G_t = 0$, $\varkappa_t = \varkappa$, and $R_t = 1$. The flexible prices allocation is the optimal allocation for the preferences and technology described in Proposition 3.5. Also, for $v_t \neq v$, $G_t = 0$, $\varkappa_t = \varkappa$, and $R_t > 1$, the flexible prices allocation is the optimal allocation for those preferences and technology, if the interest rate changes with the velocity shocks so that R_t' is state independent.*

Proof: When $R_t = 1$ the economy is insulated from velocity shocks and therefore $R_t = R_t'$. When $R_t > 1$, to maintain a constant cost of holding money the nominal interest rate has to react to the velocity shocks so that R_t' is constant. In both cases the conditions on preferences and technology in Proposition 3.5 are sufficient to guarantee that the right hand side of (3.4) is state independent, and the flexible prices allocation is the optimal solution.■

The following proposition states that for multiplicative preference shocks, the result of optimality of the flexible prices allocation still holds.

Proposition 3.7. *Let $G_t = 0$, $\varkappa_t \neq \varkappa$, and $u\left(C_t, 1 - N_t, \varkappa_t\right) \equiv \varkappa_t u(C_t, 1 - N_t)$. The flexible prices allocation is the optimal allocation under sticky prices for the preferences and technology described in Proposition 3.5.*

Proof: It is immediate to see that D_t does not depend directly on $\varkappa_t$. Therefore D_t is constant for the preferences and technology described in (ii) of Proposition 3.5. Also in this case the flexible price allocation does not depend on $\varkappa_t$, and therefore N_t is still constant across states for the set of preferences described by i) in Proposition 3.5.

In the following proposition and corollary we state the results when there are shocks to government expenditures:

Proposition 3.8. *Let the path of $\{R_t'\}$ be constant across states, and the same under flexible and sticky prices. If $\frac{G_t}{z_t F(N_t)}$ is state dependent, the optimal allocation under sticky prices provides higher utility than the allocation under flexible prices, for the classes of preferences and technologies described in Proposition 3.5.*

Proof: If the cost of holding money, $\{R_t'\}$, is constant across states, then the optimality of the flexible prices allocation implies that D_t is constant across states. It is easy to verify that D_t is not constant when $\frac{G_t}{z_t F(N_t)}$ varies across states for the flexible prices labor allocation. For preferences i), N_t in the flexible prices allocation is now state dependent and D_t depends on N_t. For preferences ii), D_t depends on $\frac{G_t}{z_t F(N_t)}$ and since this ratio is state dependent D_t is also state dependent. Therefore if $G_t \neq 0$, in general the optimal solution does not coincide with the flexible prices allocation. Since the flexible prices allocation is feasible and implementable under sticky prices, it must be the case that the optimal policy under sticky prices dominates, in welfare terms, the optimal policy under flexible prices.■

Since in order to achieve the optimal allocation the interest rates must be constant, $R_t' = R_t = 1$, the corollary follows directly.

Corollary 3.9. *If $\frac{G_t}{z_t F(N_t)}$ is state dependent, the optimal allocation under sticky prices provides higher utility than the optimal allocation under flexible prices, for the classes of preferences and technologies described in Proposition 3.5.*

When the cost of holding money $\{R_t'\}$ is constant and high enough, then the optimal allocation under sticky prices can be attained under flexible prices with a different path for $\{R_t'\}$. This no longer the case for interest rates at the lower bound, $R_t' = R_t = 1$. If the optimal allocation under sticky prices requires variable social mark ups, this can only be achieved under sticky prices.

Corollary 3.9 implies that, in general, the optimal sticky prices allocation dominates in terms of welfare the optimal allocation under flexible prices. However, because the optimal allocation under sticky prices can only be achieved at the Friedman rule, the optimal allocation is only one in a large set of feasible allocations that includes the flexible prices allocation. Still it is possible to claim that sticky prices allow the planner to achieve higher utility than flexible prices. This is stated in the following proposition:

Proposition 3.10. *Sticky prices allow the government to improve upon the optimal allocation under flexible prices.*

Proof: Let us consider the optimal allocation under flexible prices, where the monetary policy is the Friedman rule and let the preferences, technology and shocks be such that the optimal allocation under sticky prices differs from the one under flexible prices (conditions of Corollary 3.9). At the Friedman rule, for a given money supply policy the sticky prices allocation is indeterminate. However, as the mapping from nominal interest rates into equilibrium allocations is continuous, there is a sufficiently small interest rate such that $R_t - 1 > 0$ and the optimal allocation under sticky prices provides higher utility than the optimal flexible prices allocation (at the Friedman rule). For strictly positive interest rates, the allocation can be determined by using the money supply.■

Our results resemble the ones obtained by Zhu (1992) when studying the optimal financing of the government with labor and capital taxes. There, the optimal taxation of labor depends mainly on the level of employment, and when labor changes over time, (outside the balanced growth path) optimal labor taxation changes over time. In our set up, for preferences consistent with balanced growth, when the flexible prices allocation is characterized by an employment level that reacts to shocks, the objective of smoothing off the distortions across states is not attained by smoothing off the proportionate wedges across states. The optimal allocation is associated with social mark ups that vary with the state. The additional power of money in the sticky prices environment allows for the decentralization of the allocations associated with these state dependent mark ups. The lower bound on the interest rate implies that the optimal state dependent social mark ups cannot be decentralized in the flexible prices environment.

4. Concluding Remarks

We analyze a simple environment with short run non neutrality of money and determine principles for the conduct of monetary policy as stabilization policy. The environment consists of an underlying real business cycles model without capital to which we add three sources of distortions: monopolistic competition; a cash-in-advance restriction on households' transactions; and a restriction on firms that prices must be set in advance.

In the RBC economy with money and flexible prices, as in Cooley and Hansen (1989), monetary policy affects the allocation solely through the path of nominal interest rates. There is a large set of money supply policies consistent with that path of nominal interest rates, and characterized by different paths for the price levels. If prices are set in advance, there is also a large set of money supply policies consistent with the given path for the nominal interest rates. However, those different money supply policies are associated with different real allocations, that in general deviate from the allocation under flexible prices. These deviations from flexible prices, for a particular money supply policy under sticky prices, are our definition of gaps.

The same friction that creates the gaps in the economy provides the government with an additional policy tool, namely, a short run money supply policy, that can be conducted separately from the nominal interest rate policy. Interestingly enough, the government can always undo the distortions (close the gaps) created by the price stickiness using this additional policy instrument. It is even more interesting though, that under many circumstances, a benevolent government will not want to close those gaps, meaning that it will achieve a better allocation, from a welfare point of view, than the flexible prices allocation.

We use the methodological device of taking as given a path for the nominal interest rates, and show how, under sticky prices, the additional policy instrument ought to be used by a welfare maximizer government. This way we are able to compare the economy under flexible prices that is distorted by an interest rate path, and the same economy under sticky prices, with the same interest rates, but where the money supply policy can be used to replicate the flexible prices allocation or, possibly, to achieve a better allocation. In a second stage, we determine the optimal interest rate policy and compare the optimal allocation under sticky prices to the optimal allocation under flexible prices.

The first major result of the paper is that the Friedman rule must be followed in order to achieve the optimal allocation. This is a general result and thus extends the result in Ireland (1996), that shows that it is optimal to follow the Friedman rule when the utility function is separable, linear in leisure and logarithmic in consumption. A second major result of our paper is that the optimal allocation under sticky prices is in general different from the one under flexible prices, and that it is also better. Because of the zero bound, the optimal allocation cannot be achieved under flexible prices.

If there were only technological shocks in this economy, and the nominal interest rates were constant across states, then, for classes of preferences commonly used in macroeconomics, it would be optimal to replicate the flexible prices allocation, eliminating the distortions arising from sticky prices. Under the optimal interest rate policy, i.e. under the Friedman rule, the only remaining distortion would be the constant mark up resulting from monopolistic competition. These is a not general results, though. As soon as we consider other preference or production structures, and allow for shocks to government expenditures, that result vanishes. If there are only shocks to velocity, these are ineffective at the Friedman rule, so that the optimal allocation is the one under flexible prices. If the production function exhibits decreasing returns to labor, then in order for the result of optimality of the flexible prices allocation to hold further restrictions may have to be imposed on the production function. If there are shocks to government expenditures, it is no longer the case that the flexible prices allocation is optimal. In general it is optimal to set varying social mark ups across states.

The varying mark ups that characterize the optimal allocation under sticky prices cannot be achieved under flexible prices, since they would imply negative interest rates. However, that allocation is only one in a set of possible equilibrium allocations. The fact that at the Friedman rule, the cash in advance constraint is not binding implies that it is not possible to use money supply to uniquely pin down one particular allocation. Still, it is true that under sticky prices it is possible to achieve higher utility than under flexible prices. The argument is a continuity argument. There is a small interest rate, constant across states, for which the optimal allocation under sticky prices provides higher utility than the optimal allocation under flexible prices. For a strictly positive interest rate path, the path of the money supply uniquely determines the allocation.

If the government was able to subsidize production, and raise lump sum taxes for that purpose, then it would be possible to eliminate the effects of the three restrictions to the standard RBC environment. The optimal monetary policy would set nominal interest rates to zero, eliminating the distortion arising from the requirement that transactions must use money, and money supply would be conducted so that prices would not have to react to contemporaneous information. The optimal allocation would be the RBC flexible prices allocation. This is the approach taken in a good part of the literature on optimal monetary policy.

In this paper we focus on optimal monetary policy as stabilization policy. A different approach to the one we take here would be to consider the monetary policy instruments as part of a larger set of fiscal instruments in the context of a full Ramsey problem. The natural assumption would be to consider that lump sum taxes would not be available and that consumption and income taxes could be used to affect the allocations. If no restrictions were imposed on those fiscal instruments, then our results suggest that the monetary instruments could be redundant. Still it would not be possible to achieve the first best allocation where all three restrictions are undone, but the second best could be achieved with fiscal instruments only and thus, trivially, the optimal allocations under sticky and flexible prices would coincide. The relevant policy issue would be one of assessing the flexibility with which the different policy instruments can react to contemporaneous shocks.

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5. Appendix A

We want to show that

$$D_t = -\frac{u_{C,C}(t)C_t}{u_C(t)}\frac{Y_t}{C_t} - 1 + \frac{u_{C,1-N}(t)}{u_C(t)}\varpi(t) + \varpi'(t) - \frac{u_{1-N,1-N}(t)}{u_{1-N}(t)}\varpi(t) + \frac{u_{C,1-N}(t)}{u_{1-N}(t)}C_t\frac{Y_t}{C_t}$$

is invariant to monotonic transformations F of the utility function u.

Let us define $V = F(u)$.

Then

$$\begin{aligned} V_C &= F'u_C \\ V_{CC} &= F''u_C^2 + F'u_{CC} \\ V_{C,1-N} &= F''u_C u_{1-N} + F'u_{C,1-N} \\ V_{1-N} &= F'u_{1-N} \\ V_{1-N,1-N} &= F''u_{1-N}^2 + F'u_{1-N,1-N} \end{aligned}$$

and

$$\begin{aligned} D^V = & -\frac{(F''u_C^2 + F'u_{CC})C\frac{Y}{C}}{F'u_C} + \frac{(F''u_C u_{1-N} + F'u_{C,1-N})\varpi}{F'u_C} + \varpi' - \\ & -\frac{(F''u_{1-N}^2 + F'u_{1-N,1-N})\varpi}{F'u_{1-N}} + \frac{(F''u_C u_{1-N} + F'u_{C,1-N})C\frac{Y}{C}}{F'u_{1-N}} \end{aligned}$$

or

$$D^V = D - \frac{F''u_C C\frac{Y}{C}}{F'} + \frac{F''u_{1-N}\varpi}{F'} - \frac{F''u_{1-N}\varpi}{F'} + \frac{F''u_C C\frac{Y}{C}}{F'} = D$$

6. Appendix B

In this appendix, we show that any solution of the Ramsey problem, with $R_t > 1$, can be decentralized by an appropriate monetary policy, which we also characterize.

Take an allocation $(C_t^*(s^t), N_t^*(s^t))$ that satisfies the problem of the social planner. There are price levels $P_t^*(s^t)$ and money balances $M_t^{s*}(s^t)$ that satisfy

$$P_t^*(s^t)C_t^*(s^t) = M_t^{s*}(s^t)v_t$$

and

$$\frac{u_C\left(C_t^*(s^t), N_t^*(s^t)\right)}{P_t^*(s^t)}\frac{R_t}{R_t'} = k(s^{t-1}), \text{ for } k \geq 0$$

for all s^t. For these prices and money supplies, the intertemporal first order condition of the households can be written as

$$\frac{u_C\left(C_t^*(s^t), N_t^*(s^t)\right)}{P_t^*(s^t)} = \beta R_t' \frac{u_C(C_{t+1}^*(s^{t+1}), N_{t+1}^*(s^{t+1}))}{P_{t+1}^*(s^{t+1})}\frac{R_{t+1}}{R_{t+1}'}.$$

i.e. without the conditional expectations operator.

Observe that this vector of prices and money supplies depend on the current state and therefore cannot be part of the sticky prices economy equilibrium. From this vector of nominal variables we construct another vector with predetermined prices which are part of the equilibrium. Let us call this new pair $P_t(s^{t-1})$ and $M_t^s(s^t)$. This pair is defined as,

$$\frac{P_t^*(s^t)\gamma(s^t)}{P_t(s^{t-1})} = 1, \ \gamma(s^t) \in \mathcal{R}^{++}, \text{ for all } s^t$$

and

$$P_t(s^{t-1})C_t^*(s^t) = M_t^s(s^t)v_t, \text{ for all } s^t \tag{6.1}$$

Observe that this price, $P_t(s^{t-1})$, is a price firms could choose at date $t-1$ since the allocation is consistent with the firms' first order condition (2.17).

Next, we compute the value for $\gamma(s^t)$. The intertemporal first order condition of households can be rewritten as

$$\frac{u_C\left(C_t^*(s^t), N_t^*(s^t)\right)}{\gamma(s^t)P_t^*(s^t)} = \beta R_t' \sum_{s^{t+1}|s^t \in S^{t+1}} \Pr(s^{t+1}|s^t)\frac{u_C(C_{t+1}^*(s^{t+1}), N_{t+1}^*(s^{t+1}))}{\gamma(s^{t+1})P_{t+1}^*(s^{t+1})}\frac{R_{t+1}}{R_{t+1}'},$$

or

$$\frac{u_C\left(C_t^*(s^t),N_t^*(s^t)\right)}{\gamma(s^t)P_t^*(s^t)}=\beta R_t'\frac{u_C(C_{t+1}^*(s^{t+1}),N_{t+1}^*(s^{t+1}))}{P_{t+1}^*(s^{t+1})}\frac{R_{t+1}}{R_{t+1}'}\sum_{s^{t+1}|s^t\in S^{t+1}}\frac{\Pr(s^{t+1}|s^t)}{\gamma(s^{t+1})}.$$

This implies that,

$$\gamma(s^t)=\left[\sum_{s^{t+1}|s^t\in S^{t+1}}\frac{\Pr(s^{t+1}|s^t)}{\gamma(s^{t+1})}\right]^{-1},\text{ for all }s^t$$

and that the price vector

$$P_t(s^{t-1})=P_t^*(s^t)\left[\sum_{s^{t+1}|s^t\in S^{t+1}}\frac{\Pr(s^{t+1}|s^t)}{\gamma(s^{t+1})}\right]^{-1},\text{ for all }s^t$$

or

$$P_{t+1}(s^t)=P_t(s^{t-1})P_t^*(s^t)^{-1}\left[\sum_{s^{t+1}|s^t\in S^{t+1}}P_{t+1}^*(s^{t+1})\Pr(s^{t+1}|s^t)\right],\text{ for all }s^t.$$

There is freedom in the choice of the first price level in the economy, and thus we may choose $P_0=P_0^*$, for instance. Given the initial price level, the remaining price levels are obtained according to the equation above.

The monetary policy that implements the allocation $(C_t^*(s^t),N_t^*(s^t))$ is given by (6.1). Additionally we describe the money growth rate associated with the optimal allocation. From the equation above we get

$$\frac{P_{t+1}(s^t)}{P_t(s^{t-1})}=\frac{\left[\sum_{s^{t+1}|s^t\in S^{t+1}}P_{t+1}^*(s^{t+1})\Pr(s^{t+1}|s^t)\right]}{P_t^*(s^t)},$$

using the cash-in-advance conditions we can rewrite it as

$$\frac{v_{t+1}M_{t+1}^s(s^{t+1})}{v_tM_t^s(s^t)}=\frac{C_{t+1}^*(s^{t+1})}{C_t^*(s^t)}\frac{\left[\sum_{s^{t+1}|s^t\in S^t}P_{t+1}^*(s^{t+1})\Pr(s^{t+1}|s^t)\right]}{P_t^*(s^t)},$$

or as

$$\frac{v_{t+1}M_{t+1}^s(s^{t+1})}{v_tM_t^s(s^t)}=\frac{C_{t+1}^*(s^{t+1})}{C_t^*(s^t)}\left[\beta R_t\sum_{s^{t+1}|s^t\in S^t}\Pr(s^{t+1}|s^t)\frac{u_C(C_{t+1}^*(s^{t+1}),N_{t+1}^*(s^{t+1}))}{u_C\left(C_t^*(s^t),N_t^*(s^t)\right)}\frac{R_{t+1}}{R_{t+1}'}\right].$$

The value of $M_0^s=P_0^*C_0^*$, and M_t^s for $t>0$ given by the equation above.