DISCUSSION PAPER SERIES

No. 2627

PERSONAL REDISTRIBUTION AND THE REGIONAL ALLOCATION OF PUBLIC INVESTMENT

Ramón Caminal

PUBLIC POLICY

Centre for Economic Policy Research

www.cepr.org

ISSN 0265-8003

PERSONAL REDISTRIBUTION AND THE REGIONAL ALLOCATION OF PUBLIC INVESTMENT

Ramón Caminal, Institut d'Anàlisi Econòmica, CSIC, Barcelona, and CEPR

Discussion Paper No. 2627
November 2000

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 2627

November 2000

ABSTRACT

Personal Redistribution and the Regional Allocation of Public Investment*

Should the geographic allocation of public investment aim at reducing regional inequalities or should it exclusively be concerned with the maximization of aggregate output? This paper studies the potential role of public investment in reducing personal welfare inequality, in combination with distortionary taxation. The case for public investment as a significant redistribution device seems weak. If the tax code is constrained to be uniform across regions then it is optimal to distort the allocation of public investment in favour of the poor regions, but only to a limited extent. The reason for this is that poor individuals are relatively more sensitive to public transfers, which are maximized by allocating public investment efficiently. If the tax code can vary across regions then the result is ambiguous and it might even be the case that the optimal policy involves an allocation of public investment distorted in favour of the rich regions.

JEL Classification: D31, H41
Keywords: public investment, redistribution, regional policy

Ramón Caminal
Institut d'Anàlisi Econòmica, CSIC
Universitat Autonoma de Barcelona
Bellaterra
E-08193 Barcelona
SPAIN
Tel: (34 93) 580 6612
Fax: (34 93) 580 1452
Email: caminal@cc.uab.es

*Partial financial support from DGCYT Grants PB95-0130 and PB98-0695, and from the project 'Políticas autonómicas y equilibrio territorial' (Generalitat de Catalunya and Fundación BBVA) is gratefully acknowledged.

Submitted 04 October 2000

NON-TECHNICAL SUMMARY

In most countries governments pursue active policies that aim at changing the relative economic conditions of certain regions. The range of these ‘regional policies’ is very wide but usually involves the allocation of public capital (particularly, infrastructure). The motivation behind these policies is likely to be complex, but very often they are justified on equity grounds: governments seem compelled to ‘reduce regional differences in income per capita’ or, similarly, to ‘help poorer regions to catch up’. In the case of Spain, de la Fuente (1996) has studied the determinants of the regional allocation of public investment and concluded that policy-makers have deviated from the efficient allocation (equalization of marginal returns), and have contributed to reducing regional differences in income per capita.

Clearly, concerns over income distribution are legitimate but, nevertheless, it is not obvious why governments should pay any attention at all to the regional distribution of income. Usually equity concerns in a political union refer exclusively to the distribution of welfare over individuals (or households). Nevertheless, regions with lower than average income per capita tend to host a larger proportion of low productivity individuals (low income-generating capacities). In this case, the Government may be able to improve the economic opportunities of these individuals by reallocating public investment from rich to poor regions.

Distorting the allocation of public investment may not, however, be the most efficient way of reducing interpersonal welfare differences. The merits of such a device must be compared with those of other standard mechanisms. The goal of this paper is to study precisely the potential role of the regional allocation of public capital in reducing individual welfare inequality when a particular tax-transfer scheme is also available.

The Paper builds a very specific and simple model with the aim of highlighting the crucial trade-off. The main ingredients of the analysis are the following. First, public capital is modelled as an input of the aggregate production function that enhances individual productivity. In particular, it is assumed that an increase in the stock of public capital in a certain region multiplies the productivity of all the residents by the same factor. Thus, in absolute terms more productive individuals benefit more than less productive ones. Secondly, the proportional tax on income system is considered and a uniform transfer to each individual (these policy parameters may or may not differ across regions). Taxes are assumed to be distortionary and hence the optimal policy balances the gains from redistribution and the efficiency costs.

In most of the Paper the analysis has a normative nature: the decision-maker is endowed with a social welfare function, which is assumed to exhibit constant elasticity of substitution over individual utilities. Hence, the intensity of the equity motive is exclusively reflected in the parameter measuring the elasticity of substitution.

The main results can be summarized as follows. First, a higher degree of redistribution is associated with a lower provision of public capital. The reason is that a more egalitarian social planner puts more weight on the welfare of the low productivity individuals, which are the ones that benefit less from public investment. These individuals prefer higher transfers and less public capital than the average individual. In addition, more redistribution involves higher tax rates and thus a larger dead-weight loss (the marginal cost of public investment increases with the level of redistribution).

Secondly, if tax rates and lump-sum transfers are constrained to be uniform across regions then the optimal policy involves an allocation of public investment distorted in favour of the poor region. The contribution of public investment to individual equity can, however, only be modest. The reason is that there exist two countervailing effects. If the efficient allocation of public investment is distorted in favour of the poor region then, on the one hand, for a fixed transfer, the distribution of personal income becomes more equitable but, on the other hand, the tax base shrinks, which implies that transfers must be cut down, hurting particularly those with a low income-generating capacities.

The balance between these two effects induces a non-monotonic relationship between the distortion in the regional allocation of public investment and the intensity of the equity motive. At one extreme of the parameter space, if the social planner does not care about equity (if the social welfare function exhibits infinite elasticity of substitution) then the optimal fiscal policy includes an efficient allocation of public investment. At the other extreme, if the social planner only cares about the welfare of the poorest (the social welfare function exhibits zero elasticity of substitution), again, the optimal policy includes an efficient allocation of public investment, although with lower levels of aggregate investment. For intermediate cases, however, the optimal allocation of public capital is such that the marginal product in the rich region is higher than in the poor region (public investment is redistributed in favour of the poor region). In other words, in order to reach a certain level of personal redistribution it is efficient to use a combination of both distortionary taxes and a distorted allocation of public capital.

Thirdly, if the tax code can vary across regions, then it might be the case that the optimal policy distorts the efficient allocation of public investment in favour of the rich regions! The reason for such a counterintuitive result is that the residents of the poor regions benefit from a higher lump-sum transfer, which has to be partially compensated by a less favourable allocation of public capital.

Finally, if we let citizens choose between an efficient rule for allocating public investment (equalization of marginal returns) and an egalitarian rule (equalization of levels of public capital) then the former would be supported by the residents of the rich regions and by the low productivity individuals of the poor regions.

1. Introduction

In most countries regional productivity differences are quantitatively significant. Governments may be tempted to reduce these differences, in particular, through the allocation of public capital. In doing so, policy-makers may be motivated by efficiency considerations. For example, persistent regional inequalities may induce massive migration flows which may cause negative externalities to both the destination as well as the origin regions. Also, policy-makers may be concerned about equity. However, in this case it is not clear why one should take the region as the functional unit of analysis. In other words, including the reduction of regional inequality as an additional policy goal requires some justification.

Suppose society cares only about the distribution of welfare over individuals (or households). Since regions typically have different distributions of individual characteristics the geographic allocation of public investment could still be a useful device to reduce interpersonal inequalities. However, under this approach, we must compare the costs and benefits of such an instrument to those of alternative devices; in particular, personal taxes and transfers. The goal of this paper is precisely to study the potential role of the regional allocation of public capital in reducing individual welfare inequality, when a particular tax-transfer scheme is also available.

The main ingredients of the analysis are the following. First, I model public capital as an input of the aggregate production function that enhances individual productivities. In particular, I assume that an increase in the stock of public capital in a certain region multiplies the productivity of all the residents by the same factor. Thus, in absolute terms more productive individuals benefit more than less productive ones. Second, the government has access to an alternative mechanism to influence the personal welfare distribution. In particular, I consider a proportional tax on income and a uniform transfer to each individual (these policy parameters may or may not differ across regions). Taxes are assumed to be distortionary and hence the optimal policy balances the gains from redistribution and the efficiency costs. I also make various simplifying assumptions to keep the analysis tractable, like considering only two periods and two regions of equal size. In most of the paper the analysis has a normative nature: the decision maker is endowed with a social welfare function, which is assumed to exhibit constant elasticity of substitution over individual utilities. Hence, the intensity of the equity motive is exclusively reflected in the parameter measuring the elasticity of substitution.

I pose three questions. Two of them concern the optimal policy. First, how is the aggregate level of public investment affected by the degree of interpersonal redistribution? The second question is the main focus of the paper: should the regional allocation of public investment be distorted in favor of the poor region? The third question is about the political determinants of fiscal policy. In particular, under majority voting, what kind of coalitions can be formed in support of different regional allocation rules ?

The main results can be summarized as follows. First, a higher degree of redistribution is associated with a lower provision of public capital. The reason is that a more egalitarian social planner puts more weight on the welfare of the low productivity individuals, which are the ones that benefit less from public investment. These individuals prefer higher transfers and less public capital than the average. In addition, more redistribution involves higher tax rates and thus a larger dead-weight loss (the marginal cost of public investment increases with the level of redistribution).

The answer to the second question depends crucially on the characteristics of the tax code. If tax rates and lump-sum transfers are uniform across regions then, under mild conditions on the distributions of personal characteristics, the optimal policy is such that the marginal return of public investment in the rich region is higher than in the poor; that is, the allocation of public investment is distorted in favor of the poor region.[1] Thus, the allocation of public investment across regions can help the tax-transfer mechanism in improving the distribution of personal welfare. However, the contribution of public investment to individual equity can only be modest. The reason is that more egalitarian policy makers, on the one hand, are more willing to distort the regional allocation of public investment in order to reduce interregional productivity differences and achieve a more equitable income distribution; but, on the other hand, they prefer larger public transfers, since low productivity individuals only get a low direct benefit from public investment and their welfare is relatively more sensitive to public transfers. It turns out that public transfers can be increased by allocating public investment more efficiently, which expands the tax base and increases revenue.

The balance between these two effects induces a non-monotonic relationship between the distortion in the regional allocation of public investment and the intensity of the equity motive. At one extreme of the parameter space, when the social welfare function

[1] Redistribution in favor of the poor region does not necessarily imply that the latter obtains more investment funds (relative to the size of the region). In fact, if the equity motive is sufficiently strong or sufficiently weak, then the rich region gets more funds than the poor region.

exhibits infinite elasticity of substitution, the goal is to maximize average utility (utilitarian social planner), which implies the maximization of the present value of output and hence an efficient allocation of public investment. At the other extreme, the social welfare function exhibits zero elasticity of substitution and hence the goal is to maximize the welfare of the least productive individuals (Rawlsian social planner). If the pre-tax income of the least productive individuals is unaffected by public investment (like in the case of retired or severely handicapped people), then the optimal allocation of public investment is also efficient.[2] The reason is that fiscal policy only influences the utility of the least productive individuals through the lump-sum transfer. As a result, the optimal policy maximizes tax revenue, which implies again the maximization of second period output. However, for intermediate cases, the optimal allocation of public capital is such that the marginal product in the rich region is higher than in the poor region (public investment is redistributed in favor of the poor region). Therefore, there is a non-monotonic relationship between the intensity of the redistribution motive and the efficiency of the regional allocation of public capital.

Such a distortion in the allocation of public capital can also be explained as follows. If public capital is efficiently allocated then, for a given amount of public investment, total output is maximized but personal income inequality is exacerbated (there are more highly productive individuals in the rich region, whose income is boosted by public investment). Hence, an efficient allocation of investment will require more redistribution through the tax-transfer system, which involves an efficiency cost (the dead-weight loss associated to distortionary taxation). Instead, if one unit of public capital is redistributed from the rich to the poor region, the negative effect on total output is of second order magnitude, but the associated reduction in distortionary taxation has a first order effect. In other words, in order to reach a certain level of personal redistribution it is efficient to use a combination of both distortionary taxes and a distorted allocation of public capital.

The limited role of public investment as a redistribution device seems to depend crucially on the assumption of a uniform tax code across regions. If the tax code can also vary across regions then results change dramatically. In fact, in the particular example I work out in Section 6 the optimal policy tends to distort the efficient allocation of public investment in favor of the rich region! The reason is that

[2] If fiscal policy has a positive but small effect on the pre-tax income of the least productive individuals then a Rawlsian social planner also distorts the allocation of public investment in favor of the poor region although such a distortion is also small.

individuals of the poor region benefit from a higher lump-sum transfer which has to be partially compensated by a less favorable allocation of public capital.

Third, switching to the political economy question, suppose that citizens must choose between an efficient rule for allocating public investment (equalization of marginal returns) and an egalitarian rule (equalization of levels of public capital). An efficient rule would be supported by the residents of the rich region and by the low productivity individuals of the poor region. The intuition is exactly the same as above. The pre-tax income of those individuals with low productivities is almost unaffected by the allocation of public investment. Hence, they prefer an allocation rule that maximizes the tax base and thus the transfers they receive from the government.

After the seminal paper by Aschauer (1989), the literature on the effect of public capital accumulation on the productivity of private inputs has grown considerably.[3] Despite of some methodological difficulties there is sufficient evidence of the positive contribution of at least some categories of public capital (like infrastructures).

With respect to the regional dimension most of the empirical studies have focused on the measurement of regional spillovers.[4] On the equity-efficiency trade-off, De la Fuente (1996) has studied for the case of Spain the determinants of the regional allocation of public investment.[5] The evidence suggests that policy makers have deviated form the efficient allocation, and have aimed at reducing regional differences in income per capita.

A strand of the literature has extensively analyzed the normative and positive implications of productive public spending in dynamic general equilibrium models[6], but to the best of my knowledge, the relationship between the regional allocation of public investment and interpersonal redistribution has not been explored. In very different frameworks Michel et al. (1983) and Takahashi (1998) have taken a normative approach and studied the optimal allocation of public spending, although they abstract from intraregional income inequality. Similarly, Persson and Tabellini (1994b), Lookwood (1996) and Cheikbossian (1997) present political economy models to study the regional allocation of public spending, but again without looking at intraregional

---

[3] See, for instance, the early survey by Gramlich (1994).

[4] See Hulten and Schwab (1997) for a discussion and references.

[5] See also De la Fuente and Vives (1995).

[6] See, for instance, Barro (1990), Baxter and King (1993), and Turnovsky and Fischer (1995).

income inequality.[7] Finally, Persson and Tabellini (1996a and b) study the design of fiscal federations as optimal risk sharing arrangements. They focus on inter-regional transfers and worry about personal redistribution, but they do not consider productive public expenditure.

In the next section I present the baseline model. As a preliminary exercise in Section 3 I characterize the optimal policy for the single region case. The main result for the two region case is presented in Section 4. Sections 5 and 6 deal with robustness issues and Section 7 worries about alternative characterizations of both public spending and aggregation of individual preferences. Finally, some concluding remarks close the paper.

2. The baseline model

Consider a two-region and two-period economy. Regions are indexed by i, i = A, B, and periods by t, t = 1, 2. Each region is populated by a continuum of agents of equal mass. Individuals are heterogeneous and characterized by a parameter θ, which can be interpreted as an index of their productivity. Individual productivities are distributed in region i according to the density function $h^i(\theta)$, which takes strictly positive values in the interval $\left[0, \bar{\theta}\right]$. The cumulative distribution function is denoted by $H^i(\theta)$. A resident of region i, endowed with parameter θ, enjoys an income equal to θ in the first period and $\theta\ f(g^i)$ in the second, where g_i is the first period public investment in region i, and f is an increasing and concave function, which also satisfies the Inada conditions. Thus, public investment increases individual productivity by the same relative amount. Hence, those agents with a higher value of θ benefit more in absolute terms. The government, as well as private agents, have access to a perfect bond market at an exogenous interest rate, r.

Such a set up can be interpreted as the reduced form of a small open economy model with perfect capital mobility, walrasian markets, and where individuals are endowed

[7] In fact, Persson and Tabellini (1994b) consider two alternative models. In one of them, there are no intraregional productivity differences, and the regional allocation of public spending is determined as the outcome of a lobbying game. In the other model, individual characteristics vary within each region and fiscal policy is concerned about redistribution. However, fiscal policy consists of linear taxes and a lump-sum transfer; moreover, the central government is constrained to set the same transfer in both regions.

with different amounts of labor (in efficiency units) and public capital is an input of the production function.[8]

Let $\hat{\theta}^j$ and $\hat{\theta}$ be the average of θ in region j and in the entire economy, respectively, i.e., $\hat{\theta} = \dfrac{\hat{\theta}^A + \hat{\theta}^B}{2}$. Regions have different average productivities. I denote region A as the relatively richer region, i.e., $\hat{\theta}^A > \hat{\theta}^B$. Also, throughout the paper I maintain the following hypothesis:

A.1) Let $\eta(\theta) \equiv \dfrac{h^A(\theta)}{\hat{\theta}^A} - \dfrac{h^B(\theta)}{\hat{\theta}^B}$. There exist a $\theta_0 \in (0, \bar{\theta})$ such that $\eta(\theta) < 0$ for all $\theta \in [0, \theta_0]$, and $\eta(\theta) > 0$ for all $\theta \in [\theta_0, \bar{\theta}]$.

This is a sufficient condition for some of the results although it is not necessary. It literally says that the two density functions weighted by their own averages cross only once, and that there are sufficiently more low productivity individuals in region B than in region A.

Private agents derive utility only from consumption. Since the interest rate is exogenous, individual welfare depends exclusively on the present value of disposable income and hence we do not need to consider explicitly the consumer's optimization problem.

Tax policy

The government taxes income in both regions at the same rate τ_t. Tax revenue can be either distributed as a lump-sum transfer to individuals (independently of their region), T,[9] or can be used to finance public investment, $g_A + g_B$. If we denote by $u^i(\theta)$ the utility of an agent located in region i and endowed with parameter θ, and we let β be the discount factor, $\beta \equiv \dfrac{1}{1 + r}$, we can write:

$$u^i(\theta) = (1 - \tau_1)\,\theta + T + \beta\,(1 - \tau_2)\,f(g^i)\,\theta \tag{2.1}$$

[8] Thus, the underlying aggregate production function of region i is of the following type: $Y_i = F(g_i, L_i) = f(g_i)L_i$, where L_i denotes aggregate labor in efficiency units. Considering private capital would considerably complicate the analysis unless we make trivial assumptions about the joint distribution of labor endowments and holdings of private capital.

[9] Alternatively, in Section 4 we consider the case that the tax code can be region specific.

I assume (but I do not model explicitly) that taxation is distortionary. This is captured by assuming that the marginal revenue of an extra unit of income is $\tau_t - \frac{(\tau_t)^2}{2}$, that is there is a dead-weight loss of $\frac{(\tau_t)^2}{2}$ times the tax base.[10] Then, the government's budget constraint can be written as:

$$\left(\tau_1 - \frac{(\tau_1)^2}{2}\right)\hat{\theta} + \beta\left(\tau_2 - \frac{(\tau_2)^2}{2}\right)\frac{\hat{\theta}^A f(g^A) + \hat{\theta}^B f(g^B)}{2} \geq T + \frac{g^A + g^B}{2} \quad (2.2)$$

The assumption of perfect capital markets and exogenus interest rates implies that the timing of transfers is irrelevant. Thus, T denotes the present value of transfers. However, since taxes are distortionary the timing of revenues does matter.

If taxation were non-distortionary then the marginal revenue of an extra unit of income would be just τ_t, and the government's budget constraint would become:

$$\tau_1 \hat{\theta} + \beta \tau_2 \frac{\hat{\theta}^A f(g^A) + \hat{\theta}^B f(g^B)}{2} \geq T + \frac{g^A + g^B}{2} \quad (2.3)$$

Social welfare

The government is endowed with a social welfare function with constant elasticity of substitution, $\frac{1}{\sigma}$, $\sigma \geq 0$:

$$W = \frac{1}{1-\sigma}\left\{\int u^A(\theta)^{1-\sigma} dH^A(\theta) + \int u^B(\theta)^{1-\sigma} dH^A(\theta)\right\} \quad (2.4)$$

A value of $\sigma = 0$ implies that the government has linear preferences with respect to individual payoffs (utilitarian). In the limit as σ goes to ∞, social welfare depends only on the lowest individual utility (Rawlsian). In general, a higher value of σ reflects a higher preference for equity.

---

[10] Such an specification is very convenient and rather common in the literature (See, for instance, Bolton and Roland, 1997). All qualitative results would be identical in case we model explicitly the distortions associated to taxation. In the current formulation, the maximum of the Laffer curve is reached at τ =1. Again, this is not substantial. Also, throughout the paper I assume that potential tax revenue is sufficient to finance the first best level of public investment.

3. Preliminaries: The single region case

Let us consider the case that both tax codes and government spending have to be uniform across regions, which is equivalent to the single region case. Thus, individuals are exclusively identified by their productivity parameter θ, and their utility is given by:

$$u(\theta) = (1 - \tau_1)\,\theta + T + \beta\,(1 - \tau_2)\, f(g)\,\theta \tag{3.1}$$

We can denote by $H(\theta)$ the overall distribution of productivities, i.e., $H(\theta) \equiv H^A(\theta) + H^B(\theta)$. Finally, we can write the social welfare function as:

$$W = \frac{1}{1-\sigma}\int u(\theta)^{1-\sigma} dH(\theta) \tag{3.2}$$

3.1. *Non-distortionary taxes*

With non-distortionary taxation, the budget constraint is analogous to equation (2.3):

$$\tau_1\,\hat{\theta} + \beta\,\tau_2\,\hat{\theta}\, f(g) \geq T + g \tag{3.3}$$

The optimal policy consists of choosing (τ_1, τ_2, T, g) in order to maximize (3.2) subject to (3.1) (3.3) and $T \geq 0$, and is characterized in the following proposition.

<u>Proposition 3.1</u>
With non-distortionary taxes, for all $\sigma > 0$, the optimal policy includes confiscatory tax rates, $\tau_1 = \tau_2 = 1$, and a level of public investment that maximizes the present value of aggregate consumption, i.e., $\beta\,\hat{\theta}\, f'(g) = 1$.

With non-distortionary taxes there is no trade-off between efficiency and redistribution. As a result, for all $\sigma > 0$, the optimal policy implies an egalitarian distribution of disposable income and the first best level of public investment. In other words, an efficient spending policy is compatible with complete income redistribution.

3.2. *Distortionary taxes*

In this case the government budget constraint is:

$$\left(\tau_1 - \frac{(\tau_1)^2}{2}\right)\hat{\theta} + \beta\left(\tau_2 - \frac{(\tau_2)^2}{2}\right)\hat{\theta}\, f(g) \geq T + g \qquad (3.4)$$

and results drastically change:

Proposition 3.2
With distortionary taxes, the optimal policy satisfies $\tau_1 = \tau_2 \equiv \tau$, $0 < \tau < 1$, and $\beta\,\hat{\theta}\, f'(g)\left(1 - \tau + \frac{\tau^2}{2}\right) = 1$. As σ increases τ also increases (and approaches 1 as σ goes to infinity) and thus g decreases.

In order to get some intuition on these results, let us characterize the most preferred fiscal policy of an individual with productivity θ. Provided the constraint $T \geq 0$ is not binding[11], the individual utility function can be written as:

$$u(\theta) = (1 - \tau)\left[1 + \beta f(g)\right]\theta + \left\{\left(\tau - \frac{\tau^2}{2}\right)\left[1 + \beta f(g)\right]\hat{\theta} - g\right\}$$

Thus, individual welfare depends on the sum of two terms: the present value of after-tax income and the present value of transfers. If we concentrate exclusively on the first term, then an increase in tax rates or a decrease in public investment both decrease the individual's payoff. It is important to notice that such an effect is proportional to the productivity parameter θ. Fiscal policy also affects individual welfare through the lump-sum transfer, but such an effect is the same for everyone. The preferred policy parameters of an individual with productivity θ balance these two effects and are given by:

$$\tau = 1 - \frac{\theta}{\hat{\theta}} \qquad (3.5)$$

$$\beta f'(g)\left[(1 - \tau)\theta + \left(\tau - \frac{\tau^2}{2}\right)\hat{\theta}\right] = 1 \qquad (3.6)$$

Thus, the preferred level of g and τ increase and decrease respectively with θ. On the one hand, a higher value of θ implies a lower demand for redistribution and thus a

[11] There exist a θ_0 below the average, such that for all $\theta < \theta_0$ the non-negativity constraint on T is not binding. The most preferred policy of all individuals with $\theta > \theta_0$ is the same policy than that of θ_0, which consists on a positive tax rate and an amount of public investment below the first best level.

lower tax rate. On the other hand, the direct effect of public investment increases with the level of individual productivity and thus an agent with low productivity prefers higher direct transfers and lower investment.

These remarks should help interpreting Proposition 3.2. As σ increases, the social planner puts more weight on the utility of low productivity individuals, and hence it chooses higher tax rates and lower levels of public investment.

The characteristics of the optimal policy can also be interpreted as follows. Distortionary taxation creates a trade-off between efficiency and redistribution. As a result, as σ increases the social planner is willing to pay a higher efficiency cost to achieve a more equitable distribution of individual utilities, which involves a higher tax rate and larger transfers. Public investment also decreases with σ for two complementary reasons. First, a higher value of σ implies higher tax rates and hence a higher marginal efficiency cost of tax revenue (financing an extra unit of public investment involves a larger dead-weight loss). Second, higher public investment exacerbates income inequality which requires more redistribution and hence higher tax rates and lump-sum transfers, which in turn involves a larger dead-weight loss.

With a different mechanism for aggregating individual preferences results would be analogous. Suppose for instance that fiscal policy is chosen by majority voting. By plugging (3.5) into (3.6) we get:

$$\beta\, f'(g) \left[\frac{\hat{\theta}}{2} + \frac{\theta^2}{2\,\hat{\theta}} \right] = 1 .$$

Thus, an individual with a productivity below the average will prefer inefficiently low levels of public investment. Thus, provided the distribution of productivities is skewed to the right (the median voter is below the average) there is a demand for redistribution and public investment would be inefficiently low. Moreover, as the median voter becomes less productive with respect to the average, the political support for public investment weakens. This result is analogous to those in Alesina and Rodrick (1994) and Persson and Tabellini (1994a), among others, in the sense that it predicts that those societies with a more equitable income distribution tend to invest more in output-enhancing activities.

4. Optimal policy in the two-region case

Let us now consider the two-region model. In the case of non-distortionary taxation the results are analogous to those in Proposition 3.1, i.e., for any $\sigma > 0$, the optimal policy consists of $\tau_1 = \tau_2 = 1$ and $\hat{\theta}^A f'(g^A) = \hat{\theta}^B f'(g^B) = \frac{1}{\beta}$. Thus, there is no conflict between redistribution and efficiency. The tax-transfer system takes care of redistribution and public investment responds exclusively to efficiency considerations. More precisely, the optimal policy consists of a level of public investment in each region that maximizes the present value of aggregate consumption, which implies the equalization of marginal returns.

In the case of distortionary taxation, the government's optimization problem consists of choosing $\left\{\tau_1, \tau_2, T, g^A, g^B\right\}$ in order to maximize (2.4) subject to (2.1), (2.2) and $T \geq 0$. Since the results on tax rates and aggregate government spending are analogous to the single region case (Proposition 3.2) here I focus on the regional allocation of public investment.

<u>Proposition 4.1</u>
If $\sigma = 0$ public investment is efficiently allocated across regions, i.e., $\hat{\theta}^A f'(g^A) = \hat{\theta}^B f'(g^B)$. For any σ, $0 < \sigma < \infty$, public investment is redistributed in favor of the poor region, i.e., $\hat{\theta}^A f'(g^A) > \hat{\theta}^B f'(g^B)$. Finally, in the limit as σ goes to ∞, public investment is also efficiently allocated across regions.

If $\sigma = 0$, fiscal policy is designed to maximize the present value of aggregate consumption. Thus, any amount of public investment must be efficiently allocated across regions, otherwise second period output could be increased by reallocating public investment across regions.

If σ goes to infinity, the social planner is exclusively concerned about the welfare of the poorest individuals, i.e., those with $\theta = 0$. Thus, since public investment has no direct effect on their welfare, fiscal policy aims at maximizing the lump-sum transfer, which implies again that public investment must be efficiently allocated.

However, for intermediate values of σ, public investment is used as a complementary redistribution device. The reason is that the tax and transfer system balances the gains from redistribution and the efficiency losses: As a result, a certain degree of welfare inequality remains. Moreover, if public investment is efficiently allocated across regions then interpersonal income inequalities are exacerbated, since individuals in the rich region benefit from a higher level of public investment. If the government reallocates one unit of investment from the rich to the poor region, the loss of output is only second order but it results in a more equitable income distribution, which allows for a reduction of tax rates, which has a first order effect on efficiency.

Summarizing, the extent to which the regional allocation of public investment must be used as a redistribution device is non-monotone with respect to the intensity of the equity motive (parametrized by σ). At the same time, these results suggest that the redistribution role of public investment is somewhat limited. The reason is that low productivity individuals in both regions have a strong preference for an efficient distribution of public investment, because it maximizes tax revenues and hence the lump-sum transfer. A numerical example can illustrate in more detail the characteristics of the optimal policy.

EXAMPLE

Consider the case in which θ can take two values: 0 and 1. The proportion of high productivity individuals ($\theta = 1$) in region i is μ^i, i.e., $\hat{\theta}^i = \mu^i$ Also take $f(g) = g^{\lambda}$. Figure 1 plots the optimal policy for the case $\lambda = 0.5$, $\mu^A = 0.7$, $\mu^B = 0.3$, $\beta = 1$. The functions that relate the policy parameters (g^i, τ_t, T) to the intensity of the equity motive, σ, have the expected shape. The redistribution role of public investment is captured by the variable Z:

$$Z = \frac{\hat{\theta}^A f'(g_A)}{\hat{\theta}^B f'(g_B)}.$$

Thus, depending on whether $Z = 1$, $Z > 1$, or $Z < 1$, the regional allocation of public investment is efficient, distorted in favor of the poor region, or distorted in favor of the rich region, respectively. In this particular example, as well as in all the parameter values considered[12], Z is higher than one, and has a bell shape, skewed to the right. It is interesting to notice that in this particular example, the ratio of average income in the two regions is as high as 2.333, and the maximum value of Z is 1.025. In other words, the average productivity of the rich region is 133% higher than the one of the poor region, but in the optimal policy the marginal return of public investment is only 2,5% higher in the rich region.

[12] See Appendix for details.

5. Alternative assumptions about income distribution

In this section I investigate whether Proposition 4.1 extends to alternative assumptions about regional differences in income distribution. In particular, I consider two alternative assumptions. First, I study the optimal policy when the direct effect of public investment on the income of the poorest individuals is strictly positive. Second, I consider the case in which most of the poorest individuals are located in the rich region, and hence assumption A.1 is violated.

5.1. *Strictly positive productivities*

Consider the case in which the support of θ is bounded away from zero, i.e. θ is distributed in the interval $\left[\underline{\theta},\ \bar{\theta}\right]$ with $\underline{\theta} > 0$, but sufficiently small. In this case public investment affects the utility of the poorest individuals not only through T but also by changing second period pre-tax income. In such a case the most preferred tax rate of the poorest individuals would be:

$$\tau_t = 1 - \frac{\underline{\theta}}{\hat{\bar{\theta}}} < 1,\ t = 1,2$$

In the limit as σ goes to infinity, the government cares only about the poorest individuals. Since optimal tax rates are strictly less than one, the allocation of public investment across regions will affect the welfare of the poorest individuals in each of the two regions. Under an efficient allocation of public investment the poorest individuals in the poor region would be strictly worse than those in the rich region. Hence, the government will distort the efficient allocation in favor of the poor region, although such a distortion is closely related to the size of $\underline{\theta}$ (there is no discontinuity at $\underline{\theta}$ = 0). More formally:

<u>Proposition 5.1</u>
If $\underline{\theta} > 0$, as σ goes to ∞ then public investment is redistributed in favor of the poor region, i.e., $\hat{\theta}^A f'(g^A) > \hat{\theta}^B f'(g^B)$. As $\underline{\theta}$ goes to zero the allocation becomes efficient.

Thus, Proposition 4.1 is robust to changes in the lower bound of the support of θ, although as $\underline{\theta}$ increases a Rawlsian government will increase the distortion in the allocation of public investment.

5.2. *The poorest individuals in the rich region*

If $\underline{\theta} > 0$ then it is easy to construct examples in which assumption A.1 is violated and the optimal policy redistributes public investment in favor of the region with the highest average productivity.

Consider the following discrete income distributions (D). The variable θ can take three values. ε, 1 and 2, with ε > 0, but very small. In region B all agents have θ = 1, and in region A a proportion λ have θ = 2 and a proportion (1 - λ) have θ = ε. Region A is still the rich one:

$$\hat{\theta}^A = (1 - \lambda)\, \varepsilon + 2\, \lambda > 1 = \hat{\theta}^B$$

but average productivity differences are assumed to be small. In this case I obtain the following result:

<u>Proposition 5.2</u>

In case of the income distributions (D), as σ goes to ∞ public investment is redistributed in favor of the rich region, i.e., $\hat{\theta}^A f'(g^A) < \hat{\theta}^B f'(g^B)$.

The intuition is analogous to that of Proposition 5.1. As σ goes to infinity, the social planner maximizes the welfare of the poorest group of citizens which happens to be located in region A (which has the highest average productivity). If public capital is efficiently allocated then the size of the transfer is maximized, but the poorest individuals in the rich region are worse off than the poorest in the poor region (since the optimal policy involves τ < 1). By shifting one unit of public capital from region B to region A, the effect on the transfer is only second order but it has a first order effect on the after tax income of the poorest group of agents.

6. Region-specific tax codes

In previous sections I have considered fiscal policies with a peculiar asymmetric characteristic: tax rates and lump-sum transfers were constraint to be equal across regions, and the only fiscal variable that was allowed to vary regionally was public investment. In this section we ask about the redistribution role of public investment when the government can set different tax codes in different regions.[13]

Suppose that the government has a larger set of instruments: $\left\{\tau_t^i,\ T^i,\ g^i\right\}$, t = 1, 2 and i = A, B. Its budget constraint becomes:

$$\sum_{i=A,B}\left(\tau_1^i - \frac{\left(\tau_1^i\right)^2}{2}\right)\hat{\theta}^i + \beta\left(\tau_2^i - \frac{\left(\tau_2^i\right)^2}{2}\right)\hat{\theta}^i f(g^i) \geq \sum_{i=A,B}\left(T^i + g^i\right) \qquad (6.1)$$

and individual utilities must be written as:

$$u^i(\theta) = \left(1 - \tau_1^i\right)\theta + T^i + \beta\,(1 - \tau_2^i)\, f(g^i)\,\theta \qquad (6.2)$$

The optimal policy is the solution to maximizing (2.4) subject to (6.1), (6.2), and $T^i \geq 0$. It turns out that Proposition 4.1 only holds for extreme values of σ:

<u>Proposition 6.1</u>
If σ = 0 and in the limit as σ goes to ∞, public investment is efficiently allocated across regions, i.e., $\hat{\theta}^A f'(g^A) = \hat{\theta}^B f'(g^B)$.

This result is a straight forward extension of Proposition 4.1[14] If $\sigma = 0$, the social planner is only concerned about efficiency and hence the regional allocation of public investment must also be efficient. In fact, an utilitarian social planner will set $T^A = T^B = 0$ (no redistribution) and $\tau^A = \tau^B$, in order to minimize the dead-weight loss of financing investment. As σ goes to infinity the social planner only cares about the utility of the poorest individuals, and hence tax revenue should be maximized, which implies that $\tau^A = \tau^B = 1$, and that public investment must be efficiently allocated across regions. Finally, proceeds must be equally distributed: $T^A = T^B$.

---

[13] One may also wonder about more general tax-transfer schemes, generalizing the linear scheme considered here. In particular, with more progressive taxation it looks like the poor may appropriate a higher proportion of the returns from public investment and thus the optimal policy may involve higher levels of public investment than under the linear scheme. However, a more general tax-transfer scheme would clearly require a more detailed analysis of the distortions caused by taxation.

[14] The optimal policy includes constant tax rates over time: $\tau_1^i = \tau_2^i = \tau^i$, i = A, B.

Proposition 6.1 is silent about the regional allocation of public investment for intermediate values of σ. In fact, by means of an example, it can be shown that in the optimal policy Z can be higher or lower than one. Let us consider again the example of Section 4 with discrete income distributions and an exponential production function. Figure 2 plots the optimal policy for the case $\lambda = 0.5$, $\mu^A = 0.7$, $\mu^B = 0.3$, $\beta = 1$. Except for very small values of σ, it turns out that $Z < 1$, that is public investment in general must be redistributed in favor of the rich region. The reason has to do with the fact that "regional redistribution" is essentially conducted through (T^B - T^A), which notice that is positive for all values of σ.

In other words, the main message of this section is that whenever the tax code can vary across regions it is not clear whether the social planner uses public investment as a redistribution device. It may be the case that in the optimal policy public investment is distorted in favor of the rich region to partially compensate the differential in the lump-sum transfers. In fact, for the above specification of the production function and income distributions, and for all the parameter values considered[15], the same pattern of Figure 2 is observed, i.e., as long as σ is not too small, then $Z < 1$.

7. Political support for alternative allocation rules

It would be interesting to study alternative public decision mechanisms, like majority voting. In our context this is quite complex, given the multidimensionality of the problem. In this subsection I do not attempt to provide a full analysis but only to give some intuition on the type of coalitions that can arise when determining the regional allocation of public investment.

Suppose that the regional allocation of public investment must be chosen after all other items in the budget are already fixed, including total public investment. In this case there is no chance of forming cross-border coalitions since all individuals of region i prefer to maximize the share of public investment allocated to their own region. What do we expect to happen in such a situation? Different answers have been provided in the literature. For instance, Lookwood (1996) argues that region representatives may bargain efficiently over the distribution of the pie. In contrast, Persson and Tabellini (1994) model this situation as a lobbying game.

[15] See the Appendix for details.

Let us consider a different situation. Let us go back to the model outlined in Section 2 and suppose that tax rates are already set, but the level of transfers is still to be determined. How would people vote on the level and allocation of public investment? The payoff function of a voter is given by:

$$u^i(\theta) = \left(\tau_1 - \frac{(\tau_1)^2}{2}\right)\hat{\theta} + \beta\left[\left(\tau_2 - \frac{(\tau_2)^2}{2}\right)\frac{\hat{\theta}^i f(g^i) + \hat{\theta}^j f(g^j)}{2}\right] - \frac{g^i + g^j}{2} +$$

$$+ \left[(1 - \tau_1) + \beta(1 - \tau_2) f\left(g^i\right)\right]\theta \qquad i, j = A, B \qquad i \neq j$$

Hence the optimal level of (g_i, g_j) for such an individual are given by:

$$\left[\tau_2 - \frac{(\tau_2)^2}{2}\right]\frac{\hat{\theta}^i f\left(g^i\right)}{2} - \frac{1}{2\beta} + (1 - \tau_2) f'\left(g^i\right) = 0$$

$$\left[\tau_2 - \frac{(\tau_2)^2}{2}\right]\frac{\hat{\theta}^j f\left(g^j\right)}{2} - \frac{1}{2\beta} = 0$$

The first observation is that no voter wishes a 100% share of public investment for its region, since public investment in the other region expands the tax base and increases the transfer (fiscal spillovers). Second, an individual with $\theta = 0$ will prefer an efficient allocation of resources, $\hat{\theta}^A f'(g^A) = \hat{\theta}^B f'(g^B)$, since the direct effect of public investment is null and only cares about maximizing the transfer. Third, as θ increases voters prefer a more biased distribution of public investment in favor of their own region.

This suggest that cross-border coalitions can arise in determining the regional allocation of public investment. In order to fix ideas, suppose that voters can only choose between two alternative rules::

(i) efficiency rule: $\hat{\theta}^A f'(g^A) = \hat{\theta}^B f'(g^B)$,

(ii) egalitarian rule: $g^A = g^B$.

Then the following result obtains:

<u>Result 7.1</u>

The efficient rule is supported by a coalition of all region A's voters and region B's voters with θ in the non-degenerated interval $[0, \theta_0]$.

In words, an efficient allocation rule would be supported by the coalition of all the voters of the rich region plus the poorest segment of the poor region.

8. Concluding remarks

The analysis has been conducted in a highly simplified framework. Although the main insights are likely to survive as we modify the model in certain directions, some of the maintained hypothesis of the paper deserve further attention. Here, I will briefly comment on two of them: the lack of mobility across regions and the absence of regional spillovers.

In this paper, I have assumed that agents to do not move across regions, or at least they do not do it in response to fiscal policy. In practice, some agents do decide to migrate whenever taxes or the provision of public goods in their region become sufficiently unfavorable. Clearly, as more people is willing to move for a given regional fiscal gap, more difficult is for the government to discriminate across regions, either through the tax code or through the allocation of public investment. Also, for a government that is concerned about equity it becomes crucial how economic and non-economic incentives interact in migrants' decisions. Will the poor or the rich be more willing to move if by doing so they obtain the same proportional increase in their disposable income?

Although most public investment projects have a large local effect, in most cases their benefits reach citizens in other regions (and other countries). This is an important consideration which is at the core of the literature on fiscal federalism.[16] Clearly, it would also affect the policy design issue I have been discussing in this paper. With asymmetric regions, spillovers from public investment are also likely to be asymmetric. Consequently, the problem of allocating public investment across regions may drastically change depending on the size and direction of such asymmetric spillover effects.

These two issues are left for future research.

[16] See, again, Hulten and Schwab (1997).

References

Alesina, A. and D. Rodrick (1994), Distributive Politics and Economic Growth, *Quarterly Journal of Economics,* May, pp 465-490.

Aschauer, D (1989), Is public expenditure productive? *Journal of Monetary Economics* 23, 177-200.

Barro, R. (1990), Government Spending in a Simple Model of Endogenous Growth, *Journal of Political Economy* 98 (5), pp. 103-125.

Baxter, M and R. King (1993), Fiscal Policy in General Equilibrium, *American Economic Review* 83(3), pp. 315-334.

Bolton, P. and G. Roland (1997), The Break-up of Nations: A Political Economy Analysis, *Quarterly Journal of Economics*, pp. 1057-1090.

Cheikbossian, G. (1997), Fiscal Federalism, Political Equilibrium and Distributive Politics, DELTA, mimeo.

De la Fuente, A. (1996), Inversión pública y redistribucion regional: el caso de España en la década de los ochenta, *Papeles de Economía Española*, 67, pp.238-56.

De la Fuente, A. and X. Vives (1995), Infrastructure and education as instruments of regional policy: evidence from Spain, *Economic Policy* 20, pp. 13-51.

Gramlich, E.M. (1994), Infrastructures Investment: A Review Essay, *Journal of Economic Literature*, pp. 1176-1196.

Hulten, Ch. and R. Schwab (1997), A fiscal federalism approach to infrastructure policy, *Regional Science and Urban Economics* 27, 139-159.

Lookwood, B. (1996), Inter-Regional Insurance with Asymmetric Information, University of Exeter, mimeo.

Michel, P., P. Pestieau, and J.F. Thisse (1983), Regional Allocation of Investment with Distributive Objectives, *Journal of Regional Science* 23 (2), 199-209.

Persson, T. and G. Tabellini (1996a), Federal Fiscal Constitutions: Risk Sharing and Moral Hazard, *Econometrica* 64 (3), 623-646.

Persson, T. and G. Tabellini (1996b), Federal Fiscal Constitutions: Risk Sharing and Redistribution, *Journal of Political Economy* 104(5), 979-1009.

Persson, T. and G. Tabellini (1994a), In Inequality Harmful for Growth?, *American Economic Review* 84 (3), 600-621.

Persson, T. and G. Tabellini (1994b), Does centralization increase the size of the government?, *European Economic Review* 38, 765-773.

Takahashi, T. (1998), On the optimal policy of infrastructure provision across regions, *Regional Science and Urban Economics* 28, 213-235.

Turnovsky, S. and W. Fischer (1995), The Composition of Government Expenditure and its Consequences for Macroeconomic Performance, *Journal of Dynamics and Control* 19, 747-786.

Appendix

Proof of Proposition 3.1

The first order conditions of an interior solution are:

$$\frac{dW}{d\tau_1} = \frac{dW}{d\tau_2} = \int u(\theta)^{-\sigma}\left(-\theta + \hat{\theta}\right) dH(\theta) = 0$$

$$\frac{dW}{dg} = \int u(\theta)^{-\sigma}\left[\beta\left(1 - \tau_2\right) f'(g)\,\theta + \beta\,\tau\,\hat{\theta}\, f'(g) - 1\right] dH(\theta) = 0$$

The second order conditions are satisfied.

Suppose either τ_1 or τ_2 are strictly lower than 1. Then $u(\theta)^{-\sigma}$ is strictly decreasing with θ and hence $\frac{dW}{d\tau_t} < 0$. Therefore, $\tau_1 = \tau_2 = 1$, which implies that the optimal level of g is given by:

$$\beta\,\hat{\theta}\, f'(g) = 1 .$$

Finally, the budget constraint implies that:

$$T = \hat{\theta}\left[1 + \beta f(g)\right] - g > 0 . \qquad \text{QED}$$

Proof of Proposition 3.2

The first order conditions are (δ is the Lagrange multiplier associated to $T \geq 0$):

$$\int u(\theta)^{-\sigma}\left[-\theta + (1-\tau_1)\hat{\theta}\right] dH(\theta) = -\delta(1-\tau_1)\hat{\theta}$$

$$\int u(\theta)^{-\sigma}\left[-\theta + (1-\tau_2)\hat{\theta}\right] dH(\theta) = -\delta(1-\tau_2)\hat{\theta}$$

$$\int u(\theta)^{-\sigma}\left\{\beta(1-\tau_2)\theta f'(g) + \beta\left[\tau_2 - \frac{(\tau_2)^2}{2}\right]\hat{\theta} f'(g) - 1\right\} dH(\theta) =$$

$$= -\delta\left\{\beta\left[\tau_2 - \frac{(\tau_2)^2}{2}\right]\hat{\theta} f'(g) - 1\right\}$$

The second order conditions also hold. The first two equations imply that $\tau_1 = \tau_2 = \tau$. Taking this into account, and solving the first equation for δ and plugging into the last equation the system can be rewritten:

$$\int u(\theta)^{-\sigma}\left[-\theta + (1-\tau)\hat{\theta}\right] dH(\theta) = -\delta(1-\tau)\hat{\theta}$$

$$\beta\,\hat{\theta}\, f'(g)\left(1 - \tau + \frac{\tau^2}{2}\right) = 1$$

Totally differentiating the system:

$$\frac{d\tau}{d\sigma} > 0, \quad \frac{dg}{d\sigma} < 0.$$

In the limit as s goes to infinity τ goes to 1. In order to check this, the first order condition can be written as:

$$\int_0^{(1-\tau)\hat{\theta}} \frac{u(\theta)^{-\sigma}}{u\left[(1-\tau)\hat{\theta}\right]^{-\sigma}}\left[-\theta+(1-\tau)\hat{\theta}\right]dH(\theta)+$$

$$\int_{(1-\tau)\hat{\theta}}^{\hat{\theta}} \frac{u(\theta)^{-\sigma}}{u\left[(1-\tau)\hat{\theta}\right]^{-\sigma}}\left[-\theta+(1-\tau)\hat{\theta}\right]dH(\theta)=-\delta(1-\tau)\hat{\theta}$$

Suppose that $\tau < 1$ in the limit as σ goes to infinity. Then the left hand side of the equation goes to infinity. Therefore, $\tau = 1$. QED

Proof of Proposition 4.1

The first order conditions are:

$$\frac{dW}{d\tau_1}=\int u^A(\theta)^{-\sigma}\left[-\theta+(1-\tau_1)\hat{\theta}\right]dH^A(\theta)+\int u^B(\theta)^{-\sigma}\left[-\theta+(1-\tau_1)\hat{\theta}\right]dH^B(\theta)=$$

$$=-\delta(1-\tau_1)\hat{\theta}$$

$$\frac{dW}{dg_i}=\int u^i(\theta)^{-\sigma}\left\{\beta(1-\tau_2)\theta f'\left(g^i\right)+\beta\left[\tau_2-\frac{(\tau_2)^2}{2}\right]\frac{\hat{\theta}^i f'\left(g^i\right)}{2}-\frac{1}{2}\right\}dH^i(\theta)+$$

$$+\int u^j(\theta)^{-\sigma}\left(\beta\left[\tau_2-\frac{(\tau_2)^2}{2}\right]\frac{\hat{\theta}^i f'\left(g^i\right)}{2}-\frac{1}{2}\right)dH^j(\theta)=$$

$$=-\delta\left(\beta\left[\tau_2-\frac{(\tau_2)^2}{2}\right]\frac{\hat{\theta}^i f'\left(g^i\right)}{2}-\frac{1}{2}\right)$$

$$i, j = A, B, \; i \neq j$$

Consider first the case $\sigma = 0$. The first order conditions can be rewritten:

$$-2\tau_1 + \delta(1-\tau_1) = 0$$

$$\tau_1 = \tau_2 \equiv \tau$$

$$\beta\hat{\theta}^i \; f'\left(g^i\right)\left[1 - \tau + \frac{\tau^2}{2}\right]=1$$

Moreover, the non-negativity constraint is binding. Suppose not, i.e., $\delta = 0$. In this case $\tau = 0$, but since g_i are positive, the government budget constraint is violated. Hence, T = 0. Consider now the limit as σ goes to infinity. The first order condition with respect to τ_1 can be written as:

$$\int_0^{(1-\tau_1)\hat\theta}\left[\frac{u^A(\theta)}{u^A\left[(1-\tau_1)\hat\theta\right]}\right]^{-\sigma}\left[-\theta+(1-\tau_1)\hat\theta\right]dH^A(\theta)+$$

$$+\int_{(1-\tau_1)\hat\theta}^{0}\left[\frac{u^A(\theta)}{u^A\left[(1-\tau_1)\hat\theta\right]}\right]^{-\sigma}\left[-\theta+(1-\tau_1)\hat\theta\right]dH^A(\theta)+$$

$$+\left\{\frac{u^B\left[(1-\tau_1)\hat\theta\right]}{u^A\left[(1-\tau_1)\hat\theta\right]}\right\}^{-\sigma}\left\{\int_0^{(1-\tau_1)\hat\theta}\left[\frac{u^B(\theta)}{u^B\left[(1-\tau_1)\hat\theta\right]}\right]^{-\sigma}\left[-\theta+(1-\tau_1)\hat\theta\right]dH^B(\theta)+\right.$$

$$\left.+\int_{(1-\tau_1)\hat\theta}^{0}\left[\frac{u^B(\theta)}{u^B\left[(1-\tau_1)\hat\theta\right]}\right]^{-\sigma}\left[-\theta+(1-\tau_1)\hat\theta\right]dH^B(\theta)\right\}=-\frac{\delta(1-\tau_1)\hat\theta}{\left\{u^A\left[(1-\tau_1)\hat\theta\right]\right\}^{-\sigma}}$$

As σ goes to infinity, if $\tau_1 < 1$, the first and third integral of the left hand side go to infinity, and the second and the fourth go to zero. Also the factor multiplying the third and fourth integrals goes either to infinity or to zero. Thus, the left hand side is positive and equation does not hold. Hence, $\tau_1 = 1$. The argument to show that $\tau_2 = 1$ is analogous.

Suppose $\delta = 0$. The first order condition with respect to g_i imply that:

$$\hat\theta^i \; f'\left(g^i\right) = \frac{2}{\beta}$$

The government budget constraint becomes:

$$T = \frac{1}{2}\left[\hat\theta + \beta\frac{\hat\theta^A f\left(g^A\right)+\hat\theta^B f\left(g^B\right)}{2}\right]-\frac{g^A+g^B}{2}$$

Thus, $T > 0$.

Finally, let us consider the case $0 < \sigma < \infty$. By direct inspection of the first order conditions τ_1 and τ_2 are strictly less than 1. Next, I write the first order condition with respect to g_i:

$$2\;\beta\;(1-\tau_2)\;\hat\theta^i\;f'\left(g^i\right)\;\psi^i - \left[1-\beta\left(\tau_2-\frac{(\tau_2)^2}{2}\right)\hat\theta^i\;f'\left(g^i\right)\right]Z = 0$$

where

$$\psi^i \equiv \int u^i(\theta)^{-\sigma}\frac{\theta}{\hat\theta^i}\;dH^i\;(\theta)$$

$$Z \equiv \int u^A\left(\hat\theta\right)^{-\sigma}\;dH^A(\theta)+\int u^B(\theta)^{-\sigma}dH^B(\theta)+\delta$$

Notice that $\hat\theta^A \; f'\left(g^A\right)$ is higher, equal or lower than $\hat\theta^B \; f'\left(g^B\right)$ if and only if ψ^A is lower, equal or higher than ψ^B, respectively.

Suppose that $\hat{\theta}^A \ f'\left(g^A\right) \leq \hat{\theta}^B \ f'\left(g^B\right)$. Then $g^A > g^B$ and $u^A(\theta)^{-\sigma} < u^B(\theta)^{-\sigma}$ for all θ. Thus,

$$\psi^A \equiv \int u^A(\theta)^{-\sigma} \frac{\theta}{\hat{\theta}^A} dH^A(\theta) < \int u^B(\theta)^{-\sigma} \frac{\theta}{\hat{\theta}^A} dH^A(\theta) <$$

$$< \int u^B(\theta)^{-\sigma} \frac{\theta}{\hat{\theta}^B} dH^B(\theta) \equiv \psi^B$$

The last inequality comes from the fact that $u^B(\theta)^{-\sigma}$ is strictly decreasing and from Assumption A.2. Thus, we reach a contradiction and hence conclude that $\hat{\theta}^A \ f'\left(g^A\right) \geq \hat{\theta}^B \ f'\left(g^B\right)$. QED

<u>Proof of Proposition 5.1</u>

The proof follows most of the steps of the previous one. The first order conditions are obviously the same. Similarly, as σ goes to infinity then:

$$\tau_t = 1 - \frac{\underline{\theta}}{\hat{\theta}} < 1.$$

Hence, $u^i(\theta)$ strictly increases with θ. Next, let us consider the first order condition with respect to g_i. Consider the following notation:

$$\Omega^i \equiv \frac{\beta}{2}\left[\tau_2 - \frac{(\tau_2)^2}{2}\right] \hat{\theta}^i \ f'\left(g^i\right) - \frac{1}{2}.$$

$$\Phi^i(\theta) \equiv \beta \left(1 - \tau_2\right) \theta \ f'\left(g^i\right) + \Omega^i$$

Notice that $\Omega^i \leq 0$. The reason is that otherwise the left hand side is strictly positive and the right hand side is non-positive. Also, for all $\theta \geq \underline{\theta}$, $\Phi^i(\theta) \geq 0$, otherwise, we can divide the first order condition by $u^i(\theta_0)^{-\sigma}$, where θ_0 is given by $\Phi^i(\theta_0) = 0$,. As σ goes to infinity, the left hand side goes to minus infinity but the right hand side is non-negative.

Suppose that $u^A(\underline{\theta}) \leq u^B(\underline{\theta})$. This implies that $g^A \leq g^B$, and hence $\hat{\theta}^A \ f'\left(g^A\right) > \hat{\theta}^B \ f'\left(g^B\right)$. If we define θ_1 as given by $u^A(\theta_1) = u^B(\underline{\theta})$, then if we divide the first order condition with respect to g_A and g_B by $u^A(\theta_1)^{-\sigma}$ and take the limit as σ goes to infinity, then we must have:

$$\Omega^B = 0$$

$$\Phi^A(\underline{\theta}) \leq 0.$$

However, $\Phi^A(\underline{\theta}) > \Omega^A > \Omega^B = 0$. The first inequality comes from the differential in marginal returns and the second by definition. Hence, we reach a contradiction.

Suppose that $\hat{\theta}^A\ f'\left(g^A\right) \leq \hat{\theta}^B\ f'\left(g^B\right)$. Similarly, define θ_2 as given by $u^B(\theta_2) = u^A(\underline{\theta})$, then if we divide the first order condition with respect to g^A and g^B by $u^B(\theta_2)^{-\sigma}$ and take the limit as σ goes to infinity, then we must have:

$$\Omega^A = 0$$

$$\Phi^B(\underline{\theta}) \leq 0.$$

However, $\Phi^B(\underline{\theta}) > \Omega^A > \Omega^B = 0$. We reach a contradiction.

Finally, as $\underline{\theta}$ goes to zero, then $\Phi^B(\underline{\theta}) - \Omega^B$ goes to zero, and hence $\Omega^A - \Omega^B$ must also go to zero, which implies that $\hat{\theta}^A\ f'\left(g^A\right) = \hat{\theta}^B\ f'\left(g^B\right)$. QED

<u>Proof of Proposition 5.2</u>

Analogously to the proof of Proposition 5.1, it has to be the case that in the limit as σ goes to infinity:

$$\tau_1, \tau_2 < 1$$

$$\beta\left[\tau_2 - \frac{(\tau_2)^2}{2}\right]\hat{\theta}^i\ f'\left(g^i\right) < 1..$$

$$\beta\left(1 - \tau_2\right)\theta\ f'\left(g^i\right) + \beta\left[\tau_2 - \frac{(\tau_2)^2}{2}\right]\frac{\hat{\theta}^i\ f'\left(g^i\right)}{2} - \frac{1}{2} > 0$$

Suppose that $u^A(\varepsilon)^{-\sigma} < u^B(1)^{-\sigma}$. If we divide the first order condition with respect to g_A by $u^B(1)^{-\sigma}$, then the left hand side goes to minus infinity while the right hand side is non-negative. Similarly, if $u^A(\varepsilon)^{-\sigma} > u^B(1)^{-\sigma}$ we divide the first order condition with respect to g_B by $u^A(\varepsilon)^{-\sigma}$, then the left hand side goes to minus infinity while the right hand side is non-negative. Hence, $u^A(\varepsilon) = u^B(1)$, which implies that g^A is much higher than g^B. Since average productivity differences are relatively small, then $\hat{\theta}^A f'(g^A) < \hat{\theta}^B f'(g^B)$. QED

<u>Proof of Proposition 6.1</u>

If we denote the Lagrange multiplier associated with the constraint $T^i \geq 0$, by δ^i, the first order conditions are:

$$\tau_1^i = \tau_2^i \equiv \tau^i$$

$$\int u^i(\theta)^{-\sigma}\left[-\theta + \left(1 - \tau^i\right)\theta^i\right] dH^i(\theta) = -\delta^i\left(1 - \tau^i\right)\theta^i$$

$$\int u^A(\theta)^{-\sigma}\ dH^A(\theta) + \delta^A = \int u^B(\theta)^{-\sigma}\ dH^B(\theta) + \delta^B$$

$$\beta\ \hat{\theta}^i\ f'\left(g^i\right)\left(1 - \tau^i + \frac{\left(\tau^i\right)^2}{2}\right) = 1.$$

If $\sigma = 0$ and in the limit as σ goes to infinity we can follow the argument used in the proof of Prooposition 4.1 and conclude that the regional allocation of public investment must be efficent. QED

Example

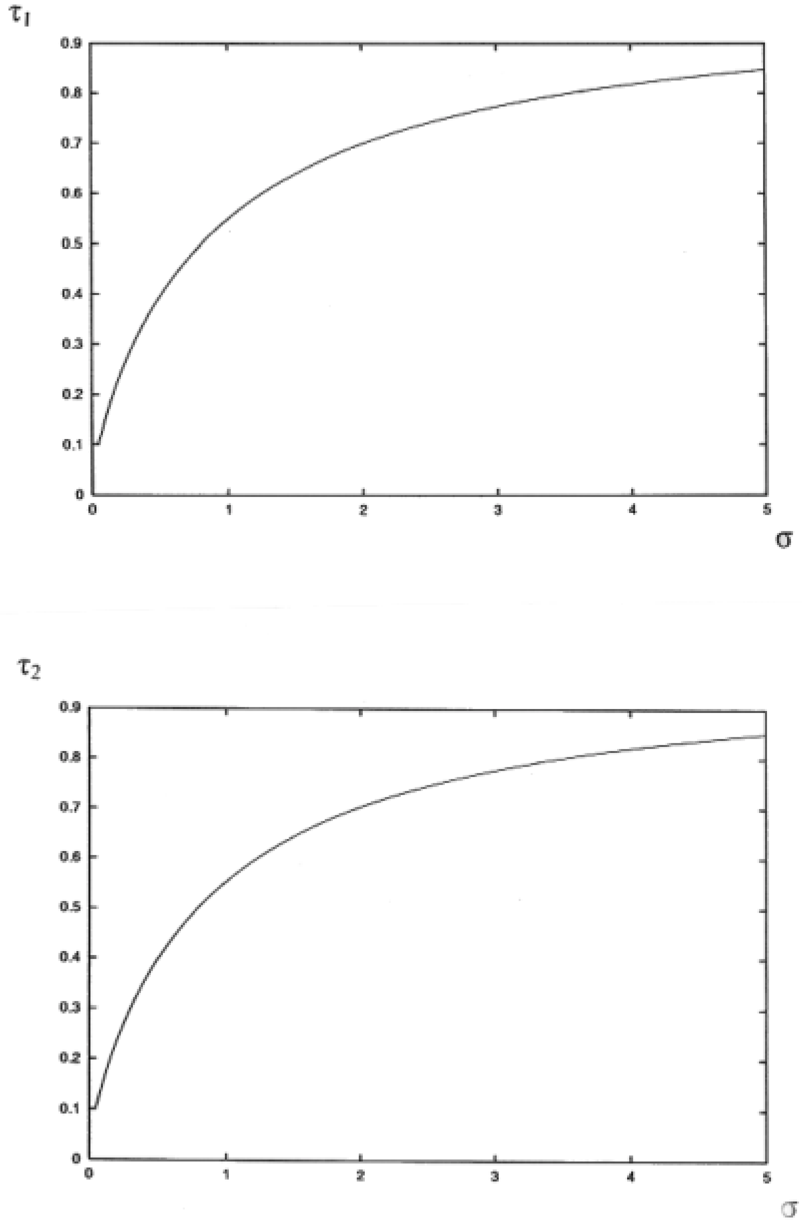
I have computed the optimal policy in the case θ can take two values: 0 and 1, with frequencies $(1 - \mu^j)$ and μ^j, respectively and $f(g) = g^{\lambda}$. I have considered the following combinations of parameter values:

$$\mu^A \in \{0.55,\ 0.6,\ 0.65,\ 0.7,\ 0.75,\ 0.8\}$$
$$\mu^B = 1 - \mu^A$$
$$\lambda \in \{0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8\} .$$
$$\beta \in \{0.5,\ 1\}$$

This implies that the total number of simulations has been 84. The fact that $\mu^A + \mu^B = 1$ is only a normalization. Also, as λ and μ^A approach 1 (i.e., the production function becomes linear and relative productivity differences explode) computations become much harder.

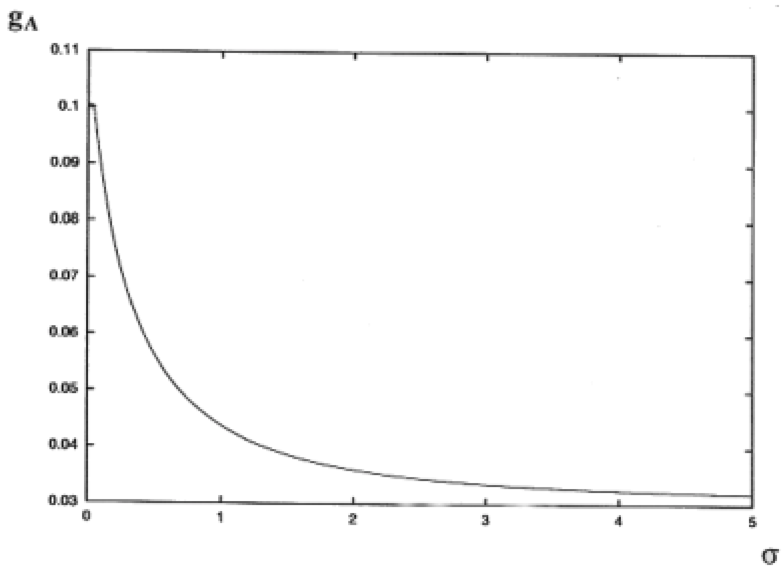
For all these parameter values I have computed the optimal policy with and without uniform tax codes. The same qualitative properties illustrated in Figures 1 and 2 are sistematically observed. In particular, in the case of region-specific tax codes $Z > 1$ provided σ is not too small.

Figure 1.a



τ_1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
σ
τ_2
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
σ

Figure 1.b


gA
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0
1
2
3
4
5
σ

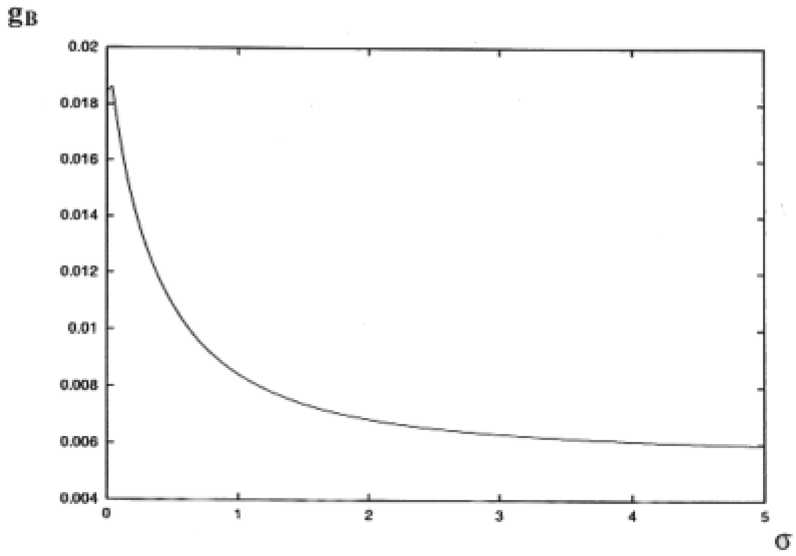
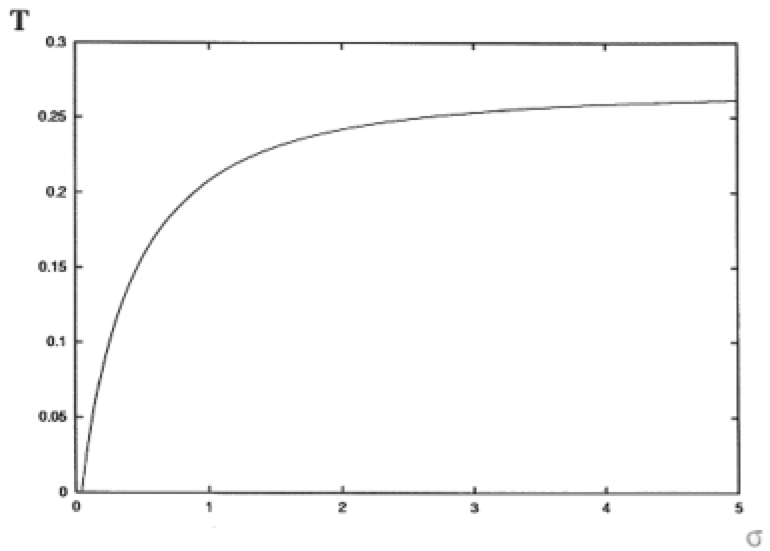

gB
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0
1
2
3
4
5
σ

Figure 1.c


T
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
σ

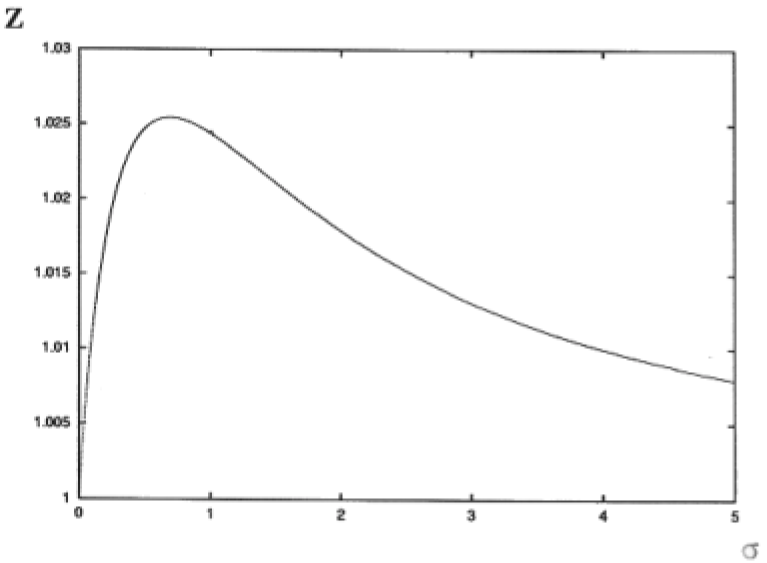
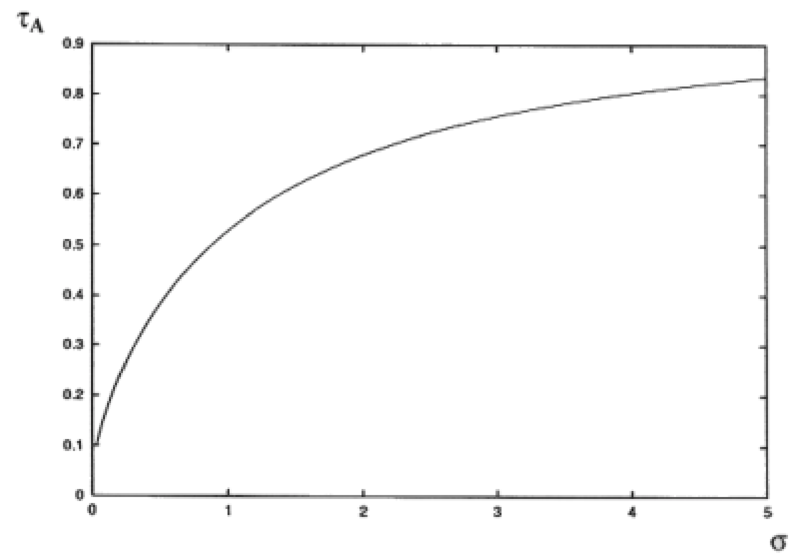
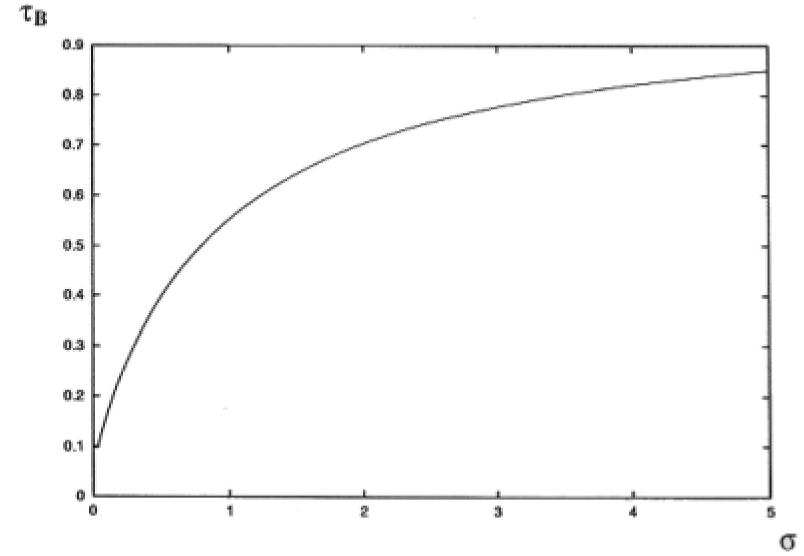

Z
1.03
1.025
1.02
1.015
1.01
1.005
1
0
1
2
3
4
5
σ

Figure 2.a

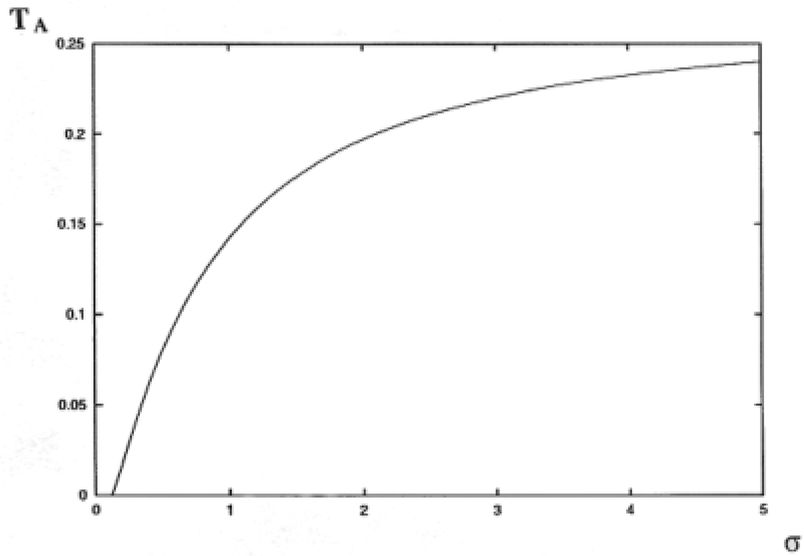


τ_A
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
σ



τ_B
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
σ

Figure 2.b



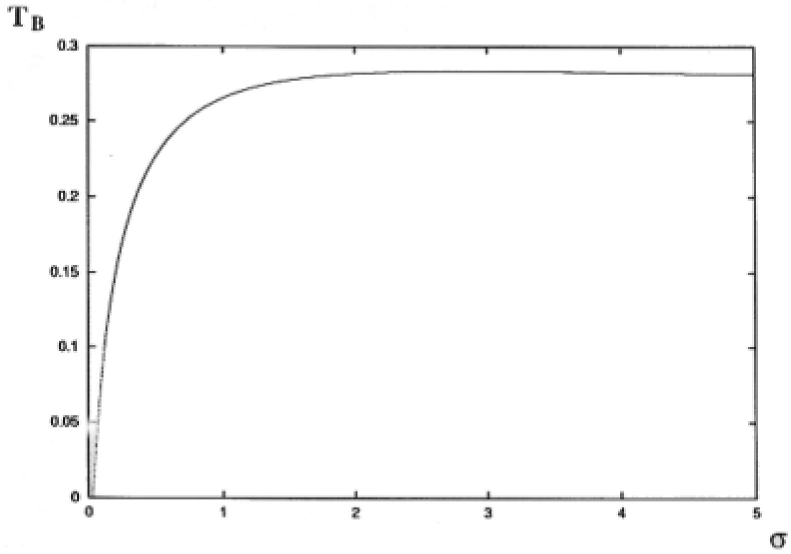
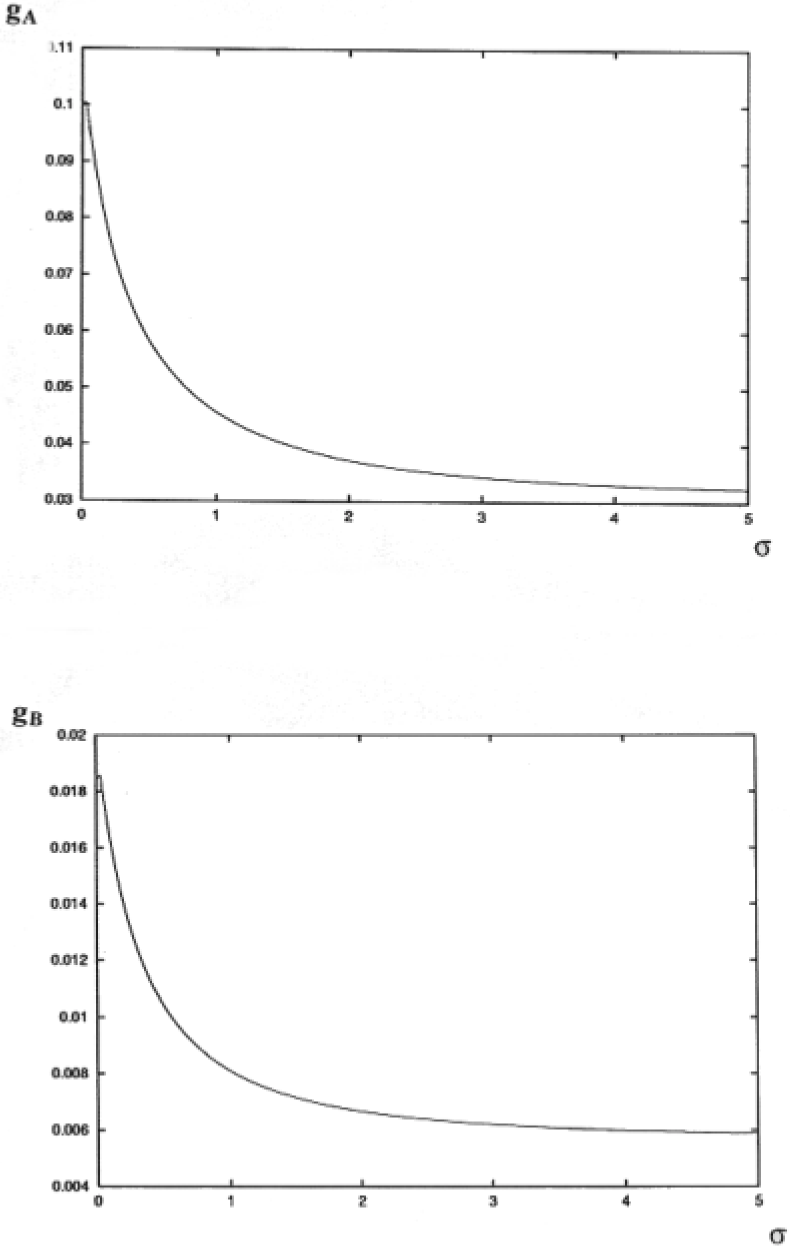


Figure 2.d



g_A
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0
1
2
3
4
5
σ
g_B
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0
1
2
3
4
5
σ

Figure 2.e