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ABSTRACT

Imperfect Competition, Market Size and Firm Turnover*

This paper is motivated by the empirical regularity that industries differ greatly in the level of firm turnover, and that entry and exit rates are positively correlated across industries. Our objective is to investigate the effect of sunk costs and, in particular, market size on entry and exit rates. We analyse a stochastic dynamic model of a monopolistically competitive industry. Each firm's marginal cost (or, alternatively, perceived quality) is assumed to follow a Markov process. We show existence and uniqueness of a stationary equilibrium with simultaneous entry and exit: efficient firms survive while inefficient ones leave the market and are replaced by new entrants. We perform comparative statics with respect to the level of sunk costs: entry costs are positively, and fixed production costs negatively, related to entry and exit rates. Another empirical prediction of the model is that the level of firm turnover is increasing in market size. The intuition is as follows. In larger markets price-cost margins are smaller since the number of active firms is larger. This implies that the marginal surviving firm has to be more efficient than in smaller markets. Hence, in larger markets, the expected life span of firms is shorter. Moreover, firm profits and values are more skewed in larger markets. In the empirical part, the prediction on market size and firm turnover is tested on industries where firms compete in well-defined geographical markets of different sizes. We use data on local services (driving schools) in Sweden in the 1990s. The empirical results provide some support for the prediction that hazard rates are increasing with market size.

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NON-TECHNICAL SUMMARY

Many empirical studies in industrial organization and labour economics have shown that industries differ substantially in the level of firm turnover and gross job reallocation. These differences are stable over time and similar across countries. Most of the reallocation of inputs and outputs can be accounted for by reallocation within narrowly defined industries. The open research agenda is to explain these industry differences in turnover levels and relate them to observable industry characteristics.

This paper studies the determinants of market turbulence and relates the level of entry and exit rates to observable industry characteristics: sunk costs and, in particular, market size. To this end, we analyze a stochastic dynamic model of an imperfectly competitive industry. There is a continuum of atomistic firms that offer differentiated products so that each firm faces a downward-sloping demand curve. Moreover, firms differ in their marginal costs (or, alternatively, perceived qualities), which are subject to idiosyncratic shocks. The Markov process governing the evolution of incumbents' costs exhibits persistency: the more efficient the firm, the more likely it is to be efficient in the future. To enter the market, new firms have to pay some entry fee. Furthermore, active firms have to pay a fixed production cost in each period: this provides an incentive for inefficient firms to leave the market. To allow for a large class of models describing competition in the output market, we do not specify firms' strategic variables (prices or quantities) nor the details of the demand system. Instead, each firm's equilibrium profit is summarized by a reduced-form profit function. The assumptions we impose on the profit function are satisfied by a large class of oligopoly models. In a stochastic dynamic model, it is more convenient to work with a continuum of (monopolistically competitive) firms rather than with a finite number of (oligopolistic) players: (i) there are no integer constraints; (ii) idiosyncratic uncertainty washes out at the aggregate level; and (iii) the set of equilibria is greatly reduced.

In the stationary equilibrium, incumbent firms use a simple exit policy: firms with a cost draw below a certain threshold stay in the market, while the other firms leave the market and are replaced by new entrants. If entry costs are not too large, there exists a unique stationary equilibrium exhibiting simultaneous entry and exit. One central empirical prediction of the model is that the rate of firm turnover is positively related to the size of the market. For a given distribution of active firms, an increase in market size raises firms' sales and profits proportionally. But the distribution of firms is endogenous: free entry implies that it increases with market size. An increase in the distribution of firms causes price-cost margins to fall. This hurts all firms. The fractional reduction in profits is, however, larger the less efficient the firm. This implies that, in larger markets, efficient firms are better off and inefficient firms worse off, while the profit of the average entrant remains unchanged. Hence, firms have to be more efficient in larger markets in order to survive, and the probability of failure is larger. It should be pointed out that the negative relationship between market size and firm turnover would be absent in a model with perfect competition (such as in Hopenhayn (1992)) or in a Dixit-Stiglitz type model. In these models, firms' output or pricing decisions are independent of the number of rivals. Hence, in a free entry equilibrium, prices are independent of market size and the number of active firms rises proportionally with market size. This is in contradiction to the empirically well-documented regularity (e.g. Bresnahan and Reiss (1991)) that the number of active firms rises less than proportionally with market size (due to a decline in price-cost margins as the number of firms increases).

In this paper, we also investigate the effects of changes in sunk costs on turbulence: the equilibrium turnover rate is decreasing with entry costs and increasing with fixed or opportunity costs. In an extension of the model, we show that both entry and exit rates should tend to increase over time in growing markets, and decrease in declining markets. The paper provides a number of results in addition to those on market turbulence. In particular, the model predicts that, in larger markets, firms are more efficient on average, and the distribution of profits and firm values are more skewed than in smaller markets.

In the empirical application of the paper, we set out to test the model's prediction on market size and firm turnover. The idea of the empirical test is to study industries where firms compete in well-defined geographical ('local') markets of different sizes. This should avoid the usual problems with cross-industry studies since we compare turnover rates within the same industry. Specifically, we use data on driving schools in Sweden that were active in the 1990s. The empirical results provide some support for the prediction that exit rates are increasing with market size.

1 Introduction

Much empirical research in industrial economics has been conducted on firm entry and exit. Several empirical regularities have emerged from this literature.[1] First, a finding of virtually all studies is that the gross entry rates of an industry (e.g. measured as the number of new entrants) is a multiple of the net entry rate (e.g. measured as the net increase in the number of active firms). A related line of research in labour economics, concerned with gross job creation and destruction, has shown that the rate of gross job reallocation is much higher than the rate of net employment growth.[2] These observations imply that there is much simultaneous entry and exit going on at the industry level, and among the surviving firms there is significant mobility. Even within the same sector, there is considerable variation in firm turnover ("turbulence") across industries. Second, and more surprisingly, there is a strong positive correlation between entry and exit levels across industries. The correlation persists when rates are measured at intervals that are much shorter than a business cycle. That is, industries with high entry rates are likely to exhibit high exit rates. Third, rates of entry and exit are positively correlated over time. Finally, the ranking of industries by turnover rates appears to be similar from one country to another.[3]

In short, empirical evidence suggests that there are "high turnover industries" and "low turnover industries". To explain these cross-industry differences in firm turnover is one of the important research agendas in the field of industrial market structure. For example, Dunne, Roberts, and Samuelson (1988) conclude their study as follows.

> The high correlation between entry and exit across industries indicates that industries differ substantially in their degree of firm turnover. One area for further study is then to identify the characteristics of industry technology and demand that give rise to across industry differences in turnover.

It is probably fair to say that little progress has been made so far. Although there are some notable exceptions (discussed below), there appears to be a scarcity of theories which make empirically testable predictions regarding the determinants of firm turnover. This paper considers observable industry characteristics – sunk costs and, in particular, market size – and explores their effects on entry and exit rates in an imperfectly competitive industry.

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[1] Caves (1998) provides a recent survey of the empirical literature on turnover and mobility of firms. See also Sutton (1997a) and Cabral (1997).

[2] See Davis and Haltiwanger (1999) for a survey of the literature on job flows.

[3] For example, in their study of gross job flows in the U.S. and Canadian manufacturing sectors, Baldwin, Dunne and Haltiwanger (1998) report a correlation coefficient of 0.83 between U.S. and Canadian job reallocation rates.

To this aim, we analyse a stochastic dynamic model of a monopolistically competitive industry. Firms are heterogenous and subject to idiosyncratic shocks to their "efficiencies" (marginal costs or perceived qualities). Upon entry, a new firm gets a random draw of its efficiency while an incumbent's efficiency level follows a Markov process exhibiting persistence: expected future efficiency is increasing in the firm's current efficiency level. We show existence and uniqueness of a stationary equilibrium with simultaneous entry and exit.[4] In equilibrium, firms follow a threshold exit policy: efficient firms survive while inefficient ones leave the market and are replaced by new entrants. We perform comparative statics with respect to the level of sunk costs: entry costs are positively and fixed production costs negatively related to entry and exit rates.

The central new prediction of the model is that the level of firm turnover is increasing in market size. The intuition is the following. In a free entry equilibrium, a rise in market size causes the population of active firms to increase. This, in turn, leads to lower price-cost margins. Hence, there are two opposing effects on firms' profits: lower price-cost margins and larger sales. For the average entrant, these two effects cancel each other out. The overall effect is positive for more efficient firms, and negative for less efficient firms. The marginal surviving firm which is just indifferent between exiting and staying in the market is less efficient than the average entrant: it has the same (zero) value but its entry costs are already sunk. This implies that the marginal surviving firm has to be more efficient in larger markets. Hence, in larger markets, the expected life span of firms is shorter, and the rate of firm turnover larger. Another implication of the stronger "selection effect" in larger markets is that firms are on average more efficient and the distributions of profits and firm values more skewed than in smaller markets.

Our result on market size and firm turnover relies on a well-known property of most models of oligopolistic competition: equilibrium prices and, hence, price-cost margins fall with an increase in the population of firms. This, in turn, implies the following empirical prediction: in a free entry equilibrium, the number of active firms should rise less than proportionally with market size. This prediction has been tested empirically by Bresnahan and Reiss (1991) in their seminal paper, and more recently by Asplund and Sandin (1999a) and Campbell and Hopenhayn (1999). The available empirical evidence strongly supports the prediction. In contrast, under perfect competition and in Dixit-Stiglitz type models, equilibrium prices are independent of the number of firms. Hence, these models predict the number of active firms to rise proportionally with market size, contradicting the empirical evidence.

In an extension of the model, we investigate the time-series pattern of entry and exit rates when market size changes over time. Perhaps surprisingly, we show that both entry and exit rates will tend to increase over time in growing markets, and decrease in declining markets. The main results on turnover in stationary markets hold for a large

[4]Note that we study the properties of a *stationary* industry equilibrium; we do *not* analyse the life-cycle of an industry as in Klepper (1996).

class of Markov processes. By imposing stronger assumptions, one can obtain additional results, which are consistent with a number of empirical regularities. For instance, the probability of firm exit is decreasing in firm size and age.

In our model, we make quite general assumptions on the reduced-form profit function, which are consistent with a large class of oligopoly models, including the Cournot model. However, we do depart from the assumption of oligopolistic competition in that we assume that there is a continuum of firms, i.e. each firm is atomistic, but faces a downward-sloping demand curve; this is the assumption of monopolistic competition. In a stochastic dynamic model, it is more convenient to work with a continuum of (monopolistically competitive) firms rather than with a finite number of (oligopolistic) players. First, if firms are atomistic, we do not have to worry about integer constraints. In a free entry equilibrium, the value of a new entrant is exactly equal to its outside option. Second, with a continuum of firms, idiosyncratic uncertainty washes out at the aggregate level. Hence, if uncertainty enters at the individual level only, all aggregate variables are deterministic. Third, the assumption of monopolistic competition greatly reduces the set of equilibria. With a finite number of firms and oligopolistic competition there would be two sources of multiplicity of equilibria. First, the identity or type of an exiting firm are not uniquely determined. There are often equilibria in which an efficient firm exits even though (or, rather, precisely because) a less efficient firm decides to stay in the market. Second, if players are sufficiently patient, the Folk theorem applies and more or less "collusive" equilibria are sustainable. Clearly, it would be desirable to study the case of oligopolistic competition as well. But, at the very least, our dynamic stochastic model of monopolistic competition should provide us with a useful benchmark.

How can our predictions be tested empirically? The magnitude of the underlying fluctuations in the pattern of demand (or technology) is likely to vary greatly across industries. As pointed out by Sutton (1997a), this factor may be of primary importance, but it is very difficult to measure it or to control for its impact empirically. This causes a serious problem for any empirical test of cross-industry predictions on firm turnover. Fortunately, an attractive feature of the explanatory variable "market size" is that this problem can be largely circumvented. The idea is to study turnover rates in geographically independent ("local") markets of different sizes but within the same industry. This should control for all those factors (except market size) that would differ across industries. This is the route taken in the empirical part of the paper, where we set out to test the predictions of the model empirically, using Swedish data on driving schools. The empirical results provide some support for the prediction that hazard rates are increasing with market size.

The starting point of the recent literature on stochastic dynamic industry equilibria with heterogenous firms is the seminal paper by Jovanovic (1982). Jovanovic considers a perfectly competitive industry where firms have different but time-invariant efficiency levels. Firms only gradually learn their types over time by observing their "noisy" cost realisations. Firms which learn that they are efficient grow and survive, while firms that obtain consistently negative information decline and eventually leave the market. The model produces a rich array of empirical predictions on the relationship between firm growth and survival on the one hand and firm age and size on the other. However, all firms eventually learn their efficiency level; hence, there is no firm turnover in the long run. Lambson (1991) considers another model with atomistic price takers. In his paper, there are no idiosyncratic shocks but instead common shocks to input price (and demand), i.e. there is aggregate uncertainty. In equilibrium, firms may choose different technologies and hence be affected differently by the common shocks. The model predicts that variability of firm values is negatively related to the level of sunk costs. Some empirical evidence for this prediction is given in Lambson and Jensen (1998). Ericson and Pakes (1995) analyse a stochastic dynamic oligopoly model. There are two sources of uncertainty in the model: the outcomes of firms' investments in "quality" are stochastic and firms are subject to (negative) aggregate shocks. The equilibrium distribution of qualities at any time is itself stochastic (and ergodic): the industry is in a constant flux. Few analytic restrictions can be placed on equilibrium outcomes. Instead, the authors have developed a simulation package; see Pakes and McGuire (1994).[5]

Hopenhayn (1992) is closely related to our model. The key difference between his model and ours is the assumed form of competition: Hopenhayn considers a perfectly competitive industry. The main prediction of his model is that firm turnover is negatively related to entry costs. Market size, however, has no effect on entry and exit rates. In fact, the equilibrium price is independent of market size. This is in contrast to the predictions of standard oligopoly models with free (but costly) entry. There are a few other papers that build on Hopenhayn's framework. Hopenhayn and Rogerson (1993) apply a general equilibrium version of the model to study the effect of changes in firing costs on total employment and welfare. Bergin and Bernhardt (1999) consider business cycle effects in a model of perfect competition. Das and Das (1997) and Melitz (1999) both introduce monopolistic competition *à la* Dixit-Stiglitz into the model. Das and Das analyse the effect of entry adjustment costs on the convergence path to the stationary state. Melitz (1999) considers the impact of trade in a general equilibrium version of Hopenhayn's model. In his model, the efficiency of incumbents does not vary over time, and the death rate of incumbents is exogenously given. Entrants may decide to exit immediately after entry (and before production takes place). There are no market size effects on firm turnover.[6]

[5] The passive learning model by Jovanovic (1982) differs from a number of other models (such as Ericson and Pakes (1995), Hopenhayn (1992), and our model) in that the stochastic process generating the size of a firm is non-ergodic. This is explored in Pakes and Ericson (1998).

[6] Indeed, as pointed out above, in Dixit-Stiglitz type models with a continuum of firms markups are independent of the mass of competitors (thus violating the main condition of our central prediction on market size). Consequently, price-cost margins are independent of market size and the mass of active firms rises proportionally with market size. Market size has no impact on firms' exit policies, and

The plan of the paper is as follows. In Section 2, we present the basic model. This is followed, in Section 3, by the equilibrium analysis, including our existence and uniqueness results. In Section 4, we investigate the comparative statics properties of the stationary equilibrium, which lie at the heart of the paper. This section contains the central prediction on the relationship between market size and firm turnover. We extend the model in Section 5 by analysing the case of growing and declining markets. We provide results on the comovements of entry and exit rates and market size. The robustness of the main results is discussed in Section 6. In the empirical part of the paper, Section 7, we test our prediction on market size and firm turnover, using Swedish data on driving schools. Finally, we conclude in Section 8.

2 The Model

We consider a stochastic dynamic model of a monopolistically competitive industry. There are a continuum of consumers and a continuum of (potential) firms. Firms' products are differentiated. Although each firm is atomistic, it does not take price as given since it faces a downward-sloping demand curve. Firms differ in their "efficiency levels", which are subject to idiosyncratic shocks. Under our leading interpretation, the shocks directly affect firms' marginal costs. In this case, the marginal cost of each new entrant is independently drawn from the continuous cumulative distribution function $G(\cdot)$ with support $[0, 1]$. An incumbent's marginal cost in period t, c_t, is given by

$$\begin{aligned} c_t &= c_{t-1} \text{ with probability } \alpha, \\ c_t &\sim G(\cdot) \text{ otherwise,} \end{aligned} \tag{1}$$

where $\alpha, \alpha \in [0, 1)$, measures the persistence of an incumbent's costs. The "shocks" to incumbents' efficiencies are assumed to be firm specific. Under our alternative interpretation, all firms face the same constant level of marginal cost, but their products differ in the level of perceived quality; see Example 2 below. In this case, a firm's perceived quality is negatively related to firm type c. In the remainder of the paper, we will refer to c as a firm's marginal cost, but the reader may keep in mind the alternative interpretation. The simple stochastic process (1) allows us to obtain closed-form solutions and to perform comparative statics with respect to the persistence parameter α. In Section 6, we show that the main predictions of the paper remain unchanged if we replace (1) by a large class of Markov processes. Regarding sunk costs, we assume that a firm has

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therefore none on the turnover rate. We believe this to be an unwarranted feature of Dixit-Stiglitz type models. As an *example* of our model of monopolistic competition with properties similar to oligopolistic models, we propose instead the linear demand model with a continuum of products. This example is potentially of great interest for trade and growth theorists as an alternative to the widely used Dixit-Stiglitz model.

to pay a (sunk) entry fee ϵ, $\epsilon > 0$, when it enters the market. Additionally, a firm faces a fixed production (or opportunity) cost of ϕ, $\phi > 0$, per period.

Time is discrete and indexed by t. Firms have an infinite horizon and maximise the discounted sum of profits. The common discount factor is denoted by δ, $\delta \in [0, 1)$. In each period, the timing is as follows.

1. Entry stage. The potential entrants decide whether to enter the market or not.
2. Learning stage. The new entrants and incumbents observe the realisation of their current costs, c_t.
3. Exit stage. The new entrants and incumbents decide whether to leave the market forever or not.
4. Output stage. The active firms play some (unmodelled) "market game", pay a fixed production cost ϕ, and receive profits. The inactive firms obtain a payoff normalised to zero.

Let us make two remarks on the sequence of moves. First, potential entrants decide whether or not to enter the market before knowing their current efficiency. This assumption is common to most dynamic industry models. It allows us to avoid assumptions about the size of the pool of potential entrants other than that it is sufficiently large to ensure there is always a positive mass of firms which do not enter in equilibrium. This is of particular importance in our model as we are interested in the effects of market size. Arguably, the number of potential entrants may not be independent of market size. Fortunately, with the assumed sequence of moves, we can remain agnostic about this relationship. Second, new entrants are treated as incumbents at the exit stage, which takes place after firms have learnt their current types. This sequence of moves gives rise to a convenient mathematical structure, which is different from existing models. In Section 6, we show that the assumption is not essential for the results.

Formally, the model can be described as an *anonymous sequential game*; see Jovanovic and Rosenthal (1988). Let $\mathcal{M}$ denote the set of Borel measures on $[0, 1]$, and μ, $\mu \in \mathcal{M}$, the measure of firms' cost levels at the output stage. That is, for any Borel set A, $A \subset [0, 1]$, $\mu(A)$ gives the mass of active firms with costs in A. In period t, stage 1, a potential entrant, knowing the measure ("distribution") of active firms in period $t-1$, μ_{t-1}, selects an action $e_t(\mu_{t-1}) \in \{0, 1\}$, where $e_t(\mu_{t-1}) = 1$ means that the firm decides to enter the market. Similarly, at stage 3 of the same period, a firm, knowing its own cost c_t, the distribution of last period's firms, μ_{t-1}, and the mass of new entrants, M_t, takes its exit decision $x_t(c_t, \mu_{t-1}, M_t) \in \{0, 1\}$: a firm chooses $x_t(c_t, \mu_{t-1}, M_t) = 1$ if it decides to leave the market. Although firms' efficiency levels follow a stochastic process, there is no aggregate uncertainty in our model.[7] This follows from the large number of firms and the assumption that shocks are "firm specific".[8] Hence, the evolution of the industry is deterministic.

To allow for a large class of models describing competition at the output stage, we do not model the "market game" explicitly. In particular, we do not specify firms' strategic variables (prices or quantities) nor the details of the demand system. Instead, each firm's equilibrium profit is summarised by a reduced-form profit function. A firm's equilibrium profit depends on its own type, on market size, and on the endogenous distribution of active firms. The equilibrium profit (gross of fixed costs) of a type-c firm is given by

$$S\pi(c;\mu), \tag{2}$$

where S, $S>0$, is a measure of market size (e.g., the mass of consumers in the market), and $\pi(\cdot;\cdot):[0,1]\times\mathcal{M}\rightarrow[0,\infty)$. By writing a firm's profit as in (2), we make a number of implicit assumptions. First, firms differ only in their types; they are symmetric in all other respects. Hence, if c denotes marginal costs, then the aggregate demand system is symmetric and competition non-localised. Second, an increase in market size means a replication of the population of consumers, leaving unchanged the distribution of preferences and income. Moreover, firms' marginal production costs are independent of output levels. This implies that market size enters (2) in a multiplicative way. Of course, in equilibrium, the distribution μ will depend on market size, but it is taken as given at the output stage.

To be more specific, let us consider the case where c denotes marginal costs. Denote by $SD(\cdot;\mu)$ and $P(\cdot/S;\mu)$ the demand and inverse demand functions, respectively, faced by an individual firm in equilibrium when the measure of active firms is given by μ. In this case, equilibrium gross profit can be written as

$$\begin{aligned} S\pi(c;\mu) &\equiv [p(c;\mu)-c]\,SD\,(p(c;\mu);\mu) \\ &= [P(q(c;\mu,S)/S;\mu)-c]\,q(c;\mu,S), \end{aligned} \tag{3}$$

where $p(c;\mu)$ and $q(c;\mu,S)$ are equilibrium price and output.

Throughout the paper, we make the following assumptions on the reduced-form profit function.

C.1 *The reduced-form profit function* $\pi(\cdot;\mu)$ *is strictly decreasing in* c *on* $[0,\overline{c}(\mu)]$, *and* $\pi(c;\mu)=0$ *for all* $c\in(\overline{c}(\mu),1]$, *where* $\overline{c}(\mu)\in[0,1]$.

[7]Since there is no aggregate uncertainty and each firm is atomistic, the requirements on firms' information can be weakened substantially. For instance, incumbents may or may not condition their exit decisions on the current mass of entrants.

[8]See Feldman and Gilles (1985) and Uhlig (1996) for a precise statement of the conditions under which a law of large numbers can be justified for a continuum of random variables.

That is, firms with lower marginal costs have higher profits. We allow for the possibility that some inefficient firms (the types above $\overline{c}(\mu)$) may not sell their products even when offered at marginal cost and hence make zero gross profit.

Since the distribution of active firms is endogenous, we have to compare different distributions. For this purpose, we define a partial ordering, denoted by $\succeq$, on the set $\mathcal{M}$. Formally, let

$$\mu' \succsim \mu \Leftrightarrow \forall c \in [0,1],\ \pi(c;\mu') \leq \pi(c;\mu),$$

and

$$\mu' \succ \mu \Leftrightarrow \forall c \in [0, \overline{c}(\mu)),\ \pi(c;\mu') < \pi(c;\mu).$$

Note that the ordering implies $\overline{c}(\mu') \leq \overline{c}(\mu)$ for $\mu' \succsim \mu$. Distribution μ' is said to be (weakly) *larger* than μ if $\mu' \succeq \mu$. Distributions μ' and μ are said to be equivalent if $\mu' \sim \mu$. We define the *equivalence class* of measure μ as the set of Borel measures μ' in $\mathcal{M}$ such that $\mu' \sim \mu$.

C.2 *If $\mu'([0,z]) \geq \mu([0,z])$ for any $z \in (0,1]$, then $\mu' \succsim \mu$. If the inequality is strict and $\pi(0;\mu) > 0$, then $\mu' \succ \mu$.*

The distribution of active firms is larger if the mass of active firms is larger and the population of firms more efficient. We can remain completely agnostic about the effect of a more efficient but smaller population of firms on profits.

C.3 *The set $(\mathcal{M}, \succeq)$ is completely ordered.*

Complete ordering of $(\mathcal{M}, \succeq)$ is an implication of symmetry and non-localised competition. The assumption rules out that some firm makes larger profits when the distribution of active firms is given by μ rather than μ', while some other firm is better off under μ'. If firms differ only in their types and are symmetric otherwise, then all types should have the same ranking of distributions.[9]

C.4 *The reduced-form profit function $\pi(c;\mu)$ is continuous.*[10]

For two of our main comparative statics results, we have to impose further structure on the reduced-form profit function. The following two conditions summarise properties of a large class of oligopoly models with heterogenous firms. (These properties appear to have remained widely unnoticed in the literature.)

A.1 *For $\mu' \succ \mu$, the profit difference $|\pi(c;\mu) - \pi(c;\mu')|$ is strictly decreasing in c on $[0, \overline{c}(\mu))$.*

A.2 *For $\mu' \succ \mu$, the profit ratio $\pi(c;\mu')/\pi(c;\mu)$ is strictly decreasing in c on $[0, \overline{c}(\mu'))$.*

Consider an increase in the distribution of active firms. Clearly, this causes the gross profit of any firm to decrease, provided the firm makes a positive profit in the first place.

[9] Actually, the assumption is stronger than necessary. It suffices to assume that $(\mathcal{M}^*, \succeq)$ is completely ordered, where $\mathcal{M}^* \equiv \{\mu \in \mathcal{M} \mid \forall z \in [0,1], \mu([0,z]) = kG(\min\{z, c^*\}), k > 0, c^* \in (0,1]\}$. Alternatively, we could assume the following. There exist functions $h : \mathcal{M}^* \to \mathbb{R}$ and $\widehat{\pi} : [0,1] \times \mathbb{R} \to \mathbb{R}_+$ such that $\pi(c;\mu) \equiv \widehat{\pi}(c; h(\mu))$ for all $c \in [0,1]$ and $\mu \in \mathcal{M}^*$, where $\widehat{\pi}$ is strictly decreasing in its second argument. Hence, by definition, $h(\mu') \geq h(\mu)$ if and only if $\mu' \succeq \mu$.

[10] *We endow $\mathcal{M}$ with the topology of weak* convergence.*

Assumption A.1 says that efficient firms suffer more than inefficient firms in terms of the absolute decrease in profit. Assumption A.2 is the condition for our central result on the relationship between market size and market turbulence. It says that the percentage decrease in gross profit is larger for inefficient firms than for efficient ones.

To improve our understanding of assumptions A.1 and A.2, the following proposition shows that they are equivalent to well-known properties of equilibrium prices and quantities.

Proposition 1 *Suppose firm type c denotes marginal costs, and firms compete either in prices or in quantities. That is, equilibrium profit is given by (3). Assume also that the demand and inverse demand functions faced by an individual firm in equilibrium, $SD(\cdot;\mu)$ and $P(\cdot/S;\mu)$, are differentiable.*

1. *Assumption A.1 holds if and only if equilibrium output $q(c;\mu) = SD(p(c;\mu);\mu)$ is decreasing in μ; that is, if and only if $q(c;\mu') < q(c;\mu)$ $\forall c \in [0, \bar{c}(\mu))$, $\mu' \succ \mu$.*
2. *Assumption A.2 holds if and only if the equilibrium price $p(c;\mu) = P(q(c;\mu)/S;\mu)$ is decreasing in μ; that is, if and only if $p(c;\mu') < p(c;\mu)$ $\forall c \in [0, \bar{c}(\mu'))$, $\mu' \succ \mu$.*

Proof. The assertions can be shown by taking the derivative of the profit difference and profit ratio with respect to c and applying the envelope theorem. ■

By definition, an increase in the distribution of active firms causes the profits of firms (with positive sales) to fall. For this to hold, the equilibrium price or quantity of any given firm must fall. If firm type c denotes marginal costs, assumptions A.1 and A.2 say that both quantities and prices must fall in equilibrium. Proposition 1 may be of independent interest. Since its proof does not make use of the fact that μ summarises the distribution of active firms, it can be applied to any shift in consumers' tastes or incomes which uniformly reduces firms' profits.

Assumptions A.1 and A.2 (as well as the other conditions we impose on the reduced form profit function) are satisfied by a wide class of oligopoly models where firms differ in their marginal costs. Examples include the Cournot model (with homogenous products) and the linear demand model (with a finite number of differentiated products and either price or quantity competition); in Appendix A, this is shown for the Cournot model.[11] As to models of monopolistic competition with a continuum of firms, the widely used Dixit-Stiglitz model satisfies most of our assumptions, including A.1, but not A.2. In fact, in the Dixit-Stiglitz model, firms use a simple markup pricing rule in which the markup is a function of some substitutability parameter in the utility function, but not of the mass of active firms. That is, the Dixit-Stiglitz model does not capture the empirically well-documented regularity that price-cost margins are falling with the number of firms in the

[11] Of course, in the case of oligopoly models with a finite number of firms, $\mu(A)$ has to be interpreted as the *number* of firms with costs in A.

market. In the following, we give two examples of models of monopolistic competition that satisfy our assumptions.

Example 1 (The linear demand model with a continuum of firms.) *There is a continuum of S identical consumers whose utility U is defined over a continuum of substitute goods and a Hicksian composite commodity. Specifically,*

$$U(\mathbf{x}; H) = \int_0^n \left(x(i) - x^2(i) - 2\sigma \int_0^n x(j)x(i)dj \right) di + H,$$

where $x(i)$ is the consumption of variety i, and H the consumption of the Hicksian composite commodity.[12] *The parameter σ, $\sigma \in (0,1)$, measures the substitutability between different varieties. Suppose each active firm in the industry produces a unique variety. Denote by $p(i)$ and $c(i)$ the price and marginal cost of variety i, respectively, and by Y consumer income. Normalising the price of the Hicksian composite commodity to 1, $H = Y - \int_0^n p(i)x(i)di$. Let us now re-label firms in increasing order of marginal costs, i.e. $c(j) > c(i) \Rightarrow j > i$. Let m, $m \in (0, n]$ denote the least efficient producer with positive sales in equilibrium. Then, if firms compete in prices, equilibrium profit of firm i, $i \in [0, m]$, is given by*

$$\frac{S}{8} \left(\frac{2 + \sigma \int_0^m c(j)dj}{2 + m\sigma} - c(i) \right)^2 .$$

That is, profit is of the form $S\pi(c;\mu) = S(h(\mu) - bc)^2$, with $h(\mu) > bc$. It is straightforward to check that the equilibrium profit function satisfies our assumptions.□

Example 2 (The linear demand model with perceived qualities.) *This example is similar to the preceding one. All firms now have the same constant marginal cost, normalised to zero. The utility of the representative consumer is given by*

$$U(\mathbf{x}; \mathbf{u}; H) = \int_0^n \left(x(i) - \frac{x^2(i)}{u^2(i)} - 2\sigma \int_0^n \frac{x(j)}{u(j)} \frac{x(i)}{u(i)} dj \right) di + H,$$

where $u(i)$, $u(i) \geq 1$, is the perceived quality of variety i.[13] *Utility is strictly increasing in quality $u(i)$, provided that $x(i) > 0$. Re-label firms in decreasing order of quality, i.e. $u(i) > u(j) \Rightarrow i < j$. Let m again denote the marginal producer. Firm i's equilibrium profit under price competition can then be written as*

$$\frac{S}{8} \left(u(i) - \frac{\sigma}{2 + m\sigma} \int_0^m u(j)dj \right)^2 ,$$

which is again of the form $S\pi(c;\mu) = S(h(\mu) - bc)^2$, using, for example, the transformation $u = 2 - c$.□

[12] This is the continuum version of the quadratic utility function which goes back to Bowley (1924). The associated demand system is widely used in oligopoly models; see Vives (1999).

[13] This is the continuum version of the utility function in Sutton (1997b) .

3 Stationary Equilibrium

The aim of this section is to characterise the industry equilibrium, and to show existence and uniqueness of equilibrium. In our equilibrium analysis, we confine attention to a free entry stationary equilibrium, in which firms' strategies and the distribution of firms are stationary. We defer the analysis of the comparative statics properties, which lie at the heart of this paper, to the next section.

In a stationary equilibrium, the value of an incumbent of type c at the start of the exit stage, $V(c)$, can be written as

$$V(c) = \max\left\{0, [S\pi(c;\mu) - \phi] + \delta\left[\alpha V(c) + (1-\alpha)\int_0^1 V(z)G(dz)\right]\right\}. \tag{4}$$

If the firm exits the market, it gets a zero payoff. If, on the other hand, it stays in the market, its current profit is $S\pi(c;\mu) - \phi$; in the following period, it will be of the same efficiency with probability α, and obtain a new draw from $G(\cdot)$ with probability $1-\alpha$. Let c^* be defined by

$$\begin{aligned} c^* &\equiv \sup\{c \in [0,1] \mid V(c) > 0\}, \\ \text{or } c^* &\equiv 1 \text{ if } V(c) > 0 \text{ for all } c \in [0,1]. \end{aligned}$$

The value function $V(c)$ is continuous on $[0,1]$, and constant on $[\bar{c}(\mu), 1]$. If $\pi(0;\mu) > 0$, then standard arguments from dynamic programming imply that $V(c)$ is strictly decreasing on $[0, \min\{c^*, \bar{c}(\mu)\}]$. Hence, a firm's equilibrium exit strategy takes the form of a simple threshold rule, c^*, according to which the firm exits ($x = 1$) if and only if $c > c^*$. Let M denote the mass of entering firms in each period. In a stationary equilibrium,

$$\begin{aligned} V(c^*) &\geq 0, \\ \text{with } V(c^*) &= 0 \text{ if } M > 0. \end{aligned} \tag{5}$$

The value of an entrant at stage 1, V^e, is given by

$$V^e = \int_0^1 V(c)G(dc) - \epsilon. \tag{6}$$

Free entry implies that

$$\begin{aligned} V^e &\leq 0, \\ \text{with } V^e &= 0 \text{ if } M > 0. \end{aligned} \tag{7}$$

Suppose there is simultaneous entry and exit in the stationary equilibrium, i.e. $M > 0$ (or, equivalently, $c^* < 1$). Using equations (4), (6), and (7), we can compute the value functions in closed form as

$$V(c) = \frac{1}{1-\alpha\delta}\left\{[S\pi(c;\mu) - \phi] + \delta(1-\alpha)\epsilon\right\},\ c \in [0, c^*), \tag{8}$$

and

$$V^e = \frac{1}{1-\alpha\delta}\left\{\int_0^{c^*} [S\pi(c;\mu) - \phi + \delta(1-\alpha)\epsilon]\, G(dc) - (1-\alpha\delta)\epsilon\right\} = 0.$$

To simplify the exposition, let us neglect in this section the non-generic case where $\phi = \delta(1-\alpha)\epsilon$. If $c^* \in (0,1)$, the entry and exit conditions can be re-written as

$$\psi(c^*;\mu) \equiv \int_0^{c^*} [S\pi(c;\mu) - \phi + \delta(1-\alpha)\epsilon]\, G(dc) - (1-\alpha\delta)\epsilon = 0, \tag{E}$$

$$S\pi(c^*;\mu) - \phi + \delta(1-\alpha)\epsilon = 0,\ c^* \in (0,1), \tag{X}$$

or simply as

$$\psi(c^*;\mu) = 0, \tag{E'}$$

and

$$\frac{\partial}{\partial c^*}\psi(c^*;\mu) = 0. \tag{X'}$$

Note that $\psi(c;\mu)$ is proportional to the value of an entrant who uses exit policy c in the period of entry and behaves optimally thereafter. Next, observe that the exit condition is simply the derivative of the entry condition with respect to the exit policy. This mathematical structure is a consequence of the assumed sequence of moves. Since any new entrant is treated as an incumbent at the exit stage, the equilibrium exit policy of an incumbent, c^*, must maximise the value of an entrant. Hence, both the value of a new entrant and the derivative of this value with respect to the exit policy have to be zero. From exit condition (X), it can be seen that the current net profit of the marginal incumbent firm, $S\pi(c^*;\mu) - \phi$, is negative. The last term in (X) captures the option value from staying in the market. Indeed, with probability $1-\alpha$, the firm gets a new draw from $G(\cdot)$. In a stationary free entry equilibrium with simultaneous entry and exit, the value of a new draw must be equal to the entry cost ϵ. Moreover, since $\phi \neq \delta(1-\alpha)\epsilon$, a necessary condition for a stationary equilibrium with entry and exit is $\phi > \delta(1-\alpha)\epsilon$, which means that the current gross profit of the marginal incumbent firm must be strictly positive, i.e. $c^* < \overline{c}(\mu)$.

Let us consider the properties of the function $\psi(c;\mu)$, which are illustrated in Figure 1. Note first that $\psi(\cdot;\mu)$ is (generically) single-peaked on $[0,1]$.[14] Either $\psi(\cdot;\mu)$ is

[14] Any generic result refers to $\phi \neq \delta(1-\alpha)\epsilon$.

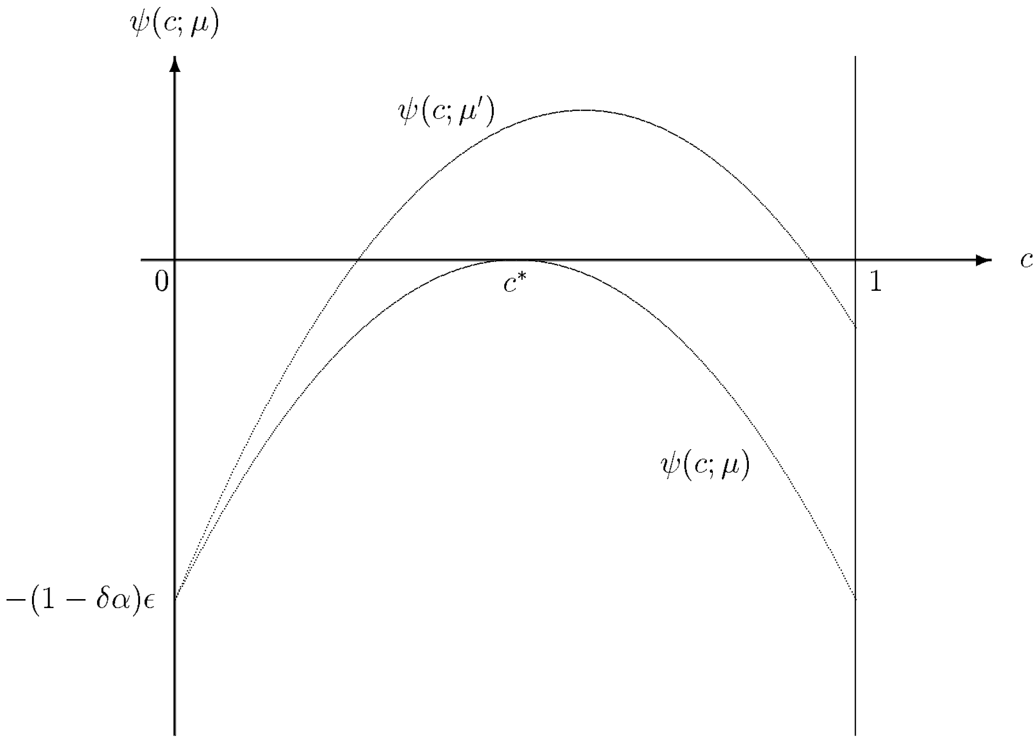

Figure 1: The effect of an increase in the distribution of firms ($\mu \succ \mu'$) on the value of an entrant with exit policy c.

increasing in c on $[0,1]$, decreasing on $[0,1]$, or there exists a unique $\widehat{c}(\mu)$ such that $\psi(\cdot;\mu)$ is increasing in c on $[0,\widehat{c}(\mu)]$ and decreasing on $[\widehat{c}(\mu),1]$.[15] Furthermore, ψ is continuous, and continuously differentiable with respect to its first argument. Observe also that $\psi(0;\mu) = -(1-\alpha\delta)\epsilon < 0$. Hence, if μ gives the cost distribution in the stationary equilibrium, and c^* the equilibrium exit policy, then $\psi(\cdot;\mu)$ takes its unique maximum at $\widehat{c}(\mu) = c^*$. This implies that if there exists a unique stationary distribution, there can be at most one stationary equilibrium with simultaneous entry and exit. Note finally that, for all $c \in (0,1]$, $\psi(c;\cdot)$ is decreasing in μ (as long as $\overline{c}(\mu) > 0$).

The distribution of active firms is endogenous. Let $\mu[c^*, M]$ denote the invariant measure of firms' efficiencies at stage 4 if all firms follow exit policy c^*, $c^* \in (0,1)$, and

[15] In the non-generic case $\phi = \delta(1-\alpha)\epsilon$, $\psi(\cdot;\mu)$ is increasing in c on $[0,\overline{c}(\mu)]$ and constant on $[\overline{c}(\mu),1]$.

the mass of entrants in each period is M. This measure is uniquely defined by

$$\mu[c^*, M]([0,z]) = \frac{M}{(1-\alpha)(1-G(c^*))} G(\min\{z, c^*\}), \ \forall z \in [0,1]. \tag{D}$$

The stationary distribution has thus the shape of $G(\cdot)$, truncated at c^*, and scaled by factor $M/[(1-\alpha)(1-G(c^*))]$. The stationary equilibrium with simultaneous entry and exit can now be defined as the triple (μ, M, c^*) satisfying equations (E), (X), and (D).

In a stationary equilibrium without simultaneous entry and exit, we have $M = 0$, $c^* = 1$, and the stationary distribution is given by

$$\mu_\lambda([0,z]) = \lambda G(z), \ \lambda > 0, \ \forall z \in [0,1]. \tag{9}$$

The stationary equilibrium without entry and exit is given by $(\mu_\lambda, 0, 1)$ satisfying (5), (7), and (9). Note that *any* stationary equilibrium distribution must be an element of $\mathcal{M}^*$, $\mathcal{M}^* \equiv \{\mu \in \mathcal{M} \mid \forall z \in [0,1], \ \mu([0,z]) = kG(\min\{z, c^*\}), k > 0, c^* \in (0,1]\}$.

Before we turn to the issues of existence and uniqueness of equilibrium, let us pause for a moment. The stationary equilibrium with entry and exit has a few features which are consistent with stylised facts on industry dynamics. The probability of immediate exit of a new entrant is $1-G(c^*)$, whereas an incumbent leaves the market with a smaller probability, namely $(1-\alpha)(1-G(c^*))$. Empirical studies have indeed shown that new firms are the ones most likely to exit the market. Moreover, the model implies that new entrants are on average more efficient than exiting firms, but less efficient than surviving incumbents. This is again consistent with the empirical evidence. Finally, if firm size (e.g. measured by output) decreases with marginal cost c, the simple stochastic process given by (1) implies that firm growth is negatively related to firm size, as found by Evans (1987) and others. In Section 6, we re-consider the model and allow for a large class of Markov processes. This permits the model to be consistent with a number of other empirical findings.

The following lemma will prove helpful in showing existence and uniqueness.

Lemma 1 *Consider any two positive measures μ and μ' in $\mathcal{M}^*$ such that $\pi(0;\mu)$ and $\pi(0;\mu')$ are positive. Then, there exists a unique h, $h > 0$, such that measures $h\mu$ and μ' are equivalent, i.e. $h\mu \sim \mu'$.*

Proof. Since μ and μ' are elements of $\mathcal{M}^*$, we have

$$\mu'([0,z]) = k'G(\min\{z, l'\}), k' > 0, l' \in (0,1]$$

and

$$\mu([0,z]) = kG(\min\{z, l\}), k > 0, l \in (0,1]$$

for $z \in [0,1]$. Assume w.l.o.g. that $l \leq l'$. Then, from condition C.2, $h\mu \prec \mu'$ if $hk < k'$, and $h\mu \succ \mu'$ if $hkG(l) > k'G(l')$. Continuity of π (C.4) and C.2 imply that there exists a unique h such that $h\mu \sim \mu'$. ■

Corollary 1 *Consider any positive measure μ' in $\mathcal{M}^*$ such that $\pi(0;\mu') > 0$, and fix an arbitrary exit policy c^* in $(0,1)$. Then, there exists a unique mass of entrants, M, such that the stationary distribution $\mu[c^*,M]$, generated by c^* and M, and defined by (D), is equivalent to μ', i.e. $\mu[c^*,M] \sim \mu'$.*

Proof. This is an immediate implication of Lemma 1 and equation (D). ■

To simplify the proof of existence and uniqueness, we impose a technical condition on the reduced-form profit function π. We assume that the profit of a firm can be made arbitrarily small by making the distribution of efficient firms sufficiently large. Similarly, in the limit as the measure of active firms tends to zero, the expected profit of an entrant becomes positive. The condition may be formally expressed as follows.

C.5 *Fix any positive measure μ in $\mathcal{M}^*$, and let $k > 0$ be some scaling parameter. Then,*

$$\lim_{k\to\infty} \pi(c;k\mu) = 0,\ \ c \in (0,1],$$

and

$$\lim_{k\to 0} \int_0^1 S\pi(c;k\mu)G(dc) > \phi + (1-\delta)\epsilon.$$

The second part of condition C.5 implies that any stationary equilibrium distribution must be positive. We are now in a position to state and prove our existence result.

Proposition 2 *There always exists a stationary equilibrium. Moreover, if a stationary equilibrium with simultaneous entry and exit exists, it is (generically) unique.*

Proof. The proof proceeds in several steps.

Step one. Consider any positive measure μ' in $\mathcal{M}^*$ such that $\psi(1;\mu') = 0$. Conditions C.4 (continuity) and C.5 ensure that μ' exists. Since $\psi(\cdot;\mu')$ is single-peaked, there are two possibilities.

(i) $\psi(c;\mu') < 0$ for all $c \in [0,1)$,

(ii) $\psi(c;\mu') > 0$ for some $c \in (0,1)$.

Step two. In case (i), there does not exist a stationary equilibrium with simultaneous entry and exit. To see this, suppose otherwise that there exists a stationary equilibrium with simultaneous entry and exit. Denote the associated stationary distribution by μ'', and the exit policy by c''. Since $\psi(c;\mu)$ is decreasing in μ, condition (E) implies that $\mu'' \prec \mu'$. It follows that $\psi(1;\mu'') > \psi(c'';\mu'') = 0$. Since $\psi(\cdot;\mu'')$ is single-peaked, we thus have $\partial\psi(c'';\mu'')/\partial c > 0$, which contradicts condition (X). Although there does not exist a stationary equilibrium with simultaneous entry and exit, there does exist at least one without entry and exit. Indeed, Lemma 1 implies that there exists a positive number λ' such that $\mu_{\lambda'} \sim \mu'$, where $\mu_{\lambda'}$ is the stationary distribution defined by (9). It is easy to check that $(\mu_{\lambda'}, 0, 1)$ satisfies conditions (5), (7), and (9). Note that there may exist a multiplicity of stationary equilibria. More precisely, there exists a nonempty interval of λ-values, $[\underline{\lambda}, \overline{\lambda}]$, with $\overline{\lambda} \geq \underline{\lambda}$, such that $(\mu_\lambda, 0, 1)$, $\lambda \in [\underline{\lambda}, \overline{\lambda}]$, forms a stationary equilibrium.

Step three. Consider case (ii), which can only arise if $\phi > \delta(1-\alpha)\epsilon$. There exists a unique stationary equilibrium, which involves simultaneous entry and exit. We denote the stationary equilibrium by (μ, M, c^*). Existence and uniqueness can be shown as follows. C.4 (continuity), C.5, and single-peakedness of $\psi(\cdot;\mu)$ imply that we can increase the distribution of active firms from μ' to μ'' such that $\psi(\cdot;\mu'')$ assumes a unique maximum at c^*, and $\psi(c^*;\mu'') = 0$. (It is easy to see that the exit policy c^* is unique.) Next, applying Corollary 1, there exists a unique M such that $\mu[c^*, M] \sim \mu''$. The unique equilibrium distribution is then given by $\mu \equiv \mu[c^*, M]$.[16] ■

In our proof, we do *not* simultaneously determine the equilibrium values of c^* and M, using a fixed-point argument. Instead, we construct the equilibrium sequentially. First, we neglect condition (D) and find the equilibrium exit policy c^* by varying the distribution of firms in $\mathcal{M}^*$ until (E) and (X) are satisfied. (If such a c^* does not exist in $(0,1)$, then the stationary equilibrium does not exhibit entry and exit.) Since $\psi(c;\cdot)$ is strictly decreasing in μ for $c \in (0,1]$, conditions (E) and (X) also pin down the equivalence class of the stationary distribution. Only then do we consider the stationary distribution generated by c^* and M, as given by (D). From Corollary 1, we know that there exists a unique mass M of entrants such that the stationary distribution generated by M and the exit policy c^* is equivalent to the distribution determined in the first part of the proof. Our method of proof differs considerably from Hopenhayn's (1992). It should be of rather general interest, as it is applicable also to models of perfect competition, where firms are price-takers.

It is straightforward to find conditions under which the unique stationary equilibrium involves simultaneous entry and exit.

Proposition 3 *If the entry cost ϵ is sufficiently small, there exists a unique stationary equilibrium with simultaneous entry and exit.*

Proof. Let

$$\xi(c^*;\mu) \equiv \int_0^{c^*} \left[S\pi\left(c;\mu\right) - \phi\right] G(dc).$$

[16]If $\phi = \delta(1-\alpha)\epsilon$ (which occurs with zero probability if the parameters are drawn from some continuous distribution), two cases can arise: either $\overline{c}(\mu') = 1$ or $\overline{c}(\mu') < 1$. The former case is given by (i), i.e. there exists a stationary equilibrium, but it does not exhibit entry and exit. In the latter case, $\psi(c;\mu') < 0$ for all $c \in [0, \overline{c}(\mu'))$ and $\psi(c;\mu') = 0$ for all $c \in [\overline{c}(\mu'), 1]$. In this case, there exists both a stationary equilibrium without entry and exit ($c^* = 1$) as well as a continuum of equilibria with simultaneous entry and exit, where $c^* \in [\overline{c}(\mu'), 1)$ and $\mu[c^*, M] \sim \mu'$.

Conditions C.4 (continuity) and C.5 ensure that there exists a positive measure $\widehat{\mu}$ in $\mathcal{M}^*$ such that $\xi(1;\widehat{\mu}) = 0$. C.1 then implies that $S\pi(1;\widehat{\mu}) < \phi < S\pi(0;\widehat{\mu})$. It follows that there exists an exit policy $\widehat{c}$, $\widehat{c} \in (0,1)$, such that

$$\xi(\widehat{c};\widehat{\mu}) > 0 = \xi(1;\widehat{\mu}).$$

Then, for ϵ sufficiently small, we have

$$\psi(\widehat{c};\widehat{\mu}) = \xi(\widehat{c};\widehat{\mu}) - [(1-\delta\alpha) - \delta(1-\alpha)G(\widehat{c})]\,\epsilon > 0,$$

and

$$\psi(1;\widehat{\mu}) = \xi(1;\widehat{\mu}) - (1-\delta)\epsilon < 0.$$

Hence, if we define measure μ', $\mu' \in \mathcal{M}^*$, again by $\psi(1;\mu') = 0$, we obtain $\psi(\widehat{c};\mu') > 0$. That is, we are in case (ii) of the proof of Proposition 2. As we have already shown there, this implies that there exists a unique stationary equilibrium which involves simultaneous entry and exit, i.e. $c^* \in (0,1)$. ■

Since incumbents have already sunk their entry costs, they may be less efficient than the average entrant but optimally decide to stay in the market. However, for small entry costs, this efficiency wedge is small, so that the least efficient incumbents must exit the market in equilibrium.

In the remainder of the paper, we will focus on the case of sufficiently small entry costs so that the stationary equilibrium exhibits firm turnover.

4 Comparative Statics: Market Size and Sunk Costs

The aim of this section is to analyse the comparative statics properties of the stationary equilibrium. We begin by defining a measure of firm turnover. Then, we analyse the effect of changes in the level of the entry cost ϵ and the fixed cost ϕ on the equilibrium level of firm turnover. Next, we turn to the main concern of the paper, namely the relationship between market size S and firm turnover. Finally, we look at changes in the level of persistence, α.

The most natural measure of (relative) firm turnover is the ratio between the mass of new entrants and the total mass of active firms in each period. This suggests defining the turnover rate θ as

$$\theta \equiv \frac{M}{\mu\left([0,c^*]\right)},$$

where the denominator is the mass of active firms at stage 4, and the numerator is the mass of firms that have entered at stage 1 of the same period. Alternatively, we could use the ratio $G(c^*)M/\mu\left([0,c^*]\right) = G(c^*)\theta$ as the turnover rate, which does not keep track of the entrants that decide to leave the market in the same period. Using (D), the turnover rate in the stationary equilibrium can be written as

$$\theta = (1-\alpha)\frac{1-G(c^*)}{G(c^*)}. \tag{10}$$

That is, given persistence α, there is a monotonically decreasing relationship between the equilibrium exit policy c^* and the turnover rate θ.[17] The monotonicity of the relationship not only holds for the simple Markov process (1) considered here, but for a general class of Markov processes, as we will show in Section 6.

Having defined the measure of firm turnover, we can now analyse the effects of changes in the parameters of the model on the turnover rate. Let us begin by looking at the effect of an increase in the level of entry costs.

Proposition 4 *An increase in the entry cost ϵ leads to an increase in exit policy c^*, and hence to a lower turnover rate θ. Furthermore, it causes the distribution of active firms, μ, and the mass of entrants per period, M, to decrease.*

Proof. Starting from a stationary equilibrium with simultaneous entry and exit, denoted by (μ_0, M_0, c_0^*), we consider an increase in the entry cost from ϵ_0 to ϵ_1, $\epsilon_1 > \epsilon_0$. Let us assume that there is still a positive turnover rate in the new stationary equilibrium (μ_1, M_1, c_1^*), i.e. $\theta_1 > 0$. From (E), it is easy to see that

$$\psi\left(c^*; \mu_0; \epsilon_1\right) < \psi\left(c^*; \mu_0; \epsilon_0\right) \text{ for all } c^* \in [0,1],$$

abusing notation by inserting argument ϵ into the function ψ, as defined by (E). Hence, for condition (E) to hold in the new stationary equilibrium, we must have $\mu_1 \prec \mu_0$. Since $\mu_1 \prec \mu_0$ and $\epsilon_1 > \epsilon_0$, we must have $c_1^* > c_0^*$ for condition (X) to hold:

$$\begin{aligned} S\pi(c_0^*; \mu_1) - \phi + \delta(1-\alpha)\epsilon_1 &> S\pi(c_0^*; \mu_0) - \phi + \delta(1-\alpha)\epsilon_0 \\ &= 0 \\ &= S\pi(c_1^*; \mu_1) - \phi + \delta(1-\alpha)\epsilon_1. \end{aligned}$$

¿From (10), we thus obtain $\theta_1 < \theta_0$. (This would hold trivially if there is no turnover in the new stationary equilibrium.) To see that $M_1 < M_0$, notice that $\mu_1 \prec \mu_0$ and $c_1^* > c_0^*$ imply that $\mu_1\left([0, c_0^*]\right) < \mu_0\left([0, c_0^*]\right)$. Using (D), the result follows. ■

The assertion of the proposition may be roughly explained as follows. Both the marginal incumbent (with cost level c^*) and the new entrant have a value of zero in equilibrium. Now, since the marginal incumbent has already sunk the entry cost, the

[17] This monotonic relationship also holds between c^* and the alternative turnover measure $\widehat{\theta} = G(c^*)\theta$. Hence, the main results of the paper are not sensitive to the choice of the turnover measure.

average entrant has to be more efficient than the marginal incumbent. Clearly, this wedge in efficiency is increasing in the level of entry costs. That is, exit policy c^* increases with ϵ. This, in turn, implies, that the hazard rate of incumbents is negatively related to the level of entry costs. A more formal explanation is the following. For any exit policy c^* and distribution μ, an increase in entry cost ϵ reduces the value of an entrant; that is, the curve ψ in Figure 1 shifts downwards. For the entry condition (E) to hold in the new equilibrium, the distribution of active firms must be smaller. This implies that the net profit $S\pi(c;\mu)-\phi$ of any type c goes up. Hence, the value of the marginal incumbent in the old equilibrium is now positive. This shows that the marginal incumbent is less efficient in the new equilibrium.[18]

Next, we analyse the effect of a change in ϕ, which may be interpreted as a fixed production cost or as an opportunity cost. The following proposition summarises our results.

Proposition 5 *Suppose assumption A.1 holds. Then, an increase in the fixed (or opportunity) cost ϕ leads to a decrease in exit policy c^*, and hence to a higher turnover rate θ. Furthermore, it causes the distribution of active firms, μ, and the total mass of active firms, $\mu([0,c^*])$, to decrease.*

Proof. Starting from a stationary equilibrium with simultaneous entry and exit, denoted by (μ_0, M_0, c_0^*), we consider an increase in ϕ from ϕ_0 to ϕ_1, $\phi_1 > \phi_0$. Assume that there is still a positive turnover rate in the new stationary equilibrium (μ_1, M_1, c_1^*), i.e. $\theta_1 > 0$. From entry condition (E), it follows that

$$\psi(c^*;\mu_0;\phi_1) < \psi(c^*;\mu_0;\phi_0) \text{ for all } c^* \in (0,1],$$

and $\psi(0;\mu_0;\phi_1) = \psi(0;\mu_0;\phi_0)$. This implies that we must have $\mu_1 \prec \mu_0$ for (E) to hold again in the new stationary equilibrium. We now claim that $c_1^* < c_0^*$. To see this, suppose otherwise that $c_1^* \geq c_0^*$. According to condition (X),

$$S\pi(c_1^*;\mu_1) - \phi_1 + \delta(1-\alpha)\epsilon = 0 = S\pi(c_0^*;\mu_0) - \phi_0 + \delta(1-\alpha)\epsilon,$$

which implies that $S\pi(c_0^*;\mu_1) - \phi_1 \geq S\pi(c_0^*;\mu_0) - \phi_0$. Assumption A.1 then ensures that

$$S\pi(c;\mu_1) - \phi_1 > S\pi(c;\mu_0) - \phi_0 \text{ for all } c \in [0, c_0^*). \tag{11}$$

Thus, we obtain

$$\begin{aligned} \psi(c_1^*;\mu_1;\phi_1) &\geq \psi(c_0^*;\mu_1;\phi_1) \\ &> \psi(c_0^*;\mu_0;\phi_0) \\ &= 0, \end{aligned}$$

[18]The same result has been obtained by Hopenhayn (1992) in a model of perfect competition.

where the first inequality follows from the fact that $\psi(\cdot;\mu_1;\phi_1)$ assumes a maximum at c_1^*, and the second inequality from (11). Now, $\psi(c_1^*;\mu_1;\phi_1) > 0$ cannot hold as it is in contradiction with (E). That is, we must have $c_1^* < c_0^*$. Finally, notice that, from (10), the turnover rate decreases monotonically with c^*, holding α fixed. Let us now show that we must indeed have $\theta_1 > 0$ (as assumed above), given that $\theta_0 > 0$. Define μ_0' and μ_1' by $\psi(1;\mu_0';\phi_0) = 0$ and $\psi(1;\mu_1';\phi_1) = 0$, respectively. (In the proof of Proposition 2, we have already shown that such measures exist.) It is easy to see that $\phi_1 > \phi_0$ implies $\mu_1' \prec \mu_0'$. Since $\theta_0 > 0$ by assumption, we have $\psi(c^*;\mu_0';\phi_0) = 0$ for some $c^* \in (0,1)$. From assumption A.1, $\mu_1' \prec \mu_0'$, and $\phi_1 > \phi_0$, we get $\psi(c^*;\mu_1';\phi_1) > \psi(c^*;\mu_0';\phi_0)$ for all $c^* \in (0,1)$. This concludes the proof of the assertion on turnover. The result on the total mass of active firms follows immediately from $\mu_1 \prec \mu_0$ and $c_1^* < c_0^*$. ■

Holding fixed the distribution of active firms, an increase in the fixed cost ϕ shifts the curve ψ downwards: the value of an entrant decreases. Since the equilibrium value of an entrant is zero, this implies that the distribution μ is smaller in the new equilibrium. Hence, there are two opposing effects on an incumbent's net profit $S\pi(c;\mu) - \phi$. The increase in ϕ reduces the net profit all types by the same amount. In contrast, A.1 implies that a decrease in μ (which, by assumption, raises the profit of all types) increases the profit of more efficient firms by a larger amount than the profit of less efficient firms. In particular, if there exists a type for which the value remains unchanged, then the value of all superior types increases, while the opposite holds for all inferior types. Moreover, for the entry condition (E) to hold, the value of an incumbent cannot increase for all types in $[0, c^*]$. It follows that the value of the marginal incumbent in the old equilibrium, conditional on staying in the market, is now negative. This shows that the exit policy c^* decreases with ϕ, which implies the predicted negative relationship between the fixed cost ϕ and turnover rate θ.

We now turn to our major concern, namely the relationship between market size and firm turnover. The central prediction of this paper is summarised in the following proposition.

Proposition 6 *Suppose assumption A.2 holds. Then, an increase in market size S leads to a decrease in exit policy c^*, and hence to a rise in the turnover rate θ. Furthermore, an increase in market size causes the distribution of active firms, μ, and the mass of entrants per period, M, to rise.*

Proof. Starting from a stationary equilibrium with simultaneous entry and exit, denoted by (μ_0, M_0, c_0^*), we consider an increase in the size of the market from S_0 to S_1, $S_1 > S_0$. Let us assume that there is still a positive turnover rate in the new stationary equilibrium (μ_1, M_1, c_1^*), i.e. $\theta_1 > 0$. (It is straightforward to show that turnover must indeed be positive in the new equilibrium, given that $\theta_0 > 0$. The argument is similar to that in the proof of Proposition 5, replacing assumption A.1 by A.2.) The proof proceeds in several steps. First, notice that

$$\psi\left(c^*;\mu_0;S_1\right) > \psi\left(c^*;\mu_0;S_0\right) \text{ for all } c^* \in (0,1],$$

and $\psi\left(0;\mu_0;S_1\right) = \psi\left(0;\mu_0;S_0\right)$. For entry condition (E) to hold in the new equilibrium, we thus need $\mu_1 \succ \mu_0$. Second, suppose there exists a y, $y \in (0,\overline{c}(\mu_1))$ such that $S_1\pi(y;\mu_1) = S_0\pi(y;\mu_0)$. Assumption A.2 then implies that $S_1\pi(c;\mu_1) > S_0\pi(c;\mu_0)$ for all $c \in [0,y)$, and the reverse inequality for all $c \in (y,\overline{c}(\mu_1))$. Third, assume the assertion of the proposition does not hold, such that $c_1^* \geq c_0^*$. Then,

$$\begin{aligned} S_1\pi(c_1^*;\mu_1) &= S_0\pi(c_0^*;\mu_0) \\ &\geq S_0\pi(c_1^*;\mu_0), \end{aligned}$$

where the equality follows from condition (X). From A.2 we then obtain

$$S_1\pi(c;\mu_1) > S_0\pi(c;\mu_0) \text{ for all } c \in [0,c_1^*) \tag{12}$$

Consequently,

$$\begin{aligned} \psi\left(c_1^*;\mu_1;S_1\right) &\geq \psi\left(c_0^*;\mu_1;S_1\right) \\ &> \psi\left(c_0^*;\mu_0;S_0\right) \\ &= 0, \end{aligned}$$

where the first inequality follows from the fact that $\psi\left(\cdot;\mu_1;S_1\right)$ is maximised at c_1^*, and the second inequality from (12). Thus, entry condition (E) cannot hold in the new equilibrium: a contradiction. That is, we must indeed have $c_1^* < c_0^*$, and hence $\theta_1 > \theta_0$. It is worth mentioning that y exists and is in $(0,c_0^*)$; otherwise (E) would be violated. Finally, let us consider the effect of the increase in market size on the mass of firms that enter each per period. Since $\mu_1 \succ \mu_0$ and $c_1^* < c_0^*$, we obtain $\mu_1\left([0,c_1^*]\right) > \mu_0\left([0,c_1^*]\right)$, and hence, using (D), $M_1 > M_0$. ■

The result may be explained as follows. In a free entry equilibrium, the distribution of active firms, μ, is positively related to market size. Holding the distribution of active firms fixed, an increase in market size raises the value of firms for any exit policy. Graphically, this means that the curve ψ in Figure 1 shifts upwards. Free entry then implies that the distribution of firms has to increase with market size. This shifts the curve ψ downwards. Hence, there are two opposing effects on a firm's gross profit $S\pi(c;\mu)$. On the one hand, the rise in market size S increases the profits of all firms proportionally. This can be thought of as an increase in output levels, holding prices fixed. On the other hand, as the distribution of firms increases, prices (and, hence, price-cost margins) fall. A.2 implies that the percentage decrease in profit from an increase in μ is greater the less efficient is the firm. Hence, if there is some type for which the value remains unchanged, then the value of all better types increases, and that of all worse types decreases. Since the value of an entrant remains unchanged (and equal to zero), there must be some types in $[0, c^*]$, which are worse off in the larger market. In particular, the marginal incumbent in the old equilibrium would now have a negative value if it decided to use the same exit policy as before. This implies that the marginal surviving firm has to be more efficient in larger markets. That is, exit policy c^* is decreasing with market size. The hazard rate of the average incumbent firm is therefore higher in larger markets: firm turnover and market size are positively correlated.

It is important to point out that our prediction on market size and firm turnover (Proposition 6) would not obtain in a model of perfect competition (as in Hopenhayn (1992)) or in a Dixit-Stiglitz type model of monopolistic competition. The reason is that, in these models, price-cost margins are independent of market size; this is in contrast to many standard models of oligopolistic competition. Let us illustrate this in a model of a homogenous goods industry with perfect competition. In such a model, the gross profit of a type-c firm may be written as $\pi(c; p)$, where p is the equilibrium price. Market size enters the profit function only indirectly through p. The value of a type-c incumbent at stage 4 may then be denoted by $V(c; p)$. Under weak assumptions, $V(c; p)$ is strictly increasing in p for all $c \in [0, c^*]$, where c^* is the optimal exit policy, given price p. Furthermore, $V(c; p) > 0$ for all $c \in [0, c^*)$, and $V(c; p) = 0$ for all $c \in [c^*, 1]$. The entry condition for a stationary equilibrium is given by

$$\int_0^1 V(c; p) G(dc) - \epsilon = 0.$$

Hence, the entry condition uniquely determines the equilibrium price p, which is independent of market size. Given p, the exit threshold is uniquely determined by the exit condition $V(c^*; p) = 0$. Consequently, in a model of perfect competition, exit policy and turnover rate do not vary with market size.

The reasoning in the proof of Proposition 6 shows that efficient firms make higher profits in larger markets, and hence are more valuable. In contrast, less efficient firms are better off in smaller markets. That is, the distribution of profits and firm values is more skewed in larger markets. The effect on profits is illustrated graphically in Figure 2.

Corollary 2 *The range of profits and firm values across active firms in the same market is increasing with market size. Formally,* $\Delta\pi(S) \equiv S\pi(0; \mu) - S\pi(c^*; \mu)$ *and* $\Delta V(S) \equiv V(0) - V(c^*)$ *are increasing with* S.

Proof. Exit condition (X) implies that $S\pi(c^*; \mu) = \phi - \delta(1 - \alpha)\epsilon$, and hence $\Delta\pi(S_1) - \Delta\pi(S_0) = S_1\pi(0; \mu_1) - S_0\pi(0; \mu_0)$, using the same notation as in the proof of Proposition 6. For $S_1 > S_0$ (and, therefore, $\mu_1 \succ \mu_0$), the last expression is strictly positive since $y > 0$. Similarly, $V(c^*) = 0$, and hence $\Delta V(S) = V(0)$. Using (8) and $y > 0$, one obtains the result. ■

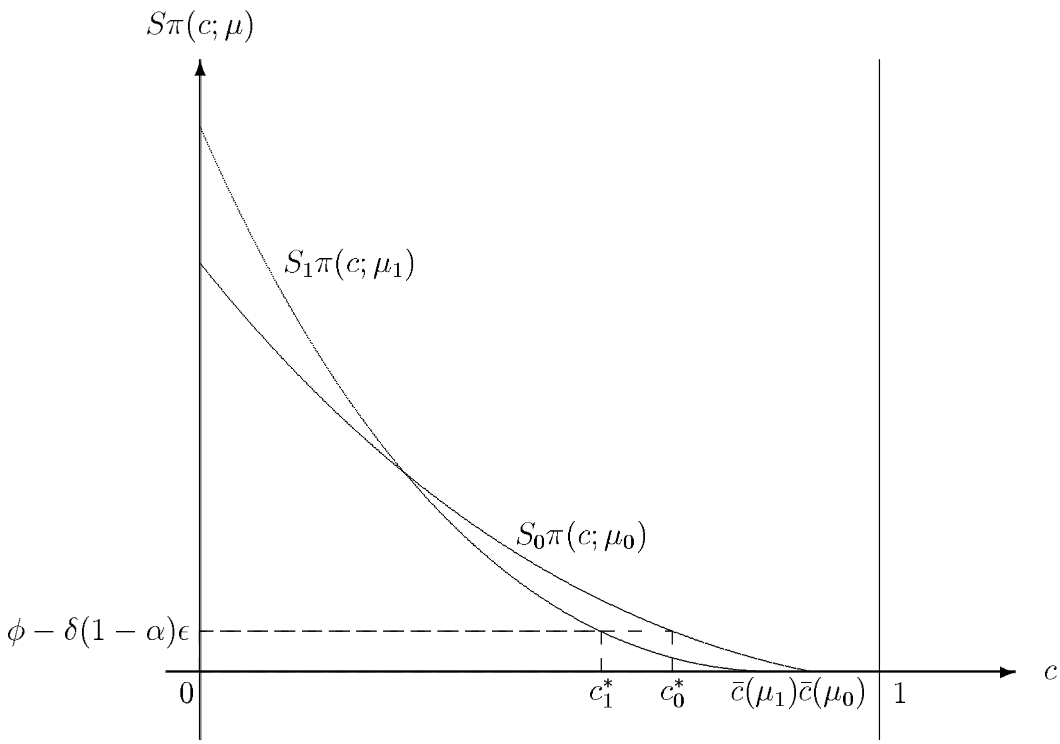

Figure 2: The effect of an increase of market size on gross profits: $S_1 > S_0$, and hence $\mu_1 \succ \mu_0$.

Our model implies that the measure of active firms μ increases with market size S in terms of our ordering on measures. Without imposing further assumptions, it does not, however, imply that the mass ("number") of active firms, $\mu([0, c^*])$, increases with market size. Since the "average efficiency" of firms increases with market size (as c^* is negatively related to S), the total mass of active firms may actually decrease with an increase in market size. Empirically, this selection effect does not appear to be strong enough to generate a non-monotonic relationship between market size and the number of active firms; see Bresnahan and Reiss (1991), and Campbell and Hopenhayn (1999).

In many industries, one may expect fixed costs (e.g. rents) to be higher in larger markets. In contrast, the costs of setting up a firm are often independent of market size. The following reassuring result is an immediate implication of Propositions 5 and 6.

Corollary 3 *Suppose entry and fixed costs are functions of market size. Specifically, let $\epsilon(S) = h(S)\overline{\epsilon}$ and $\phi(S) = f(S)\overline{\phi}$. Assume that $f(S)/h(S)$ and $S/h(S)$ are increasing in market size. Then, turnover rate θ is increasing with market size S.*

Hence, as long as entry costs do not rise linearly with market size, and not faster than fixed costs, the empirical prediction of the paper holds: firm turnover is higher the larger is the market.

Let us now consider a change in the persistence of a firm's efficiency.

Proposition 7 *An increase in the persistence parameter α leads to a decrease in the exit policy c^*. The overall effect on turnover rate θ is ambiguous, but the effect on the alternative turnover measure $G(c^*)\theta$ is unambiguously negative.*

Proof. Starting from a stationary equilibrium with simultaneous entry and exit, denoted by (μ_0, M_0, c_0^*), we consider an increase in the level of persistence from α_0 to α_1, $\alpha_1 > \alpha_0$. Let us assume that there is still a positive turnover rate in the new stationary equilibrium (μ_1, M_1, c_1^*); otherwise, the result holds trivially. The first step in the proof consists in showing that $\mu_1 \succ \mu_0$. This follows immediately from condition (E) and the fact that

$$\psi\left(c^*; \mu_0; \alpha_1\right) > \psi\left(c^*; \mu_0; \alpha_0\right) \text{ for all } c^* \in [0, 1),$$

and $\psi\left(1; \mu_0; \alpha_1\right) = \psi\left(1; \mu_0; \alpha_0\right)$. The second step consists in proving that $c_1^* < c_0^*$. To see this, notice that

$$S\pi(c_0^*; \mu_1) - \phi + \delta(1 - \alpha_1)\epsilon < S\pi(c_0^*; \mu_0) - \phi + \delta(1 - \alpha_0)\epsilon = 0.$$

The claim then follows from condition (X) and assumption C.1. In the final step of the proof, we show that $G(c_1^*)\theta_1 < G(c_0^*)\theta_0$. From $\mu_1 \succ \mu_0$ and $c_1^* < c_0^*$., we get

$$\eta_1 \equiv \int_0^{c_1^*} \left[S\pi(c; \mu_1) - \phi\right] G(dc) < \int_0^{c_0^*} \left[S\pi(c; \mu_0) - \phi\right] G(dc) \equiv \eta_0.$$

Condition (E) implies

$$\begin{aligned}\psi\left(c_1^*;\mu_1;\alpha_1\right) &\equiv \eta_1 - G(c_1^*)\theta_1\delta\epsilon - (1-\delta)\epsilon \\ &= 0 \\ &= \eta_0 - G(c_0^*)\theta_0\delta\epsilon - (1-\delta)\epsilon \equiv \psi\left(c_0^*;\mu_0;\alpha_0\right).\end{aligned}$$

Hence, the result obtains. ■

The prediction that more persistence implies less turnover seems to be obvious, but it is not. The reason is that the marginal incumbent, who is less efficient than the average entrant, is only in the market because of the prospect of lower costs in the future ("option value effect"); indeed, the marginal incumbent's current net profit, $S\pi(c^*;\mu)-\phi$, is negative. An increase in the persistence of costs decreases the value of this option; that is, the marginal incumbent has to be more efficient. This implies that there are two opposing effects on the turnover rate θ: the first term in (10), $(1-\alpha)$, clearly decreases with α, but the second term, $\left[1-G(c^*)\right]/G(c^*)$, is positively correlated with α. The overall effect on θ is ambiguous. Nevertheless, one can show that the overall effect is unambiguous if we use the alternative turnover measure $G(c^*)\theta$, which does not keep track of those entrants that exit immediately after learning their current efficiency.

In empirical applications, it may be feasible to control for cross-industry differences in sunk costs. But it is probably extremely difficult to control for differences in the stochastic process governing the evolution of firms' efficiencies (or consumers' tastes), such as differences in our persistence parameter α. Turnover rates are likely to be affected by such differences. This indicates that any cross-industry study involving industries with very different products and technologies is likely to face major problems. In contrast, in our empirical test, we analyse turnover rates across independent local markets within the same industry. This methodology should keep differences in the level of α as small as possible.

5 Growing and Declining Markets

Intuitively, one may expect entry (exit) rates to be high (low) in periods of market growth and low (high) in periods of decline. This intuition is false. The aim of this section is to analyse entry and exit rates when market size changes over time. In nonstationary markets, entry and exit rates will, in general, be different. Hence, our previous analysis does not carry over. In this section, we show that both entry and exit rates tend to be positively correlated with market size.

The exogenous evolution of market size is summarised by the deterministic sequence $\{S_t\}$, which is common knowledge to all firms. All other exogenous variables remain constant over time. To make the analysis more tractable, we assume throughout in this section that changes in market size and entry costs are sufficiently small so that there is simultaneous entry and exit in each period along the equilibrium path. Moreover, let us simplify the analysis further by assuming $\alpha = 0$; that is, in each period, all incumbents get a new draw from distribution $G(\cdot)$.

The value function of an incumbent in period t, stage 3, is denoted by $V_t(\cdot)$. It is given by

$$V_t(c) = \max\left\{0, S_t\pi(c;\mu_t) - \phi + \delta\int_0^{c_{t+1}^*} V_{t+1}(z)G(dz)\right\},$$

where μ_t is the distribution of active firms in period t, stage 4, and c_{t+1}^* the exit policy in $t+1$. The value of an entrant at the start of period t, which is denoted by V_t^e, can be written as

$$V_t^e = \int_0^{c_t^*} V_t(c)G(dc) - \epsilon.$$

Since we assume entry costs and the changes in market size to be small, there is simultaneous entry and exit in each period, i.e. $c_t^* \in (0,1)$ and $M_t > 0$ for all t. This implies that the value of an entrant is always zero, as is the value of the marginal incumbent: $V_t^e = 0$ and $V_t(c_t^*) = 0$ for all t. The entry and exit conditions can then be expressed as

$$\int_0^{c_t^*} \left[S_t\pi(c;\mu_t) - \phi + \delta\epsilon\right] G(dc) - \epsilon = 0, \tag{E$_t$}$$

and

$$S_t\pi(c_t^*;\mu_t) - \phi + \delta\epsilon = 0, \tag{X$_t$}$$

respectively. Given that all firms follow exit policy $\{c_t^*\}$ and the mass of new entrants is given by $\{M_t\}$, we obtain the following relationship between the distribution of active firms in period t and the total mass of active firms in $t-1$:

$$\mu_t([0,z]) = \left[M_t + \mu_{t-1}\left(\left[0, c_{t-1}^*\right]\right)\right] G\left(\min\{z, c_t^*\}\right), \forall z \in [0,1]. \tag{D$_t$}$$

As before, the equilibrium distribution is an element of $\mathcal{M}^*$. Given the mass of initially active firms, equilibrium is described by the sequence $\{\mu_t, M_t, c_t^*\}$ satisfying equations (E$_t$), (X$_t$), and (D$_t$) for all t.

We define the entry rate to be "forward looking", and the exit rate to be "backward looking". The period-t entry rate, η_t, is the ratio between period-t-entrants and the mass of active firms at the output stage of the same period. Formally,[19]

$$\eta_t \equiv \frac{M_t}{\mu_t([0,c_t^*])}.$$

[19] Again, we could use $\widehat{\eta}_t \equiv G(c^*)\eta_t$ as an alternative measure, which does not keep track of those entrants that exit immediately after learning their current efficiency.

The period-t exit rate, χ_t, is the share of firms active in period $t-1$, which have left the market in period t. That is,

$$\begin{aligned} \chi_t &\equiv \frac{[1-G(c_t^*)]\, \mu_{t-1}\left(\left[0, c_{t-1}^*\right]\right)}{\mu_{t-1}\left(\left[0, c_{t-1}^*\right]\right)} \\ &= 1-G(c_t^*). \end{aligned}$$

Note that the future enters conditions (E_t) and (X_t) only through the option value of staying in the market, which is constant over time and given by $\delta\epsilon$, provided (as assumed) that there is positive gross entry in all future periods. In contrast, the future does not directly enter condition (D_t). The only way the past enters the equilibrium conditions is in (D_t) through the total mass of firms active in the previous period. As a result, the dynamic entry and exit conditions, (E_t) and (X_t), are equal to the static ones, (E) and (X), except for the fact that $\alpha = 0$, and S and μ are indexed by t. Now, from our previous analysis (proof of Proposition 2), we know that the entry and exit conditions uniquely determine the equilibrium exit policy and the "equivalence class" of the stationary distribution. That is, c_t^* and the equivalence class of μ_t are uniquely defined by (E_t) and (X_t). In particular, they are independent of future and past values of market size. Since the stationary distribution in period t is, according to (D_t), given by distribution function $G(\cdot)$, truncated at c_t^*, and scaled by $M_t+\mu_{t-1}\left(\left[0, c_{t-1}^*\right]\right)$, conditions ($\mathrm{E}_t$) and ($\mathrm{X}_t$) also pin down the scaling factor, and hence the total mass of active firms, $\mu_t\left([0, c_t^*]\right)$. Thus, condition ($\mathrm{D}_t$) uniquely determines the mass of new entrants, M_t, as a function of $\mu_{t-1}\left(\left[0, c_{t-1}^*\right]\right)$ and S_t. More specifically, since (E_t) and (X_t) pin down $M_t + \mu_{t-1}\left(\left[0, c_{t-1}^*\right]\right)$, any change in the mass of last period's active firms will be exactly offset by a change in this period's mass of entrants (again, provided there is some entry and exit in all periods).

We are now in the position to state our two main results on firm turnover when market size changes over time. First, we consider a given sequence $\{S_t\}$, and analyse the effect of changing market size in a single period. Then, we analyse the co-movements between $\{S_t\}$, $\{\eta_t\}$, and $\{\chi_t\}$.

Proposition 8 *Consider two sequences of market size, $\{S_t\}$ and $\{S_t'\}$, where $S_r < S_r'$, and $S_t = S_t'$ for all $t \neq r$. Then, the resulting equilibrium sequences of entry rates are such that $\eta_t = \eta_t'$ for all $t < r$ and $t > r+1$, $\eta_r < \eta_r'$, and $\eta_{r+1} > \eta_{r+1}'$ if and only if $\mu_r\left([0, c_r^*]\right) < \mu_r'\left([0, c_r^{*\prime}]\right)$. Equilibrium exit rates are characterised by $\chi_t = \chi_t'$ for all $t \neq r$, and $\chi_r < \chi_r'$.*

Proof. As pointed out above, the future enters the equilibrium conditions only through the constant option value $\delta\epsilon$ in (E_t) and (X_t). Consequently, the change in market size in period r has no effect on endogenous variables in periods before r. Moreover, again from the discussion above, c_t^* and the equivalence class of μ_t are independent of future and past values of market size. The same applies to $M_t + \mu_{t-1}\left(\left[0, c_{t-1}^*\right]\right)$. Using our comparative statics result on market size, Proposition 6, we then obtain the following characterisation of endogenous variables: $c_t^* = c_t^{*\prime}$ for all $t \neq r$, and $c_r^* > c_r^{*\prime}$, $\mu_t = \mu_t'$ for all $t \neq r$, and $\mu_r \prec \mu_r'$, $\eta_t = \eta_t'$ for all $t < r$ and $t > r+1$, $M_r < M_r'$, and $M_{r+1} > M_{r+1}'$ if and only if $\mu_r\left([0, c_r^*]\right) < \mu_r'\left([0, c_r^{*\prime}]\right)$. The prediction on the evolution of exit rates follows immediately. Let us now show that $\eta_r < \eta_r'$. Condition (D$_t$), $\mu_{r-1} = \mu_{r-1}'$, and $c_{r-1}^* = c_{r-1}^{*\prime}$ imply

$$\frac{\left[M_r' + \mu_{r-1}\left(\left[0, c_{r-1}^*\right]\right)\right] G\left(c_r^{*\prime}\right)}{\mu_r'\left([0, c_r^{*\prime}]\right)} = 1 = \frac{\left[M_r + \mu_{r-1}\left(\left[0, c_{r-1}^*\right]\right)\right] G\left(c_r^*\right)}{\mu_r\left([0, c_r^*]\right)}.$$

Since $M_r < M_r'$, one obtains

$$\frac{M_r' G\left(c_r^{*\prime}\right)}{\mu_r'\left([0, \widehat{c}_r^*]\right)} > \frac{M_r G\left(c_r^*\right)}{\mu_r\left([0, c_r^*]\right)},$$

and hence

$$\frac{M_r'}{\mu_r'\left([0, \widehat{c}_r^*]\right)} > \frac{M_r}{\mu_r\left([0, c_r^*]\right)},$$

which is the desired result.[20] The remaining characterisation of the evolution of entry rates follows from the discussion above. ■

Corollary 4 *Consider an arbitrary sequence of market size, $\{S_t\}$. In equilibrium, exit rates are positively correlated with current market size. More precisely, χ_t is increasing in S_t, and independent of S_r, $r \neq t$. Equilibrium entry rates are positively related to current market size, too, but only if controlling for last period's market size: holding S_{t-1} fixed, entry rate η_t is increasing in S_t, and independent of S_r, $r \neq t, t-1$. Moreover, holding S_t fixed, η_t is negatively correlated with the mass of active firms in $t-1$.*

Let us point out that the predictions of Proposition 8 and Corollary 4 would remain unchanged if we assumed that market size followed some stochastic process, and the realisation of current market size became common knowledge at the start of each period (prior to entry decisions). The results of this section provide us with a useful benchmark. But it should be kept in mind that they have been obtained under the assumption that incumbents' cost draws are i.i.d., i.e. $\alpha = 0$. A more general treatment of growing and declining markets is left for future research.

[20] Observe that the first inequality implies that the result also holds when using the alternative entry measure $\widehat{\eta}_t = G(c^*)\eta_t$.

6 Robustness of Results

In this section, we explore the robustness of our results along two dimensions, namely the sequence of moves and the evolution of firms' efficiencies. So far, we have assumed that exit decisions take place *after* incumbents and entrants learn their current type. This gave rise to a simple mathematical structure. Moreover, it captured the observation that, in many markets, a subset of new entrants make initial investments but never reach the production stage and, hence, exit again. The obvious question is whether our results are sensitive to the sequence of moves. The timing we want to analyse here is as in Hopenhayn's (1992) model. At the first stage, entry and exit decisions take place. At the second stage, the new entrants and surviving incumbents learn their current type. Finally, the active firms play a market game and receive profits.

In the main part of this paper, we assumed that the evolution of an incumbent's efficiency is governed by equation (1). This stochastic process allowed us to obtain simple closed-form solutions for firms' value functions and the stationary distribution, expressed only in terms of the exit policy c^* and the mass of entrants M. We now want to show that the main results of this paper hold more generally for an interesting class of Markov processes. This is of particular interest since the simple stochastic process considered so far cannot account for some of the stylised facts regarding the relationship between firm size and age and firm growth and survival. For instance, if a firm's output $q(c;\mu)$ is decreasing in c, (1) implies that the probability of exit, $(1-\alpha)(1-G(c^*))$, is independent of an incumbent's size. This is inconsistent with the empirical finding that firm size and failure are negatively correlated; see Evans (1987), Hall (1987) , and Dunne, Roberts and Samuelson (1989). With the class of Markov processes considered here, this and other stylised facts are implied by or consistent with our model. For brevity of exposition, we change both the timing and the stochastic process in our model, rather than discussing each modification in turn. Incorporating the new Markov process into the model of Section 2 proceeds in a very similar fashion.

Consider an incumbent of type c. The probability that, in the next period, his marginal cost is less than or equal to c' is given by $F(c'|c)$. For any $c' \in (0,1)$, we assume that $F(c'|c)$ is strictly decreasing in c. This means that a currently efficient firm is more likely to be efficient tomorrow than a currently less efficient firm; this is formally expressed in terms of first-order stochastic dominance. Note that if $q(c;\mu)$ is decreasing in c, and $c^* < 1$, then the probability of survival is increasing with firm size. For convenience, we assume that $F(\cdot|c)$ is strictly increasing on $[0,1]$, and $F(c'|c)$ is continuous in c' and c. Moreover, we posit that the distribution of entrants' efficiencies is given by the distribution of a particular type of incumbent. That is, there exists a cost level $\widehat{c} \in (0,1)$ such that $G(\cdot) \equiv F(\cdot|\widehat{c})$. For some results, we impose a further technical condition, which will be discussed below.

At the entry and exit stage (stage 1), the value of an incumbent with previous cost level c can be written as

$$V(c;\mu,S) \equiv \max\left\{0, \overline{V}(c;\mu,S)\right\},$$

where $\overline{V}(c;\mu,S)$ is the value of the incumbent conditional on staying in the market and behaving optimally thereafter. This conditional value is given by

$$\overline{V}(c;\mu,S) \equiv \int_0^1 W(c';\mu,S)F(dc'|c)$$

where $W(c';\mu,S) \equiv S\pi(c';\mu) - \phi + \delta V(c';\mu,S)$ is the value of a firm *after* learning that its new type is c'. Similarly, prior to learning his current type, the value of a new entrant is given by

$$\begin{aligned} V^e(\mu,S) &\equiv \int_0^1 W(c;\mu,S)G(dc) - \epsilon \\ &= \overline{V}(\widehat{c};\mu,S) - \epsilon. \end{aligned}$$

Standard arguments in dynamic programming imply that the conditional value $\overline{V}(c;\mu,S)$ is strictly decreasing in marginal cost c, provided the conditional value of the most efficient firm is positive, i.e. $\overline{V}(0;\mu,S) > 0$. Furthermore, under the same condition, $\overline{V}(c;\mu,S)$ is strictly decreasing in the distribution μ and strictly increasing in market size S.

In a stationary equilibrium with simultaneous entry and exit, i.e. $c^* \in (0,1)$,

$$V^e(\mu,S) = 0 \tag{E_R}$$

and

$$\overline{V}(c^*;\mu,S) = 0. \tag{X_R}$$

Consequently, if $c^* \in (0,1)$, the "average entrant" $\widehat{c}$ is more efficient than the marginal incumbent c^*: $\widehat{c} < c^*$.

Exit policy c^* and the mass of entrants per period, M, induce a unique stationary distribution. The mass of active firms with costs in $[0,z]$ is given by

$$\mu([0,z]) = MG(z) + \int_0^{c^*}\int_0^z F(dc'|c)\mu(dc),\ \forall z \in [0,1]. \tag{D_R}$$

A stationary equilibrium is a triple $\{\mu, M, c^*\}$ satisfying equations (E_R), (X_R), and (D_R). Proving existence and uniqueness of a stationary equilibrium is beyond the scope of this section. Instead, we focus on the main comparative statics results.

The turnover rate is again defined by $\theta \equiv M/\mu([0,1])$. For the case of the simple Markov process, we showed earlier that the equilibrium turnover rate θ is strictly decreasing in exit policy c^*. This is easily generalised for the large class of Markov processes considered here. In a stationary equilibrium, we can interpret the total mass of firms active at the output stage in period t as being the survivors amongst the firms that entered in periods t, $t-1$, $t-2$, $t-3$, and so on. Decomposing the mass of active firms into different cohorts of surviving firms, we obtain

$$\mu\left([0,1]\right) = M\left[1 + \sum_{s=1}^{\infty}\sigma_s(c^*)\right],$$

where $\sigma_s(c^*)$ is the probability of surviving s periods. This probability is given by

$$\sigma_s(c^*) \equiv \int_0^{c^*}\int_0^{c^*}...\int_0^{c^*}\int_0^{c^*} F(dc_s|c_{s-1})F(dc_{s-1}|c_{s-2})...F(dc_2|c_1)G(dc_1)$$

Since $d\sigma_s(c^*)/dc^* > 0$, the equilibrium turnover rate θ is strictly decreasing with exit policy c^*. Hence, if we are interested in the effect of changes in the underlying parameters on the equilibrium turnover rate, it suffices to consider the effect on the equilibrium exit policy.

We are now in the position to analyse the effects of sunk costs and market size on market turbulence. Throughout, we assume that there is simultaneous entry and exit in the initial stationary equilibrium. First, we consider an increase in entry costs from ϵ_0 to ϵ_1. Holding the distribution of active firms fixed at μ_0, this reduces the value of a new entrant:

$$\overline{V}(\widehat{c};\mu_0,S) - \epsilon_1 < \overline{V}(\widehat{c};\mu_0,S) - \epsilon_0 = 0.$$

For the entry condition (E_R) to hold in the new equilibrium, the distribution of active firms must decrease, i.e. $\mu_1 \prec \mu_0$. This raises the conditional value of any firm. In particular, the conditional value of the marginal incumbent in the initial equilibrium is now positive:

$$\overline{V}(c_0^*;\mu_1,S) > \overline{V}(c_0^*;\mu_0,S) = 0.$$

The exit condition (X_R) then implies that the marginal incumbent is less efficient in the new equilibrium: $c_1^* > c_0^*$. Hence, the rise in entry costs causes the equilibrium turnover rate θ to decrease, as predicted by Proposition 4.

Next, we consider the effect of an increase in market size from S_0 to S_1. As before, we assume A.2. In addition, we impose a technical condition on the Markov process. Specifically, the conditional (cumulative) distribution function $F(c'|c)$ can be decomposed as a weighted average of two distribution functions:

$$F(c'|c) = a(c)\underline{F}(c') + [1 - a(c)]\,\overline{F}(c'), \tag{13}$$

where the weight $a(c)$, $a(c) \in (0,1)$, is continuous and strictly decreasing in c. The (continuous) distribution functions $\underline{F}(\cdot)$ and $\overline{F}(\cdot)$ have support $[0,\varphi]$ and $[\varphi,1]$, respectively, where $\varphi \in (0,1)$. That is, $\underline{F}(\cdot)$ gives the distribution of "good types", and $\overline{F}(\cdot)$ the distribution of "bad types". The more efficient the firm is currently, the more "likely" it is to get a draw from the good distribution in the future: $F(c'|c)$ is decreasing in c. We assume that φ is not too large so that $\varphi < \overline{c}(\mu_0)$.

For a given distribution of active firms, the increase in market size raises the value of a new entrant:

$$V^e(\mu_0, S_1) > V^e(\mu_0, S_0) = 0.$$

Hence, the distribution of active firms is increasing with market size; that is, $\mu_1 \succ \mu_0$. Assumption A.2 implies that there exists a type $y \in (0, c_0^*)$ such that all better types have higher current profits in the larger market, while all worse types have lower profits (provided they make profits at all in the smaller market). Moreover, our assumption on the stochastic process ensure that

$$\int_0^{\varphi} S_1 \pi(c;\mu_1)\underline{F}(dc) > \int_0^{\varphi} S_0 \pi(c;\mu_0)\underline{F}(dc),$$

and

$$\int_{\varphi}^{1} S_1 \pi(c;\mu_1)\overline{F}(dc) < \int_{\varphi}^{1} S_0 \pi(c;\mu_0)\overline{F}(dc).$$

It should be clear that the two inequalities cannot point the same way: if they did, the conditional value of *all* types would rise (fall) with market size, which is inconsistent with the entry condition (E_R). Since the value of an entrant can be written as

$$V^e(\mu, S) = a(\widehat{c}) \int_0^{\varphi} W(c;\mu,S)\underline{F}(dc) + [1 - a(\widehat{c})] \int_{\varphi}^{1} W(c;\mu,S)\overline{F}(dc),$$

we obtain

$$\int_0^{\varphi} W(c;\mu_1,S_1)\underline{F}(dc) > \int_0^{\varphi} W(c;\mu_0,S_0)\underline{F}(dc)$$

and

$$\int_{\varphi}^{1} W(c;\mu_1,S_1)\overline{F}(dc) < \int_{\varphi}^{1} W(c;\mu_0,S_0)\overline{F}(dc).$$

That is, conditional on obtaining a draw from the good distribution, the conditional value of a firm is larger in the larger market, while the opposite relationship holds conditional on getting a draw from the bad distribution. For an entrant, these two effects cancel each other out in expectation. Now, the marginal incumbent firm in the smaller market, c_0^*, is less efficient than the average entrant $\widehat{c}$, and hence obtains a bad draw with a larger probability. This implies that the conditional value of type c_0^* is negative in the larger market, i.e. $\overline{V}(c_0^*;\mu_1,S_1) < 0$. It follows that $c_1^* < c_0^*$. The efficiency of the marginal incumbent rises with market size. This implies the positive relationship between market size and firm turnover predicted by Proposition 6. The analysis of the effect of a change in the fixed cost ϕ proceeds analogously. It is omitted for brevity. The results of this section should be reassuring. The main predictions of this paper are quite robust to changes in the sequence of moves and the stochastic process governing the evolution of incumbents' efficiencies.

7 Empirical Application: Driving Schools in Sweden

To provide a first empirical test of the theory, we need an industry that contains many geographical submarkets of different sizes.[21] The Swedish driving school industry satisfies this condition and also has the attractive feature that very few firms are active in more than one local market. Moreover, there are no national chains.[22] In line with the model's assumptions, entry involves some sunk setup costs and there are certainly some fixed costs of production. Sharply at odds with the theoretical framework, however, is the fact that most markets contain less than a handful of firms. Despite such clear departure from the theoretical model, we believe the industry captures one essential aspect - competition in large markets is more intense.[23]

7.1 The Data

Our information on surviving, exiting, and entering firms are derived from the 1990, 1995, and 2000 editions of the Yellow Pages. A firm present in the Yellow Pages is referred to as "active". The firms that we consider are offering driving lessons with primarily car, motor cycle, and/or light truck, and were located in the Yellow Pages under the heading *Trafikskolor* (Traffic Schools). The market definition follows the Swedish standard municipal classification (288 areas). The market size, S, is the population aged 16-24 in 1990, 1995 and 1999. In 1995 the mean S (in thousands) was 3.42 with a standard deviation of 6.14. We will assume that demand per capita in a given year is the same across markets. Hence, the population variable will capture relative differences

[21]As noted in the previous sections, the problem with cross-industry data lies in controlling for differences in e.g. entry costs, fixed costs, and the nature of the stochastic shocks.

[22]In industries where some firms are active in several markets, the decision to enter (or exit) one market is likely to be related to the decisions taken for some neighbouring markets. Furthermore, in such industries it is often the case that firms have several subsidiaries in the same neighbourhood. To empirically control for such complex strategies is, to say the least, problematic.

[23]This is supported by two previous studies of the industry. Asplund and Sandin (1999a) found a (monotone) concave relation between the number of firms and market size. The argument, advanced by Bresnahan and Reiss (1991), is that an increase in market size permits a larger number of firms, but this is partly offset by more intense competition (lower price-cost margins) such that each firm needs a greater market size to cover fixed costs. Using price data, Asplund and Sandin (1999b) found a negative relation between price levels in a market and the number of active firms.

in market size and, thereby, differences in the number of firms, N. Here, the number of firms N corresponds to the mass of active firms, $\mu([0, c^*])$, in our theoretical model. In 1995, the mean N was 2.55 with a standard deviation of 4.39. This approximation appears to be reasonable, as evidenced by a correlation between S and N above 0.95 in each of the three years. For those firms which were active in non-urban markets (municipals outside the three largest towns with surrounding suburbs) in 1995, the data also include the number of cars each firm operated.

Table 1 shows the definition (by their dates of activity) of the six different "types" of firms in the data, and their frequencies.[24] As seen in the bottom row, the number of firms is almost identical in 1990 and 1995, whereas the number is substantially lower in 2000. To some extent, this can be attributed to a demographic shift which reduced the population aged 16-24 by about 10 percent between 1995 and 1999. However, the population in the age group also shrank by some 10 percent in the previous five years.[25] Hence, while the population of young people provides a good measure of the relative market sizes, there may have been shifts in the demand for driving lessons per capita that affected all markets. We consider the first five years, 1990-1995, to be a period with stable demand. The years 1995-2000 and 1990-2000 provide an opportunity to examine entry and exit patterns in declining markets.

Table 1 also reveals the amount of firm turnover in the industry. In total, there were 977 active firms, out of which 462 survived over the entire sample period. In the 1990-1995 period, 157 out of 740 firms in 1990 exited, and 154 firms entered. The raw numbers suggest that the exit probability is about 5 percent per year: $(1 - 0.05)^5 \approx 0.77 \approx (740 - 157)/740$. Over the last five years, 184 firms out of 737 exited, and only 83 entered.

7.2 Econometric Tests

We suggest two tests of Proposition 6. First, by taking the firms active at $t - 1$ as the sample, we estimate the probability that a firm has exited before t as a function of market characteristics. Second, by using the firms active at t as the sample, we estimate the probability that a firm has entered after $t - 1$. Proposition 6 predicts that both probabilities should be increasing with market size. To identify the effect, we need to assume that the stochastic shocks have the same distribution across markets, and that entry costs and fixed costs are the same. Below, we discuss how we control for the fact

[24] To decide whether a firm has survived between $t - 1$ and t we use the firm's name. (Minor changes in a firm's name, such as adding or deleting a suffix/first name/family name are ignored.)

[25] In relative terms, the decrease in population was about the same across markets in both periods. The median S_{1995}/S_{1990} was 0.88 and the 10th and 90th percentiles were 0.82 and 0.97, respectively. The corresponding numbers for S_{1999}/S_{1995} were 0.89, 0.83, and 0.97. In comparison, the median S_{1990}/S_{1985} was 1.00 and the 10th and 90th percentiles 0.91 and 1.09.

Table 1: The frequency of firms of different types, and the total number of firms in 1990, 1995, and 2000.

	Active 1990	Active 1995	Active 2000	# Firms
Type_1	Yes	Yes	Yes	462
Type_2	Yes	Yes	No	121
Type_3	No	Yes	Yes	91
Type_4	Yes	No	No	157
Type_5	No	Yes	No	63
Type_6	No	No	Yes	83
# Firms	740	737	636	

that the tests are performed on data from markets which do not satisfy all assumptions of the proposition.

Let the dummy variable $EXIT_j$ take the value 1 if firm j, active in market i at $t-1$, exits between $t-1$ and t. Correspondingly, $ENTRY_j$ takes the value 1 if firm j, active in market i at t, has entered between $t-1$ and t. We assume that the probabilities to exit and enter are given by

$$\Pr(EXIT_j) = \Phi(\beta^{EX} X_i^{EX}) \tag{14}$$

$$\Pr(ENTRY_j) = \Phi(\beta^{EN} X_i^{EN}) \tag{15}$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution, X_i^{EX} and X_i^{EN} vectors of market characteristics, and β^{EX} and β^{EN} parameter vectors to be estimated. In (14) we also add some firm-specific characteristics, X_j.

Most importantly, in (14), X_i^{EX} includes the market size at t, $S_{i,t}$, to capture across market differences in market size. Even if the data do not conform fully to the theoretical model we predict that firms in relatively larger markets should have higher exit probabilities.

Section 5 dealt with firm turnover in declining markets, as reflected in the data by the periods 1995-2000 and 1990-2000. It was shown (Proposition 8 and Corollary 4) that exit probabilities tend to be decreasing over time in declining markets. However, the results were derived under the assumption of positive entry and exit rates in each period. Moreover, we considered only a rather special stochastic process exhibiting no persistence over time. In a more general setting, it is conceivable that the intuition of higher (lower) exit probabilities in declining (growing) markets holds. Empirically, we test whether exit probabilities are influenced by changes in market size by including $S_{i,t}/S_{i,t-1}$.

Throughout the previous sections, we assumed that markets are, in each period, in the stationary equilibrium. This need not be the case when the number of firms is small and idiosyncratic uncertainty no longer washes out at the aggregate level. Effectively, this means that the number of firms in market i at t, $N_{i,t}$, may be above or below the (average) number in the stationary equilibrium, $N^*_{i,t-1}$. It seems plausible that exit probabilities are higher (lower) for firms in markets with too many (few) firms. In order to capture this possibility, we add $N_{i,t-1}/N^*_{i,t-1}$ in X_i^{EX}.[26]

To operationalise $N^*_{i,t}$, we associate it with the expected number of firms given the market size, $N^*_{i,t} = E[N_{i,t}|S_{i,t}]$. Although there are several ways to estimate $N^*_{i,t}$, we choose a count data model: the Poisson.[27] The conditional expectation is parameterised as

$$\lambda_{i,t} = E[N_{i,t}|S_{i,t}] = \exp(\delta_0 + \delta_1 \sqrt[2]{S_{i,t}} + \delta_2 S_{i,t}), \tag{16}$$

where N is Poisson distributed with

$$f(N|\lambda) = \frac{\lambda^N \exp(-\lambda)}{N!}. \tag{17}$$

The estimated coefficents in (16) are reported in Table 4 in Appendix B.

Finally, there are some firm-specific characteristics, X_j. For a more general Markov process (for example, of the class discussed in Section 6), the exit probability depends on the incumbent's characteristics. Empirical studies of firm survival tend to find that hazard rates are lower for (i) old firms, and (ii) large firms. To test (i), we use the incumbents in 1995, market size in 2000, and let the dummy variable $ACTIVE\_t - 2$ take the value 1 if the firm was also active in 1990. The prediction is that firms that were active already in 1990 have lower exit probability as they are older. To test (ii), we again use the set of incumbents in 1995, but restrict attention to firms in non-urban markets. As noted above, the data contains information on the number of cars each firm operates in 1995, which provides the measure of firm size, $FIRMSIZE$. The prediction is that exit probabilites are decreasing with firm size.

[26]To see that this intuition may also be misleading, note that even when there are, for example, too many firms in a market at t, the exit probability of *any given* incumbent may be unaffected. Instead, it is a lower number of entrants at $t+1$ that provides the tendency for the number of firms to revert to the stationary equilibrium.

[27]In Bresnahan and Reiss (1991), an ordered probit model was used to estimate the relationship between the number of firms and market size. In Asplund and Sandin's (1999a) study of Swedish driving schools, the results from an ordered probit model were found to be almost identical to those from Tobit and Poisson models. For more on count data models, see Cameron and Trivedi (1998) and Winkelmann (1997).

Next, let us turn to the entry probability (15) and the variables to include in X_i^{EN}. Clearly, the market size at t, $S_{i,t}$, is expected to have a positive effect on entry probabilities. We use $S_{i,t}/S_{i,t-1}$ to control for the possibility that changes in market size have an impact upon entry probabilities. Finally, $N_{i,t-1}/N^*_{i,t-1}$ is included to capture the intuition that an "excess number" of active firms at $t-1$ should reduce the probability that a firm active at t is an entrant. In contrast to (14), we do not have any firm-specific variables to include in the entry equation.

7.3 Results

Table 2 shows the probit regressions for incumbents' exit probabilities. We focus on the period 1990-1995, during which the total number of firms was roughly constant. The coefficient on market size is positive and statistically significant at the one percent level. Hence, the exit probability is increasing with market size, as predicted by Proposition 6.[28] Firms in growing (or less shrinking) markets tend to have lower hazard rates, but the effect is not statistically significant. To some extent this might be explained by the small variation in $S_{i,t}/S_{i,t-1}$. Firms in markets with too many firms (relative to the stationary equilibrium) have a significantly higher exit probability.[29]

Repeating the exercise for the periods 1990-2000 and 1995-2000 (columns 2 and 3), periods in which the number of firms declined, shows that the market size coeffificent is no longer significant.[30] In fact, for the 1995-2000 period, the point estimate is negative. The relative change in market size does not have a significant effect. But as in the first column, the relationship between the actual and expected number of firms at $t-1$ remains a highly significant determinant for exit probabilities.

Finally, results for the firm-specific variables in the 1995-2000 regressions conform to previous studies. Firms that were active at $t-2$ have far lower exit probabilities (column 4). For the subsample of firms in non-urban markets, we find that larger firms (measured by the number of cars) have lower exit probabilities.

Table 3 gives the results for the entry probabilities. Most importantly, market size is not statistically significant in any of the periods. The change in market size never shows

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[28] With only a constant and $S_{i,1995}$, market size is positive and significant. The estimated probit regression is (standard errors in parenthesis): -0.887 (0.0639) + 0.00572 (0.00231) $S_{i,1995}$ with LogL=–379.2.

Adding the squared market size, $S^2_{i,1995}$, gives joint significance of the market size variables: -0.853 (0.0838) - 0.000781 (0.0108) $S_{i,1995}$ + 0.0000921 (0.000149) $S^2_{i,1995}$ with LogL=–379.0.

[29] In the reported regressions we used a continuous measure of $N^*_{i,t}$; $N^*_{i,t} = \exp(\hat{\delta}_0 + \hat{\delta}_1 \sqrt[2]{S_{i,t}} + \hat{\delta}_2 S_{i,t})$. However, the results do not change with other measures such as the integer part of the left hand side, or rounding it to the nearest integer. (Note that the latter two estimates of $N^*_{i,t}$ can be equal to zero even when $N_{i,t} > 0$. In practice this turned out to be a problem only for a handful of markets)

[30] The difference in the number of observations is due to missing information on the market size variable.

Table 2: Probability that a firm active at $t-1$ has exited before t.

$t-1/t$	Probit 1990/1995	Probit 1990/2000	Probit 1995/2000	Probit 1995/2000	Probit 1995/2000
Constant	-1.01*	-0.569	-1.51	-1.17	-0.656
	(0.555)	(0.386)	(0.991)	(1.01)	(1.27)
S_t	0.00776***	0.00454	-0.00119	-0.00157	-0.00408
	(0.00290)	(0.00291)	(0.00324)	(0.00330)	(0.0147)
S_t/S_{t-1}	-0.225	-0.205	0.419	0.561	0.479
	(0.610)	(0.438)	(1.03)	(1.05)	(1.40)
N_{t-1}/N^*_{t-1}	0.242**	0.293***	0.419***	0.333***	0.204
	(0.111)	(0.100)	(0.118)	(0.121)	(0.140)
$ACTIVE\_t-2$				-0.562***	-0.458***
				(0.118)	(0.139)
$FIRMSIZE$					-0.147***
					(0.0481)
LogL	-376.6	-483.1	-408.6	-397.4	-291.4
LogL (R)	-381.9	-487.9	-413.9	-413.9	-305.8
Nobs	738	737	736	736	540

Notes: LogL (R) refers to loglikelihood value with only a constant. N^*_t estimated by Poisson.

up significant. Notice, however, that for a given period the sign of coefficients on market size and the change in market size are the same in Tables 2 and 3. The only significant variable is $N_{i,t-1}/N^*_{i,t-1}$, with the opposite sign from that in Table 2. This shows that firms tend to enter the markets with too few incumbent firms.

As noted above, in the driving school industry most markets contain few firms whereas the theoretical model was in a monopolistic competition setting. As a robustness test we have estimated (14) and (15) for subsamples of larger markets, where there are more firms. Overall, the results are quite similar to those reported in (2) and (3). For instance, restricting the sample to markets with a 1990 population in the age group exceeding 5000, exit probabilities for the period 1990-1995 are significantly increasing in market size. The coefficient on market size in the entry equation, in the same period, is positive but not statistically significant.

While not conclusive, the results give some support for the main proposition in the paper: in stationary environments, hazard rates are increasing with market size.

Table 3: Probability that a firm active at t has entered after $t-1$.

$t-1/t$	Probit 1990/1995	Probit 1990/2000	Probit 1995/2000
Constant	0.486	-0.0983	-1.97
	(0.621)	(0.430)	(1.29)
S_t	0.000526	0.00106	-0.000133
	(0.00325)	(0.00334)	(0.00385)
S_t/S_{t-1}	-0.661	-0.130	1.18
	(0.691)	(0.496)	(1.35)
N_{t-1}/N^*_{t-1}	-0.618***	-0.355***	-0.218*
	(0.123)	(0.120)	(0.133)
LogL	-362.2	-366.7	-241.0
LogL (R)	-377.3	-371.3	-244.1
Nobs	735	633	634

Notes: LogL (R) refers to loglikelihood value with only a constant. N^*_t estimated by Poisson.

8 Conclusion

Many empirical studies in industrial organisation and labour economics have shown that industries differ substantially in the level of firm turnover and gross job reallocation. These differences are stable over time and similar across countries. Most of the reallocation of inputs and outputs can be accounted for by reallocation within narrowly defined industries. The open research agenda is to explain these industry differences in turnover levels and relate them to industry characteristics.

This paper has examined the determinants of market turbulence, in particular the relationship between market size and entry and exit rates. To this end, we have analysed a stochastic dynamic model of a monopolistically competitive industry. Firms are heterogenous and subject to idiosyncratic shocks to their efficiencies. The stationary equilibrium exhibits simultaneous entry and exit: currently efficient firms survive while firms with bad cost draws leave the market and are replaced by new entrants. The main prediction of the model is that the rate of firm turnover is positively related to the size of the market. For a given distribution of active firms, an increase in market size raises firms' sales and profits proportionally. But the distribution of firms is endogenous: free entry implies that it increases with market size. An increase in the distribution of firms causes price-cost margins to fall. This hurts all firms. However, the fractional reduction in profits is smaller the more efficient is the firm. This implies that, in larger markets, efficient firms are better off and inefficient firms worse off, while the profit of the average entrant remains unchanged. It follows that firms have to be more efficient in larger markets in order to survive, and the probability of failure is larger. Somewhat informally, our model provides an underpinning for the common claim that "more intense product market competition fosters efficiency": in larger markets, price-cost margins are narrower and the marginal firm more efficient.

In this paper, we have also investigated the effects of changes in sunk costs on turbulence: the equilibrium turnover rate is decreasing with entry costs and increasing with fixed or opportunity costs. In an extension of the model, we have shown that both entry and exit rates should tend to increase over time in growing markets, and decrease in declining markets. The paper provides a number of results in addition to those on market turbulence. In particular, the model predicts that, in larger markets, firms are more efficient on average, and the distributions of profits and firm values are more skewed than in smaller markets. Our results may have important implications not only for industrial market structure. For instance, an increase in market size may be interpreted as the opening of an industry (or country) to trade, e.g. as a move from two closed economies to a fully integrated economy. Such a move has no effect on average efficiency and turnover levels in Melitz' (1999) trade model, which uses a specification *à la* Dixit-Stiglitz. In contrast, our model predicts that average firm efficiency and turnover levels should rise as a result of economic integration (or opening to trade). The merger wave currently taking place in the European Union may be interpreted in this light.

In the empirical part of the paper, we set out to test the predictions of the model. The idea of the empirical test has been to study an industry where firms compete in well-defined local markets of different sizes. This should avoid the usual problems with cross-industry studies since we compare turnover rates within the same industry. Specifically, we used data on driving schools in Sweden that were active in the 1990s. The empirical results provide some support for the prediction that hazard rates are increasing with market size.

Appendix A

Properties of the Profit Function in a Cournot Model. We want to show that assumptions A.1 and A.2 hold in a homogenous goods Cournot model, where firms differ in their (constant) marginal costs. Let $P(Q/S)$ denote inverse demand when aggregate output is Q and market size is S. We assume that the demand function is downward-sloping, i.e. $P'(\cdot) < 0$. In equilibrium, aggregate output Q will be some (possibly complicated) function of the vector of firms' marginal costs, i.e. $Q = Sf(\mathbf{c})$, where $\mathbf{c}$ is the vector of firms' marginal costs. It is therefore convenient to consider changes in aggregate output which reflect changes in the underlying distribution of firms' efficiencies. Conditional on aggregate output Q, the equilibrium output of a firm with constant marginal cost c is denoted by $Sq(c;Q)$. (The function $Sq(c;\cdot)$ is sometimes called the backward-reaction function.) The first-order condition for profit maximisation is given by

$$P\left(Q/S\right) - c + q(c;Q)P'\left(Q/S\right) = 0, \tag{18}$$

which implies

$$Sq(c;Q) = -S\frac{P(Q/S) - c}{P'(Q/S)}.$$

The associated second-order condition is given by

$$2P'(Q/S) + q(c;Q)P''(Q/S) < 0. \tag{19}$$

It is straightforward to show that (18) and (19) imply that a firm's equilibrium equilibrium profit $S\pi(c;Q)$ is strictly decreasing in industry output Q. Any change in the underlying distribution of efficiencies which reduces firms' profits must induce an increase in industry output Q. Hence, an increase in Q is equivalent to an increase in the distribution of firms as defined in the main text. Moreover, the assumption of complete ordering of distributions (C.3) is satisfied. Conditional on industry output Q, the equilibrium profit of a type-c firm can be written as

$$S\pi(c;Q) = -S\frac{\left(P(Q/S) - c\right)^2}{P'(Q/S)}.$$

We now consider an increase in the distribution of active firms: suppose aggregate output increases from Q to Q', $Q' > Q$. Assumption A.2 says that the profit ratio $\pi(c; Q')/\pi(c; Q)$ is decreasing in c for all c such that $q(c; Q) > 0$. It is immediate to see that this condition holds if and only if $P(Q'/S) < P(Q/S)$, which is clearly satisfied since the inverse demand function is downward-sloping. Assumption A.1 requires that the profit difference $\pi(c; Q') - \pi(c; Q)$ is increasing in c for all c such that $q(c; Q) > 0$. Taking the derivative with respect to c, we obtain that A.1 holds if and only if $q(c; Q') < q(c; Q)$, i.e. a firm's equilibrium output is decreasing in the distribution of active firms. This inequality is satisfied if

$$P'(Q/S) + q(c; Q)P''(Q/S) < 0,$$

which is equivalent to the assumption that quantities are strategic substitutes (firms' reaction curves are downward-sloping). It is a rather weak (and standard) assumption in Cournot models. Hence, in a Cournot model with homogenous products and constant marginal costs, A.1 and A.2 hold under fairly general conditions on demand.[31]

Appendix B

Table 4: Poisson estimates of $\lambda_{i,t} = E\left[N^*_{i,t} | S_{i,t}\right] = \exp\left(\delta_0 + \delta_1\sqrt{S_{i,t}} + \delta_2 S_{i,t}\right)$.

t	Poisson 1990	Poisson 1995
Constant	-1.32	-1.06
	(0.133)	(0.121)
$\sqrt{S_{i,t}}$	1.24	1.16
	(0.0758)	(0.0713)
$S_{i,t}$	-0.0728	-0.0692
	(0.00821)	(0.00782)
LogL	-410.0	-421.8
LogL (R)	-877.5	-829.2

[31]It is easy to check that assumptions C.1 to C.4 are satisfied as well. The first part of C.5 holds if $\lim_{Q\to\infty} P(Q) = 0$. The second part of C.5 is difficult to interpret with a discrete number of firms. But, essentially, it says that fixed costs and entry costs are sufficiently small so as to make entry of at least one firm profitable.

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