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LOCAL LABOUR MARKETS, JOB MATCHING AND URBAN LOCATION

Jan K Brueckner, Jacques-Francois Thisse and Yves Zenou

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Jan K Brueckner, University of Illinois
Jacques-François Thisse, CERAS, Paris, CORE, Université Catholique de Louvain and CEPR
Yves Zenou, Southampton University and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Local Labour Markets, Job Matching and Urban Location*

We present a new way of modelling local labour markets by linking the space of workers' skills and the physical space of cities. The key lesson of our analysis is that firms exploit workers in these two spaces by setting wages that are below the competitive level. The degree of monopsony power depends on the elasticity of the firm's labour pool, which is inversely related to the costs workers incur in commuting and acquiring skills. Our analysis thus shows how socio-economic ghettos emerge as workers with poor skill matches are also those who incur the highest commuting costs.

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Keywords: imperfect labour markets, spatial mismatch, system of cities

Jan K Brueckner
Department of Economics
University of Illinois
1206 South Sixth Street
Champaign, IL 61820
USA
Tel: (1 217) 333 4557
Fax: (1 217) 244 6678
Email: jbrueckner@uiuc.edu

Jacques-François Thisse
CORE
Université Catholique de Louvain
34 Voie du Roman Pays
B-1348 Louvain-la-Neuve
BELGIUM
Tel: (32 10) 474312
Fax: (32 10) 474301
Email: thisse@core.ucl.ac.be

Yves Zenou
Department of Economics
University of Southampton
Southampton SO17 1BJ
UK
Tel: (44 8059) 3264
Fax: (44 8059) 3858
Email: yz@soton.ac.uk

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NON-TECHNICAL SUMMARY

The aim of this Paper is to propose a new way of modelling urban labour markets and to show how individual skills and urban locations play a fundamental role in the development of local labour markets. We indeed believe that workers' heterogeneity in terms of both individual skill and urban location is necessary to understand the nature of the specific interactions existing in local (urban) labour markets. In such a context, it becomes clear that the labour market and the land market are strongly connected, and that this connection is central to the understanding of the socio-economic forces at work in our modern cities.

To reach this objective, we consider a model linking two separate 'spaces': the skill space of workers and the physical space of cities. We show how heterogeneity in the skill space is mirrored in the residential-location choices of workers, drawing a connection between outcomes in the land and labour markets.

In our model, firms with different skill needs and different urban locations compete for mobile workers who are differentiated by a continuous skill measure. A firm is identified with a job, while workers with heterogeneous skills choose among only a few jobs. In such a context firms have different job requirements because they have incentives to differentiate their job offers in order to obtain market power in the labour market, thus allowing them to set lower wages. This implies that the labour market is an oligopsony in which firms compete to attract workers. In terms of urban economics, each firm is considered as a company town attracting workers who also choose to reside in this firm's vicinity. Firms are separated in the physical space because this allows them to enjoy market power over the workers situated in their vicinity. As a consequence, our setting may be viewed as a system of cities in which each firm/city competes to attract workers who are also residents. Each city is monocentric with a central business district (CBD) formed by a firm (which may stand for a group of identical firms behaving collusively) competing with firms located in other cities. The fact that each firm is anchored in a distinct location is the main reason for the emergence of local labour markets.

When workers and firms are heterogeneous, it is clear that the information available to firms about workers matters in the process of wage formation. In this Paper, it is assumed that firms cannot observe the cost of training a worker but that the worker knows this cost. Hence, workers must bear the training cost that allows them to erase any mismatch between their innate skills and the skill needs of their employer. As a result, the net wage is lower for workers whose 'skill distance' from their employer is large.

In the ultimate spatial equilibrium, commuting distance is perfectly correlated with a worker's skill distance from the firm. This relationship arises because

the low net wage earned by a worker with a large skill distance translates into a low value of time, which allows the worker to tolerate a long commute. Thus, the equilibrium residential location of workers is governed by the quality of their match in the labour market. Knowledge of the connection between skill and commute distances affects the firm's interaction with its rivals as it competes for labour, in a manner made clear in the analysis. The upshot is that the equilibrium wage depends on the commuting cost in a non-standard way, yielding a link between the urban spatial element in the model and the labour market. By exploring these interconnections, our Paper therefore shows how to construct models with income heterogeneity and interaction between the land and labour markets.

A distinctive feature of our setting is that low-skill workers have long commute trips, which yields a low wage net of training and commuting costs. Low skill workers are therefore distant from firms in both the skill and urban spaces implying that such workers are sorted similarly in both the skill space and the urban space. Because such workers thus live on the urban periphery, our model provides a rationale for the existence and the emergence of socio-economic ghettos as workers with poor skill matches are also those who incur the highest commuting costs.

1 Introduction

The labor market is not a global market in which the labor force is homogeneous. Quite the opposite. One witnesses an increasing heterogeneity of the labor force as well as a thinner segmentation of this market into sub-markets characterized by a fairly weak mobility between segments. For example, the existence of regional/urban labor markets is a well-established fact: workers and firms interact only in local labor markets whose size is much smaller than that of the national market, and few people move from one market to another (Armstrong and Taylor, 1993; Bartik, 1996; Hughes and McCormick, 1994; Topel, 1986). Yet, in the standard neoclassical model, economic agents do not choose with whom they exchange goods or labor. They are supposed to operate in an impersonal market where nobody has to know the identity of the other party in the transaction. Therefore, explaining the existence of local labor markets is beyond the reach of the standard paradigm. A new approach is thus required that explicitly accounts for the possibility of local markets pulling sub-groups of agents together. Such an extension should also allow for the determination of the size of these markets since it is precisely their geographical extension that limits the reality of the global market.

It is our contention that *the force inducing the formation of local labor markets finds its origin in the skill and geographical heterogeneity of workers.* Indeed, once the heterogeneity of the labor force is recognized, it is reasonable to think that the restriction to a sub-market allows firms to acquire more market power over their potential workers, while facilitating at the same time the matching process between jobs and workers. In the same vein, geographical separation gives firms market power over the workers residing in their vicinity, who attach themselves to the firm in order to reduce their commuting costs. Stated differently, we believe that workers' heterogeneity in terms of both individual skill and urban location is necessary to understand the nature of the specific interactions existing in local (urban) labor markets. In such a context, it becomes clear that the labor market and the land market are strongly connected, and that this connection is central to the understanding of the socio-economic forces at work in our modern cities.

The aim of this paper is thus to propose a new way of modeling urban labor markets and to show how individual skills and urban locations play a fundamental role in the development of local labor markets. To reach this objective, we consider a model linking two separate "spaces": *the skill space of workers* and *the physical space of cities.* We show how heterogeneity in the skill space is mirrored in the residential-location choices of workers, drawing a connection between outcomes in the land and labor markets.

In our model, firms with different skill needs and different urban locations compete for mobile workers who are differentiated by a continuous skill measure. In the spirit of Sattinger (1993), a firm is identified with a job, while workers with heterogeneous skills choose among only a few jobs. As argued by Stevens (1994), in such a context firms have different job requirements because they have incentives to differentiate their job offers in order to obtain market power in the labor market, thus allowing them to set lower wages. This implies that the labor market is an *oligopsony* in which firms compete to attract workers. In terms of urban economics, each firm is considered as a company town attracting workers who also choose to reside in this firm's vicinity. Firms are separated in the physical space because this allows them to enjoy market power over the workers situated in their vicinity (Gabszewicz and Thisse, 1986). As a consequence, our setting may be viewed as *a system of cities in which each firm/city competes to attract workers who are also residents.* Each city is monocentric with a central business district (CBD) formed by a firm (which may stand for a group of identical firms behaving collusively) competing with firms located in other cities. The fact that each firm is anchored in a distinct location is the main reason for the emergence of local labor markets.

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In the ultimate spatial equilibrium, commuting distance is perfectly correlated with a worker's skill distance from the firm. This relationship arises because the low net wage earned by a worker with a large skill distance translates into a low value of time, which allows the worker to tolerate a long commute. Thus, *the equilibrium residential location of workers is governed by the quality of their match in the labor market.* Knowledge of the connection between skill and commute distances affects the firm's interaction with its rivals as it competes for labor, in a manner made clear in the analysis. The upshot is that *the equilibrium wage depends on the commuting cost in a nonstandard way*, yielding a link between the urban spatial element in the model and the labor market. By exploring these interconnections, our paper therefore shows how to construct models with income heterogeneity and interaction between the land and labor markets.

A distinctive feature of our setting is that low-skill workers have long commute trips, which yields a low wage net of training and commuting costs. Low skill workers are therefore distant from firms in both the skill and urban spaces. Because such workers thus live on the urban periphery, our model provides a rationale for the existence of *socio-economic ghettos*, occupied by workers who are "socially" and physically distant from their employers (Akerlof, 1997). This twofold segregation is found in a number of European cities, where high-income residents reside near the center and lower income workers live on the outskirts of the city (see, e.g., Brueckner, Thisse and Zenou, 1999).

Despite some differences in the spatial configurations, our results are also consistent with the spatial mismatch hypothesis (Kain, 1968), which aims to explain the high rates of poverty among African-American inner city residents. Residing in segregated areas distant from and poorly connected to major centers of employment growth (located in general in the US suburbs), African Americans are said to face strong geographic barriers to finding and keeping well-paid jobs. Even though in our model the poor reside in peripheral locations, the story is the same because what matters in the spatial mismatch hypothesis is the distance to jobs. Stated differently, *people who have low skills are those who live far away from jobs.*

To the best of our knowledge, these two characteristics, individual skills and urban locations, have not been combined in existing models. Indeed, the typical general-equilibrium model of urban land-use suppresses such complexity. Models either assume that urban residents are identical (e.g., Wheaton, 1974) or that they can be divided into a handful of distinct income classes (e.g., Hartwick, Schweizer and Varaiya, 1976). Only Beckmann (1969), who analyzes a model with a continuum of income groups, attempts to capture the true diversity of real-world populations in an urban spatial model. A related criticism applies to imperfectly competitive models of wage determination in a world with imperfect labor markets (see, e.g. the survey by Boal and Ransom, 1997, as well as Hamilton, Thisse and Zenou, 2000). While worker heterogeneity is one of their key features, these models suppress the spatial side of the labor market, ignoring the commuting phenomenon that plays a central role in urban land-use models. As a result, urban models suppress worker heterogeneity, and the above labor-market models suppress the journey to work, thus ignoring the heterogeneity of residence locations around the business district.

Our paper bears a close connection to several earlier studies. First, the idea of spatially modeling urban labor markets is already present in Smith and Zenou (1997) and Wasmer and Zenou (1999). In these two papers, workers are heterogeneous in only one dimension (the geographical space), whereas here two dimensions of heterogeneity are present. Second, our way of capturing labor force heterogeneity through a spatial device borrowed from product differentiation theory goes back to Kim (1989). In later work, the same author elaborates his model to incorporate a land market in the context of the monocentric city (Kim, 1991). However, Kim assumes a bargaining process between firms and individual workers that fits a world of complete information, while ours corresponds to a situation of asymmetric information in which firms do not observe a worker's skill type before hiring. As a result, Kim obtains a flat wage rate, unrelated to a worker's skill, which leads to a population that is homogeneous in terms of income. By contrast, our approach leads to a heterogeneous income distribution, which in turn generates an unconventional model of urban spatial structure.[1] Helsley and Strange (1990) and Abdel-Rahman and Wang (1995, 1997) consider similar frameworks in order to study the formation of systems of cities. In these models, cities have no spatial extension, so that the intraurban location problem does not arise. Another related paper by Fujita and Thisse (1986) attempts to unify spatial competition theory and urban economics by assuming that firms choose their location, anticipating the corresponding residential equilibrium of their customers. However, they assume no price competition on the product market, while all consumers have the same income. Lastly, the idea of studying competition in multi-characteristics space has recently been investigated by Irmen and Thisse (1998). However, while we consider the interaction between two different markets/spaces, they work within the context of a single market/space which has several dimensions.

The plan of the paper is as follows. Section 2 gives the setup of the model. In section 3, we analyze equilibrium in the land market, taking the wage as given. One result from this analysis, namely the positive association between skill and commuting distance, is then used in section 4's analysis of wage determination, as explained above. Section 5 concludes.

2 The model

Consider n firms producing a homogeneous good c which is sold on a competitive market (we take this good as the numéraire). There is a continuum of mass 1 of workers with heterogeneous skills. Workers are heterogeneous in the type of work they are best suited for, but there is no ranking in any sense of these types of work. Each worker supplies a fixed amount of labor provided that the resulting wage net of all costs is positive. Firms are heterogeneous both in terms of their skill needs and their urban locations. Each type of heterogeneity is now described.

[1] Note also that Kim's analysis of the bargaining process is incomplete in that he fails to account for the presence of competing firms in the negotiation between a firm and a worker (see Hamilton, Thisse and Zenou, 2000, for a more precise analysis of such a mechanism).

The type (or skill) of a worker is given by his/her location y, which is uniformly distributed along a circle C of unit length, called the *skill space*. The skill requirement by firm i $(= 1, \cdots, n)$ is unique and given by $y_i \in C$. Firms have different skill requirements because a firm's market power increases when it chooses a technology requiring a specific training that protects its location against infringements by rival employers (Stevens, 1994).[2] We suppose that firms' skill requirements are evenly spaced along C so that the skill distance between two adjacent firms is $1/n$. As suggested by Economides (1989) and Kats (1995) in their analysis of the product market, this is likely to be an equilibrium of a game in which firms strategically choose their technologies prior to their wages. The reason is that this configuration endows firms with the strongest market power in the labor market by relaxing wage competition.

If firm i hires a worker whose skill y differs from y_i, the cost of training the worker to meet the firm's skill requirement is a function of the distance $d \equiv |y - y_i|$ and is given by $s|y - y_i|$, where $s > 0$ expresses the efficiency of the training process. As explained in the introduction, firms do not observe the workers' types. This seems to be a fairly natural assumption in the case of a thick market in which many people apply for a few jobs, thus making it very difficult for firms to observe workers' abilities. However, workers know their own types and observe the firms' skill needs. In order to induce the appropriate set of workers to take jobs with the most suitable firm, workers must pay at least some part of the cost of training. In addition, since the labor supply of a worker is inelastic, firms cannot offer a wage menu, so that the worker must pay for all the costs of training that are not observable to the firm (hence resolving the adverse selection problem).[3] Consequently, each firm i offers the same wage to all workers, conditional on the worker having been trained to the skill y_i. In a nonspatial context, each worker then compares the wage offers of firms and the required training costs, choosing to work for the firm offering the highest wage net of training costs. After training, all workers are identical from the firms' viewpoint since their *ex post* productivity is observable and equal to $q > 0$. We assume that q is large enough so that in equilibrium all workers take a job. In section 4, we derive explicitly the condition that ensures equilibrium in the labor market. Each firm is then free to set its wage, and it hires all workers who want a job. Indeed, since workers pay the training cost, the firm correctly anticipates that workers choose the most suitable employer.

[2] Mills and Smith (1996) also show that firms have a strategic incentive to differentiate their technologies to gain more market power in the product market. Other reasons for the heterogeneity of technologies are also surveyed by Mills and Smith (1996).

[3] See Hamilton, Thisse and Zenou (2000) for more details.

Regarding the second type of heterogeneity, each firm is described by its location in the physical space. As in Salop (1979), we assume that firm i is located at $y_i \in C$, where the circle C is now interpreted as the physical substratum for the urban activities. Note that we assume, without loss of generality, that firm addresses in skill space are the same as their addresses in physical space. Although both spaces are identically described for notational simplicity, it should be clear that they are distinct and governed by different mechanisms. In such a context, it is natural to consider a firm as the CBD of the corresponding city where jobs are offered, recognizing that the firm represents the employment center in a city in which it is the only employer. Our physical space C thus contains n CBDs evenly spaced at a distance of $1/n$, implying that the n firms can be viewed as a system of cities competing to attract workers. Each worker chooses a location in C at a distance x from the firm in which he/she works. In this context, it is natural to define a local labor market as the set of workers hired by the same firm and commuting to the same CBD.

As in Fujita and Thisse (1986), the interaction between firms and workers is modeled as a two stage process, reflecting the fact that firms have more market power than workers. In the first stage, firms simultaneously choose their wages at a Nash equilibrium; in the second, individuals decide where to live and how much of the consumption good c to consume at the residential equilibrium. Stated differently, we model the interaction between two markets, assuming that the labor market is *strategic* while the land market is *competitive.*

In our framework, a *market equilibrium* requires solving a two-stage game involving, first, a wage-setting problem whose solution is referred to as a *labor market equilibrium* and, then, a location and land rent problem whose solution is described by a *residential equilibrium.* In other words, a market equilibrium is a subgame perfect Nash equilibrium. As usual, the model is solved by backward induction. To ease the burden of notation, we will describe only the equilibrium path.

Formally, let w denote the common equilibrium wage, and let $\Phi(w)$ denote the corresponding residential equilibrium, whose elements are defined below. Then, $[w, \Phi(w)]$ is a market equilibrium.

Though a complete analysis of the labor market is given in section 4, we find it useful to briefly summarize the results that are needed to describe the urban structure emerging in the second stage. Since workers must bear the training costs in order to take a job in the nearest firm along the circle C, they earn different net wages. More precisely, the net wage is decreasing in skill distance: an employed worker situated at a distance d from the nearest firm in the skill space, called a *type-d worker*, earns a wage equal to $w(d) \equiv w - sd$.

Finally, it remains to say that we do not tackle the issue of agglomeration (Fujita and Thisse, 1996). Agglomeration of employment is ruled out by our assumption that firms are immobile, and residential agglomeration is precluded by our assumption that all workers consume one unit of land (see section 3). These hypotheses are imposed for simplicity in order to focus on local labor markets. It should be kept in mind, indeed, that the existence of local labor markets is the consequence of distinct and geographically separated employment centers.

3 Urban equilibrium structure

As said above, the circle showing firm and worker locations simultaneously represents both the skill and physical spaces. Since we assume that the circle has unit width and that workers each consume one unit of land, the entire unit mass of workers can be accommodated on the physical land area of the circle.

Under symmetry, the workers within a distance of $1/2n$ on either side of a firm will commute to it. A key element of the residential equilibrium, as formalized below, is a mapping that assigns the worker at a given location in physical space to a particular point in skill space. The question is the relationship between the commuting distances of these workers and their distance from the firm in skill space. Below, we show that, *in equilibrium, these distances are the same*. In other words, a worker's physical distance from the firm coincides with his/her skill distance.

To generate a location pattern by skill type, we introduce a key assumption that links commuting costs to the wage paid. In a more general model, this link is achieved through a labor-leisure choice, which implies that a unit of commuting time is valued at the wage rate (see, for example, Fujita, 1989, Chapter 2). However, such a model is cumbersome to analyze, and it is likely not to yield additional insights beyond those available from our simpler approach. This one is based on a particular formulation of the labor-leisure choice, which is consistent with the empirical literature that shows that the time cost of commuting increases with the wage (see, e.g. Small, 1992, and Glaeser, Kahn and Rappaport, 2000).

We assume that each worker consumes one unit of land in the city in which he/she lives, while providing a fixed amount of labor time T. With land consumption fixed, the worker's utility depends solely on the quantity of a consumption good c and on the time L available for leisure.

In a model with labor-leisure choice, work hours are adjusted until the marginal value of leisure time equals the wage rate. While the explicit incorporation of flexible work hours would seriously complicate the model, we incorporate the spirit of this standard result by assuming that utility can be approximately by a function that values leisure time at the wage rate. In particular, we assume that workers consume a composite good z defined as follows:

$$z = c + w\,L \tag{1}$$

The marginal utility of L is thus equal to the worker's wage, as would be the case if work hours were adjusted optimally in a more general setting.

Recall that while x gives commute distance, d gives a worker's distance from the firm in skill space. The worker purchases the good c produced and sold at the corresponding CBD and incurs τx in monetary commuting costs when he/she lives at distance x from the CBD. Letting R denote rent per unit of land, the budget constraint of a type-d worker at distance x can be written as follows:

$$w(d)T = c + R + \tau x \tag{2}$$

where T, the amount of working hours, is assumed to be the same and constant across workers, an assumption that agrees with most jobs in the vast majority of developed countries. Furthermore, commuting time from distance x is equal to tx, where $t > 0$ is time spent per unit of distance. Hence, the time constraint of a type-d worker at distance x is given by

$$1 = T + L + tx \tag{3}$$

in which the total amount of time is normalized to 1 without loss of generality.

Substituting (2) and (3) in (1), the utility of a type-d worker at distance x is then given by:

$$\begin{aligned} z &= w(d)T - R - \tau x + w(d)(1 - T - tx) \qquad (4) \\ &= w(d) - R - [tw(d) + \tau]\,x \end{aligned}$$

Therefore, the time cost of commuting for a type-d worker residing at a distance x from the CBD is $tw(d)x$; in accordance with empirical observation, it increases with the income $w(d)T$. As usual, $w(d)$ in (4) does not stand for the worker's actual income (which is equal to $w(d)T < w(d)$) but for the income that would accrue to an individual working all the time ($T = 1$).

Rearranging (4) yields:

$$R = w(d) - z - [tw(d) + \tau]\,x \tag{5}$$

This equation can be used to generate the bid-rent functions for workers with different skill levels. Recalling that a type-d worker earns a net wage of $w(d) \equiv w - sd$ and substituting in (5) yields the bid-rent function for the full employment case:

$$R\left(d, x, z(d)\right) = (w - sd) - z(d) - \left[t(w - sd) + \tau\right] x \tag{6}$$

which gives the land-rent payment for a type-d worker located at distance x that is consistent with utility $z(d)$.

Inspection of (6) shows that, as usual, the bid-rent function is decreasing in x, with $\partial R/\partial x < 0$. In the present model, this reflects the combined influence of the time cost of commuting and the monetary cost. Further inspection shows that, at a given x, an increase in d makes the bid-rent slope less negative ($\partial^2 R/\partial d \partial x > 0$). This means that low-$d$ workers have steeper bid-rent curves than high-d workers. The intuitive reason is that *an extra mile of commuting reduces income more for a low-d worker than for a high-d worker*, a consequence of the higher net wage. Therefore, the low-d worker requires a larger decline in land rent than a high-d worker to maintain a given utility level.

In comparing the residential locations of two groups, it is well known that the group with the steeper bid-rent curve locates closer to the CBD (see, for example, Fujita, 1989, Chapter 2). In the present model, this implies that *low-d workers locate closer to the CBD than high-d workers.* To formalize this notion, we introduce the definition of residential equilibrium:

Definition 1 *A residential equilibrium Φ consists of a mapping $d(\cdot)$ that assigns a worker of skill type $d(x)$ to a location x, a set of utility levels $z^*(d)$, and a land rent curve $\Psi(x)$ such that:*

(i) at each $x \in [0, 1/2n]$:

$$\begin{aligned} \Psi(x) &= R\left(d(x), x, z^*(d(x))\right) \\ &= \max_d R\left(d, x, z^*(d)\right) \end{aligned} \tag{7}$$

(ii)

$$\Psi(\frac{1}{2n}) = 0 \tag{8}$$

Equation (7) says that land rent $\Psi(x)$ at location x equals the maximum of the bid rents across skill types, and that the skill type offering the highest bid at x is equal to $d(x)$. Note that, as in other urban models with heterogeneous consumers, the type of individual living at a given location is the highest bidder for land at that location. Because of our assumptions of a fixed lot size and of no vacant land, the land rent at the edge of the city is undetermined. Since the value of this constant does not affect our results, we say in equation (8) that it equals the opportunity cost of land, which is assumed to be zero without loss of generality.

The residential equilibrium is characterized as follows:

Proposition 1 *When*

$$t < 2n \tag{9}$$

there exists a unique residential equilibrium characterized by:

$$d(x) = x \qquad \textit{for} \quad 0 \leq x \leq 1/2n \tag{10}$$

$$z^*(d) = w\left(1 - \frac{t}{2n}\right) - \left(sd + \frac{\tau}{2n}\right) + \frac{st}{2}\left(\frac{1}{4n^2} + d^2\right) \ \textit{for}\ 0 \leq d \leq 1/2n \tag{11}$$

which is decreasing and convex in d*, and*

$$\Psi(x) = (tw_f + \tau)(\frac{1}{2n} - x) - \frac{st}{2}(\frac{1}{4n^2} - x^2) \qquad \textit{for}\ 0 \leq x \leq \frac{1}{2n} \tag{12}$$

which is decreasing and convex in x*.*

Proof. See the Appendix.

Equation (10) formalizes the claim above that low-d workers locate closer to the CBD than high-d workers. Indeed, the mapping between the physical and skill distances of workers involves perfect correlation between these distances: in other words, *the two distances are the same.* It should be noted that this result depends on the assumption of a fixed lot size. As is well known, variable land consumption can overturn the present inverse association between residential distance and the time cost of commuting (see, for example, Fujita, 1989, Chapter 2). With variable consumption, however, additional conditions could be imposed to guarantee that the two distances remain perfectly correlated.

Condition (9) is a natural requirement because it says that commute time from the edge of the city, which equals $t/2n$, is less than the total time available (unity). As shown in the appendix, (9) ensures that utility in (11) is a decreasing function of d. The equilibrium utility levels themselves at different d values are generated by the requirement that the highest bid for land at distance x is offered by a worker of type $d = x$. Once these bids are derived, they can be used to generate the equilibrium land rent function in (12), which is the upper envelope of the bid rents. Note that (11) is positive for a type-d worker when w exceeds the worker's training and commuting costs, a condition that must hold for all d.

4 Labor market equilibrium

We now study the first stage of the model in which firms set wages, anticipating the resulting residential equilibrium. The labor market game is described as follows: (i) the players are the n firms; (ii) the strategies are the wages $(w_1, ..., w_n)$ that are chosen simultaneously by firms; (iii) the payoffs are firms' profits. We may then define the labor market equilibrium as follows:

Definition 2 *A labor market equilibrium w is a symmetric Nash equilibrium of the labor market game in which profits are strictly positive.*

The previous section assumed that all firms pay the same wage, which led to a symmetric system of cities. However, the equilibrium wage emerges from a process of strategic interaction among firms. In this process, a firm evaluates the gains from allowing its wage to deviate from those of nearby firms (cities). When a firm increases its wage, workers at the fringe of adjacent cities find it attractive to switch their employment location, enlarging the given firm's labor pool. Equilibrium is achieved when such wage deviations are not profitable. To study this process, we assume that *all* workers earn a wage net of training cost large enough for them to take a job in equilibrium.

Consider the marginal worker, whose physical and skill locations are denoted by $\overline{y}_1$, and who is indifferent between taking a job in firms $i-1$ and i. This worker is located at a distance $y_i - \overline{y}_1$ from firm i and at a distance $\overline{y}_1 - y_{i-1}$ from firm $i-1$. The value of $\overline{y}_1$ that makes the worker indifferent between the two firms is the solution to the following equation:

$$\begin{aligned} & w_{i-1} - s(\overline{y}_1 - y_{i-1}) - \{t[w_{i-1} - s(\overline{y}_1 - y_{i-1})] + \tau\}(\overline{y}_1 - y_{i-1}) \\ = \; & w_i - s(y_i - \overline{y}_1) - \{t[w_i - s(y_i - \overline{y}_1)] + \tau\}(y_i - \overline{y}_1) \end{aligned}$$

Two important aspects of this equation should be noted. First, the training costs are based on skill distances from the two firms that are equal to physical distances, reflecting Proposition 1. Second, firms know that the marginal worker must be located at the edges of the corresponding cities (here $i-1$ and i), thus paying the same land rent at the residential equilibrium regardless of which CBD is chosen. Land rent equality is indeed a necessary feature of the land market equilibrium. As one city's wage is varied, starting, say, at the symmetric equilibrium, an asymmetric equilibrium obtains, with rents again equal at the boundary of cities $i-1$ and i. Since boundary rents are always equal at all residential equilibria, they play no role in determining the location of the marginal worker, whose location depends entirely on wages net of commuting cost.

Solving for $\overline{y}_1$ yields

$$\overline{y}_1 = \frac{w_{i-1} - w_i + [s + \tau + st(y_{i-1} - y_i)](y_{i-1} + y_i) + t(w_{i-1}y_{i-1} + w_i y_i)}{2[s + \tau + st(y_{i-1} - y_i)] + t(w_{i-1} + w_i)} \tag{13}$$

Similarly, it can be shown that the location of the individual $\overline{y}_2$ indifferent between firms i and $i+1$ is given by

$$\overline{y}_2 = \frac{w_i - w_{i+1} + [s + \tau + st(y_i - y_{i+1})](y_i + y_{i+1}) + t(w_i y_i + w_{i+1}y_{i+1})}{2[s + \tau + st(y_i - y_{i+1})] + t(w_i + w_{i+1})} \tag{14}$$

The *labor pool* of firm i is given by the interval $(\overline{y}_1, \overline{y}_2)$ whereas the corresponding mass of workers is equal to $\overline{y}_2 - \overline{y}_1$.

Firm i's profit function is then written as follows:

$$\Pi_i = (q - w_i)(\overline{y}_2 - \overline{y}_1) \tag{15}$$

The firm chooses w_i to maximize (15) subject to (13) and (14), taking w_{i-1} and w_{i+1} as given. The following result characterizes the symmetric equilibrium, where all wages are identical:

Proposition 2 *When*

$$t < n \tag{16}$$

and

$$q > \frac{3}{2n}s + \frac{4n}{(2n-t)^2}\tau \tag{17}$$

there exists a unique symmetric market equilibrium in which the common wage is equal to:

$$w = q - \frac{2s(1 - t/n)}{2n + t} - \frac{2(tq + \tau)}{2n + t}\ , \tag{18}$$

and the utilities $z^*(d)$ and the land rent $\Psi(x)$ are respectively given by (11) and (12) with (18) substituted in place of w.

Proof. See the Appendix.

The following comments are in order. First, the condition (16), which is sufficient for concavity of the profit function, holds when transport costs are sufficiently low. Though more restrictive than (9), it is a weak requirement since it implies that the time cost must eat up less than half of the wage net of training cost for the most distant worker. Second, the condition (17), which guarantees that all workers take a job, holds when q is large enough and/or the unit costs s, t and τ are low enough.[4] Third, the wage (18) equals the marginal productivity of labor ($w = q$) when all the cost parameters s, t and τ are zero, namely when there are no labor and urban heterogeneities. In addition, w monotonically increases when the number of firms rises because their market power weakens. As n approaches infinity, w converges to the competitive wage, which equals the marginal productivity q.

Using (16) and (17), it can be verified that:

$$\frac{\partial w}{\partial q} > 0, \qquad \frac{\partial w}{\partial \tau} < 0, \qquad \frac{\partial w}{\partial t} < 0, \qquad \frac{\partial w}{\partial s} < 0, \qquad \frac{\partial w}{\partial n} > 0 \tag{19}$$

Firms, which are *oligopsonists* in the labor market, thus find that their oligopsony power (as reflected in the magnitude of w) changes as parameters vary. Oligopsony power rises with s, for example, causing the firm to reduce w. The reason is that the aggregate labor supply to the firm, $\overline{y}_2 - \overline{y}_1$, becomes *less elastic* as s rises, as can be demonstrated easily. As is well-known, the market power of an oligopsonist is greater the steeper (i.e., less elastic) is the factor supply curve he/she faces. The same conclusion holds for t and τ. As a result, by reducing the elasticity of aggregate labor supply, an increase in any of these parameters reduces the equilibrium wage paid by the firm.

Intuitively, each firm is able to use its proximity advantage in the skill space in order to pay a lower wage to its workers, subtracting an amount equal to $2s(1 - t/n)/(2n + t)$ relative to the competitive wage q. This in turn implies a decrease in the value of commuting time, which allows firms to further reduce the wage by an amount equal to $2(tq + \tau)/(2n + t)$ while still compensating workers for the cost of their journey to work. In other words, increasing t or τ strengthens firms' market power, thus leading them to pay a wage that is further reduced below the competitive wage. All of this implies that *firms exploit workers in both the skill space and the urban space,* with this exploitation leading to a wage lower than the competitive wage. The wage cut rises with the three cost parameters but, as expected, falls with the productivity q.

Lastly, after substituting (18) in (11), it is readily verified, using (19), that the equilibrium utility level of the type-d worker is affected by the parameters in the same way as w, except that a change in s has an ambiguous impact on $z^*(d)$:

[4] If condition (17) does not hold, then two market equilibrium configurations may emerge: either some workers do not take a job so that unemployment prevails in equilibrium or labor pools just touch and all workers are hired. This is in the same spirit as Salop (1979). Since the analysis of the latter two cases are especially cumbersome, we have chosen to focus on the most meaningful case in which (17) holds.

$$\frac{\partial z^*(d)}{\partial q} > 0, \qquad \frac{\partial z^*(d)}{\partial \tau} < 0, \qquad \frac{\partial z^*(d)}{\partial t} < 0, \qquad \frac{\partial z^*(d)}{\partial s} \gtrless 0, \qquad \frac{\partial z^*(d)}{\partial n} > 0$$

5 Conclusion

This paper has shown how the labor and land markets interact under conditions of skill heterogeneity. Because a worker's skill level is private information, each individual must pay his/her own training cost, a burden that leads to heterogeneity in net wages. This heterogeneity in turn generates a separation of workers by skill type in each city's physical space, mimicking their separation in skill space. Firms exploit this physical separation in setting wages, inducing a connection between the equilibrium wage and the commuting-cost parameters of the model. The analysis shows how *socio-economic ghettos emerge as workers with poor skill matches are also those who incur the highest commuting costs.* In this sense, such workers are sorted similarly in both the skill space and the urban space.

Although a few previous models, most notably Kim (1991), have explored the interaction between land and labor markets, our paper is the first to do so when workers are heterogeneous. Further exploration of models of this type is likely to generate new insights into the operation of local labor markets, as well as more understanding of the forces shaping the spatial structure of cities. Our analysis could be extended along the following lines. First, one should investigate the possible existence of asymmetric equilibria. Such equilibria are especially interesting here because there would be associated with the emergence of an urban hierarchy. Second, different cities could have different productivities ($q_i > 0$), for example because of different endowments in natural resources and amenities. In this case, cities and labor markets end up with different equilibrium sizes. Third, one could assume that firms produce different goods so that cities would specialize and would export to other cities. Such extensions should allow us to build a theory of systems of cities as in Henderson (1974, 1987) but where firms (or cities) behave strategically in the labor market.

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APPENDIX

Proof of Proposition 1

Existence: We establish existence by construction. In order for

$$R\left(d(x), x, z^*(d(x))\right) = \max_d R\left(d, x, z^*(d)\right),$$

it must be true that:

$$\frac{\mathrm{d}R(d, x, z^*(d))}{\mathrm{d}d}|_{d=d(x)} = 0 \tag{20}$$

We use this equation to solve for the unknown function $z^*(d)$, which ensures that the bid rent of a type $d(x)$ worker is maximal at location x. Using (6) to compute the above derivative, (20) becomes:

$$-s - z^{*\prime}(d(x)) + tsx = 0 \tag{21}$$

Since we want to solve for $z^*(d)$, this equation must be rewritten in terms of d. Since our assumed mapping $d(x) = x$ implies $x(d) = d$, we have:

$$-s - z^{*\prime}(d) + tsd = 0 \tag{22}$$

and rearranging, we obtain:

$$z^{*\prime}(d) = dst - s \tag{23}$$

Equation (23) constitutes a differential equation involving the unknown function $z^*(\cdot)$. Integrating, the solution is:

$$z^*(d) = (st/2)d^2 - sd + K \tag{24}$$

where K is a constant of integration.

To verify that the above solution indeed maximizes $R(\cdot)$, we need to check that the second-order condition holds at any solution to the first-order condition (21). This condition requires that $\mathrm{d}^2R/\mathrm{d}d^2 < 0$, which implies from (21) that $z''(\cdot) > 0$. This inequality is verified by using (24).

Substituting $d(x) = x$ and $z^*(d(x)) = z^*(x)$ from (24) into (7), we get the equilibrium land rent at the given x, which equals:

$$\Psi(x) = w - K - (tw + \tau)x + \frac{stx^2}{2} \tag{25}$$

The constant K in (25) is determined by the condition (8). Solving for K yields

$$K = w\left(1 - \frac{t}{2n}\right) - \frac{\tau}{2n} + \frac{st}{8n^2}$$

Substituting in (24) and (25), it is easily verified that the equilibrium utility is given by (11) and the city's equilibrium land-rent function by (12).

Since the land rent function is the upper envelope of downward-sloping bid-rent curves, we know that it must be downward sloping and convex. Convexity of $\Psi(\cdot)$ is clear from inspection of (25), and the slope is negative when the derivative of (25) at the city-edge is negative, i.e. $(2nw-s)t+2n\tau > 0$. Since by assumption all workers take a job, it must be that $w - sx > 0$ holds for any $x \leq 1/2n$. Therefore, $2nw - s > 0$ holds, ensuring that the first inequality above is satisfied and confirming that $\Psi'(\cdot) < 0$.

Lastly, since $x = d$, the equilibrium z consumption of a type-d worker is (from (5)) given by:

$$z(d) = w - sd - \Psi(d) - [t(w - sd) + \tau]d \tag{26}$$

which is equal to (11) after substituting for $\Psi(d)$. While the net wage $w - sd$ is decreasing in the skill type, we want to know whether utility from (24) is similarly decreasing in d. Using (24), $z'(d) < 0$ requires

$$d < \frac{1}{t} \tag{27}$$

Since $d \leq 1/2n$, this inequality will hold if $t < 2n$ (equation (9)), a condition that must hold for commute time from the edge of the city $(t/2n)$ to be less than the total time available, which equals unity. Since $z''(\cdot) > 0$, the equilibrium utility level is a convex function of the skill type.

Uniqueness: To show uniqueness of the equilibrium, suppose that skill types d_0 and $d_1 < d_0$ reside at distances x_0 and x_1 where $x_1 > x_0$. This pattern differs from the mapping in (10). For workers of skill type d_0 to reside at the close-in location x_0, they must outbid workers of type d_1 for land at this location. But since the bid rent curve of type d_0 is flatter than that of type d_1, it follows that type d_0 will also outbid type d_1 for land at the more-distant location x_1, where that type is assumed to live. This is a contradiction, and it rules out any location pattern in which skill type and location distance are not perfectly correlated. ■

Proof of Proposition 2

The function Π_i defined by (15) is continuous with respect to all wages. We now show that it is strictly concave in w_i. Applying the first order condition, we have:

$$\frac{\partial \Pi_i}{\partial w_i} = -\frac{1}{n} + (q - w_i)\left[\frac{\partial \overline{y}_2}{\partial w_i} - \frac{\partial \overline{y}_1}{\partial w_i}\right] = 0 \tag{28}$$

so that

$$\frac{\partial \overline{y}_2}{\partial w_i} - \frac{\partial \overline{y}_1}{\partial w_i} > 0$$

must hold at the solution.

For the second order condition

$$\frac{\partial^2 \Pi_i}{\partial w_i^2} = -\left[\frac{\partial \overline{y}_2}{\partial w_i} - \frac{\partial \overline{y}_1}{\partial w_i}\right] + (q - w_i)\frac{\partial}{\partial w_i}\left[\frac{\partial \overline{y}_2}{\partial w_i} - \frac{\partial \overline{y}_1}{\partial w_i}\right] < 0$$

to hold, we have to show, at $w_{i-1} = w_{i+1} = \widehat{w}$ and $\widehat{w} \neq w_i$, that:

$$\frac{\partial}{\partial w_i}\left[\frac{\partial \overline{y}_2}{\partial w_i} - \frac{\partial \overline{y}_1}{\partial w_i}\right] < 0$$

Since

$$\frac{\partial}{\partial w_i}\left[\frac{\partial \overline{y}_2}{\partial w_i} - \frac{\partial \overline{y}_1}{\partial w_i}\right] = -B\frac{2t}{\left[2\left(s + \tau - st/n + t(w_i + \widehat{w}\right)\right]^3} < 0$$

where $B \equiv 2\left(s + \tau - st/n\right)\left(2 - t/n\right) + t^2\widehat{w}\left[2y_i + y_{i+1} + y_{i-1}\right] > 0$, any solution to firm i's first order condition is a local maximizer (note that $t < n$ from (16) is used). This implies that Π_i is strictly concave in w_i when firms $i-1$ and $i+1$ set the same wage. Consequently, a symmetric Nash equilibrium in wages exists.

Furthermore, at the symmetric Nash equilibrium ($w_{i-1} = w_{i+1} = w_i$), the first order condition (28) can be written as:

$$(q - w_i)(2n - t) = 2\left(s + \tau - st/n + tw_i\right) \tag{29}$$

meaning that each firm's first-order condition is linear in w_i. Thus, there is a unique symmetric Nash equilibrium.

Solving (29) yields (18). It remains to check two conditions. First, the equilibrium wage is positive. This leads to (16). Second, the utility (11) is such that all workers are willing to take a job. For that, it must be that the farthest worker from the CBD (located at $x = 1/2n$) has a positive income net of training and transport costs, so that:

$$w - \frac{s}{2n} - [t(w - \frac{s}{2n}) + \tau]\frac{1}{2n} > 0$$

which is equivalent to (17). Under this condition, the utility (11) is always positive. ■

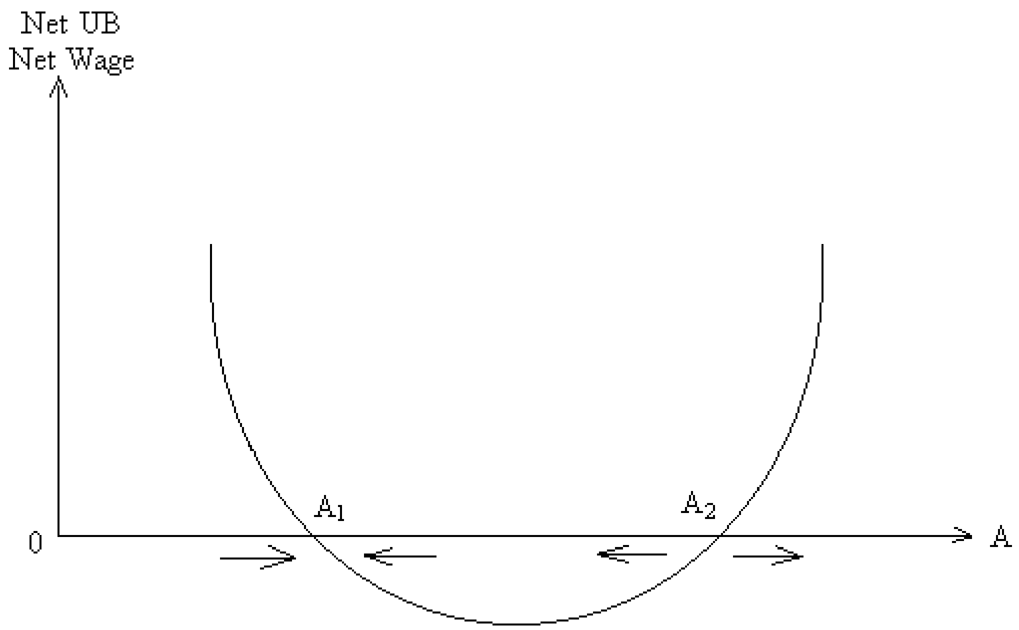

Figure 1