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TRUE MULTILATERAL INDEXES FOR INTERNATIONAL COMPARISONS OF REAL INCOME: THEORY AND EMPIRICS

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ABSTRACT

True Multilateral Indexes for International Comparisons of Real Income: Theory and Empirics*

This paper introduces a utility-consistent benchmark for international comparisons of real income, the GAIA (Geary-Allen International Accounts) System. It coincides with the Geary method (which underlies the Penn World Table) when preferences are Leontief and with the EKS method (favoured by OECD) when preferences are homogeneous quadratic. The Geary method seems preferable since it gives a (possibly poor) approximation to a consistent set of international comparisons, whereas the EKS method gives a good approximation to an inconsistent set. An illustrative empirical application, using estimates of a QUAIDS (quadratic almost ideal demand system), suggests that both methods impose excessive 'convergence' on the data.

JEL Classification: C80, D10, F00
Keywords: exchange rates, GAIA system, Geary method, index numbers of prices and real incomes, international comparisons of real incomes, Penn world table, purchasing power parities, QUAIDS (quadratic almost ideal demand system)

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NON-TECHNICAL SUMMARY

How should we compare price levels and real incomes between countries? The question is of great importance, since the demand for such comparisons is enormous. Apart from their intrinsic interest, they are essential for testing hypotheses about comparative growth performance. Indeed, such tests have themselves become a major growth industry in recent years. This reflects, in part, the revival of interest in growth theory and the development of models of endogenous growth. It also reflects the relatively recent availability of comparative data on real incomes for a wide range of countries and years, of which the major source is the Penn World Table – an enormous data set that originates from the United Nations International Comparison Project.

The basic question of how comparable measures of real income should be calculated, however, remains unresolved. Some method must be used to correct for the fact that prices are not equalized between countries, and so international comparisons that use current exchange rates give a misleading picture of the extent of divergences in real incomes. On average, poorer countries have lower price levels, so international comparisons that do not correct for deviations from common price levels tend to exaggerate the degree of income inequality between nations.

A great many methods of calculating real incomes have been proposed. In practice, the two most widely used are the 'Geary method', devised by the Irish statistician Roy Geary in 1958, and the 'EKS method', named after its originators Eltetö and Köves (1964) and Szulc (1964). The Geary method has many practical advantages, most notably that it leads to a consistent set of world accounts which can be disaggregated by country and commodity. For that reason it was adopted in the International Comparison Project and so forms the basis for the Penn World Table. The method lacks a secure theoretical foundation, however, and has been heavily criticized by theorists – especially by Erwin Diewert, who argues for the EKS method in his authoritative surveys. This method is a multilateral extension of the Fisher 'Ideal' index, and is used by the OECD and by Eurostat (the Statistical Office of the European Union) to produce purchasing-power-parity-corrected real income data for their member countries.

The Paper re-examines the theoretical foundations for international comparisons of purchasing power and real incomes. It suggests that the claims of Diewert and others for the EKS method, based on the Fisher index, do not hold up. The Paper also shows that Fisher-type indexes have desirable properties for *bilateral* comparisons that do not extend to the *multilateral* case.

More positively, the Paper proposes a new set of 'true' indexes for international comparisons, which combine the desirable aggregation property of the Geary method with a firm foundation in economic theory. This proposed system is called the GAIA (‘Geary-Allen International Accounts’) System. Like the Geary method, it yields a consistent set of real incomes that can be consistently disaggregated by commodity and country. It also draws on the economic theory of index numbers, however, and, in particular, on the work of RGD Allen. He showed how to compute a bilateral real income index which takes account of the behaviour of consumers, who typically respond to price differences by substituting away from more expensive goods towards cheaper ones. Neglecting this substitution yields biased estimates of price and real income indexes. The GAIA system extends this approach to multilateral international comparisons. Like any index of the Allen type, it provides an answer to the question ‘How well-off would a given reference consumer be in different countries?’ The distinctive feature of the GAIA system is that the reference consumer chosen is the hypothetical consumer whose consumption bundle most closely approximates the observed consumption patterns of the world as a whole.

The GAIA system has two kinds of uses: theoretical and empirical. At a theoretical level, it can be used as a benchmark to evaluate the performance of empirical indexes, which are easy to calculate, such as the Geary and EKS indexes. The paper shows that the GAIA system is identical to the EKS index if tastes exhibit a particularly strong form of ‘homotheticity’. This means that consumption patterns do not vary with income: rich and poor consumers spend the same fraction of their budget on food, for example. By contrast, the GAIA system is identical to the Geary system if there is no substitutability in demand. This means that consumption levels do not vary with price: unless their real incomes change, consumers do not reduce their consumption of goods that have risen in price. Both of these assumptions, homotheticity and zero substitutability, are highly unrealistic. If forced to choose, however, it is less implausible to assume that there is no substitutability (i.e., consumption levels do not vary with price) rather than to assume that tastes are homothetic (i.e., consumption patterns do not vary with income). The Paper argues that, for practical purposes, the Geary method is to be preferred to the EKS method and its variants, since it gives an approximation, though not necessarily a very good one, of an appropriate ideal procedure, whereas the EKS method yields a set of inconsistent multilateral comparisons.

The second use to which the GAIA index can be put is to estimate it directly. This requires estimating the behaviour of the hypothetical reference consumer, whose behaviour is as close as possible to the consumption behaviour of the world as a whole. This is done in the Paper by estimating a variety of demand systems from the QUAIDS (Quadratic Almost Ideal Demand System) family. This is a general assumption about demand behaviour, and, by looking at special cases, it is shown how different assumptions about income and price responsiveness affect the performance of the Geary and EKS indexes relative to the empirical GAIA index. The empirical application is of interest in itself, because it uses data from the International Comparison Project that underlies the Penn World Table, covering 11 categories of consumption expenditure in 60 countries in 1980. As well as illustrating the pitfalls and potential of estimating true multilateral indexes, this section turns up two key empirical findings. First, in accordance with the theoretical results, homothetic tastes rationalize the EKS index, whereas with non-homothetic tastes there is little basis for choosing between the EKS and Geary indexes. Second, both the EKS and Geary indexes compress the distribution of world income much more than the acceptable true indexes. This suggests that conclusions about international convergence of real incomes based on either index have to be treated with caution.

One empirical application in the Paper illustrates the importance of ‘excessive’ convergence and also the quantitative significance of choosing between different indexes. This takes as its starting point the UN target for foreign aid by rich countries of 0.7% of GNP. When applied to the OECD countries in the sample, this target implies a transfer to poor countries of over $30 billion. Different index numbers, however, give significantly different answers. The EKS gives the lowest, with the Geary index $0.3 billion greater. The true GAIA indexes, estimated using the Paper's empirical demand systems, imply a transfer fully $1 billion more than that implied by the EKS. Just as the Boskin Commission found that the choice of price index number formula was crucially important for calculating the US federal deficit, this finding shows that the choice of multilateral real income formula is critical for measures which are cumulated over many countries.

TRUE MULTILATERAL INDEXES FOR INTERNATIONAL COMPARISONS OF REAL INCOME: THEORY AND EMPIRICS*

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ABSTRACT

I introduce a utility-consistent benchmark for international comparisons of real income, the GAIA ("Geary-Allen International Accounts") System. It coincides with the Geary method (which underlies the Penn World Table) when preferences are Leontief and with the EKS method (favoured by OECD) when preferences are homogeneous quadratic. The Geary method seems preferable since it gives a (possibly poor) approximation to a consistent set of international comparisons, whereas the EKS method gives a good approximation to an inconsistent set. An illustrative empirical application, using estimates of a QUAIDS demand system, suggests that both methods impose excessive "convergence" on the data.

Keywords: International Comparisons of Real Incomes; Exchange Rates; Purchasing Power Parities; Index Numbers of Prices and Real Incomes; Geary Method; Penn World Table; GAIA System; QUAIDS (Quadratic Almost Ideal Demand System).
JEL Classification Nos.: D1, C8, F0

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How should we compare price levels and real incomes between countries? The question is of great importance, since the demand for such comparisons is enormous. Apart from their intrinsic interest, they are essential for testing hypotheses about comparative growth performance. Indeed, such tests have themselves become a major growth industry in recent years. This reflects in part the revival of interest in growth theory and the development of models of endogenous growth. It also reflects the relatively recent availability of comparative data on real incomes for a wide range of countries and years, of which the major source is the Penn World Table, an enormous data set which originates from the United Nations International Comparison Project (ICP). (See Kravis (1984) and Summers and Heston (1991).)

However, a paradox lies at the heart of the Penn World Table. The basic method it uses to construct internationally comparable data on real incomes relies on a method for computing "purchasing-power-parity-corrected" exchange rates, devised by the Irish statistician Roy Geary (1958). This method has many practical advantages, most notably that it leads to a consistent set of world accounts which can be disaggregated by country and commodity. However, the method lacks a secure theoretical foundation and has been heavily criticised by theorists, especially by Erwin Diewert, who argues for alternative approaches in his authoritative surveys (1987, pp. 776-778; 1999, pp. 14-16).[1] The best-known of these alternative methods is the "EKS" index, named after its originators Eltetö and Köves (1964) and Szulc (1964) (though Diewert (1987) notes that it was earlier proposed by Gini). This is a multilateral extension of the Fisher "Ideal" index, and has been used by the OECD and by Eurostat (the Statistical Office of the European Union) to produce purchasing-power-

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[1] See also the dismissive remarks by Samuelson and Swamy (1974, p. 591), Caves, Christensen and Diewert (1982, p. 83), Samuelson (1984, p. 277 and 1994, p. 212) and Officer (1989).

parity-corrected real income data for their member countries.[2]

This paper reexamines the theoretical foundations for international comparisons of purchasing power and real incomes. I suggest that the claims of Diewert and others for methods based on the Fisher index do not hold up. Essentially, the Fisher-type indexes have desirable properties for *bilateral* comparisons which do not extend to the *multilateral* case. More positively, I propose a new set of true indexes for international comparisons which combine the desirable aggregation property of the Geary method with a firm foundation in economic theory. I also show that, under a wide class of assumptions about demand behaviour, my true indexes yield international comparisons of real income relative to a hypothetical country whose income is an average of world incomes in an appropriate sense. Finally, I argue that, for practical purposes, the Geary method is to be preferred to the EKS method and its variants, since it gives an approximation, though not necessarily a very good one, to an appropriate procedure, whereas the EKS method yields a set of inconsistent multilateral comparisons.

Section 1 sets up the problem and introduces the three multilateral indexes which will be compared in the paper, the EKS index, the closely-related CCD index of Caves, Christensen and Diewert (1982) and the Geary index. Section 2 reviews some relevant results from the theory of index numbers, paying particular attention to the specific issues which arise in multilateral cross-section comparisons. Section 3 considers the results of Konüs and Byushgens (1926) and Diewert (1976) which provide a theoretical justification for Fisher-type

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[2] These two organisations convened a conference in 1989, at which their expert advisors failed to agree on whether the Geary or EKS methods should be adopted. As a result, the OECD now publishes annually two complete tables of real income indexes for its member countries. However, "Eurostat requires that only one set of results be recognised as the official results of the Community," so that based on the EKS method is released a year before that based on the Geary method. See OECD (1990).

indexes and shows that they do not extend to multilateral comparisons. Section 4 introduces my own proposed index, which I call the GAIA ("Geary-Allen International Accounts") System. I note its theoretical properties, show how it relates to the Geary method and draw on the theory of linear aggregation to explain the world prices which it implies. Finally, Section 5 presents an empirical application. By estimating a variety of demand systems from the QUAIDS ("Quadratic Almost Ideal Demand System") family, I show how different assumptions about income and price responsiveness affect the performance of the Geary and EKS indexes relative to the empirical GAIA index.

1. Preliminaries

1.1 The Problem

Suppose that, for each of m countries, labelled $j = 1, \ldots m$, we have observations on the prices (expressed in national currencies) and quantities consumed (expressed in common units) of n commodities, labelled $i = 1, \ldots n$. Price and quantity vectors in country j are denoted p^j and q^j, with typical elements p_{ij} and q_{ij}, respectively. Total expenditure in domestic prices is denoted $z_j = p^j.q^j$. (Following standard practice, I use "income" as shorthand for total expenditure.) Each commodity is assumed to be identical in quality worldwide but, because of transport costs, imperfect competition or other barriers to arbitrage, prices are not equalised internationally. Hence, official exchange rates are not appropriate for comparing price levels or real incomes between countries. What we seek is a set of index numbers which express the real income of each country j relative to every other country k: $\{Q_{jk}, \forall\ j,k\}$.[3]

[3] Many more methods for multilateral comparisons have been proposed than are considered here. See, for example, Balk (1996), Diewert (1996) and Hill (1997). However, the EKS and Geary methods are by far the most commonly used in practice.

1.2 The EKS Index

The simplest way of making multilateral comparisons of real incomes is the so-called "star" method, which revalues each country's consumption vector in terms of the prices of a single reference country. This amounts to constructing a set of Laspeyres quantity indexes, with the country at the centre of the star as base. But this method is clearly arbitrary since the results are sensitive to the choice of base country. Moreover, it leads to a well-known bias, the "Gerschenkron Effect": a country's measured real income is higher the more the base country's prices differ from its own.[4] Even in bilateral comparisons these problems are usually avoided by adopting some compromise between the base-weighted Laspeyres index and the current-weighted Paasche index, of which the most widely-used is their geometric mean, the Fisher "Ideal" index:

$$\ln Q^F_{jk} \quad = \quad \tfrac{1}{2}\left\{ \ln \frac{p^k.q^j}{p^k.q^k} + \ln \frac{p^j.q^j}{p^j.q^k} \right\}. \qquad (1)$$

The Fisher index has many desirable properties but (as we shall see in the next section) it is not suited to multilateral comparisons. The EKS index avoids the drawbacks of the Fisher index by extending it to the multilateral context. It equals the geometric mean of the ratios of all *m* bilateral Fisher indexes, taking each of the *m* countries in turn as base:

$$\ln Q^{EKS}_{jk} \quad = \quad \frac{1}{m} \sum_{l=1}^{m} \left\{ \ln Q^F_{jl} - \ln Q^F_{kl} \right\}. \qquad (2)$$

Since the Fisher index is reflexive ($Q^F_{jj}=1$) and symmetric ($Q^F_{jk}.Q^F_{kj}=1$), this may be rewritten as:

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[4] See Gerschenkron (1951), Nuxoll (1994) and Neary and Gleeson (1997). The Gerschenkron Effect is a consequence of utility maximisation if preferences are homothetic, but may not arise, even with a single utility-maximising consumer, if preferences are non-homothetic. Neary and Gleeson develop and implement a test of the Effect.

$$\ln Q_{jk}^{EKS} \quad = \quad \frac{1}{m} \left[2 \ln Q_{jk}^{F} \; + \sum_{l=1,\, l \neq j,k}^{m} \left\{ \ln Q_{jl}^{F} \; - \; \ln Q_{kl}^{F} \right\} \right]. \tag{3}$$

which reduces to the Fisher index when $m=2$. Thus the EKS index is indeed an appropriate multilateral generalisation of the Fisher index.

1.3 The CCD Index

Caves, Christensen and Diewert (1982) have proposed an alternative to the EKS index which resembles it in many respects but has superior theoretical properties. Its starting point is the bilateral Törnqvist index,[5] defined as:

$$\ln Q_{jk}^{T} \quad = \quad \frac{1}{2} \sum_{i} \; (\omega_{ij} + \omega_{ik}) \; \ln (q_{ij} / q_{ik}) \,, \tag{4}$$

where ω_{ij} is the budget share of good i in country j. The CCD index extends the Törnqvist index to multilateral comparisons in the same way as the EKS index extends the Fisher index:

$$\ln Q_{jk}^{CCD} \quad = \quad \frac{1}{m} \sum_{l=1}^{m} \left\{ \ln Q_{jl}^{T} \; - \; \ln Q_{kl}^{T} \right\}. \tag{5}$$

Caves, Christensen and Diewert have applied this index to international comparisons of output and productivity and Prasada Rao, Selvanathan and Pilat (1995) have applied the corresponding price index to international comparisons of consumer prices.

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[5] This is sometimes called the Divisia index. See for example, the extensive literature on monetary aggregates surveyed by Barnett, Fisher and Serletis (1992). However, strictly speaking, the Divisia index is defined in continuous time and the Törnqvist index (4) is a discrete approximation to it.

1.4 The Geary Method

The Geary method proceeds in a very different way to the other two indexes.[6] It first postulates the existence of "world" prices π and "true" exchange rates ε. The true exchange rates are Laspeyres price indexes, which compare the world prices with the prices of each country in turn:[7]

$$\epsilon_j = \frac{\sum_i \pi_i q_{ij}}{\sum_i p_{ij} q_{ij}}, \qquad j = 1, \ldots m. \tag{6}$$

Put differently, each country's real income is the same, whether valued at world prices ($\Sigma_i \pi_{ij} q_{ij}$) or valued at domestic prices, converted at the true exchange rates ($\varepsilon_j \Sigma_i p_{ij} q_{ij}$). As for the world prices themselves, they are implicitly defined by the requirement that total world spending on commodity i is the same whether valued at its world price ($\pi_i \Sigma_j q_{ij}$) or at domestic prices converted at the true exchange rates ($\Sigma_j \varepsilon_j p_{ij} q_{ij}$):

$$\pi_i = \frac{\sum_j \epsilon_j p_{ij} q_{ij}}{\sum_j q_{ij}}, \qquad i = 1, \ldots n. \tag{7}$$

Solving simultaneously for ε and π, it is then straightforward to calculate the real income of each country at world prices:

$$z_j^G = \epsilon_j z_j = \sum_i \pi_i q_{ij}, \qquad i = 1, \ldots m. \tag{8}$$

These in turn imply real income indexes, $Q^G_{jk} = z^G_j / z^G_k$, $\forall\, j,k$. Thus the Geary method is a star

[6] For a geometric exposition, see Neary (1997).

[7] The ICP defines true exchange rates or "purchasing power parities" as the inverse of (6), following the U.S. convention of measuring exchange rates. I follow Geary in using the U.K. convention, since it facilitates the matrix derivations.

system with the hypothetical country (the "world") whose prices are π as centre.

2. Criteria for Choosing between Index Numbers[8]

How can we choose between the different real-income indexes which have been introduced in the last section? There are two distinct approaches which can be taken to this problem. The "test" or "axiomatic" approach, following Fisher (1922), treats prices and quantities as independent variables and assesses the extent to which different indexes satisfy certain desirable, though not necessarily mutually consistent, properties. By contrast, the "economic" approach assumes that prices and quantities arise from optimising behaviour and explores how closely empirical indexes approximate to the "true" indexes based on economic theory.

Consider first the test approach. While there are a great many tests which a satisfactory index number formula might be expected to satisfy, four in particular are especially relevant to multilateral comparisons:[9]

1. *Base-Country Invariance*: It is intuitively desirable that indexes of real income should not be sensitive to the choice of base or reference country.

2. *Transitivity* or *Circularity*: A satisfactory index number formula should provide a unique cardinal ranking of the real incomes of the countries considered. Thus, the real income of country j relative to country k should be the same whether the two are compared directly or via an arbitrary intermediate country l: $Q_{jk} = Q_{jl}.Q_{lk}$.

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[8] Overviews of the vast literature on index numbers may be found in Pollak (1971) and Diewert (1981) and (1987). A more extended but non-technical treatment of the issues specific to international comparisons of real income is given in Neary (1996).

[9] For alternative perspectives on the test approach applied to the choice of index numbers for international comparisons, see Diewert (1987, Section 9, 1988 and 1996) and Eichhorn and Voeller (1990).

3. *Characteristicity* or *Independence of Irrelevant Countries*: The comparison between two countries should as far as possible depend only on variables which characterise them and not on variables characteristic of other countries. Thus, country *j*'s real income relative to country *k*'s should ideally be unaffected by changes in third countries.

4. *Matrix Consistency*: Finally, the usefulness of a set of real income indexes is much enhanced if they can be consistently disaggregated by commodity as well as by country.

How well do the index numbers considered in the last section meet these criteria? It is clear that the "star" and bilateral Fisher index numbers do not satisfy either base-country invariance or transitivity, whereas all three multilateral systems do. As for characteristicity, the EKS index exhibits this to a high degree by construction, since it is the solution to the problem of finding a transitive index which minimises the sum of squared deviations from the bilateral (and non-transitive) Fisher indexes. (See Drechsler, 1973, p. 28.) However, both the EKS and CCD indexes fail to satisfy matrix consistency, whereas, because of its linear structure, the Geary system does satisfy this test. It was primarily for this reason that the Geary system was used in the ICP and subsequently as the foundation for the Penn World Table.

The test approach is a useful starting point in choosing between competing index numbers. However, ever since Frisch (1936), it has been criticised on a number of grounds. At a practical level, different tests often turn out to be mutually inconsistent. For example, we have seen above that some trade-off is necessary in practice between the criteria of transitivity and characteristicity. At a theoretical level, the test approach does not require that the indexes have any basis in economic theory; in particular, little or no intercommodity substitution may be allowed. Finally, at a conceptual level, all empirical index number formulae are open to the devastating criticism of Afriat (1977) that they provide no more than "answers without questions": in the absence of a clear conceptual framework no meaning can be attached to the concept of "real income" which empirical indexes purport to measure.

The "economic" approach to index numbers avoids these difficulties by explicitly starting from maximising behaviour. In the context of international comparisons, the data are assumed to be generated by the utility-maximising behaviour of a representative consumer in each country, with identical tastes worldwide. The assumption of identical tastes should not be thought of as a naive and almost certainly false restriction on behaviour. Rather, it is a conceptual framework within which Afriat's criticism can be answered: the tastes assumed are those of a particular (real or hypothetical) consumer, and the resulting index answers the well-defined question of what real standard of living would that consumer attain in each country.

One immediate difficulty with the economic approach is that it does not imply a unique ideal real income index. In addition to the need to select a particular reference consumer, there are alternative ways of comparing living standards across countries. Even confining attention for the present to bilateral comparisons, at least three distinct measures of real income have been proposed:

1. *The Allen Quantity Index,* Q^A_{jk}: This equals the ratio of the expenditure functions of the two countries evaluated at a common reference price vector p^r:

$$\ln Q^A_{jk}(p^r) \quad = \quad \ln e(p^r, u_j) \; - \; \ln e(p^r, u_k) \,. \tag{9}$$

Since the expenditure function gives the minimum cost of attaining a given utility level facing given prices, this index allows for intercommodity substitution and so avoids the Gerschenkron Effect bias of fixed-weight indexes.

2. *The Konüs Quantity Index,* Q^K_{jk}: A problem with the Allen index is that it is not in general consistent with the true price or cost-of-living index due to Konüs. An alternative

index which meets this criterion by construction is the Konüs quantity index:

$$\ln Q^K_{jk}(u_r) \quad = \quad [\ln(p^j.q^j) - \ln(p^k.q^k)] - [\ln e(p^j,u_r) - \ln e(p^k,u_r)] \,. \qquad (10)$$

This index equals the ratio of actual expenditures in the two countries divided by the Konüs price index, evaluated at a reference utility level u_r.

3. *The Malmquist Quantity Index,* Q^M_{jk}: Finally, a difficulty with both the Allen and Konüs indexes is that they are not homogeneous of degree one in quantities. An index which meets this desirable criterion is that of Malmquist:

$$\ln Q^M_{jk}(u_r) \quad = \quad \ln d(q^j,u_r) - \ln d(q^k,u_r) \,. \qquad (11)$$

This is defined not in terms of the expenditure function but of the distance function, $d(q,u_0) \equiv Max_\delta \{\delta : u(q/\delta) \geq u_0\}$. Like the Konüs index, it is evaluated at a reference utility level u_r.

If tastes are homothetic, all three indexes reduce to the ratio of utility levels, u_j/u_k (since the expenditure function and distance function become $e(p,u)=u\varepsilon(p)$ and $d(q,u_0)=u(q)/u_0$ respectively). But otherwise the three indexes differ among themselves and the value of each one depends on the reference price vector or utility level chosen. This underlines the fact that there is no such thing as a *unique* measure of real income.

Finally, what can be said about the different empirical index numbers introduced in the last section in the light of the economic approach to index numbers? As far as the Geary method is concerned, the consensus appears to be that it has no basis in economic theory.[10] By contrast, the EKS and CCD indexes have obtained considerable support from results of Konüs and Byushgens (1926) and Diewert (1976) which relate the bilateral Fisher and

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[10] An exception to this rule is Marris (1984), who compares the Geary method with a set of multilateral Allen indexes, though without discussing how the world prices may be calculated. Geary himself did not provide any theoretical justification for his system, other than remarking in passing: "if the entities π_i and ε_j exist, they could scarcely be defined reasonably in any other terms."

Törnqvist indexes to particular specifications of preferences. In the next section I review these results and consider their relevance to multilateral comparisons.

3. Superlative Indexes and Multilateral Comparisons of Real Income

The first result relating true to empirical indexes is the following:

Result 1 [Konüs and Byushgens (1926)]: *The Fisher index is exact when the utility function is a homogeneous quadratic: $u=(q'Aq)^{1/2}$, A symmetric.*

Since tastes are homothetic in this case, saying that the Fisher index is exact means simply that it equals the ratio of the utility levels in the two countries:

$$\ln Q^F_{jk} \quad = \quad \ln u_j - \ln u_k . \tag{12}$$

As Diewert (1976) has noted, the quadratic utility function is a *flexible* functional form, i.e., it provides a second-order approximation to an arbitrary twice-differentiable linearly homogeneous utility function. He argues strongly for the use of index numbers which are *superlative,* in the sense that they are exact for flexible functional forms, and Result 1 shows that the Fisher index is superlative.

Result 1 would appear to justify the use of the EKS index for multilateral comparisons, since the EKS is an appropriate generalisation of the bilateral Fisher index. However, my first proposition throws doubt on this:

Proposition 1: The EKS index is exact when the utility function is a homogeneous quadratic.

Proof: The proposition follows immediately on substituting from (12) into the expression for the EKS index (2):

$$\ln Q_{jk}^{EKS} = \frac{1}{m}\sum_{l=1}^{m}\left\{(\ln u_j - \ln u_l) - (\ln u_k - \ln u_l)\right\}, \tag{13}$$

$$= \ln u_j - \ln u_k . \qquad \square \tag{14}$$

At first sight, Proposition 1 appears to justify the use of the EKS method. Since the quadratic utility function is a flexible functional form, Proposition 1 implies that the EKS method is superlative. The difficulty with Proposition 1 is that it goes too far. It shows that, as far as economic theory is concerned, there is nothing to be gained by using the EKS procedure over the bilateral Fisher index. While the EKS index is exact for the quadratic utility function, it is also redundant, since it actually equals the bilateral Fisher index in that case. Of course, the EKS index by construction yields a transitive ranking of income levels, unlike the Fisher index. However, this is a statistical property and does not imply that the EKS method approximates an underlying transitive preference ordering when the utility function is not a homogeneous quadratic.

What if tastes are not homothetic? The quadratic utility function does not generalise to this case. However its logarithmic equivalent, the translog, does. Consider first a result of Diewert's which deals with the homogeneous translog:

Result 2 [Diewert (1976)]: *The Törnqvist index, Q_{jk}^{T}, is exact when the utility function is a homogeneous translog*:

$$\ln u(q) = a_0 + a'\ln q + \tfrac{1}{2}(\ln q)'A\ln q . \tag{15}$$

The translog is also a flexible functional form but is more general than the quadratic. Result 2 therefore suggests that the Törnqvist index is even more "superlative" than the Fisher index.

This in turn has been interpreted to justify the use of the CCD Index, which as we saw in Section 1 is the appropriate multilateral extension of the Törnqvist index. However, my next proposition shows that it fares no better than the EKS index:

Proposition 2: The CCD index is exact when the utility function is a homogeneous translog.

The proof is identical to that of Proposition 1. Hence, like the EKS, the CCD index is redundant when it is exact.

Consider finally the extension of the translog to the non-homogeneous case, which leads to the translog distance function:[11]

$$\ln \boldsymbol{d}(q,u) \;=\; \boldsymbol{a}_0 \;+\; \boldsymbol{a}'\ln\boldsymbol{q} \;+\; \tfrac{1}{2}(\ln\boldsymbol{q})'\boldsymbol{A}\ln\boldsymbol{q} \;+\; \boldsymbol{b}_0\ln u \;+\; (\ln u)\,\boldsymbol{b}'\ln\boldsymbol{q} \;+\; \tfrac{1}{2}c_0(\ln u)^2 . \qquad (16)$$

The appropriate bilateral index number corresponding to this non-homothetic specification of preferences is given by another result of Diewert's:

Result 3 [Diewert (1976)]: *The Törnqvist index Q^T_{jk} equals the Malmquist index Q^M_{jk} (and so is exact) if the distance function is a general (non-homogeneous) translog and Q^M_{jk} is evaluated at the geometric mean of the two countries' utilities, $(u_j u_k)^{0.5}$.*

However, the difficulty with this result is that the Malmquist index is evaluated at a particular utility level which is specific to the two countries being compared. This suggests that the corresponding multilateral index, the CCD index, aggregates in general over *m* inconsistent

[11] The vector *b* is the source of non-homogeneity, which may be seen from the equations for the budget shares: $\omega = \partial\ln d(q,u)/\partial\ln q = a + A\ln q + b\ln u$. When *b* is zero, (16) reduces to (15) by setting $d(q,u)=1$ and (without loss of generality) normalizing $b_0=-1$ and $c_0=0$.

bilateral comparisons. This is confirmed by the next proposition:

Proposition 3: The CCD index deviates systematically from the Malmquist index wherever the latter is evaluated, if the distance function is a general (non-homogeneous) translog.

Proof: Suppose first that the Malmquist index is evaluated at the geometric mean of the utilities of the two countries being compared, denoted by $\bar{u}^{jk} \equiv (u_j u_k)^{0.5}$. It is then straightforward to calculate the bias of the CCD index explicitly:

$$\ln Q_{jk}^{CCD} - \ln Q_{jk}^{M}(\bar{u}^{jk}) = \tfrac{1}{2}(\ln u^*) b'(\ln q^j - \ln q^k) - \tfrac{1}{2}(\ln u_j - \ln u_k) b' \ln q^*, \quad (17)$$

where u^* and q^* are the geometric means of *all* m countries' utility levels and quantity vectors, respectively:

$$\ln u^* \equiv \frac{1}{m} \sum_l \ln u_l \qquad \text{and} \qquad \ln q^* \equiv \frac{1}{m} \sum_l \ln q^l. \quad (18)$$

Alternatively, and perhaps more naturally in a multilateral context, suppose that the Malmquist index is itself evaluated at u^*. The bias then becomes:

$$\ln Q_{jk}^{CCD} - \ln Q_{jk}^{M}(u^*) = \tfrac{1}{2}(\ln u_j - \ln u^*) b'(\ln q^j - \ln q^*) - \tfrac{1}{2}(\ln u_k - \ln u^*) b'(\ln q^k - \ln q^*). \quad (19)$$

Of course, the CCD index is also biased, and in a less symmetric way, if Q_{jk}^{M} is evaluated at any other utility level.

□

Equation (19) shows that the CCD index evaluated at u^* is exact only when tastes are homothetic (i.e., $b=0$), when from Proposition 2 it is redundant, or when the two countries compared deviate symmetrically from average. Proponents of the CCD index can perhaps draw some consolation from the fact that, in any sample of countries, the bias of a given bilateral comparison depends only on the deviations of utilities and quantities in the two countries being compared from the corresponding worldwide averages. However, it should be noted that neither the benchmark u^* nor the biases in (17) and (19) is invariant with respect to non-proportional transformations of the utility function.

In conclusion, note that all the propositions in this section give only sufficient conditions for the various multilateral indexes to be exact (or not, in the case of Proposition 3). In this respect they are no weaker than the results for bilateral comparisons which I have quoted, and which underlie the enormous theoretical and empirical regard in which the Fisher and the Törnqvist indexes are held. (Witness for example the official "Divisia" indexes of the UK money supply published by the Bank of England.)

4. The GAIA System

The propositions in the last section throw doubt on the claims that the EKS and CCD indexes have a firm basis in economic theory when applied to multilateral comparisons. By contrast, the Geary method at least uses a consistent set of world prices to compare real incomes. However, it suffers from the drawback of all fixed-weight indexes that it does not allow any substitution in consumption. In this section I propose a new set of true indexes which overcome this drawback while preserving the spirit of the Geary method.

4.1 True Multilateral Indexes

The first step is to select a particular reference consumer, whose tastes are represented by an expenditure function $e(p,u)$. Note that this is not the same as assuming identical tastes worldwide, although this interpretation is used in Section 4.3 to rationalise the world prices.

Next, replace the fixed-weight Laspeyres formula in the Geary exchange rates (6) with their true equivalents, which I call Geary-Konüs exchange rates:

$$\mathrm{E}_j \quad = \quad \frac{e(\Pi, u_j)}{e(p^j, u_j)} \quad = \quad \frac{\sum_i \Pi_i q_{ij}^*}{\sum_i p_{ij} q_{ij}}, \qquad j = 1, \ldots m. \tag{20}$$

Here the q_{ij}^* denote the "virtual" or imputed quantities which the reference consumer would choose if it had country *j*'s level of utility and faced the world prices Π:

$$q_{ij}^* \quad = \quad e_i(\Pi, u_j), \qquad i = 1, \ldots n; \quad j = 1, \ldots m. \tag{21}$$

Comparing the prices of all countries with a common world price vector is unremarkable in itself. The final step is to require that the world prices satisfy aggregation conditions of the Geary type. They cannot do so in terms of actual quantities consumed[12] but they can in terms of virtual quantities. In place of (6), this leads to the following world prices:

$$\Pi_i \quad = \quad \frac{\sum_j \mathrm{E}_j p_{ij} q_{ij}}{\sum_j q_{ij}^*}, \qquad i = 1, \ldots n. \tag{22}$$

The corresponding measures of real income can be expressed in three equivalent ways:

$$z_j^* \quad = \quad \mathrm{E}_j z_j \quad = \quad \sum_i^n \Pi_i q_{ij}^* \quad = \quad e(\Pi, u_j), \qquad j=1, \ldots m. \tag{23}$$

These in turn imply Geary-Allen true indexes of real income:

In words, z_j^*, the real income of country *j*, is the expenditure needed to give the reference

[12] After the first version of this paper was written, I came across Prasada Rao and Salazar-Carillo (1988), who propose a system which does this. The motivation and approach of their paper is very similar to mine. However, instead of my (20) and (22), they propose a hybrid combination of (7) and (20). Summing (7) over commodities and (20) over countries (as in (25) below) shows that these two sets of equations are inconsistent in general.

$$Q_{jk}^{*} = \frac{z_j^{*}}{z_k^{*}} = \frac{e(\Pi, u_j)}{e(\Pi, u_k)}, \qquad \forall\, j,k. \tag{24}$$

consumer the same standard of living (i.e., level of utility) at world prices as it would attain at country *j*'s own prices. Country *j*'s true exchange rate is the ratio of its real income z_j^* to its income at domestic prices. Finally, the world price of good *i* equates the value of total world virtual consumption of good *i* with the sum of each country's actual spending on that good converted at the true exchange rates.

The advantages of this proposed system are that it combines the best features of the economic approach to index numbers and the Geary method. Like the former, it is firmly based on the microeconomic theory of the consumer and allows for the possibility of inter-commodity substitution. Like the latter, it satisfies matrix consistency, albeit in terms of virtual rather than actual consumption levels: the q_{ij}^* can be consistently aggregated across countries and across commodities using the world prices and true exchange rates. Hence it is appropriate to use the acronym GAIA ("Geary-Allen International Accounts") for the system. Finally, the system presented here avoids the conflict between bilateral Allen and Konüs quantity indexes noted in Section 2: each exchange rate E_j is a Konüs true price index, while the real income indexes Q_{jk}^* are Allen true quantity indexes, using Π as the reference prices.

4.2 Groping from Geary to GAIA

Can we be sure that non-negative GAIA exchange rates and world prices exist? Are they unique? And what is the relationship between the original Geary system and the "ideal" GAIA system proposed here? The answers to these questions are provided in the following

Proposition:[13]

Proposition 4: Assume $q_{ij}>0$ and $p_{ij}>0$, $\forall$ i,j. Then: (a) there exists a solution to equations (20) to (22) with all E_j, q^*_{ij} *and* Π_i *strictly positive; and (b) there exists a unique tâtonnement path from the Geary to the GAIA prices.*

Proof: As a preliminary step, we need to choose an appropriate normalisation. This is because the $m+n$ equations (20) and (22) are linearly homogeneous in E and Π and are not independent, since from matrix consistency they both imply the same aggregate equation (summing (20) over countries and (22) over commodities):

$$\Sigma_i \Pi_i \left(\Sigma_j q^*_{ij}\right) \quad = \quad \Sigma_j E_j \left(\Sigma_i p_{ij} q_{ij}\right). \tag{25}$$

A similar equation applies to the Geary exchange rates and world prices, ε and π. In this section, the normalisation chosen is to require both the π and Π vectors to lie on the unit simplex: $\Sigma_i \pi_i = \Sigma_i \Pi_i = 1$. The remainder of the proof is in three parts.

(i) The first step is to prove that the Geary prices and real incomes are unique and strictly positive. To do this first rewrite the key equations (6) and (7) in matrix notation as follows:[14]

---

[13] The proof of part (i) is adapted from Balk (1996).

[14] All vectors are column vectors; z denotes the m-by-one vector of total expenditures by country, with typical element $z_j = p^j.q^j$; Z denotes the n-by-m matrix of expenditures by commodity and country, with typical element $z_{ij} = p_{ij} q_{ij}$; Q denotes the n-by-m matrix of quantities by commodity and country, with typical element q_{ij}; and q denotes the n-by-one vector of world consumption levels of each commodity, with typical element $q_i = \Sigma_j q_{ij}$. Finally, a prime (′) denotes a transpose; and a circumflex (^) over a vector denotes a diagonal matrix formed by placing on the principal diagonal the corresponding elements of the vector. Note that q can be written as $Q\iota$, where ι is an m-by-one vector of ones.

$$z^G = \hat{z}\epsilon = Q'\pi, \tag{26}$$

$$\pi = \hat{q}^{-1}Z\epsilon. \tag{27}$$

These two equations can be combined in a single equation for the Geary real incomes:[15]

$$z^G = Mz^G, \quad \text{where:} \quad M \equiv Q'\hat{q}^{-1}W, \tag{28}$$

and the typical element of the *m*-by-*m* matrix *M* is:

$$m_{jk} = \Sigma_i \left\{ \frac{q_{ij}\,\omega_{ik}}{\Sigma_l q_{il}} \right\}. \tag{29}$$

The matrix *M* is known and all its elements are strictly positive by assumption. Hence, by the Perron-Frobenius theorem, its largest eigenvalue is real and positive and corresponds to a positive eigenvector which is unique (up to a constant of proportionality). Moreover, the matrix *M* is column-stochastic: $\iota'M=q'\hat{q}^{-1}W=\iota'W=\iota'$; i.e., $\Sigma_k m_{jk}=1$. Hence, from a corollary of the Perron-Frobenius theorem due to Solow (1952), its largest eigenvalue equals one. (See Takayama (1985), p. 388, for discussion and further references.) It follows from (28) that z^G itself is the corresponding eigenvector. Hence, z^G is unique and strictly positive and so, from (27) and (6), the required π and ε vectors are also unique and strictly positive.

(ii) Next, apply exactly analogous derivations to the GAIA exchange rates (20) and world prices (22), to obtain:

$$z^* = M^*z^* \quad \text{where:} \quad M^* \equiv Q^{*\prime}(\hat{q}^*)^{-1}W. \tag{30}$$

Of course, unlike (28), this is a highly non-linear equation, since the M^* matrix depends on the unknown Π. Assume a particular Π^0 is given. Then, from (21), with the utility levels

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[15] $W \equiv Z\hat{z}^{-1}$ is the matrix of world budget shares (in domestic prices), with typical element $\omega_{ij} = z_{ij}/z_j$.

u_j given, the virtual quantities q^*_{ij} are uniquely determined and so from (30) M^* is also uniquely determined. Now, applying part (i) of the proposition, we may solve for a strictly positive $z^*(\Pi^0)$. Finally, substituting in the definition of Π (i.e., the equation which is the GAIA analogue of (27)) we may solve for a strictly positive $\Pi(\Pi^0)$ vector, which we may normalise such that $\Sigma_i \Pi_i = 1$. All this defines a continuous mapping from the unit simplex into itself and so Brouwer's Fixed Point Theorem implies that it must have a fixed point: $\Pi(\Pi^0)=\Pi^0$, for some Π^0. This proves statement (a) of the Proposition.

(iii) Part (ii) proves that GAIA prices always exist but not that they are unique. Now, consider equation (30) as defining not a continuous mapping which must have a fixed point, but rather the following discrete algorithm. First, calculate the Geary world incomes z^G using (28). Next, use (27) to solve for the Geary world prices π and then use the Hicksian demand functions (21) to calculate first-round estimates of q^*_{ij}. Then, from the second equation in (30), calculate the implied $M^*(\pi)$ matrix and use it in the first to calculate second-round estimates of z^*. From part (ii), repeating this algorithm must converge to a solution for the GAIA prices. Moreover, from part (i), the prices at each step must be unique and positive, and so the final price vector Π must also be unique and positive. This proves statement (b) of the Proposition. (Note I have only proved that there exists a unique path from the Geary prices to *some* GAIA price vector, not that this value is itself unique.)

□

The algorithm proposed in the Proposition is not very efficient from a computational point of view. (Computational issues are discussed further in Appendix 1.) However, it shows clearly that the GAIA prices can be viewed as the outcome of a tâtonnement process which adjusts prices at each stage to ensure worldwide virtual commodity balance. Hence, it gives a further justification for the Geary method, as providing a first-round approximation to the true but unobservable GAIA prices and real incomes.

Finally, a different link between the two systems is that they coincide when there are no substitution possibilities in consumption:

Proposition 5: If prices in different countries are unrestricted (subject only to $p_{ij}>0$, $\forall\ i,j$*), then the GAIA world prices and real incomes coincide with those from the Geary system if and only if preferences are of the Leontief (fixed-coefficients) kind.*

Proof: Leontief preferences are equivalent to zero substitution effects: $e_{pp}=0$. (Note that this does not imply that preferences are homothetic.) It follows immediately by inspection that (28) and (30) can have the same solution if and only if $q^*_{ij}=q_{ij}$, $\forall\ i,j$. With no restrictions on prices (other than that they are positive) this is equivalent to Leontief preferences.

□

Leontief preferences are not very realistic. Nevertheless, it is useful to have a benchmark case where the Geary and GAIA systems coincide and plausible that it is precisely the case where the Gerschenkron Effect does not arise. Of course, an alternative benchmark is provided by homothetic preferences. From (23) it is immediate that the GAIA real income index reduces to the ratio of utility levels when tastes are homothetic and so coincides with all three true real income measures introduced in Section 2. In addition, it follows from Proposition 1 that it coincides with the EKS index when the utility function is a homogeneous quadratic and with the CCD index when the utility function is a homogeneous translog.

4.3 Interpretation of the GAIA World Prices

The GAIA system satisfies base-country invariance and matrix consistency only because it chooses a particular set of world prices. This raises the question: to which countries, if any, do the world prices correspond? In the ICP, the Geary prices have been found to come closest to the prices of middle-income countries such as Hungary or Italy. (See, for example, Nuxoll (1994).) In this section, I show why this must be so for the GAIA prices, under a wide class of preferences.

This issue is most easily addressed when the equations defining the world prices and true exchange rates are reexpressed in terms of budget shares. Consider first the Geary case. Let θ_j^G denote the share of each country in world income, measured at world prices:

$$\theta_j^G \equiv \frac{z_j^G}{\Sigma_k z_k^G} = \frac{\epsilon_j p^j.q^j}{\Sigma_k \epsilon_k p^k.q^k}. \tag{31}$$

Define the *world budget share* of commodity *i* as the average of each country's actual budget share ω_{ij}, weighted by the θ_j^G:

$$\bar{\omega}_i^G \equiv \sum_j \theta_j^G \omega_{ij}, \qquad i = 1, \ldots n. \tag{32}$$

Substituting (6) into (7) and dividing by world income $\Sigma_k z_k^G$ shows that (32) may alternatively be expressed as the share of commodity *i* in world spending at world prices:

$$\bar{\omega}_i^G = \Sigma_j \theta_j^G \omega_{ij}^G = \frac{\pi_i \sum_j q_{ij}}{\sum_h \pi_h \sum_j q_{hj}}, \qquad i = 1, \ldots. n, \tag{33}$$

where the ω_{ij}^G are budget shares in world prices, $\pi_i q_{ij}/z_j^G$.

Though the equivalence of (32) and (33) is a neat implication of the Geary aggregation conditions, it yields no additional insights by itself, because the budget shares ω_{ij}^G have no behavioral significance: quantities are chosen facing prices p^j but aggregated using prices π.

The same is not true, however, of the corresponding equation for the GAIA system:

$$\bar{\omega}_i^* \equiv \sum_j \theta_j^* \omega_{ij} = \sum_j \theta_j^* \omega_{ij}^*, \qquad i = 1, ... n. \tag{34}$$

Here θ_j^*, $\bar{\omega}_i^*$ and ω_{ij}^* are defined analogously to the corresponding terms in (31), (32) and (33), except using GAIA rather than Geary world prices: $\theta_j^* \equiv z_j^* / \Sigma_k z_k^*$, $\bar{\omega}_i^* \equiv \Pi_i \Sigma_j q_{ij} / \Sigma_h \Pi_h \Sigma_j q_{hj}$ and $\omega_{ij}^* \equiv \Pi_i q_{ij}^* / z_j^*$. Now the quantities underlying the budget shares at world prices ω_{ij}^* are both generated by and aggregated by the same world prices. They therefore have a behavioral interpretation which links our results with the theory of linear aggregation developed by Gorman (1953) and Muellbauer (1975). The key result is the following:

Proposition 6: If preferences exhibit Generalized Linearity, then world demand patterns would be generated by a hypothetical country facing the GAIA world prices and with an income equal to a weighted quasi-linear mean of the individual countries' incomes.

Proof: Generalized Linearity is a specification of preferences introduced by Muellbauer (1975) which implies an expenditure function of the following form:

$$e(p,u) = f[a(p), b(p), u], \tag{35}$$

where the functions a and b are linearly homogeneous in prices p and the function f is linearly homogeneous in *(a,b)*.[16] Muellbauer shows that the budget shares implied by Generalized Linearity are:

[16] Generalized Linearity is equivalent to the "no-torsion" condition of Freixas and Mas-Colell (1987) and to demands being of rank two for the general definition of demand rank due to Lewbel (1991).

$$\omega_i^{GL}(p,z) \quad = \quad \phi(p,z)\,.\,A_i(p) \;+\; B_i(p)\,, \tag{36}$$

where A_i and B_i are commodity-specific functions which are independent of income. Hence, if preferences exhibit Generalized Linearity, country j's budget shares evaluated at world prices are:

$$\omega_{ij}^* \quad = \quad \omega_i^{GL}(\Pi, z_j^*) \quad = \quad \phi(\Pi, z_j^*)\,.\,A_i(\Pi) \;+\; B_i(\Pi)\,. \tag{37}$$

Weighting by country size and aggregating over countries gives:

$$\sum_j \theta_j^* \omega_{ij}^* \quad = \quad \omega_i^{GL}(\Pi, \tilde{z}^*)\,, \tag{38}$$

where "average" world income $\tilde{z}^*$ is defined implicitly by:

$$\phi(\Pi, \tilde{z}^*) \quad = \quad \sum_j \theta_j^* \, \phi(\Pi, z_j^*)\,. \tag{39}$$

Given Π, $\tilde{z}^*$ is a *symmetric mean*, or more specifically a *weighted quasi-linear mean*, of the z_j^*.[17] Combining (34) and (38) yields the desired result:

$$\bar{\omega}_i^* \quad = \quad \omega_i^{GL}(\Pi, \tilde{z}^*)\,. \tag{40}$$

Equation (40) thus states that world expenditure patterns, in the sense of the world budget shares at world prices, would be generated by a hypothetical country which faces the same prices and whose income is an appropriate average of the individual countries' incomes.

□

Generalized Linearity is an extremely general specification of preferences, which nests many of the most widely-used demand systems. Proposition 6 is significantly strengthened when we specialise to some of these sub-cases:

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[17] A symmetric mean is a function which is symmetric in the z_j^* and which equals z_0 when $z_j^*=z_0$, $\forall\, j$. See Chew (1983) and Diewert and Nakamura (1993, chap. 14).

A. Price-Independent Generalized Linear ("PIGL") Preferences

In this case, also due to Muellbauer (1975), the expenditure function specialises to a CES form:

$$e(p,u) \quad = \quad [(1-u)\, a(p)^{\alpha} \; + \; u\,.\,b(p)^{\alpha}]^{\frac{1}{\alpha}}\,, \qquad \alpha > 0\,, \tag{41}$$

and the income function in the budget shares is independent of prices (whence the name):

$$\omega_i^{PIGL}(p,z) \quad = \quad z^{-\alpha}.A_i(p) \; + \; B_i(p)\,. \tag{42}$$

It follows that the average income level which generates world spending patterns at the GAIA prices is also independent of prices and equals a CES mean of individual countries' incomes:

$$\bar{\omega}_i^* \quad = \quad \omega_i^{PIGL}(\Pi, \tilde{z}^*) \qquad \text{where:} \qquad \tilde{z}^* \quad = \quad \left\{ \sum_j \theta_j^* (z_j^*)^{-\alpha} \right\}^{-\frac{1}{\alpha}}. \tag{43}$$

B. Price-Independent Generalized Logarithmic ("PIGLOG") Preferences

This system is the limit of the PIGL system as α approaches zero. A special case, due to Lewbel (1989), nests in turn both the exactly aggregable translog of Christensen, Jorgenson and Lau (1975) and the AIDS ("Almost Ideal Demand System") model of Deaton and Muellbauer (1980). The expenditure function takes a Cobb-Douglas form:

$$\ln e(p,u) \quad = \quad (1-u)\ln a(p) \; + \; u \ln b(p)\,, \tag{44}$$

and the budget shares depend linearly on lnz. Hence the average world income which generates world budget shares at the GAIA prices is a weighted geometric mean of individual countries' incomes:

$$\bar{\omega}_i^* \quad = \quad \omega_i^{PIGLOG}(\Pi, \tilde{z}^*) \qquad \text{where:} \qquad \ln \tilde{z}^* \quad = \quad \sum_j \theta_j^* \ln z_j^*\,. \tag{45}$$

C. The Gorman Polar Form

A different special case of PIGL, obtained by setting α equal to one, is the Gorman Polar Form, which nests the Linear Expenditure System corresponding to the Stone-Geary utility function. The expenditure function is now:

$$e(p,u) \quad = \quad (1-u)\, a(p) \; + \; u \,.\, b(p)\,, \tag{46}$$

and the budget shares are linear in the reciprocal of income:

$$\omega_i^{GPF}(p,z) \quad = \quad z^{-1} \,.\, A_i(p) \; + \; B_i(p)\,. \tag{47}$$

Now, the simple average of world incomes generates world consumption patterns at the GAIA prices:

$$\bar{\omega}_i^{*} \quad = \quad \omega_i^{GPF}(\Pi, \bar{z}^{*}) \qquad \text{where:} \qquad \bar{z}^{*} \quad = \quad \frac{1}{m} \sum_j z_j^{*}\,. \tag{48}$$

In addition, the demand patterns aggregate not just in the sense of yielding the same budget shares but in the much stronger sense of yielding the same *levels* of world expenditure on each commodity:

$$\Pi_i \sum_j q_{ij} \quad = \quad \sum_j z_j^{*} \omega_{ij} \quad = \quad m \bar{z}^{*} \omega_i^{GPF}(\Pi, \bar{z}^{*})\,. \tag{49}$$

Thus, when preferences exhibit the Gorman Polar Form, the GAIA world prices would generate actual world demands if world income was equally distributed (or, since expenditure is linear in utility from (46), if world utility was equally distributed).

5. An Empirical Application

So far, I have discussed the theoretical properties of the GAIA system, and used it as a benchmark to infer the implicit assumptions underlying the Geary and EKS indexes.

However, like any true index, the GAIA system can also be implemented empirically if a particular specification of preferences is selected. In principle, any specification of preferences could be chosen: the investigator's, perhaps, or that of the representative consumer in one country, if such exists. However, in multilateral applications such choices seem totally arbitrary. The approach adopted here is to choose as reference consumer that specification of preferences which comes closest to generating the observed world consumption data. In this section, I show how to calculate GAIA real income indexes using estimates of a complete system of consumer demand equations, and compare their performance and policy implications with those of the standard indexes.

5.1 A First Look at the Data

The data are taken from the International Comparison Project for 1980. (See Appendix 2 for details.) Price and quantity observations are available for *per capita* consumer expenditure on 11 commodity groups in 60 countries. Table 1 and Figure 1 rank the countries by total expenditure valued at current exchange rates, taking the poorest country in the sample (Ethiopia) as reference.[18] Five empirically based real income indexes are shown: the EKS, CCD and Geary indexes, and the maximum and minimum of the sixty different Laspeyres star indexes for each country. The latter two define a range or corridor (analogous to the Laspeyres-Paasche interval in bilateral comparisons) within which any index which equals an average of bilateral fixed-weight comparisons must lie. Moreover, if tastes are

[18] The actual values of *per capita* expenditure (in U.S. dollars) range from $8919.51 for (West) Germany to $112.56 for Ethiopia.

homothetic, the GAIA index must lie within this corridor.[19]

Figure 1 shows clearly that all five real income indexes make an enormous difference to the absolute levels of real income, reducing by 50 to 70% their range of variation relative to the variation in expenditures at current exchange rates. They also affect the rankings across countries in similar ways: for example, the U.S. and Canada jump from fifth and tenth to first and second places respectively for all five real income indexes. In addition, there are significant differences between the indexes (except for the EKS and CCD indexes, which are empirically indistinguishable). In particular, the Geary index compresses the distribution rather more than the EKS, lowering the coefficient of variation from 0.821 to 0.799 and the Gini coefficient from 0.458 to 0.448.

5.2 Specifying Consumer Preferences

The next step is to select an empirical specification of preferences. From Section 4, we seek a specification which rationalises the consumption behaviour of all 60 countries in terms of utility-maximising behaviour. Recalling the conditions given in Section 4.2 for the EKS and Geary indexes to be exact, we would also like to be able to relax separately the responsiveness of demand to prices and income. Since our representative world consumer is hypothetical rather than actual, we are not concerned with *testing* the hypothesis of utility maximisation, nor are we concerned with intra-national income distribution. The estimates which follow should not be seen as attempting to provide a full explanation for international differences in demand patterns. Rather, they attempt to answer the question "Which representative-agent parameterisation of consumer behaviour is most consistent with the data?"

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[19] The proof uses the standard result from bilateral comparisons that, if tastes are homothetic, the Allen index is unique and lies between the Laspeyres and Paasche bounds. See for example, Neary and Gleeson (1997), Section 1.2.

A suitable framework within which to do all this is the Quadratic Almost Ideal System ("QUAIDS") of Banks, Blundell and Lewbel (1997).[20] The expenditure function for this system is:

$$\ln e(p,u) \quad = \quad \ln \alpha(p) \ + \ \frac{u\,\beta(p)}{1 - u\,\lambda(p)}, \tag{50}$$

where:

$$\begin{aligned} \ln \alpha(p) \ &\equiv \ \alpha_0 \ + \ \Sigma_i\, \alpha_i \ln p_i \ + \ \tfrac{1}{2} \Sigma_i \Sigma_h\, \gamma_{ih} \ln p_i \ln p_h, \\ \ln \beta(p) \ &\equiv \ \Sigma_i\, \beta_i \ln p_i, \qquad \lambda(p) \ \equiv \ \Sigma_i\, \lambda_i \ln p_i. \end{aligned} \tag{51}$$

The budget shares are quadratic in $\ln y$ (whence the name), where $\ln y \equiv \ln z - \ln\alpha(p)$; i.e., the log of nominal expenditure deflated by the "subsistence" price index $\alpha(p)$:

$$\omega_i \ = \ \alpha_i \ + \ \sum_h \gamma_{ih} \ln p_h \ + \ \beta_i \ln y \ + \ \frac{\lambda_i}{\beta(p)} (\ln y)^2. \tag{52}$$

For consistency with consumer theory, the parameters must satisfy some restrictions. Homogeneity or absence of money illusion requires that $\Sigma_{i=1}\alpha_i=1$ and $\Sigma_i\beta_i=\Sigma_i\lambda_i=\Sigma_h\gamma_{ih}=\Sigma_i\gamma_{ih}=0$; while negativity or symmetry of the Slutsky substitution matrix requires that $\gamma_{ih}=\gamma_{hi}$. With further restrictions, the QUAIDS system also nests cleanly two important sub-cases. Setting $\lambda_i=0,\ \forall i$, gives the AIDS model mentioned in Section 4.3, while setting $\beta_i=0$ *and* $\lambda_i=0,\ \forall i$, gives the Homothetic AIDS (HAIDS) model.

5.3 Imposing Negativity

The homogeneity restrictions are easily imposed in the estimation procedure by dropping

[20] An earlier pilot project by Neary and Gleeson (1997) used the linear expenditure function. However, the wide range of variation in expenditure levels in the sample pulls the estimated subsistence parameters in the Stone-Geary utility function close to zero, which is tantamount to imposing homotheticity. I am grateful to Anton Barten for this point.

the budget share equation for the *n*th good and expressing all prices relative to that good's price. However, the negativity restriction is more tricky. To impose it, I use an approach pioneered by Lau (1978) and applied to the AIDS model by Moschini (1998).

For any demand system, the typical term in the Slutsky substitution matrix equals:

$$e_{ih} = \frac{z}{p_i p_h}\left\{\frac{\partial^2 \ln e(p,u)}{\partial \ln p_i \, \partial \ln p_h} - \delta_{ih}\omega_i + \omega_i \omega_h\right\}, \tag{53}$$

where δ_{ih} is the Kronecker delta. For the QUAIDS system, the first term on the right-hand side becomes:

$$\frac{\partial^2 \ln e(p,u)}{\partial \ln p_i \, \partial \ln p_h} = \gamma_{ih} + \beta_i \beta_h \ln y + \frac{\beta_i \lambda_h + \beta_h \lambda_i}{\beta}(\ln y)^2 + \frac{2\lambda_i \lambda_h}{\beta^2}(\ln y)^3. \tag{54}$$

To impose negativity at a point, choose units so that $z{=}p_i{=}1$, $\forall i$ (and so $\ln y{=}0$ and, from (52), $\omega_i{=}\alpha_i$, $\forall i$) at that point. The Slutsky substitution term (53) then reduces to a simple function of parameters only:

$$\bar{e}_{ih} = \gamma_{ih} - \delta_{ih}\alpha_i + \alpha_i \alpha_h. \tag{55}$$

(This is the same as in the AIDS case, though because of the additional quadratic and cubic terms in (54), the restriction is more likely to be violated away from the point where it is imposed.) Finally, negativity is imposed by estimating not $\bar{E}$, the $(n-1)$-by-$(n-1)$ matrix of the $\bar{e}_{ih}$ from (55), but rather its Cholesky decomposition, the upper-triangular matrix T, where $\bar{E}=-T'T$. In practice, I chose units to impose negativity at the sample mean, estimated the α_i, β_i and λ_i along with the elements of T, and then used (55) to recover the implied estimates of γ_{ih}.[21]

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[21] Moschini gives detailed formulae for the γ_{ih} in the AIDS case. In the present application, the estimation was programmed in a matrix language, which avoided the need to use such formulae.

5.4 Restricting Price Responsiveness: The Semiflexible QUAIDS

Even with the restrictions of homogeneity and negativity imposed, the QUAIDS model has $\frac{1}{2}(n-1)(n+6)$ parameters ($n-1$ each of the α_i, β_i and λ_i; and $\frac{1}{2}n(n-1)$ of the γ_{ih}).[22] With 11 commodity groups, this gives a total of 85 parameters. More seriously, most of the parameters (all but $n-2$ of the λ_i) enter every budget share equation, so there are 76 [$=\frac{1}{2}(n^2+3n-2)$] parameters in each equation but only 60 observations. The conventional wisdom is that this makes maximum likelihood infeasible.[23] Previous authors have either resorted to approximations (replacing $\alpha(p)$ by an empirical price index such as the "Stone" index, $\ln P^S = \Sigma_i\, \omega_i \ln p_i$, following Deaton and Muellbauer) or used the two-step (and hence less efficient) minimum distance estimator rather than maximum likelihood. However, the fact that utility maximisation is a maintained hypothesis in the present application allows this problem to be overcome in a more satisfactory manner.

The estimation method I use extends to the QUAIDS model an approach developed by Diewert and Wales (1988) and applied to the AIDS model by Moschini (1998). This involves restricting the degree of price responsiveness of a flexible functional form, leading to a "semiflexible" system. Using the Cholesky decomposition, this is accomplished by setting the last $n-k-1$ rows of T to zero. As a result, the rank of T (and hence of $\bar{E}$) is k, and so the number of parameters to be estimated is reduced (to $3(n-1)+\frac{1}{2}k(2n-k-1)$ for the whole system and $\frac{1}{2}(2n-1)(k+2)-\frac{1}{2}k^2$ per equation).

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[22] This does not include α_0. In practice it has proved difficult to estimate this parameter with precision and previous writers have used one of two methods to impose its value. Deaton and Muellbauer (1980) and Banks, Blundell and Lewbel (1997) set α_0 just below the lowest expenditure level in the sample, while Moschini (1998) sets it equal to zero. Given the range of variation in the data, these two methods are equivalent in the present application.

[23] Deaton and Muellbauer (1980) and Deaton (1986, p. 1784) attribute this piece of oral tradition to Teun Kloek.

Previous authors have used the semiflexible approach to estimate systems of reduced rank only. However, it has a further computational advantage. For any value of k above zero, the coefficient estimates from the *preceding* value of k can be used as starting values.[24] This procedure allowed maximum likelihood estimation of both the AIDS and QUAIDS systems for *all* values of k (including the full rank case of $k=n-1$), without the need to resort to approximations.[25]

This procedure has the added attraction in the present context that restricting price responsiveness is of interest in itself. Recalling from Proposition 5 that with zero price responsiveness the Geary method is identical to the GAIA, it seems reasonable to conjecture that increasing the degree of price responsiveness should make the Geary index less attractive.

5.5 Econometric Estimates

In other respects, the estimation procedure was standard. Thirty-three different specifications were estimated, one for each value of k from 0 to 10 and for each of the HAIDS, AIDS and QUAIDS systems. For each specification, budget share equations for the first ten commodity groups were estimated by maximum likelihood. The budget share equations (52) are linear given $\beta(p)$ and $\alpha(p)$, so an iterative approach was used. The starting

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[24] Of course, estimating the system for $k=k_0$ gives estimates of only some of the τ_i parameters needed in the $k=k_0+1$ case. Starting values of 0.1 were assigned to the remaining τ_i: starting values of zero did not work, presumably because, since all the τ_i are squared, the numerical derivatives are all zero.

[25] The fact that, for high values of k, estimation was only possible using the parameter values from the previous value of k as starting values might cause concern that a global maximum of the likelihood function has not been found. Some reassurance on this point is provided by the results of a serendipitous programming error. In preliminary runs, the equations given by (52) were estimated with both $\ln y$ and (erroneously) λ_i deflated by the Stone price index. This converged to estimates similar to those in Figure 2 from arbitrary starting values (where, because of the difficulty noted in footnote 24, the "arbitrary" starting values used were typically $\alpha_i=\beta_i=\lambda_i=0$ and $\tau_i=0.1$).

values were used to construct estimates of $\beta(p)$ and $\alpha(p)$, and the resulting parameter values were then used to recalculate these functions for the next iteration.

Table 2 gives the values of the log likelihood for the different specifications estimated and Figure 2 illustrates them in a "likelihood tree", with the number of parameters in each specification given on the horizontal axis. Each "branch" of the tree corresponds to a different specification of the relationship between budget shares and real expenditure: none in the case of HAIDS, linear in the case of AIDS and quadratic in the case of QUAIDS. Each "leaf" on a given branch corresponds to a different value of k, the rank of the Slutsky substitution matrix, as the subsistence price index $\alpha(p)$ varies between a Cobb-Douglas ($k=0$) and a translog ($k=n-1$) specification.[26] Hence, moving to a higher branch implies a greater degree of responsiveness to income, while moving rightwards along a branch implies a greater degree of responsiveness to price.

As well as allowing a convenient visual presentation of the results, the likelihood tree diagram has an additional advantage that various test criteria for comparing pairs of likelihood values can be illustrated directly. The three loci in the lower right-hand corner of Figure 2 show how this can be done for a hypothetical log likelihood value given by point A. The three criteria illustrated are the Akaike information criterion (AIC) and the χ^2 likelihood ratio test at the 5% and 10% significance levels. The AIC attaches equal weight to identical increments in the log likelihood and the number of parameters, whereas the likelihood ratio typically trades off increases in the log likelihood more generously against increases in the number of parameters. All points below a given locus can be rejected in favour of point A by the test criterion in question. Hence, the significance of any particular specification

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[26] The HAIDS specification with $k=0$ (so all $\gamma_{ih}=0$) corresponds to the Cobb-Douglas utility function, with budget shares independent of price as well as income. The AIDS specification with $k=0$ is considered and estimated by Deaton (1978).

against alternatives can be assessed visually by moving the three loci so that point A coincides with the point representing the specification of interest. Of course, some of the possible comparisons between branches are non-nested. (Specifically, for any given point, this is true for all comparisons with points that are on a *higher* branch but have a *lower* value of *k*, and conversely). Even for the nested comparisons, the significance levels cannot be taken too seriously, both because they are asymptotic only and because of the sheer number of possible comparisons: with 32 independent bilateral nested comparisons, we would expect some to be significant even with random data. Nevertheless, the results have a degree of regularity which seems to justify some tentative conclusions.[27]

The first conclusion to be drawn is the overwhelming importance of allowing for some relationship between the budget shares and income. For each value of *k*, the HAIDS specification is clearly dominated by AIDS. However, the pay-off to including quadratic terms in income is not major: QUAIDS does not do much better than AIDS for given *k*. Next, at least in the AIDS and QUAIDS cases, allowing for additional price responsiveness does not contribute very much to the likelihood. Indeed, on purely statistical grounds, the single specification which is to be preferred overall is the AIDS case with $k=0$ (henceforth abbreviated in an obvious way as "A0").

5.6 Comparisons between the GAIA and Empirical Indexes

The final step is to use the estimated demand parameters to calculate the corresponding GAIA index. Appendix 1 gives details of how the GAIA indexes are calculated; Tables A1

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[27] I concentrate on likelihood function comparisons for choosing between systems. Of course, every "leaf" in Figure 2 represents a complete system of demand equations, each with its own parameter estimates, implied income and price elasticities, equation-by-equation diagnostics and observation-by-observation checks for negativity. Details are available on request.

to A6 present the GAIA real incomes and world prices for all thirty-three true indexes; Figure 3 illustrates a selection; and Figures 4 to 6 compare their summary statistics with those of the EKS, CCD and Geary indexes.[28]

The first conclusion to be drawn from these tables and figures is that homothetic tastes rationalise the EKS index. The indexes conditional on the HAIDS systems are almost indistinguishable from the EKS index in Figure 3 and are much more correlated with the EKS than with the Geary index in Figure 4. Conversely, non-homothetic tastes are more favourable to the Geary index, QUAIDS more so than AIDS, although even they are slightly more highly correlated with the EKS than with the Geary index.

Homotheticity or its absence also affects the responsiveness of the GAIA indexes to changes in k, the rank of the Slutsky matrix. Consider first the GAIA world prices in Tables A2, A4 and A6. These are highly implausible in all the HAIDS cases and also for A0 and Q0. By contrast, in the AIDS and QUAIDS cases they are reasonably stable for all values of k above zero, and are fairly strongly correlated with the Geary world prices. This alone justifies rejecting the A0 and Q0 indexes, which are outliers in all four figures. (In Figure 3, A0 is implausible low, while Q0 is omitted since it is implausibly high.) Ignoring these cases, increases in k have little effect on the correlation with the EKS index, but (at least for increases from 0 to 2) tend to raise the correlation with the Geary index, contrary to our earlier conjecture. (Table A7 shows that the rank of the different indexes is extremely stable across countries, except for very low-income countries.)

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[28] None of the figures illustrates indexes for values of k higher than 7, since for each of HAIDS, AIDS and QUAIDS, every single real income index was unaffected to three decimal places as k increased from 7 to 10. Figure 3 also omits many other indexes: Q0 is implausibly high, as discussed in the text; Hk for all k, A1 and Q1 are indistinguishable from the EKS index; A4 and A5 are indistinguishable from A3; A6 and A7 are indistinguishable from Q7, and Q6 is very close to it; Q3 is indistinguishable from Q5; and Q4 lies between A2 and Q5.

Figure 3 suggests that increases in *k* tend to raise the absolute dispersion of the true indexes, and this impression is confirmed by Figure 5, which shows the coefficient of variation and the Gini coefficient for all indexes. (The two measures of dispersion are not far from linearly related to each other.) Figure 5 shows that, ignoring the rogue A0 and Q0 cases, all the GAIA indexes imply a dispersion in real incomes greater than either the EKS or Geary indexes; this dispersion is greater the greater the degree of income responsiveness; and for the AIDS and QUAIDS indexes it is close to monotonically increasing in *k*.

Finally, the high correlations in Figure 3 might suggest that the choice between any of these indexes is of little practical consequence. This ignores the fact, familiar for intertemporal comparisons from studies such as the U.S. Boskin Commission, that small differences between index numbers can have significant implications for policy issues, when their effects are cumulated. Figure 6 illustrates this in the present context, taking as its starting point the U.N. target for foreign aid donations by high-income countries of 0.7% of GNP. To operationalise this, I calculate for each index the implied total transfer in billions of U.S. dollars (taking 0.7% of consumption expenditure grossed up by population) from countries which were members of the OECD in 1980 (denoted by an asterisk in Table 1). This measure is sensitive to the choice of reference country, and with a high-income country (the U.S.) selected as reference, the implied transfer is inversely related to the dispersion of the OECD countries relative to the rest of the sample. Figure 6 shows that, despite its lower coefficient of variation, the Geary index implies a transfer which is over $0.3 billion greater than that implied by the EKS index; the H*k* indexes imply considerably lower transfers; while the A*k* and Q*k* indexes imply much greater transfers, over $1 billion more than the $31.0 billion implied by the EKS index.

To conclude, we need to address two key questions: which of the candidate "true" indexes is preferable?; and what do the true indexes imply about the absolute and relative merits of the EKS and Geary indexes? As far as the first question is concerned, we have seen that statistical considerations alone rule out the HAIDS indexes. Within the AIDS and QUAIDS sub-groups, the superior statistical performance of the two systems with the least degree of price responsiveness (A0 and Q0) is outweighed by their implausible implications for the GAIA real incomes and world prices. But from Figure 2 if we accept increases in k there is no statistical justification for stopping at $k=1$. Indeed, on economic grounds, it may be desirable to allow as much flexibility as possible to both income and price. All this suggests that at least Q2, and possibly Q7 (which it may be recalled is indistinguishable from the full QUAIDS case Q10) is the preferred GAIA series.

As for the Geary and EKS indexes, I have already emphasised the key finding that homotheticity alone justifies preferring the latter. Otherwise, there is little advantage to either. Moreover, both compress the distribution considerably more (and thus underestimate the degree of inequality across countries) than the true AIDS and QUAIDS based indexes.

6. Conclusions

Many researchers have worked with the Penn World Table,[29] but not many have asked what exactly the numbers mean, and those who have considered the question have mostly advocated very different methods for calculating real or purchasing-power-corrected incomes. In this paper I have reexamined the conceptual framework of international comparisons, proposed a new benchmark for them, and considered how well it is approximated by existing methods, including the Geary method which underlies the Penn World Table.

[29] Heston and Summers (1996) quote an anonymous claim that over 20,000 regressions have been estimated using the Penn World Table. Sala-i-Martin (1997) alone exceeds this (in combination with other data sources) by a factor of almost 100!

At a conceptual level, I have argued that there are two distinct questions which must be faced in choosing a desirable index for multilateral comparisons. First, what are we trying to measure? Second, given a particular choice of true index, which of its nature is unobservable, how can it best be approximated in practice? The importance of the first question is inadequately recognised in the literature, because it does not arise when tastes are homothetic, a special and highly unrealistic case which is unduly emphasised in both theoretical and empirical analyses. (See, for example, Diewert (1996) and Dowrick and Quiggin (1997) respectively.) Except in that case, there is a threefold infinity of candidates for the "true" index. Real incomes could be measured by either the Allen, Konüs or Malmquist indexes, each one of which in turn can be evaluated at any of an infinite number of reference points (a reference price vector in the Allen case or a reference utility level in the other two cases). Hence, as emphasised by Allen (1949), there is no unique "true" measure of real income. For different purposes, different reference vectors may be preferred: for example, a Swiss multinational wishing to calculate local allowances for its executives might want to use a "star" system based on Swiss prices rather than the method proposed here. However, for researchers interested in world growth patterns or non-economists interested in international comparisons of living standards, there seems little justification for privileging one country in this way.

The answer proposed in this paper to the first question above, the GAIA system, is a set of Allen real income indexes, defined with reference to a vector of "world" prices which ensure consistent aggregation across commodities and countries. Under a wide class of demand systems, the world prices would generate actual world demand patterns. This system combines the best features of the economic approach to index numbers and of the Geary method. It meets all of the tests discussed in Section 2, except that of "characteristicity"; it allows for inter-commodity substitution; and it relates directly to the theory of linear aggregation.

As for the second question posed above, the bottom line of this paper is that the Geary method which underlies the Penn World Table is an acceptable, though not necessarily a particularly good, approximation to an appropriate ideal system. By contrast, the EKS and CCD methods provide good (second-order) approximations to an *inconsistent* set of multilateral comparisons. The results of this paper suggest that the EKS and CCD methods can only be recommended if tastes are close to homothetic, whereas the Geary method is satisfactory if substitutability is weak. Neither assumption is attractive. However, if forced to choose, it seems more reasonable to assume that spending patterns are invariant to price changes than that they are invariant to income changes. Of course, an appreciation of the theoretical underpinnings of the Geary method draws attention to potential pitfalls in applying it. For example, since the only case where the Geary method has been shown to be exact is when preferences are of the fixed-coefficient type, it would not be appropriate to use the Penn World Table to *test* hypotheses concerning the degree of inter-commodity substitutability.

Two drawbacks of the benchmark proposed here, one genuine and one illusory, should be mentioned. A genuine objection to the GAIA system is that it draws on consumer theory alone and hence relates only to comparisons of household expenditure patterns. Appropriate theoretical foundations for making international comparisons of investment and government spending must await further research. Of course, an alternative approach is to make international comparisons of real *output* only but such comparisons have no implications for *income* or *living standards*.[30]

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[30] Comparisons of real output corrected for deviations from purchasing power parity may also give highly misleading estimates of relative *productive potential*, if international differences in technology are significant. See Honohan (1997).

A second but in my view spurious objection to the approach adopted here is that it assumes that tastes are identical worldwide. This confuses the question of whether a *representative* consumer exists with the need to select a *reference* consumer in order to make international comparisons of real income. None of the paper's results, except those in Section 4.3, relies on the assumption of identical tastes worldwide. The empirical application in Section 5 sets out to find that parameterisation of preferences which most closely approximates the data, but does not require that a world representative consumer actually exist. By contrast, selecting a reference consumer is a necessary requirement for making international comparisons in the first place. Insofar as data on real income have any meaning, it is that they provide an answer to the question: "How well-off would the same reference consumer be in different countries?" Of the multitude of candidate reference consumers, it seems sensible for economists to focus on the hypothetical consumer whose consumption patterns mimic world consumption behaviour as closely as possible.

Turning to the empirical application in Section 5, this showed how the GAIA index can be calculated, using estimates of a complete set of demand equations, based on data from the International Comparison Project which underlies the Penn World Table. As well as illustrating the pitfalls and potential of estimating true multilateral indexes, this section turns up two key empirical findings. First, in accordance with the theoretical results, homothetic tastes rationalise the EKS index, whereas with non-homothetic tastes there is little basis for choosing between the EKS and Geary indexes. Second, both the EKS and Geary indexes compress the distribution of world income much more than the acceptable true indexes, suggesting that conclusions about "convergence" based on either index have to be treated with caution.[31] It would be very desirable to estimate the GAIA index on a panel data set, to investigate whether this over-compression effect imparts some spurious convergence to intertemporal comparisons.[32]

Appendix 1: Calculating the Geary and GAIA World Prices

The solution algorithm given in Section 4.2 is not very efficient, since it requires solving (30), a matrix characteristic equation of order *m*, at each step. Moreover, it is not guaranteed to converge, in spite of Proposition 4 which guarantees that a solution exists. The difficulty is that this Proposition only holds if world prices and virtual quantities remain strictly positive at all times. This is reasonable for actual consumers but it is not guaranteed for the QUAIDS system. Neary and Gleeson (1997) used a different algorithm, iterating on the alternative solution to (20) and (22): $\Pi=(\hat{q}^{*})^{-1}WQ^{*\prime}\Pi$. This is only of order *n* and it converged rapidly for the linear expenditure system. However, it failed to converge in the present case, because calculating virtual quantities for the QUAIDS system requires dividing the virtual budget shares by prices at each iteration, and from (52) the budget shares are not defined if even a

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31 This section also makes some methodological contributions. It estimates a semiflexible QUAIDS model; it introduces the "likelihood tree" diagram (Figure 2) as a convenient summary of a nested specification search; and, by using the estimates from round *k* as starting values in round *k+1*, it overcomes the "Kloek critique" (see Section 5.4), which has hitherto been assumed to restrict the feasible dimensionality of estimated demand systems.

32 The approach proposed in this paper has a further implication for time-series comparisons. The requirement of "characteristicity" is often taken for granted in a time-series context: why should the estimated growth rate of real income between 1990 and 1991 change when data on 1999 become available? But once it is recognised that there is no such thing as the "true" growth rate of real income, this question loses its paradoxical character. For some purposes, a consistent time series expressed in terms of the prices of a central year or even of a consistent average of different years, as in the GAIA system, may be more appropriate. As the results of Section 3 have shown, superlative indexes have desirable properties only in bilateral contexts or when tastes are homothetic. Since almost all comparisons in economics are multilateral, this calls into question the standard practice of "chaining" together a set of annual bilateral comparisons to produce a multi-period time series.

single price is negative.

In practice, the world prices were calculated by solving the equations in (34) non-linearly.[33] Positive prices were ensured by seeking values for the square root of the Π vector, rather than Π itself. This method worked well for all specifications except HAIDS[0] (the case of Cobb-Douglas preferences), for which the calculated Π vector was not well-determined. This was not surprising, since Cobb-Douglas budget shares are independent of prices. Even in this case, the implied z^* vector was identical to that calculated by an alternative route. This relies on the fact that relative GAIA indexes for all QUAIDS models can be derived explicitly:

$$\ln z_j^* - \ln z_k^* \quad = \quad \frac{\beta(\Pi)\,.\,(u_j - u_k)}{\{1 - u_j \lambda(\Pi)\}\,\{1 - u_k \lambda(\Pi)\}} \qquad (56)$$

In the HAIDS case, this expression can be used to calculate the relative indexes directly, without the need to calculate Π. With $\beta(\Pi)=1$ and $\lambda(\Pi)=0$, (56) reduces to:

$$\begin{aligned} \ln z_j^* - \ln z_k^* \quad &= \quad (u_j - u_k) \\ &= \quad \{\ln z_j - \ln \alpha(p^j)\} - \{\ln z_k - \ln \alpha(p^k)\}\,, \end{aligned} \qquad (57)$$

which is just the ratio of nominal expenditures deflated by a Cobb-Douglas function of domestic prices, where the weights are the estimated budget shares.

Appendix 2: The Data

The raw data are taken from Phase IV (1980) of the United Nations International Comparisons Project (ICP), available in hard copy as United Nations (1986). For the 60

[33] As explained in Section 5.3, the parameters of the demand systems were estimated using prices and expenditure scaled by sample means. For consistency, therefore, the solution to (34) used correspondingly scaled quantities.

countries in 1980, data on 11 categories of personal consumption expenditure were used: food; beverages; tobacco; clothing and footwear; gross rents; fuel and power; house furnishings, appliances and operations; medical care; transport and communication; recreation and education; and miscellaneous goods and services.

Table 6 of United Nations (1986) gives data on per capita expenditure in national currencies, $z_{ij} = p_{ij} .q_{ij}$ where p_{ij} is the price of good *i* in country *j* and q_{ij} is the quantity of good *i* in country *j*. Table 8 gives the purchasing power parities which are the national currency expenditures from Table 6 divided by expenditure in international prices. These international prices are produced by the Geary method of aggregation used in the ICP. Therefore the entries in Table 8 are: $(p_{ij} .q_{ij}) / (\pi_i .q_{ij}) = p_{ij} / \pi_i$, where π_i is the international price of good *i*. Dividing each entry by the corresponding entry for the United States, p_{i1} / π_i, gives prices in country *j* relative to prices in the United States: p_{ij} / p_{i1}. Dividing each entry in Table 6, $p_{ij} .q_{ij}$, by the corresponding relative price, p_{ij} / p_{i1}, gives quantities in country *j* measured in U.S. prices, $p_{i1} .q_{ij}$. This gives p_{ij} / p_{i1} and $p_{i1} .q_{ij}$, which are the price and quantity data required to calculate the various real income indexes.

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Table 1: Alternative Indexes of Real Income, 1980

	Country	Expenditure	Max. Lasp.	Min. Lasp.	EKS	CCD	Geary
1	Germany*	79.242	35.979	22.638	30.017	29.859	25.503
2	Denmark*	78.833	36.438	20.822	28.995	28.645	24.699
3	Belgium*	73.600	32.618	21.932	29.085	28.982	24.851
4	France*	73.282	35.337	20.916	29.338	29.319	25.021
5	U.S.A.*	70.273	44.789	26.716	36.120	35.653	30.624
6	Luxembourg*	69.860	38.703	22.369	30.066	30.016	26.114
7	Netherlands*	68.652	32.509	20.829	28.265	28.013	23.993
8	Norway*	63.528	26.880	17.236	23.571	23.445	20.591
9	Austria*	62.537	33.509	18.800	26.636	26.607	22.905
10	Canada*	62.280	41.345	25.125	34.171	34.033	29.230
11	U.K.*	56.530	31.368	17.669	25.304	25.251	21.614
12	Finland*	52.658	27.797	15.663	22.089	22.000	19.061
13	Japan*	49.079	26.722	15.356	21.630	21.597	19.102
14	Italy*	43.309	31.265	18.221	24.784	24.755	21.596
15	Spain*	36.987	24.896	16.087	21.070	21.196	18.226
16	Ireland*	33.789	18.917	12.288	16.978	17.217	14.638
17	Argentina	31.926	14.896	9.387	11.752	11.903	10.451
18	Israel	30.784	25.323	12.603	18.791	18.685	16.525
19	Hong Kong	30.640	25.132	18.700	22.450	22.530	20.783
20	Greece*	27.082	19.566	12.907	16.531	16.601	14.297
21	Uruguay	24.203	15.727	12.193	14.497	14.645	13.226
22	Venezuela	20.852	18.175	10.980	14.085	14.161	12.566
23	Portugal*	18.026	18.162	10.322	13.741	13.887	12.340
24	Yugoslavia	16.669	14.116	8.056	11.011	10.876	9.821
25	Chile	16.536	12.017	8.461	10.755	10.811	9.396
26	Poland	13.733	14.982	8.736	12.156	12.198	10.545
27	Brazil	13.598	12.901	8.875	11.347	11.297	10.045
28	Costa Rica	13.555	10.980	8.275	10.270	10.264	9.115
29	Hungary	10.454	16.031	9.658	13.285	13.152	12.017
30	Panama	10.439	8.232	6.510	8.026	8.066	7.142
31	Paraguay	9.473	9.027	6.032	7.576	7.554	6.686
32	Korea	9.454	8.038	5.774	6.679	6.834	6.491
33	Dominican Rep.	8.698	7.980	6.210	7.635	7.561	6.820
34	Colombia	8.443	11.193	8.036	9.813	9.862	8.758
35	Ecuador	8.406	7.644	6.212	7.161	7.172	6.719
36	Tunisia	8.094	6.888	5.045	6.201	6.149	5.583
37	Guatemala	7.845	9.223	7.054	8.624	8.470	8.077
38	Côte d'Ivoire	7.114	3.983	2.675	3.410	3.396	2.924
39	Peru	6.968	8.161	6.444	7.685	7.592	7.065
40	Bolivia	6.178	5.073	3.721	4.464	4.493	4.172
41	Nigeria	5.690	2.468	1.983	2.355	2.375	2.120
42	Botswana	5.615	3.761	3.129	3.658	3.676	3.285
43	Morocco	5.599	4.161	3.289	3.896	3.870	3.520
44	Cameroon	5.313	3.071	2.303	2.803	2.804	2.509
45	El Salvador	5.178	5.243	4.108	4.804	4.824	4.413

Table 1: Alternative Indexes of Real Income, 1980 (cont.)

	Country	Expenditure	Max. Lasp.	Min. Lasp.	EKS	CCD	Geary
46	Philippines	4.646	6.194	4.308	5.891	5.907	5.096
47	Honduras	4.398	4.987	3.204	4.079	4.051	3.642
48	Zimbabwe	4.245	2.825	2.273	2.674	2.711	2.371
49	Senegal	3.907	2.645	2.070	2.442	2.438	2.266
50	Zambia	3.369	1.948	1.257	1.620	1.629	1.421
51	Indonesia	2.753	3.431	2.459	2.958	2.946	2.802
52	Madagascar	2.652	2.196	1.704	1.974	1.963	1.862
53	Pakistan	2.619	4.772	3.114	4.175	3.983	3.624
54	Kenya	2.603	2.051	1.857	2.052	2.055	1.938
55	Sri Lanka	2.023	5.902	3.252	4.377	4.402	4.246
56	Tanzania	1.906	1.298	1.030	1.186	1.187	1.135
57	India	1.526	1.891	1.412	1.716	1.730	1.602
58	Mali	1.512	1.345	0.917	1.187	1.128	1.073
59	Malawi	1.302	1.502	1.083	1.260	1.282	1.208
60	Ethiopia	1.000	1.000	1.000	1.000	1.000	1.000
Mean		23.358	14.754	9.355	12.370	12.345	10.841
Standard Deviation		25.014	12.528	7.351	10.153	10.105	8.660
Coefficient of Variation		1.071	0.849	0.786	0.821	0.819	0.799
Correlation with EKS†		0.9210	0.9937	0.9948	1.0000	0.9999	0.9989
Correlation with Geary†		0.9107	0.9921	0.9962	0.9989	0.9990	1.0000
Gini Coefficient		0.5620	0.4720	0.4406	0.4581	0.4574	0.4479
Implied Transfer**		33.674	30.675	30.699	30.997	31.203	31.327

* Denotes an OECD member in 1980

† Squared correlation coefficients

** In billions of US$; see text for details.

Table 2: Number of Parameters and Value of Log Likelihood for Different Specifications

k	HAIDS		AIDS		QUAIDS	
	# Params.	LF Value	# Params.	LF Value	# Params.	LF Value
0	10	435.464	20	457.913	30	464.094
1	20	437.432	30	460.174	40	464.756
2	29	448.354	39	468.280	49	473.098
3	37	452.837	47	474.616	57	479.289
4	44	455.592	54	477.625	64	483.785
5	50	456.738	60	478.922	70	485.071
6	55	457.408	65	479.861	75	485.987
7	59	457.416	69	480.054	79	486.567
8	62	457.416	72	480.054	82	486.567
9	64	457.416	74	480.054	84	486.567
10	65	457.416	75	480.054	85	486.567

Table A1: GAIA-HAIDS[k] Indexes of Real Income, 1980

	Country	EKS	Geary	0	1	2	3	4	5	6	7-10
1	Germany	30.017	25.503	30.027	30.579	30.571	30.538	30.181	30.146	30.137	30.128
2	Denmark	28.995	24.699	28.665	29.243	29.275	29.273	28.937	28.906	28.903	28.894
3	Belgium	29.085	24.851	29.665	30.165	30.133	30.159	29.807	29.769	29.762	29.754
4	France	29.338	25.021	29.446	30.017	30.006	30.036	29.684	29.644	29.640	29.631
5	U.S.A.	36.120	30.624	37.142	37.403	37.418	37.314	37.109	37.071	37.047	37.039
6	Luxembourg	30.066	26.114	29.593	30.065	30.077	30.265	29.926	29.947	29.938	29.928
7	Netherlands	28.265	23.993	28.863	29.271	29.279	29.286	28.939	28.889	28.883	28.875
8	Norway	23.571	20.591	23.178	23.531	23.630	23.559	23.368	23.344	23.341	23.335
9	Austria	26.636	22.905	26.439	26.956	26.980	26.936	26.630	26.617	26.616	26.610
10	Canada	34.171	29.230	34.955	35.150	35.121	35.021	34.788	34.779	34.866	34.859
11	U.K.	25.304	21.614	25.267	25.827	25.784	25.710	25.412	25.402	25.402	25.396
12	Finland	22.089	19.061	22.165	22.445	22.482	22.398	22.144	22.137	22.148	22.145
13	Japan	21.630	19.102	21.229	21.638	21.651	21.557	21.332	21.328	21.388	21.383
14	Italy	24.784	21.596	24.673	25.131	25.117	25.086	24.802	24.795	24.793	24.791
15	Spain	21.070	18.226	20.963	21.355	21.338	21.321	21.073	21.044	21.039	21.034
16	Ireland	16.978	14.638	17.526	17.692	17.668	17.607	17.402	17.443	17.468	17.472
17	Argentina	11.752	10.451	11.408	11.522	11.556	11.524	11.403	11.452	11.442	11.439
18	Israel	18.791	16.525	18.377	18.742	18.710	18.651	18.436	18.447	18.449	18.444
19	Hong Kong	22.450	20.783	22.123	22.177	22.189	22.223	21.978	22.088	22.105	22.098
20	Greece	16.531	14.297	16.254	16.662	16.660	16.658	16.468	16.462	16.462	16.457
21	Uruguay	14.497	13.226	14.149	14.362	14.348	14.295	14.132	14.192	14.223	14.218
22	Venezuela	14.085	12.566	13.840	13.744	13.759	13.839	13.809	13.821	13.819	13.817
23	Portugal	13.741	12.340	13.398	13.551	13.637	13.583	13.462	13.482	13.497	13.496
24	Yugoslavia	11.011	9.821	11.008	11.160	11.207	11.127	11.030	11.098	11.102	11.101
25	Chile	10.755	9.396	10.559	10.801	10.798	10.781	10.666	10.664	10.666	10.664
26	Poland	12.156	10.545	12.276	12.265	12.311	12.239	12.113	12.175	12.179	12.180
27	Brazil	11.347	10.045	11.507	11.592	11.574	11.593	11.457	11.495	11.489	11.485
28	Costa Rica	10.270	9.115	10.145	10.318	10.328	10.290	10.191	10.199	10.216	10.214
29	Hungary	13.285	12.017	13.358	13.460	13.473	13.379	13.235	13.314	13.320	13.318
30	Panama	8.026	7.142	7.966	8.145	8.152	8.128	8.039	8.044	8.044	8.042
31	Paraguay	7.576	6.686	7.524	7.588	7.603	7.640	7.567	7.581	7.600	7.598
32	Korea	6.679	6.491	6.724	6.566	6.665	6.637	6.581	6.630	6.647	6.654
33	Dominican R	7.635	6.820	7.631	7.719	7.724	7.698	7.643	7.644	7.665	7.665
34	Colombia	9.813	8.758	9.726	9.731	9.781	9.746	9.672	9.680	9.739	9.738
35	Ecuador	7.161	6.719	7.004	7.083	7.081	7.092	7.017	7.043	7.049	7.047
36	Tunisia	6.201	5.583	6.176	6.309	6.321	6.303	6.239	6.243	6.241	6.240
37	Guatemala	8.624	8.077	8.537	8.515	8.544	8.533	8.459	8.472	8.522	8.522
38	Côte d'Ivoire	3.410	2.924	3.400	3.463	3.468	3.457	3.419	3.419	3.419	3.419
39	Peru	7.685	7.065	7.529	7.653	7.670	7.644	7.578	7.577	7.591	7.589
40	Bolivia	4.464	4.172	4.489	4.520	4.537	4.515	4.484	4.476	4.492	4.492
41	Nigeria	2.355	2.120	2.411	2.459	2.458	2.455	2.432	2.433	2.434	2.434
42	Botswana	3.658	3.285	3.603	3.690	3.685	3.674	3.638	3.635	3.634	3.633
43	Morocco	3.896	3.520	3.836	3.938	3.936	3.927	3.889	3.886	3.886	3.885
44	Cameroon	2.803	2.509	2.755	2.794	2.798	2.797	2.777	2.778	2.778	2.777
45	El Salvador	4.804	4.413	4.727	4.738	4.745	4.728	4.702	4.709	4.729	4.728

Table A1: GAIA-HAIDS[k] Indexes of Real Income, 1980 (cont.)

Country	EKS	Geary	0	1	2	3	4	5	6	7-10
46 Philippines	5.891	5.096	5.978	6.081	6.085	6.066	6.005	6.009	6.012	6.012
47 Honduras	4.079	3.642	4.028	4.064	4.076	4.076	4.037	4.039	4.055	4.054
48 Zimbabwe	2.674	2.371	2.649	2.730	2.726	2.714	2.687	2.687	2.686	2.685
49 Senegal	2.442	2.266	2.409	2.449	2.444	2.443	2.418	2.415	2.420	2.420
50 Zambia	1.620	1.421	1.573	1.601	1.600	1.595	1.588	1.587	1.587	1.587
51 Indonesia	2.958	2.802	2.939	2.948	2.992	2.971	3.031	3.027	3.027	3.027
52 Madagascar	1.974	1.862	1.917	1.972	1.970	1.965	1.944	1.942	1.943	1.942
53 Pakistan	4.175	3.624	4.341	4.371	4.410	4.398	4.352	4.360	4.360	4.361
54 Kenya	2.052	1.938	2.016	2.061	2.058	2.056	2.038	2.036	2.036	2.036
55 Sri Lanka	4.377	4.246	4.554	4.324	4.503	4.474	4.436	4.481	4.483	4.485
56 Tanzania	1.186	1.135	1.259	1.225	1.228	1.242	1.236	1.259	1.258	1.257
57 India	1.716	1.602	1.752	1.741	1.765	1.757	1.746	1.751	1.755	1.755
58 Mali	1.187	1.073	1.214	1.243	1.243	1.244	1.229	1.230	1.232	1.232
59 Malawi	1.260	1.208	1.234	1.249	1.248	1.244	1.231	1.229	1.234	1.234
60 Ethiopia	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Mean	12.370	10.841	12.352	12.513	12.527	12.508	12.384	12.391	12.398	12.396
Standard Dev.	10.153	8.660	10.210	10.366	10.361	10.355	10.248	10.239	10.240	10.237
Coef. of Var.	0.821	0.799	0.827	0.828	0.827	0.828	0.828	0.826	0.826	0.826
Corr. with EKS*	1.0000	0.9989	0.9993	0.9994	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995
Corr. with Geary*	0.9989	1.0000	0.9973	0.9972	0.9973	0.9973	0.9973	0.9975	0.9975	0.9975
Gini Coefficient	0.4581	0.4479	0.4600	0.4609	0.4603	0.4606	0.4603	0.4599	0.4597	0.4597
Transfer (US$b.)	30.997	31.327	30.449	30.643	30.633	30.651	30.552	30.557	30.581	30.581

* Squared correlation coefficients

Table A2: World Prices from HAIDS Estimates

k:	Geary	0*	1	2	3	4	5	6	7-10
1 Food	1.068	0.667	0.536	0.528	0.425	0.389	0.343	0.312	0.309
2 Beverages	0.732	0.356	0.149	0.178	0.156	0.287	0.149	0.147	0.144
3 Tobacco	0.894	2.529	0.294	0.225	0.273	0.273	0.222	0.411	0.406
4 Clothing & Footwear	1.010	0.000	0.553	0.576	0.592	0.321	0.397	0.471	0.471
5 Gross Rents	0.870	14.410	0.735	0.720	0.702	0.630	0.411	0.447	0.436
6 Fuel and Power	0.968	0.950	1.694	1.906	1.280	9.512	2.976	4.732	4.596
7 House Furnishings	1.109	0.693	1.625	1.666	1.276	0.388	0.585	0.683	0.684
8 Medical Care	1.012	0.365	0.822	0.770	0.853	0.405	0.353	0.462	0.456
9 Transport & Comms.	0.940	0.003	1.023	0.915	0.804	1.436	0.951	1.250	1.241
10 Recreation & Ed.	1.209	0.000	1.219	1.181	1.065	0.483	0.603	0.440	0.440
11 Misc.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Corr. with Geary prices†		-0.331	0.576	0.525	0.625	-0.039	0.083	0.004	0.007

* Estimates of Π for k=0 are not well-determined. See Appendix 1.

† Simple correlation coefficient between GAIA world prices $\Pi[k]$ and Geary world prices π

Table A3: GAIA-AIDS[k] Indexes of Real Income, 1980

	Country	EKS	Geary	0	1	2	3	4	5	6	7-10
1	Germany	30.017	25.503	19.051	30.618	34.831	38.818	38.515	38.284	40.645	40.785
2	Denmark	28.995	24.699	18.703	29.802	34.061	37.993	37.725	37.511	39.828	39.968
3	Belgium	29.085	24.851	18.772	30.182	34.276	38.125	37.836	37.614	39.907	40.047
4	France	29.338	25.021	18.806	30.223	34.250	38.108	37.812	37.591	39.912	40.054
5	U.S.A.	36.120	30.624	21.663	35.378	40.538	44.913	44.854	44.570	47.386	47.557
6	Luxembourg	30.066	26.114	19.207	31.046	35.166	39.081	38.801	38.585	40.948	41.106
7	Netherlands	28.265	23.993	18.079	28.726	32.578	36.281	36.001	35.796	37.964	38.096
8	Norway	23.571	20.591	15.673	23.919	27.292	30.536	30.365	30.277	32.028	32.143
9	Austria	26.636	22.905	17.621	27.822	31.580	35.280	35.033	34.869	36.990	37.122
10	Canada	34.171	29.230	21.980	35.898	40.682	45.353	45.194	45.310	48.210	48.415
11	U.K.	25.304	21.614	16.792	26.168	29.790	33.494	33.219	33.070	35.081	35.207
12	Finland	22.089	19.061	15.478	23.582	26.877	30.218	29.993	29.854	31.576	31.695
13	Japan	21.630	19.102	15.277	23.248	26.531	29.784	29.591	29.472	31.232	31.352
14	Italy	24.784	21.596	16.657	25.935	29.451	32.877	32.631	32.479	34.434	34.565
15	Spain	21.070	18.226	14.245	21.442	24.429	27.344	27.128	26.995	28.530	28.631
16	Ireland	16.978	14.638	12.536	18.027	20.598	23.130	22.934	22.858	24.236	24.356
17	Argentina	11.752	10.451	8.718	11.687	13.354	15.004	14.899	14.937	15.647	15.704
18	Israel	18.791	16.525	12.897	18.890	21.594	24.219	24.023	23.943	25.295	25.388
19	Hong Kong	22.450	20.783	15.196	22.417	25.629	29.149	28.988	28.924	30.674	30.787
20	Greece	16.531	14.297	11.792	16.986	19.406	21.792	21.618	21.533	22.684	22.773
21	Uruguay	14.497	13.226	10.533	14.554	16.740	18.802	18.635	18.642	19.625	19.698
22	Venezuela	14.085	12.566	10.508	14.278	16.336	18.598	18.505	18.454	19.397	19.463
23	Portugal	13.741	12.340	10.064	13.826	15.832	17.784	17.701	17.647	18.591	18.665
24	Yugoslavia	11.011	9.821	8.472	10.721	12.757	14.301	14.294	14.286	14.997	15.055
25	Chile	10.755	9.396	8.163	10.871	12.404	13.904	13.809	13.769	14.409	14.464
26	Poland	12.156	10.545	9.394	12.408	14.268	15.987	15.862	15.876	16.684	16.750
27	Brazil	11.347	10.045	8.778	11.674	13.333	15.008	14.892	14.905	15.620	15.669
28	Costa Rica	10.270	9.115	7.836	10.304	11.760	13.162	13.068	13.021	13.647	13.692
29	Hungary	13.285	12.017	10.204	13.524	15.751	17.657	17.538	17.589	18.492	18.563
30	Panama	8.026	7.142	6.369	8.076	9.176	10.221	10.138	10.127	10.546	10.578
31	Paraguay	7.576	6.686	5.939	7.408	8.446	9.397	9.368	9.374	9.759	9.788
32	Korea	6.679	6.491	5.316	6.247	7.074	7.841	7.783	7.801	8.181	8.239
33	Dominican R	7.635	6.820	6.009	7.510	8.508	9.457	9.402	9.369	9.782	9.811
34	Colombia	9.813	8.758	7.561	9.770	11.135	12.450	12.382	12.337	13.018	13.066
35	Ecuador	7.161	6.719	5.678	6.980	7.885	8.810	8.759	8.781	9.126	9.163
36	Tunisia	6.201	5.583	5.131	6.304	7.088	7.834	7.777	7.757	8.044	8.068
37	Guatemala	8.624	8.077	6.566	8.213	9.344	10.434	10.345	10.310	10.847	10.885
38	Côte d'Ivoire	3.410	2.924	3.056	3.388	3.782	4.132	4.095	4.081	4.194	4.218
39	Peru	7.685	7.065	6.032	7.562	8.569	9.527	9.459	9.438	9.848	9.876
40	Bolivia	4.464	4.172	4.012	4.624	5.136	5.768	5.728	5.701	5.886	5.909
41	Nigeria	2.355	2.120	2.159	2.330	2.492	2.652	2.630	2.625	2.674	2.679
42	Botswana	3.658	3.285	3.250	3.662	4.052	4.408	4.373	4.363	4.486	4.498
43	Morocco	3.896	3.520	3.366	3.859	4.253	4.621	4.582	4.569	4.698	4.712
44	Cameroon	2.803	2.509	2.339	2.539	2.803	2.980	2.958	2.945	3.006	3.014
45	El Salvador	4.804	4.413	3.672	4.167	4.632	5.086	5.055	5.034	5.225	5.239

Table A3: GAIA-AIDS[k] Indexes of Real Income, 1980 (cont.)

Country	EKS	Geary	0	1	2	3	4	5	6	7-10
46 Philippines	5.891	5.096	4.693	5.606	6.419	7.082	7.027	7.012	7.256	7.280
47 Honduras	4.079	3.642	3.308	3.628	4.049	4.392	4.348	4.335	4.487	4.509
48 Zimbabwe	2.674	2.371	2.622	2.839	3.096	3.414	3.383	3.363	3.446	3.457
49 Senegal	2.442	2.266	2.329	2.528	2.743	2.917	2.896	2.896	2.955	2.964
50 Zambia	1.620	1.421	1.600	1.571	1.727	1.846	1.850	1.845	1.873	1.876
51 Indonesia	2.958	2.802	2.736	2.829	3.088	3.576	3.633	3.618	3.695	3.706
52 Madagascar	1.974	1.862	1.808	1.889	1.991	2.110	2.090	2.084	2.113	2.120
53 Pakistan	4.175	3.624	3.501	3.824	4.455	4.917	4.876	4.855	4.995	5.007
54 Kenya	2.052	1.938	1.901	2.005	2.137	2.245	2.228	2.217	2.252	2.255
55 Sri Lanka	4.377	4.246	3.318	3.472	3.984	4.398	4.358	4.371	4.482	4.495
56 Tanzania	1.186	1.135	0.989	0.929	0.962	0.944	0.939	0.935	0.936	0.939
57 India	1.716	1.602	1.456	1.440	1.524	1.636	1.623	1.611	1.624	1.626
58 Mali	1.187	1.073	1.202	1.196	1.226	1.259	1.248	1.240	1.244	1.247
59 Malawi	1.260	1.208	1.471	1.452	1.532	1.606	1.599	1.604	1.622	1.626
60 Ethiopia	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Mean	12.370	10.841	8.725	12.550	14.273	15.928	15.823	15.771	16.633	16.694
Standard Dev.	10.153	8.660	6.314	10.531	12.019	13.447	13.363	13.302	14.166	14.219
Coef. of Var.	0.821	0.799	0.724	0.839	0.842	0.844	0.845	0.843	0.852	0.852
Corr. with EKS*	1.0000	0.9989	0.9937	0.9984	0.9986	0.9983	0.9984	0.9983	0.9983	0.9983
Corr. with Geary*	0.9989	1.0000	0.9952	0.9969	0.9974	0.9975	0.9976	0.9975	0.9975	0.9975
Gini Coefficient	0.4581	0.4479	0.4106	0.4676	0.4700	0.4719	0.4719	0.4715	0.4756	0.4757
Transfer (US$b.)	30.997	31.327	33.036	32.152	32.011	32.188	32.074	32.112	32.062	32.064

* Squared correlation coefficients

Table A4: World Prices from AIDS Estimates

k:	Geary	0	1	2	3	4	5	6	7-10
1 Food	1.068	0.000	0.943	0.935	0.935	0.932	0.957	0.918	0.898
2 Beverages	0.732	0.115	0.665	0.660	0.668	0.693	0.716	0.689	0.667
3 Tobacco	0.894	0.853	0.903	0.880	0.908	0.893	0.904	1.085	1.003
4 Clothing & Footwear	1.010	0.989	1.028	1.009	0.976	0.963	0.990	1.041	1.011
5 Gross Rents	0.870	0.000	0.941	0.915	0.918	0.916	0.952	0.943	0.876
6 Fuel and Power	0.968	0.247	1.162	1.138	1.145	1.029	1.077	1.120	1.092
7 House Furnishings	1.109	0.906	1.207	1.134	1.105	1.106	1.129	1.214	1.147
8 Medical Care	1.012	0.978	1.087	1.039	1.036	1.049	1.097	1.217	1.148
9 Transport & Comms.	0.940	0.000	1.069	1.050	1.055	1.025	1.047	1.098	1.086
10 Recreation & Ed.	1.209	0.000	1.267	1.229	1.204	1.211	1.229	1.149	1.154
11 Misc.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Corr. with Geary prices*		0.123	0.852	0.852	0.821	0.892	0.875	0.686	0.753

† Simple correlation coefficient between GAIA world prices $\Pi[k]$ and Geary world prices π

Table A5: GAIA-QUAIDS[k] Indexes of Real Income, 1980

	Country	EKS	Geary	0	1	2	3	4	5	6	7-10
1	Germany	30.017	25.503	67.720	29.444	32.568	36.668	35.695	37.016	40.203	41.221
2	Denmark	28.995	24.699	66.496	28.826	32.009	36.060	35.094	36.382	39.499	40.533
3	Belgium	29.085	24.851	66.238	28.962	31.986	35.940	34.988	36.275	39.373	40.367
4	France	29.338	25.021	66.555	29.057	32.046	36.015	35.041	36.333	39.427	40.449
5	U.S.A.	36.120	30.624	79.930	34.022	37.811	42.316	41.660	43.284	47.216	48.385
6	Luxembourg	30.066	26.114	69.323	30.060	33.135	37.160	36.236	37.588	40.935	42.021
7	Netherlands	28.265	23.993	62.706	27.478	30.338	34.130	33.219	34.420	37.295	38.222
8	Norway	23.571	20.591	53.092	23.273	25.795	29.155	28.547	29.567	31.924	32.710
9	Austria	26.636	22.905	61.552	26.915	29.726	33.540	32.660	33.843	36.661	37.596
10	Canada	34.171	29.230	82.531	34.804	38.319	43.248	42.400	44.488	48.587	49.818
11	U.K.	25.304	21.614	57.509	25.256	27.939	31.750	30.934	32.032	34.675	35.537
12	Finland	22.089	19.061	52.358	22.941	25.417	28.865	28.193	29.165	31.485	32.260
13	Japan	21.630	19.102	51.658	22.673	25.131	28.489	27.852	28.870	31.240	32.003
14	Italy	24.784	21.596	57.238	25.124	27.761	31.291	30.509	31.584	34.166	35.011
15	Spain	21.070	18.226	46.537	20.777	23.006	25.996	25.405	26.239	28.249	28.930
16	Ireland	16.978	14.638	39.428	17.594	19.523	22.119	21.647	22.312	24.108	24.674
17	Argentina	11.752	10.451	24.205	11.469	12.738	14.420	14.134	14.557	15.437	15.798
18	Israel	18.791	16.525	40.907	18.401	20.435	23.130	22.630	23.337	25.113	25.706
19	Hong Kong	22.450	20.783	50.856	21.705	24.099	27.712	27.172	28.105	30.519	31.267
20	Greece	16.531	14.297	36.376	16.551	18.370	20.807	20.376	20.993	22.477	23.022
21	Uruguay	14.497	13.226	31.206	14.208	15.864	17.957	17.591	18.168	19.381	19.853
22	Venezuela	14.085	12.566	30.758	13.790	15.338	17.604	17.345	17.839	19.035	19.473
23	Portugal	13.741	12.340	29.334	13.470	14.981	16.949	16.739	17.193	18.375	18.805
24	Yugoslavia	11.011	9.821	23.049	10.451	11.972	13.504	13.524	13.868	14.772	15.082
25	Chile	10.755	9.396	22.017	10.620	11.804	13.331	13.090	13.413	14.194	14.498
26	Poland	12.156	10.545	26.454	12.043	13.448	15.172	14.908	15.322	16.378	16.729
27	Brazil	11.347	10.045	24.549	11.493	12.780	14.499	14.217	14.626	15.517	15.843
28	Costa Rica	10.270	9.115	20.327	9.928	11.044	12.440	12.209	12.493	13.207	13.483
29	Hungary	13.285	12.017	29.719	13.153	14.859	16.772	16.509	17.048	18.212	18.620
30	Panama	8.026	7.142	15.502	7.903	8.763	9.810	9.614	9.822	10.312	10.511
31	Paraguay	7.576	6.686	13.892	7.217	8.032	8.977	8.815	9.025	9.459	9.627
32	Korea	6.679	6.491	11.721	6.053	6.723	7.480	7.324	7.443	7.863	8.000
33	Dominican Re	7.635	6.820	14.159	7.264	8.055	8.998	8.883	9.066	9.505	9.683
34	Colombia	9.813	8.758	19.416	9.439	10.498	11.813	11.608	11.895	12.598	12.864
35	Ecuador	7.161	6.719	13.192	6.842	7.565	8.503	8.344	8.532	8.930	9.092
36	Tunisia	6.201	5.583	11.573	6.171	6.822	7.564	7.461	7.598	7.943	8.075
37	Guatemala	8.624	8.077	15.615	7.829	8.710	9.775	9.589	9.793	10.330	10.541
38	Côte d'Ivoire	3.410	2.924	5.488	3.342	3.652	4.002	3.964	4.005	4.148	4.228
39	Peru	7.685	7.065	14.244	7.317	8.129	9.079	8.899	9.096	9.532	9.719
40	Bolivia	4.464	4.172	7.992	4.511	4.939	5.543	5.407	5.479	5.693	5.767
41	Nigeria	2.355	2.120	3.137	2.263	2.408	2.559	2.490	2.502	2.544	2.576
42	Botswana	3.658	3.285	6.068	3.656	3.985	4.341	4.280	4.330	4.465	4.533
43	Morocco	3.896	3.520	6.280	3.800	4.130	4.494	4.427	4.479	4.623	4.693
44	Cameroon	2.803	2.509	3.620	2.489	2.701	2.870	2.849	2.863	2.929	2.966
45	El Salvador	4.804	4.413	6.538	3.909	4.283	4.706	4.604	4.656	4.807	4.890

Table A5: GAIA-QUAIDS[k] Indexes of Real Income, 1980 (cont.)

Country	EKS	Geary	0	1	2	3	4	5	6	7-10
46 Philippines	5.891	5.096	9.662	5.372	6.013	6.655	6.526	6.626	6.888	6.997
47 Honduras	4.079	3.642	5.929	3.533	3.879	4.207	4.114	4.150	4.275	4.376
48 Zimbabwe	2.674	2.371	4.412	2.815	3.038	3.353	3.282	3.303	3.403	3.461
49 Senegal	2.442	2.266	3.741	2.535	2.722	2.894	2.852	2.888	2.963	2.998
50 Zambia	1.620	1.421	2.138	1.592	1.719	1.842	1.872	1.879	1.909	1.927
51 Indonesia	2.958	2.802	4.309	2.681	2.913	3.336	3.283	3.306	3.405	3.466
52 Madagascar	1.974	1.862	2.451	1.854	1.947	2.062	2.017	2.026	2.058	2.082
53 Pakistan	4.175	3.624	6.333	3.668	4.200	4.633	4.543	4.588	4.736	4.798
54 Kenya	2.052	1.938	2.698	1.992	2.109	2.212	2.185	2.191	2.226	2.247
55 Sri Lanka	4.377	4.246	5.522	3.183	3.630	3.996	3.906	3.958	4.087	4.140
56 Tanzania	1.186	1.135	0.961	0.915	0.945	0.922	0.924	0.918	0.926	0.931
57 India	1.716	1.602	1.668	1.375	1.463	1.557	1.513	1.508	1.527	1.532
58 Mali	1.187	1.073	1.313	1.180	1.211	1.240	1.215	1.213	1.223	1.232
59 Malawi	1.260	1.208	1.923	1.492	1.567	1.642	1.633	1.653	1.678	1.691
60 Ethiopia	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Mean	12.370	10.841	26.552	12.161	13.467	15.152	14.832	15.303	16.427	16.809
Standard Dev.	10.153	8.660	24.274	10.162	11.255	12.738	12.430	12.939	14.128	14.500
Coef. of Var.	0.821	0.799	0.914	0.836	0.836	0.841	0.838	0.846	0.860	0.863
Corr. with EKS*	1.0000	0.9989	0.9972	0.9979	0.9981	0.9977	0.9977	0.9977	0.9977	0.9977
Corr. with Geary*	0.9989	1.0000	0.9959	0.9966	0.9970	0.9971	0.9972	0.9970	0.9969	0.9968
Gini Coefficient	0.4581	0.4479	0.5069	0.4660	0.4668	0.4701	0.4688	0.4725	0.4795	0.4808
Transfer (US$b.)	30.997	31.327	31.733	32.268	32.171	32.351	32.174	32.132	31.996	32.000

* Squared correlation coefficients

Table A6: World Prices from QUAIDS Estimates

k:	Geary	0	1	2	3	4	5	6	7-10
1 Food	1.068	0.000	1.006	1.000	0.989	0.995	1.017	0.923	0.884
2 Beverages	0.732	0.087	0.724	0.717	0.712	0.747	0.770	0.686	0.648
3 Tobacco	0.894	0.191	0.966	0.939	0.964	0.952	1.051	1.111	0.966
4 Clothing & Footwear	1.010	0.505	1.075	1.049	1.012	0.995	1.071	1.039	0.959
5 Gross Rents	0.870	0.020	0.951	0.925	0.924	0.929	1.015	0.880	0.797
6 Fuel and Power	0.968	6.197	1.104	1.089	1.093	1.008	1.193	1.080	0.981
7 House Furnishings	1.109	0.026	1.277	1.201	1.167	1.160	1.216	1.204	1.103
8 Medical Care	1.012	0.000	1.107	1.054	1.050	1.055	1.226	1.203	1.090
9 Transport & Comms.	0.940	0.000	1.053	1.036	1.040	1.012	1.092	1.098	1.055
10 Recreation & Ed.	1.209	0.000	1.274	1.244	1.221	1.212	1.227	1.173	1.189
11 Misc.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Corr. with Geary prices*		-0.045	0.914	0.929	0.906	0.944	0.792	0.718	0.828

* Simple correlation coefficient between GAIA world prices $\Pi[k]$ and Geary world prices π

Figure 1: Alternative Indexes of Real Income, 1980 (Ethiopia = 1.0)

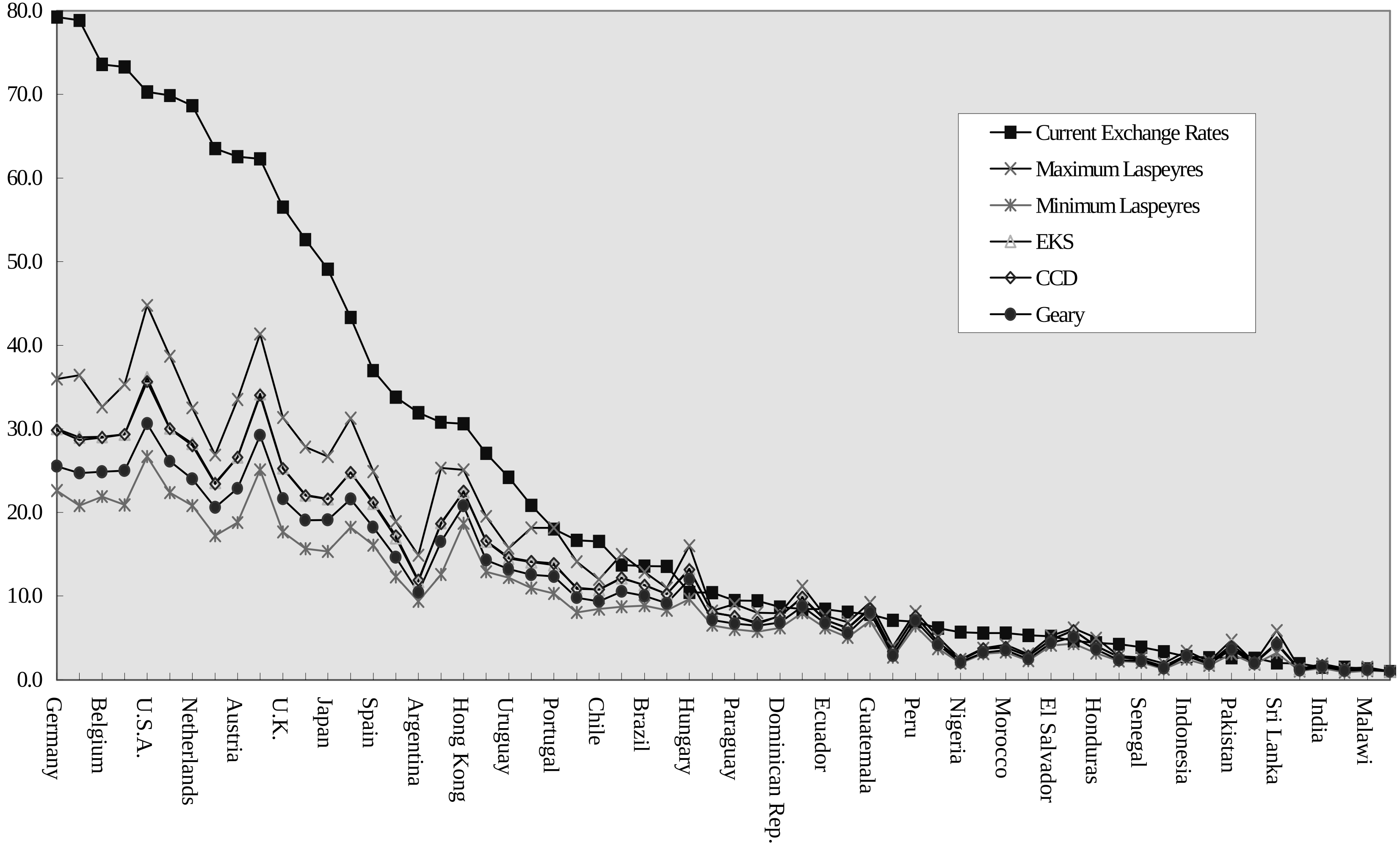

Figure 2: Value of Log Likelihood for Different Specifications, 1980

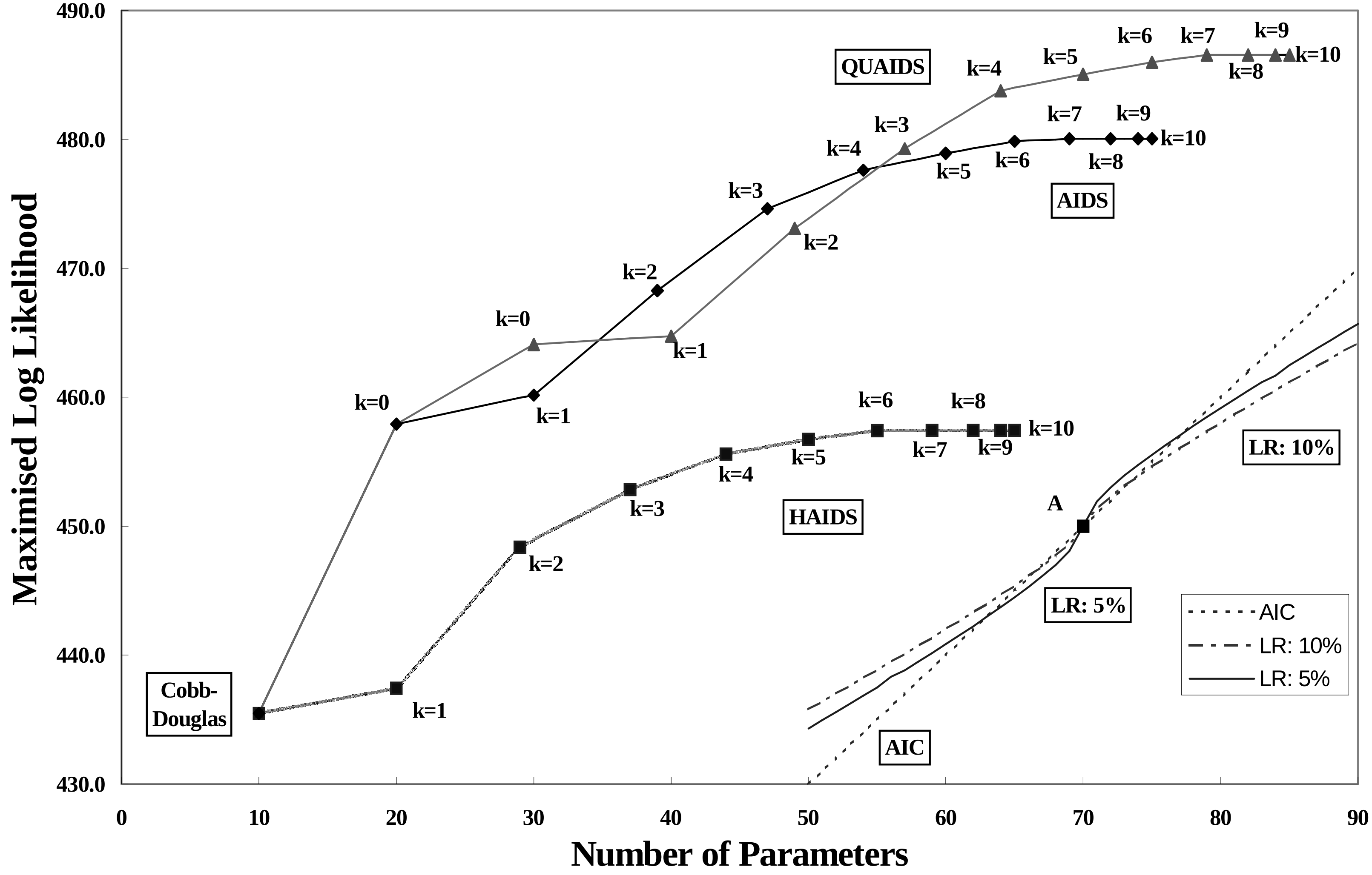

Figure 3: Alternative True Indexes of Real Income, 1980

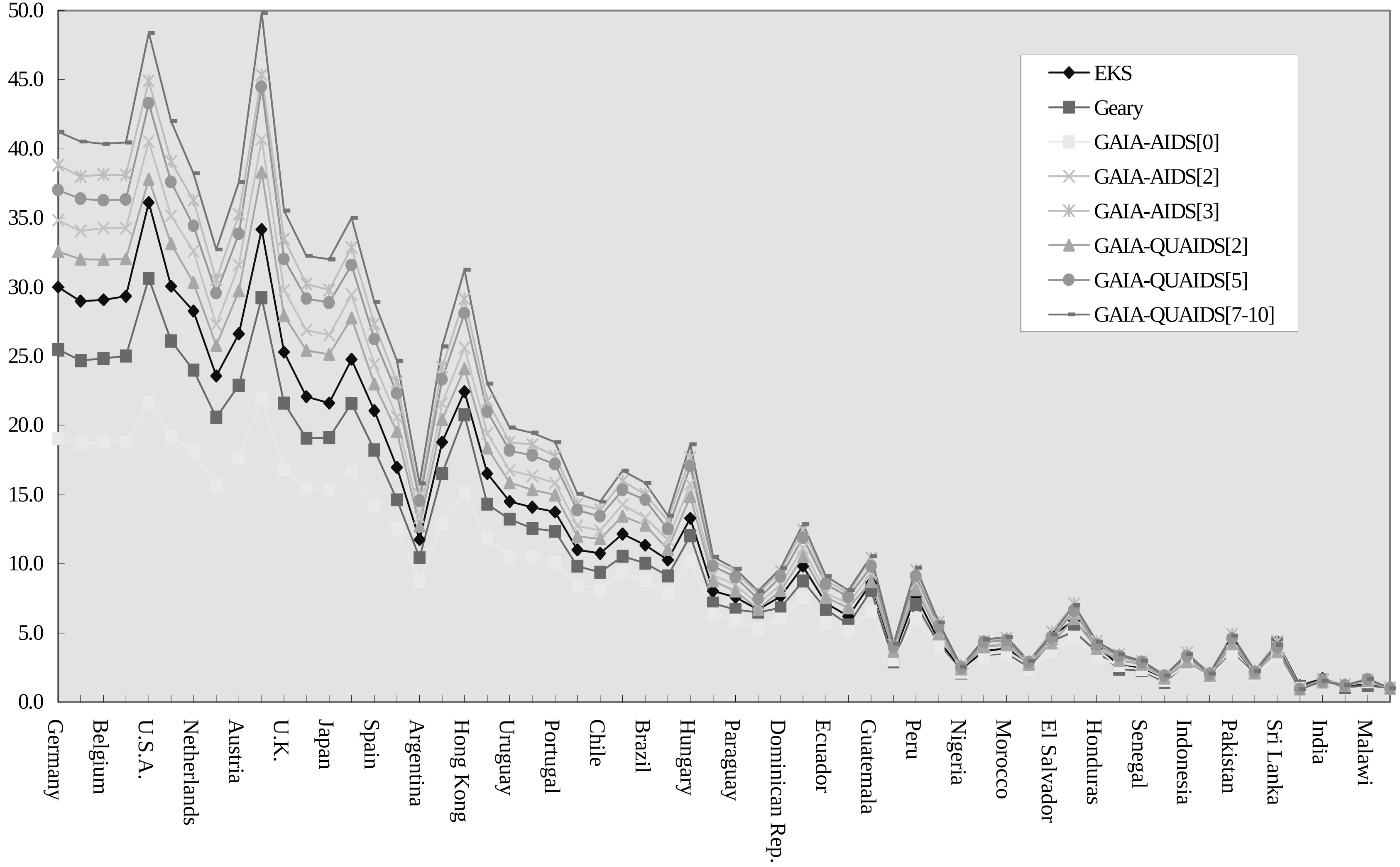

Figure 4: Correlations with EKS and Geary Indexes

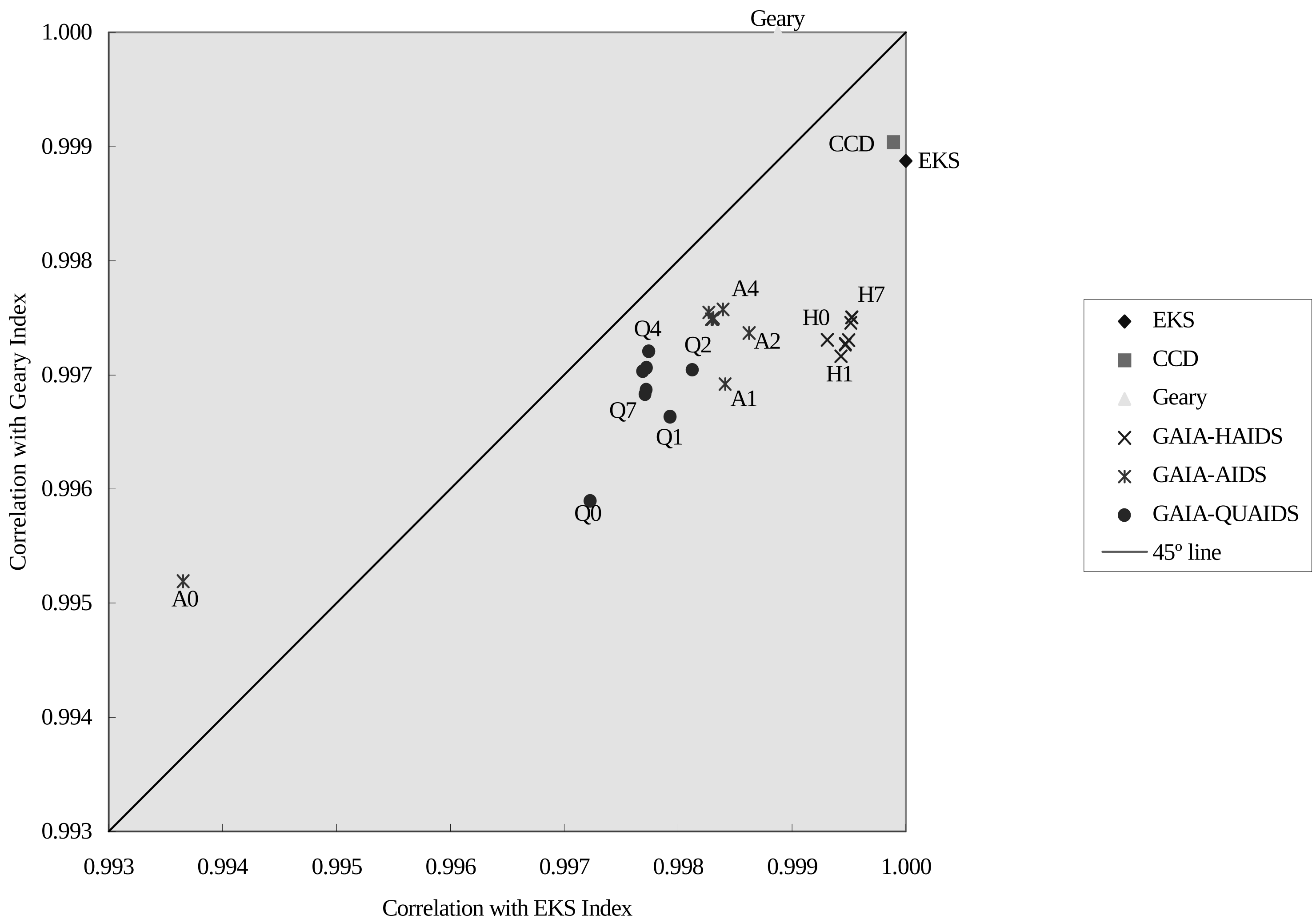

Figure 5: Gini Coefficient and Coefficient of Variation

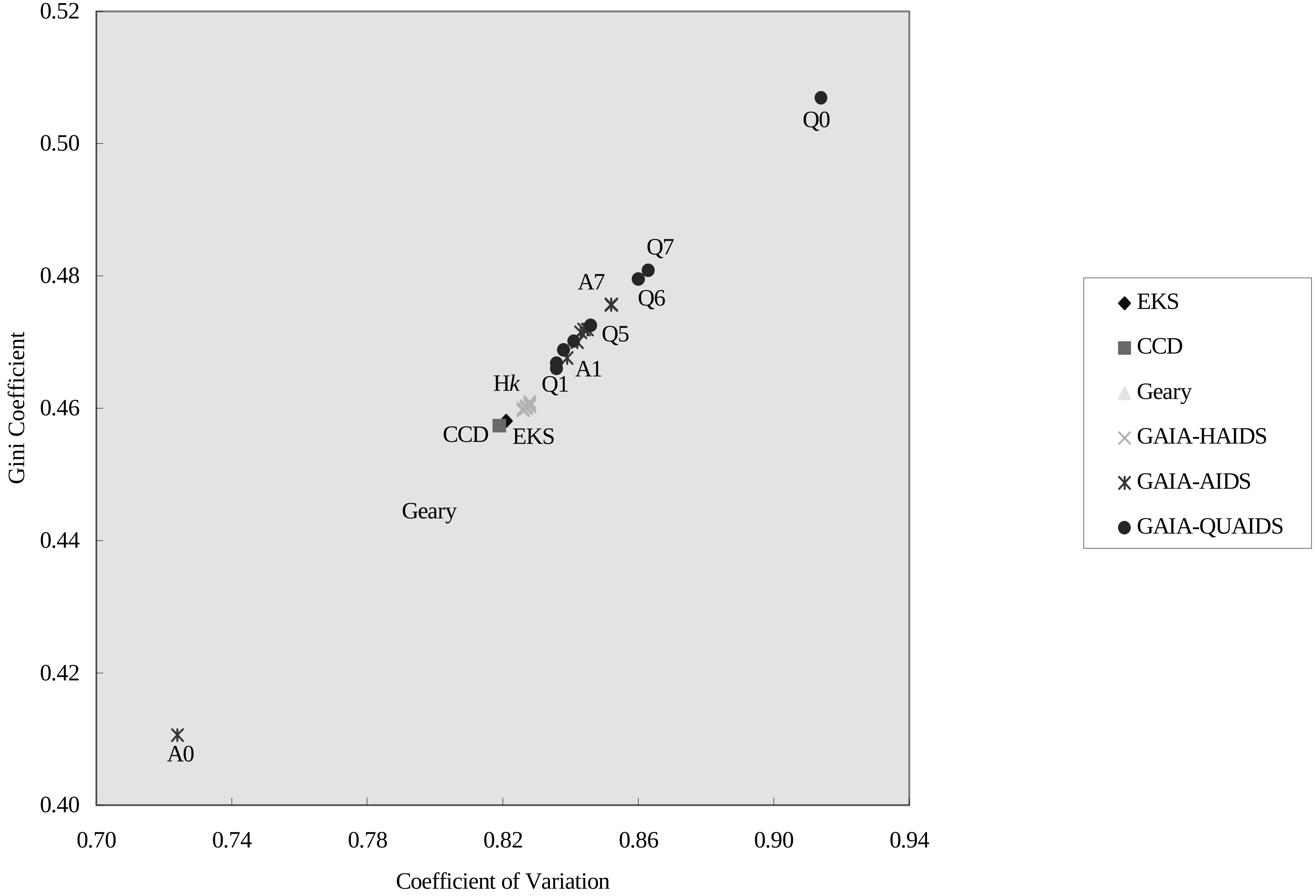

Figure 6: Gini Coefficient and Implied Transfer