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ABSTRACT

Location and Education in South African Cities Under and After Apartheid*

This paper models a South African city during Apartheid (in which both schooling and mobility are restricted on the basis of race) and after Apartheid (in which no restrictions are imposed). It is shown first that the inequality between blacks and whites decreases when Apartheid laws are removed. In fact, blacks are better off because of human capital externalities due to the possibility of mixing with white students whereas whites are worse off due to negative human capital externalities and intensified land market competition. It is also shown that, after Apartheid, reducing the commuting costs of black children always increases the utility of black families and may even increase that of whites.

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NON-TECHNICAL SUMMARY

In 1994, Nelson Mandela became the first black president of South Africa, ending half a century of Apartheid policy. South Africa's society has been literally shaped by the segregation policy of the former regime which created appalling disparities between communities. In particular, Apartheid has had a tremendous impact on the workings of South African cities, leading to great inequalities in the distribution of income and human capital among spatially divided population groups. A key feature under Apartheid was that urban non-whites were discriminated against, concerning both their residential location and their access to schooling: they were forced to reside in peripheral locations with segregated and low quality education, far away from whites and central jobs.

The case of Cape Town perfectly illustrates these patterns so typical of South African cities. In the 'Mother City', which expands beyond 25km and encompasses over 2.5 million inhabitants (among which one quarter are white), the segregation between communities is still almost complete. In 1991, the black-white index of dissimilarity (that is the percentage of people of one group that would have to be removed in order to obtain a perfectly homogeneous distribution of the population in the city) was above 97%. In 1996, the index of dissimilarity still exceeded 93%. Asians, coloureds and blacks were forced to live in peripheral townships far away from jobs, leading to a strong spatial mismatch between the places of residence of non-whites and their places of employment. Most centres of opportunity are clustered around one edge of the city – the CBD (central business district) and its closed surroundings. More than 80% of formal employment is located in the CBD or its vicinity and most whites, but less than 40% of the city's total population, live there. This induces long and costly commuting trips for those non-whites who are lucky enough to hold a job. This situation is even more exacerbated since whites and non-whites do not use the same transportation modes. In 1992, 87% of whites used their cars to commute yet 79% of blacks resorted to a very inefficient public transportation network. For example, in 1990, a resident commuting to Cape Town's city centre from Khayelitsha (one of the city's black townships) had to face two hours and 40 minutes of transport, excluding waits at connection points.

In addition, access to education was (and still is) very unequal across communities. Under Apartheid, education was completely segregated by law and for every four Rands spent on a white child, only three Rands were spent on an Asian child, two Rands on a coloured child and one Rand on a black child. The whole system was meant to prevent non-whites from having access to a fair education. In 1991, 88% of white pupils reached the final year of high school whereas only 14% of black pupils managed to do so. Not surprisingly, this resulted in extreme imbalances between communities. In 1995, 69% of

Cape Town's white households had a monthly income above 3,500 Rands yet 74% of black households had an income lower than 1,500 Rands.

The aim of this paper is to model the South African urban situation during and after Apartheid and to analyse its consequences in terms of education, housing and mobility. It is our contention that the mechanisms of school desegregation (after Apartheid) can only be fully understood at a local and spatial level. The main question is whether ending the restrictions on residential location and school choices can significantly contribute to the reduction of human capital imbalances.

Keeping this issue in mind, we study two types of situations. In the Apartheid regime, whites live near the city centre where all jobs are located whereas blacks (or non-whites) are forced to reside at the outskirts of the city. White children go to the centrally located white school while black children attend a black peripheral school. Because of Apartheid, there is no competition between groups either in education (black and white children do not mix so there are no human capital externalities) or in the land market (there are two separate markets). On the contrary, in the post-Apartheid regime, there are no land-use or education restrictions so everybody can live or study anywhere in the city.

This paper is the first attempt to model education in a spatial urban context in South Africa under and after Apartheid. Though the model may seem quite stylized, we believe that it captures the basic features of most South African post-Apartheid cities. The key features of our model are: (i) the historical difference in human capital; (ii) the fact that historically disadvantaged families have to pay a cost of adjustment to the school norms to attend the high-quality school; and (iii) the difference in commuting costs. If we observe that these factors are present in other countries then our results could apply to contexts outside South Africa where minority students have been historically discriminated against (for example blacks in the US).

The Paper reaches the following results:

1. Even though the land-use restrictions have been removed, the general structure of cities remains the same after Apartheid as it was under Apartheid: whites still reside close to the centres of opportunity whereas blacks live far away from these centres.
2. When Apartheid racial laws are removed, the inequality between blacks and whites can only decrease because blacks benefit from local human capital externalities by mixing with white students, and because whites are worse off since both their human capital decreases and the land rent increases.
3. In this paper the emergence of a new phenomenon widely recognized in post-Apartheid South Africa is observed – the fact that some black students

accept very long commutes in order to attend better schools located in white central locations.

Our model also has strong implications for the policies that can be implemented in post-Apartheid South African cities, namely the use of buses and transport-to-school subsidies. Indeed, we have shown that a policy aiming at reducing the transport costs of black children has an ambiguous effect on the utility of whites but is always beneficial to blacks since it enables them to go to the best school in the city.

From the above discussion, it should be clear that space and education are central to the analysis of South African cities. Space and education were the two main criteria driving discrimination as Apartheid was designed to prevent blacks and whites from interacting, especially in schools, and the spatial separation of communities was the most efficient way to ensure that no such interaction would ever happen. Our analysis shows that post-Apartheid integration can be promoted through education even though the spatial separation between communities remains.

1 Introduction

In June 1998, the South African newspaper *the Star* told the story of Nomakhazi Mdakane, a 46 year-old single mother who lives in the suburbs of Cape Town and commutes everyday to a job located in a central part of the city. Up before dawn, she makes a 1 km walk to a taxi rank and often waits an hour before being able to board her only transport to work, at about 6 am. If she could afford it, she would buy a car but half of her wages are spent on transport already and the rest goes to fees for her daughters who attend an inner-city school.

Such stories are common in South African cities and illustrate some of the major problems that have been inherited from the past policy of Apartheid. Indeed, South Africa's former segregation policy has had a tremendous impact on the workings of South African cities, leading to great inequalities in the distribution of income and human capital among spatially divided population groups. A key feature under Apartheid was that urban non-whites were discriminated on both their residential location and access to schooling: they were forced to reside in peripheral locations with segregated and low quality education, far away from whites and central jobs.

The aim of this paper is to model the South African urban situation during and after Apartheid and to analyze its consequences in terms of education, housing and mobility. It is indeed our contention that the mechanisms of school desegregation (after Apartheid) can only be fully understood at a local and spatial level. *The main question is whether ending the restrictions on residential location and school choices will lead to more equality or larger disparities.*

Keeping this issue in mind, we study two types of situations. In the Apartheid regime, whites live near the city center where all jobs are located whereas blacks (or non-whites) are forced to reside at the outskirts of the city. Moreover, white children go to the centrally located white school while black children attend a black peripheral school. Because of Apartheid, there is no competition between groups neither in education (black and white children do not mix so there are no human capital externalities) nor in the land market (there are two separate markets). There is however within-group competition.

In the post-Apartheid regime, there are no land-use nor education restrictions so everybody can live or study anywhere in the city. We show that white families still reside at the vicinity of the city center and bid away black families to the outskirts of the city. This corresponds to the structure of many post-Apartheid South African cities (such as, for instance, Cape Town or Durban). In this context, as illustrated by the Star's article, black

parents still have long commuting trips to work in the city center whereas whites have much shorter commuting trips. However, in the post-Apartheid regime, there is a certain amount of educational mixing since some blacks are able to study at the central school (the former ‘white school’). If, for black students, there is an integration cost to study in a white school (because, for instance, they have to adapt to the English language or to the new norms), and if blacks are heterogeneous with respect to this cost, we show that, on average, those who attend the central school live relatively closer to the city center and have a low integration cost. The interesting feature of the post-Apartheid situation is that black families are induced to send their children to the central school, which leads to educational mixing.

We then compare these two regimes. For whites, it should be clear that they incur a utility loss because they face more competition in the land market (leading to higher land prices) and because their level of education decreases due to their mixing with black students (who inherited a lower initial human capital). Concerning blacks (as a whole), there is a trade-off. On one hand, they gain because of human capital externalities, but on the other hand, they lose because of fiercer competition in the land market. We show that, as a net effect, black families are better off and inequality (as measured by the difference between the utilities of whites and blacks) decreases when restrictions are removed (as in the post-Apartheid regime).

The rest of the paper is as follows. The next section gives some stylized facts about South African cities that fit with our model. Section 3 describes our model while section 4 focuses on the Apartheid equilibrium. In section 5, we determine the post-Apartheid equilibrium and section 6 compares the two types of equilibria. Finally, section 7 concludes.

2 Some facts about South African cities

The aim of this section is to describe the South African situation under and after Apartheid. We focus on different elements that are relevant to our model, namely the structure of cities, transportation costs, inequality and education.

2.1 The structure of South African cities

It is necessary to recall that Apartheid was implemented for almost half a century and resulted in tremendous disparities between communities, one of the main features being the *land-use restrictions* that were imposed on all communities. Under Apartheid, only whites could live close to the city center

where most jobs were located. The non-white labor force (i.e. ‘Asians/Indians’, ‘coloureds’ or ‘blacks/Africans’ according to the former racial classification established by the Apartheid regime) could only live in peripheral locations of cities, sometimes very far from the city center (see e.g. Smith [27]). Racially homogeneous townships separated by buffer zones were created in order to prevent people from interacting with individuals from other communities. Typically, Asian and coloured townships were distant but relatively closer to the city center whereas black townships were located as far away as possible from the center. Even though a certain amount of residential desegregation started occurring at the end of the 1980s, these spatial patterns of segregation still prevail in the 1990s. In Cape town for instance, the black-white index of dissimilarity was above 97% in 1991 (Christopher [12]) and still exceeded 93% in 1996.¹ As Table 1 shows, there is a very high level of segregation in all South African metropolitan areas and between all communities. These values are extremely high since, even in highly segregated American cities such as Detroit, the dissimilarity index there is ‘only’ in the range of 70% (Cutler, Glaeser and Vigdor [13]).

Table 1: Indices of dissimilarity for the five major metropolitan areas in 1991

	Johannesburg	Cape Town	Durban	Pretoria	Port Elizabeth
Asian-Black	94.5	98.0	90.9	91.5	99.2
Asian-Coloured	88.6	94.0	89.1	96.4	82.1
Asian-White	90.0	90.6	98.3	96.3	93.4
Black-Coloured	93.5	97.5	91.0	90.4	95.0
Black-White	89.6	97.3	66.6	87.1	98.2
Coloured-White	93.9	96.3	94.7	92.4	97.7

Source: Christopher [12].

Not surprisingly, one of the main effects of racial zoning and segregation was to break down cities into very contrasted urban zones. In the Cape Metropolitan Area for instance, an urban area that expands beyond 25 km and encompasses over 2.5 million inhabitants, most centers of opportunity are clustered around one edge of the city — the central business district and

¹Considering two communities, blacks and whites for example, the dissimilarity index is equal to $\frac{1}{2} \sum_i \left| \frac{Blacks_i}{Blacks} - \frac{Whites_i}{Whites} \right|$ where i refers to neighborhoods. This index gives the percentage of individuals of a given type who would have to relocate in order to produce a homogeneous distribution of the population within the city. A dissimilarity index of less than 30% is considered low, between 30% and 60%, medium, and above 60%, high (see Cutler, Glaeser and Vigdor [13]).

its closed surroundings (Mail & Guardian [20]). Indeed, the job rich central parts of the city contrast with the townships and the poor peripheral informal housing areas. The central parts comprise of centers of employment laid out along ‘corridors’ extending outward from the port and city center. They mainly host middle and higher income people. On the contrary, townships are inhabited by middle to lower income people with poor access to activities and services while peripheral informal housing areas mainly consist of high density slums. Needless to say that the main problem caused by the lay-out of South African cities is obviously the *separation of workplace and residence*. In Cape Town, more than 80% of formal employment is located in the CBD or along the ‘corridors’ whereas less than 40% of the population lives there (Cape Metropolitan Council [8] and [9]). Therefore, these spatial patterns result in a considerable amount of commuting (Cape Times [10], Naude and Crous [24]).

2.2 Travel to work and the corresponding commuting costs

Mainly because of their respective locations within cities, the different communities experience very distinct commuting patterns in terms of distances traveled, time costs and transportation modes. These differences are very detrimental to non-whites: in South African cities, the average commuting distance for blacks, over 15 km, is twice as long as for whites who travel less than 7 km to go to work (Vines Mikula Associates [36]). In 1990, a resident commuting to Cape Town’s city center from Khayelitsha (one of the city’s black townships) had to face 2 hours and 40 minutes of transport, excluding waits at connection points (Urban Problems Research Unit [33] and [34]). Table 2 shows that, in 1992, 87% of whites and 57% of Asians used their cars to commute whereas 79% of blacks and 53% of coloureds resorted to public transportation. Observe that the use of taxis was quite frequent, especially among black commuters (46%).² In 1999, this is even more true for blacks but also for other non-white communities and taxis are used by 65% of urban commuters (Cape Argus [7]).

²A taxi or minibus-taxi is a cheap and quite unsafe means of transportation that can board up to 15 people. The wide use of taxis among blacks is a recent phenomenon that started in the 1980s as a response to the inadequate public transport from townships and informal settlement areas (Cape Argus [6]).

Table 2: Mode of transport of metropolitan commuters in 1992

	Whites	Asians	Coloureds	Blacks
Public Transport	7%	33%	53%	79%
bus	4%	22%	10%	20%
taxi	—	8%	8%	46%
train	3%	3%	35%	13%
Car	87%	57%	36%	9%
Walking	2%	4%	7%	11%
Other	4%	6%	4%	1%
Average monthly cost (Rands ³)	221	134	101	67
Average travel time (mn)	25	36	44	51

Source: Vines Mikula Associates [36].

2.3 Inequality and education

The spatial patterns of South African cities have contributed to create and maintain significant disparities between population groups, notably in terms of income and education. In Cape Town, a city in which there are almost as many whites as blacks (23% and 27% of the local population respectively) and where coloureds are in majority (about one half of the local population)⁴, 69% of white households have a monthly income above 3500 Rands whereas 74% of black households have an income inferior to 1500 Rands (see Table 3).

Table 3. Estimated monthly household income distribution in Cape Town in 1995

	Whites	Asians	Coloureds	Blacks
0-1500 Rands	12%	27%	36%	74%
1501-2500 Rands	9%	18%	21%	14%
2501-3500 Rands	10%	13%	15%	5%
Above 3500 Rands	69%	42%	27%	6%

Source: Moving Ahead [21].

The stark disparities in income compare well with education imbalances in South African cities. For many years, in accordance with Apartheid's

³1 Rand = 0.1635 US dollar (July 9, 1999).

⁴Asians accounts for the rest of the city's population (3%). See Cape Metropolitan Council [8].

logic, there was a separation of educational provision on the basis of race. Prior to the 1994 elections that marked the end of Apartheid, education was operated by departments organized by race and geographic location. The system was very unequal, the objective being to limit the post-school opportunities of non-white children, essentially to menial occupations (World Bank [38]). Table 4 shows the discrepancies in the quality of education that was delivered to the different communities. In the early 1990s, just before the end of Apartheid, while Africans accounted for 75% of the country's population, they only received 47% of recurrent government expenditure on education. In short, for every 4 Rands spent on a white child, only 3 Rands were spent on an Asian child, 2 Rands on a coloured child and 1 Rand on a black child (Thomas [32]). Moreover, classes in black schools were overcrowded (there were on average 42 students per teacher) and only 14% of black students studied through the end of high school (grade 12) whereas as much as 88% of white children were able to graduate from high school.

Table 4 - Inequality in Education in South Africa in 1991

	Whites	Asians	Coloureds	Blacks
Population share	13%	3%	9%	75%
Share of expenditure	34%	5%	14%	47%
Pupils per teacher	19	22	24	42
Academic survival (up to grade 12)	88%	53%	20%	14%
Average years of education (men)	9.5	7.9	5.5	3.9

Sources: Case and Deaton [11], Thomas [32] and World Bank [38].

In view of these striking figures, one should not be surprised at the resulting discrepancies in the levels of education across population groups. White and black schooling systems were 'two limiting cases in terms of the opportunities that they afforded' (Fedderke, Kadt and Luiz [15]) and, obviously, the impact of such a system has been strongly discriminating. If we compare with the US, there is the same gap in terms of schooling levels between blacks and whites born in the early 1970s in South Africa and those born in the 1930s in America (Thomas [32]).

In the context of Apartheid's abolition, the key issue is whether school desegregation can significantly contribute to the reduction of human capital imbalances. It should be noted that, even though limited desegregation began in state schools in the 1980s and the 1990s,⁵ unrestricted formal desegregation was only decided in 1995 and still comes up against many difficulties.

⁵The inefficiency of segregation, the need for qualification, and the *de facto* emergence of 'grey areas' (mixed neighborhoods) boosted desegregation in schools even before the

Sociologists have pointed out the particularities associated with school desegregation in South Africa which boils down to ‘integrating a majority group into a privileged minority culture’ (Penny et al. [25]). In particular, black children bear a cost to integration, either because they have to adapt to the language (English or Afrikaans) or because they must adapt to new norms at school (Naidoo [23], Vally [35], Zafar, [39] and [40]). In South African cities particularly, these social obstacles to integration also involve spatial considerations. Indeed, parents who care about education may wish to reside close to good (central) schools and thus may be willing to pay higher housing prices (Case and Deaton [11]). Those who cannot afford moving, might send their children on long and expensive commuting trips as it is currently observed. If moving and commuting costs are prohibitive, then the neighborhood location of schools reinforces the polarization in education by limiting the exposure of pupils to the world beyond their immediate community, and thus aggravates inequality in education (Smit and Hennessy [26]).

The facts presented in this section have stressed the crucial role of space in order to understand patterns of education and inequality by race. Therefore, taking into account the inherited inequality, the problems of access to schooling and the workings of cities, the role of this paper is to gauge the conditions under which school integration will succeed in South African cities.

3 The model

The city is closed, linear, with absentee landlords⁶. There are two centers. The first one, the Central Business and Educational District (CBED hereafter), is at the city center (taken as the origin of the line) whereas the other one, the Suburban Educational District (SED hereafter), is exactly at the city fringe. The CBED has all the jobs (and thus all the firms)⁷ as well as one big representative school whereas the SED only has one big representative school and no jobs at all.

There is a continuum of families uniformly distributed along the linear city and they all consume the same amount of land (normalized to 1). The density of residential land parcels is taken to be unity so that there are exactly x units of housing within a distance x from the CBED. The families belong

abolition of Apartheid laws. For an interesting presentation of school desegregation and subsequent issues in South Africa, see Naidoo [22] and [23], Tikly [30] and [31], Penny et al. [25].

⁶All these assumptions are very standard in urban economics and relaxing them does not alter our main conclusions (see Fujita [16]).

⁷Observe that our model could incorporate informal employment since, in South-African cities, a very large part of informal jobs is located in the city center.

to two different types, non-whites or blacks (type B) and whites (type W) whose respective masses are given by \bar{N}_B and \bar{N}_W (with $\bar{N}_B + \bar{N}_W = \bar{N}$). The only difference between these two types lies in the initial human capital endowments of adults which is lower for blacks, i.e. $\bar{h}_B < \bar{h}_W$, where \bar{h}_i ($i = B, W$) is the initial amount of human capital of a type i parent. This assumption reflects the fact that, because of South Africa's history, levels of human capital between blacks and whites are unequal (see Table 4 and the discussion in section 2.3).

A family consists of one parent and one child. We use a static model which captures our main message about location and educational choices. Parents are working while children are studying.

After Apartheid, a certain amount of integration is bound to take place within schools. We assume that there is a cost θ to integration (see e.g. Akerlof [1], for a discussion of peer pressures in black communities facing integration). In South Africa, this cost reflects the fact that it is the non-white learner who must adjust to the school norms, developing for instance competency in the English language (see Zafar [39] and our discussion at the end of section 2.3). Observe that white children being the reference group in the central school, they do not incur any interaction cost with black children other than human capital externalities. We thus assume that only black children bear the cost of integration when attending the central former white school. Moreover, all black children do not have the same ability to integrate and interact with the other group. Formally, the integration cost, denoted by θ , is uniformly distributed on $[0, 1]$ among the group of black children.

The timing of the model is as follows. In stage 1, families choose their location in the city. Then, in stage 2, they decide which school to send their children to after the revelation of their integration cost.⁸ In stage 3, children participate to the educational process and, in stage 4, families consume.

4 The equilibrium during Apartheid

In this section, we develop a model where blacks are discriminated in both the housing market and the access to the central school. The first hypothesis means that blacks cannot reside in central locations nearby the CBED, a portion of the urban space exclusively reserved for whites. This is indeed a well documented fact that under the laws of Apartheid white locations were central whereas those of non-whites were peripheral (see Smith [27] and our

⁸Note that under Apartheid, there is no such choice since black and white children are forced to attend separate schools. The fact that the integration cost is revealed after the location decision is discussed at the beginning of section 5.

discussion in section 2.1). The second assumption is in accordance with the first one since under Apartheid blacks did not have access to white (central) schools.

In this context, all workers (black or white parents) commute to the CBED while white students travel to the CBED and black students to the SED. Families endogenously decide their optimal residence within their allocated group area, i.e. between the CBED (normalized to zero) and \overline{N}_W for whites, and between \overline{N}_W and the city fringe $\overline{x}_f \equiv \overline{N}$ for blacks. As a result, there is no competition for land between blacks and whites but families compete within their respective group areas.

Since land consumption is normalized to 1, each parent located at a distance x from the CBED has the following (realized) utility function and budget constraint:

$$u_i = z_i + \beta h_i \quad i = B, W \quad (1)$$

$$\overline{y}_i = z_i + t_i x + \tau_i |x - \widehat{x}_i| + R(x) \quad i = B, W \quad (2)$$

where β is a positive parameter, z_i denotes the consumption of the non-spatial composite good (whose price is taken as the numeraire) of a type i family, h_i is the human capital level of a type i child, \overline{y}_i is the income of a type i parent, t_i and τ_i , are the respective transport costs per unit of distance for a type i parent and a type i child, \widehat{x}_i is the location of the school so that, under Apartheid, we have $\widehat{x}_W = 0$ and $\widehat{x}_B = \overline{N}$. Finally, $R(x)$ denotes the equilibrium land rent at a distance x from the CBED.

The following comments are in order. First, equation (1) assumes that parents care about the human capital of their children (β represents the parents' degree of altruism) which affects positively their utility. Parents are thus altruist and children' levels of human capital depend on parental choices as, for example, in Glomm [17]. The main difference with the standard human capital literature is that the present model does not only focus on explicit human capital investment but also on location choices as in Benabou [2] and [3] to the extent that location choices interact with educational choices. If there were no restrictions on location and educational choices, the preference for a school would depend on a trade-off between quality and proximity. We will elaborate further on this in the next section when we explore the unrestricted market equilibrium.

Second, we assume that the *total* unit commuting costs of blacks are lower than those of whites, i.e. $t_B + \tau_B < t_W + \tau_W$ and that black parents have higher unit transport costs than their children, i.e. $t_B > \tau_B$. The first assumption is easy to justify since blacks have a lower human capital

and income than whites, and thus a lower opportunity cost of time, which in turn implies lower generalized unit commuting costs. This justification is also related to the fact that, in general, blacks and whites do not use the same transport modes (see Table 2). Whites mainly use cars whereas blacks resort to public transportation, which, in the South African context, means that whites face higher monetary commuting costs per unit of distance than blacks.⁹ The second assumption is also easy to understand since many black children use public transportation or even walk to school. Therefore, if commuting cost is related to the parents' opportunity cost of time, then it can be argued that transporting black children is cheaper than transporting black adults because no parental time is used.

Third, a child's level of human capital is determined by the quality of the school he/she attends. The general educational output h_j of a school j ($j = C, S$ where C stands for *central school* and S for *suburban school*) is given by:

$$h_j = \frac{N_{Wj}}{N_{Wj} + N_{Bj}} \bar{h}_W + \frac{N_{Bj}}{N_{Wj} + N_{Bj}} \bar{h}_B \quad (3)$$

where N_{ij} is the number of type i children ($N_{iC} + N_{iS} \equiv \bar{N}_i$, $i = B, W$) in school j ($j = C, S$), and \bar{h}_i ($i = B, W$) is the contribution of a type i child ($i = B, W$) to the educational process, as measured by his/her parent's inherited level of human capital. The following comments are in order. First, equation (3) takes into account *human capital externalities or spillovers* so that each student, regardless of his/her ethnic origin, acquires the school's average human capital contribution. In other words, education is determined by the peer group effect only.¹⁰ Second, the linearity of (3) implies that there are no aggregate gains nor losses from the mixing of children with different abilities. Third, each child's contribution is measured by his or her parent's level of human capital \bar{h}_i (which can be considered as a form of social capital that captures the quality of the home learning environment). Fourth, education is not priced in our model. This is quite different from Epple and Romano [14] who show that when pupils have different contributions,

⁹In Cape Town for instance, a recent study [5] evaluates the per kilometer commuting cost of cars to 1.52 Rands while for public transportation (train, minibus taxi or bus) it is less than 0.15 Rands.

¹⁰Other more general educational production functions have been used in the literature, all of them putting a strong emphasis on the key role of peer group effects. See for example Summers and Wolfe [29], Henderson, Mieszkowski and Sauvageau [19] or more recently the survey by Hanushek [18]. In the present paper, the focus is on the interaction between education and location and we have tried to keep the model tractable by using a simple form of education production technology. Introducing educational spending in our model would have complicated matters without altering our main message.

it is optimal to subsidize the education fees of high achievers because of spillover effects. Our framework does not enable us to raise this issue because education is not priced. Observe however that, since $\bar{h}_B < \bar{h}_W$, the mixing of blacks with white pupils (which will be possible in the post-Apartheid regime) is always harmful to whites and beneficial to blacks, so that even if education were priced competitively, some blacks would still mix with whites, and this externality would still exist.

Under Apartheid however, educational choices are restricted and there is no interaction between blacks and whites at school and thus no inter-group education externalities. Then, by using the fact that under Apartheid, $N_{WC} = \bar{N}_W$ and $N_{BS} = \bar{N}_B$, we have :

$$h_W \equiv h_C = \bar{h}_W \quad (4)$$

for white students and

$$h_B \equiv h_S = \bar{h}_B \quad (5)$$

for black students. This means that, under Apartheid, parents and children have the same human capital so that this policy prevents the possibility of intergenerational improvement for blacks in terms of both location choices and human capital. In other words, under Apartheid, parents' education fully determines the schooling of their children (see e.g. Wilson [37]).

By using (1) and (2), we obtain:

$$u_B = \bar{y}_B - t_B x - \tau_B(\bar{N} - x) - R(x) + \beta \bar{h}_B, \quad x \in [\bar{N}_W, \bar{N}]$$

$$u_W = \bar{y}_W - (t_W + \tau_W)x - R(x) + \beta \bar{h}_W, \quad x \in [0, \bar{N}_W]$$

In this framework, the only choice of parents is to determine the family's optimal location by maximizing their utility. Considering that, in equilibrium, all families of type $i = B, W$ obtain the same utility levels v_B^A and v_W^A for blacks and whites respectively, we are now able to write the families' bid rents.¹¹ They are equal to:¹²

$$\Psi_B^A(x, v_B^A) = \bar{y}_B - t_B x - \tau_B(\bar{N} - x) + \beta \bar{h}_B - v_B^A \quad (6)$$

for blacks and

$$\Psi_W^A(x, v_W^A) = \bar{y}_W - (t_W + \tau_W)x + \beta \bar{h}_W - v_W^A \quad (7)$$

¹¹Bid rents are functions $\Psi_i(x, v)$ defined as the maximum rent that a family of type i would be willing to pay at a given location x so as to reach a given level of utility v .

¹²All variables with superscript A refer to Apartheid.

for whites. Moreover, we have:

$$\frac{\partial \Psi_B^A(x, v_B^A)}{\partial x} = -(t_B - \tau_B) < 0$$

$$\frac{\partial \Psi_W^A(x, v_W^A)}{\partial x} = -(t_W + \tau_W) < 0$$

Our comments are the following. First, bid rents are always linear and the trade-off is between land rents and commuting costs. Second, the bid rent of whites is always decreasing with distance from the CBED while for blacks this is true only if $t_B > \tau_B$, an assumption that we made and which is compatible with South African cities in which the commuting of black children involves little parental time (see our discussion above). Observe that both white parents and children commute to the city center whereas black parents commute to the city center while their children travel to the suburbs (see Figure 1 for an illustration of the Apartheid city).

By normalizing the agricultural land rent (outside the city) to zero, we have the following definition:

Definition 1 *An Apartheid Equilibrium (AE) is a triple $(v_B^A, v_W^A, R^A(x))$ such that:*

$$R^A(x) = \begin{cases} \Psi_W^A(x, v_W^A) & \text{for } 0 \leq x \leq \bar{N}_W \\ \Psi_B^A(x, v_B^A) & \text{for } \bar{N}_W < x \leq \bar{N} \\ 0 & \text{for } x > \bar{N} \end{cases} \quad (8)$$

$$\Psi_W^A(\bar{N}_W, v_W^A) = 0 \quad (9)$$

$$\Psi_B^A(\bar{N}, v_B^A) = 0 \quad (10)$$

By solving (9) and (10), we easily obtain the following equilibrium utilities for blacks and whites:

$$v_B^A = \bar{y}_B - t_B \bar{N} + \beta \bar{h}_B \quad (11)$$

$$v_W^A = \bar{y}_W - (t_W + \tau_W) \bar{N}_W + \beta \bar{h}_W \quad (12)$$

The following comments are in order. First, in the Apartheid equilibrium, the two communities are totally separated so that there are two distinct housing markets (see Figure 1). This urban configuration is typical of South African cities under Apartheid, as initially modelled by Brueckner [4]. Second, an increase in the income of one group raises its utility whereas an increase in the transport cost decreases its utility. Moreover, the overall city

size negatively affects black families whereas white families are only affected by the size of their own community. In other words, in the Apartheid regime, whites are not affected by the presence of blacks. Lastly, in this context τ_b does not affect the utility of blacks since in equilibrium we determine their utility at a location where distance to school is zero. It is indeed easily verified in Figure 1 that starting from \bar{N} and moving inward any black family experiences an increase in the child's transport cost and a decrease in the parent's commuting cost. Since $t_B > \tau_B$ it causes a decrease in the family's overall transport cost which is nevertheless exactly compensated by a higher land rent.

[Insert Figure 1 here]

The black-white inequality in this economy is equal to:

$$\begin{aligned} I^A &\equiv v_W^A - v_B^A \\ &= \bar{y}_W - \bar{y}_B - (t_W + \tau_W) \bar{N}_W + t_B \bar{N} + \beta (\bar{h}_W - \bar{h}_B) \end{aligned} \quad (13)$$

so that

Proposition 1 *In the Apartheid Urban Equilibrium, utilities and inequality are respectively given by (11), (12) and (13). Inequality increases with:*

- the difference in human capital between black and white children,
- the difference in the income of parents,
- the black parent's unit commuting cost.

Inequality decreases with the commuting costs of white parents and white children.

5 The equilibrium after Apartheid

After Apartheid, there are no more spatial nor educational restrictions (due to the removal of Apartheid laws) and workers commute to the CBED while students can either travel to the CBED or to the SED. Families endogenously decide their optimal residence between the CBED and the city fringe $\bar{x}_f \equiv \bar{N}$ and choose the school attended by their child. Because it is truly the case in post-Apartheid South African cities, we assume that white parents do not send their children to the former black school located in the township. Different reasons can be given to justify this assumption. The more realistic

one is that white parents are afraid of the high level of crime in black areas and are therefore very reluctant to send their children there.¹³

We also assume that schools, in particular the former white school, are not capacity constrained.¹⁴

It is important to recall the timing of the model. In the first stage, all families choose their location in the city without knowing the type θ of the child (the cost of integration incurred by a black child studying in a former white school) but anticipating (with rational expectations) the number of black children who will attend the central school. In other words, black families base their location decision on expected utility, anticipating the proportion of black children that will go to the central school. In the second stage, types are revealed and black families choose their school depending on their previously determined location. The assumption that types are revealed only after location is chosen takes into account the relative inertia of the housing market compared to educational mobility. In stage 3, children obtain different human capital levels and then, in stage 4, families consume the composite good. Therefore, one of the main differences between the Apartheid and the post-Apartheid equilibria is that, in the latter, the initial type of children matters, whereas in the former, children were not given the opportunity to make use of their ability to integrate.

5.1 Housing versus education

In the post-Apartheid regime, the choice of a school for blacks depends on a trade-off between location, transport costs (distance to the school) and *human capital externalities*. Indeed, as mentioned in the previous section, we assume that there are *local spillovers* or *peer group effects* in the production of education in the sense that studying among students that have a high human capital contribution increases one's education. In particular, since by assumption the inherited human capital of black parents \bar{h}_B is lower than that of white parents \bar{h}_W , equation (3) implies that the quality of a school

¹³Instead of just assuming it, we could have easily found a condition ensuring that in equilibrium white pupils do not attend black schools. In the context of our model, this is because, as we will see below, after Apartheid, whites still reside close to the city center so that going to the suburban black school implies both higher commuting costs and lower human capital. This assumption is very realistic in post-Apartheid South African cities and we have adopted it for simplicity.

¹⁴This is consistent with the assumption that education is not priced in our model. It should be clear that if school sizes were constrained, and if households bid for spots in a similar manner to the formation of bid rents, it would then be quite conceivable that education in the post-Apartheid regime could be somewhat closer to education under Apartheid, even if there were some unused capacity in the central school.

increases with the relative number of white students. This means that blacks have an incentive to send their children to the central former white school in order to benefit from human capital externalities. Therefore, we can consider two types of black families: those who send their child to the central school C (type BC), whose mass is given by N_{BC} , and families whose child remains in the suburban school S after Apartheid (type BS), whose mass is given by N_{BS} ($N_{BC} + N_{BS} \equiv \overline{N}_B$). This implies that human capital outputs in each school are given by $h_{BS} \equiv h_S$ and $h_{BC} = h_W \equiv h_C$. In this context, the utilities of whites and blacks of type BS are still given by (1). However, since only the black children who attend the central school incur an integration cost θ , their families have the following (realized) post-Apartheid utility function:

$$u_{BC} = z_{BC} + \beta h_{BC} - \theta \quad (14)$$

Observe that the integration cost of black students takes the form of a disutility when they attend the former white school so that this disutility is increasing in the type θ . Note that θ can also be interpreted as an inverse index of personal learning abilities to the extent that it may be less difficult for a high ability children to be efficient in an environment that he or she is not used to. Furthermore, the budget constraint of a family of type $i = BS, BC, W$ is equal to:

$$\overline{y}_i = z_i + t_i x + \tau_i |x - \hat{x}_i| + R(x) \quad (15)$$

where school locations are given by $\hat{x}_{BC} = \hat{x}_W = 0$ and $\hat{x}_{BS} = \overline{N}$, and transport costs remain unchanged so that $t_{BS} = t_{BC} = t_B$ and $\tau_{BS} = \tau_{BC} = \tau_B$.

Since families decide to locate before knowing the number of black pupils that will attend the central school, they base their decision on their (rational) expectations of the human capital output of this school denoted by h_C^e . Using (3), we have:

$$h_C^e = \frac{\overline{N}_W}{\overline{N}_W + N_{BC}^e} \overline{h}_W + \frac{N_{BC}^e}{\overline{N}_W + N_{BC}^e} \overline{h}_B \quad (16)$$

where N_{BC}^e is the expected number of black children going to the central school. By using (1) and (15), we obtain the following expected utility for a white family located in x :

$$U_W = \overline{y}_W - (t_W + \tau_W) x - R(x) + \beta h_C^e \quad (17)$$

In order to compute their expected utility, black families take into account the probability that they will send their child to the central or the peripheral

school after the child's type is revealed. They do not know their type when computing their expected utility. However, for a given type θ and location x , if they decide to send their child to the central school, they will obtain the following utility (using (14)):

$$U_{BC} = \bar{y}_B - (t_B + \tau_B)x - R(x) + \beta h_C^e - \theta \quad (18)$$

On the contrary, if they choose to send their child to the suburban school after the revelation of their type, the child will not bear any integration cost, and using (1), the utility of the family will amount to:

$$U_{BS} = \bar{y}_B - t_B x - \tau_B(\bar{N} - x) - R(x) + \beta h_S \quad (19)$$

where $h_S = \bar{h}_B$ is the human capital output of the suburban school.

Therefore, a black family located at a distance x from the CBED will decide to send its child of type θ to the central school if U_{BC} is greater than U_{BS} . This is the case when θ is smaller than a threshold value $\tilde{\theta}(x, N_{BC}^e)$ making a black family located in x indifferent between sending its child to either one of the two schools. It is given by:

$$\begin{aligned} \tilde{\theta}(x, N_{BC}^e) &= \beta (h_C^e - h_S) - \tau_B (2x - \bar{N}) \\ &= \beta (\bar{h}_W - \bar{h}_B) \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^e} - \tau_B (2x - \bar{N}) \end{aligned} \quad (20)$$

In particular, the incentives for a particular black family to send its child to the central school depend on the location of that family in the city. More precisely, everything else being equal, people living further away from (closer to) the CBED are less (more) likely to send their child to the central school. This is because, due to the presence of commuting costs, $\tilde{\theta}$ is a decreasing function of x , the distance to the CBED, so that children living far away need to have a very low θ to go to the central school while those living closer to the CBED can have a higher θ and still go to the central school. Figure 2 illustrates this point: a child of a given type θ will decide (ex post) to go to the central school if his or her family lives close to the CBED ($x < \bar{x}$) whereas the same child of type θ will go to the suburban school if his or her family resides far away from the CBED ($x \geq \bar{x}$).

[Insert Figure 2 here]

Moreover, simple comparative statics on formula (20) yields the following results:¹⁵

Proposition 2 *Black families are induced to send their children to the central former white school if:*

- *the inherited difference in human capital between blacks and whites is large,*
- *the expected number of black children attending the central school is low,*
- *the number of white students in the city is large.*

Black families living relatively closer to (further from) the CBED have more incentives (less incentives) to send their children to the central school when the unit transport cost of black children is high.

Proof. See the Appendix.

This proposition summarizes important results that will help us later in the equilibrium analysis. In particular, the incentives to go to the central school negatively depend on the expected number of black children going to the central school, N_{BC}^e . In other words, there is a *negative group externality* on education incentives since, when N_{BC}^e increases, the expected human capital output of the central school h_C^e decreases, which in turn reduces the incentives to go to the central school.

We further assume that the following condition is met:

$$\tau_B \frac{\bar{N}^2}{\bar{N}_W} < \beta (\bar{h}_W - \bar{h}_B) < 1 - \tau_B (\bar{N}_B - \bar{N}_W) \quad (21)$$

which guarantees that $\tilde{\theta}(x, N_{BC})$ is always strictly interior, i.e. that $\tilde{\theta}(x, N_{BC})$ is always comprised between 0 and 1.¹⁶

¹⁵In order to compute the results of Proposition 2, we have held N_{BC}^e constant when varying any generic variable. This means that, when a parameter varies (for example commuting costs), individuals do not change their expectations on the distribution (here the expected number of black pupils that go to the white school). In other words, they consider that this variation has not impact on the distribution.

¹⁶Condition (21) expresses the requirement that $\tilde{\theta}(x, N_{BC}) \in]0, 1[$ for all $(x, N_{BC}) \in [\bar{N}_W, \bar{N}] \times [0, \bar{N}_B]$. This condition corresponds to the assumption that blacks still occupy peripheral locations after Apartheid as will be proven below (see Proposition 3). Condition (21) is quite intuitive since it means that the difference in inherited human capital between

We are now able to determine the expected utility of a black family residing at a distance x from the CBED, before the revelation of its θ . It is given by:

$$\begin{aligned}
EU_B(x) &= \int_0^{\tilde{\theta}} U_{BC} d\theta + \int_{\tilde{\theta}}^1 U_{BS} d\theta \\
&= \int_0^{\tilde{\theta}} [\bar{y}_B - (t_B + \tau_B)x - R(x) + \beta h_C^e - \theta] d\theta \\
&\quad + \int_{\tilde{\theta}}^1 [\bar{y}_B - t_B x - \tau_B(\bar{N} - x) - R(x) + \beta \bar{h}_B] d\theta \\
&= \bar{y}_B - t_B x + \tau_B x(1 - 2\tilde{\theta}) - \tau_B \bar{N}(1 - \tilde{\theta}) - R(x) \\
&\quad + \beta \left[(\bar{h}_W - \bar{h}_B) \tilde{\theta} \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^e} + \bar{h}_B \right] - \int_0^{\tilde{\theta}} \theta d\theta
\end{aligned}$$

where h_C^e is defined by (16). Observe that the location decision is based on this expected utility which is computed before the revelation of the child's type. After it is revealed, school attendance is decided but location will remain unchanged.

We now obtain the whites' bid rent by inverting equation (17). We thus have:¹⁷

$$\Psi_W^P(x, v_W^P) = \bar{y}_W - (t_W + \tau_W)x + \beta h_C^e - v_W^P \quad (22)$$

where v_W^P is the equilibrium utility of white families. Moreover, by inverting the expected utility of blacks, we obtain their bid rent:

$$\begin{aligned}
\Psi_B^P(x, v_B^P) &= \bar{y}_B - t_B x + \tau_B x(1 - 2\tilde{\theta}(x)) - \tau_B \bar{N}(1 - \tilde{\theta}(x)) - v_B^P \quad (23) \\
&\quad + \beta \left[(\bar{h}_W - \bar{h}_B) \tilde{\theta}(x) \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^e} + \bar{h}_B \right] - \int_0^{\tilde{\theta}} \theta d\theta
\end{aligned}$$

where v_B^P is the equilibrium utility of blacks.

Let us now determine the location of all agents in the city (see Figure 3). We have:

blacks and whites ($\bar{h}_W - \bar{h}_B$) must take a medium value. In other words, if $\bar{h}_W - \bar{h}_B$ were very large, then some black families residing close to whites would always send their child to the central school no matter the value of their integration cost since the θ associated with their locations would tend to be greater than 1. On the contrary, a very low value of $\bar{h}_W - \bar{h}_B$ would discourage some black students residing at the city's periphery from attending the white school, even if their integration cost turns out to be zero, because the $\tilde{\theta}$ associated with those locations would tend to be less than 0. Assuming that $\bar{h}_W - \bar{h}_B$ takes a medium value thus ensures that, before the type is revealed, any black child in the city stands a chance to attend or not to attend the former white school.

¹⁷All variables with superscript P refer to the post-Apartheid equilibrium.

Proposition 3 *After Apartheid, the structure of the city remains the same as under Apartheid: black families reside at the outskirts of the city (between \bar{N}_W and \bar{N}) whereas white families locate at the vicinity of the city center (between 0 and \bar{N}_W).*

Proof. See the Appendix.

[Insert Figure 3 here]

This result is quite interesting since, even after Apartheid when blacks and whites are free to choose where to live, we obtain a residential equilibrium (Figure 3) similar to the Apartheid one (Figure 1): whites still live close to the city center whereas blacks reside in peripheral locations. In our model, because of low unit commuting costs, blacks have a ‘natural’ tendency to live on the periphery, hence Apartheid did not distort the natural location pattern. This urban configuration broadly corresponds to most South African post-Apartheid cities such as Cape Town or Durban. A notable exception is Johannesburg since, after Apartheid, whites *and* jobs have abandoned the city center to relocate in the suburbs. Brueckner [4] addresses the location reversal in Johannesburg by assuming that blacks consume less land than whites so that, in that model, Apartheid suppresses a natural tendency of blacks to live at the center.¹⁸ In our view, since there has been a dramatic decentralization of jobs in Johannesburg, our model can also address the specific case of Johannesburg just by flipping the city and thus locating the new CBED in the suburbs. In this particular context, blacks reside close to the historical city center and whites in the suburbs. It should thus be clear that our results remain valid for a city such as Johannesburg since what matters is the distance between communities and centers of opportunity.

Observe that, in our model, the main difference between the two equilibria (before and after Apartheid) is that, after Apartheid, the spatial lay-out of the city is not imposed by law any more but results from a bidding process between blacks and whites. As a consequence, competition in the land market is fiercer and land prices are higher after Apartheid than during Apartheid.

5.2 The equilibrium

As stated above, once location is decided, all types θ are revealed and expected values are observed (recall that all families have rational expectations). Therefore, we have:

¹⁸In the standard urban economics model, the lower the consumption of land, the steeper the bid rent.

Definition 2 A *Post-Apartheid Equilibrium (PAE)* with rational expectations is a quadruple $(N_{BC}^P, v_W^P, v_B^P, R^P(x))$ such that:

$$R^P(x) = \begin{cases} \Psi_W^P(x, v_W^P) & \text{for } 0 \leq x \leq \bar{N}_W \\ \Psi_B^P(x, v_B^P) & \text{for } \bar{N}_W < x \leq \bar{N} \\ 0 & \text{for } x > \bar{N} \end{cases}$$

$$\Psi_W^P(\bar{N}_W, v_W^P) = \Psi_B^P(\bar{N}_W, v_B^P) \quad (24)$$

$$\Psi_B^P(\bar{N}, v_B^P) = 0 \quad (25)$$

$$N_{BC}^P = \int_{\bar{N}_W}^{\bar{N}} \tilde{\theta}(x, N_{BC}^P) dx \quad (26)$$

Equation (24) indicates that, at the border \bar{N}_W , the bid rents of blacks and whites are equal. Equation (25) states that the land rent paid at the city fringe is equal to the outside land rent normalized to 0. Equation (26) says that, under rational expectations, the expected number of black children going to the central school (denoted in equilibrium by N_{BC}^P) has to be the average number of black children that attend the central school. If we denote by $\hat{\theta}$ the equilibrium proportion of black children attending the central school (i.e. $\hat{\theta} \equiv N_{BC}^P/\bar{N}_B$), equation (26) is equivalent to:

$$\hat{\theta} = \frac{1}{\bar{N} - \bar{N}_W} \int_{\bar{N}_W}^{\bar{N}} \tilde{\theta}(x, N_{BC}^P) dx$$

where the RHS is the average proportion of black children whose θ is lower than $\tilde{\theta}(x, N_{BC}^P)$ (when types are randomly distributed across space) and thus attend the central school.

We have:

Proposition 4 Under condition (21), there exists a unique *Post-Apartheid Equilibrium (PAE)* with rational expectations $(N_{BC}^P, v_W^P, v_B^P, R(x))$.

Proof. See the Appendix.

Observe that, in the first stage of our timing, each black family *randomly* chooses its location between \bar{N}_W and \bar{N} because, whatever the location, the expected utility level is the same and equals v_B^P . It is only after location has been decided that types are revealed and families choose their child's school. In equilibrium, families who reside closer to the CBED are on average more likely to send their children to the central school whereas those located closer to the city fringe \bar{N} tend to send their children to the suburban school (see our comments on formula (20) above). In other words, black families must stick

to their initial location, which strongly affects the human capital attainment of their children.

We are now able to determine the equilibrium value N_{BC}^P . By developing (26), we obtain the following second degree equation:

$$(N_{BC}^P)^2 + [(1 + \tau_B \bar{N}_B) \bar{N}_W] N_{BC}^P + \bar{N}_W \bar{N}_B [\tau_B \bar{N}_W - \beta (\bar{h}_W - \bar{h}_B)] = 0$$

There are two solutions to this equation with one obviously being negative so that the other solution gives the value of N_{BC}^P . Using (21), it is indeed easily verified that this solution is always comprised between 0 and \bar{N}_B .

In this context, there is a unique equilibrium $\tilde{\theta}^P(x)$ given by (using (20)):

$$\tilde{\theta}^P(x) = \beta (\bar{h}_W - \bar{h}_B) \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^P} - \tau_B (2x - \bar{N}) \quad (27)$$

and a unique equilibrium human capital output for the central school equal to:

$$h_C^P = \frac{\bar{N}_W \bar{h}_W + N_{BC}^P \bar{h}_B}{\bar{N}_W + N_{BC}^P} \quad (28)$$

with

$$\frac{\partial h_C^P}{\partial N_{BC}^P} = - \frac{(\bar{h}_W - \bar{h}_B) \bar{N}_W}{(\bar{N}_W + N_{BC}^P)^2} < 0 \quad (29)$$

Equation (29) is quite intuitive since it means that when there are more black children who attend the central school, the educational outcome of this school decreases.

Now, since we know the equilibrium values of N_{BC}^P , h_C^P and $\tilde{\theta}^P(x)$, we can derive the equilibrium utility levels of blacks and whites. By using (27), (25) and (24), we obtain:

$$v_B^P = \bar{y}_B - t_B \bar{N} + \beta \bar{h}_B + \frac{1}{2} [\tilde{\theta}(\bar{N})]^2 \quad (30)$$

and

$$\begin{aligned} v_W^P &= \bar{y}_W - (t_W + \tau_W) \bar{N}_W - (t_B - \tau_B) \bar{N}_B + 2\tau_B^2 \bar{N}_B \bar{N}_W \\ &+ \beta \left[(1 - 2\tau_B \bar{N}_B) (\bar{h}_W - \bar{h}_B) \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^P} + \bar{h}_B \right] \end{aligned} \quad (31)$$

6 Implications for post-Apartheid policies

In this section, we initiate a welfare analysis, derive some comparative statics results and discuss the implications of education mixing. Let us first determine the changes in the utilities of blacks and whites and the resulting change in inequality between the two equilibria (i.e. during and after Apartheid). It is easily verified that, for blacks, we have:

$$\Delta v_B \equiv v_B^P - v_B^A = \frac{1}{2} \left[\tilde{\theta}(\bar{N}) \right]^2 > 0 \quad (32)$$

This is because, as a net effect, blacks benefit from the removal of restrictions in the land market and the educational system. Indeed, the average human capital of blacks always increases because of peer group effects but land prices are higher due to fiercer competition for central locations. However, the first effect dominates the second one. For whites, we have:

$$\begin{aligned} \Delta v_W &\equiv v_W^P - v_W^A \\ &= \Delta z_1 + \Delta z_2 + \beta \Delta h < 0 \end{aligned} \quad (33)$$

where

$$\begin{aligned} \Delta z_1 &= -(t_B - \tau_B) \bar{N}_B < 0 \\ \Delta z_2 &= 2\tau_B \bar{N}_B \bar{N}_W [\tau_B - \beta(\bar{h}_W - \bar{h}_B) / (\bar{N}_W + N_{BC}^P)] < 0 \\ \Delta h &= (h_C^P - \bar{h}_W) < 0 \end{aligned}$$

In this equation, Δz_1 is the net income loss for whites due to the entry of blacks in the competition for land, Δz_2 is the net income loss for whites due to the intensification of the competition for land caused by human capital externalities and Δh denotes the sheer human capital loss of white children.¹⁹ It is quite obvious why whites incur a loss in utility since land prices are higher and their human capital is lower ($h_C^P < \bar{h}_W$) after Apartheid. This can easily be seen by analyzing (33). The removal of land use restrictions negatively affects the net income of whites ($\Delta z_1 < 0$) due to fiercer competition in the land market. The removal of school segregation leads to a decrease in their human capital ($\Delta h < 0$) and also has an indirect effect on the price of land (which increases) since the city center becomes more attractive for blacks who are attracted by the positive human capital externalities, which further reduces the net income of whites ($\Delta z_2 < 0$).

¹⁹It is easily verified that, by using (21), $\tau_B < \beta(\bar{h}_W - \bar{h}_B) / (\bar{N}_W + N_{BC}^P)$ so that Δz_2 is negative.

Let us denote by $I^P \equiv v_W^P - v_B^P$ the inequality after Apartheid. We have:

$$\Delta I \equiv I^P - I^A = \Delta v_W - \Delta v_B < 0 \quad (34)$$

The following result summarizes our discussion:

Proposition 5 *When Apartheid is removed, whites are worse off whereas blacks are better off and thus the inequality between blacks and whites decreases.*

Let us now continue with the following result:

Proposition 6 *After Apartheid, the equilibrium number of black students attending the central former white school increases when:*

- *the initial human capital difference between blacks and whites is higher,*
- *the number of whites is larger,*
- *their unit transportation cost is lower.*

The human capital output of the central school varies in the opposite direction.

Proof. See the Appendix.

The results in Proposition 6 are quite intuitive and corresponds to the inducement for blacks to go to the central school (see Proposition 2). In particular, in equilibrium, when the transport cost of black children is lower, the overall number of blacks attending the central school (the former white school) rises and the general level of human capital in the central school decreases.

Since the transport cost of black children τ_B is a key variable affecting N_{BC}^P , it is interesting to examine its impact on the utility levels of blacks and whites. We have the following result:

Proposition 7 *In the post-Apartheid city, reducing the unit transport cost of black children increases the utility of blacks but has an ambiguous effect on the utility of whites. This means that a reduction in the unit transport cost of black children may possibly improve the utilities of both groups.*

Proof. See the Appendix.

For blacks, the intuition runs as follows. We can easily identify two opposite forces at work. On the one hand, when τ_B decreases, there is a *direct and positive* effect on the utility of blacks since the transport cost of black children becomes cheaper and accessibility is improved. On the other hand, this yields a negative group externality: when τ_B decreases, there is an *indirect and negative* effect on the utility of blacks since more (black) children attend the central school (N_{BC}^P increases), which in turn tends to reduce utility (it is easily checked that $\partial v_B^P / \partial N_{BC}^P < 0$). As shown in the Appendix, the first effect is always dominant so that the overall effect is positive.

For whites, when τ_B decreases, their utility tends to decrease since more black families are able to send their children to the central school, which reduces h_C^P (the human capital output of the central school). However, this reduction leads to a weaker competition in the land market since central locations become less attractive for blacks and land rents decrease at the core of the city. This tends to increase the utility of whites. It is not clear which effect is dominant.

In order to complete our analysis, we now present some simulations that aim at highlighting the direction and magnitude of utility changes associated with the removal of Apartheid laws (since it is quite cumbersome to determine it analytically). In Table 5, we have reported the results of our analysis. In the base case (see the first column), 75% of the city's residents are non-whites and 25% are whites. The latter have three times as much inherited human capital as blacks. When Apartheid is removed, the mean level of human capital for blacks increases by 14%. Indeed, among black children, 8.8% attend the central school, which enables them to raise their human capital by as much as 158% in comparison with what they would have obtained under Apartheid. Attending the former white school involves longer commuting trips and higher overall transport costs for black children which increase by 4.7% on average.²⁰ Observe that after Apartheid, central locations have become more attractive for blacks and land rent in the black neighborhood increase by 16.5% on average. The resulting effect of these changes is a 3.5% increase in the utility of blacks. For whites, their decreasing human capital (-14%) as well as the increasing land rents they face (+100.9% on average in the white neighborhood) are detrimental to their utility, which is reduced by 14.9%. Consequently, the inequality between blacks and whites decreases by 20.2% but the surplus of the population, which we define as

²⁰Of course, those black children who actually attend the central school face a much higher increase in their transportation costs.

the weighted sum of the utilities of blacks and whites, decreases by 7.6%. In fact, to complete our surplus analysis, we must also take into account the utility of absentee landlords. Since utility (see (1) and (2)) is measured in units of consumption, we can add total utility to land rent. In this context, the overall surplus during and after Apartheid is defined by:

$$S^k \equiv \bar{N}_B v_B^k + \bar{N}_W v_W^k + TLR^k \quad k = A, P$$

where TLR^k denotes the equilibrium total land rent own by absentee landlords in regime $k = A, P$. From Table 5, we see that absentee landlords gain much from the removal of Apartheid since rents increase by an average 53.4% in the city as a whole, so that the total surplus S is only reduced by 2.3%.²¹

In the second column of Table 5, we have changed the ethnic composition of the city by considering the case in which there are as many blacks and whites in the city. The key lesson is that, when the city accommodates a lower proportion of blacks, relatively more school integration takes place. This is both because blacks reside closer to the white school and because there is a large pool of white students, which is very attractive for blacks. Indeed, when blacks represent only 50% of the city's population, 9.7% of black families send their children to the central school and the gain in human capital for these children amounts to 182.3%. In spite of a greater proportion of blacks going to the central school, the proportion of black children within that school is small because whites are much more numerous than under the base case. Thus, the human capital loss of whites only amounts to 5.9%. Moreover, land competition is not as fierce in the white neighborhood where land rents only increase by 34.1% on average, while it becomes a little more fiercer in the black neighborhood where rents increase by 18.6% on average. The overall effect is that landlords gain less while the reduction in the population surplus and the total surplus are less dramatic. Inequality still significantly decreases.

In the third column, we show that larger inherited disparities in human capital ($\bar{h}_W = 5\bar{h}_B$) strongly induce black children to attend the central school (15.7%) where they can obtain a huge human capital gain (272%). This causes a significant reduction in inequality (-30.3%) which is nevertheless harmful to the total surplus (-5.7%). The last column of Table 5 presents quite similar effects when agents put twice as much weight on the human capital of their children than on composite good consumption ($\beta = 2$).

²¹Observe that, when we compare the post-Apartheid equilibrium to the Apartheid equilibrium, the total surplus is always reduced because agents face integration costs and higher transportation costs while the total human capital in the economy remains constant. This result would not hold with a production function involving gains associated with heterogeneity.

Table 5: Simulation analysis for changes between the Apartheid and the post-Apartheid equilibria (%)

	<i>BASE</i>	$\bar{N}_B = 50\%$	$\bar{h}_W = 5\bar{h}_B$	$\beta = 2$
N_{BC}^P/\bar{N}_B	8.8	9.7	15.7	15.7
$\Delta h_B^{mean}/\bar{h}_B$	14.0	17.7	42.7	21.4
$\Delta h_{BC}/\bar{h}_B$	158.0	182.3	272.0	136.0
$\Delta h_W/\bar{h}_W$	-14.0	-5.9	-25.6	-21.3
$\Delta TC_{B,Children}/TC_{B,Children}^A$	4.7	18.5	9.2	9.2
$\Delta R_W/R_W^A$	100.9	34.1	112.6	112.6
$\Delta R_B/R_B^A$	16.5	18.6	30.1	30.1
$\Delta v_W/v_W^A$	-14.9	-9.4	-23.3	-20.4
$\Delta v_B/v_W^B$	3.5	5.1	13.6	7.3
$\Delta PS/PS^A$ (<i>population surplus</i>)	-7.6	-6.4	-11.3	-8.2
$\Delta TLR/TLR^A$ (<i>landlord surplus</i>)	53.4	32.2	66.2	66.2
$\Delta S/S^A$ (<i>total surplus</i>)	-2.3	-1.5	-5.7	-4.2
$\Delta I/I^A$ (<i>inequality</i>)	-20.2	-14.4	-30.3	-30.3

Base case: $\beta = 1$, $t_B = .05$, $\tau_B = .025$, $t_W = .1$, $\tau_W = .075$, $\bar{h}_W = .18$,
 $\bar{h}_B = .06$, $\bar{y}_W = .18$, $\bar{y}_B = .06$, $\bar{N}_W = .25$, $\bar{N}_B = .75$.

Δ expresses changes between the post-Apartheid and the Apartheid equilibria.

Whereas Table 5 focused on the possible gains resulting from the removal of Apartheid, the following table stresses the impact of a variation in the transport cost τ_B on post-Apartheid welfare. Starting from the same base case as used in Table 5, we consider the impact of a 20% increase in τ_B and a 20% decrease in τ_B . First, observe that τ_B is negatively (positively) correlated with blacks' (whites') utility. This indicates that for whites, when τ_B decreases (see equation (39)), the negative effect (negative group externality) dominates the positive one (lower land rent). Moreover, when there is a twofold increase in τ_B , 1.2% of black children attending the central school will shift to the suburban school. This leads to a higher population surplus (since the gains for whites outweigh the losses for blacks), but a lower total surplus (because of a deflating effect on land rents) and more inequality. On the contrary, halving τ_B reduces both the population surplus and inequality while increasing the total surplus. This suggest a positive correlation between transport cost for black children and post-Apartheid inequality.

**Table 6: Simulation analysis
for changes in τ_B after Apartheid (%)**

	$\tau_B = 0.03$ (+20%)	$\tau_B = 0.02$ (-20%)
$\Delta v_B^P/v_B^P$	-.5	.5
$\Delta v_W^P/v_W^P$	1.2	-1.2
$\Delta N_{BC}^P/N_{BC}^P$	-1.2	1.2
$\Delta I^P/I^P$	1.9	-1.9
$\Delta PS^P/PS^P$.5	-.5
$\Delta S^P/S^P$	-1.0	1.0

Base case: $\beta = 1$, $t_B = .05$, $\tau_B = .025$, $t_W = .1$, $\tau_W = .075$, $\bar{h}_W = .18$,
 $\bar{h}_B = .06$, $\bar{y}_W = .18$, $\bar{y}_B = .06$, $\bar{N}_W = .25$, $\bar{N}_B = .75$.

Δ expresses changes in the post-Apartheid regime following variations in τ_B

7 Conclusion

This paper is the first attempt to model education in a spatial urban context in South Africa under and after Apartheid. Though the model may seem quite stylized, we believe that it captures the basic features of most South African post-Apartheid cities. The key features of our model are: (i) the historical difference in human capital, (ii) the fact that historically disadvantaged families have to pay a cost of adjustment to the school norms to attend the high-quality school and (iii) the difference in commuting costs. Observe that these factors are present in other countries and our results could apply to contexts outside South Africa where minority students have been historically discriminated against (for example blacks in the U.S.).

Our results are the following. First, we obtain that after Apartheid, even though the land-use restrictions have been removed, the general structure of cities remains the same as under Apartheid: whites still reside close to the centers of opportunity whereas blacks live far away from these centers. Second, when Apartheid racial laws are removed, the inequality between blacks and whites can only decrease because blacks benefit from local human capital externalities by mixing with white students, and because whites are worse off since both their human capital decreases and the land rent increases. Third, we explain the emergence of a new phenomenon widely recognized in post-Apartheid South Africa: the fact that some black students accept to bear very long commutes in order to attend better schools located in white central locations.

Our model also has strong implications for policies that can be implemented in post-Apartheid South African cities, namely busing and transport-to-school subsidies. Indeed, we have shown that a policy aiming at reducing

the transport costs of black children has an ambiguous effect on the utility of whites but is always beneficial to blacks since it enables them to go to the best school in the city.

From the above discussion, it should be clear that space and education are central to the analysis of South African cities. Space and education were the two main criteria driving discrimination since Apartheid was designed to prevent blacks and whites from interacting, especially in schools, and the spatial separation of communities was the most efficient way to ensure that no such interaction would ever happen. Our analysis shows that post-Apartheid integration can be promoted through education even though the spatial separation between communities remains.

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APPENDIX

Proof of Proposition 2:

By differentiating (20), we easily obtain:

$$\frac{\partial \tilde{\theta}}{\partial (\bar{h}_W - \bar{h}_B)} > 0 \quad , \quad \frac{\partial \tilde{\theta}}{\partial \bar{N}_W} > 0 \quad , \quad \frac{\partial \tilde{\theta}}{\partial x} < 0 \quad , \quad \frac{\partial \tilde{\theta}}{\partial N_{BC}^e} < 0$$

$$\frac{\partial \tilde{\theta}}{\partial \tau_B} \geq 0 \iff x \leq \frac{\bar{N}}{2}$$

The economic justifications for these effects are quite straightforward and discussed in the core text of the paper. The number of white pupils and the difference in inherited human capital induce blacks to attend the white school because the larger this difference or the higher the number of whites, the greater the potential human capital gains for blacks. On the contrary, distance to the white school and the expected number of blacks attending the white school have a negative impact on the willingness of black children to attend the central school because of transport costs and the negative peer group externality. Finally, an increase in the unit transport cost of black children strengthens the families' incentives to send their child to the closest school. So, since $\bar{N}/2$ is exactly the middle point between the two schools, when indifferent families reside on the right (on the left) of $\bar{N}/2$, a rise in τ_B induces them to send their child to the suburban (central) school.

Proof of Proposition 3:

In this proof, we calculate the bid rent slopes for whites and blacks respectively and show that whites have a steeper bid rent, so that, in equilibrium, whites do indeed bid away blacks from central locations.

From equation (22) we have:

$$\frac{\partial \Psi_W^P(x, v_W^P)}{\partial x} = -(t_W + \tau_W) < 0$$

From equation (23), we can write

$$\begin{aligned} \frac{\partial \Psi_B^P(x, v_B^P)}{\partial x} &= -t_B + \tau_B \left[1 - 2\tilde{\theta}(x, N_{BC}^e) \right] \\ &\quad + \frac{\partial \tilde{\theta}(x, N_{BC}^e)}{\partial x} \left[\beta (\bar{h}_W - \bar{h}_B) \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^e} - \tau_B (2x - \bar{N}) - \tilde{\theta}(x, N_{BC}^e) \right] \end{aligned}$$

Since, by definition of $\tilde{\theta}$, we have $\tilde{\theta}(x, N_{BC}^e) = \beta (\bar{h}_W - \bar{h}_B) \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^e} - \tau_B (2x - \bar{N})$, the term in brackets cancels out, so that we have:

$$\frac{\partial \Psi_B^P(x, v_B^P)}{\partial x} = -t_B + \tau_B \left[1 - 2\tilde{\theta}(x, N_{BC}^e) \right]$$

Since by assumption $t_B > \tau_B$, then because $\tilde{\theta} > 0$, $t_B > \tau_B \left[1 - 2\tilde{\theta}(x, N_{BC}^e) \right]$ and thus the bid rent $\Psi_B^P(x, v_B^P)$ is always downward sloping.

Furthermore, it is easy to verify that the bid rent of whites is always linear whereas the bid rent of blacks is strictly convex since:

$$\frac{\partial^2 \Psi_B^P(x, v_B^P)}{\partial x^2} = -2\tau_B \frac{\partial \tilde{\theta}(x, N_{BC}^e)}{\partial x} = 4\tau_B^2 > 0$$

The latter result means that, in absolute terms, the slope of blacks' bid rent depends positively on $\tilde{\theta}(x, N_{BC}^e)$. This is because the higher $\tilde{\theta}$, the more attractive the city center and thus the steeper the bid rent. Finally, since we have assumed that $t_W + \tau_W > t_B + \tau_B$ and since $\tilde{\theta} > 0$, it is easily checked that,

$$\forall x \in [0, \bar{N}] \ , \quad \left| \frac{\partial \Psi_W^P(x, v_W^P)}{\partial x} \right| > \left| \frac{\partial \Psi_B^P(x, v_B^P)}{\partial x} \right|$$

so that whites always reside closer to the city center than blacks. In equilibrium, white families live between 0 and \bar{N}_W whereas black families locate between \bar{N}_W and \bar{N} .

Proof of Proposition 4:

First, we know that in standard urban economics models, an urban equilibrium always exists and is unique (see Fujita, 1989, ch.4, who shows that the existence and the uniqueness of an urban equilibrium is guaranteed as soon as bid rents can be ranked in order of relative steepness, here blacks and whites). Second, in our model, we also need to show that there exists a uniquely determined N_{BC}^P with $0 < N_{BC}^P < \bar{N}_B$. Let us consider the following function which implicitly determines N_{BC}^P :

$$\begin{aligned}\Phi(N_{BC}^P) &\equiv \int_{\bar{N}_W}^{\bar{N}} \tilde{\theta}(x, N_{BC}^P) dx - N_{BC}^P \\ &= \bar{N}_W \bar{N}_B \left[\beta \frac{\bar{h}_W - \bar{h}_B}{\bar{N}_W + N_{BC}^P} - \tau_B \right] - N_{BC}^P\end{aligned}\quad (35)$$

It is easily verified that this continuous function is always decreasing in N_{BC}^P and that, by using (21), we have $\Phi(0) > 0$ and $\Phi(\bar{N}_B) < 0$, so that there always exists a unique N_{BC}^P comprised between 0 and \bar{N}_B . Consequently, there exists a unique equilibrium.

Proof of Proposition 6:

Let us start with the comparative statics on N_{BC}^P . By using (35), simple differentiation provides for any generic exogenous variable k :

$$\frac{\partial N_{BC}^P}{\partial k} = - \frac{\partial \Phi(N_{BC}^P) / \partial k}{\partial \Phi(N_{BC}^P) / \partial N_{BC}^P} \quad (36)$$

which has the sign of $\partial \Phi(N_{BC}^P) / \partial k$ since $\partial \Phi(N_{BC}^P) / \partial N_{BC}^P < 0$. Using the definition (35) of $\Phi(N_{BC}^P)$, the comparative statics on N_{BC}^P follows immediately for parameters $(\bar{h}_W - \bar{h}_B)$, \bar{N}_W and τ_B .

We have:

$$\frac{\partial N_{BC}^P}{\partial(\bar{h}_W - \bar{h}_B)} > 0 \quad , \quad \frac{\partial N_{BC}^P}{\partial \bar{N}_W} > 0 \quad , \quad \frac{\partial N_{BC}^P}{\partial \tau_B} < 0$$

Concerning the comparative statics on h_C^P , we have:

$$\frac{\partial h_C^P}{\partial k} = \frac{\partial h_C^P}{\partial N_{BC}^P} \frac{\partial N_{BC}^P}{\partial k}$$

so that, by using (29), we have:

$$\frac{\partial h_C^P}{\partial(\bar{h}_W - \bar{h}_B)} < 0 \quad , \quad \frac{\partial h_C^P}{\partial \bar{N}_W} < 0 \quad , \quad \frac{\partial h_C^P}{\partial \tau_B} > 0$$

Proof of Proposition 7:

By totally differentiating (30), we obtain:

$$\frac{\partial v_B^P}{\partial \tau_B} = -\tilde{\theta}^P(\bar{N}) \left[\frac{\partial N_{BC}^P}{\partial \tau_B} \frac{\beta(\bar{h}_W - \bar{h}_B)\bar{N}_W}{[\bar{N}_W + N_{BC}^P]^2} + \bar{N} \right] \quad (37)$$

Applying (36) with $k = \tau_B$, we have that:

$$\frac{\partial N_{BC}^P}{\partial \tau_B} = -\frac{\bar{N}_W \bar{N}_B}{\frac{\beta(\bar{h}_W - \bar{h}_B)\bar{N}_W \bar{N}_B}{[\bar{N}_W + N_{BC}^P]^2} + 1} < 0 \quad (38)$$

Finally, plugging (38) into (37), we obtain:

$$\frac{\partial v_B^P}{\partial \tau_B} = -\tilde{\theta}^P(\bar{N}) \left[\frac{\bar{N}_W \bar{N}_B^2 \beta(\bar{h}_W - \bar{h}_B) + \bar{N}(\bar{N}_W + N_{BC}^P)^2}{\bar{N}_W \bar{N}_B \beta(\bar{h}_W - \bar{h}_B) + (\bar{N}_W + N_{BC}^P)^2} \right] < 0$$

Next, by differentiating (31), we have:

$$\begin{aligned} \frac{\partial v_W^P}{\partial \tau_B} &= \bar{N}_B (1 + 4\tau_B \bar{N}_W) \\ &\quad - \frac{\beta(\bar{h}_W - \bar{h}_B)\bar{N}_W}{(\bar{N}_W + N_{BC}^P)^2} \left[\frac{\partial N_{BC}^P}{\partial \tau_B} (1 - 2\tau_B \bar{N}_B) + 2\bar{N}_B (\bar{N}_W + N_{BC}^P) \right] \geq 0 \end{aligned} \quad (39)$$

which sign is ambiguous since the first term is positive and the second term can be shown to be negative (using (38)).

Figure 1. Apartheid Equilibrium (AE)

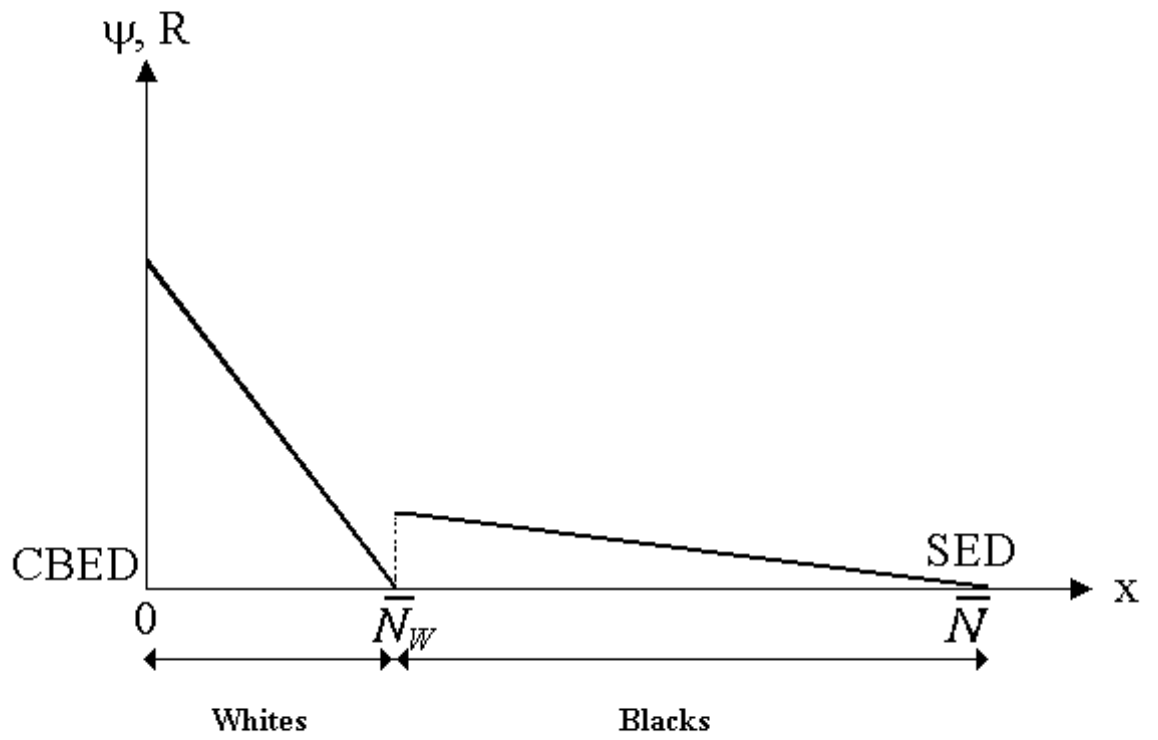


Figure 2. School choices

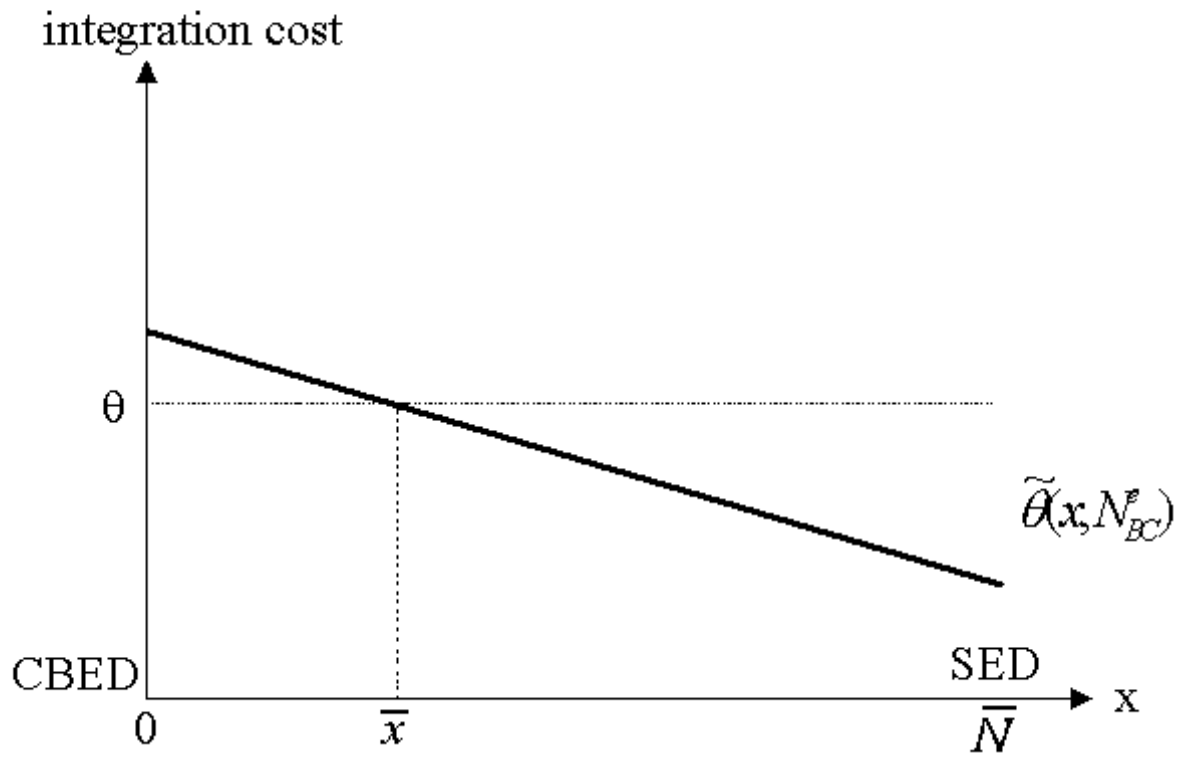


Figure 3. Post-Apartheid Equilibrium (PAE)

