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ABSTRACT

Indeterminacy in a Small Open Economy Ramsey Growth Model*

This Paper presents a small open economy version of the Benhabib and Farmer [2] two-sector optimal growth model with production externalities. It is shown that indeterminacy is considerably easier to obtain under a regime of perfect world capital markets than in the closed economy variant. Furthermore, the result is not dependent on high labour supply elasticity since that input is fixed. The Paper also examines a variant that takes into account external borrowing constraints and it is shown that the qualitative results on indeterminacy remain basically unaffected by this extension.

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NON-TECHNICAL SUMMARY

Economic policy advice periodically urges less-developed and developing countries to lift trade barriers and actively engage in international trade. The underlying reasoning for this recommendation is perhaps as old as the Economics discipline and stresses of one of its most central statements. Nevertheless, a number of countries (or their leaders) remain resistant to this advice or, as in the case of some Asian leaders, try to blame internationally operating investors for experienced turmoil. The present Paper reflects on the suitability of policy switching towards the opening up of markets. In particular, we show that economies which have adopted unlimited international trading may in fact find themselves in a conundrum of a different kind: international capital movements and trading may have important implications of belief formation on the fragility of an economy. That is, economies may be subject to fluctuations that are purely consequential to self-fulfilling expectations.

Recent advances in macroeconomics have highlighted the importance of such self-fulfilling prophecies in explaining economic instability. Models identified with this 'indeterminacy literature' are able to account for business cycles and other macroeconomic phenomena without having to rely on shocks to fundamentals. In fact, this multiplicity of solution arises in otherwise perfectly working market-clearing rational expectation environments. Thus, *crowd behaviour* and turbulent markets endogenously originate from optimizing behaviour. Furthermore, it has been demonstrated that the occurrence of indeterminacy is not restricted to assumptions that are *a priori* unrealistic. Moreover, very recent empirical work has demonstrated that aggregate scale economies are perhaps increasing but close to constant. These estimated values are simply too low, however, to give a number of existing indeterminacy models a sufficient empirical foundation.

In this context, the intent of this Paper is to develop a two-sector open economy model with sector-specific externalities in production. It is shown that indeterminacy is obtained not only at lower returns to scale than in the closed economy case but also at insignificant levels thereof. The reason for indeterminacy in the present model is that perfect capital markets allow the smoothing of consumption via international lending and borrowing at the world interest rate. Accordingly, the implied irrelevance of utility curvature makes it easier to construct alternative investment paths – the need to curtail consumption as a consequence of investment bunching disappears. Indeterminacy still arises from a correct path of prices in the presence of externalities; however, these can be minimal in size. Unlike the closed economy variant, the desire to smooth consumption must not be offset by a sufficient amount of increasing returns.

The assumption of a constant external interest rate can be justified as long as the country is small compared to the world market. However, there are many

situations where the rate of interest does depend on the amount of debt. We shall therefore also consider an open economy that faces an imperfect capital market. It will be shown that our qualitative results remain unchanged even when the economy is facing constraint lending: indeterminacy is possible at essentially constant returns to scale.

1 Introduction

Recent advances in macroeconomics have highlighted the importance of self-fulfilling prophecies in explaining economic instability. Models identified with this "indeterminacy literature" are able to account for business cycles and other macroeconomic phenomena without having to rely on shocks to fundamentals (see Benhabib and Farmer [3] for an extensive survey). Furthermore, it has been demonstrated that the occurrence of indeterminacy is not restricted to assumptions that are *a priori* unrealistic. Indeed non-uniqueness of equilibria can arise straightforwardly in dynamic general equilibrium settings once the hypothesis of perfect markets and constant returns in production is abandoned.

In this context, the intent of this paper is to develop a two sector open economy model with externalities in production. In particular, we study an international economy version of Benhabib and Farmer [2]. It will be shown here that indeterminacy is obtained not only at lower returns to scale than in the closed economy case but also at insignificant levels thereof. This aspect of the model is of importance since recent empirical work has demonstrated that aggregate scale economies are close to constant (see for example Basu and Fernald [1], Burnside [7], and Harrison [8]). Moreover, these estimates have pointed to values that are too low to give a number of existing indeterminacy models a sufficient empirical foundation.

The reason for indeterminacy in the present model is that perfect capital markets allow the smoothing of consumption via international lending and borrowing at a constant world interest rate. Accordingly, the implied irrelevance of utility curvature makes it easier to construct alternative investment paths – the need to curtail consumption as a consequence of investment bunching disappears. Indeterminacy still arises from a correct path of prices in the presence of externalities, however, these can be minimal in size. Unlike the closed economy variant, the desire to smooth consumption must not be offset by a sufficient amount of increasing returns.

The assumption of a constant external interest rate can be justified as long as the country is small compared to the world market. However, there are many situations where the rate of interest does depend on the amount of debt. We shall therefore also consider an open economy which faces an imperfect capital market. It will be shown that the qualitative results remain unchanged even when the economy is facing constraint lending.

In a related work, Lahiri [11] establishes that in a (perfect market) small

open economy endogenous growth model indeterminacy arises more straight-forwardly than in closed economy versions. The model that is constructed here is less abstractly formulated than Lahiri's and consequently allows for a more elaborate specification of imperfections, e.g. increasing returns. That is, we can quantify returns to scale in a way that we can draw plausible inferences from empirical work. Furthermore, our model structure is well embedded in the formulation most recently used in the indeterminacy literature. Thus, a comparison to closed economy versions can easily be undertaken. Meng and Velasco [12] specify a two sector open economy along the lines of Benhabib and Nishimura [4]. That is, they allow for decreasing internal and constant overall returns to scale in production. In their case, increasing returns may come from fixed costs rather from a declining marginal costs schedule. Here we specify only differing externalities so as to isolate the importance of these effects while assuming constant returns at the firm level. Finally, both Lahiri's and the Meng-Velasco works do not consider imperfect capital markets which will be done here

The remainder of this paper is organized as follows. Section 2 presents the model. The equilibrium dynamics and indeterminacy are discussed in Section 3. An economic interpretation of the main result is offered in Section 4. Section 5 extends the model and considers the case of external borrowing constraints. Section 6 concludes.

2 Model with perfect capital markets

The economy is based on Benhabib and Farmer [2]. We use this two sector model since it is a fairly general specification of an economy with imperfections. However, it still enables these departures from constant returns to scale and perfect markets to be quantified in a way in which theoretical results can be confronted with empirical work. The novel aspect here is that agents are allowed to borrow and lend internationally.

In the first model we assume that the economy is too small to affect the world interest rate and the rate facing the country is parametric. Households are represented by a single agent that maximizes lifetime utility. The agents own the stock of capital and receive income from wages and the rental of capital services to the firm sector. Firms produce either the consumption good or the investment good. Each firm has access to an externally increasing (or decreasing) returns to scale technology. All markets are perfectly competi-

tive and factors of production are completely mobile between the economy's sectors. Consumer goods are tradeable whereas it is assumed that capital goods are nontradeables.

2.1 Technology

The production technology of a typical firm in the investment good sector is

$$y_t^I = \left[(K_t^I)^\alpha (L_t^I)^{1-\alpha}\right]^{\theta_I} \left[K_t^\alpha L_t^{1-\alpha}\right]^{\sigma_I} (k_t^I)^\alpha (l_t^I)^{1-\alpha} \qquad \alpha \in (0,1)$$

while that of the typical producer of the consumption good is given by

$$y_t^C = \left[(K_t^C)^\alpha (L_t^C)^{1-\alpha}\right]^{\theta_C} \left[K_t^\alpha L_t^{1-\alpha}\right]^{\sigma_C} (k_t^C)^\alpha (l_t^C)^{1-\alpha}.$$

Here k_t^I and l_t^I denote the capital and labor services used by the individual firm in the investment good producing sector. K_t^I is the average stock of capital in this sector and K_t stands for the stock of capital in the country. θ_I indicates sector-specific externalities and σ_I denotes the degree of economy-wide externalities for the I-branch of the firms. The remaining variables are defined respectively with the index C denoting the consumption good producing sector. We assume that tradeable consumption is the numeraire good. Let w_t and r_t denote the competitive wage and capital rental rates. The relative price of investment goods is given by p_t. In symmetric equilibrium, the firms will hire capital and labor to satisfy the equalities:

$$w_t = (1-\alpha)p_t \kappa_t^{\theta_I} \left[K_t^\alpha L_t^{1-\alpha}\right]^{1+\theta_I+\sigma_I} L_t^{-1} = (1-\alpha)(1-\kappa_t)^{\theta_C} \left[K_t^\alpha L_t^{1-\alpha}\right]^{1+\theta_C+\sigma_C} L_t^{-1}$$

and

$$r_t = \alpha p_t \kappa_t^{\theta_I} \left[K_t^\alpha L_t^{1-\alpha}\right]^{1+\theta_I+\sigma_I} K_t^{-1} = \alpha(1-\kappa_t)^{\theta_C} \left[K_t^\alpha L_t^{1-\alpha}\right]^{1+\theta_C+\sigma_C} K_t^{-1}.$$

Here we have already used the definition for the relative factor intensities

$$\kappa_t \equiv \frac{K_t^I}{K_t} = \frac{L_t^I}{L_t}.$$

The equations can be combined to yield the relative price

$$p_t = (1-\kappa_t)^{\theta_C} \kappa_t^{-\theta_I} \left[K_t^\alpha L_t^{1-\alpha}\right]^{\theta_C+\sigma_C-\theta_I-\sigma_I}. \tag{1}$$

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Further, output in the two sectors simplifies to

$$Y_t^I = \kappa_t^{1+\theta_I} \left[K_t^\alpha L_t^{1-\alpha} \right]^{1+\theta_I+\sigma_I} \tag{2}$$

and

$$Y_t^C = (1 - \kappa_t)^{1+\theta_C} \left[K_t^\alpha L_t^{1-\alpha} \right]^{1+\theta_C+\sigma_C} . \tag{3}$$

2.2 Preferences

The preferences of the representative agent are depicted by the utility integral

$$U_0 = \int_0^\infty U(C_t) \, e^{-\rho t} dt$$

where C_t and $\rho > 0$ stand for consumption and the time discount factor. $U(C_t)$ has the usual properties. We normalize the fixed labor supply to unity. The representative individual owns the capital stock and lends its services to the firms. The agent's intertemporal budget constraint dictates

$$\dot{D}_t = \varrho D_t + C_t + p_t I_t - w_t - r_t K_t \tag{4}$$

where D_t is the amount of foreign debt and ϱ is the world interest rate. We further impose the standard assumption for small open economy models that the external interest rate is stationary and equal to the utility discount rate: $r_t - \delta = \varrho = \rho$. The capital accumulation technology is given by

$$\dot{K}_t = I_t - \delta K_t \tag{5}$$

with $\delta > 0$ the rate of capital depreciation.

We can analyze the agent's optimization problem by formulating the Hamiltonian

$$\mathcal{H}_t = [U(C_t) + \lambda_t \left(\rho D_t + C_t + p_t I_t - w_t - r_t K_t \right) + \Lambda_t \left(I_t - \delta K_t \right)] \, e^{-\rho t}$$

where Λ_t and λ_t are the shadow prices associated with the two constraints (4) and (5) subject to initial conditions $D_0 = D(0)$ and $K_0 = K(0) > 0$. The first order conditions are

$$U'(C_t) = \lambda_t \tag{6}$$

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$$\Lambda_t = \lambda_t p_t \tag{7}$$

$$\dot{\lambda}_t = 0 \tag{8}$$

$$\dot{\Lambda}_t = (\rho + \delta)\Lambda_t - r_t \lambda_t \tag{9}$$

$$\lim_{t \to \infty} -\lambda_t D_t \; e^{-\rho t} = 0 \tag{10}$$

$$\lim_{t \to \infty} \Lambda_t K_t \; e^{-\rho t} = 0. \tag{11}$$

Equation (8) indicates a constant shadow value λ along the optimal path. This is a standard result in small open economy versions of the Ramsey growth model (see for example Blanchard and Fischer [6]). Then, (6) means that consumption is constant which implies that complete consumption smoothing is realized through international lending and borrowing.

3 Equilibrium dynamics

3.1 Dynamics

When formulated, the reduced form of the model is not of the block-recursive structure that is required to analyze local dynamics. In particular, when local techniques are applied to derive dynamics (8) implies a zero eigenvalue, thus making it impossible to solve the model in this way. Consequently, the next step will be to show that (i) when reformulated, the model observes a block-recursive structure in capital and in the relative price and (ii) given the evolution of capital and relative price, there exists an unique initial level of consumption consistent with no-Ponzi and transversality conditions.

To begin with, we must reformulate the model so that the steady state λ must not be determined. It is then that consumption dynamics (and debt dynamics for that matter) no longer appear in the dynamic equations that we consider. To this end, let us rewrite the system in capital and the relative $p_t = p(\lambda, \Lambda_t)$. Now equations (7), (8) and (9) imply

$$\frac{\dot{p}_t}{p_t} = (\rho + \delta) - \alpha \kappa_t^{\theta_I} K_t^{\alpha(1+\theta_I+\sigma_I)-1}. \tag{12}$$

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Similarly, (2) and (5) yield

$$\frac{\dot{K}_t}{K_t} = \kappa_t^{1+\theta_I} K_t^{\alpha(1+\theta_I+\sigma_I)-1} - \delta. \tag{13}$$

Finally, the price equation (1) implies $\kappa_t = \kappa(p_t, K_t)$:

$$p_t = (1 - \kappa_t)^{\theta_C} \kappa_t^{-\theta_I} K_t^{\alpha(\theta_C+\sigma_C-\theta_I-\sigma_I)}. \tag{14}$$

It is easily seen that the last three equations form a two-dimensional system of differential equations in two variables: K_t and p_t. The budget constraint (4) and the transversality condition (10) now determine the unique consumption level. To see this, integrate over the budget constraint (together with production function 3) and apply the transversality condition to obtain

$$\int_0^\infty C_t e^{-\varrho t} dt = \int_0^\infty Y_t^C(\kappa(p_t, K_t), K_t) e^{-\varrho t} dt - D_0 \equiv H_0 - D_0$$

or

$$C_0 = \varrho\left[H_0 - D_0\right]. \tag{15}$$

Given that D_0 is an initial condition, the choice of C_0 is a linear, thus unique function of the difference in the present value of consumption output and initial debt, $H_0 - D_0$. By observing the block-recursive character of (12) through (14), we note that H_0 is independent of consumption. Thus, once H_0 is determined, we can derive the initial choice C_0 from (15) (see also Turnovsky [13]).

Let us return to equations (12) to (14) which can be used to determine local capital and price dynamics. The unique stationary state implies

$$\kappa = \frac{\alpha\delta}{\delta + \rho}.$$

The nonlinear system is Taylor-approximated around the steady state. This yields

$$\left[\begin{array}{c} \dot{\log p_t} \\ \dot{\log K_t} \end{array}\right] = \mathbf{J} \left[\begin{array}{c} \log p_t - \log p \\ \log K_t - \log K \end{array}\right], \tag{16}$$

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where the Jacobian matrix $\mathbf{J}$ is

$$\mathbf{J} \equiv \begin{bmatrix} \frac{(\delta+\rho)\theta_I}{\theta_I+\theta_C\kappa/(1-\kappa)} & -(\delta+\rho)\left[\alpha(1+\theta_I+\sigma_I)-1+\frac{\alpha\theta_I(\theta_C+\sigma_C-\theta_I-\sigma_I)}{\theta_I+\theta_C\kappa/(1-\kappa)}\right] \\ \frac{\delta(1+\theta_I)}{\theta_I+\theta_C\kappa/(1-\kappa)} & \delta\left[\alpha(1+\theta_I+\sigma_I)-1+\frac{\alpha(1+\theta_I)(\theta_C+\sigma_C-\theta_I-\sigma_I)}{\theta_I+\theta_C\kappa/(1-\kappa)}\right] \end{bmatrix}.$$

Given the structure of (16), we define perfect foresight equilibria and indeterminacy as follows:

Definition 1 *(Perfect foresight equilibrium) In the model economy, a perfect foresight equilibrium is a path of capital and the relative price, initial stocks of capital $K(0) > 0$ and debt $D(0)$ satisfying optimality conditions. In addition, markets clear and the resource constraints and transversality condition hold*

Definition 2 *(Indeterminacy) The equilibrium is indeterminate if there exists an infinite number of perfect foresight equilibrium paths.*

K_t is a predetermined variable and evolves continuously. The costate p_t is a non-predetermined variable, that is, its initial value is not given by history and it may jump instantaneously in response to new information. The steady state is indeterminate if both eigenvalues of $\mathbf{J}$ are negative. Since the trace of the matrix is the sum of its eigenvalues and the determinant is the product of the eigenvalues, indeterminacy can be restated as

$$\mathrm{Tr}\mathbf{J} < 0 < \mathrm{Det}\mathbf{J}.$$

In this case, the first order conditions and the transversality conditions are not sufficient to determine a unique solution path. Essentially, multiplicity of this sort indicates that the rational expectations equilibria involve random variables that are unrelated to the economy's fundamentals simply because agents believe it to be so.

3.2 Indeterminacy and scale economies

In this section the size of returns to scale that is needed to generate indeterminacy will be analyzed. If all four externalities are zero, the economy collapses to the one sector model, the equilibrium is unique and the dynamics are degenerate. Throughout the paper we will assume that the following holds.

Assumption *The level of increasing returns from all sources is modest. That is, $\alpha(1 + \theta_j + \sigma_j) < 1$ for $j = C, I$.*

The assumption of modesty includes values of the externality that are empirically plausible given evidence in Basu and Fernald [1] and Harrison [8]. Note that increasing returns are not high enough to induce endogenous growth.

We start with the parameter configuration that has received empirical confirmation: the presence of sector specific externalities. For example, Harrison [8] finds modest increasing returns in the investment goods sector whereas she reports that consumption goods are produced under constant or decreasing returns. When $\sigma_C = \sigma_I = 0$, the trace of the Jacobian **J** is given by

$$\mathrm{tr}\mathbf{J} = \frac{\alpha\delta\rho\theta_C + \rho\theta_I(\rho + (1-\alpha)\delta) + \alpha\delta\theta_C\theta_I(\rho + \delta)}{\alpha\delta\theta_C + \delta\theta_I - \alpha}$$

while its determinant is

$$\mathrm{Det}\mathbf{J} = \frac{\delta(\rho + (1-\alpha)\delta)(\delta + \rho)(1 - (1 + \theta_I)\alpha)}{\alpha\delta\theta_C + (\rho + (1-\alpha)\delta)\theta_I}.$$

Figure 1 about here

Table 1		
α	δ	ρ
0.40	0.10	0.05

Figure 1 reports nonzero (θ_C, θ_I) combinations that imply indeterminacy. The figure is drawn assuming the parameter values summarized in Table 1. These choices are standard in *Real Business Cycle* calibration. They imply a labor share of sixty percent, an annual capital depreciation rate of ten percent, an annual utility discount rate of five percent, a capital to output ratio of 2.6 and a consumption share on output of seventy-five percent. The dark cone in Figure 1 indicates stable equilibria, that is indeterminacy. Above this region, the model is unstable. Below the cone, the dynamics are of saddletype, thus, the model is determinate. Let us further investigate

analytical indeterminacy conditions. From our Assumption, the numerator in Det**J** is always positive. Thus, nonsaddlepoint behavior occurs for

$$\theta_C > \left(\frac{\rho + (1-\alpha)\delta}{\alpha\delta}\right)\theta_I.$$

A necessary condition for multiplicity is that the sector-specific externalities are of opposite sign. From Figure 1, we can also see that as θ_C increases, the dynamics change from a sink (the standard indeterminacy classification) to a source. In other words, there exists a function $\theta_C^* = \theta_C^*(\theta_I^*)$ such that the equilibrium has a simple pair of imaginary eigenvalues and a Hopf bifurcation takes place.[1] In this parametric neighborhood, the equilibrium is stable for $\theta_C < \theta_C^*$ and unstable for $\theta_C > \theta_C^*$. The trace vanishes when

$$\theta_I^* = \frac{\alpha\delta\rho}{-\rho(\rho + (1-\alpha)\delta)\theta_C^{*-1} - \alpha\delta\rho - \alpha\delta^2}.$$

By continuity, if the negative (positive) externality parameter θ_C is infinitesimally close to zero, a positive (negative) value for θ_I will always exist to render indeterminacy possible: indeterminacy arises at virtually zero departures from the constant returns to scale case. Moreover, the equilibrium becomes a saddle along the straight line: $\theta_C^{saddle} = -\frac{\rho+(1-\alpha)\delta}{\alpha\delta}\theta_I^{saddle}$. One eigenvalue passes through minus infinity at this bifurcation point. Indeterminacy is thus satisfied close to the bifurcation boundary. Finally, it should be noted that given the results by Harrison [8], it is plausible to restrict $\theta_C < \theta_I$. In the closed economy variant of the two sector model with sector-specific externalities and perfectly elastic labor supply, indeterminacy is obtained at

$$0 < \frac{\alpha^2\delta}{\rho + (1-\alpha^2)\delta} < \theta = \theta^C = \theta^I$$

(see Harrison and Weder [9]). Given the calibration in Table 1, the closed economy version of the two sector model requires increasing returns of at least $\theta^{\min} = 0.119$. The following Proposition summarizes the results.

Proposition 1 *The small open economy two sector model with sector-specific externalities generates indeterminacy at virtually constant returns*

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[1]The corresponding (global) analysis of deterministic cycles is beyond the scope of this paper.

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to scale. For plausible parameter constellations, the overall economy exhibits decreasing returns to scale.

Let us consider two special cases. First, we limit sector specific externalities to the investment sector: $\theta_C = \sigma_C = \sigma_I = 0$. Now the trace of $\mathbf{J}$ becomes $\rho > 0$, thus, the steady state can never be a sink. The determinant is given by

$$\text{Det}\mathbf{J} = \frac{\delta\,(\rho + \delta)\,(1 - (1 + \theta_I)\alpha)}{\theta_I}.$$

Given our Assumption, sign $\text{Det}\mathbf{J}$ =sign θ_I. Hence, positive sectoral externalities in the investment sector imply that the steady state is a source, otherwise the equilibrium would be a saddle point and would be unique. Second, when we limit sector specific externalities to the consumption sector $(\theta_I = \sigma_C = \sigma_I = 0)$, the trace of $\mathbf{J}$ again becomes $\rho > 0$. The determinant is given by

$$\text{Det}\mathbf{J} = \frac{(1 - \alpha)\,(\rho + \delta)\,(\rho + (1 - \alpha)\delta)}{\alpha\theta_C}.$$

It follows that sign $\text{Det}\mathbf{J}$ =signθ_C. Hence, positive sectoral externalities in the consumption good sector imply that the steady state is a source, otherwise the equilibrium would be a saddle-point. The next Proposition restates the results.

Proposition 2 *For indeterminacy to occur, sector specific externalities must be present in both sectors. Moreover, sector specific diseconomies of some sort are necessary for nonsaddle equilibria to be stable.*

If departures from constant returns are limited to the investment sector, the trace of the matrix $\mathbf{J}$ becomes

$$\text{tr}\mathbf{J} = \rho - \frac{\alpha\delta\sigma_I}{\theta_I}$$

and the determinant is

$$\text{Det}\mathbf{J} = \frac{\delta\,(\rho + \delta)\,(1 - (1 + \theta_I + \sigma_I)\alpha)}{\theta_I}.$$

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Given our Assumption, a necessary condition for indeterminacy is $\theta_I > 0$, otherwise the determinant of the Jacobian is negative. Further, the trace is negative for

$$\theta_I < \frac{\alpha\delta}{\rho}\sigma_I.$$

The following proposition summarizes the result.

Proposition 3 *Suppose the production function in the consumption good sector is constant returns to scale. Indeterminacy requires positive sector-specific and positive aggregate externalities in the investment goods producing sector. Accordingly, the steady state is a sink if $\theta_I/\sigma_I < \alpha\delta/\rho$.*

If the investment good sector operates under constant returns and externalities are present in the consumption good sector, the trace and determinant become

$$\text{tr}\mathbf{J} = \frac{\sigma_C(\rho + (1-\alpha)\delta) + \rho\theta_C}{\theta_C}$$

and

$$\text{Det}\mathbf{J} = \frac{(1-\alpha)\,(\rho+\delta)\,(\rho+(1-\alpha)\delta)}{\alpha\theta_C}$$

respectively. A positive determinant is given for all $\theta_C > 0$. Indeterminacy is obtained for

$$\sigma_C < -\frac{\rho + (1-\alpha)\delta}{\rho}\theta_C.$$

Thus, a stable steady state requires negative aggregate externalities. Let us summarize our findings.

Proposition 4 *Suppose the production function in the investment good sector is constant returns to scale. Indeterminacy requires positive sector-specific and negative aggregate externalities in the consumption goods producing sector. The steady state is a sink if $\sigma_C/\theta_C < (1-\alpha)\delta/\rho - 1$.*

4 Interpretation

It has been shown in the last section that indeterminacy can be obtained at plausible parameter values. The mechanism that led to indeterminacy differs slightly from the one operating in the closed economy model. In closed economies if agents believe in realizing capital gains by reallocating resources over time, they will start investing today and accordingly purchase more consumption goods in the future. If returns to scale in the investment good sector are sufficiently high, the relative price will first fall (as a result of the sector-specific positive externalities) and then rise. The expected price path is self-fulfilling. Nevertheless, the effect of increasing returns on relative prices must be sufficiently strong to overcome the agent's desire to smooth consumption. Unlike in the closed economy, in the small open economy there are no costs of foregone consumption along alternative paths. Perfect international capital markets allow the smoothing of consumption: the shadow price of wealth λ is a constant. Therefore, if agents want to increase investment, the realization of consumption is unaffected. Even when utility has curvature, alternative equilibria can easily be constructed. Indeterminacy arises from an appropriate sequence of price effects that appear in the presence of externalities. In a small open economy model such as the present, the externalities must be only minimal in size to prompt this price path. However, if only sector specific externalities are considered, indeterminacy requires that the model must possess some form of a dampening effect similar to the Howitt and McAfee [10] search externality model. Howitt and McAfee assume two externalities (one positive, one negative) in their model. Unless the negative effect is present, capital gains could be forwarded *ad infinitum* and the economy would eventually depart from its stationary equilibrium forever. In the case of the small open economy, (9) is the relevant Euler equation and utility curvature is no longer part of the argument. When investment increases, p_t falls as a result of the positive sector specific externalities. Intertemporal equilibrium now requires that the shadow price of capital appreciates. However, the decline of p_t spurs further investment activity, hence, the process would apply over and over again. To reverse the infinite appreciation and to rule out an ultimate violation of the transversality conditions, some form of negative externalities must be set into motion. This is the result of Proposition 1 and essentially it is present in all other cases as well. Thus, intertemporal equilibrium eventually requires a depreciation and the economy moves back towards the steady state, giving rise to

an alternative equilibrium trajectory.

5   Model with external borrowing constraints

In the preceding sections, it has been assumed that the external interest is constant regardless of the amount the country borrows or lends. In the following we consider an open economy which faces an imperfect capital market.[2] There are many situations where the rate of interest does depend on the amount of debt. Widely reported is the positive relationship between foreign debt and the spread that banks charge to LDCs. However, an upward sloping supply of debt does not necessarily imply that the analysis must concentrate on less developed economies. As *Moody's* 1998 downgrading of Japanese government debt evinces, even industrialized countries whose outstanding debt surges can suddenly be confronted with worsening credit conditions.

The model that we consider is basically the same as the one that was developed in Section 2. The only difference is that here the external interest rate is an increasing function of the amount of the country's international debt. Accordingly, the agent's intertemporal budget constraint now becomes

$$\dot{D}_t = \varrho(\frac{D_t}{K_t})D_t + C_t + p_t I_t - w_t - r_t K_t. \qquad (17)$$

In particular we assume that the interest rate follows

$$\varrho\left(\frac{D_t}{K_t}\right) = \overline{r} + r\left(\frac{D_t}{K_t}\right) \qquad r'(.) > 0, \quad r''(.) > 0.$$

The representative agent takes the interest rate $\varrho(.)$ as given. The upward sloping supply function reflects the influence of aggregate debt. We assume that it is outside the control of an individual investor, thus, each agent does not take account of its decision on the prevailing interest rate. Since the external rate becomes variable, it follows that the shadow value of wealth is no longer a constant. As a consequence, we must now assume a specific

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[2]I would like to thank the associate editor Jess Benhabib for suggesting to examine an upward sloping interest rate schedule. See Bhandari, Haque and Turnovsky [5] for a model with a variable debt function.

functional for utility. For simplicity, we consider the standard logarithmic case.

In symmetric equilibrium, the economy is described by the following set of equations:

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - \varrho\left(\frac{D_t}{K_t}\right) \tag{18}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho + \delta - \alpha\kappa_t^{\theta_I} K_t^{\alpha(1+\theta_I+\sigma_I)-1} \tag{19}$$

$$\dot{D}_t = \varrho(\frac{D_t}{K_t})D_t + \frac{1}{\lambda_t} - (1-\kappa_t)^{1+\theta_C} K_t^{\alpha(1+\theta_C+\sigma_C)} \tag{20}$$

and

$$\frac{\dot{K}_t}{K_t} = \kappa_t^{1+\theta_I} K_t^{\alpha(1+\theta_I+\sigma_I)-1} - \delta. \tag{21}$$

We also have a condition that relates the costates to the relative price of capital goods:

$$\Lambda_t = \lambda_t(1-\kappa_t)^{\theta_C}\kappa_t^{-\theta_I} K_t^{\alpha(\theta_C+\sigma_C-\theta_I-\sigma_I)}. \tag{22}$$

Given the analytical structure of the model we examine the local dynamics. The model boils down to the linear system

$$\begin{bmatrix} \dot{\log\lambda_t} \\ \dot{\log\Lambda_t} \\ \dot{\log D_t} \\ \dot{\log K_t} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \log\lambda_t - \log\lambda \\ \log\Lambda_t - \log\Lambda \\ \log D_t - \log D \\ \log K_t - \log K \end{bmatrix} \tag{23}$$

where $\mathbf{J}$ is the 4×4 Jacobian. K_t and D_t are predetermined variables and evolve continuously. The costates λ_t and Λ_t are non-predetermined variables. Thus, the steady state is indeterminate if at least three of the eigenvalues of $\mathbf{J}$ are negative.

Considering the empirical character of the recent debate on the plausibility of indeterminacy models, a numerical solution offers itself. More

importantly, it turns out that the eigenvalues of the above four-dimensional dynamical system cannot be solved to obtain tractable expressions for parametric indeterminacy regions. We have therefore decided to calibrate the model along the lines of Table 1 and to report relevant qualitative results on indeterminacy.

The inspection of the loglinearized version shows that two additional steady state variables must be calibrated. In particular, we must formulate assumptions on the (semi-) elasticity of the interest rate, $\eta \equiv \varrho' D/K$, and the debt-to-capital-ratio, $\mu \equiv D/K$. These values vary greatly across countries. We therefore assume combinations of values that appear to limit the reasonable parameter space in determining our results relative to the extreme case without any constraints in lending. To this end, we study four combinations associated with $(\eta^{\min}, \eta^{\max}, \mu^{\min}, \mu^{\max})$ as given in Table 2. For example, a country with $\eta^{\max}$ and $\mu^{\max}$ could be interpreted as a nation that faces a very upward sloping supply of debt schedule for which the reasoning could be a large level of existing external debt. An economy with $\eta^{\min}$ and $\mu^{\min}$ resembles the one that was considered in the first part of the paper. Other combinations have similar interpretations.

Table 2			
$\eta^{\min}$	$\eta^{\max}$	$\mu^{\min}$	$\mu^{\max}$
0.001	1	0.001	1

5.1   Indeterminacy and scale economies

Next, the size of returns to scale that is needed to generate indeterminacy will be analyzed. All computations are conducted along the calibration that is outlined in Tables 1 and 2. We limit the presentation here to sector-specific externalities.

Figures 2-5 about here.

Figures 2 and 3 show that if the effect of debt on the rate of interest is small (the case of $\eta^{\min}$), then the model with credit constraints operates very similarly to the perfect market case. Again we need to assume the presence of both positive and negative sector-specific externalities to generate indeterminacy. These, however, can be essentially zero again. The picture

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changes slightly once the country faces a strong response to its credit conditions. Figures 4 and 5 study η^{max} cases. Indeterminacy still arises and the mechanism does not appear to change. Increasing returns may be again very small indeed. Furthermore if η^{max} (Figure 5), the region that corresponds to indeterminacy apparently becomes smaller. We can now formulate our last Proposition.

Proposition 5 *The presence of borrowing constraints does not change the qualitative results for open economies. If the interest rate does not respond too strongly to increases in debt, the departure from constant returns that implies indeterminacy can be infinitesimally small.*

It should be noted that the two model structures that were discussed here converge as η goes to zero. Considering these results, it appears that it is not only the (almost) constant interest rates that drive the results but also the ability to use international credit markets to disconnect savings and investment decisions (or consumption and consumption production for that matter) which is the essential element that materializes in indeterminacy.

6   Summary

This paper has presented a small open economy version of the Benhabib and Farmer [2] two sector optimal growth model with production specific externalities. It has been shown that indeterminacy is considerably easier to obtain under a regime of perfect world capital markets than in the closed economy variant. Furthermore, the result is not dependent on a high labor supply elasticity as that input is fixed. The paper has also considered a variant which takes into account external borrowing constraints and has shown that the qualitative results on indeterminacy remain unaffected by this extension unless the imperfections are very strong. In sum, economies become more vulnerable to sunspot equilibria, hence to endogenous cycles, once trading across borders becomes available.

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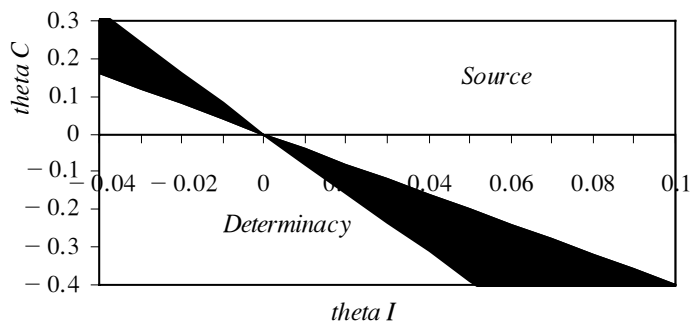
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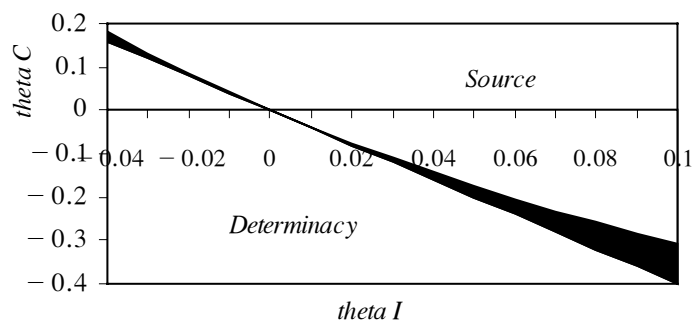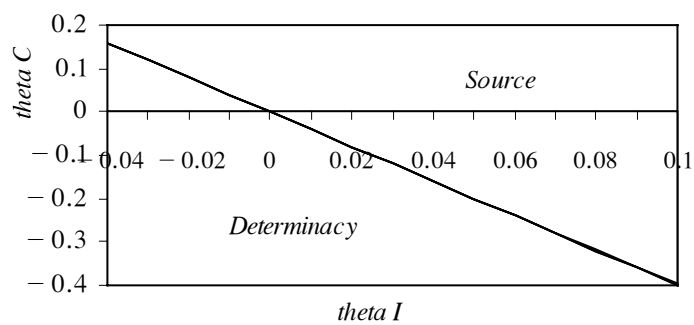[13] S. J. Turnovsky, "Methods of Macroeconomic Dynamics," MIT Press, Cambridge, 1995.
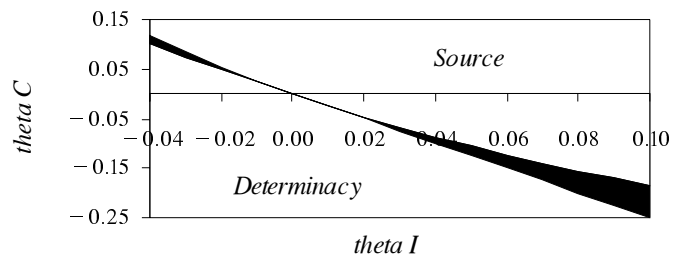
Figure 1
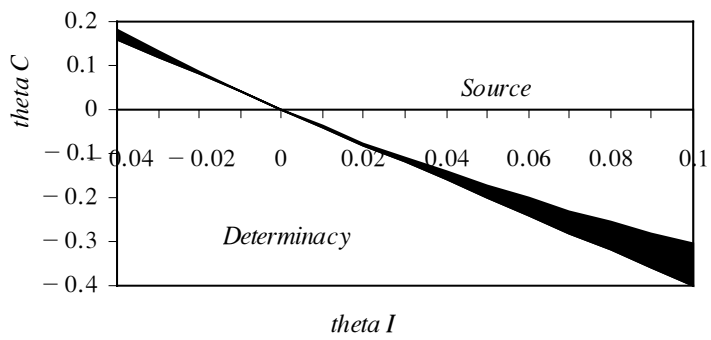
Figure 2

Figure 3

Figure 4

Figure 5