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GROSS CREDIT FLOWS

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ABSTRACT

Gross Credit Flows*

This Paper contributes to the empirical and theoretical knowledge of gross credit flows: the simultaneous process of credit expansion and contraction associated with a net change in the aggregate quantity of credit. Empirically, the Paper summarizes heterogeneity in the banking industry by estimating gross credit flows for the entire US banking system between 1979 and 1999. The empirical exercise shows that sizeable gross flows coexist at any phase of the cycle, even within narrowly defined regional units and bank size categories. Furthermore, the Paper finds that aggregate credit contraction is a concentrated series, which implies that a burst in credit contraction is followed by prolonged periods of low contraction. Theoretically, the Paper proposes a matching model in which financiers have to spend time and resources to expand credit to heterogeneous entrepreneurs. The outcome of the model resulting from the combination of idiosyncratic shocks and asymmetric adjustment to positive and negative aggregate shocks appears consistent with the empirical properties of aggregate credit flows.

JEL Classification: E40 and E50
Keywords: flows and matching models

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NON-TECHNICAL SUMMARY

Aggregate changes in the quantity of credit arguably represent one of the most important macroeconomic variables for policy-makers, and a large literature has studied the dynamic adjustment of net credit. By definition, the change in the quantity of credit is the result of two different processes: the extension of new loans, and the cancellation of expired and non-performing loans. In a given period, banks are active on both sides, screening new applicants to reduce information asymmetries, and investing time and resources to recover non-performing loans. These activities generate simultaneous aggregate gross flows of credit expansion and credit contraction, shaping the observed dynamics of net credit flows. Surprisingly, the empirical and theoretical knowledge of gross credit flows is still very limited. This Paper is an attempt to fill this gap.

Empirically, the Paper uses bank level data on the entire US banking system to construct aggregate statistics that summarize the process of credit expansion and contraction between 1979 and 1999. More specifically, it summarizes the process of credit expansion (contraction) by aggregating positive (negative) changes in credit across individual banks, using *The Report of Condition and Income Database*. It constructs aggregate flows at quarterly and yearly frequencies and shows that sizeable gross flows coexist at any phase of the cycle, even when banks are bunched together in terms of regional and/or size characteristics. On average, gross credit flows appear four times larger than net credit flows, and heterogeneity across states and bank size groups can only partially account for the large rates of credit reallocation, where the latter is defined as the sum of aggregate gross credit expansion and contraction. Furthermore, credit contraction is a *concentrated series*, which implies that a burst in one period is followed by prolonged periods of low contraction.

Understanding the magnitude and the dynamic of gross credit flows has important policy and macroeconomic implications. Indeed, the Paper argues that the transmission of aggregate shocks (including monetary policy shocks) over the gross margins is likely to be asymmetric, with immediate consequences for the dynamic response of credit to aggregate shocks. Further, the Paper suggests that reallocative shocks to liquidity represent a propagation mechanism of aggregate changes in net credit. Indeed, the realization of reallocative liquidity shocks may raise credit contraction and, with some delays, may raise credit expansion, with a temporary fall in aggregate credit.

The Paper explores also the theoretical implication stemming from the dynamic asymmetric behaviour between credit contraction and expansion. Specifically, the Paper proposes and solves a dynamic matching model, where homogeneous financiers actively search to establish investment relationships with *ex ante* heterogeneous entrepreneurs. The model is used to study the dynamic interaction of aggregate and idiosyncratic shocks on entrepreneurs-financiers matches, and to analyse the response of the creation and the destruction margins to stochastic shocks to the financier's outside options, which can be interpreted as changes in money market rates. Indeed, a dynamic stochastic version of the proposed model generates gross credit flows that replicate the main dynamic properties of credit creation and destruction.

1 Introduction

Net changes in the aggregate level of bank lending are the result of two endogenous gross flows: the extension of new loans and the cancellation of expired and non-performing loans. Banks are active on both margins, screening new applicants to reduce informational asymmetries, and investing time and resources to recover non-performing loans. While both activities are costly and time-consuming, they are intrinsically different, with potentially important consequences for the dynamic behavior of aggregate credit. Although a large literature has studied the dynamic adjustment of net credit,[1] the empirical and theoretical knowledge on gross credit flows is still very limited. This paper is an attempt to fill that gap. First, starting from the empirical evidence that banks behave heterogeneously and that such heterogeneity can be only partially explained by regional and structural differences, it constructs new aggregate time series of gross credit expansion and contraction. Second, it analyzes the cross sectional and dynamic behavior of the constructed gross credit flows, and shows that credit expansion and contraction feature asymmetric time-series properties. Finally, it presents a new matching model of bank lending that mimics relatively well the dynamic characteristics of the constructed series.

The obvious question that arises is what we do miss by ignoring the magnitude and the dynamics of gross credit flows. This paper suggests two answers. First, the transmission of aggregate shocks (including, possibly, policy shocks) over the gross margins is likely to be asymmetric, with immediate consequences for the dynamic response of credit to aggregate shocks. Second, the emphasis on gross flows suggests that reallocative shocks to liquidity can represent a propagation mechanism of aggregate changes in net credit. Indeed, the paper argues that as long as credit expansion is time consuming, a reallocation of liquidity across heterogeneous banks is associated with an aggregate contraction of credit.

Empirically, we use bank level data on the entire U.S. banking system to construct aggregate statistics that summarize the process of credit expansion and contraction between 1979 and 1999. Our methodology, which to our knowledge has never been applied to the study of aggregate credit, has been extensively applied by Davis and Haltiwanger for studying the aggregate consequences of heterogeneous labor adjustments (Davis Haltiwanger and Schuh, 1996). More specifically, we summarize the process of credit expansion and contraction by aggregating positive (negative) changes in credit across individual banks, using the *The Report of Condition and Income database* (Call Report Files) for the U.S. banking system

[1] See Friedman and Kuttner (1993) and references therein.

between 1979 and 1999.

We construct aggregate gross flows at quarterly and yearly frequencies, and show that sizable gross flows coexist at any phase of the cycle, even when banks are bunched together in terms of regional and/or size characteristics. On average, gross credit flows appear four times larger than net credit flows, and heterogeneity across states and bank size groups can only partially account for the large rates of credit reallocation.

Dynamically, credit expansion (contraction) rises (falls) during net expansion and falls (rises) during net contraction (expansion). Furthermore, we detect an asymmetry in the dynamic behavior of gross flows. In particular, credit contraction is a concentrated series, which implies that a burst in credit contraction is followed by prolonged periods of low aggregate contraction. This property does not find a correspondence in the process of credit expansion.

The asymmetric dynamic behavior of credit expansion and contraction is novel, and suggests that a matching model of the credit market, where credit expansion is costly and time consuming, would neatly rationalize the dynamic properties of gross credit flows. Thus, we propose and solve a stochastic matching model, where homogeneous financiers actively search to establish investment relationships with *ex-ante* heterogeneous entrepreneurs. The model allows us to study the dynamic interaction of aggregate and idiosyncratic shocks on entrepreneurs-financiers matches, and to analyze the response of the creation and the destruction margins to stochastic shocks to the financiers' outside option. Indeed, a dynamic stochastic version of our model generates gross credit flows that replicate reasonably well the main dynamic properties of credit creation and destruction.

In reality, there are several reasons why lending may be a time consuming process, especially in intermediated capital markets in which asymmetric information is pervasive, screening is costly and time consuming, and good investment opportunities may be difficult to find.[2] In this paper, however, we do not deal directly with informational asymmetries, even though we do model an aggregate form of credit-rationing. Specifically, we assume that there is a positive probability that financiers' liquidity and idle projects *do not succeed* in finding each other in a given period. In other words, we assume that financial relationships can profitably take place only after a financier and an entrepreneur have been randomly matched.[3] This over-simplification, while extreme from the perspective of microeconomic

[2]In the case of existing bank-client relationships, these problems are potentially less severe. However, financial institutions still need to evaluate the profitability of increasing exposure vis-a-vis an existing client.

[3]Den Haan, Ramey, and Watson (1999), Wasmer and Weil (1999), and Dell'Ariccia and Garibaldi (1998) apply the theoretical ideas of the matching literature to capital market issues.

theory, is meant to capture in an aggregate model the time consuming character of credit formation. In real life capital markets, credit contraction is also costly, as banks may incur liquidation costs when cancelling a loan. However, at the aggregate level, credit contraction may not involve significant time delays, since a sizable component of aggregate bank lending is represented by lines of credit, which banks can recall immediately. Whereas bank lending expansion is time consuming and recovering non-performing loans is costly, buying and selling money market funds (or t-bills) can take place at will and without delays. As a result, the speed at which lending opportunities become available, and the cost incurred for recalling liquidity are important determinants of the dynamic behavior of aggregate bank lending.

The paper proceeds as follows. Section 2 describes the empirical methodology and defines credit expansion and contraction. Section 3 describes the cross sectional characteristics of gross credit flows, with particular emphasis on the role of regional shocks and bank size. Section 3 presents also the dynamic properties of gross flows, emphasizing the asymmetries between credit expansion and contraction. Section 4 introduces concepts and notation of our theoretical framework, while section 5 solves the steady state model. Section 6 extends the analysis to a full dynamic setting, and shows that the implications of our model are consistent with the dynamic properties of credit expansion and contraction. Section 7 discusses the relevance of our work, and concludes.

2 Empirical Methodology

This section briefly describes the data used in this study and introduces the methodology to construct gross credit flows.

2.1 Data

The Report of condition and Income database (Call Report Files) represents an ideal data set for studying heterogeneous behavior in the U.S. banking system.[4] The database contains bank level balance sheet information for all banks regulated by the Federal Reserve System, Federal Deposit Insurance Corporation, and the Controller of the Currency. Complete balance sheets are available from 1976:1 to 1999:4, but constraints on the availability of merger information limit the sample that we consider to the period 1979:3 to 1999:3. The number

[4]The database is available on-line on the Federal Reserve Bank of Chicago server at address: http://www.frbchi.org/RCRI/rcri_database.html

of banks we study is reported in Table 1 and the corresponding total number of observations is approximately 1,300,000.

During the period of our investigation, the U.S. banking system went through an intense concentration process, leading to a reduction in the number of banks from more than 15,000 to less than 10,000. Table 1 summarizes in few aggregate statistics the dynamic evolution of the banking system, and the distribution of credit across banks.[5] The first three columns of Table 1 highlight the dynamics of the two components of the aggregate credit: the number of banks and the size of the average bank. Clearly, as the former dramatically fell, the growth of aggregate credit was driven by a large increase in the latter. Indeed, the value of the real loans of the average bank more than tripled over the period 1979-1998, while aggregate credit doubled. The last three columns of Table 1 summarize the dynamics of the dispersion of the credit distribution across banks. The coefficient of variation and the Herfindahl index feature a u-shape dynamics, with both measures falling until 1991, and rising again toward the end of the period. The Gini coefficient, conversely, rose steadily over the last twenty years, indicating a mild decrease in the inequality in the bank size distribution. These measures suggest that despite the dramatic fall in the number of banks, the overall dispersion in the distribution of real loans across banks has not changed markedly.

2.2 Constructing Gross Credit Flows

Our measure of aggregate credit expansion and contraction draws on a methodology successfully applied by Davis, Haltiwanger, and Schuh (1996) to construct job flows data. The basic logic of the methodology is as follows. Starting from individual bank data, we say that a bank expands (contracts) credit in a given period if the net change in the value of real credit is positive (negative). Then, at the aggregate level, gross credit expansion is proxied by the sum of all positive credit changes across banks; gross credit contraction is the sum of the absolute value of negative credit changes across banks. Finally, dividing the aggregate gross flows by a measure of aggregate credit, we obtain gross rates of expansion and contraction. While working with real credit appears sensible, we also construct our gross measures with nominal data, and show that the main properties of the two series are very similar.

Traditionally, there are two main shortcomings associated with this procedure. The first problem refers to the obvious underestimation of gross flows, due to the fact that this methodology (and our data) cannot identify simultaneous expansion and contraction within

[5]See Berger, Kashyap, and Scalise (1995) for a detailed analysis on the evolution of the banking system in the '80s and '90s.

Table 1: Dynamic Evolution of the U.S. Banking System

Year[a]	Num. [b] Banks	Av. [c] Bank.	Agg. [d] Loan	Coeff. [e] Var.	Herf. [f]	Gini [g]
1979	14946	100	100	12.42	1.04	0.836
1980	15410	98.19	101.24	14.69	0.99	0.89
1981	15372	102.74	105.6	14.93	1.00	0.894
1982	15412	108.93	112.3	14.68	0.99	0.895
1983	15410	113.75	117.3	13.92	0.89	0.892
1984	15270	126.2	128.9	13.15	0.79	0.895
1985	15270	137.7	140.7	12.39	0.70	0.897
1986	15109	156.9	158.6	11.84	0.64	0.902
1987	14649	164.3	161.9	11.17	0.57	0.908
1988	14086	175.0	164.9	10.74	0.53	0.913
1989	13674	184.3	168.6	10.90	0.54	0.916
1990	13311	183.0	163.0	11.02	0.56	0.917
1991	12887	190.9	164.6	10.98	0.55	0.917
1992	12502	192.0	160.6	11.72	0.63	0.918
1993	12057	204.8	165.2	11.71	0.63	0.921
1994	11541	227.3	175.5	11.74	0.63	0.927
1995	11001	253.7	186.7	12.06	0.67	0.933
1996	10550	280.0	197.7	13.24	0.80	0.939
1997	10090	308.5	208.2	14.56	0.97	0.942
1998	9639	334.6	215.8	12.24	1.18	0.922

[a] Data Refer to December
[b] Number of banks with non zero real value of loans
[c] Index for the loan value of the average bank
[d] Index for the aggregate value of loans
[e] Coefficient of Variation of the the value of loans
[f] Herfindahl index (*100) for the U.S. banking system
[g] Gini Coefficient for the loan distribution.
Source: Authors' calculation.

the smallest unit of observation (the individual bank in this paper). The second problem refers to the risk of overestimating gross flows, by recording spurious gross credit flows that are due, in reality, to merger and acquisitions. There is nothing we can do to correct for the first problem in the absence of data on individual loans.[6] However, data on mergers and acquisitions allow us to correct for the second problem.

In the case of gross credit flows, the bias introduced by mergers and acquisitions is particularly serious, as the U.S. banking system experienced a marked reduction in the number of banks since the mid eighties (Table 1). Fortunately, we are able to clean the data from spurious expansion and contraction by using a second database from the Federal Reserve.[7] This "merger file" contains information that can be used to identify all bank

[6]The same underestimation problem exists in the gross job flows data compiled by Davis and Haltiwanger. However, in our case there is also the possibility of overestimating gross flows due to the trading of loans among financial institutions.

[7]Also this database is available on-line on the Federal Reserve Bank of Chicago server at address: http://www.frbchi.org/RCRI/rcri_database.html

acquisitions and mergers that have occurred between 1976 and 1999.[8] These data can be merged with those from the Call Report files by using the bank identity code variables.

We start from the raw data on gross total loans as defined in variable RCFD 1400 of the Call Report Files. This series reports total loans at the bank level for each bank in the database, and corresponds to the variable traditionally used by the scholars of the credit view, and was recently exploited by Kashyap and Stein (2000) in their empirical test of the credit channel of monetary policy.[9] First, we express the data in real terms by dividing them by the CPI.[10] Next, for each bank i and period t, we consider the change in total real loans

$$\Delta l_{it} = l_{i,t} - l_{i,t-1}, \tag{1}$$

where l_{it} is a measure of real loans of bank i at time t. Then, we proceed to correct this raw changes for the actual mergers that occurred in the sample.

Consider a merger occurring between time t and time $t-1$, between bank i (surviving bank) and bank j (non surviving bank). In period t, the total credit of bank j will be zero; while the total credit of bank i will be equal to its own credit in $t-1$, plus the net change in its own credit, plus the credit of bank j in period $t-1$, plus the net change in the credit of bank j. The first difference of the raw data overestimates both credit creation and credit destruction, as the whole credit of bank j in period $t-1$ would be improperly counted as credit destruction and credit creation. To avoid this measurement error, in period t, we subtract the credit of bank j in period $t-1$ from the raw difference in equation (1) for bank i, and add it to the difference for bank j.[11] More formally, let's consider the following function:

$$\phi_{ij}(t) = \begin{cases} 1 & \text{if bank } i \text{ acquires bank } j \text{ between } t \text{ and } t-1, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

[8] Because of possible errors due to changes in the classification of bank assets, we chose to limit the analysis to the period 1979-1999.

[9] The series RCFD1400 reports the aggregate gross book value of total loans (before deduction of valuation reserves) at the bank level. It includes all bank's loan, regardless of the maturity and the borrower type. It includes also commercial paper issued by non financial institutions. Since, in reality, the liquidity of such claims varies across firms we decided to leave such assets in our definition of total loans. A more detailed description of the series can be found at: www.frbchi.org/RCRI/dictionary.html

[10] We also estimate flows working with nominal differences.

[11] There were two exceptions to this methodology. First, in a number of cases the non-surviving bank was split among numerous surviving banks. In that case, we assumed that each surviving bank absorbed an equal share of the credit of the non-surviving institution. Second, in 13 cases (merger code 5) the acquired bank j was split rather than absorbed, and it continued to exist after the merger. In those cases, we assumed that $\Delta l_{jt} = 0$; and subtracted $(l_{j,t-1} - l_{j,t})$ from the total credit of the acquiring bank.

which keeps track of all the banks acquired by bank i between time $t-1$ and t. Further, we let $\psi_i(t)$ be an indicator function that keeps track of the banks that are accrued between t and $t-1$, and whose expression reads

$$\psi_i(t) = \begin{cases} 1 & \text{if bank } i \text{ is acquired between } t-1 \text{ and } t \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

Hence, making use equations (1) and (3) we obtain the adjusted real difference in credit $\tilde{\Delta l_{it}}$, whose expression reads

$$\tilde{\Delta l_{it}} = \Delta l_{it} - \sum_{k=1}^{N} \phi_{ik}(t) l_{k,t-1} - \psi_i(t)\Delta l_{it}. \quad (4)$$

With this methodology we are able to correct for 90 percent of the mergers that occurred over the period of our sample.[12]

The final step to obtain aggregate gross measures requires a simple cross section aggregation of positive and negative changes. The aggregate creation rate between time t and $t-1$ (POS_t) is simply the relative sum of the individual banks' "adjusted differences", $\tilde{\Delta l_{it}}$ that resulted positive (divided by a measure of aggregate credit). More formally, POS_t is

$$POS_t = \frac{\sum_{i|\tilde{\Delta l_{it}} \geq 0}^{N} \tilde{\Delta l_{it}}}{\sum_{i=1}^{N} l_{i,t-1}}. \quad (5)$$

Similarly, gross credit contraction is the sum of the absolute values of the adjusted differences of equation (4), where the summation is taken over all and only those banks whose "modified differences" were negative between t and $t-1$ (divided by a measure of aggregate credit). Thus, its formal expression is

$$NEG_t = \frac{\sum_{i|\tilde{\Delta l_{it}} < 0}^{N} \tilde{\Delta l_{it}}}{\sum_{i=1}^{N} l_{i,t-1}}. \quad (6)$$

¿From these measures of gross flow rates, it is immediately possible to obtain NET_t, the growth rate in net credit as

$$NET_t = POS_t - NEG_t. \quad (7)$$

Finally we introduce two measures of credit reallocation. First, we indicated with SUM_t the simple sum of gross credit flows

$$SUM_t = POS_t + NEG_t, \quad (8)$$

[12] For the other mergers, missing data and other mismatches prevented us from doing the correction.

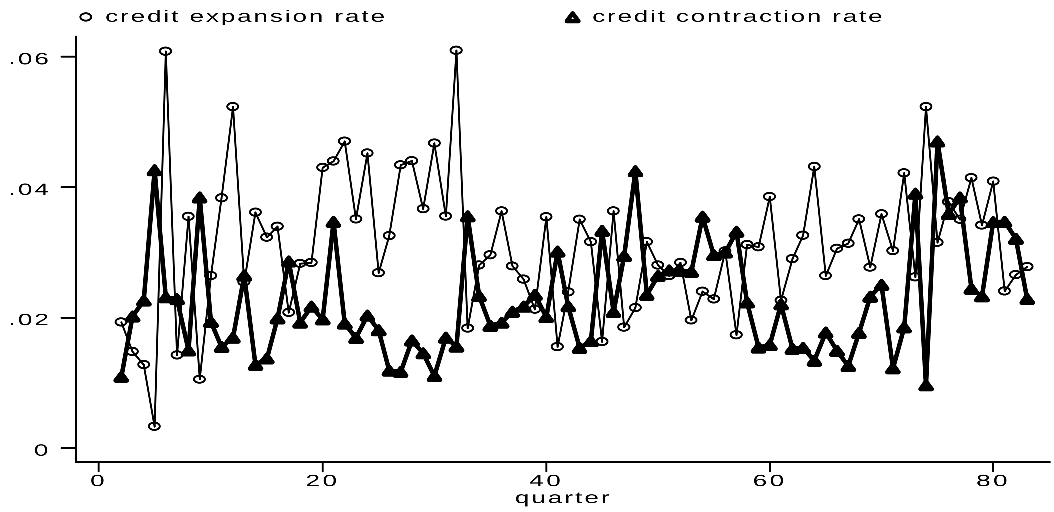

Figure 1: Gross Credit Flows in the U.S. Banking System, Quarterly 1979:3-1999:3

or the total reallocation of credit in a given period. Second, we indicate with EXC_t the reallocation of credit in excess of the net credit change, and its expression reads

$$EXC_t = POS_t + NEG_t - |NET_t|. \tag{9}$$

EXC_t a measure of credit reshuffling across banks, and represents our estimate of the extant by which credit is reallocated across banks. The next section presents and discusses the aggregate measures introduced in this section.

3 Aggregate Credit Flows: Magnitude, Composition and Cyclicality

The availability of quarterly observations allows us to obtain estimates of gross flows at two different frequencies: quarterly and annual. Figure 1 plots in a single figure quarterly credit expansion (POS) and credit contraction (NEG). Table 2 reports summary statistics of the time series plotted in figures 1 and for the same gross measures obtained with yearly frequencies.

3.1 Cross Sectional Properties

The first finding of Table 2 concerns the relative magnitude of gross and net credit flows, and the observation that the latter are remarkably larger than the former. Roughly speaking,

Table 2 shows that the average quarterly net growth of 1 percent is the result of a simultaneous quarterly gross expansion of 3 percent, and a quarterly gross contraction of 2 percent. If we work with nominal data, the net growth in credit in the typical quarter is about 2 percent, and such growth is associated with a 4 percent of gross credit expansion and a 2 percent of credit contraction. When measured at yearly frequencies, net credit grows at an average rate of almost 4 percent, with a corresponding gross credit expansion of almost 9 percent and credit contraction of 5 percent. Excess credit reallocation, the expansion and contraction in excess of net changes, is about 4 percent per quarter (or 8 percent per year), implying that in a given quarter (year) four (eight) percent of the existing credit is reshuffled across individual banks. In addition, we find that gross credit flows are large at all times, and sizable credit destruction exists also in periods of very large net creation. In 1985 and 1986, when net credit flows grew by almost 10 percent per year, gross credit contraction was still around 3 percent. These numbers neatly summarize the heterogeneity in the banking system.

An obvious rationalization of the magnitude of credit reallocation at the aggregate level is the existence of sizable regional shocks. Indeed, excess credit reallocation may simply mask the fact that credit is moving across different U.S. states. To this purpose, Table 3 constructs measures of credit expansion and contraction within each U.S. state, and shows that credit expansion and contraction coexist within each of the 50 states, albeit the average level of excess credit reallocation falls from 4 percent at the aggregate level to some 2 percent at the state level. In what follows, we analyze quantitatively the extent by which the aggregate process of credit expansion and contraction reflects credit reshuffling *across states* or simultaneous expansion and contraction *within states.* Thus, we construct an index of within state credit reshuffling as

$$within_t = 1 - \frac{\sum_j^J |NET_{jt}|}{\sum_j^J SUM_{jt}}, \tag{10}$$

where J is the total number of categories (states) in the sample. If equation (10) is equal to 0, than credit shifts occurs entirely across states, while a value of 1 reflects reallocation occurring entirely within states. Table 3 shows that the average values of the index *within* calculated at the quarterly frequency is equal to 0.42, suggesting that state shocks can account only for about 55 percent of the aggregate process of credit expansion and contractions.

Structural differences across banks may provide a second rationale for credit reallocation. In particular, a large literature has underlined the heterogeneous behavior of banks in terms

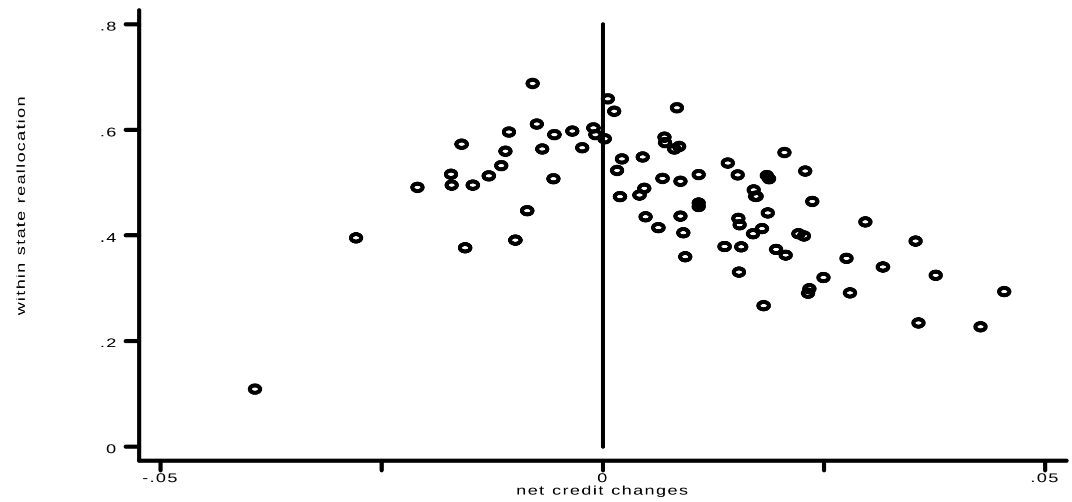

Figure 2: Credit Reallocation within states and aggregate change in credit

of size (see Bernanke and Lown, 1991, Kashyap and Stein, 2000 and references therein). Roughly speaking, small and large banks are likely to react to shocks very differently. Thus, it may well be that an important component of total credit reallocation is accounted for by credit expansion and contractions across banks of different size. If that was indeed the case, we would expect that gross and net credit flows within narrowly defined classes of banks would turn out to be very similar. In Table 4 we report the estimates of gross credit flows calculated by dividing the distribution of average credit over bank size in deciles. A given bank is assigned to a given decile according to its relative positions in the bank's size distribution, where bank credit is calculated as the average credit across all quarters in which the bank was active. Results in Table 4 suggests that the gross credit flows coexisting within each decile are remarkably large, with a value of excess credit reallocation falling from 5.4 percent in first decile to 3.7 in the largest decile. Indeed, the index *within* calculated in terms of bank size yields an average value of 0.3, suggesting that 70 percent of credit reshuffling occur within banks of similar size.

3.2 Cyclical Properties

Next, we look at the cyclical behavior of the constructed macroeconomic series. In Figure 1, the 1979 and 1990-91 recessions are immediately notable, with gross credit contraction being larger than credit expansion for more than one quarter. If we take NET changes as a measure of the aggregate economic activity, the correlations of Table 5 can be used

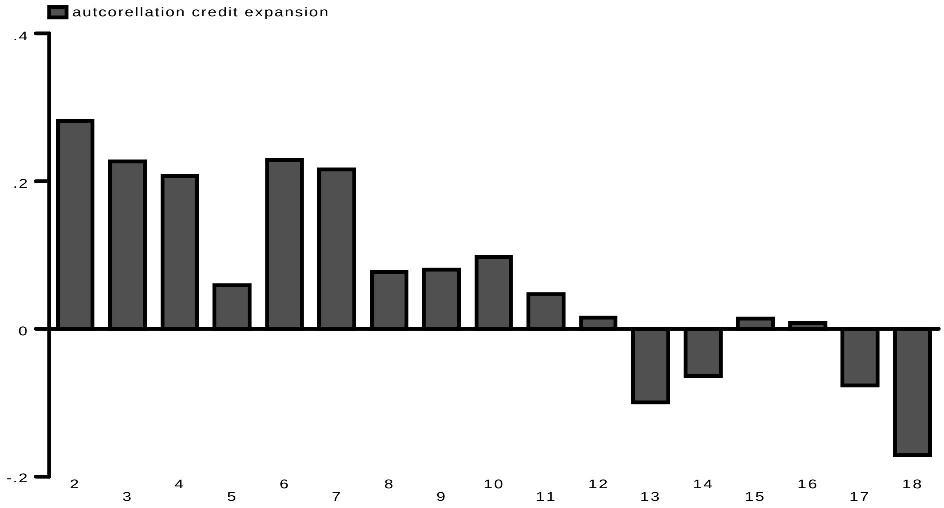

Figure 3: Correlogram of Credit Expansion, Seasonal Adjusted

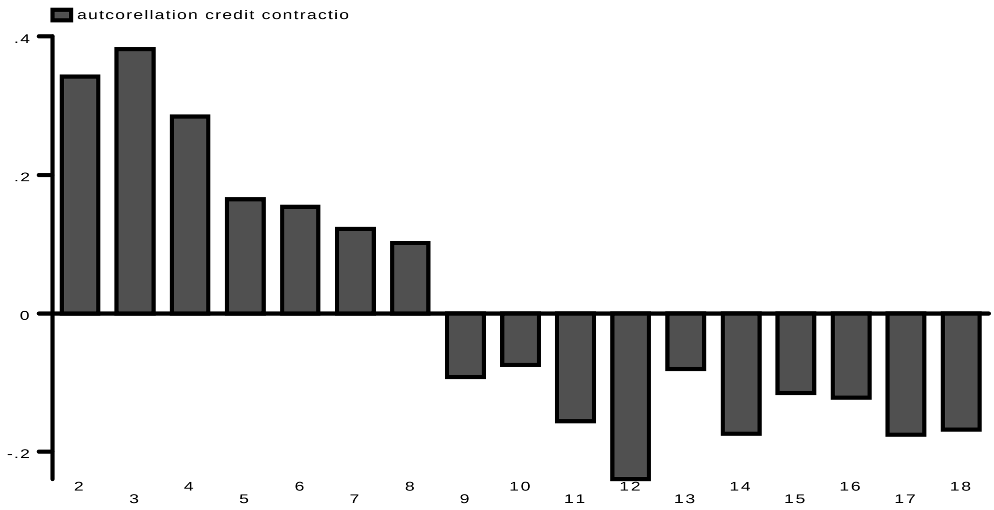

Figure 4: Correlogram of Credit Contraction, Seasonal Adjusted

to study the behavior of our gross measures over the cycle. At both quarterly and yearly frequencies, credit creation appears strongly positively correlated with net credit changes, while credit destruction is negatively correlated with net credit changes. Credit expansion and credit contraction are also negatively and significantly correlated, but with an actual correlation value that is much smaller than the correlation between gross and net flows. We believe this latter findings to be important, since a correlation less than one indicates that the dynamic behavior of the two gross time series is not just the mirror image of each other, and suggests the existence of additional information encoded in the gross flows. In any event, the difference in the dynamic behavior of credit expansion and contraction becomes apparent in the final part of this section, where we show that credit contraction is a concentrated time series. Credit reallocation, the sum of credit creation and destruction, appears positively correlated with net credit changes, but with a correlation coefficient that is significant only at quarterly frequencies. In order to test the robustness of our simple correlation, Table 5 reports also the results of the correlation of gross flows with net flows in a small panel data of gross flows at the state level. Overall, the results in Table 5 suggest that even when controlling for state effects and time effects, the correlation between gross and credit flows is the same as the one obtained in the simple aggregate statistics.

Table 2: Gross Credit Flows: Summary Statistics

	NET	POS	NEG	SUM	EXC
Quarterly "Real" Flows:					
Average	0.008	0.031	0.023	0.053	0.038
Standard Deviation	0.016	0.01	0.008	0.010	0.012
Minimum	-0.039	0.003	0.009	0.030	0.006
Maximum	0.045	0.06	0.046	0.083	0.07
Coefficient of Variation	2.01	0.32	0.34	0.25	0.31
Annual "Real" Flows:					
Average	0.035	0.090	0.054	0.14	0.100
Standard Deviation	0.042	0.026	0.024	0.023	0.037
Minimum	-0.033	0.048	0.026	0.11	0.055
Maximum	0.127	0.153	0.117	0.20	0.18
Coefficient of Variation	1.2	0.28	0.44	0.2	0.034
Quarterly "Nominal" Flows:					
Average	0.024	0.042	0.017	0.059	0.034
Standard Deviation	0.017	0.013	0.006	0.013	0.012
Minimum	-0.006	0.016	0.002	0.026	0.004
Maximum	0.093	0.104	0.037	0.114	0.07
Coefficient of Variation	0.70	0.30	0.35	0.22	0.035
Source: Authors' calculation.					

Table 2 reports summary statistics on the volatility of credit expansion and contraction. In absolute terms, credit expansion is slightly more volatile than credit contraction, as indicated by the standard deviation of POS and NEG. However, the comparison of the standard

Table 3: Gross Flows Within States

State	NET	EXC	State	NET	EXC
AK	0.011	0.013	MS	0.011	0.016
AL	0.016	0.011	MT	0.003	0.023
AR	0.011	0.026	NC	0.020	0.009
AZ	0.016	0.019	ND	0.007	0.02
CA	0.006	0.027	NE	0.009	0.024
CO	0.01	0.024	NH	0.020	0.021
CT	0.007	0.015	NJ	0.013	0.018
DC	0.005	0.017	NM	0.008	0.021
DE	0.043	0.036	NV	0.023	0.015
FL	0.020	0.025	NY	0.009	0.043
GA	0.021	0.033	OK	0.006	0.026
HI	0.015	0.008	OR	0.013	0.022
IA	0.005	0.022	PA	0.008	0.018
ID	0.014	0.009	RI	0.012	0.016
IL	0.008	0.037	SC	0.019	0.009
IN	0.008	0.018	SD	0.014	0.025
KS	0.007	0.027	TN	0.013	0.018
KY	0.012	0.015	TX	0.008	0.031
LA	0.008	0.021	UT	0.016	0.021
MA	0.019	0.019	VA	0.014	0.014
MD	0.012	0.014	VT	0.007	0.011
ME	0.009	0.013	WA	0.014	0.021
MI	0.008	0.013	WI	0.010	0.018
MN	0.009	0.019	WV	0.008	0.018
MO	0.010	0.022	WY	0.003	0.027

Average Value of *Within* State Excess: 0.46
See equation 10
Source: Authors' calculation.

deviation, a measure of absolute variability, does not take into account the fact that credit expansion is, on average, twice as large as credit contraction. Thus, Table 2 reports also the coefficient of variation, and shows that in relative terms credit contraction is marginally more volatile then credit expansion at both quarterly and yearly frequencies. Figure 2 plots net credit changes versus the *within* states component of credit reallocation. While there is an overall negative correlation between the within index and net credit changes, an interesting asymmetry emerges by separating periods of positive net changes to periods of negative net changes, as we did in Figure 4. In tranquil times, when net credit changes are close to zero, credit appears to be reallocated mainly within states. Conversely, when net credit is growing fast or contracting very fast, credit is reallocated mainly across states. While we do not exploit this finding in the present paper, we believe that this phenomenon is novel and interesting, and should be exploited in future research.

As Hall (1999) has recently argued, if heterogeneity is important and microeconomic adjustment can be instantaneous, aggregate gross flows should feature the concentration property, which implies that a burst in microeconomic adjustment is followed by prolonged

Table 4: Gross Flows Within Size Categories

Decile	NET	POS	NEG	SUM	EXC
1st	0.008	0.041	0.033	0.075	0.054
2nd	0.008	0.035	0.026	0.062	0.044
3rd	0.009	0.034	0.025	0.060	0.043
4th	0.009	0.033	0.024	0.057	0.041
5th	0.009	0.031	0.021	0.053	0.037
6th	0.009	0.031	0.021	0.052	0.037
7th	0.010	0.031	0.020	0.052	0.035
8th	0.009	0.032	0.023	0.055	0.040
9th	0.009	0.034	0.025	0.059	0.044
10th	0.008	0.030	0.022	0.053	0.037

Average Value of *Within* Size Excess: 0.71

See equation 10

Source: Authors' calculation.

Table 5: Gross Credit Flows: Cyclical Characteristics

	Annual	Quarterly "Real"			Nominal
Correlation Between	Simple	Simple	Regression[a]	Seasonal [b]	Simple
POS-NET	0.837	0.779	0.623	0.86	0.93
p-value	0.000	0.000	0.000	0.00	0.00
NEG-NET	-0.803	-0.731	-0.376	-0.78	-0.63
p-value 0.000	0.000	0.000	0.000	0.00	0.00
SUM-NET	0.089	0.084	0.24	0.20	0.65
p-value	0.724	0.450	0.00	0.06	0.00
POS-NEG	-0.347	-0.442	-0.590	-0.36	-0.31
p-value	0.158	0.000	0.000	0.00	0.00

[a] Regressions of on net flows, with time and country effects:

$POS_{it} = \alpha_i + \beta NET_{it} + \gamma t + u_{it}; \quad i = 1..50; t = 1..79$

[b] Seasonal Adjusted with a regression on seasonal dummies.

Source: Authors' calculation.

periods of low adjustment. Thus, it seems natural to ask whether, with respect to gross credit flows, concentration is detected along the expansion and/or the contraction margin. In time-series jargon, a process is said to be concentrated when its current value is likely to be lower if its values over a span of time up to the recent past have been higher. If d_t is a covariance stationary time series, the concentration function with lag τ and window N is defined as

$$c_{\tau,N} = -\frac{E[d_t \,|d_{t\ -\tau} + ...d_{t\ -\tau-N+1}]}{\frac{1}{N}(d_{t\ -\tau} + ...d_{t\ -\tau-N+1})}, \tag{11}$$

or as the negative of the coefficient of the regression of d_t on the lagged moving average $\frac{1}{N}(d_{t\ -\tau} + ...d_{t\ -\tau-N+1})$. For a window one observation wide, the concentration function is simply the negative of the autocorrelation function. In the moving average representation of a concentrated series, the coefficients on longer lags of the innovation are negative. An innovation causes a unit increase in the series when it occurs, but causes lower future val-

Table 6: Concentration of Gross Credit Flows

Flows	Coefficient[a]	p-value	structure [b]
CREDIT CONTRACTION			
"Real Flows"	0.55	0.03	3,9
"Real Seasonal Adjusted"	0.44	0.02	3,9
"Nominal Flows"	0.21	0.14	3,9
"Nominal Seasonal Adjusted"	0.21	0.09	3,11
CREDIT EXPANSION			
"Real Flows"	0.17	0.26	3,11
"Real Seasonal Adjusted"	0.06	0.61	3,11

[a] Regression of gross flows on the lagged moving average,
[b] Refer to windows N, and lagged value τ. See equation 11.
Source: Authors' calculation.

ues. While a concentration coefficient can be estimated as the negative of the coefficient in the regression of d_t on $\frac{1}{N}(d_{t\ -\tau} + ...d_{t\ -\tau-N+1})$, the correlogram is a useful tool for diagnose concentration. The correlagrams of the series combining the driving process and the concentration effect have the shape characteristics of a concentrated series except for the low-order correlations. Estimation of a concentration coefficient with an appropriate lag τ should reveal the effect of concentration even though it is obscured by the dynamics of the driving process for the low-order coefficients. The detection of concentration is most effective if the lag τ is chosen just high enough to avoid contamination by the short-run dynamics of the driving process, and window width, N, is chosen not so high as to extend the window into ranges where the concentration effect has disappeared.

Our estimates suggests that only gross credit contraction is a concentrated series. Figure 4 shows the correlogram of credit contraction at quarterly frequencies. The first few autocorrelation are positive, presumably reflecting the short-run dynamics of the driving force. The preliminary inspection of the correlogram suggests a fairly long lag, indicating a fairly persistent behavior of the driving force. At lag 9, the autocorrelation turns negative and remains consistently negative through lag 20. The concentration coefficient with lag 9 and width 3 is 0.55 with a standard deviation of 0.23, providing statistical unambiguous evidence of concentration ($p = 0.03$). Figure 3 shows the correlogram of credit creation, and suggests that there is no tendency for the autocorrelation of credit expansion to turn negative after a few quarters. Indeed, a regression of credit expansion (pos_t) on its lagged moving average fails to find any significant concentration coefficient. To test the robustness of our findings, Table 6 reports estimates of the concentration coefficients calculated on different time series: real gross flows, nominal gross flows and seasonal adjusted flows. In all such series, gross credit contraction features the concentration property, even though with nominal data the lag τ is marginally longer. Conversely, there is no evidence on concentration in credit expansion. Further, we also tried to understand whether the concentration of aggregate credit is detected across banks of different sizes. Regressions similar to that reported in Table 1 for different bank sizes suggest that aggregate concentration is mainly linked to the behavior of large banks.

3.3 Implications

There are two main messages from our empirical analysis. First, the banking system exhibits a degree of heterogeneity that can not be fully accounted for by changes in aggregate credit across states, and across banks of different size. Second, our decomposition of credit changes into aggregate credit expansion and contraction has shown that the dynamic behavior of the gross flows is very different and asymmetric, as indicated by the existence of concentration in the credit contraction series. This, in turn, implies that heterogeneity is important, and that along the contraction margin banks can act on their stock of credit. Such property is absent over the creation margin, suggesting that expansion and contraction are different, and that such difference is relevant at the aggregate level.

In the rest of the paper we provide an analytical framework for thinking about the "churning lending market", and for rationalizing the asymmetric dynamic behavior of credit expansion and contraction. In the theoretical model that we propose, the combination of a symmetric aggregate shock and an idiosyncratic shock generate a dynamic environment that is consistent with a simultaneous process of credit reallocation across narrowly defined units. Furthermore, our theoretical analysis features an asymmetric dynamic behavior of credit expansion and contraction that appears consistent with the dynamics of gross flows described in the present section.

4 The Theoretical Environment and the Model

We consider an economy populated by a continuum of risk-neutral financiers and risk-neutral entrepreneurs. Entrepreneurs are endowed with projects of different qualities and seek project financing. Financiers are endowed with liquid funds and seek investment opportunities. Entrepreneurs have no private source of funds, and the financiers are their only source of external capital. Financiers, however, may invest their capital in two alternative assets: money market bonds and project-loans. For simplicity, we assume that each entrepreneur is endowed with an indivisible project requiring an initial investment of \$1 that is productive only when it is matched to a unit of financial funds. Since our focus is on financial contracts, we assume that each financier is endowed with a single unit of liquidity. As a result, we abstract from issues related to market structure in the financial system.

The entrepreneur population has mass 1. Entrepreneurs are heterogenous in the quality of their projects. Each project is characterized by a pair (x, λ), where $x \in [\underline{x}, \overline{x}]$,is the project's return, and λ is the instantaneous probability of the project being hit by a shock, a Poisson process that measures the project's idiosyncratic risk. When hit by an idiosyncratic shock, a project changes its type according to the distribution function $F(x)$, which is a continuous distribution function defined over the support $[\underline{x}, \overline{x}]$. Finally, each project can be in two different states, depending on whether or not it is matched to a financier's capital. A financed project is active and it produces its idiosyncratic dividend x while an un-financed project is idle and does not yield any dividend.

The financier population has also mass 1. Each financier can invest her indivisible unit of liquidity in two different assets: money market funds or project loans. We let the money-market investment be risk-free, and we let r_d indicate its instantaneous return. Financiers are exposed to their own idiosyncratic shocks that strikes with instantaneous probability ξ. When hit by such shock, the individual financier is forced to liquidate her investment. The point here is to introduce shocks to the financial system that may interact with the orderly functioning of the real economy.[13] We assume that such liquidation has no consequences when the financier's capital is invested in the fully liquid money-market; while it involves a loss B, when the capital is used to finance entrepreneurial projects. We want the shocks ξ to represent a reallocation of liquidity across the banking system, and we thus assume that in each instant a flow ξ of liquidity comes into the economy. This, in turn, ensures that aggregate liquidity (and the mass of financiers) is constant at all times.

We model credit expansion as a time-consuming process. When time elapses in a continuous way, as we assume in the rest of the analysis, money market investment can be undertaken immediately, whereas loan expansion is time consuming. Similarly, we assume that credit contractions involve real liquidation costs, while money market disinvestment are costless.

Formally, an analytically convenient way to model credit formation as a time-consuming process can be borrowed from the traditional matching literature (Diamond, 1982, and Mortensen and Pissarides, 1994). In what follows, we assume that the number of credit

[13] One possible interpretation of this liquidity shocks is that they represent the reduced form of bank runs. More on this later.

applications that are fully screened and evaluated in a given interval of time is described by a unique function of few aggregate variables: the stock of capital in the money market, and the mass of idle projects.[14] In other words, we are assuming that in the economy, during a short period of time δt, there is a positive probability $1 - \alpha\delta t$ that a unit of liquidity and an idle project do not succeed in finding each-other.[15] As a result of this assumption, the economy is characterized by *aggregate (and stochastic) credit rationing.*[16] Formally, we do not need to specify whether banks meet entrepreneurs randomly over time, or whether banks find new projects at an infinite speed but their screening technology is intrinsically time-consuming. In either case aggregate credit formation is time consuming, and the parameter α captures this property in a simple way.

When a unit of financial liquidity and an idle project match, all ex-ante uncertainty is resolved, and the financier immediately learns the type of the project. An active project, however, remains subject to idiosyncratic risk at rate λ. For the entrepreneur, the realization of the shock represents an immediate change in its productivity. For the financier, a shock to a financed project brings changes to the income generated by the associated loan. A shock may be large enough to make a project economically not viable (negative surplus), and separation takes place, but at a cost T. This captures the idea that bankrupt firms have assets that can potentially be liquidated, but only via a costly process. A second cause of contract termination is an idiosyncratic liquidity shock to the financier. When such shock strikes, the financier is forced to withdraw the liquidity, and the project, independent of its type x, goes back to its idle state.

The existence of credit rationing as a result of a finite α generates a *pure economic rent* to be split between entrepreneurs and financiers that successfully match. As a result, to formally close the model, we need a sharing rule that determines the interest rate charged to different projects. We follow the standard matching literature and assume that the total surplus generated by an active project is continuously shared in fixed proportions, and we let β represents the financier's share. Thus, the financial contract of this model, which we often improperly call "loan" for purposes of exposition, does not see the entrepreneur as the residual claimant. Note that, because of the liquidation cost, the outside option for the

[14]This assumption implies that the amount of capital invested in existing loans does not affect the number of applications screened. Relaxing this assumption would make the analytic of the model much more cumbersome, but it would not alter its conclusions.

[15]Formally, this can be model as a matching function. Our framework is similar to the simple form of constant return to matching in the two-sided search model of Burdett and Wright (1998).

[16]We can also say that the banking system issues new loans with an average waiting time $\frac{1}{\alpha}$.

financier changes once her capital has been invested in a specific project. For that reason financiers will keep alive projects of a type that they would not finance from an ex-ante point of view, meaning before sinking their investment.

Financiers choose a search strategy that maximizes the expected value of their capital: they select a decision rule that describes whether to finance a specific project, whenever it becomes available. Since the present value of financing each entrepreneur is monotonic in x, the financier decision rule satisfies a "reservation" property. We show that in equilibrium each bank selects an ex-ante cut-off quality x_c, such that for projects of quality lower than x_c financiers prefer to keep their funds in the money market, and an ex-post cut-off quality x_d, such that all active projects that fall below x_d are liquidated at cost T. Obviously, for $T \geq 0$, we will have $x_c \geq x_d$. The equilibrium of the model will be represented by a couple (x_c, x_d) resulting from the financiers' maximization problem. In the next section, we describe the model in greater detail.

5 Steady State Analysis

This section presents and solves the steady state model, with an exogenous and time invariant money-market interest rate, r_d.

In what follows, we will indicate with $V(x)$ and $J(x)$ the present discounted values to an entrepreneur of type x of an idle project and an active project, respectively. The discount rate, ρ, is assumed constant over time and equal for financiers and entrepreneurs. Let's start with $V(x)$. Even though idle projects do not yield any dividend, their present discounted value may still be positive, by virtue of the expected capital gain associated with successful matching. An additional capital gain term reflects the fact that the project may change type with instantaneous probability λ.[17] We can thus write

$$(\rho + \alpha + \lambda)\, V(x) = \alpha \max\left[J(x) - \Delta; V(x)\right] + \lambda \int_{\underline{x}}^{\overline{x}} V(z)dF(z) \tag{12}$$

[17] We will show that in equilibrium some types are not financed. We interpreted the matching function as the reduced form of a time consuming stochastic screening process. However, as some types have zero incentive to search, the equilibrium is not generic, meaning that an epsilon positive searching cost would lead such types out of the market conditionally on the banks always screening. In that situation, the only equilibrium would be one in mixed strategies where banks mix between screening and not screening applicant borrowers and entrepreneurs with low type idle projects mix between applying and not applying for credit. This is result is fully consistent with the Diamond paradox, and is common to many search models.

where the *max* operator indicates that entrepreneurs have always the option to leave their projects inactive. The term Δ is a transfer that takes into account the different outside option of the financier before and after having sunk her investment into the project (its function will be clearer in a moment). Active, or financed, projects yield an instantaneous dividend equal to their type x. In addition, their discounted value contains two capital gain/loss terms. The first is introduced by the probability of a change in their productivity/type; the second by the probability that their financier suffers a liquidity shock and is forced to recall the loan. We can write the value of an active project of type x as

$$(\rho + \lambda + \xi)\, J(x) = x - r\,(x) + \lambda \int_{\underline{x}}^{\overline{x}} \max\left[J(z); V(z)\right] dF(z) + \xi V(x)\ , \tag{13}$$

where, again, the max operator indicates that conditional on λ striking, entrepreneurs have always the option of going back to the idle state.

Now, look at the financier side. In what follows, we will indicate with D and $C(x)$ the present discounted values to a financier of deposit on the money market and of loan extended to a type x project, respectively.

The discounted value of a unit of capital invested in the money market is determined by the risk-free interest rate, r_d, and by a capital gain term introduced by the probability of a successful matching with an entrepreneurial project. We can write

$$(\rho + \alpha + \xi)\, D = r_d + \alpha \int_{\underline{x}}^{\overline{x}} \max\left[C(z) + \Delta; D\right] dF(z)\ , \tag{14}$$

where, again, the term Δ is included to keep into account the different outside option of the bank ex-ante and ex-post; while the *max* operator indicates that financiers have the choice to refuse credit to applicant borrowers. Note that the capital gain term is expressed as an average, as financiers know only the distribution of applicant borrowers and discover their individual type only after the screening has successfully taken place. Active projects of type x, pay an interest rate $r(x)$. In addition, their discounted value contains two capital gain/loss terms. The first term is introduced by the probability of a change in the type of the project; the second by the probability that the financier itself is forced to liquidate the loan. Then, we can write the value of a unit of capital invested in a loan of type x as

$$(\rho + \lambda + \xi)\, C(x) = r\,(x) + \lambda \int_{\underline{x}}^{\overline{x}} \max\left[C(z); D - T\right] dF(z) - \xi B \ , \tag{15}$$

where the *max* term indicates that the financier has always the option to liquidate the project at the cost T, and cut losses after a shock occurs.

For the financiers, creating a financial relationship with an entrepreneur is optimal as long as $C(x) + \Delta \geq D$; while it is optimal to maintain a previously established relationship as long as $C(x) \geq D - T$. Similarly, for entrepreneurs, activating an idle project is optimal as long as $J(x) - \Delta \geq V(x)$; while it is worth to maintain it as long as $J(x) \geq V(x)$. The surplus ex-ante, meaning when the investment has not been sunk and outside option is D, can be written as

$$S_0(x) = \left[J(x) - V(x)\right] + \left[C(x) - D\right]. \tag{16}$$

The surplus ex-post, meaning when the loan has been granted and liquidating the project would involve the cost T, can be written as

$$S_1(x) = \left[J(x) - V(x)\right] + \left[C(x) - D + T\right],$$

obviously, we have

$$S_1(x) = S_0(x) + T \ . \tag{17}$$

As long as $T > 0$, there will be a difference between the outside option of the financier before and after sinking the loan investment. Consequently, there will be a difference between the value of a match ex-ante and ex-post, with repercussions on the allocation of the associated surplus between investor and financier. Since $C(x)$ is the value of a financial relationship ex-post, equation (14) introduces a lumpsum transfer Δ for taking into account the fact that upon meeting, and only upon meeting, the financiers obtain $C(x)$ plus a side transfer. To solve the model, we need an exogenous sharing rule that determines how financiers and entrepreneurs divide the surplus generated by the match. Assuming that the rule is such that financiers get a fixed share β of the surplus, we have

$$\beta S_0(x) = C(x) - D + \Delta; (1-\beta)\, S_0(x) = J(x) - D - \Delta \ ; \tag{18}$$

$$\beta S_1(x) = C(x) - D + T; (1-\beta)\, S_1(x) = J(x) - D;$$

$$\Delta = (1-\beta)T. \tag{19}$$

Equation (18) reflects two features characteristic of matching models. First, it is profitable for the financier and the entrepreneur to establish (maintain) a credit relationship as long as the total surplus is positive. Second, there is agreement between the financier and the entrepreneur over which projects should be financed. Note that, in this model, there is an additional cause of project destruction represented by liquidity shocks to the financiers. The interpretation of such shocks is discussed in detail in the last section of the paper.[18] Finally, equation (19) shows that the entrepreneur transfers upfront a side payment identical to its share of the liquation cost. This shows that the liquidation cost, even if technically paid by the financier, is actually split in fixed proportions. We are now in a position to define a stationary equilibrium.

A *Stationary Equilibrium* is defined as a set of two reservation margins (x_c, x_d), an interest rate rule $r(x)$, a side payment Δ, and a set of balance flow conditions
such that

- financiers and entrepreneurs solve the financial entry decision;
- financiers and entrepreneurs solve their separation decision;
- the distribution of types is time invariant.

In the appendix, we prove that both surplus functions are increasing in x, and we show that a simple restriction on the parameters guarantees the existence of a unique equilibrium.[19] In equilibrium, no project of type $x < x_c$ is ever financed, and no project of type $x < x_d$ is ever kept active. On the plan (X_c, X_d) the equilibrium can be identified as the intersection of two functions representing the locus $S_0(x_c) = 0$, and the locus $S_0(x_d) = 0$, respectively. Formally, (x_c, x_d) is the solution of the system

$$\begin{aligned} x_c - r_d - \lambda\beta T - \xi B + H\left(x_c, x_d\right) &= 0 \ , \\ x_d - r_d + (\rho + \xi)T - \xi B + H\left(x_c, x_d\right) &= 0 \ , \end{aligned}$$

[18]In brief, we introduced these shocks as a reduced form of a model where financiers have both an asset and a liability side. In such a model, financiers would have first to collect funds from depositors and, then, engage in lending. To the extent that depositors and entrepreneurs would not be able to bargain directly, this kind of framework would entail inefficient destruction (the destruction of financially viable projects) if depositors had to withdraw their funds for liquidity needs.

[19]The restriction is $\Re$: $\alpha < \rho + \xi$.

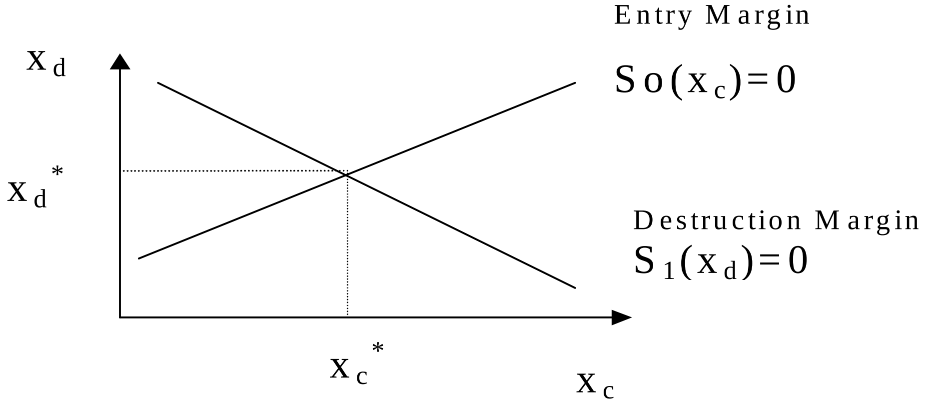

Figure 5: Equilibrium Entry Margin (x_c) and Destruction Margin (x_d)

where

$$H\left(x_c, x_d\right) = \frac{\lambda}{\rho+\lambda+\xi}\int_{x_d}^{x_c}\left(1-F(z)\right)dz + \frac{\lambda-\alpha\beta}{\rho+\lambda+\xi+\alpha\left(1-\beta\right)}\int_{x_c}^{\overline{x}}\left(1-F(z)\right)dz.$$

¿From which it follows immediately that

$$x_d - x_c = (\rho+\xi+\lambda)T,$$

or that $x_d < x_c$ as long as $T > 0$. Since the entry margin entry margin is upward sloping and contraction margin is downward sloping (Figure 5), the equilibrium is unique.

Note that in equilibrium, the continuum of projects can be divided in four categories according to their type, x, and their state, active or idle. First, there is a mass j_g of *Good Active Projects*. They represent entrepreneurs that are financed and active, and whose idiosyncratic type is above the entry margin, or $x > x_c$. Second, there is a mass j_b of *Bad Active Projects*. These are entrepreneurs that are currently active, but would not be refinanced if freshly met on the market. When originally financed, these were highly productive projects with a type $x > x_c$, and later deteriorated because of negative idiosyncratic shocks, such that their current type lies between x_c and x_d. This group of projects is kept alive because of the cost of liquidation T and the associated loans can be considered "stuck liquidity". Third, there is a mass v_g of *Good Idle Projects*. These are entrepreneurs that would get liquidity if they met a financier, and are such that their current idiosyncratic type x is above the entry margin, or $x > x_c$. Finally, there is a mass v_b of *Bad Idle Projects*. These are idle projects that in

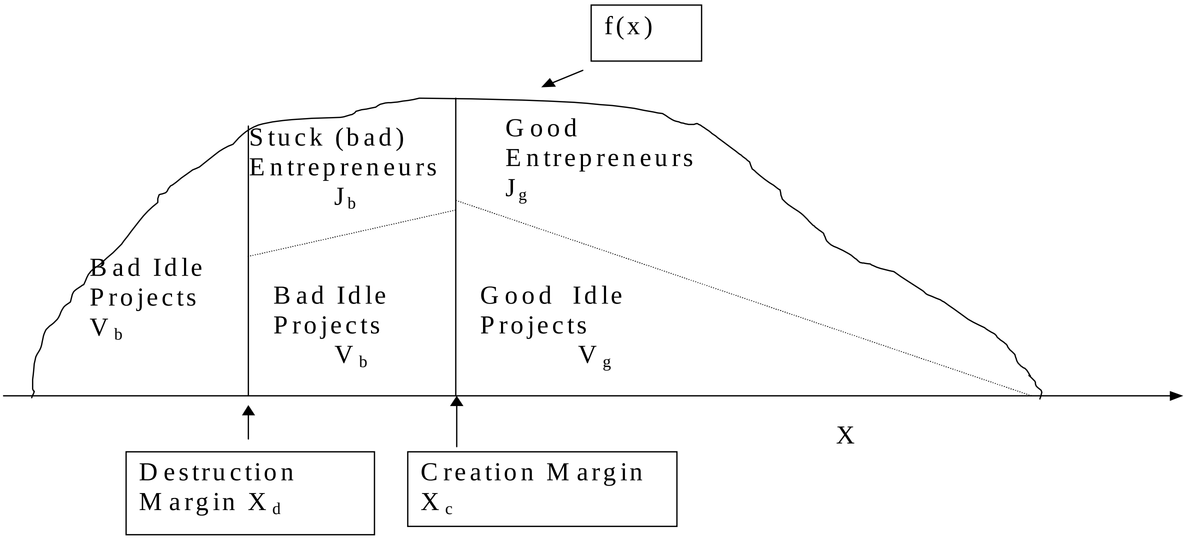

Figure 6: Endogenous Partition of the Distribution of Projects

their current state would be turned down by financiers, but could become a profitable match if they were hit by a positive idiosyncratic shock. Formally, they correspond to all the idle entrepreneur whose type is below the entry margin, or $x < x_c$. Symmetrically, there will be a mass of financial liquidity $d = v_b + v_g$ invested on the money market, a mass $c_g = j_g$ lent to good entrepreneurs, and a mass $c_b = j_b$ stuck on non-performing loans. Obviously, $v_b + v_g + j_b + j_g = d + c_b + c_g = 1$.

In steady state the mass of projects in each type/state is constant. Then, we can derive the steady-state values of v_b, v_g, j_b, and j_g by imposing that inflows and outflows in each type/state are balanced. The balance flow condition for bad entrepreneurs is

$$[\lambda F(x_d) + \lambda(1 - F(x_c)) + \xi]j_b = [\lambda F(x_c) - \lambda F(x_d)]j_g \ , \tag{20}$$

where the left hand side is the outflow and the right hand side is the inflow. Outflows consist of bad projects that transit into bad idle projects because of a negative idiosyncratic shock or a shock to their financier, and of projects that become good because of a positive idiosyncratic shock. Inflows correspond to good entrepreneurs that are hit by a negative shock to their productivity and end up with a type $x \in [\ x_d, x_c)$. Next consider the flows for the bad idle projects. The balance condition yields

$$\lambda F(x_d)v_g + [\lambda F(x_d) + \xi]j_b = \lambda[1 - F(x_c)]v_b \ . \tag{21}$$

The left hand side simply says that active projects (good and bad) flow into the bad vacancy state when hit by a negative shock that brings their type below the exit threshold or, for bad projects only, when their financier is hit by a liquidity shock. Such inflows into the bad idle state are balanced by the outflows consisting of project leaving the bad idle state because of sufficiently large positive idiosyncratic shocks. Projects stop being "good idle projects" when successfully matched with a financier or when hit by a sufficiently negative idiosyncratic shock; new good idle projects consist of bad idle projects hit by positive shocks and by good active projects that turn idle in response to a shock to their financier. The balance condition yields

$$\alpha v_g + \lambda F(x_c) v_g = \lambda[1 - F(x_c)] v_b + \xi j_g \ . \tag{22}$$

Finally, the balance condition for the good active projects reads

$$\alpha v_g + \lambda(1 - F(x_c)) j_b = [\lambda F(x_c) + \xi] j_g \tag{23}$$

where outflows correspond to good active projects which are hit by negative shocks and turn into bad active projects or are liquidated. Inflows are new matches and bad active projects turning good. The system consisting of three equations among (20), (21), (22), (23), and the condition that their sum equals one yields unique solution for j_g , j_b, v_b, and v_g.

5.1 Comparative Statics

We are now in the position to analyze a number of comparative static results. In particular, we are interested in the relationship between the entry and exit thresholds and the parameters that define the general economic environment of the model.

Let us consider changes in the outside option of the financiers, the money-market rate r_d. First, let us define

$$\begin{aligned}
a(x_{d,}x_c) &= \frac{\rho + \xi + \lambda F(x_d)}{\rho + \lambda + \xi} \\
b(x_{d,}x_c) &= [\frac{\lambda}{\rho + \lambda + \xi} - \frac{\lambda - \alpha\beta}{\rho + \lambda + \xi + \alpha(1-\beta)}](1 - F(x_c)) \\
c(x_{d,}x_c) &= \frac{\lambda(1 - F(x_d))}{\rho + \lambda + \xi}
\end{aligned}$$

which are all strictly positive. Then, by totally differentiating the two margins with respect to r_d and applying Kramer's rule, we obtain

$$\frac{\partial x_c}{\partial r_d} = \frac{\partial x_d}{\partial r_d} = \frac{1}{a(x_{d,}x_c)(1 + b(x_{d,}x_c)) + b(x_{d,}x_c)c(x_{d,}x_c)} > 0 \ .$$

An increase of the money-market rate raises both thresholds. The intuition is straightforward and goes as follows. When the return on the alternative investment increases, financiers become more choosy with respect to entrepreneurial projects and require higher returns both to start a lending relationship and to keep an existing one alive.

Now consider the effect of a reduction of the liquidation cost T. Such changes could be interpreted as improvements to the bankruptcy law that allowed financiers to recover their investment more swiftly and cheaply. Again, by totally differentiating the two margins with respect to T and applying the Kramer's rule, we obtain

$$\begin{aligned}\frac{\partial x_c}{\partial T} &= \frac{a(.)\lambda\beta - (\rho+\xi)c(.)}{a(.)(1+b(.))+b(.)c(.)} =? \\ \frac{\partial x_d}{\partial T} &= -\frac{[1+b(.)](\rho+\xi)+b(.)\lambda\beta}{a(.)(1+b(.))+b(.)c(.)} < 0\end{aligned}$$

As expected the exit threshold decreases with T. Hence, improvements to the bankruptcy law would reduce the amount of "stuck liquidity" facilitating the relocation of funds from project that are no longer profitable to new investment opportunities. However, the relationship between T and the entry threshold is ambiguous. The net effect is the result of the negative effect of T on surplus, that would raise x_c, and the indirect effect on the financier's immediate gain, through Δ, that decreases x_c. It can be shown that the sign of the derivative is the same as for $\beta - (1 - F(x_d))$.

Finally, consider changes in B, the loss suffered by financiers when they are hit by a liquidity shock while their funds are invested in illiquid projects. Following the same procedure as above we obtain

$$\frac{\partial x_c}{\partial B} = \frac{\partial x_d}{\partial B} = \frac{\xi}{a(.)(1+b(.))+b(.)c(.)} > 0 \ .$$

Not surprisingly, an increase in B raises both thresholds. As the loss associated to a liquidity shock increases, financiers require a higher premium to hold illiquid assets relative to the fully liquid money-market investment.

6 Aggregate Stochastic Shocks and Simulations

This section shows that our theoretical framework is consistent with most dynamic characteristics of gross credit flows presented in the empirical section of the paper. To this purpose, this section extends the stationary model solved in the previous sections into a fully dynamic stochastic model with aggregate shocks, which we choose to model as stochastic changes in the financiers' outside option. However, we first provide an intuitive explanation of why our model is consistent with the cyclical behavior of gross flows.

Credit contraction is governed by the endogenous destruction margin, x_d and the exogenous reallocative shocks ξ. Formally, the credit contraction rate is defined as

$$cc = \lambda F(x_d) + \xi \tag{24}$$

where $\lambda F(x_d)$ is the rate at which financial relationships are interrupted. Credit expansion is governed by the probability that an idle project has instantaneous return grater than the entry margin. From the discussion in the previous section it is clear that such probability depends from the ratio of good to bad idle projects, and its formal expression reads

$$prob[V(x) > V(x_c)] = \frac{v_g}{v_g + v_b}$$

where v_g and v_b are defined by the balance conditions (21) and (22). The credit expansion rate reads

$$ce = \alpha(\frac{v_g}{v_g + v_b})[\frac{1-c}{c}]. \tag{25}$$

Let us consider what happens in the aftermath of a change in r_d. Larger r_d values lead to a lower stock of financial contracts c, thus indicating a fall in *net* flows. Our main interest is linked to the behavior of *gross* flows. Our comparative static result has shown that larger values of r_d increase the destruction margin x_d. Thus, credit destruction increases and moves countercyclically. Conversely, a larger r_d makes financiers "more choosy", and reduces the proportion of good idle projects in the economy leading, through equation (25), to lower credit expansion. Thus, credit expansion moves with the aggregate stock of credit.

Further inspection of equations (25) and (24) suggests that our model implies interesting dynamic asymmetries. First, ce is likely to be a relatively persistent process, since its dynamic behavior is linked to that of aggregate credit. Second, credit contraction is a jumping variable. Consider what happens when there is an increase in r_d. Equation (24) suggests that there is an immediate burst in credit contraction, since the reservation value jumps, and all financial relationships between the new and the old reservation values are liquidated. Thereafter, credit contraction stays high, but at a lower value than the one reached in the aftermath of the change in r_d. This burst in credit contraction, followed by lower values of credit contraction, suggests that credit contraction is likely to be a concentrated process. The next section shows that a dynamic version of our model is able to reproduce a correlogram similar to that featured by the aggregate gross credit flows. Note that the dynamic behavior of credit contraction does not find counterpart in the behavior of credit expansion in the aftermath of a fall in r_d. Indeed, inspection of equation (25) suggests that credit expansion rises, and thereafter continues to increase, albeit at a slower rate. As a consequence, credit expansion should not feature the concentration property, in a way consistent with what was observed in the empirical exercise.

6.1 Aggregate Stochastic Shocks

We now looks at the dynamics of gross flows in a more formal way. This requires solving an aggregate dynamic stochastic version of the stationary model solved in the previous sections. To this end, we need to consider an explicit aggregate driving force, which we assume to be a stochastic change in the financiers' outside option r_d. In this section, we assume that aggregate conditions move stochastically across n states, indexed by the level of the money market rate r_d^i with $r_d^i > r_d^{i+1}$. Aggregate shocks are described by the elements π_{ij} of a nxn stochastic matrix that keeps track of the probability that the financiers' outside option jumps from state i to state j. From the analysis of the previous section, it is clear that for each value of the outside option r_d^i, the system is characterized by two reservation values $[x_c^i$ $x_d^i]$ with $x_c^i < x_d^i$. In the appendix, we sketch the methodology for solving for pairs x_c^i x_d^i, $i = 1....n$.

The comparative static results of the previous section let us infer that, in general, since $r_d^i > r_d^{i+1}$, $x_c^i > x_c^{i+1}$ and $x_d^i > x_d^{i+1}$. Further, for analytical simplicity we assume that $x_d^n < x_c^1$ so that the $2n$ reservation productivities sort along the distribution F in a monotonic fashion, with all the exit margin preceding the entry margins. This conjecture must then be confirmed numerically in equilibrium. Since we assume that aggregate shocks are anticipated, we need to spell out a set of aggregate state contingent value functions for the various states. If we indicate with $C^i(x)$ the value to the financier of a contract with idiosyncratic return x when the aggregate state is i, its value function reads

$$\begin{aligned}(\eta + \Sigma_i \pi_{ij})\, C^i(x) &= r^i(x) + \lambda \int_{\underline{x}}^{\overline{x}} \max\left[C^i(z); D^i - T\right] dF(z) - \xi B + \\ &\quad + \Sigma_i \pi_{ij} \max\left[C^j(x); D^j - T\right] \;, \end{aligned} \tag{26}$$

where, for analytical simplicity, we have indicated $\eta = \rho + \lambda + \xi$. In equation (26) the aggregate stochastic shocks introduce an additional source of credit contraction, since in the aftermath of a change in r_d, the financier may find optimal to sever the relationship and put its funds in a money market deposit. The value of the latter, when aggregate conditions are i reads

$$(\eta + \Sigma_i \pi_{ij}) D^i = r_d^i + \alpha \int_{\underline{x}}^{\overline{x}} \max \left[C(z) + \Delta; D \right] dF(z) + \Sigma_i \pi_{ij} D^j,$$

where the aggregate realization of the shock does not imply any new element of choice for the financiers, since they simply alter the value of investing in the deposit. Similarly, the value function for an idle entrepreneur $V^i(x)$ reads

$$(\eta + \Sigma_i \pi_{ij}) V^i(x) = \alpha \max \left[J(x) - \Delta; V(x) \right] + \lambda \int_{\underline{x}}^{\overline{x}} V(z) dF(z) + \Sigma_i \pi_{ij} V^j(x)$$

while the value of a financed project $J^i(x)$ reads

$$\begin{aligned}(\eta + \Sigma_i \pi_{ij}) J^i(x) &= x - r^i(x) + \lambda \int_{\underline{x}}^{\overline{x}} \max \left[J(z); V(z) \right] dF(z) + \xi V(x) + \\ &+ \Sigma_i \pi_{ij} \max \left[J^j(x); V^j(x) - T \right],\end{aligned}$$

where the latter capital gain term reflects the fact that upon the realization of the aggregate shock the entrepreneur may be better off idle. Proceeding as in section 4, there will be now two surpluses for each aggregate state i, $S_o^i(x)$ and $S_1^i(x)$, where, as in the steady state, the difference between the two surpluses is a simple function of the liquidation cost T. The $2n$ cut off are obtained as a solution of a non linear system of $2n$ equations $S_o^i(x_c^i) = 0$, $S_1^i(x_d^i) = 0$ for $i = 1...n$. The appendix sketches a two steps procedure for obtaining such solution.

6.2 Numerical Simulations

In the rest of this section, we present a numerical simulation of the aggregate stochastic dynamic version of the model. This forces us to work with discrete time, and to make a specific set of assumptions on the sequence of events. In what follows, we assume that aggregate conditions are determined at the beginning of each period and stay constant throughout. Thus, following the realization of the aggregate shock, the entry and the exit margins are determined. At this points new matches are formed and unprofitable matches are severed.

Finally, the new stocks are determined. If c_{t-1}^{j} is the stock of credit at end of period $t-1$ with aggregate conditions j, we indicate with $x_{d,t}^{i}$, and with $x_{c,t}^{i}$ the reservation values at time t, after the aggregate conditions i are realized. Credit expansion reads

$$ce_t = \alpha \frac{v_{g,t}^{i}}{v_{g,t}^{i} + v_{b,t}^{i}} \frac{1 - c_{t-1}^{j}}{c_{t-1}^{j}}$$

where the proportion of good idle projects depends only on the productivity relevant at time t,$(x_{d,t}^{i}, x_{c,t}^{i})$. Credit contraction reads

$$cc_t = \xi + \lambda F(x_{d,t}^{i}) + \phi_t \int_{x_{d,t-1}^{j}}^{x_{d,t}^{i}} j(z)dz \tag{27}$$

where ϕ_t is an indicator function that takes the value of 1 if $x_{d,t}^{i} > x_{d,t-1}^{j}$ and 0 otherwise. This last term is one of the key mechanism of our theoretical framework and reflects the one off adjustment of credit contraction in the aftermath of an adverse aggregate shock. In equation (27), $j(z)$ is a measure of the financial contracts at productivity z, and its expression is governed by the dynamic counterpart of the balance flow equation presented in the previous section.

In the simulations, to make the mechanism of our model as transparent as possible, we assume that there are only two states: aggregate conditions switch between r^1 and r^2, where state 1 indicates the "tight" regime and state 2 the "easy" regime. The rest of the parameters are indicated in Table 7. Chart 7 shows a time profile of two hundreds periods of credit expansion and contraction. Spikes in credit contraction are observed in few periods, while credit expansion appears a much more persistent process, with increases in expansion followed by a slow adjustment process back to the steady state values.

Now, let us compare these simulated series with the gross flows constructed in the empirical section of the paper. First, Table 7 shows that credit expansion (contraction) move procycically (countercyclically) in a way similar to that observed in gross and net credit flows. Furthermore, the correlation between credit expansion and contraction is negative, although lower than the correlation between the observed gross flows and net flows. Second, the most important results of our simulations, concerning the asymmetric dynamic behavior between credit expansion and contraction. Figures 8 and 9 plot the correlogram of credit expansion and contraction from our simulated series. Whereas the autocorrelation of credit expansion is positive for 13 lags, the dynamic structure of credit contraction suggests the existence of concentration. After the first few lags, the autocorrelation of credit contraction turns negative, and stays negative throughout. Thus, in the model, credit contraction is a

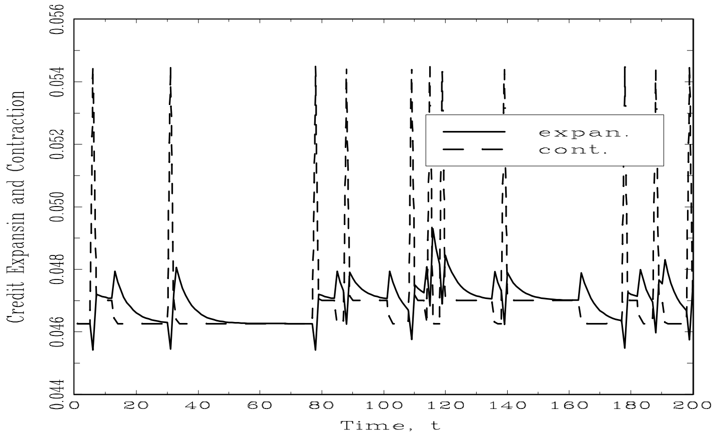

Figure 7: Credit Expansion and Credit Contraction from the model.

concentrated series. This asymmetric dynamic behavior suggests that our theoretical analysis is fully consistent with the dynamic behavior of gross flows in the U.S. banking system.

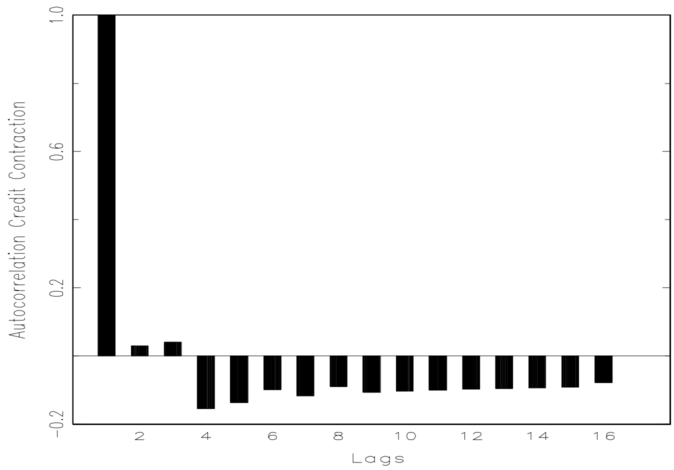

Figure 8: Autoccorelation Function of Credit Contraction from the Model

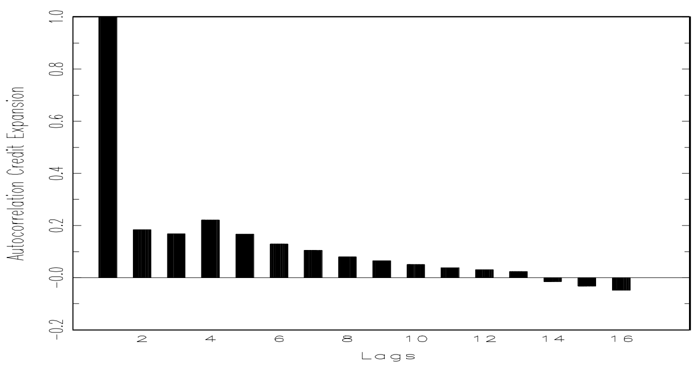

Figure 9: Autocorrelation Function of Credit Expansion

Table 7: Simulation Statistics

Variables	Notation	Value
matching probability	α	0.06
pure discount rate	ρ	0.04
reallocative shock	ξ	0.04
arrival rate idiosyncratic shock	λ	0.10
arrival rate liquidity shock	ψ	0.04
liquidation cost	T	4.00
bankruptcy cost	B	10.00
int. rate tight	r^1	0.08
int. rate easy	r^2	0.06
Aggregate Statistics		
Average Net Flows	Net	-0.0001
Average Expansion	Pos	0.0470
Standard Dev. Expansion	σ_{POS}	0.0006
Average Contraction	Neg	0.0470
Standard Dev. Contraction	σ_{NEG}	0.0019
Corr. Net Pos		0.61
Corr. Net Neg		-0.97
Corr. Pos Neg		-0.39

F is uniform with $\overline{x} = 2$ and $\underline{x} = 0$

Source: Authors' calculation

7 Discussion and Conclusions

We have presented theory and evidence on gross credit flows, the simultaneous process of credit expansion and contraction associated with a net change in the aggregate level of credit. Empirically, we have summarized heterogeneity in the U.S. banking system by constructing aggregate measures of credit expansions and contractions. Our empirical exercise has shown that sizable gross credit flows coexist at any phase of the cycle, even within narrowly defined regional units and within small partition of the bank size distribution. Furthermore, credit contraction is a concentrated series, which implies that a burst in one period is followed by prolonged periods of low values. Theoretically, we have proposed a matching model in which financiers have to spend time and resources to expand credit to heterogeneous entrepreneurs. The combination of idiosyncratic and aggregate shocks, and asymmetric adjustment between credit expansion and contraction appears consistent with the characteristics of aggregate credit flows. In this section, we emphasize the relevance of our theoretical and empirical frameworks, and we suggest few directions for future research.

The concentration of aggregate credit contraction has important policy implications. If the aggregate shocks to the money-market rate emphasized by our theoretical analysis are interpreted as policy innovations, our framework provides new insights on the dynamic effects of monetary policy on aggregate credit. Indeed, the propagation of interest rate changes over the banking system depends crucially on the reaction of two different margins: the banking system's ability to find and screen new projects, and the banking system's ability to recall existing loans. Since these processes are inherently different, bank lending will respond asymmetrically to positive and negative innovations. Hence, the design and implementation of monetary policy should take into account the asymmetric lag structure between policy contractions and policy expansions, and react consequently. In particular, as a significant part of the stock of credit may consist of unused credit lines, it is likely that banks are able to contract credit much faster than they can expand it. Indeed, such dynamic asymmetry appears fully consistent with the concentration of aggregate credit contraction. In addition, Dell'Ariccia and Garibaldi (1998) estimate the impact of monetary policy shocks on net flows, and find evidence of asymmetry: aggregate credit reacts more sharply to policy contractions than to policy expansions.

Our theoretical analysis suggests that reallocative shocks to liquidity can be an additional source of dynamics in aggregate credit. In our model, liquidity is reallocated across the banking system at the instantaneous rate ξ. Following the realization of a liquidity shock ξ, the liquidity allocated to a financier ceases to be available, and forces the financier into bankruptcy whenever such liquidity is invested in entrepreneurial projects. Thus, the realization of reallocative liquidity shocks may raise credit contraction and, with some delays, may raise credit expansion. Since credit expansion is time consuming, aggregate credit temporarily falls. Furthermore, the size of the impact of reallocative shocks on aggregate credit depends on the percentage of liquidity invested in credit relationships, and is thus likely to be more important in periods of fast net credit expansion. In the real world, interbank markets are likely to alleviate the burden of reallocative shocks, by giving to financiers that have illiquid, but solvent, positions the opportunity to borrow against their assets. However, we know that some banks, and those of lower size in particular, have not unlimited access to the interbank market. Thus, we are convinced that the source of aggregate credit dynamics emphasized by our paper is important, and future research should try to quantify its empirical relevance.

In addition, as long as the interbank market provides only imperfect insurance against liquidity shocks, our theoretical analysis suggests also that reallocative shocks lead to inefficient separations. Following a ξ shock, otherwise active, entrepreneurs cease production and remain idle, and are forced to undergo a new round of screening. In such circumstances, there would be obvious room for an exogenous injection of liquidity, and a roll-over of the financial relationships that came to an abrupt halt.

Future research should try to address efficiency considerations related to gross credit flows. The magnitude of gross credit reallocation depends clearly on the average duration of financial contracts, and it is probably difficult to establish a link between rates of credit reallocation and efficiency measures in the banking system. However, we may still ask whether, for a given average duration of financial contracts, larger rates of credit reallocation are indeed desirable. Our model suggest that liquidation costs limit the banks' ability to recall the invested capital, and leads to lower credit reallocation in equilibrium. Thus, measures aimed at speeding up the liquidation process (e.g. improvements in bankruptcy laws, increased enforcement of property rights) are likely to be welfare improving. Other things equal, the lower is the liquidation costs, the higher is credit reallocation and the higher is the ability of the banking system to reallocate liquidity from less productive to more productive projects. One immediate consequence is for the analysis of banking crises. Countries characterized by higher liquidation costs will suffer more when hit by aggregate and/or reallocative shocks. Indeed, the amount of liquidity allocated to unproductive projects would be higher, with direct consequences on banks' health and aggregate output.

Appendix

A Equilibrium Existence

The surplus function implied by

$$S_0(x) = \frac{1}{\rho+\lambda+\xi}\left\{x - r - \lambda\beta T - \xi B + \lambda\int_{\underline{x}}^{\overline{x}} \max\left[S_0(z)+T;0\right]dF(z)\right. \tag{28}$$

$$\left. -\alpha\beta\int_{\underline{x}}^{\overline{x}} \max\left[S_0(z);0\right]dF(z) - \alpha(1-\beta)\max[S_0(x)\right\};0\right] . \tag{29}$$

does not satisfy the Blackwell conditions for a contraction. However, we can prove equilibrium existence and uniqueness directly by applying the Contraction Mapping Theorem. Equation (28) implies as mapping function $T : S \to S$; where S is a space of bounded real functions with a metric d defined by

$$d(W,Y) = \|W-Y\| = \int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\,dF(z). \tag{30}$$

In addition, define a sort of discount factor, $\psi = \frac{\lambda+\alpha}{\rho+\lambda+\xi}$, depending on the actual discount factor, ρ, and other parameters of the model. We can, then, state the following proposition that guarantees the existence of a unique fixed point:

Proposition 1 *If $\psi < 1$, for any W, $Y \in R$, $\exists\ \varphi \in (0,1)$ such that $\|T(W) - T(Y)\| \leq \varphi \|W - Y\|$.*
Proof. *¿From Eq. (28) we can write for any x*

$$\begin{aligned}|T(W(x)) - T(Y(x))| &= \frac{1}{\rho+\lambda+\xi}\left|\lambda \int_{\underline{x}}^{\overline{x}} \{\max[W(z)+T;0] - \max[Y(z)+T;0]\}\, dF(z)\right. \\ &+\alpha\beta \int_{\underline{x}}^{\overline{x}} \{\max[Y(z);0] - \max[W(z);0]\}\, dF(z) \\ &\left.+\alpha(1-\beta)\{\max[Y(x);0] - \max[W(x);0]\}\right|\end{aligned}$$

by applying the triangular inequality, we obtain

$$\begin{aligned}|T(W(x)) - T(Y(x))| &\leq \frac{\lambda}{\rho+\lambda+\xi}\left|\int_{\underline{x}}^{\overline{x}} \{\max[W(z)+T;0] - \max[Y(z)+T;0]\}\, dF(z)\right| \\ &+\frac{\alpha\beta}{\rho+\lambda+\xi}\left|\int_{\underline{x}}^{\overline{x}} \{\max[Y(z);0] - \max[W(z);0]\}\, dF(z)\right| \\ &+\frac{\alpha(1-\beta)}{\rho+\lambda+\xi}\left|\max[Y(x);0] - \max[W(x);0]\right|.\end{aligned}$$

Now consider the first term on the rhs. By applying the triangular inequality again we can write

$$\left|\int_{\underline{x}}^{\overline{x}} \{\max[W(z)+T;0] - \max[Y(z)+T;0]\}\, dF(z)\right| \leq \int_{\underline{x}}^{\overline{x}} |\max[W(z)+T;0] - \max[Y(z)+T;0]|\, dF(z)\ .$$

Furthermore, we know that $\forall z$

$$|\max[W(z)+T;0] - \max[Y(z)+T;0]| \leq |(W(z)+T) - (Y(z)+T)| = |W(z) - Y(z)|$$

that means

$$\left|\int_{\underline{x}}^{\overline{x}} \{\max[W(z)+T;0] - \max[Y(z)+T;0]\}\, dF(z)\right| \leq \int_{\underline{x}}^{\overline{x}} |W(z) - Y(z)|\, dF(z)\ ;$$

similarly, for the second term we have

$$\left|\int_{\underline{x}}^{\overline{x}} \{\max[Y(z);0] - \max[W(z);0]\}\, dF(z)\right| \leq \int_{\underline{x}}^{\overline{x}} |W(z) - Y(z)|\, dF(z)\ ;$$

and for the third term,

$$|\max[Y(x);0] - \max[W(x);0]| \leq |W(x) - Y(x)|\,.$$

Finally, putting the three terms back together, we can write

$$
\begin{aligned}
\|T(W)-T(Y)\| \leq & \int_{\underline{x}}^{\overline{x}} \left(\frac{\lambda}{\rho+\lambda+\xi} \int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\, dF(z) \right. \\
& + \frac{\alpha\beta}{\rho+\lambda+\xi} \int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\, dF(z) \\
& \left. + \frac{\alpha(1-\beta)}{\rho+\lambda+\xi} |W(z)-Y(z)| \right) dF(z) \ ;
\end{aligned}
$$

as $\int_{\underline{x}}^{\overline{x}} dF(z) = 1$*, it follows directly that*

$$
\begin{aligned}
\|T(W)-T(Y)\| \leq & \frac{\lambda}{\rho+\lambda+\xi} \int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\, dF(z) \\
& + \frac{\alpha\beta}{\rho+\lambda+\xi} \int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\, dF(z) \\
& + \frac{\alpha(1-\beta)}{\rho+\lambda+\xi} \int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\, dF(z) \ ,
\end{aligned}
$$

and substituting from (30),

$$
\begin{aligned}
\int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\, dF(z) \leq & \frac{\lambda}{\rho+\lambda+\xi} \int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\, dF(z) \\
& + \frac{\alpha\beta}{\rho+\lambda+\xi} \int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\, dF(z) \\
& + \frac{\alpha(1-\beta)}{\rho+\lambda+\xi} \int_{\underline{x}}^{\overline{x}} |W(z)-Y(z)|\, dF(z) \ .
\end{aligned}
$$

Then a sufficient condition for T to be a contraction mapping is

$$
\frac{\lambda}{\rho+\lambda+\xi} + \frac{\alpha\beta}{\rho+\lambda+\xi} + \frac{\alpha(1-\beta)}{\rho+\lambda+\xi} < 1,
$$

or

$$
\alpha < \rho + \xi.
$$

q.d.e. ■

B Monotonicity of the Surplus Functions

Proposition 2 *If $f(x) > 0$ for any $x \in [\underline{x}, \overline{x}]$, the surplus function $S_0(x)$ and $S_1(x)$ are increasing in x.*
Proof. *We can write*

$$\begin{aligned} S_0(x) &= C(x) - D + J(x) - V(x) \ , \\ S_1(x) &= S_0(x) + T \ , \end{aligned}$$

and substituting using Equations (12), (13), (14), and (15), becomes

$$\begin{aligned} (\rho + \lambda + \xi) S_0(x) &= x - r - \lambda T - \xi B + \lambda \int_{\underline{x}}^{\overline{x}} \max\left[S_1(z); 0\right] dF(z) \\ &\quad - \alpha\beta \int_{\underline{x}}^{\overline{x}} \max\left[S_0(z); 0\right] dF(z) \\ &\quad - \alpha(1-\beta) \max[S_0(x); 0]. \end{aligned}$$

¿From which it follows immediately that

$$S_0'(x) = \frac{1}{(\rho + \lambda + \xi)} \qquad \forall x \mid S_0(x) < 0$$

and

$$S_0'(x) = \frac{1}{(\rho + \lambda + \xi + \alpha(1-\beta))} \qquad \forall x \mid S_0(x) \geq 0$$

which shows that the surplus $S_0(x)$ an increasing function of x. Analogously, for $S_1(x)$ we have

$$S_1'(x) = \frac{1}{(\rho + \lambda + \xi)} \qquad \forall x \mid S_1(x) < T$$

and

$$S_1'(x) = \frac{1}{\rho + \lambda + \xi + \alpha(1-\beta)} \qquad \forall x \mid S_1(x) > T$$

q.d.e. ■

C Stochastic Dynamic Model

The system is described by $2n$ value functions for the surpluses $S_o^i(x)$ and $S_1^i(x)$ whose expression read

$$\begin{aligned} (\eta + \Sigma_i \pi_{ij}) S_o^i(x) &= x - r_d^i - \xi B - \lambda T + \lambda \int_{\underline{x}}^{\overline{x}} \max[S_1^i(z); 0] dF(z) \\ &\quad - \alpha\beta \int_{\underline{x}}^{\overline{x}} \max[S_0^i(z); 0] dF(z) + \Sigma_i \pi_{ij} \max[S_o^j(x); 0] \end{aligned} \quad (31)$$

and

$$(\eta+\Sigma_i\pi_{ij})S_1^i(x) = x-r_d^i-\xi B+(\rho+\xi)T+\lambda\int_{\underline{x}}^{\overline{x}}\max[S_1^i(z);0]dF(z) \quad (32)$$
$$-\alpha\beta\int_{\underline{x}}^{\overline{x}}\max[S_0^i(z);0]dF(z)+\Sigma_i\pi_{ij}\max[S_1^j(x)+T;0]$$

for $i=1...n$. The $2n$ reservation productivity are defined as $S_1^i(x_d^i)=0$ and $S_0^i(x_c^i)=0$ for $i=1...n$. Differentiating the previous expressions with respect to x shows that each surplus function is a piece wise linear increasing function of x, with n kinks corresponding to the values of the reservation productivities. The first step of the procedure is to solve the linear system obtained by differentiating equations (32) and (31) with respect to x. The solution to the linear system yields the partial derivative $\frac{\partial S_1^i}{\partial x}=d_{k,1}$, and $\frac{\partial S_1^i}{\partial x}=d_{k,o}$ for $k=1..n$. The expected values in the surplus function can then be computed as

$$\int_{\underline{x}}^{\overline{x}}\max[S_1^i(z);0]dF(z)=\int_{x_d^1}^{x_d^n}S_1^idF(z)=\sum_{k=1}^{n}d_{k,1}\int_{x_d^k}^{x_d^{k+1}}(1-F(z))d(z)$$

and

$$\int_{\underline{x}}^{\overline{x}}\max[S_o^i(z);0]dF(z)=\int_{x_c^1}^{x_c^n}S_o^idF(z)=\sum_{k=1}^{n}d_{k,o}\int_{x_c^k}^{x_c^{k+1}}(1-F(z))d(z)$$

The second step in the procedure requires solving for x_d^i-$x_c^i$the non linear system of $(2n)$ equation given by

$$0 = x_d^i-r_d^i-\xi B+(\rho+\xi)T-\xi B+\lambda\sum_{k=1}^{n}d_{k,1}\int_{x_d^k}^{x_d^{k+1}}(1-F(z))d(z)$$
$$-\alpha(1-\beta)\sum_{k=1}^{n}d_{k,1}\int_{x_c^k}^{x_c^{k+1}}(1-F(z))d(z)+\Sigma_i\pi_{ij}\max[S_1^j(x_d);0]$$

and

$$0 = x_c^i-r_d^i-\xi B-\lambda T-\xi B+\lambda\sum_{k=1}^{n}d_{k,o}\int_{x_d^k}^{x_d^{k+1}}(1-F(z))d(z)$$
$$-\alpha(1-\beta)\sum_{k=1}^{n}d_{k,1}\int_{x_c^k}^{x_c^{k+1}}(1-F(z))d(z)+\Sigma_i\pi_{ij}\max[S_o^j(x_c);0]$$

for all $i=1...n$

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