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Pedro L Marín and Antón García-Díaz

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Pedro L Marín, Universidad Carlos III de Madrid and CEPR
Antón García-Díaz, Universidad Carlos III de Madrid

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Strategic Bidding in Electricity Pools with Short-Lived Bids: An Application to the Spanish Market*

We generalize von der Fehr and Harbord's (1993) multi-unit auction model for the case of a deterministic demand allowing for any technology mix and elastic demand in order to account for demand-side bidding. We obtain a general characterization of the equilibrium and show that the Cournot model overestimates market power in pool markets. We simulate the Spanish electricity pool and show that price-cost margins substantially increased with the 1996 merger that took the industry from a six firm structure to its current four firm structure. This is almost equivalent to a nearly symmetric duopoly. The introduction of demand-side bidding is not likely to change this situation.

JEL Classification: K23, L13 and L94
Keywords: bids, electricity pools and market power

Pedro L Marín
Departamento de Economía
Universidad Carlos III de Madrid
C./ Madrid, 126
28903 Getafe (Madrid)
SPAIN
Tel: (34 91) 624 9875
Fax: (34 91) 624 9646
Email: marin@eco.uc3m.es

Antón García-Díaz
Departamento de Economía
Universidad Carlos III de Madrid
C./ Madrid, 126
28903 Getafe (Madrid)
SPAIN

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NON-TECHNICAL SUMMARY

Electricity pools are at the core of electricity deregulation processes throughout the world. After the pioneering case of England and Wales several other spot wholesale markets for electricity were created in order to introduce competition into the generation of electricity. Argentina, California, the Scandinavian region, Spain and the states of Victoria and New South Wales in Australia, currently have electricity spot markets in operation, while several are in the process of creation.

The common characteristic of all electricity pools is that generators make bids to supply a given amount of electricity at a certain price. A market operator orders these bids from highest to lowest constructing the market offer curve, and the intersection of this offer curve with a demand curve yields a price at which all trade occurs. An important characteristic of pool markets is the period of time for which bids are fixed and cannot be altered. In certain pools, such as the Argentinian pool, firms place bids every six months, in the England and Wales pool the bidding is daily, while in Spain, California and Nordpool firms submit bids for each hour. In this Paper we develop a methodology for the analysis of competition in electricity spot markets where firms make bids valid for short periods of time. We will consider bid life to be short when demand does not vary significantly during the period of time for which the bid is valid, i.e. when firms bid facing a certain demand.

In a seminal paper, Green and Newbery (1992) analyse the behaviour of firms in the England and Wales electricity pool. Their analysis assumes that bids are fixed for a period during which demand shifts in a given interval. Accordingly, they assume that firms strategic variable is their supply function and apply this methodology to simulate the England and Wales pool. However, Klemperer and Meyer (1989) show that in the absence of uncertainty any price above marginal cost can be sustained in a supply function equilibrium. Accordingly, in pools where firms make bids for periods of time in which demand varies very little the supply function approach has very limited predictive power.

Von der Fehr and Harbord (1993) model electric pools as multi-unit auctions. In our Paper we generalize Von der Fehr and Harbord’s (1993) approach for the case of deterministic demand. In particular, we allow for multiple asymmetric firms with increasing step cost functions and for a downward sloping demand function that represents the existence of demand-side bidding.

We do not model a double auction where consumers’ behaviour is determined endogenously but the model includes a positively sloped demand. This amounts to assuming price-taking behaviour by consumers. This is justified by the small size of eligible consumers with respect to total demand and the fact that distributors, although large in aggregate, supply customers that face a fixed tariff and therefore are not price responsive.

We obtain a characterization of the pure strategy equilibria for this model and find that firms' asymmetries in size and technology mix significantly affect price-cost margins. In particular, very strong asymmetries lead to a single equilibrium price with a dominant firm model flavour where small firms behave competitively, while the market leader maximizes profits given its residual demand. Also symmetric market structures lead, in general, to a single equilibrium price but to lower average price-cost margins. Intermediate situations lead to multiple equilibrium prices since any of several different firms can adopt a dominant role and set the market price.

We implement a simple algorithm to identify equilibria in a simulation for the Spanish pool. In particular, we simulate our model using 1998 data from the Spanish electricity market in order to measure market power under the current market structure and to analyse the effect of the 1996 merger that took the industry from a six to a four firm structure.

The main advantage of this approach to simulate firms' behaviour is that it exploits all the information available on how firms interact in the pool, that is, it accounts for the pool market institution. It is reasonable to believe that this will lead to closer predictions of the generating firm's strategic behaviour. For instance, when comparing our results with those derived from the Cournot model, we observe that Cournot yields significantly higher mark-ups except when demand is very elastic or the industry is very fragmented, two situations that are extremely unlikely in the electricity industry.

The results of our simulation for the Spanish case show that market power measured by price-cost margins substantially increased with the 1996 merger that took the industry from a six firm structure to its current four firm structure. In fact, our simulation shows that the current situation is equivalent in terms of market power to a nearly symmetric duopoly, with the three smallest firms merging. We also show that the introduction of elegibility and demand-side bidding may not be enough to curve market power in the Spanish spot market for electricity.

Our simulation estimates variations in market power following changes in market and cost structure. This cannot be empirically contrasted since they have not taken place, or have taken place before the pool was in operation. However, some of the results of the simulation for the current four firm structure could be contrasted with empirical observations, such as the identity of the firm that determines the system marginal price and the individual firm's hourly production shares. Unfortunately, it is not possible to compare the results of our simulation with current data for two reasons. First, while the

Spanish pool started operation in 1998, it is going through a transition period designed to allow firms to recover their stranded costs. We do not explicitly model this stranded cost recovery mechanism but rather analyse firms' behaviour in the pool once this transition period is over. Second, lack of data availability. The market operator only publishes the pool price and the total quantity dispatched for each hour. Agents' individual bids, sales and purchases, as well as the identity of the firm that determines the system marginal price are not publicly available.

1. Introduction

Electricity pools are at the core of electricity deregulation processes throughout the world. After the pioneering case of England and Wales several other spot wholesale markets for electricity where created in order to introduce competition into the generation of electricity. Argentina, California, the Scandinavian region, Spain and the states of Victoria and New South Wales in Australia, currently have electricity spot markets in operation, while several are in the process of creation.

The common characteristic of all electricity pools is that generators make bids to supply a given amount of electricity at a certain price. A market operator orders these bids from highest to lowest constructing the market offer curve, and the intersection of this offer curve with a demand curve yields a price at which all trade occurs. An important characteristic of pool markets is the period of time for which bids are fixed and cannot be altered. In certain pools, such as the Argentinian pool, firms place bids every six months, in the England and Wales pool the bidding is daily, while in Spain, California and Nordpool firms submit bids for each hour.[1] In this paper we develop a methodology for the analysis of competition in electricity spot markets where firms make bids valid for short periods of time. We will consider bid life to be short when demand does not vary significantly during the period of time for which the bid is valid, i.e. when firms bid facing a certain demand.

In a seminal paper, Green and Newbery (1992) analyze the behavior of firms

[1] Pools may differ in several other features: they may or not allow for demand side bids, they may incorporate mechanisms to remunerate firms for non variable costs (such as start-up and capacity costs), as well as to take into account technological restrictions (see Von der Fehr and Harbord 1997).

in the England and Wales electricity pool. These authors assume that firms have a continuously differentiable cost function and submit continuously differentiable bid functions to the pool and apply Klemperer and Meyer's (1989) results to obtain a range of equilibrium supply functions. Their analysis assumes that bids are fixed for a period during which demand shifts in a given interval. The equilibrium prices range from Cournot to perfect competition. They apply this methodology to simulate the England and Wales pool assuming that firms coordinate on the highest pricing equilibrium. Klemperer and Meyer (1989) show that in the absence of uncertainty any price above marginal cost can be sustained in a supply function equilibrium. Accordingly, in pools where firms make bids for periods of time in which demand varies very little the supply function approach has very limited predictive power.

Alternatively, Von der Fehr and Harbord (1993) model electric pools as multi-unit auctions where generating firms face constant marginal costs up to capacity and demand is inelastic. They analyze a specific example with two firms for deterministic and uncertain demand cases and extract some general conclusions. In this paper we generalize Von der Fehr and Harbord's (1993) approach for the case of deterministic demand. In particular, we allow for multiple asymmetric firms with increasing step cost functions. We also allow for a downward sloping demand function that represents the existence of demand side bidding. However, we do not model a double auction where consumers' behavior is determined endogenously. This amounts to assuming price-taking behavior by consumers. For the Spanish case, this is justified by the small size of elegible consumers with respect to total demand, and the fact that distributors, although large in aggregate, supply customers that face a fixed tariff and therefore are not price responsive.[2]

We obtain a characterization of the pure strategy equilibria for this model and find that firms' asymmetries in size and technology mix significantly affect price-cost margins. In particular, very strong asymmetries lead to a single equilibrium price with a dominant firm model flavor where small firms behave competitively, while the market leader maximizes profits given its residual demand. Also symmetric market structures lead, in general, to a single equilibrium price but to lower average price-cost margins. Intermediate situations lead to multiple equilibrium prices since any of several different firms can adopt a dominant role and set the market price.

We implement a simple algorithm to identify equilibria in a simulation for the Spanish pool. The object of the simulation is to identify problems of market power in the generation of electricity in Spain and in particular to quantify the effect of the 1996 merger that took the industry from a six firm structure to a four firm structure.[3]

To our knowledge, the only previous attempts to simulate firms' behavior in electricity pools with deterministic demand are those of Borenstein and Bushnell

[2] In addition, vertical integration between generators and distributors is likely to reduce distributors' incentives to bid strategically, since lower pool prices would lead to lower tariffs to elegible consumers, who can buy directly from the pool or through independent suppliers. As explained in section 2, eventually all consumers will become elegible.

[3] Electricity generators face an inter-temporal decision when competing in the context of electricity pools. This is due to the existence of start-up and no-load costs. However, as noted by Kahn (1998), this inter-temporal problem has not been dealt with in any model that attempts to predict firms' equilibrium behavior in an electricity pool. Accordingly, the results of all these models can make useful predictions about firms' market power under alternative scenarios, rather than explain absolute prices. This is also the case for our model.

(1997) and Ocaña and Romero (1998) that use the Cournot model to simulate the Californian and the Spanish pools respectively. The main drawback of this analysis is that it does not exploit all the information available on how firms interact in the pool, that is, it ignores the pool market institution.[4] It is reasonable to believe that taking into account the pool auction mechanism will lead to closer predictions of the generating firms strategic behavior. When comparing our results with those derived from the Cournot model, we observe that Cournot yields significantly higher mark-ups except when demand is very elastic or the industry is very fragmented, two situations which are extremely unlikely in the electricity industry.

The results of our simulation for the Spanish case show that market power measured by price-cost margins substantially increased with the 1996 merger that took the industry from a six firm structure to its current four firm structure. In fact, our simulation shows that the current situation is equivalent in terms of market power to a nearly symmetric duopoly, with the three smallest firms merging. We also show that the introduction of elegibility and demand-side bidding may

[4] Several justifications are given in Borenstein and Bushnell (1997) and Ocaña and Romero (1998) as to why the Cournot model could be a good approximation to firm behavior in the pool. First, Green and Newbery (1992) show that Cournot is an upper bound to prices in their model of the England and Wales pool. It must be noted that in their model prices only reach the Cournot level when demand is highest, and can be considerably below on average. Second, Wolak and Patrick's (1996) argue that firms will not use prices as their strategic variable because bids that are significantly above cost will trigger a response by the regulator, and thus firms will use capacity as an strategic variable. Yet, it seems that the regulator is likely to respond to strategic use of capacity availability declarations (as it did in the England and Wales case), and that declaring a generator unavailable is a very crude mechanism for adjusting capacity given the demand variation that might exist in a 24 hour period.

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2. The Spanish generating sector: market structure and rules of the pool

In 1997 a plan for the liberalization of the Spanish electricity market was approved. It included a gradual liberalization of the generation and supply activities while transmission and distribution remained regulated.[5]

[5] Transmission refers to high tension lines while distribution refers to low tension lines. A supplier is an agent who buys electricity in the pool, pays a fee for its transmission, and sells it

Currently there are four generating firms in Spain: Endesa Group, Iberdrola, Unión Fenosa and Hidrocantábrico. Endesa Group was formed by the merger of several public firms that were gradually privatized during the nineties. The last and one of the most important mergers took place in 1996 when Endesa took control over other two important public firms: Sevillana and FECSA. Table 1 presents the capacity shares of firms and technologies in the Spanish system.

Table 1. Capacity shares of Spanish electricity generators and distributors (1997)

	Endesa G.	Iberdrola	U. Fenosa	Hidrocant.	Total
Thermal	28.7	11.8	7.0	2.9	50.4
Nuclear	8.8	8.2	1.9	0.4	19.4
Hydro	11.2	15.1	3.1	0.8	30.2
Total Generation	48.8	35.1	12.0	4.1	100.0
Distribution	41.0	39.0	16.0	4.0	100.0

Source: Comisión Nacional del Sistema Eléctrico, *Annual Report*, 1997.

Note that the industry is very concentrated, with the two largest firms controlling 83% of the generating capacity and 80% of the distribution. Another important feature is that hydro power represents more than 30% of installed capacity. Finally, notice that there is vertical integration between generators and distributors. Vertical integration is not included in the model below but it is likely to reduce the incentives of distributors to bid strategically.[6] Even though supply has been liberalized, fixed tariff consumers have to be supplied by distributors.

to a final consumer.

[6] See footnote 2.

Independent suppliers may be created in order to serve the largest consumers, referred to as eligible.

In January 1998 the Spanish electricity pool started operating.[7] The set of agents that may participate in this market are: generating firms, distributors, suppliers, and eligible consumers. The pool is organized as a double auction where agents may submit bids to the market operator both to buy and to sell energy.[8] Each day is divided into 24 hourly bidding periods. Each generating unit submits a set of price-quantity pairs. Each of these pairs is interpreted as an offer to generate that level of production at that price or greater.

On the demand side, the liberalization of the market implies that large, eligible, consumers may submit bids to buy electricity in the pool either directly or through a supplier. By October 1999, minimum annual consumption by eligible consumers will be 1 GWh, representing approximately 50% of the demand. Distributors, suppliers and eligible consumers can submit their buying bids to the pool specifying some quantity of electricity and the maximum price that they are willing to pay for it.

In each period the market operator constructs an offer and a demand curve by ranking bids according to their prices. Supply bids are ranked from lowest to highest and demand bids are ranked from highest to lowest. A market clearing algorithm is then used to determine which generating units are to produce, how much each agent can consume and the market clearing price for each period, called the system marginal price, SMP. The SMP is determined by the highest selling

[7] Currently, the pool is going through a transition period that was designed in order to allow firms to recover their stranded costs and was expected to last at least until 2003, when it would be reviewed. This process is now subject to revision by the European Commission.

[8] See Ley del Sector Eléctrico (27/11/97), and subsequent legislation.

bid that is dispatched. Any capacity offered at a price below the SMP is accepted and the generators that make offers at this price are rationed proportionately. On the demand side, any bid to buy at a price above the SMP is accepted but the set of lowest bids among them, which may be rationed proportionately.[9] This market clearing process is complicated by the several mechanisms designed to satisfy constraints that generating units may include in their offer bids.[10]

3. The model

In this section we present a model of the Spanish electricity pool as a multiple unit first price competitive auction with complete information.[11] The following notation is introduced in order to define the strategy and payoff spaces. Let $I = \{1, ..., i, ..., f\}$ be the set of agents that operate in the pool, $U = \{1, ..., u, ..., m\}$ the set of generating units, and $U_i \subset U$ the set of generating units belonging to agent i. Let c_u, k_u, and q_u be defined as u's constant unit cost, maximum generating capacity and unit output, respectively. Without loss of generality,

[9] Notice that given the discreteness of the supply function and that the SMP is determined by a selling bid, there may be excess demand at a price above the SMP.

[10] The Spanish pool allows each generating unit to add three types of constraints to the previous offers. These are minimum revenue, non-divisible offers and maximum load gradient constraints. They have been designed to ensure that generating units have non negative profits when they face start-up costs as well as to account for technological restrictions. None of these constraints is included in the analysis below since we do not consider the possibility of start-up costs in our model.

[11] First price versus second price, because the price is set by the last bid that is *accepted.* Competitive versus discriminatory because all transactions take place at the same market clearing price. Complete information because the agents' payoff functions are common knowledge.

assume $c_u \leq c_{u+1}$. We may then define a firm's marginal cost function, $MC_i(q)$, by

$$\begin{aligned} MC_i(q) &= \min_{u \in U_i} c_u \\ s.t. &\quad \textstyle\sum_{g \in E_i(u)} k_g \geq q \end{aligned}$$

where $E_i(u) = \{g : g \in U_i \text{ and } c_g \leq c_u\}$. A firm's cost function can be defined as

$$C_i(q) = \int_0^q MC_i(x)dx$$

We should note that, following Von der Fehr and Harbord (1993), this cost structure assumes that generating units have no start-up costs and constant marginal costs up to capacity.[12]

In each period, each generating unit's strategy may be represented by a nondecreasing left continuous step function with a finite number of steps. By ordering these steps for all the generating units of a firm we obtain each firm's bid function, $b_i(q) : [0, K_i] \to [0, p^{\max}]$, where $K_i = \sum_{u \in U_i} k_u$.[13] For notational convenience we will assume $b_i(0) = 0$. Analogously we may obtain the aggregate bid curve, which determines the bid of the most expensive unit necessary to produce output q, as $\bar{b}(q) : [0, K] \to [0, p^{\max}]$, where $K = \sum_{i \in I} K_i$.[14] We will assume that all firms are

[12] As it is noted by Kahn (1998), this limitation applies to all models of electricity pools that attempt to predict equilibrium behavior. As a reflection of this limitation we use our model to predict changes in price-cost margins under alternative scenarios rather than the absolute level of prices.

[13] As in Von der Fher and Harbord (1993), $p^{\max}$ may be interpreted as maximum accepted bid price in the pool, or as a price that would trigger regulatory intervention if observed.

[14] Formally $\bar{b}(q)$ is defined from the individual firms bid functions as follows,

$$\begin{aligned} \bar{b}(q) &= \min_{x \in \Re^n} \max \left(b_1(x_1), .., b_n(x_n)\right) \\ s.t. &\quad \sum_{i \in I} x_i = q \end{aligned}$$

able to produce a strictly positive amount of electricity at a cost bellow $p^{\max}$, that is, $MC_j(q) < p^{\max}$ for some positive q and for all $j \in I$.

With respect to the demand side, two types of agents may submit bids to buy electricity in the pool. On the one hand, distributors sell energy to captive consumers subject to fixed tariffs and, therefore, non responsive to the pool price. We assume that these distributors make flat bids at a given maximum price for all the electricity that their clients are expected to consume in a given period of time.

On the other hand, either eligible consumers or their suppliers may bid downward sloping demand functions reflecting their price responsive consumption. Additionally, a large consumer might bid strategically in order to exercise its market power and get lower prices from the pool. We will not consider this possibility given the fact the demand of any individual consumer will be negligible relative to total demand, and the fraction of demand that they can modulate will be even smaller. Accordingly, we will assume that eligible consumers behave as price takers. Under this assumption, demand side bidding into the pool may be represented by the aggregate demand function, $D(p)$, which is the result of adding the non responsive vertical demand of distributors and the downward sloping demand of eligible consumers or their suppliers. We will assume that this function has an inverse which we denote by $P(q)$.

The system marginal price, $p^*(b)$, is determined by the intersection of the bid

where $x = (x_1, ..., x_n)$ is a vector of firms' outputs.

curve with the inverse demand function.[15]

$$p^*(b) = \max_q \{\bar{b}(q) : \bar{b}(q) \leq P(q)\}$$

All trade in the pool takes place at the SMP, p^*. Denoting the output that generating unit i is called to produce by $Q_i(b)$, we define firm i's profits as

$$\pi_i(b) = Q_i(b)p^*(b) - C_i(Q_i(b))$$

where $b = (b_1(q), ..., b_f(q))$ denotes the firms' bid profile.[16]

Generators that make offers at the system marginal price may be rationed. In these cases, the standard rule is to ration the marginal generating units proportionally. However, this leads to a problem of non existence of equilibrium similar to that arising in a Bertrand game with asymmetric costs. Following Von der Fehr and Harbord (1993) we assume rationing is efficient, i.e., generators with a lower marginal cost are called on to produce first. This assumption may not seem innocuous as it requires that the market operator knows the generators' cost functions. However, in the appendix we show that the set of equilibria to our game approximates the set of equilibria of a game where a proportional rationing rule is applied and firms must choose their bid prices on a finite grid, which is what occurs in actual pools.

We will now characterize the pure strategy Nash equilibria of the model. Our results generalize Von der Fehr and Harbord's (1993) theoretical results under

[15] Given that the bid curve can be discontinuous the bid and inverse demand function may not cross. In this case there will be excess demand at the SMP. We assume demand is rationed efficiently.

[16] For notational convenience, where this does not lead to confusion, we will write Q_i and p^* to mean $Q_i(b)$ and $p^*(b)$.

deterministic demand, which is the appropriate assumption for pool institutions where bids are submitted for short periods, such as Spain. The theoretical results obtained will allow us to develop a tractable search algorithm in order to find the equilibria in our subsequent simulation.

Let us suppose b is a pure strategy equilibrium that results in a SMP of p^*. We refer to a firm as *marginal* when it is bidding in some capacity at the marginal price, p^*, which is at least partially accepted. Our first result states that if a firm i is marginal then any other firm $j \neq i$ behaves as a price taker. Define $O_i(p) = \max\{q : q \in [0, K_i] \text{ and } MC_i(q) < p\}$. This is the minimum output of a price taking firm when the price is p.

Theorem 1 Let b be a pure strategy equilibria that results in a SMP of p^*. If firm i is marginal, then $Q_j \geq O_j(p^*)$ for any $j \neq i$.

Proof Let us define by M_i the output of firm i that is offered at a price of p^* *and is accepted.*[17] Formally

$$M_i = Q_i - \max\{q : b_i(q) < p^*\}$$

Suppose that $Q_j < O_j(p^*)$ for some $j \neq i$. Define $k = \min(O_j(p^*) - Q_j, M_i)$ and consider the following deviation for firm j

$$\hat{b}_j(q) = \begin{cases} p^* - \epsilon & q \leq Q_j + k \\ p^{\max} & q > Q_j + k \end{cases}$$

[17] Notice that M_i only includes the output of firm i's marginal units that is accepted, not of any unit that bids at a price below the marginal price.

This will give firm j profits of *at least* $(p^* - \epsilon)(Q_j + k) - C_j(Q_j + k)$. This can be rewritten as

$$\pi_i(\hat{b}_j, b_{-j}) = \pi_j(b) - \epsilon Q_j + \int_{Q_j}^{Q_j+k} [p^* - \epsilon - MC_j(q)]\, dq$$

given that $p^* > MC_j(q)$ for any positive $q \leq Q_j + k$,

we have that $\int_{Q_j}^{Q_j+k} [p^* - MC_j(q)]\, dq$ is equal to some constant $A > 0$. Thus

$$\pi_i(\hat{b}_j, b_{-j}) = \pi_j(b) - \epsilon(Q_j + k) + A$$

For small enough $\epsilon > 0$, $\pi_j(b) < \pi_j(\hat{b}_j, b_{-j})$. Q.E.D.

The proof is based on the fact that if any firm $j \neq i$ has unused capacity that can generate at a cost $c < p$, it can bid in this capacity at a price of $p - \epsilon$. This may lower the SMP by ϵ but it will increase firm $j's$ output by a strictly positive amount that does not depend on ϵ. There are some important implications that can be derived from Theorem 1.

Let us denote by $v_i(p)$ the max profits that firm i can achieve when the SMP is p and all other firms are acting as price takers.

$$v_i(p) = \begin{cases} p\ (D(p) - O_{-i}(p)) - C_i(D(p) - O_{-i}(p)) & if\ O_i(p) \geq D(p) - O_{-i}(p) > 0 \\ pO_i(p) - C_i(O_i(p)) & if\ D(p) - O_{-i}(p) > O_i(p) > 0 \\ 0 & otherwise \end{cases}$$

where $O_{-i}(p) = \sum_{j \neq i} O_j(p)$. The first line corresponds to a case where firm i can supply the residual demand with generating units that have marginal costs below price p. In this case $O_{-i}(p) = Q_{-i}(p)$ since any firm $j \neq i$ cannot sell any extra output above $O_j(p)$ by bidding it at price p, because of the efficient rationing rule. The second line corresponds to a case where firm i cannot supply the residual demand with generating units that have marginal costs below price p.

Accordingly, $pO_i(p) - C_i(O_i(p))$ are the maximum profits it can achieve. Notice that in this case, residual demand is not satisfied if firms supply only $\sum_{j \in I} O_j(p)$. In order for p to be the SMP, it must be the case that for at least one firm $Q_j(p) > O_j(p)$.

Corollary 1 Let b be a pure strategy equilibrium that results in a SMP of p^*. If firm i is marginal, then

$$\pi_i(b) = v_i(p^*)$$

and

$$\pi_j(b) = \max_q \{p^* q - C_j(q)\}$$

for all $j \neq i$.

Corollary 1 implies that if we are in equilibrium, the SMP and the identity of a marginal firm uniquely determine equilibrium payoffs. Note that if more than one firm is marginal then by Theorem 1 every firm behaves as a price taker with respect to p^*, i.e., $(p^*, Q_1, .., Q_f)$ constitutes a competitive equilibrium.

In order to further characterize our equilibrium we will refine our equilibrium set by eliminating any profile that involves generating units bidding below their marginal cost. For cases where this refinement does not eliminate any equilibria our assumption is innocuous, when it does it is easy to see that bidding below cost is a weakly dominated strategy for some firm.

Given our refinement of the equilibrium if a firm bids in all its capacity at a price of p it can obtain profits of at least $v_i(p)$, thus payoffs in equilibrium are bounded below by $\max_p v_i(p)$. On the other hand if firm i is marginal, its profits are given by $v_i(p^*)$, thus it must be the case that $p^* \in \arg\max v_i(p)$. Since there are f firms, and given that the $\arg\max$ of $v_i(p)$ may not be unique, this yields $f\times$

$\#(\arg\max v_i(p))$ possible candidates for SMP (and the corresponding payoffs) in a pure strategy equilibrium. Note that we are not excluding any equilibria that involves more than one firm being marginal.

The following theorem characterizes which of these SMP-payoff combinations are part of a pure strategy equilibrium.[18] Let $\hat{\pi}_i\ (p)$ be a payoff vector of the form

$$\hat{\pi}_{ii}(p) = v_i(p)$$

and

$$\hat{\pi}_{ij}(p) = \max_{q\in[0,D(p)]} pq - C_j(q)$$

for $j \neq i$.

Theorem 2 There is a pure strategy equilibrium in which firm i is marginal the SMP is $p^*$and firm´s payoffs are given by the vector $\hat{\pi}_i(p^*)$ if and only if $\max_{p\in[0,p^{\max}]} v_j(p) \leq \hat{\pi}_{ij}(p^*)$ for all $j \in I$ and $D(p^*) - O_{-i}(p^*) > 0$.

Proof: Suppose there is a pure strategy equilibrium in which firm i is marginal the SMP is $p^*$and firm´s payoffs are given by the vector $\hat{\pi}_i(p^*)$. Given the assumption that rules out bidding below marginal cost any firm $j \in I$ may deviate to selling all its capacity at a price which maximizes its residual demand and obtain profits of at least

$$\max_{p\in[0,p^{\max}]} v_j(p)$$

This proves the implication in one direction. Now suppose $\max_{p\in[0,p^{\max}]} v_j(p) \leq \hat{\pi}_{ij}(p^*)$ and $D(p^*) - O_{-i}(p^*) > 0$. We will construct a strategy profile that

[18] We do not characterize the strategies played in a pure strategy equilibrium as there may be several strategy profiles that are payoff equivalent and lead to the same system marginal price.

yields profits of $\hat{\pi}_i(p^*)$ and prove it is an equilibrium. Consider a strategy profile b where the strategy for firm i is given by

$$b_i(q) = \begin{cases} p^* & q \leq O_i(p^*) \\ MC_i(q) & q > O_i(p^*) \end{cases}$$

and all other firms except i bid all their capacity at marginal cost. Suppose that the SMP under profile b, p', is greater than p^*, this implies that

$$v_i(p') \geq p'O_i(p^*) - C_i(O_i(p^*)) > p^*O_i(p^*) - C_i(O_i(p^*)) \geq v_i(p^*)$$

which leads to a contradiction. It must be the case then that profile b results in a SMP of p^*. We have then that $\pi_j(b) = \hat{\pi}_{ij}(p^*)$ for all $j \in I$.

Under profile b firm i is making profits of $\max_{p\in[0,p^{\max}]} v_i(p)$ and its rivals are behaving as price takers thus firm i has no profitable deviation given its rival's strategies.

Suppose there exists a firm $j \neq i$ that has a profitable deviation from b. It cannot result in a system marginal price below p^* since firm j is obtaining profits of $\max_q p^*q - C_j(q)$. The deviation must result in a price above p^* and deviation profits for firm j are then bounded above by $\max_p \ v_j(p)$. Q.E.D.

The previous theorem implies that given a candidate equilibrium where firm i is marginal and sets a price of $p^* \in \arg\max v_i(p)$ the only relevant deviation by any other firm $j \neq i$, is to become marginal and set a price in $\arg\max v_j(p)$. It cannot be profitable to deviate from a price taking behaviour to become marginal if this results in a lower price. This implies that the highest of the candidates for SMP is always an equilibrium. Formally, let $p^* = \max_i\{p : p \in \arg\max_p \pi_i(p)$ and $D(p) - O_{-i}(p) > 0\}$ and suppose that the maximum is achieved for $i = i^*$, then by Theorem 2 there is a pure strategy equilibrium that results in an SMP p^* and payoffs of $\hat{\pi}_i(p^*)$. This guarantees existence of a pure strategy equilibrium for our game.

We will now compare the equilibria of our model to those resulting from a Cournot equilibrium. This is of interest since Cournot has been used to approximate firm behavior in pool markets where bids are short-lived.[19] We will prove that the price that results from a Cournot equilibrium is greater than or equal to any equilibrium price of our model. In particular, the Cournot model yields significantly higher mark-ups except when demand is very elastic or the industry is very fragmented. None of these two situations is common in electricity markets. This suggests that Cournot models clearly over estimate market power in electricity pools where firms bid facing a certain demand. Our result is consistent with Green and Newbery's (1992) model that shows the same result for electricity pools where firms bid facing an uncertain demand.

Assume that the inverse demand function for a given time period, $P(q)$, is strictly decreasing and concave. Note that from the definition of technology, $C_i(q)$ is strictly increasing, convex, and left continuous, and that all the firms are capacity constrained. Let us denote the highest SMP that can be obtained in an equilibrium of our model by $\overline{p}^*$and the minimum price that results in a Cournot equilibrium by $\underline{p}^C$.

Theorem 3 Under the previous assumptions, $\underline{p}^C \geq \overline{p}^*$.

Proof By corollary 2 if more than one firm is marginal then we obtain a perfectly competitive outcome. Let us suppose that only one firm, i, is marginal and

[19] Borenstein and Bushnel (1997) for California and Ocaña and Romero (1998) for Spain.

it is setting a marginal price of $\overline{p}^*$. We denote the production of firm i in this equilibrium by Q_i^*, and the aggregate production of its rivals by Q_{-i}^*.

Let $q' = \arg\max_q P(q + Q_{-i}^*)q - C_i(q)$and $p' = P(q' + Q_{-i}^*)$, we will begin by proving that $q' \leq Q_i^*$. Suppose that $q' > Q_i^*$, we then have that $p' < P(Q_i^* + Q_{-i}^*) \leq \overline{p}^*$ and

$$P(q' + Q_{-i}^*)q' - C_i(q') = p' \ (D(p') - Q_{-i}^*) - C_i(D(p') - Q_{-i}^*) \leq v_i(p')$$

we may then write

$$P(q'+Q_{-i}^*)q'-C_i(q') \leq v_i(p') \leq v_i(\overline{p}^*) = P(Q_i^*+Q_{-i}^*)(Q_i^*+Q_{-i}^*)-C_i(Q_i^*+Q_{-i}^*)$$

given that $P(q + Q_{-i}^*)q - C_i(q)$ is a concave function of q this leads to a contradiction.

Let us denote the Cournot equilibrium productions corresponding to $\underline{p}^C$ by q_i^C and Q_{-i}^C. Following Amir(1996) let us now define the Cournot best response function of firm i when its rivals produce Q_{-i} in terms of the aggregate industry production $z(Q_{-i})$,

$$z(Q_{-i}) = \arg\max_{q \geq Q_{-i}} \ (q - Q_{-i})P(q) - C_i(q - Q_{-i})$$

Amir (1996) shows that $z(Q_{-i})$ is a decreasing function (see proof of Theorem 2.3).[20]

Let us assume that $\underline{p}^C < \overline{p}^*$, by properties of a Cournot equilibrium $Q_{-i}^C \leq O_{-i}(\underline{p}^C)$ and thus $Q_{-i}^C \leq O_{-i}(\overline{p}^*) \leq Q_{-i}^*$. We have then that $Q_i^* + Q_{-i}^* \geq q' + Q_{-i}^* = z(Q_{-i}^*) \geq z(Q_{-i}^C) = Q_{-i}^C + q_i^C$ which leads to a contradiction. Q.E.D.

[20] It is easy to see that the objective function is strictly concave in z and thus that $z(Q_{-i})$ is single valued.

4. The Simulation

For a given period, the simulation is conducted as follows. We define G_i as the set of non-differentiable points of $v_i(p)$. G_i includes $p^{\max}$, c_u for all $u \in U_{-i}$ and all the prices where the profits are kinked due to the discontinuity of the marginal cost function of firm i. By assuming demand is differentiable from theorem 2 we obtain corollary 3.

Corollary 3 Let p_1 and p_2 be two consecutive prices in G_i. If $M_i > 0$ and $p_1 < p^* < p_2$, then p^* verifies the following first order condition

$$D'(p^*)(p - C'(D(p^*) - O_{-i}(p^*))) - O_{-i}(p^*) = 0$$

Let F_i be the set of prices that verify the previous first order condition for some $i \in I$. Payoffs for a pure strategy equilibria can be uniquely characterized by a marginal firm i and a marginal price $p^* \in G_i \cup F_i$. The first step in our simulation is to compute a $\#(G_i \cup F_i) \times f$ matrix with the payoffs for all the possible (p^*, i) pairs. We then apply theorem 2 to identify which (p^*, i) configurations and payoffs correspond to a pure strategy equilibria of the game.

4.1. Comparative statics

The previous methodology allows us to obtain some comparative static results for our model. We will explore two simple examples. Our examples involve an industry with two firms, A and B, each firm owns 100 plants that can be ranked by their marginal costs from lowest to highest. The costs of plants $2k$ and $2k - 1$ are given by k. We take $p^{\max} = 100$, this limits prices to twice the marginal cost of the less efficient generator. We calculate our equilibria for inelastic demands that range from 10% to 50% of total installed capacity, which remains unchanged and equal to 200. Our base case involves two identical firms. In particular, all plants have generating capacity equal to unity, odd plants belong to firm A and even plants belong to firm B. In the following examples we alter the size and property of the generators and see how a departure from the symmetric structure affects the equilibria. The industry technological structure, however, will remain unchanged throughout.

In figure 1 the effect of asymmetries in size are analyzed. The symmetric case is compared to two situations with the same ownership structure and total installed capacity. In these two cases, firm B's plants have 50% and 75% capacity of firm A's plants, respectively. We can observe that the initial effect of introducing capacity asymmetries is the appearance of multiplicity of equilibria: a high price equilibrium with the large firm setting the marginal price and a low price equilibrium with the small firm setting the marginal price. In our example, the difference between the highest and the lowest price rises as asymmetry grows until the asymmetry is such that the equilibrium that involves the small firm setting the marginal price disappears and uniqueness of equilibrium re-emerges. Thus, although the effect of small asymmetries on market power is ambiguous, since the symmetric equilibrium price lies between the high and the low price equilibria, large asymmetries unambiguously lead to higher prices. Furthermore, we should notice that when demand grows, prices rise but, in this specific case, because of the particular technology mix, price-cost margins remain stable.

Figure 2 studies the effect of cost asymmetries. The symmetric case is compared to a situation with the same set of plants, in terms of size and technology, but a different ownership structure. In particular, we assume that firms own sets of four consecutive plants, with firm A owning the four most efficient plants. We observe that again a departure from the symmetric case leads to multiplicity of equilibria. In general, the identity of the firm setting the highest equilibrium price depends on demand, but, on average, equilibrium prices set by the most efficient firm are above prices set by the least efficient. The symmetric case equilibrium price always lies below the equilibrium prices set by the most efficient firm and, on average, it lies below the equilibrium prices set by the least efficient firm. Accordingly, this simple example suggests that asymmetries in costs lead to higher prices on average.[21]

4.2. Simulations of the Spanish market

4.2.1. Hydro generation

This section deals with the problem of scheduling hydro reserves in the context of an electricity pool. Bushnell (1998) points out that the main feature that distinguishes hydro from other technologies is that it allows firms to shift electricity generation between different time periods, in essence it makes electricity storable. In regulated welfare maximizing environments hydro is used in periods of high demand in order to "shave" demand peaks, avoiding the need to use high marginal cost peaking units.[22]

[21] In should be noted that the cyclical behaviour in mark-ups specially evident in figure 2 is due to discontinous (step function) nature of the firms' marginal cost functions. The competitive price jumps as demand moves from one step to the next but prices do not necesarily. This leads to reductions in the price-cost margins when demand jumps from one step to the next.

[22] Hydro is also used to adjust to small unexpected shifts in demand given its high degree of modulation. This is a more technical aspect of water use that is beyond the scope of this paper.

The strategic aspects of hydro scheduling have been dealt with only in the context of Cournot models. Scott and Read (1996) analyze a multi-period model where one firm controls all the storage hydro capacity. In each period this firm interacts with a number of thermal generators in a Cournot market. For each period the authors obtain the amount of electricity that the hydro firm will produce in a Cournot equilibrium as a function of the shadow price of hydro generation. By setting this shadow price so that hydro production in equilibrium is equal to the amount of water assigned for the planning horizon the authors obtain the shadow price for water in equilibrium, and given this price, equilibrium production and prices for all periods. Bushnell (1998) extends Scott and Read's methodology to allow for multiple firms with hydro capacity and for the existence of fringe firms. Using a methodology similar to the previous two works, Ocaña and Romero (1998) also analyze hydro production in a model of Cournot competition. The main result that is derived from the previous papers with respect to the scheduling of water is that firms will depart from a competitive allocation of hydro by shaving their marginal revenue instead of demand, this leads to a flatter hydro allocation. The intuition is that firms will deviate from the competitive allocation by transferring water from periods where their market power is high to periods where their market power is low (i.e. periods where their incentives to strategically reduce quantity are lower).

A general treatment of hydro scheduling in multi-unit auction models is beyond the scope of this paper. As we have noted previously our results show that our model has a closer relation with the dominant firm model than with Cournot. In a dominant firm model fringe firms will take prices as given and, to the extent that higher demand results in higher prices, will allocate water in a peak shaving manner.[23] The dominant firm will use hydro in a strategic manner equating its marginal cost across periods. In our simulation we will assume that all firms allocate hydro production in a peak-shaving manner. In the light of the previous analysis, it seems reasonable to believe that our assumption will introduce a downward bias on our measure of market power.

4.2.2. Demand side bidding

Given observed demand for a particular period t, q_t,we need to specify a pool demand function $D_t(p)$ for this period. If there is no demand side bidding in the pool, then pool demand will simply be constant at q_t for any price. However, if there is a fraction of consumers that are allowed to engage in demand side bidding, then the demand function will be decreasing with respect to price. The problem that arises in specifying $D_t(p)$ is that it will be determined by demand responsiveness to prices, which depends on pumped storage and on eligible consumers' ability to adjust their electricity consumption in a given hour. We must compensate the lack of information that we have on demand side bidding behavior with some reasonable assumptions.

We assume that the demand function is linear and that the maximum change that may occur in demand in any given period, $D_t(0) - D_t(p^{\max})$, is a fixed percentage M of the average observed demand $\overline{q}$,

$$D_t(0) - D_t(p^{\max}) = M\overline{q}$$

[23] As is noted by Bushnell (1998) the presence of price taking firms-will create a "kink" in demand. This may lead a price setting dominant firm to choose to lower its price when demand grows. We have not observed this feature in our results. In most, if not all, cases a larger demand is associated whith a larger market price.

This, along with linearity, implies that demand takes on the form $D_t(p) = a_t - bp$ where b is constant for every period and is given by

$$\frac{M\overline{q}}{p^{\max}}$$

We further assume that the difference in demand at a given price between two periods, t and t', is constant and equal to $q_t - q_{t'}$. This implies that $a_t = q_t + \delta$. Finally we assume that aggregate demand will be constant in equilibrium regardless of the level of eligible consumers. This assumption reflects the widespread belief that aggregate demand is non responsive to prices in the short run. Thus for a given assumption on the percentage of observed demand that is responsive to price, M, we calibrate δ so that in equilibrium aggregate production is constant (if there is multiplicity we take the lowest price, this is because we are interested in a lower bound on market power).[24] Our calibration of δ controls for the 'size effect' that would result if we were to keep a_t constant and to increment the slope of the demand b.[25]

[24] Given that *argmax* $\pi_i(p)$ may have more than one element a marginal rise in δ does not necessarily imply a marginal rise in aggregate production in equilibrium, it may actually result in a discontinuous drop in production. What this implies is that the existence of a δ that keeps aggregate production constant is guaranteed but it may not be unique. Given that we are interested in a lower bound on market power, in our calibrations we find the smallest of such values of δ.

[25] The linear demand assumption would lead to very high elasticities if prices were allowed to rise. This is not the case in our model because there is a restriction on the maximum price generators may bid, $p^{\max}$.

4.2.3. Data and results

The data used in the simulations has been taken from the *Comisión Nacional del Sistema Eléctrico*, CNSE, and includes information at the plant level on fuel,[26] operation and maintenance costs as well as ownership structure. In order to account for outage rates firms' generation capacity is reduced by 12.75% for thermal generators and by 14.75% for nuclear plants (outage rates provided by CNSE). This information is presented in figure 3. Data on hourly 1998 demand was obtained from the web page of the market operator, *Compañía Operadora del Mercado Eléctrico.* We divide the range of demand in 50 identical intervals and take the median demand in each of these intervals. We run our calculations using these 50 representative demand values. When aggregating our results we take into account the different frequencies in each interval. From our observed demands we subtract net exports for 1998.

Total hydro generation was set to 33.168GWh which is the observed value for 1997[27]. Maximum hydro flows for each firm were provided by CNSE. Minimum flows where set to 5% of the firms maximum hydro flow, this is consistent with minimum daily flows for the 1995-1997 (data provided by CNSE).

Figures 4 and 5 present the results of simulating the model with the Spanish generating plants and electricity demand. We allow for different market structures and degrees of demand responsiveness to prices. In particular, scenario 1

[26] Spanish generators are subject to mandatory quotas for domestic coal consumtption. This is not taken into account in our simulation. Fuel cost for coal is taken to be international price plus transportation to the plant.

[27] We have tested the effect of using values of 25,000GWh and 35,000GWh. This has a small effect on the absolute level of price-cost margins, and their relative change across scenarios. The average annual hydro generation in the 1993-1997 period was 28,500GWh.

corresponds to the market structure before the last merger wave in 1996, i.e., the operating firms are Iberdrola, U. Fenosa, Sevillana, FECSA, Hidrocantábrico and the Endesa Group. Scenario 2 corresponds to the structure after the last merger wave in 1996, i.e., Endesa Group takes control over Sevillana and FECSA. Finally, scenario 3 corresponds to a hypothetical case in which we assume a merger among U. Fenosa, Hidrocantábrico and Iberdrola. The reason for looking at this scenario is to analyze the case of a duopoly with two firms of similar sizes.

In turn, linear demand functions are constructed from our observed demand as explained before. We run our simulations for alternative demand slope scenarios with M ranging from 0% (inelastic demand) to 40%. When there is multiplicity of equilibria, results for the minimum equilibrium price are reported. Figure 4 presents yearly average price-cost margins for all the scenarios considered. We observe that price-cost margins are lower in scenario 1, where market structure is more fragmented, while the highest price-cost margins arise in scenario 2. Scenario 3, that corresponds to a duopoly structure, leads to lower price-cost margins than scenario 2 even though it represents a more concentrated market structure. This is so because in scenario 2, in most cases, there is an unique equilibrium with the large dominant firm setting the marginal price. In scenario 3, the market structure is more symmetric leading to a greater multiplicity of equilibria, this along with the fact that the minimum equilibrium price is reported leads to this result (see figure 1 for a detailed example with a similar situation). In all the scenarios, an increase in M causes price-cost margins to fall. This effect is very small in scenario 1 where price-cost margins are lower. As M increases, price-cost margins tend to converge in all the scenarios. This suggests that changes in market concentration must be evaluated according to the value of M. In particular, the last merger that took the Spanish industry from scenario 1 to scenario 2 would be particularly worrying for a small M, since price cost margins rise by a factor of 4, while a value of the slope parameter above 25% would imply that this merger had a much smaller effect on market power. We will now deal with the problem of selecting a reasonable value of M.

A value of M greater than zero reflects the fact that a fraction of demand might be price responsive. The two elements that can lead to a price responsive demand are pumped storage and hourly modulability on the side of eligible consumers. In 1998, pumped storage represented 1.1% of total demand that was dispatched and eligibility was negligible.[28] In our model, for a given equilibrium price, p, price responsive demand as a percentage of total demand is given by $\frac{D_t(0)-D_t(p)}{D_t(p)}$. Using our model, for each value of M it is possible to calculate the percentage of price responsive demand in a year. Figure 5 allows us to relate the observed value of pumped storage demand with a value for M of 16% under the 1998 four firm structure (scenario 2).

Current eligibility represent 50% of total demand, but the percentage of eligible consumers' demand that may change with the price in a given hour is not likely to be greater than 10%. Under this assumption the maximum variability in demand, in response to price changes due to eligible consumers, will not be higher than 5% of total demand. Adding this to the value obtained for pumped storage, yields an upper bound of 21% for M.[29] Accordingly, we conclude that under any reasonable value for M, the merger of Endesa with Sevillana and FECSA

[28] Data provided by CNSE.

[29] In year 2007 eligibility will rise to include all consumers. It is not very likely that this will add much to the hourly price responsiveness of demand, since small consumers are not likely to be able to modulate demand in one hour.

had a large and significant impact on market power. Moreover, our simulation shows that the current situation is worse in terms of market power to a nearly symmetric duopoly, with the three smallest firms merging.[30] The latter result shows the limitations of a simple concentration index analysis.

Our simulation estimates variations in market power following changes in market and cost structure. This cannot be empirically contrasted since they have not taken place, or have taken place before the pool was in operation. However, some of the results of the simulation for the current four firm structure could be contrasted with empirical observations. For instance, in our simulation of the four firm structure, Endesa Group is the marginal firm in most periods and this equilibrium is more likely to be unique the more inelastic the demand function and the larger the level of demand. As a result of this, Endesa Group has a lower market share in terms of production than in terms of installed capacity. The two smallest firms are seldom marginal for the relevant values of the demand slope, and only in hours with very small levels of demand. Unfortunately, it is not possible to compare the results of our simulation with current data for two reasons. Firstly, while the Spanish pool started operations in 1998, it is going through a transition period designed to allow firms to recover their stranded costs. We do not explicitly model this stranded cost recovery mechanism but rather analyse firms' behavior in the pool once this transition period is over. Secondly, lack of data availability. The market operator only publishes the pool price and the total

[30] This is due to the fact that the duopoly structure leads to multiplicty of equilibria and we are providing results for the minimum equilibrium prices. In any case, given that the maximum equilibrium prices coincide in scenarios 2 and 3, a simple average of the maximum and minimum equilibrium prices would result in lower market power under the duopoly structure.

quantity dispatched for each hour. Agents' individual bids, sales and purchases, as well as the identity of the firm that determines the system marginal price are not publicly available.

5. Conclusions

In this paper we extend the results of Von der Fehr and Harbord (1993) to analyze an electricity pool with short bidding periods. The model also accounts for the effect of demand side bidding by including a positively sloped demand. We obtain a characterization of the pure strategy equilibria for this model and we implement a simple algorithm to identify them in a simulation for the Spanish pool. In particular, we simulate our model using 1998 data from the Spanish electricity market in order to measure market power under the current market structure and to analyze the effect of the 1996 merger that took the industry from a six to a four firm structure. We find that market power measured by price cost margins is very high in the Spanish pool and that most of this market power can be attributed to the 1996 merger.

Our theoretical results suggest that asymmetries among generating firms both in size and costs play a crucial role in determining prices, leading to higher price-cost margins and, in many cases, to multiplicity of equilibria. In the presence of strong asymmetries there is a unique equilibrium where the small firms act as price takers and the large firm maximizes profits given its residual demand as in the dominant firm model. When comparing the predictions of our model to those of a Cournot model, we find that the latter yields higher prices than the former. This coincides with the results of Green and Newbery (1992) where

Cournot is an upper bound to equilibrium prices and suggests that the Cournot model overestimates market power in pool markets.

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6. Appendix

In most pools the precision that can be used by firms in their bids is limited, i.e. firms must bid prices from a grid and when there is excess generation at the SMP firms are rationed proportionately. For reasons of tractability this has been approximated in Von der Fehr and Harbord (1993) and our own model by a game where firms set their bids on $[0, p^{\max}]$ and in case of excess generation at the SMP rationing is efficient. In this appendix we show that for a sufficiently fine grid the latter is a good approximation for the former.

Let us define S_{in} as the space of increasing step functions $b_i(q) : [0, K_i] \rightarrow G_n$ where the function is restricted to a finite number of steps. Let $\{0, p^{\max}\} \subset G_n \subset [0, p^{\max}]$ and let $\epsilon_n = \max_{x \in G_n} \inf_{y \in G_n/\{x\}} |x - y|$, we will assume that $\epsilon_n \rightarrow 0$. Analogously we define S_i as the space of increasing step functions with a finite number of steps, $b_i(q) : [0, K_i] \rightarrow [0, p^{\max}]$. Let $S_n = \times_{i \in I} S_{in}$ and $S = \times_{i \in I} S_i$. Denote by Ω_n the set of pure strategy Nash equilibria when players are restricted to strategy space S_n and the rationing is proportional. Denote by Ω the set of of pure strategy Nash equilibria when players are restricted to strategy space S and rationing is efficient.

The SMP resulting from strategy profile b will be denoted by $p^*(b)$. Define $\pi_i^E(b)$ and $\pi_i^P(b)$ as the payoff functions of agent i when the rationing is efficient and proportional respectively. Analogously define $Q_i^E(b)$ and $Q_i^P(b)$ as the quantity produced by agent i when the rationing is efficient and proportional, respectively.

Define the following distance between any two strategies

$$d(b, b') = \max_i \max_{q \in [0, K_i]} |b(q) - b'(q)|$$

Given $b_i(q)$, we may define a function that gives us the amount of generation that is offered by firm i at a price strictly bellow p,

$$o_i(p) = \max\{q : b_i(q) < p\}$$

We will denote by $w_i(b_n)$ the supremum of the payoffs that firm i can obtain by deviating from strategy profile b_n when it may choose any strategy in S

$$w_i(b_n) = \sup_{p \in [0, p^{\max}]} (D(p) - \sum_{j \neq i} o_{nj}(p))p - C_i(d(p) - \sum_{j \neq i} o_{nj}(p))$$

where $o_{nj}(p) = \max\{q : b_{ni}(q) < p\}$.

The following lemmas will be useful

Lemma 1. $d(\bar{b}, \bar{b}') \leq d(b, b')$

Proof By definition of the aggregate bid function for a given q there exists a production vector x that verifies

$$\bar{b}(q) = \max\{b_1(x_1), .., b_n(x_n)\}$$

where $\sum x_i \leq q$. We then have

$$\bar{b}'(q) \leq \max\{b_1'(x_1), .., b_n'(x_n)\} \leq \max\{b_1(x_1), .., b_n(x_n)\} + d(b, b')$$

this implies that $\bar{b}'(q) - \bar{b}(q) \leq d(b, b')$. A similar argument leads to $\bar{b}(q) - \bar{b}'(q) \leq d(b, b')$.Q.E.D.

Lemma 2. *If* $b_n \xrightarrow{d} b$ *then* $\lim w_i(b_n) \to w_i(b)$

Proof Let us define the range of b_i by the correspondence $R(b_i, K_i) = \{p : b_i(q) = p \text{ for some } q \in [0, K_i]\}$. Note that $R(\bar{b}, K) = \underset{i \in I}{\cup} R_i(b_i, K_i)$. Given that bid functions are restricted to a finite number of "jumps" $R(\bar{b}, K)$ is a finite set, and so is $\Delta R(\bar{b}, K) = \{x - x' : x \in R(\bar{b}, K) \text{ and } x' \in R(\bar{b}, K) \text{ and } x > x'\}$.

Let us define n^* as the smallest value of n that verifies $\min \Delta R(\bar{b}, K) > 3\, d(b_n, b)$.We have then that for $n > n^*$, $b_i(q) - b_j(q)$ is either zero or greater than 3 $d(b_n, b)$. This implies that if $p \in R$ then

$$\begin{aligned} \max\{q : b_i(q) < p - 2d(b_n, b)\} &= \max\{q : b_i(q) < p\} \\ &= \max\{q : b_i(q) < p + 2d(b_n, b)\} \end{aligned}$$

By the definition of the distance function we have

$$\begin{aligned} \textstyle\sum_{j \neq i} \max\{q : b_i(q) < p - 2d(b_n, b)\} &\leq \textstyle\sum_{j \neq i} \max\{q : b_{ni}(q) < p - d(b_n, b)\} \\ &\leq \textstyle\sum_{j \neq i} \max\{q : b_i(q) < p\} \\ &\leq \textstyle\sum_{j \neq i} \max\{q : b_{ni}(q) < p + d(b_n, b)\} \\ &\leq \textstyle\sum_{j \neq i} \max\{q : b_i(q) < p + 2d(b_n, b)\} \end{aligned}$$

For $n > n^*$ this implies that if $p \in R$ then

$$\begin{aligned} \textstyle\sum_{j \neq i} o_i(p - 2d(b_n, b)) &= \textstyle\sum_{j \neq i} o_{ni}(p - d(b_n, b)) \\ &= \textstyle\sum_{j \neq i} o_i(p) \\ &= \textstyle\sum_{j \neq i} o_{ni}(p + d(b_n, b)) \\ &= \textstyle\sum_{j \neq i} o_i(p + 2d(b_n, b)) \end{aligned}$$

It is clear that $p^*(b) \in R$, define $\hat{p}_n \in \arg\min_{x \in G_n} |p^*(b) - x|$. Note that $|p^*(b) - \hat{p}_n| \leq d(b_n, b)$ we then have that for $n \geq n^*$

$$w_{ni}(b_n) \quad \geq (D(\hat{p}_n) - \textstyle\sum_{j \neq i} o_{ni}(\hat{p}_n))\hat{p}_n - C_i(D(\hat{p}_n) - \textstyle\sum_{j \neq i} o_{ni}(\hat{p}_n))$$

Taking limits we have $\lim w_{in}(b_n) \geq w_i(b)$. Analogously define $\breve{p}_n \in \arg\min_{x \in R} |p^*(b_n) - x|$ it is clear that $|p^*(b_n) - \breve{p}_n| \leq d(b_n, b)$

$$w_i(b) \quad \geq (D(\breve{p}_n) - \textstyle\sum_{j \neq i} o_i(\breve{p}_n))\breve{p}_n - C_i(D(\breve{p}_n) - \textstyle\sum_{j \neq i} o_i(\breve{p}_n))$$

taking limits we have $\lim w_{in}(b_n) \leq w_i(b)$.Q.E.D.

The following theorems prove the desired result.

Theorem 4 If $b_n \xrightarrow{d} b$ and $b_n \in \Omega_n$ then $b \in \Omega$

Proof: Note that $\lim \sum_{i \in I} \pi_i^E(b_n) \geq \sum_{i \in I} \pi_i^E(b)$, this is so because $\sum_{i \in I} \pi_i^E(\cdot)$ is either continuous at b or it jumps up, i.e. excess supply at p^* is allocated so as to minimize costs. We then have that

$$\lim \sum_i w_i(b_n) - \pi_i^E(b_n) \geq \sum_{i \in I} w_i(b) - \pi_i^E(b)$$

This implies that if $b_n \to b$, $\epsilon_n \to 0$ and b_n is an ϵ_n-equilibrium for the game with strategy space S and an efficient rationing rule, then b is an equilibrium of this game.

We will now prove that if $b_n \in \Omega_n$ then b_n is a ϵ_n-equilibrium for the game with strategy space S and efficient rationing for some $\epsilon_n \to 0$. Let $b_n \in \Omega_n$,define $h_{ni} \in S_i$ as a strategy such that $\pi_i^E(h_{ni}, b_{n-i}) + \frac{1}{n} \geq w_i(b_n)$. Consider the following strategy for firm i that aproximates h_{ni} in S_{ni}

$$\hat{h}_{ni}(q) = \begin{cases} \hat{p} & q \leq Q_i^E(h_{ni}, b_{n-i}) \\ p^{\max} & q > Q_i^E(h_{ni}, b_{n-i}) \end{cases}$$

where $\hat{p} = \max\{p \in G_n : p < p^*(h_{ni}, b_{n-i})\}$. Note that $Q_i^E(h_{ni}, b_{n-i}) = Q^P(\hat{h}_{ni}, b_{n-i})$. We then have $\pi_i^E(h_{ni}, b_{n-i}) - \pi_i^P(\hat{h}_{ni}, b_{n-i}) \leq K_i(p^*(h_{ni}, b_{n-i}) - \hat{p}) \leq K_i\epsilon_n$, which leads to

$$w_i(b_n) - \pi_i^P(b_n) \leq \epsilon_n K_i + \frac{1}{n}$$

and thus

$$\sum_i w_i(b_n) - \pi_i^P(b_n) \leq \epsilon_n K + \frac{f}{n}$$

and since $\sum_i \pi_i^P(b_n) \leq \sum_i \pi_i^E(b_n)$ this leads to the desired result

$$\sum_i w_i(s_n) - \pi_i^E(b_n) \leq \sum_i \epsilon_n K + \frac{f}{n}$$

Q.E.D.

Theorem 5 If $b \in \Omega$ then there exist $b_n \xrightarrow{d} b$ and $\epsilon_n \to 0$ where b_n is an ϵ_n-equilibrium for the game with strategy space R_n and proportional rationing.

Proof: Consider the following sequence of bid functions

$$b_{ni}(q) = \begin{cases} \underline{p}_n(q) & 0 \leq q \leq Q_i^E(b) \\ \overline{p}_n(q) & Q_i^E(b) < q \leq K_i \end{cases}$$

where $\underline{p}_n(q) = \max(\{p \in G_n : p < b_i(q)\} \cup \{0\})$ and $\overline{p}_n(q) = \min(\{p \in G_n : p > b_i(q)\} \cup \{p^{\max}\})$. It is clear that $b_n \xrightarrow{d} b$. Note that if $q > \sum Q_i^E(b)$ then $b_n(q) > b(q) > P(q)$. This implies $p^*(b_n) = \underline{p}_n(\sum Q_i^E(b))$, and thus

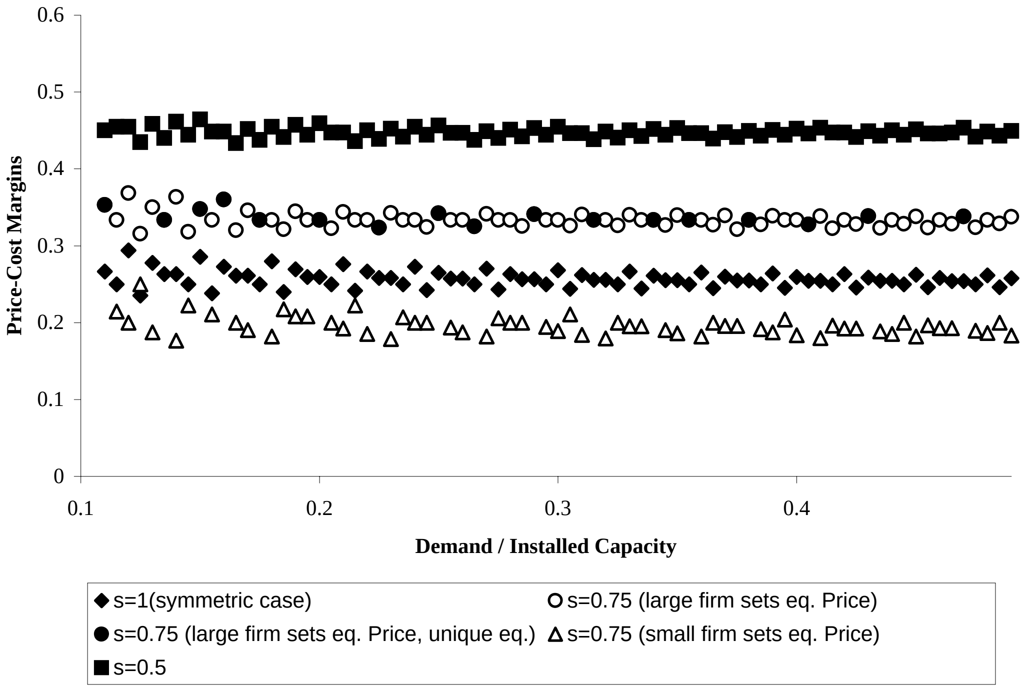
$$\pi_i^P(b_n) = Q_i^E(b)\underline{p}_n(\sum Q_i^E(b)) - c_i(Q_i^E(b))$$

We have then that $\pi_i^P(b_n) \to \pi_i^E(b)$. This along with lemma 2 allows us to write

$$\sum_i (w_i(b_n) - \pi_i^P(b_n)) \to \sum_i (w_i(b) - \pi_i^E(b))$$

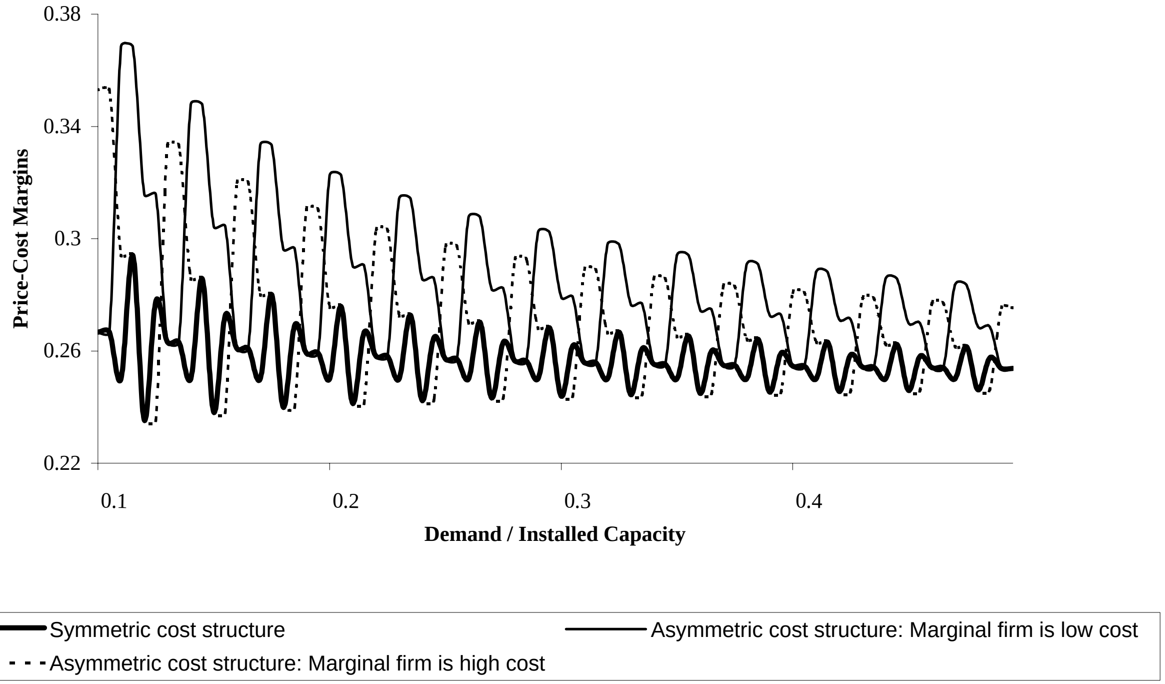
we have $\sum_i (w_i(b_n) - \pi_i^P(b_n)) \to 0$ Q.E.D.

Figure 1. The effect of size differences on price-cost margins



This example involves inelastic demand, two firms, 100 plants each and three alternative market structures. A given firm's plants are of the same capacity and can be ranked by marginal costs from lowest to highest and plant number j has a constant marginal cost of j. In each scenario contemplated total capacity is the same and firm 2's capacity is s times firm 1's capacity.

Figure 2. The effect of cost differences on price-cost margins



This example involves inelastic demand, two firms, 100 plants each and two alternative cost structures. All the plants are of the same capacity, can be ranked by marginal costs from lowest to highest. Plants $2j$ and $2j-1$ have a constant marginal cost of j.
The symmetric structure involves firm 1 owning the even numbered plants and firm 2 having odd numbered plants.The asymmetric structure involves firms owning sets of four consecutive plants with firm 1 having the four most efficient.

Figure 3. Spanish market and cost structure (thermal generation)

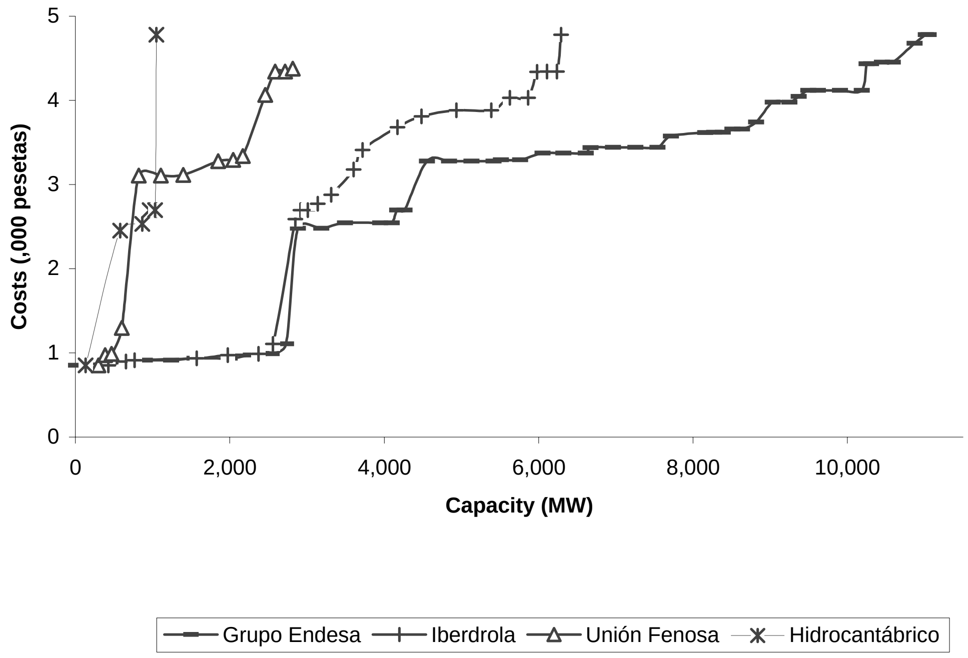
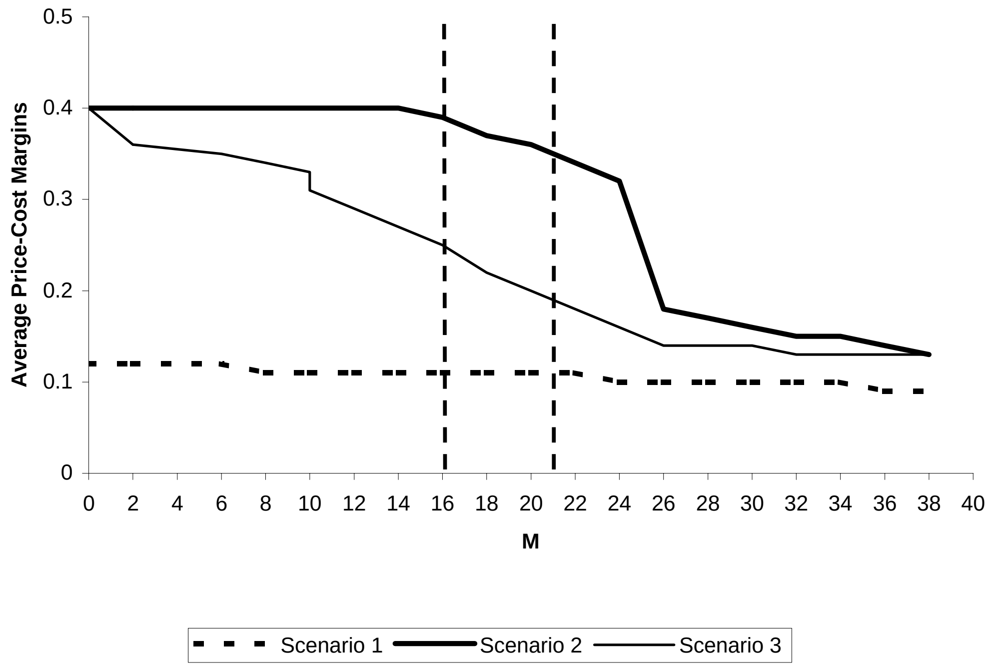
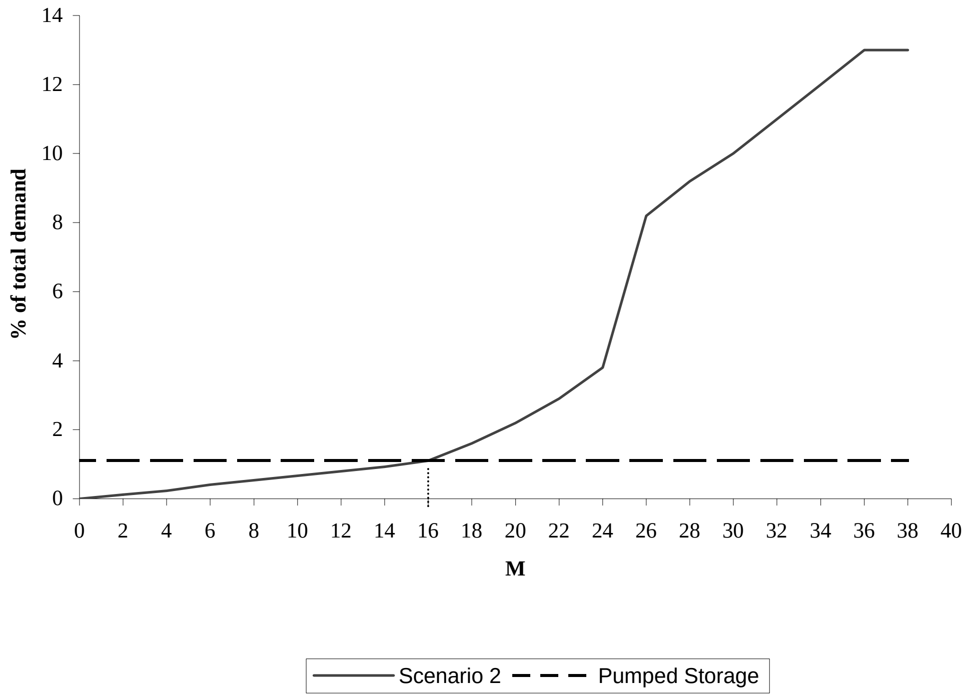


Figure 4. The effect of demand responsiveness on average price-cost margins



Scenario 1 corresponds to the pre-1996 six firm structure, scenario 2 corresponds to the current four firm structure and scenario 3 shows the minimum equilibrium mark-ups of a hypothetical duopoly.

Figure 5. Percentage of Demand that is Price Responsive



The dashed line shows actual pumped storage consumption as a percentage of total demand for 1997. The continuous line shows the price responsive demand as a percentage of total demand at the hourly equilibrium prices for each value of M (in scenario 2).