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ABSTRACT

Reallocation of Corporate Resources and Managerial Incentives in Internal Capital Markets*

One distinguishing feature of internal capital markets is their ability to reallocate funds in favour of the most profitable divisions (winner-picking). Yet, diversified firms often trade at a discount with respect to their focused counterparts. The literature has tried to explain the apparent misallocation of resources with lobbying activities or power struggles. We show that the diversification discount can be explained even in a model where resources are efficiently allocated *ex post*. When managers obtain utility from the funds under their purview, moving funds across divisions may diminish their incentives. The *ex ante* reduction in managerial incentives can more than offset the increase in firm value due to the *ex post* efficient reallocation of funds.

If headquarters have some commitment power, it is in general optimal to commit not to reallocate at least a fraction of funds. As a result, the investment in a given division is (optimally) more sensitive to the division's cash flow than to other divisions' cash flow, as confirmed by the empirical studies on internal capital markets.

Our theory complements the view that links the diversification discount to the inefficient functioning of internal capital markets.

JEL Classification: G32 and G34
Keywords: internal capital markets, managerial incentives and winner-picking

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NON-TECHNICAL SUMMARY

The analysis of the functioning of internal capital markets, that is the allocation of funds among different divisions of a conglomerate firm, is a relatively young topic. The general theme coming both from the theoretical and the empirical literature is that diversified firms destroy value. Diversified firms trade on average at a discount relative to a portfolio of focused firms in the same industries. Moreover, during the 1980s, a process of dismantling of diversified firms has occurred, driven by the idea that the divisions would be more efficiently managed as stand-alones. But if there is by now a wide consensus on the idea that diversification is value-decreasing, it is much less clear why this is the case. To address this issue, one needs to understand how internal capital markets work and how they differ from external capital markets. One major difference between internal and external capital markets has been pointed out by Stein (1997). In an internal capital market the headquarters can create value by reallocating scarce funds across projects. For example, the cash flow generated by one division's activities may be taken and spent on investment in another division, where the returns are higher. Individual projects must compete for scarce funds, and headquarters' job is to pick winners and losers in this competition. Stein denotes this activity of headquarters in a conglomerate firm as 'winner-picking'.

Contrary to the empirical findings, Stein's model suggests that an internal capital market should create value and thus a premium for diversified firms. One possible way to solve this apparent paradox is to argue that the discount of diversified firms is due to misallocation of resources in internal capital markets. For instance, funds can be misallocated in an internal capital market because of lobbying activities or power struggles.

In this paper we argue that in order to explain the discount of diversified firms we do not need to assume any misallocation of funds in internal capital markets. Conglomerates can trade at a discount even if resources are *ex post* perfectly allocated in an internal capital market. If managers derive utility from the funds under their purview, the possibility of implementing a 'winner-picking' policy, while optimizing resources allocation *ex post*, reduces managerial *ex ante*. Consider the case where headquarters reallocate the cash flow generated in one division to finance the project of another division. Taking away from the manager the cash flow his division has generated has the negative implication of reducing the *ex ante* incentives for division managers to spend effort to generate the cash flow. The reduced managerial incentives can more than offset the gains of reallocating funds to the most profitable divisions. In other words, the possibility of implementing a 'winner-picking' policy is both the bright side and the dark side of internal capital markets.

This observation has the following implications. First, a profit-maximizing headquarters will face a classic time inconsistency problem. Once the funds are generated, headquarters would like to exercise ‘winner picking’ to the highest extent. If commitment is not possible, this *ex post* maximizing behaviour by headquarters will damage *ex ante* incentives at the divisional level, and it may cause a loss of value for the corporation, leading the diversified corporation to trade at a discount with respect to a comparable portfolio of stand alone firms. The discount is particularly severe when divisions are *ex post* similar in terms of investment opportunities, i.e. when ‘winner-picking’ has a limited potential for creating value. In this case the negative effect on the reduced managerial incentives will be the dominating effect. On the other hand, when *ex post* diversity in investment opportunities is large, so that the profits generated from reallocation are also large, then the advantages of ‘winner-picking’ dominate over the reduced managerial incentives, and the diversified firm trades at a premium. Conversely, diversity in the *ex ante* profitability of divisions increases the likelihood that a conglomerate trades at a discount. If one division has a very high probability of being the most profitable one, the incentives for the manager of the less profitable division are seriously reduced. Therefore the cash flow that can be reallocated to the most profitable division is also reduced, limiting the gains of the ‘winner-picking’ policy. Second, a policy in which headquarters commit to let divisions retain a part of the cash flow generated independently of the investment opportunities may well be a policy that maximizes expected profit. Allowing firms to retain part of their funds may be an efficient way to provide incentives to divisional managers. This is in line with the evidence reported by Shin and Stulz (1998). They find that the investment in a given division is more sensitive to the division's cash flow than to other divisions' cash flow. In other words, divisions are given more funds for investment if they produced more cash. While this finding is generally interpreted as evidence that conglomerates do not allocate funds efficiently across divisions according to our model it is consistent with the efficient functioning of internal capital markets. Linking investment to the cash flow produced and not only to investment opportunities restores the right balance between the goal of allocating resources efficiently and the goal of providing high power incentives to division managers.

Reallocation of Corporate Resources and Managerial Incentives in Internal Capital Markets

Sandro Brusco* Fausto Panunzi§

July 2000

Abstract

One distinguishing feature of internal capital markets is their ability to reallocate funds in favor of the most profitable divisions (*winner-picking*). Yet, diversified firms often trade at a discount with respect to their focused counterparts. The literature has tried to explain the apparent misallocation of resources with lobbying activities or power struggles. We show that the diversification discount can be explained even in a model where resources are efficiently allocated *ex post*. When managers obtain utility from the funds under their purview, moving funds across divisions may diminish their incentives. The *ex ante* reduction in managerial incentives can more than offset the increase in firm value due to the *ex post* efficient reallocation of funds.

If headquarters have some commitment power, it is in general optimal to commit not to reallocate at least a fraction of funds. As a result, the investment in a given division is (optimally) more sensitive to the division's cash flow than to other divisions' cash flow, as confirmed by the empirical studies on internal capital markets.

Our theory complements the view that links the diversification discount to the inefficient functioning of internal capital markets.

1 Introduction

The analysis of the functioning of internal capital markets, that is the allocation of funds among different divisions of a conglomerate firm, is a relatively young topic. The general theme coming both from the theoretical and the

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empirical literature is that diversified firms destroy value. Diversified firms trade on average at a discount relative to a portfolio of focused firms in the same industries, as reported by Berger and Ofek (1995), Servaes (1996) and Lins and Servaes (1999). But if there is by now a wide consensus on the idea that diversification is value-decreasing, it is much less clear why this is the case. To address this issue, one needs to understand how internal capital markets work and how they differ from external capital markets. One major difference between internal and external capital markets has been pointed out by Stein (1997). In an internal capital market the headquarters can create value by reallocating scarce funds across projects. In Stein's words, "*For example, the cash flow generated by one division's activities may be taken and spent on investment in another division, where the returns are higher... Simply put, individual projects must compete for scarce funds, and headquarters' job is to pick winners and losers in this competition.*" Stein denotes this activity of headquarters in a conglomerate firm as 'winner-picking'.

Contrary to the empirical findings, Stein's model suggests that internal capital market should create value and thus a premium for diversified firms. One possible way to solve this apparent paradox is to argue that the discount of diversified firms is due to misallocation of resources in internal capital markets. For instance, funds can be misallocated in an internal capital market because of lobbying activities (Scharfstein and Stein 2000) or power struggles (Rajan, Servaes and Zingales 2000).

In this paper we argue that in order to explain the discount of diversified firms we do not need to assume any misallocation of funds in internal capital markets. Conglomerates can trade at a discount even if resources are (*ex post*) perfectly allocated in an internal capital market. If managers derive utility from the funds under their purview, the possibility of implementing a 'winner-picking' policy, while optimizing resources allocation *ex-post*, reduces managerial incentives *ex-ante*. To fix ideas, consider the example used by Stein where headquarters reallocate the cash flow generated in one division to finance the project of another division. Taking away from the manager the cash flow his division has generated has the negative implication of reducing the *ex ante* incentives for division managers to spend effort to generate the cash flow. The reduced managerial incentives can more than offset the gains of reallocating funds to the most profitable divisions. In other words, the possibility of implementing a 'winner-picking' policy is both the bright side and the dark side of internal capital markets.

Specifically, we consider a two-period model with two initially identical divisions and a headquarter. Division managers receive private benefits as a

proportion of the gross return of the division they run. Headquarters maximize total firm value. In the first period the two division managers have to exert a non verifiable effort to generate cash flow that will be reinvested inside the firm in the second period. Before the second period, the headquarters receive a (perfect) signal about the second period profitability of the two divisions. Then, at period two, the headquarters can reallocate cash flow across division. When divisions are separated, each division reinvests the cash flow it has generated in first period. On the contrary, in the diversified firm, the headquarters will redistribute the cash flow to the most profitable division. The redistribution has two effects: on the one hand it creates value, but on the other hand it reduces the rent for the manager of the (*ex post*) less profitable division. Anticipating the possibility of being expropriated, each division manager will reduce his effort. Consequently total cash flow to be reinvested in period two will decrease.

This observation has the following implications. First, a profit-maximizing headquarters will face a classic time inconsistency problem. Once the funds are generated, headquarters would like to exercise 'winner picking' to the highest extent. If commitment is not possible, this *ex post* maximizing behavior by headquarters will damage *ex ante* incentives at the divisional level, and it may cause a loss of value for the corporation, leading the diversified corporation to trade at a discount with respect to a comparable portfolio of stand alone firms. The discount is particularly severe when divisions are *ex post* similar in terms of investment opportunities, i.e. when 'winner-picking' has a limited potential for creating value. In this case the negative effect on the reduced managerial incentives will be the dominating effect. On the other hand, when *ex post* diversity in investment opportunities is large, so that the profits generated from reallocation are also large, then the advantages of 'winner-picking' dominate over the reduced managerial incentives, and the diversified firm trades at a premium.[1] Conversely, diversity in the *ex ante* profitability of divisions increases the likelihood that a conglomerate trades at a discount. If one division has a very high probability of being the most profitable one, the incentives for the manager of the less profitable division are seriously reduced. Therefore the cash flow that can be reallocated to the most profitable division is also reduced, limiting the gains of the 'winner-picking' policy.[2]

[1] It is not uncommon to observe diversified firms trading at a premium. Rajan, Servaes and Zingales (2000) report that 39.3 percent of diversified firms in their sample traded at a premium in 1990.

[2] This prediction is consistent with the empirical findings of Rajan *et al.* (2000).

Second, a policy in which headquarters commit to let divisions retain a part of the cash flow generated independently of the investment opportunities may well be a policy that maximizes expected profit. Allowing firms to retain part of their funds may be an efficient way to provide incentives to divisional managers. This is in line with the evidence reported by Shin and Stulz (1998). They find out that the investment in a given division is more sensitive to the division's cash flow than to other divisions' cash flow. In other words, divisions are given more funds for investment if they produced more cash. While this finding is generally interpreted as evidence that conglomerates do not allocate funds efficiently across divisions,[3] according to our model it is consistent with the efficient functioning of internal capital markets. Linking investment to the cash flow produced and not only to investment opportunities restores the right balance between the goal of allocating resources efficiently and providing high power incentives to division managers. We show that it can be optimal to allow the less profitable division to retain a higher fraction of its cash flow. In fact, the manager of the less profitable division anticipates that it is likely that funds will be allocated to the other division. To restore his incentives he must be granted the possibility of using a high fraction of the cash flow produced by his division.

The basic intuition that *ex post* interference by the principal may be harmful for incentives is of course not new. For example, Aghion and Tirole (1997) show that if the principal intervenes too often in the decisional process, it can stifle the agent's initiative. In their parlance, it may be *ex ante* optimal for the principal to leave the agent's with real authority over the decisional process, even though the agent may *ex post* use her real authority to take decisions that are suboptimal for the principal.[4] The paper closest to ours is Rotemberg and Saloner (1994), who discuss how, in the presence of incomplete contracts, firms may benefit from restricting the scope of their activities. Their basic idea is that conglomerate firms have a bigger range of projects to implement. As a result, they are more likely to implement a project that it is not ideal for providing *ex ante* incentives. Rotemberg and

[3]The underlying idea is that when internal capital markets work efficiently, the origin of internal funds should not matter: The amount of funds allocated for investment to a division should only depend on how investment opportunities present in the division compare to investment opportunities in other divisions.

[4]Stein (2000) points out that managers' incentives may be blunted when they do not have ultimate authority. However, his model addresses a different issue, namely how decentralized and hierarchical firms differ in terms of their ability to generate information about investment projects and allocate capital to these projects efficiently.

Saloner (1994) also present an application of their model to internal capital markets, showing that it may be optimal to force each division to use only funds that it has generated itself. We extend their argument pointing out the cases where internal capital markets are less likely to be beneficial. Moreover we show that allowing divisions to retain a fraction of their cash flow can be superior to both a pure internal capital market and to a narrow business strategy. Gertner, Scharfstein and Stein (1994) point out that giving control rights to capital providers in an internal capital market may be costly in that it diminishes managerial incentives. The manager of a division is more vulnerable to the opportunistic behavior by corporate headquarters than a manager of a firms receiving financing either from a bank or from an external financial market because headquarters have control rights over the division's assets. Contrary to a bank, headquarters can liquidate the assets even when the division performs well. This is the main difference with our model. In Gertner, Scharfstein and Stein (1994) the hold up problem between headquarters and divisions holds irrespective of the possibility of reallocating resources across divisions. In our model it is precisely the 'winner-picking' ability of the headquarters that blunts managerial incentives. De Motta (1999) also studies managerial incentives in internal capital markets. His point is that division managers try to influence the internal capital market's assessment of their division in order to boost their level of funding. When corporate headquarters learn the characteristics of the divisions, current performance becomes less relevant for the allocations of funds and thus managerial incentives are reduced. In his model the difference between external and internal capital markets is the informational advantage of the latter. In our model the distinctive feature is the headquarters' ability to reallocate funds across divisions and informational asymmetries do not play any role.

As a final remark, we stress that we do not want to argue that resources are indeed allocated efficiently in a internal capital market. Power struggles, influence activities etc. are surely present in most corporations, and such inefficiencies contribute to reducing the value of diversification.[5] Our

[5]For instance, Rajan *et al.* (2000) find that multi-segment firms allocate relative more than their stand-alone counterparts in 'weak' lines of business and relatively less to segments in 'strong' lines of business. Scharfstein (1998) finds that the investment of conglomerate divisions is virtually insensitive to investment opportunities, as measured by the industry q's. Lamont (1997) shows that resource allocation in diversified firms is different from that in focused firms and less sensitive to indicators of investment value such as Tobin's q.

point is that diversified firms may well trade at a discount *even if internal capital markets allocate funds efficiently.* The optimal policy *ex post* is not necessarily the optimal policy *ex ante*.

The paper is organized as follows. In section 2 we describe the basic model. Section 3 illustrates the main effects of the 'winner picking' policy, comparing the performance of a diversified corporation in which funds are allocated *ex post* efficiently with the performance of a 'stand alone' firm. We discuss the optimal funding policy in section 4. Section 5 contains the conclusions, and an appendix collects the proofs.

2 The Model

We use a slightly modified version of Scharfstein and Stein (2000). Our model has three agents: Headquarters (H) and two division managers, M_1 and M_2. Within each division there are assets in place and new investment opportunities. Each division manager derives private benefits from the assets of her division only, while headquarters are interested in total returns. The timing of the model is as follows.

- At $t = 0$ each manager i works with the assets already in place in his division. The manager chooses a level of effort e_i, and this determines the cash flow produced by division i at $t = 1$, which we denote as C_i. We assume for simplicity $C_i = e_i$. The disutility of effort e_i is $\psi(e_i) = \frac{e_i^2}{2}$.[6]

- At $t = .5$ the two managers and headquarters observe a signal s that provides information on the productivity of the investment projects available at $t = 1$ in the two divisions.

- At $t = 1$ headquarters observes the cash flow produced by the two divisions, C_1 and C_2 and redistributes funds to the divisions. The old assets in place are fully depreciated. We assume that the firm has no access to external finance, so that $C_1 + C_2$ is the total amount of funds that can be reinvested in period 2. Headquarters have the power to allocate funds across divisions in a diversified firm. We denote with K_i

[6] Most of the results in section 3 hold for any $\psi(e)$ increasing and convex. We impose a specific functional forms in order to derive more detailed predictions concerning optimal reallocation rules.

the funds assigned to division i. We assume that headquarters allocate all funds to the divisions, so that $K_1 + K_2 = C_1 + C_2$.[7]

- At $t = 2$ the investment in division i yields a cash flow[8] $K_i \widetilde{R}_i$.

For simplicity we will assume that the signal s can only take two values. If $s = s_1$ then the expected return in the first division is higher than in the second, that is $E\left(\widetilde{R}_1 \middle| s_1\right) > E\left(\widetilde{R}_2 \middle| s_1\right)$, while if $s = s_2$ the opposite occurs, that is $E\left(\widetilde{R}_2 \middle| s_2\right) > E\left(\widetilde{R}_1 \middle| s_2\right)$.[9] In order to simplify further we assume:

$$E\left(\widetilde{R}_1 \middle| s_1\right) = E\left(\widetilde{R}_2 \middle| s_2\right) = \bar{R} \qquad E\left(\widetilde{R}_2 \middle| s_1\right) = E\left(\widetilde{R}_1 \middle| s_1\right) = \underline{R}.$$

We define $\Delta = \bar{R} - \underline{R}$ and assume $\underline{R} > 0$. The assumption $\underline{R} > 0$ is without loss of generality: it just says that the divisional manager can at most squander completely the funds obtained. At last, we define $p = \Pr(s = s_1)$, and assume $p \in (0, 1)$. Given the assumption on the support of s, we have $\Pr(s = s_2) = 1 - p$.

Finally we need to specify the objective functions of the headquarters and of division managers. Headquarters maximize total returns. Concerning division managers, we follow Stein (1997) and assume that each division manager receives private benefits of control that are proportional to the gross output of its division. More precisely, we assume that in each period a division manager reaps private benefits equal to a fraction ϕ of the cash flow generated by his division; C_i is the first period cash flow, and $K_i \widetilde{R}_i$ is the second period cash flow.[10] This assumption has the following implications:

[7] We assume that all the cash flow generated is reinvested. This policy is optimal for the headquarters only if the returns of the projects are above one (assuming a zero interest rate). We will ignore this problem for simplicity.

[8] As in Scharfstein and Stein (2000), we assume that in the second period the divisional managers do not have to exert any effort. This is obviously non-essential.

[9] Note that we assume that the profitability of each division in period 2 is exogenous, that is it does not depend on the managers' effort. In a more general framework, the return on the investment of each division should be a function of both managerial effort and 'luck'. Then there would be a countervailing effect: competition for funds may boost managerial incentives. However in such a framework we should also relax the assumption that division managers are risk neutral. With managerial risk aversion, competition for funds may again stifle managerial incentives. We ignore these complications in what follows.

[10] As usual, these private benefits can be thought of in a number of ways: the usual perks, the psychic benefits from empire building, etc.

a) each manager always prefers more capital to less, but conditional on being given a certain amount of capital, each manager tries to invest it in the most profitable project available to him. In other words, managers are empire builders, but they prefer more profitable empires to less profitable empires.

b) it is the possibility of reallocating resources across divisions that may create a divergence of interests between the headquarters and the division managers. Without the possibility of 'winner-picking' there would be no conflict of interests between headquarters and divisions.

As it is common in this literature, we assume non-responsiveness of managers to monetary incentives.[11]

Formally, the utility function of the manager of division i is given by:

$$U\left(e_i, K_i, \widetilde{R}_i\right) = \phi\left(C_i + K_i\widetilde{R}_i\right) - \frac{e_i^2}{2}$$

For simplicity we assume that private benefits are psychic, that is they do not derive from 'stealing' or misusing the company's assets.[12] Finally, the risk-free interest rate is normalized to zero.

3 The Bright and the Dark Side of 'Winner-Picking'

In this section we want to highlight the basic trade-off between incentive provision and *ex post* efficiency. We therefore consider two alternative 'extreme' organizational forms: The 'stand-alone' one, in which divisions are completely separate an no internal capital market exists, and the pure internal capital market one, in which capital is entirely assigned to the *ex post* most efficient division.

In the stand-alone case, by exercising effort e_i at time zero the manager of division i obtains an amount e_i of funds for reinvestment at time 1. Given the information at time 0, the expected cash flow generated by those funds at $t = 2$ is:

$$\left[p_i\left(\underline{R} + \Delta\right) + \left(1 - p_i\right)\underline{R}\right] e_i = \left[\underline{R} + p_i\Delta\right] e_i$$

[11] Non-responsiveness to monetary incentives is a common, although extreme, assumption in this literature. One possible way to justify it is to assume that agents are infinitely risk-averse in their income.

[12] Given that we assume that private benefits are a constant fraction of the division's gross output, this assumption has no serious implications for the analysis. If private benefits were extracted at the expense of profits, we should multiply the headquarters' profit by a constant.

where $p_1 = p$ and $p_2 = 1 - p$. Since $C_i = e_i$, the problem of M_i at time zero is:

$$\max_{e_i} \ \phi e_i + \phi \left[\underline{R} + p_i \Delta\right] e_i - \psi \left(e_i\right)$$

The necessary and sufficient condition for a maximum is:

$$e_i = \phi \left(1 + \underline{R} + p_i \Delta\right) \tag{1}$$

We call e^i_{SA} the solution to this equation. The sum of the expected profit for the two divisions under the stand alone solution is given by:

$$\Pi_{SA} = \left(\underline{R} + p\Delta\right) e^1_{SA} + \left(\underline{R} + \left(1 - p\right)\Delta\right) e^2_{SA}.$$

Consider now the pure ICM case. Now headquarters observe s_i at time $t = 0.5$ and then allocate entirely the funds to division i. Therefore, division i faces a probability $1 - p_i$ of having zero funds and a probability p_i of having all funds, $C_1 + C_2 = e_1 + e_2$.

The problem for M_i is therefore:

$$\max_{e_i} \ \phi e_i - \psi \left(e_i\right) + \phi p_i \left(\underline{R} + \Delta\right) \left(e_i + e_{-i}\right)$$

and the necessary and sufficient condition for a maximum is:

$$e_i = \phi \left(1 + p_i \left(\underline{R} + \Delta\right)\right) \tag{2}$$

Let us call e^i_{ICM} the solution to this equation. The expected profit for headquarters is given by:

$$\Pi_{ICM} = \left(\underline{R} + \Delta\right) \left(e^1_{ICM} + e^2_{ICM}\right)$$

We can now compare the expected profits under the pure ICM and the stand alone form. We have:

$$\Pi_{ICM} - \Pi_{SA} = \Delta \left(\left(e^1_{ICM} - p e^1_{SA}\right) + \left(e^2_{ICM} - \left(1 - p\right) e^2_{SA}\right)\right)$$

$$-\underline{R} \left(\left(e^1_{SA} + e^2_{SA}\right) - \left(e^1_{ICM} + e^2_{ICM}\right)\right)$$

We can write:

$$e^i_{ICM} - p_i e^i_{SA} = \left(1 - p_i\right) e^i_{ICM} - p_i \left(e^i_{SA} - e^i_{ICM}\right)$$

so that we have:

$$\Pi_{ICM} - \Pi_{SA} = \Delta \left((1-p)\, e^1_{ICM} + p e^2_{ICM} \right) \tag{3}$$

$$- \left[(\underline{R} + p\Delta) \left(e^1_{SA} - e^1_{ICM} \right) + (\underline{R} + (1-p)\,\Delta) \left(e^2_{SA} - e^2_{ICM} \right) \right]$$

This can be read as follows. The term

$$\Delta \left((1-p)\, e^1_{ICM} + p e^2_{ICM} \right)$$

represents the 'winner picking' effect. With probability $(1-p)$, the second division is the more profitable one. In the SA case, this does not lead to any extra funding for the firm. In the ICM case, division 2 obtains the cash generated by division 1, that is e^1_{ICM}. Expected profit therefore increases by $\Delta\,(1-p)\, e^1_{ICM}$. A similar effect is at work when division 1 is the more profitable. This term is the bright side of internal capital markets: Resources are *ex-post* allocated to the best investment.

The key point of our paper is the second term. Notice that this term would be zero if we had $e^i_{ICM} = e^i_{SA}$. However, since:

$$\phi\,(1 + p_i(\underline{R} + \Delta)) < \phi\,(1 + \underline{R} + p_i\Delta)$$

we have $e^i_{ICM} < e^i_{SA}$. This is the 'incentive effect' denoting the reduction in expected profits as a consequence of the reduced incentives that managers have when funds are redistributed across divisions. In fact, the term $(\underline{R} + p_i\Delta)$ denotes the gross expected return on funds invested in the division, and the reduction in expected profit in division i is equal to the reduction in the amount of funds generated (that is, $e^i_{SA} - e^i_{ICM}$) times this return.

The sign of $\Pi_{ICM} - \Pi_{SA}$ depends on the parameters as follows.

Proposition 1 a) *For any given value of p and $\underline{R}$ there exists a value Δ^* such that if $\Delta < \Delta^*$ then $\Pi_{ICM} - \Pi_{SA} < 0$, while if $\Delta > \Delta^*$ then $\Pi_{ICM} - \Pi_{SA} > 0$;*

b) *For any given value of p and Δ there exists a value $\underline{R}^*$ such that if $\underline{R} < \underline{R}^*$ then $\Pi_{ICM} - \Pi_{SA} > 0$, while if $\underline{R} > \underline{R}^*$ then $\Pi_{ICM} - \Pi_{SA} < 0$;*

c) *For any given value of Δ and $\underline{R}$ the difference $\Pi_{ICM} - \Pi_{SA}$ reaches its maximum at $p = \frac{1}{2}$, and it is a decreasing function of p on the interval $\left(\frac{1}{2}, 1\right)$.*

Part a) states that when the divisions are *ex post* similar (that is, Δ is small) there is not much point in reallocating funds, so that the predominant effect of internal capital markets is the incentive reduction. In this case the diversified firm trades at a discount with respect to the stand-alone benchmark.[13] As Δ increases reallocation creates more value, and the bright side of internal capital markets eventually prevails. Part b) tells a similar story. As $\underline{R}$ approaches 0 we have $e^i_{SA} \cong e^i_{ICM}$. The reason is that in the stand-alone solution funds are basically wasted if the division is not the more profitable one. The incentives for the divisional manager are therefore almost identical under the SA and the ICM regimes: He will obtain utility from the funds generated if and only if his division turns out to be the more profitable one. The incentive reduction effect is therefore negligible, and the ICM form is more profitable because it guarantees a better allocation of funds. As $\underline{R}$ increases the incentive effect becomes more and more important, and at some point it prevails.

Part c) addresses the issue of how *ex ante* differences in profitability between the two divisions lead to a higher or lower diversification discount (or premium). The advantage of the ICM form is at its maximum when the two divisions are *ex ante* identical. As the difference between divisions increases, the SA form becomes more appealing. The total quantity of cash generated under ICM and under SA does not depend on p, since an increase in p determines an increase in the funds produced in the first division and an identical decrease in the amount of funds produced in the second division.[14] This in turn implies that Π_{ICM} does not depend on p, since the total effort $e^1_{ICM} + e^2_{ICM}$ does not depend on p and the return on each dollar of cash generated is always the same, namely $\underline{R} + \Delta$ (funds are always assigned to the more profitable division). Under SA, the total effort $e^1_{SA} + e^2_{SA}$ is also constant with respect to p. However, as p increases the probability of assigning a given dollar to the more profitable division increases. This in turn increases the average return on each dollar. The bottom line is that when the divisions become more asymmetric the profitability of the ICM form remains constant, while the profitability of the SA form increases. Thus, internal capital markets are less desirable when divisions are very diverse. The negative impact of asymmetry on diversified firm has also been pointed

[13] It is worth stressing at this point that, as all the papers in this literature, we do not analyze why there is no spin-off of the two divisions.

[14] This is a special feature of the effort function we have chosen. With a more general effort function $\psi(e)$, the total effort amount of cash produced in the first period may depend on p.

out by Rajan *et al.* (2000). They find that as diversity in opportunities among divisions increase investment becomes less efficient and firms are less valuable. The difference between their paper and ours is that in their model inefficiencies are caused by funds being transferred from divisions with good investment opportunities to divisions with poor opportunities, while in our model funds always go to the most profitable division.

4 Optimal Reallocation Rules

We pointed out in the previous section that the pure ICM case is the result of the inability to commit on the part of headquarters. Such assumption is probably extreme. In general the organizational structure can be harnessed in a way that makes (at least partial) commitment to no interference. For example, the ability of the headquarters to receive relevant information on the difference of returns between the divisions depends on the structure of relations among the headquarters and the divisions. Headquarters can decide whether divisions should report on the investment projects they have available, and it can also decide how accurate the information should be. Without timely information, it would be impossible or unprofitable for headquarters to reallocate funds across divisions. Thus, a credible commitment to partial no interference can be obtained by restricting the amount and quality of information that divisions have to send to headquarters. Alternatively, headquarters may decide that only investment projects of a certain size need superior approval, allowing divisions to spend their own cash on projects of smaller size. This again would result in giving real autorithy to divisional managers over a part of the cash flow they generate.

In this section we ignore the exact microeconomic mechanism through which commitment is attained, and we simply assume that headquarters can decide the fraction of funds that each divisional manager is allowed to retain and automatically reinvest in the division.

Allowing divisions to retain a fraction of their cash flow has costs and benefits for the headquarters. On the one hand, the funds retained by the division may be reinvested in a suboptimal project from the headquarters point of view (less 'winner-picking'); on the other hand, retaining funds boosts managerial initiative. The optimal degree of intervention optimally balances these two contrasting effects.

We assume $p > \frac{1}{2}$, so that the first division is *ex ante* more profitable.[15]

[15] Obviously assuming $p < \frac{1}{2}$ would deliver symmetric results.

We denote by γ_i the share of its cash flow C_i retained by division i,[16] so that in each period a total amount of cash $(1-\gamma_1)C_1 + (1-\gamma_2)C_2$ can be reallocated by the headquarters to the division with the highest expected return. The manager of division i maximizes:

$$\phi\left\{e_i + \gamma_i e_i\left[\underline{R} + p_i\Delta\right] + p_i\left(\underline{R}+\Delta\right)\left[(1-\gamma_i)e_i + (1-\gamma_{3-i})e^*_{3-i}\right]\right\} - \frac{e_i^2}{2}$$

so that the FOC for division i is:

$$e_i = \phi\left[1 + p_i\left(\underline{R}+\Delta\right) + \left(1-p_i\right)\gamma_i\underline{R}\right]. \tag{4}$$

We denote by $e_i(\gamma_i)$ the unique solution to this equation.

We start our analysis from the following simple observation.

Proposition 2 *Suppose that H wants to implement the same level of effort $e^* = e_1 = e_2$ in the two divisions, and let γ_1^* and γ_2^* be the two levels of fund retention that attain this goal. Then $\gamma_1^* < \gamma_2^*$.*

The proposition states that whenever the same level of effort is required in the two divisions, then the *less* profitable division is allowed to retain a *higher* share of its own cash flow. The intuition is the following: due to the reallocation of funds operated by headquarters, it is harder to motivate the manager of the less profitable division, since she anticipates a higher probability that funds will be allocated to the other division. Thus, in order to extract a given amount of effort, it is necessary to allow managers of less profitable divisions to retain and have discretion over a larger fraction of their cash flow.

Notice however that in general it will be optimal for headquarters to implement different levels of effort between the two divisions. In particular, eliciting effort from the second division is more costly than eliciting effort from the first division, since a greater share of funds has to be diverted to the less productive division (in expected value), and this leads to a decrease in the amount of effort which is optimal to require to the less profitable division. This in turn reduces the share of funds necessary to implement the desired level of effort. Depending on which one of the two effects prevail,

[16] We restrict the analysis to linear retention rules, i.e. to the case where γ_i is independent of C_i. Obviously, linear retention rules are suboptimal. However, our aim in this section is simply to show that allowing divisions to retain a fraction of the cash flow they have produced can be preferable to a policy of complete reallocation. Studying optimal retention rules is beyond the scope of our paper.

the share of funds assigned to the less profitable division may be higher or lower than the share assigned to the more profitable division.

The problem can be formally addressed as follows. Headquarters' expected profit can now be written as:

$$\begin{aligned}\Pi(\gamma_1,\gamma_2) &= p\left[\left(e_1(\gamma_1)+(1-\gamma_2)e_2(\gamma_2)\right)(\underline{R}+\Delta)+\gamma_2 e_2(\gamma_2)\underline{R}\right] \\ &\quad +(1-p)\left[\gamma_1 e_1(\gamma_1)\underline{R}+\left(e_2(\gamma_2)+(1-\gamma_1)e_1(\gamma_1)\right)(\underline{R}+\Delta)\right]\end{aligned}$$

The problem is therefore:

$$\max_{\gamma_1\in[0,1],\gamma_2\in[0,1]} \Pi(\gamma_1,\gamma_2)$$

and using the expressions for $e_1(\gamma_1)$ and $e_2(\gamma_2)$ given by (4) , the unique solution is given by:

$$\gamma_1 = \min\left\{\max\left\{\frac{(\underline{R}+\Delta)\underline{R}-\Delta(1+p(\underline{R}+\Delta))}{2(1-p)\Delta\underline{R}},0\right\},1\right\}$$

$$\gamma_2 = \min\left\{\max\left\{\frac{(\underline{R}+\Delta)\underline{R}-\Delta(1+(1-p)(\underline{R}+\Delta))}{2p\Delta\underline{R}},0\right\},1\right\}$$

More generally, observe that the optimal fraction γ_i for a division having probability p_i of being the most profitable one can be written as:

$$\gamma(p_i) = \min\left\{\max\left\{\frac{(\underline{R}+\Delta)\underline{R}-\Delta(1+p_i(\underline{R}+\Delta))}{2(1-p_i)\Delta\underline{R}},0\right\},1\right\}$$

Note that $\gamma(p_i)$ can be positive, that is it can be optimal to allow divisions to retain a fraction of the cash flow they have produced. This result sheds a different light on the findings of Shin and Stulz (1998). They show that the allocation of funds does not only depend on the investment opportunities, but also on the cash flow generated by each division. More precisely, the funds allocated to a division are more sensitive to the division's cash flow than to other divisions' cash flow. This is consistent with the finding of our model. Suppose the cash flow of division i is increased by one dollar. Then the investment of division i will be increased by $\gamma_i + (1-\gamma_i)p_i$. Each division is allowed to retain a fraction γ_i of its cash flow. The remaining fraction $(1-\gamma_i)$ will be allocated to division i only when it has the most profitable investment opportunity, i.e. with probability p_i. Suppose now the cash flow of division j is increased by one dollar. Then the investment of division i will be increased by $p_i(1-\gamma_j)$. Division i will capture a fraction

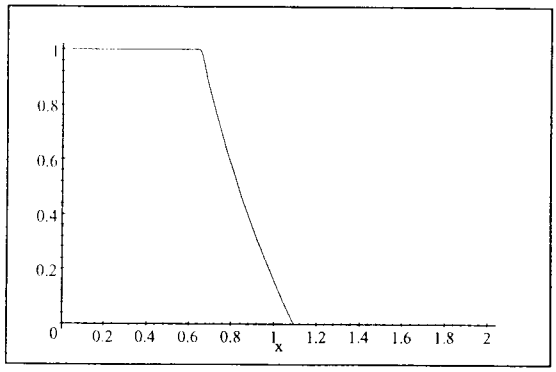
$(1-\gamma_j)$ of the cash flow of division j only when its investment opportunity is the most profitable. It is easy to note that

$$\gamma_i + (1-\gamma_i)p_i > p_i(1-\gamma_j)$$

whenever $\gamma_i > 0$ and/or $\gamma_j > 0$ and $p_i > 0$. Shin and Stulz (1998) interpret the higher sensitivity of a division's investment to its cash flow than to other divisions' cash flow as evidence that funds are not allocated efficiently in an internal capital market. According to our model, this higher sensitivity is consistent with the use of an optimal cash flow redistribution policy by the headquarters.

The general form of $\gamma(p_i)$ is the following. Take $\bar{R}$ as given. When the *ex post* difference in profitability in the second period is small (Δ is small) the gain from reallocation is limited. It is therefore optimal to provide the highest possible incentives ($\gamma = 1$). As Δ increases the gains from reallocation become large. At some point Δ^* the gains from reallocation become too large, so it is optimal to sacrifice incentives' provision. We therefore enter an intermediate range in which there is only a partial internal capital market: a fraction of the cash flow is left to the division that has produced it, in order to boost managerial effort. When the difference in *ex-post* profitability is very high, a pure internal capital market is optimal ($\gamma_1 = \gamma_2 = 0$).

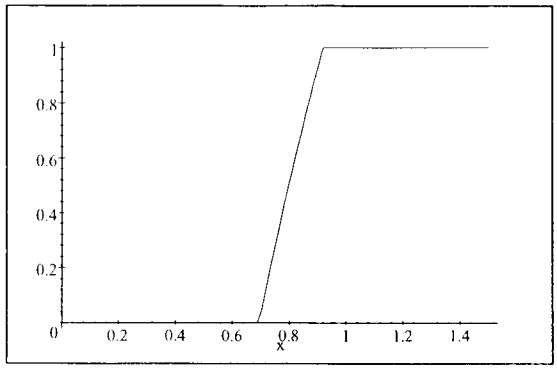
The behavior of $\gamma(p_i)$ as a function of Δ can be depicted as follows:



The fraction $\gamma(p_i)$ as a function of Δ. The parameters' values are $\underline{R} = 1.1$, $p_i = 0.55$.

Take now Δ as given. When $\underline{R}$ is close to zero, it is very costly to allow a division to retain funds. Therefore, the optimal policy will be to set

$\gamma(p_i) = 0$. As $\underline{R}$ increases it becomes less costly to provide incentives through fund retention, so that eventually $\gamma(p_i)$ becomes positive. At last, for $\underline{R}$ sufficiently high the comparative advantage of winner picking is so low that a conglomerate firm prefers to concentrate on providing incentives, setting $\gamma(p_i) = 1$.



The fraction $\gamma(p_i)$ as a function of $\underline{R}$. The parameters' values are $\Delta = 0.5$, $p_i = 0.55$.

We collect the general properties of $\gamma(p_i)$ in the following proposition.

Proposition 3 *The fraction $\gamma(p_i)$ is decreasing in Δ and increasing in $\underline{R}$.*

When Δ increases the benefits of winner picking are greater, since allocating funds to the more profitable division yields a higher return. This lowers the optimal level of γ. There is no countervailing effect, since the way in which the level of effort depends on γ is independent of Δ. In fact, the difference between the level of effort exercised at $\overline{\gamma}$ and at $\underline{\gamma}$ is:

$$e_i(\overline{\gamma}) - e_i\left(\underline{\gamma}\right) = \phi(1 - p_i)\left(\overline{\gamma} - \underline{\gamma}\right)\underline{R}$$

and it is therefore independent of Δ. This implies that the cost of decreasing γ is not affected by an increase in Δ. The net effect if therefore that a higher Δ makes a lower γ optimal.

An increase in $\underline{R}$ increases by the same amount the profitability of both retained and reallocated funds. It therefore makes the capital generated more productive, independently of its allocation. This in turn implies that it becomes convenient to increase the quantity of funds generated. The only

way to achieve this is by increasing the fraction of funds retained. This explains why γ depends positively on $\underline{R}$.

We now come to the question of how γ depends on p_i, the probability of success of the division. We first collect some preliminary results in the following proposition.

Proposition 4 *The optimal fraction $\gamma(p_i)$ is more sensitive to Δ and $\underline{R}$ as p_i increases. In particular, increased levels of Δ cause a stronger decrease in $\gamma(p_i)$ when p_i is larger, and increased levels of $\underline{R}$ cause a stronger increase in $\gamma(p_i)$ when p_i is larger.*

This points out that the optimal reaction of headquarters to an increased return on internally generated funds is quite different depending on whether this is due to an increase in Δ (more profitable divisions becoming more profitable) or $\underline{R}$ (an equal increase in profitability across division). We have already seen that in the first case there will be more 'winner picking', i.e. more funds assigned to *ex post* more profitable divisions, while in the second there will be less, i.e. higher share of retained funds across divisions. In both cases the impact will be stronger on the more profitable divisions: As Δ increases the reduction in γ is higher, while the increase in $\underline{R}$ generates a stronger increase in γ for higher values of p_i.

In general, there is a tension between two forces. Suppose Headquarters wants to generate one extra dollar from the divisions. In order to do that, it has to increase the share retained by the divisions, which in this model is the only way to generate more funds in the first period. Which division should be allowed to retain more funds? Increasing the share retained by the more profitable division is better because any dollar remaining in the division obtains a higher return. On the other hand, we have:

$$\frac{\partial e}{\partial \gamma} = \phi(1 - p_i)\underline{R}.$$

so that an increase in γ is less effective in generating more funds when p_i is high. This is the incentive effect discussed above. Providing incentives through fund retention is more effective in divisions with *ex ante* low profitability, since these divisions are more likely not to be the winner in the next period, and therefore have a lower probability of seeing again any dollar taken away for reallocation.

Our previous discussion leads to the conclusion that the first effect is the prevailing one when the reason why Headquarters wants an extra dollar is

that $\underline{R}$ has increased, while the second effect is the prevailing one when Δ has increased.

We can now finally analyze the impact of p_i on $\gamma(p_i)$.

Proposition 5 *The fraction $\gamma(p_i)$ is decreasing in p_i if the condition*

$$\Delta + \Delta^2 > \underline{R}^2$$

is satisfied, and it is increasing in p_i otherwise.

We can therefore detect two different regimes. When Δ is low and $\underline{R}$ is high then it is likely that the condition $\Delta + \Delta^2 < \underline{R}$ will be satisfied. In this case the fractions γ_1 and γ_2 of retained funds will be high, and the most profitable divisions will be allowed to retain a higher fraction of funds. These are situations in which 'winner picking' does not add much value to the firm, so incentive provision is the relevant issue. In this cases we also observe that the fraction of funds retained by the more profitable division is higher. The intuition of why it should be so comes from proposition 4: As $\underline{R}$ increases, it is optimal to increase more the fraction γ of the most profitable division, since each dollar put in the more profitable division has a higher return.

The second regime is the one in which the condition $\Delta + \Delta^2 > \underline{R}^2$ is satisfied. In this case winner picking is profitable, and the divisions are allowed to retain only a small fraction of funds, if any. However, it is the less profitable division which is allowed to retain more. Again, we can look at proposition 4 for intuition. As Δ increases the fraction of retained funds decreases for all divisions, but it is the most profitable division which suffers the highest reduction. In fact, and increase in Δ provides incentives for the divisions, since we have $\frac{\partial e_i}{\partial \Delta} = \phi p_i$. This occurs because divisional managers recognize that any dollar generated for investment will have a higher payout, and so exercise more effort. From the point of view of headquarters, this means that a lower level of γ_i is necessary to extract a given level of effort, so the optimal level of fund retention decreases. The decrease is stronger in divisions with a high p_i, since in such divisions the increase in Δ has a stronger impact on e_i.

One empirical implication of Proposition 5 is that there should be a correlation between the general level of fund retention and the (apparent) misallocation of funds. Conglomerate firms that allow high levels of fund retention will also allow more retention by the *ex ante* more profitable divisions. On the other hand, conglomerate firms that allow for low levels of

fund retention are more likely to permit a higher retention rate to divisions with poorer prospects.

5 Conclusions

The intent of this paper is to argue that one of the distinctive features of internal capital markets, that is the ability of headquarters to rellocate funds across divisions (winner-picking) is associated both with costs and benefits. The benefits derive from transferring funds to the most profitable divisions; the costs derive from the blunted managerial incentives. In other words, winner-picking is simultaneously the dark and the bright side of internal capital markets. Our theory can explain why conglomerate firms trade at a discount (or at a premium) with respect to their focused counterparts. More importantly, it does so without assuming any inefficiency in the allocation of corporate resources. We show that *ex ante* diversity in divisions' profitability increases the inefficiency of an internal capital market, confirming the findings of Rajan *et al.* (2000). Furthermore, an implication of our model is that allowing divisions to retain a fraction of the cash flow they have produced irrispectively of their investment opportunities can be an optimal policy for the headquarters since it leads to a better balance between the two conflicting goals of allocating optimally *ex post* corporate resources and providing managers with adequate *ex ante* incentives to perform. Allowing divisions to spend discretionary a fraction of their cash flow is a way of delegating real authority to division manager. As we know from Aghion and Tirole (1997), the gains of delegation in terms of boosted managerial initiative can more than compensate the loss in control of headquarters.

An important *caveat* is that we have not addressed the reasons why divisions that are very different in terms of their profitability are brought together in the same firm and why some divisions are not spun-off in those circumstances where conglomerates are inefficient. Moreover, we have assumed that all resources are internally generated, ignoring the role of external financing.

These limitations notwithstanding, we believe that the analysis of internal capital markets in terms of allocation of delegation of authority may be a promising direction for future research.

Appendix

Proof of proposition 1. Using the expression for $\Pi_{ICM} - \Pi_{SA}$ given by

(3) and the expressions for e_{SA}^{i}, e_{ICM}^{i} given by (1) and (2) we obtain:

$$\Pi_{ICM} - \Pi_{SA} = \phi \left[\Delta + 2p\left(1-p\right)\Delta^{2} - \underline{R}^{2} \right]$$

This is a strictly increasing function of Δ, a strictly decreasing function of $\underline{R}$ (over the range $\underline{R} \geq 0$). For a given pair $\underline{R}$, Δ the function reaches a maximum at $p = \frac{1}{2}$ and is decreasing in p for $p > \frac{1}{2}$. ■

Proof of proposition 2. By equating the two right hand sides of the two first order conditions, we have:

$$p\left(\underline{R}+\Delta\right) + \left(1-p\right)\gamma_{1}^{*}\underline{R} = \left(1-p\right)\left(\underline{R}+\Delta\right) + p\gamma_{2}^{*}\underline{R}$$

so that:

$$\gamma_{2}^{*} = \left(\frac{2p-1}{p}\right)\frac{\left(\underline{R}+\Delta\right)}{\underline{R}} + \frac{\left(1-p\right)}{p}\gamma_{1}^{*}.$$

The condition $\gamma_{2}^{*} > \gamma_{1}^{*}$ is equivalent to:

$$\left(\frac{2p-1}{p}\right)\frac{\left(\underline{R}+\Delta\right)}{\underline{R}} + \frac{\left(1-p\right)}{p}\gamma_{1}^{*} > \gamma_{1}^{*}$$

or:

$$\frac{\left(\underline{R}+\Delta\right)}{\underline{R}} > \gamma_{1}^{*}$$

which is always satisfied since we have restricted attention to $\gamma_{1} \in [0,1]$. ■

Proof of proposition 3. Let Δ^{*} be the value of Δ such that:

$$\frac{\left(\underline{R}+\Delta^{*}\right)\underline{R} - \Delta^{*}\left(1+p_{i}\left(\underline{R}+\Delta^{*}\right)\right)}{2\left(1-p_{i}\right)\Delta^{*}\underline{R}} = 1$$

and let Δ^{**} be the value of Δ such that:

$$\frac{\left(\underline{R}+\Delta^{**}\right)\underline{R} - \Delta^{**}\left(1+p_{i}\left(\underline{R}+\Delta^{**}\right)\right)}{2\left(1-p_{i}\right)\Delta^{**}\underline{R}} = 0$$

Then $\gamma\left(p_{i}\right) = 1$ for $\Delta \leq \Delta^{*}$ and $\gamma\left(p_{i}\right) = 0$ for $\Delta \geq \Delta^{**}$, so that the function is constant. If $\Delta \in \left(\Delta^{*}, \Delta^{**}\right)$ then $\gamma\left(p_{i}\right)$ is given by:

$$\gamma\left(p_{i}\right) = \frac{\left(\underline{R}+\Delta\right)\underline{R} - \Delta\left(1+p_{i}\left(\underline{R}+\Delta\right)\right)}{2\left(1-p_{i}\right)\Delta\underline{R}}$$

so that:

$$\frac{\partial\gamma}{\partial\Delta} = -\frac{1}{2}\frac{\alpha\Delta^{2}+\underline{R}^{2}}{\left(1-p_{i}\right)\Delta^{2}\underline{R}} < 0.$$

To prove that $\gamma(p_i)$ is increasing $\underline{R}$, let $\underline{R}^*$ be the value such that:

$$\frac{(\underline{R}^* + \Delta)\underline{R}^* - \Delta(1 + p_i(\underline{R}^* + \Delta))}{2(1-p_i)\Delta\underline{R}^*} = 0$$

and $\underline{R}^{**}$ the value such that:

$$\frac{(\underline{R}^{**} + \Delta)\underline{R}^{**} - \Delta(1 + p_i(\underline{R}^{**} + \Delta))}{2(1-p_i)\Delta\underline{R}^{**}} = 1$$

For $\underline{R} \leq \underline{R}^*$ we have $\gamma(p_i) = 0$, and for $\underline{R} \geq \underline{R}^{**}$ we have $\gamma(p_i) = 1$. If $\underline{R} \in (\underline{R}^*, \underline{R}^{**})$ then:

$$\frac{\partial\gamma}{\partial\underline{R}} = \frac{1}{2}\frac{\underline{R}^2 + \Delta + \Delta^2 p_i}{(1-p_i)\Delta\underline{R}^2} > 0. \blacksquare$$

Proof of proposition 4. When $\gamma(p_i)$ is constant (either 0 or 1) then $\frac{\partial\gamma(p_i)}{\partial\Delta\partial\alpha} = \frac{\partial\gamma(p_i)}{\partial\underline{R}\partial\alpha} = 0$. When $\gamma(p_i) \in (0,1)$ we have:

$$\frac{\partial\gamma(p_i)}{\partial\Delta\partial\alpha} = -\frac{1}{2}\frac{\Delta^2 + \underline{R}^2}{(1-p_i)^2\Delta^2\underline{R}} < 0$$

and:

$$\frac{\partial^2\gamma(p)}{\partial p\partial\underline{R}} = \frac{1}{2}\frac{\Delta^2 + \underline{R}^2 + \Delta}{(1-p)^2\Delta\underline{R}^2} > 0. \blacksquare$$

Proof of proposition 5. When γ is constant then $\frac{\partial\gamma}{\partial\alpha} = 0$. In the region in which $\gamma(p_i) \in (0,1)$ we have:

$$\frac{\partial\gamma}{\partial\alpha} = \frac{1}{2}\frac{\underline{R}^2 - \Delta - \Delta^2}{(1-p_i)^2\Delta\underline{R}}$$

so that the sign of is equal $\frac{\partial\gamma}{\partial\alpha}$ is equal to the sign of $\left(\underline{R}^2 - \Delta - \Delta^2\right)$. $\blacksquare$

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