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ABSTRACT

Inequality and Mobility*

We use a general equilibrium OLG model to analyse the relation between intergenerational social mobility and wage inequality. We show that the correlation between mobility and inequality depends on which factor caused the change in inequality. The model can thus help discriminate between different competing explanations of the recent rise in US wage inequality. Under reasonable assumptions, skill-biased technical change tends to increase upward mobility, thereby causing a positive correlation between wage inequality and mobility. Public subsidies to education reduce inequality, but the effect on mobility is ambiguous and depends on how well households with non-skilled parents can take advantage of the subsidy. The relation between subsidies and upward mobility is always concave in the short run and may also be so in the long run. Under some circumstances, the relationship between public support for education and mobility can follow an inverted U-pattern. The model can thus provide an explanation for different patterns of inequality and social mobility in Europe and the US.

JEL Classification: H52 and J62
Keywords: educational policy, inequality, intergenerational mobility and skill-bias

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NON-TECHNICAL SUMMARY

This Paper analyses the relationship, or the correlation, between inequality and mobility. By inequality, we mean wage inequality between skilled and unskilled workers and by mobility, upward intergenerational mobility. We argue that an analysis of the relationship between inequality and mobility should be undertaken within a general equilibrium framework. Since equality and mobility interact with one another, both variables should be determined simultaneously and dynamically. In particular, future inequality creates incentives for upward mobility on the one hand while, on the other, mobility also affects inequality by shifting the relative supplies of different types of labour. Thus, the equilibrium correlation between mobility and inequality is ambiguous.

Our analysis identifies two types of variables affecting the correlation between inequality and mobility. The first type is technology or productivity variables, which affect the relative demand for skilled versus unskilled workers. Changes in these variables lead to a positive short-run correlation between inequality and mobility, since an increase in inequality driven by skill-biased technological change strengthens the incentive to become skilled. We call this the ‘incentive effect’. Changes in inequality and mobility can also be due to changes in educational variables, where our focus is on public support to education. We show that such support unambiguously reduces inequality by reducing the effort requirements for becoming skilled and thus increasing the number of skilled individuals. However, public support for education has an ambiguous effect on mobility. On the one hand, public support makes it easier to become skilled, but on the other, the resulting reduction in inequality reduces the incentive to learn. We show that support for education can reduce upward mobility if the effect on downward mobility is stronger than on upward mobility, namely if children of educated parents can use the support more efficiently, due to their background. We show that the effect of public support on mobility can have an inverted U shape, increasing mobility up to some level and then reducing it. Our analysis is presented in two stages, starting with a basic model to clarify the issues. The basic model is then extended to allow for parental support for education.

In the basic model, we assume that individuals can either work as unskilled or choose to go to school and become skilled. Becoming skilled requires an effort, which varies randomly across individuals and may depend on the individual’s social background. Our measure of inequality is the wage ratio between skilled and unskilled, and it is affected both by productivity and mobility since the wage ratio depends on the ratio of skilled to unskilled in the economy. Then, we analyse the equilibrium in the short and the long run and reach the following results. An exogenous increase in the relative productivity of skilled workers raises inequality and increases upward mobility through the incentive effect. An increase in public support for education always reduces inequality, but the effect on mobility depends on the degree to which it is biased in favour of children with skilled parents.

Next, we extend the basic model in order to provide a specific intergenerational link between the choice of whether to become educated or not and the individual's social background. For this purpose, we introduce the following assumptions: First, we assume that parents derive utility by seeing their children become skilled and parents may thus choose to provide support to their children's education, which reduces the effort associated with learning and here is called 'tutoring'. Furthermore, educated parents are assumed to have an advantage in supplying tutoring for their children: they have better knowledge of which books to buy, which topics to encourage, etc. An increase in public support for education reduces tuition and all parents have more resources available for tutoring. However, we will see that children with educated parents may gain more from an across the board increase in public support for education, since educated parents tend to supply tutoring and reduce effort more effectively. Under some circumstances, this leads to a negative effect on upward mobility by public support for education.

Our Paper is related to a number of important empirical issues. The first concerns the differences in inequality and mobility between Europe and the US. Inequality is much higher in the US than in continental Europe while social mobility appears to be higher in the US than in many European countries. The low degree of social mobility in many European countries is surprising, since while most Europeans enjoy an almost free education from the cradle to university, American education is far from free. Our model can provide an explanation to these stylized facts, however. Greater support for education in European countries benefits the children of educated parents more, due to their taking advantage of general subsidies. As a result, the high level of support for education might have reached the level where it tends to reduce upward mobility.

Another issue related to our analysis is the debate on the causes for the rise in wage inequality in the US (and other countries) in recent decades. Three major alternative explanations to this phenomenon have been proposed. One is skill-biased technical change, namely the computer and communication revolution. A second explanation is trade liberalization, which increases imports from LDCs and reduces demand for unskilled labour. The third explanation is a reduction in the supply of skilled workers, probably due to declining public support for education. Our model examines all three possibilities within a unified general equilibrium framework. The first two explanations – skill-biased technical change and trade liberalization – can be interpreted as increases in the relative productivity of skilled individuals and should thus increase both wage inequality and upward mobility. The third explanation, a reduction in public support for education, indeed raises inequality in our model, but its effect on mobility is ambiguous. Hence, our model indicates that a careful examination of recent developments in upward mobility in the US is important to this debate.

1 Introduction

This paper analyzes the relationship, or the correlation, between inequality and mobility. By inequality, we mean wage inequality between skilled and unskilled workers and by mobility, upward intergenerational mobility. We argue that an analysis of the relationship between inequality and mobility should be undertaken within a general equilibrium framework. Since equality and mobility interact with one another, both variables should be determined simultaneously and dynamically. In particular, future inequality creates incentives for upward mobility on the one hand while, on the other, mobility also affects inequality by shifting the relative supplies of different types of labor. Thus, the equilibrium correlation between mobility and inequality is ambiguous.

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Next, we extend the basic model in order to provide a specific intergenerational link between the choice of whether to become educated or not and the individual's social background. For this purpose, we introduce the following assumptions: First, we assume that parents derive utility by seeing their children become skilled and parents may thus choose to provide support to their children's education, which reduces the effort associated with learning and here is called *tutoring*. Furthermore, educated parents are assumed to have an advantage in supplying tutoring for their children: they have better knowledge of which books to buy, which topics to encourage, etc. An increase in public support to education reduces tuition and all parents have more resources available for tutoring. However, we will see that children with educated parents may gain more from an across the board increase in public support to education, since educated parents tend to supply tutoring and reduce effort more effectively. Under some circumstances, this leads to a negative effect on upward mobility by public support to education.

The idea that households differ in their ability to take advantage of a general public subsidy is not new. An early and general formalization is provided by Bruno (1976), who model public expenditure and individual endowments as interacting in producing an observed distribution of income. He shows that when public expenditure and private endowments are complementary, as under our assumptions, inequality might actually be increased by an egalitarian public policy.

Our paper is related to a number of important empirical issues. The first concerns the differences in inequality and mobility between Europe and the US. Inequality is much higher in the US than in continental Europe while social mobility appears to be higher in the US than in many European countries. Eriksson and Goldthorpe (1992) construct an index of intergenerational social mobility for nine countries and find that the sample can be divided into one group with relatively low mobility, consisting of the Netherlands, France, Germany, Italy and the U.K. and another with higher mobility, consisting of Sweden, Japan, the U.S. and Australia.[1] Similarly, Ichino, Checchi and Rustichini (1999) provides a comparison between Italy and the US, showing lower inequality and mobility in Italy. The low degree of social mobility in

[1] Needless to say, this does not mean that social background is unimportant in US, see e.g., Solon (1992).

many European countries is surprising, since while most Europeans enjoy an almost free education from the cradle to university, American education is far from free.[2] Our model can provide an explanation to these stylized facts, however. Greater support to education in European countries benefits the children of educated parents more, due to their advantage taking advantage of general subsidies. As a result, the high support to education might have reached the level where it tends to reduce upward mobility.

Another issue related to our analysis is the debate on the causes for the rise in wage inequality in the US (and other countries) in recent decades. Three major alternative explanations to this phenomenon have been proposed. One is skill-biased technical change, namely the computer and communication revolution. A second explanation is trade liberalization, which increases imports from LDCs and reduces demand for unskilled labor, as discussed in e.g., Zeira (1999). The third explanation is a reduction in the supply of skilled workers, probably due declining public support to education. This argument has recently been raised both by Goldin and Katz (1999) and Card and Lemieux (1999). Our model examines all three possibilities within a unified general equilibrium framework. The first two explanations - skill-biased technical change and trade liberalization - can be interpreted as increases in the relative productivity of skilled individuals and should thus increase both wage inequality and upward mobility. The third explanation, a reduction in public support to education, indeed raises inequality in our model, but its effect on mobility is ambiguous. Hence, our model indicates that a careful examination of recent developments in upward mobility in the US is important for this debate.

Our paper is also related to a number of recent papers analyzing the dynamics of inequality and mobility along the growth path of the economy. Among these papers are Galor and Zeira (1993), Galor and Tsiddon (1996), Owen and Weil (1998), Moav and Maoz (1999) and Hassler and Mora (2000). These papers focus on how inequality and mobility change with economic growth and their effect on growth. In this paper, we are interested in different issues and instead, we assume productivity to be given exogenously focusing on the equilibrium correlation between inequality and mobility, especially on how educational subsidies affect this correlation. In this respect, our work is more related to other studies of inequality and education, such as Glomm and Ravikumar (1992) and Fernandez and Rogerson (1996), though it stresses different issues. The paper is constructed in the following way. Section 2

[2] College education is clearly not free and in private colleges and universities it is quite expensive. Elementary and high school education are formally free, but to a large extent financed by local communities.

presents the basic model and describes its equilibrium and dynamics. Section 3 presents the extension of the model to a specific analysis of parental support for education. Section 4 concludes. Proofs of results that are not given in the text can be found in the appendix.

2 Basic Model

Individuals live for two periods, the population is constant, and each generation has a mass of size one. In the first period of their life, individuals choose whether or not to go to school. They work and consume only in the second period and their utility of consumption is logarithmic.

Individuals with schooling are denoted *skilled* (of type s) and receive a wage (w^s) equal to their productivity (a^s) in the second period of their life. The productivity of *skilled* individuals is a technological parameter assumed to be exogenous. Individuals without schooling are denoted as *unskilled* (of type n), and work on a market with downward-sloping labor demand, which gives the wage

$$w_t^n = a^n N_t^{-\sigma}, \tag{1}$$

where N_t is the number of non-skilled workers in period t and $\sigma > 0$.[3]

Each individual freely chooses whether to become skilled or not, taking the equilibrium wage for non-skilled as given. Going to school requires an effort which creates disutility one-for-one and the amount of effort required to finish school is assumed to depend on innate ability. An individual with a higher ability has a lower effort requirement, denoted e, which is independent between generations and is drawn from a distribution function denoted $F(e)$ with an associated density $f(e)$.

The effort required to become skilled is not fully determined by the individual's innate ability. Instead, the government provides schooling subsidies which reduce school effort requirements. At this stage, we assume that the government can target their subsidies to children from specific social backgrounds. We denote the effort reduction for children of skilled parents by P^s, and for children of non-skilled parents by P^n. We also allow an exogeneous social handicap denoted h^n, implying an additional effort requirements for children with non-skilled parents. In later sections, we will abandon the assumption that the government can target its subsidies to specific groups.

[3]The assumption of an exogenous wage for skilled individuals is made for simplicity and is of no importance for the results, since schooling decisions are determined by the *relative* wage of skilled individuals.

Instead, we provide a mechanism whereby children with different social backgrounds have different abilities to take advantage of a subsidy even when formally available to everybody on equal grounds.

2.1 Equilibrium

Let us now use the equilibrium conditions in the model to derive the dynamic behavior of inequality and intergenerational mobility. We define inequality, denoted I_t, as the ratio of skilled and unskilled wages at time t and note that

$$I_t \equiv \frac{w^s}{w_t^n} = \frac{a_s}{a_n N_t^{-\sigma}} = aN_t^{\sigma}, \tag{2}$$

where we define $a \equiv \frac{a^s}{a_n}$. Now, note that a child i in period t with parents of type $j \in \{s, n\}$ becomes skilled only if $P^j - e^i + \ln w^s \geq \ln w_{t+1}^n$. Thus, there is a threshold level of innate effort requirements given by

$$\ln(I_{t+1}) + P^j - h^j \equiv e_t^j, \tag{3}$$

where $h^n > h^s = 0$. All individuals with parents of type j having effort requirements lower than e_t^j become skilled. Clearly, $e_t^n < e_t^s$, if $P^n < P^s$. We can now derive the law-of-motion for the number of non-skilled individuals

$$N_{t+1} = [1 - F(e_t^n)] N_t + [1 - F(e_t^s)] (1 - N_t), \tag{4}$$

which can be rewritten as

$$\begin{aligned} N_{t+1} = {} & [1 - F(\sigma \ln N_{t+1} + \ln a + P^n)] N_t \\ & + [1 - F(\sigma \ln N_{t+1} + \ln a + P^s)] (1 - N_t). \end{aligned} \tag{5}$$

This dynamic system is stable, since

$$\frac{dN_{t+1}}{dN_t} = \frac{F(e_t^s) - F(e_t^n)}{1 + \frac{\sigma}{N_{t+1}} \bar{f}_t} \in (0, 1), \tag{6}$$

where $\bar{f}$ denotes the density of marginal individuals, i.e., $\bar{f}_t \equiv f(e_t^n)N_t + f(e_t^s)(1 - N_t)$. This, together with the restriction that $F(\ln a + P^n) > 0$, allows us to focus on steady states.[4] It should also be noted that since $\frac{dN_{t+1}}{dN_t} > 0$, convergence to the steady state is monotonic.

[4]Clearly, $N_{t+1} > 0$ if $N_t = 0$. If, in addition, $F(\ln a + P^n) > 0$, then $N_{t+1} < 1$ if $N_t = 1$ which, together with (6), ensures the existence of an interior stable steady state.

2.2 Mobility and Inequality

Define (upward) mobility in period t as the share of children in period t who have non-skilled parents while themselves choosing to become skilled

$$m_t^u = F(e_t^n), \tag{7}$$

and, similarly, downward mobility as the share of children with skilled parents who become non-skilled:

$$m_t^d = 1 - F(e_t^s). \tag{8}$$

In steady state, we must have

$$m^u N = m^d (1 - N), \tag{9}$$

where variables without time subscripts denote steady state values. Using the fact that $I = aN^\sigma$ and defining the skill to unskilled ratio $s(I;a) \equiv \frac{1-N}{N} = a^{\frac{1}{\sigma}} I^{-\frac{1}{\sigma}} - 1$, we can rewrite this expression as

$$m^u(I; P^n) = m^d(I; P^s)\, s(I; a). \tag{10}$$

Note from (7) and (8) that upward mobility is positively related to inequality through e^n, while downward mobility is negatively related to inequality (through e^s). Furthermore, also $s(I;a)$ is decreasing in I, since the unskilled wage is decreasing in the number of non-skilled individuals, thereby implying a negative relation between inequality and the skilled to non-skilled ratio. Thus, the LHS of (10) is increasing in inequality. We will call this relation *the upward mobility curve*. The RHS, on the other hand, is decreasing in inequality and we will call this relation the *adjusted downward mobility curve*. Both these curves provide relations between upward mobility and inequality that must be satisfied in a steady state with strictly positive levels of mobility. The relationship given by the upward mobility curve must hold, since children with non-skilled parents are otherwise not making optimal choices. The relationship given by the adjusted downward mobility curve must hold since the number of individuals moving "down" is otherwise not the same as the number of individuals moving "up" and the economy is thus not in a steady state.

Let us consider the effect of an exogenous change in the productivity ratio a. An increase in this ratio can be thought of as skill-biased technical change. Alternatively, it may represent trade-liberalization in developed countries. Under quite general assumptions, a trade liberalization would tend to have an effect equal to an increase (decrease) in a in the country with higher (lower) proportions of skilled labor (see e.g., Zeira (1999)).[5]

It is straightforward to show that an increase in a leads to higher upward mobility and higher inequality. In Figure 1, we show this result graphically by depicting the upward mobility and the adjusted downward mobility curves. An increase in the ratio of the productivity of skilled versus non-skilled individuals (skill-biased technical change) moves the adjusted downward mobility curve up, while having no *direct* effect on the upward mobility curve. Steady state upward mobility increases as a consequence of the increase in income inequality, since the incentives to become skilled increase (downward mobility decreases for the same reason; the children with skilled parents are willing to make a larger effort in order to remain skilled). Consequently, the economy would move from A to B in Figure 1.

Let us now use the figure to analyze the effects of changes in the educational subsidies. An increase in the subsidy to children with with non-skilled parents, P^n, shifts the upward mobility curve upwards while leaving the adjusted downward mobility curve unchanged.[6] The economy moves from A to C where inequality has fallen and mobility has increased. If, on the other hand, the subsidy reduces the effort requirements of children with skilled parents (an increase in P^s), the adjusted downward mobility curve shifts upwards and inequality falls together with upward mobility. In both cases, the subsidy reduces inequality, since the subsidy must increase the inflow of skilled by an increase in upward mobility (if P^nincreases) by reducing downward mobility (if P^s increases). The reduction in inequality has a negative effect on upward mobility, which dominates unless it is due to an increase in P^n.

Summarizing our results, we have:

Result 1 *Mobility and inequality in steady state.*

1. $\frac{dm^u}{da}, \frac{dI}{da} > 0.$
2. $\frac{dm^u}{dP^n} > 0, \frac{dI}{dP^n} < 0.$
3. $\frac{dm^u}{dP^s} < 0, \frac{dI}{dPs} < 0.$

[5] Outside the scope of this paper, it would be straightforward to extend the model by introducing international trade. Comparative advantages could then, for example, be assumed to arise from differences in the cost of schooling.

[6] Clearly, a fall in h^n would have identical effects.

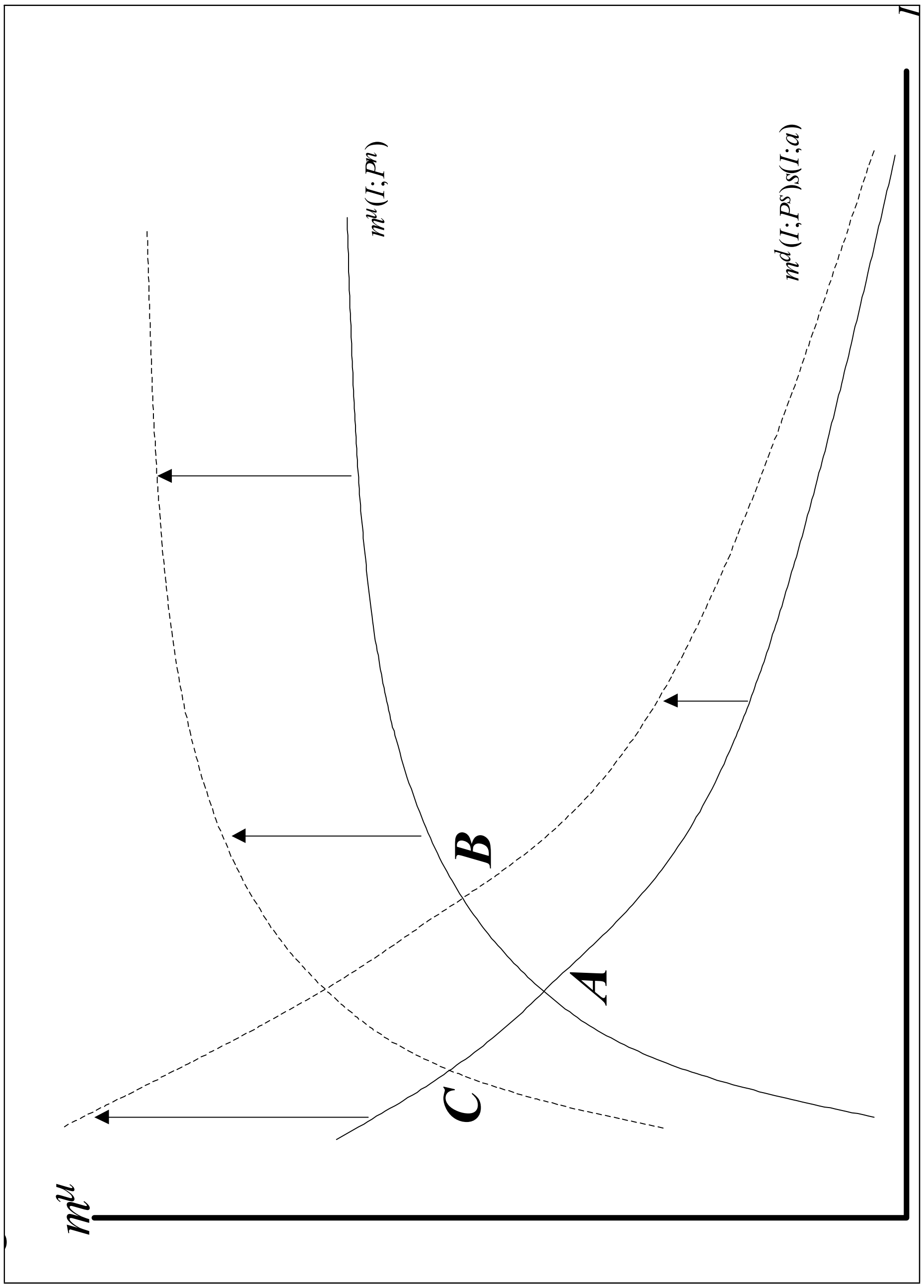

Figure 1: Mobility and inequality in the basic model

3 Parental Support to education

The analysis in the previous section suggested that if a country shows low levels of mobility and inequality, this could be a consequence of the subsidies to education being appropriated by the rich, and not the poor. We now turn to the analysis of a possible mechanism behind such a case. To this end, we extend the model by providing an intergenerational link affecting the choice of education. We assume that parents derive utility from their children becoming skille.[7] More specifically, we assume that a parent who consumes c_t receive a utility (net of the effort spent in the previous period) of $B + \ln c_t$ if her child becomes skilled and only $\ln c_t$ otherwise. Furthermore, we allow parents to influence the educational choice of their children by providing economic support to them, if they decide to go to school. The support, denoted i, paid conditional on the child going to school, pays for tuition net of public subsidies. We assume that parents pay the tuition and focus on the case when they are willing and able to do so out of their own income.

We denote tuition and public subsidies normalized by the skilled wage by T and P, respectively. We assume that bot these variables are set in terms of the skilled wage, implying that tuition net of the public subsidy is given by $(T - P)\, w^s$. The government cannot target its subsidies to particular groups. Instead, the government can only provide a general subsidy to education, reducing the (tuition) cost of attending school. The remainder of the parental support after net tuition has been paid, $i - (T - P)w^s$, is spent on measures reducing the effort requirements of the child, which may induce her to become skilled. We call these spendings *tutoring* and assume the effort reduction of a given amount of spending to be inversely proportional to the skilled wage. Of key importance for the results in this section is the fact that we assume that parents who are not skilled themselves have an informational disadvantage implying that the resources spent on effort reduction have a lower degree of effectiveness. More specifically, if a parent of type j gives an amount $i > (T - P)$ to her child, the effort reduction for the child is given by

$$\frac{d^j}{w^s}\left[i - (T - P)w^s\right], \tag{11}$$

where $d^j \geq 0$ is the efficiency of spendings on tutoring given by a parent of type j. In the remainder of this paper, we assume that children from non-skilled homes have a disadvantage manifesting itself by $d^n \leq d^s$, while $d^s > 0$.

[7] Assuming, instead, that parents care about their children's consumption unnecessarily complicates the model.

The parent uses spendings on tutoring to induce its child to become skilled and the child wants to exploit this in order to get as large subsidies as possible. To determine the amount of parental support i, we assume that the child and her parents play a non-cooperative game of asymmetric information. Furthermore, we assume that children know their innate ability (e) while parents know nothing more than the unconditional distribution. To simplify further, we assume that children make a take-it-or-leave-it offer to their parents regarding the amount of support they will receive if they go to school. This assumption implies that children can extract all surplus from their parents,[8] so that in period t, we get $\ln\left(w_j^t - i_j\right) + B = \ln w_j^t$, implying

$$i_j^t = g w_j^t, \tag{12}$$

where $g \equiv (1 - e^{-B})$.

By (12), a child today will be indifferent to whether his child will go to school next period or not. The utility of a child from a home of type j who goes to school is thus

$$-e + \frac{d^j}{w_s}\left[g w^j - (T-P)w^s\right] + \ln w^s, \tag{13}$$

where the first term is the innate effort requirement of the child, the second is the effort reduction due to tutoring and the third is second period utility. If the child does not go to school, her utility is $\ln(w_{t+1})$, implying that the threshold levels of innate effort requirements that make a child indifferent between going to school and not can be written

$$e_t^s = d^s\left[g - (T-P)\right] + \ln I_{t+1} \tag{14}$$

$$e_t^n = d^n\left[\frac{g}{I_t} - (T-P)\right] + \ln I_{t+1}. \tag{15}$$

The values $e - d^s\left[g - (T-P)\right]$ and $e - d^n\left[\frac{g}{I_t} - (T-P)\right]$ can be thought of as the *net* effort requirements for the two types of individuals. We should note that the net effort requirement for children with non-skilled parents increases (e^n falls) in I_t if $d^n > 0$, because increased inequality reduces equilibrium parental support from non-skilled parents, which constitutes the *distance*

[8] Changing the setup so that the parents make the offer makes the expression for i somewhat more complicated. In this case, a marginal condition saying that the increase in expected benefits of subsidizing the child should equal its costs determines i. If we allow parents to have information about their children's innate ability, i becomes dependent on the realized value of e, which substantially complicates the analysis, but should not qualitatively change the results.

effect. On the other hand, e^n increases in I_{t+1} because increased inequality increases the value of becoming skilled (the *incentive effect)*. If $\frac{\partial e^n(I_t,I_{t+1})}{\partial I_t} + \frac{\partial e^n(I_t,I_{t+1})}{\partial I_{t+1}} > 0$, we say that the incentive effect dominates the distance effect. In this case, an increase in steady-state inequality increases upward mobility, *ceteris paribus*.

To rule out the uninteresting case when the equilibrium non-skilled wage is higher than the skilled wage, we make the assumption that the lowest net effort requirement of children with non-skilled parents is positive, thereby implying zero upward mobility if $I = 1$.[9] In the subsequent analysis, we make the following assumption;

Assumption 1 *The support of* F *is bounded over the interval* $[\underline{e}, \bar{e}]$ *with* $\underline{e} - d^n[g - (T - P)] > 0$.

To analyze the dynamic behavior of the model, consider the law-of-motion for N:

$$\begin{aligned} N_{t+1} &= \left[1 - F\left(d^n\left[\frac{g}{N_t^\sigma a} - (T - P)\right] + \sigma \ln N_{t+1} + \ln a\right)\right] N_t \quad (16) \\ &\quad + \left[1 - F\left(d^s\left[g - (T - P)\right] + \sigma \ln N_{t+1} + \ln a\right)\right](1 - N_t), \end{aligned}$$

from which we calculate

$$\frac{dN_{t+1}}{dN_t} = \frac{F(e_t^s) - F(e_t^n) + \frac{f(e_t^n) d^n g \sigma}{I_t}}{1 + \frac{\sigma}{N_{t+1}} \bar{f}_t}. \quad (17)$$

Note that this expression is identical to (6), except for the last term in the numerator. This term is due to the distance effect, implying that an increase in N_t has a positive effect on N_{t+1} due to a reduction in parental support to children with non-skilled parents. In particular, this means that in contrast to the previous section, the system is not necessarily stable. From (17), it is clear that $\frac{f(e_t^n) d^n g}{I_t} < \frac{\bar{f}_t}{N_{t+1}}$, or

$$\frac{f(e^n)N + f(e^s)(1 - N)}{f(e^n)} > \frac{N d^n g}{I}. \quad (18)$$

which is a sufficient condition for $\frac{dN_{t+1}}{dN_t} < 1$. When the distribution of talent is rectangular, the left-hand-side of (18) is unity in any interior steady state. We know that $N, g \leq 1$ and under assumption 1 $I > 1$, implying that the system is stable unless $d^n > \frac{I}{d^n g}$.

[9] Clearly, this condition is sufficient, but not necessary, for equilibrium inequality to be positive. As we will see, this assumption has other convenient implications, however.

In steady state, the number of children with non-skilled parents who becomes skilled must equal the number of children with skilled parents who become non-skilled, thereby implying that

$$m^u(I;P,d^n) = m^d(I;P,d^s)s(I;a), \tag{19}$$

where $m^u(I;P,d^n) \equiv F(e^n)$ and $m^d(I;P,d^s) \equiv 1 - F(e^s)$, as in the previous sections. The LHS and the RHS of (19) are called the upward mobility curve and the (adjusted) downward mobility curve respectively, when expressed as functions of I. Clearly, the downward mobility curve is decreasing in I for the same reasons as in the previous section. The slope of the upward mobility curve, on the other hand, is given by $f(e^n)\left(\frac{\partial e^n}{\partial I_t} + \frac{\partial e^n}{\partial I_{t+1}}\right)$, which in general has an ambiguous sign since the *incentive* and *distance* effects oppose each other ($\frac{\partial e^n}{\partial I_t} < 0$ and $\frac{\partial e^n}{\partial I_{t+1}} > 0$). However, under assumption 1, we can show that the upward mobility curve has a non-negative slope, thus implying that the incentive effect must dominate in an interior steady state.

Result 2 *If assumption 1 is satisfied, then*

$$\begin{aligned} m^u(1;P,d^n) &= 0, \text{ and} \\ \frac{\partial m^u(I;P,d^n)}{\partial I} &\geq 0 \text{ if } I > 1, \end{aligned}$$

so that the incentive effect is dominating and there is, at most, one interior steady state.

3.1 Changes in productivity

Let us now consider the effects of changes in the productivity ratio a. As discussed above, an increase in a can be interpreted as skill-biased technical change, or trade-liberalizations. Using standard calculus, it is straightforward to show that the immediate effect of an increase in a is

$$\frac{dI_{t+1}}{da} = \frac{\frac{1}{\sigma a}N_{t+1}I_{t+1}}{\bar{f}_t + \frac{1}{\sigma}N_{t+1}} > 0, \tag{20}$$

and the steady state effect is

$$\frac{dI}{da} = \frac{m^d\frac{1}{\sigma a N}}{\frac{\partial m^u}{\partial I} - \left(\frac{\partial m^d}{\partial I}s(I) + m^d s'(I)\right)} > 0. \tag{21}$$

Increased inequality increases the incentive to become skilled, but since parental inequality is predetermined, it has no effect on parental support to their children. Thus,

$$\frac{dm_t^u}{da} = f(e_t^n)\frac{\partial e_t^n}{\partial I_{t+1}}\frac{dI_{t+1}}{da} = \frac{f(e_t^n)}{I_{t+1}}\frac{dI_{t+1}}{da} > 0, \tag{22}$$

implying that skill-biased technical change upon impact increases upward mobility. In the next period, the increased inequality reduces the support to children by non-skilled parents, which tends to reduce mobility in period m_{t+1}^u relative to m_t^u and thus further increase inequality. However, since the incentive effect is dominating, the steady state effect on mobility is always positive;

$$\frac{dm^u}{da} = f(e^n)\left(\frac{I - d^n g}{I^2}\right)\frac{dI}{da} > 0. \tag{23}$$

Let us summarize;

Result 3 *The effects of changes in relative productivity.*

1. $\frac{dI_{t+1}}{da} > 0, \frac{dm_t^u}{da} > 0.$
2. $\frac{dI}{da} > \frac{dI_{t+1}}{da}, \frac{dm^u}{da} > 0.$

As we can see, a change in productivity implies that inequality and mobility are positively related, both in the short and the long run. Let us illustrate the steady state results graphically. A skill-biased technical change or a trade liberalization, i.e., a positive shift in a, shifts the downward mobility curve upwards, as shown in Figure 2, the effect of which is to increase both inequality and upward mobility. The conclusion is that in this case, we get a positive relation between inequality and upward mobility.

3.2 Changes in school subsidies

Let us now analyze the effects of changes in general school subsidies, P. Total differentiation of (16) using $I_t = aN_t^\sigma$ yields the initial effect;

$$\frac{dI_{t+1}}{dP} = -I_{t+1}\frac{f(e_t^n)(1-N_t)d^n + f(e_t^s)N_t d^s}{\frac{1}{\sigma}N_{t+1} + \bar{f}} < 0. \tag{24}$$

The intuition for this is straightforward; an increase in school subsidies reduces the effort requirements of becoming skilled. *Ceteris paribus*, this leads to more skilled individuals and reduced inequality.

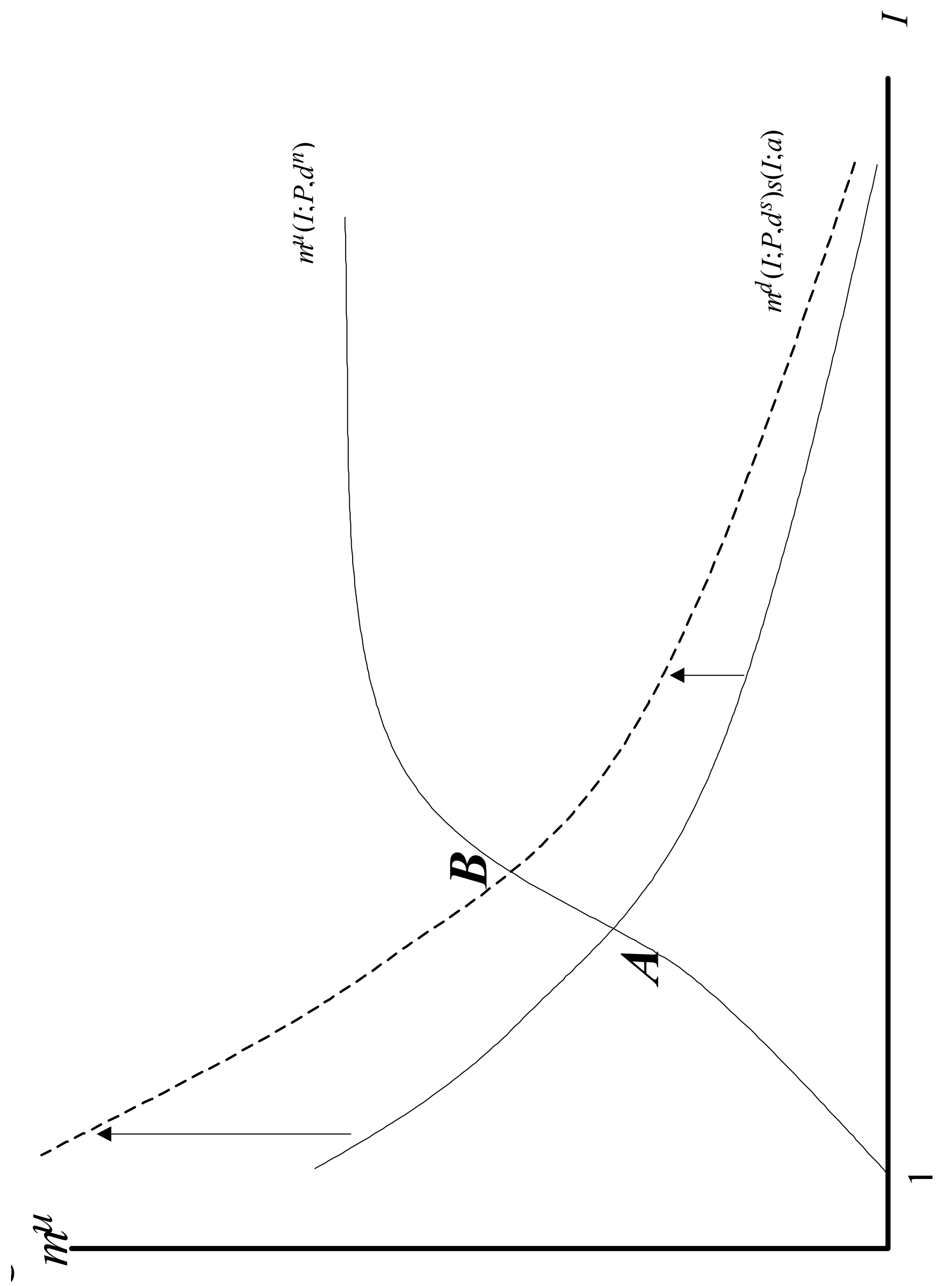

Figure 2: An increase in the relative productivity of skilled individuals.

Now, consider the initial effect on upward mobility, which is

$$\frac{dm_t^u}{dP} = f(e_t^n)\left(d^n + \frac{1}{I_{t+1}}\frac{dI_{t+1}}{dP}\right). \tag{25}$$

Clearly, this is negative if d^n is small relative to d^s. In this case, the main effect of the school subsidy is to induce more children with skilled parents to become skilled. Downward as well as upward mobility fall, the latter since the incentive to become skilled is reduced by the reduction in inequality. If, on the other hand, d_n is sufficiently large and inequality is not very sensitive to P, upward mobility might increase. For this to be the case, the subsidy should have a substantial negative effect on the effort requirements of children with non-skilled parents while the reduction in inequality is not strong enough to fully counteract this effect.

The sensitivity of inequality to changes in P depends on the densities $f(.)$. To abstract from this, consider the case when the distribution of innate effort requirements is rectangular, with a density given by ε. Then,

$$\frac{dm_t^u}{dP} = \varepsilon\left(d_n - \frac{\varepsilon}{\frac{1}{\sigma}N_{t+1} + \varepsilon}\bar{d}\right),$$

where $\bar{d} = (1 - N_t)d^n + N_t d^s$, i.e., the weighted average of the $d's$. As we can see, if $d_n = d_s = \bar{d}$, $\frac{dm_t^u}{dP}$ is always positive because in this case, the school subsidy affects both groups equally. Furthermore, increases in P reduce N_{t+1} which reduces $\frac{dm_t^u}{dP}$. In other words, mobility is concave in P and may thus be hump-shaped. The following result summarizes our findings hitherto.

Result 4 *The impact effect of school subsidies on inequality and mobility.*

1. $\frac{dI_{t+1}}{dP} < 0$.
2. *If* $d^n = 0$, $\frac{dm_t^u}{dP} < 0$.
3. *If* $d^n = d^s$, $\frac{dm_t^u}{dP} > 0$.
4. *If* $F()$ *is rectangular,* m_t^u *is concave in* P.

From result 4, it follows that the initial correlation between mobility and inequality, driven by changes in educational subsidies, is of an ambiguous sign. If the subsidy is mainly captured by children from skilled homes, the correlation is positive, which is the case when initial individual endowments are complementary to public subsidies, in the terminology of Bruno (1976).

If, on the other hand, the ability to take advantage of the subsidy is independent of social background, the correlation should be negative.

Let us now turn to the steady state effects. By totally differentiating (19), we get

$$\frac{dI}{dP} = -\frac{f(e^n)d^n + f(e^s)d^s s}{\frac{\partial m^u}{\partial I} - \left(\frac{\partial m^d}{\partial I} s + m^d s'\right)} < 0. \tag{26}$$

The steady state effect of mobility is given by

$$\frac{dm^u}{dP} = f(e^n)\left[d^n + \left(\frac{I - d^n g}{I^2}\right)\frac{dI}{dP}\right]. \tag{27}$$

From (27), we see that the effect of public subsidies on upward mobility is likely to be negative if d^n is low, if the incentive effect is strongly dominating or if $\frac{dI}{dP}$ is highly negative. The difference between (25) and (27) is that the negative effect on mobility due to reductions in inequality is weaker since in steady state, a reduction in inequality also has a positive (distance) effect on mobility. Nevertheless, as the following proposition shows, the relation between mobility and inequality, when the driving force is a change in general educational subsidies, is qualitatively the same in the short and the long run.

Result 5 *The steady state effect of school subsidies on inequality and mobility.*

1. $\frac{dI}{dP} < 0$.
2. *If* $d^n = 0$, $\frac{dm^u}{dP} < 0$
3. *When* $d^n = d^s$, $\frac{dm^u}{dP} > 0$.

As we can see, changes in school subsidies affect mobility and inequality in the same (opposite) directions when $d^n = 0$, ($d^n = d^s$). The result can also be derived graphically. For example, when d^n is close to zero, an increase in P has a small effect on e^n and thus, on the upward mobility curve. On the other hand, it has a large positive effect on e^s which shifts the downward mobility curve downwards, as shown in figure 2. The increase in P moves the steady state from A to B, where both inequality and mobility are lower.

Let us try to provide an intuitive explanation for our results, built on the results from the previous section. For this purpose, consider how a change in P changes the threshold effort requirements for the two groups of individuals:

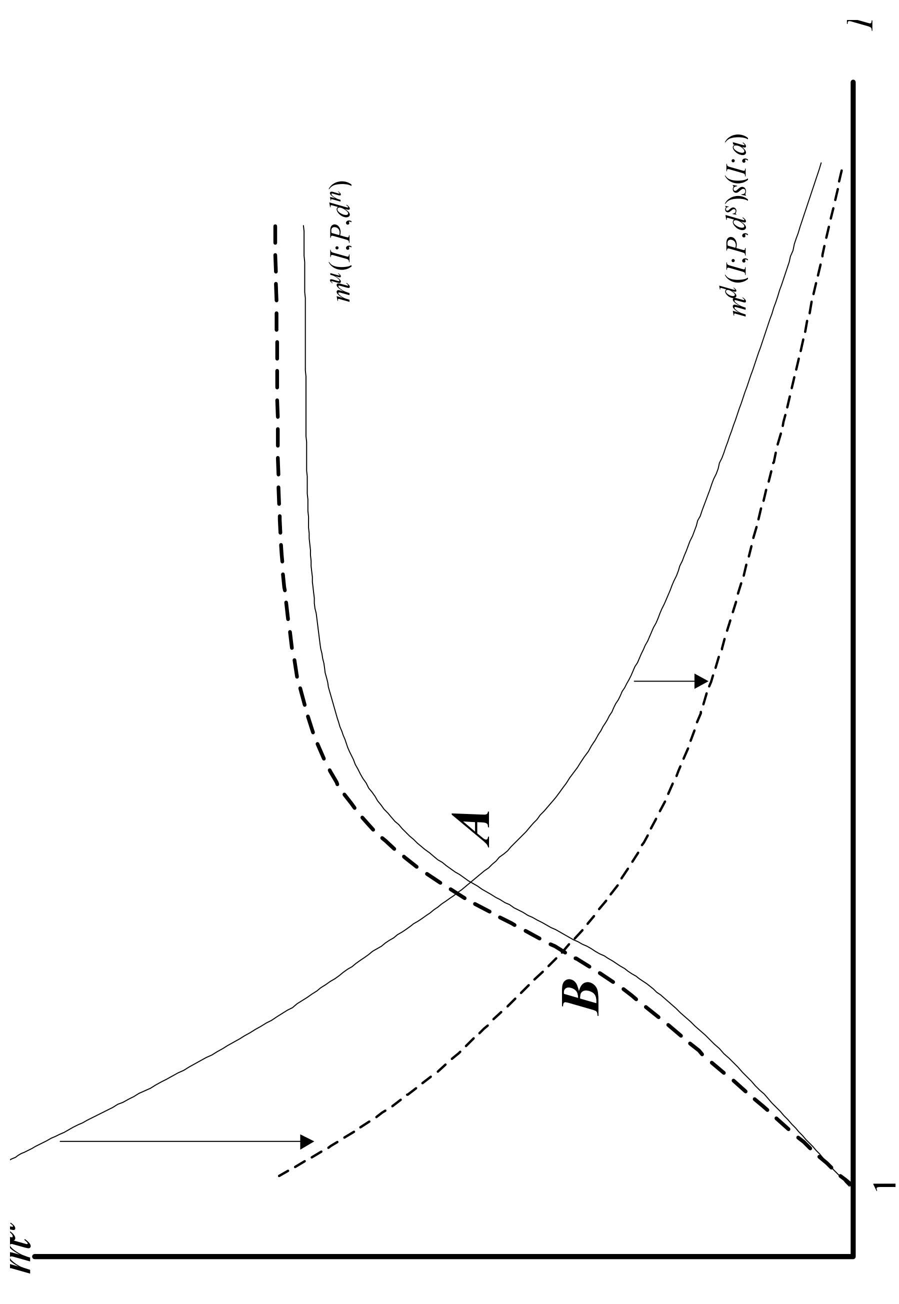

Figure 3: An increase in school subsidies.

$$\begin{aligned}\frac{de^s}{dP} &= d^s + \frac{1}{I}\frac{dI}{dP}, \\ \frac{de^n}{dP} &= d^n + \left(\frac{1}{I} - \frac{d^n g}{I^2}\right)\frac{dI}{dP}.\end{aligned} \quad (28)$$

The first term in the RHS of both equations captures the direct positive effect on the threshold by a change in P. There is also a general equilibrium effect, captured by the second term, which is negative for both groups since $\frac{dI}{dP} < 0$ and $\frac{1}{I} - \frac{d^n g}{I^2} > 0$ under assumption 1. If $d^n = 0$, the only effect of higher schooling subsidies on children from non-skilled homes is the negative general equilibrium effect. Then, higher subsidies are associated with a reduction in *both* inequality and mobility. Here, general subsidies reduce mobility by weakening the *incentive effect.*

Children of non-skilled homes are, on the other hand, less affected by the general equilibrium effect if d^n and g are both strictly positive, since a reduction in inequality then implies more support from non-skilled parents to their children. Thus, it may be the case that an increase in schooling subsidies reduces net effort requirements more for children from non-skilled homes (i.e., $\frac{de^n}{dP} > \frac{de^s}{dP}$). If $d^s = d^n$, this is necessarily the case and an increase in schooling subsidies leads to *higher* mobility and *lower* inequality. In this case, the change in schooling subsidies increases mobility by weakening the *distance effect.*

Above, we showed that the short run relation between upward mobility and P is concave, thereby implying the possibility of a non-monotonic relation between inequality and mobility. The analytical expression for the steady-state relation is more involved. We can, however, show that when $0 < d^n < d^s$, the relation is generically non-monotonic and mobility is zero both for high and low levels of inequality, but positive in between.[10] To understand the reasons for such non-monotonicity, first note that zero (low) mobility in steady state can arise in two ways. Mobility is obviously zero (low) if no (few) individual(s) become skilled (case a). Mobility can also be zero (low) if, in the steady state, the distribution of net effort requirements for the two types of individuals is non-overlapping (has a small overlap). In other words, if the most talented child with non-skilled parents has a higher effort requirement than the least talented child of skilled, we can have a perfectly stratified steady state, without intergenerational mobility (case b).

Second, note that when $d^s > d^n$ and holding I constant, an increase in P, reduces the net effort requirement *more* for children with skilled parents than

[10]See the appendix for details.

for children with non-skilled parents, since skilled parents can make better use of the subsidy. In a particular sense, an increase in subsidies thus means that individuals with different social backgrounds but with the same level of innate ability become more different.

Now, we can understand why an increase in P can have a non-monotonic effect on mobility. Suppose that for sufficiently low levels of subsidies, no individual chooses to become educated. Then, we have zero mobility for the reason described in case a. An increase in P will make it easier to become skilled and might induce positive mobility, while inequality still being relatively high. Continuing to increase P will reduce inequality further. However, it will increase the difference in net effort requirements for individuals with the same innate abilities but with different social backgrounds and might eventually imply that the distributions of net effort requirements no longer overlap. Then, mobility is back at zero, now at a low level of inequality, due to the reason described in case b.

Let us conclude the analysis by a numerical simulation, illustrating the effects on steady state inequality and mobility by changes in the public support to education. For this purpose, we parametrize the model as follows: $\sigma = 1$, $a = 4$, $T = .35$, $g = .35$ and $d^s = 1$. We assume F to be rectangular over the interval $[g, g + 1/2]$, which ensures that assumption 1 is satisfied as long as tuition net of public support is non-negative. In Figure 4, we report the steady state levels of inequality and mobility for public support levels between 0 and 100 percent of the tuition for three different levels of d^n, namely 1, .68 and 0.

Figure 4 shows that inequality is falling in the rate of public subsidies. However, inequality is more sensitive to public subsidies when d^n is high. This is straightforward, when d^n is high, the aggregate effect of subsidies on effort reduction is high, since everyone can make efficient use of the resources released as net tuition falls. This leads to a large increase in the number of skilled and a large reduction in inequality. When d^n is low, on the other hand, only children from skilled homes can make efficient use of the released resources and the increase in the number of skilled individuals is more limited.

We also see that increased public support to education increases (reduces) mobility if d^n is high (low). In the intermediate case, mobility is largely unaffected by the subsidy. A more careful look reveals that mobility is hump-shaped in the subsidy, reaching a maximum when P is 50% of T.

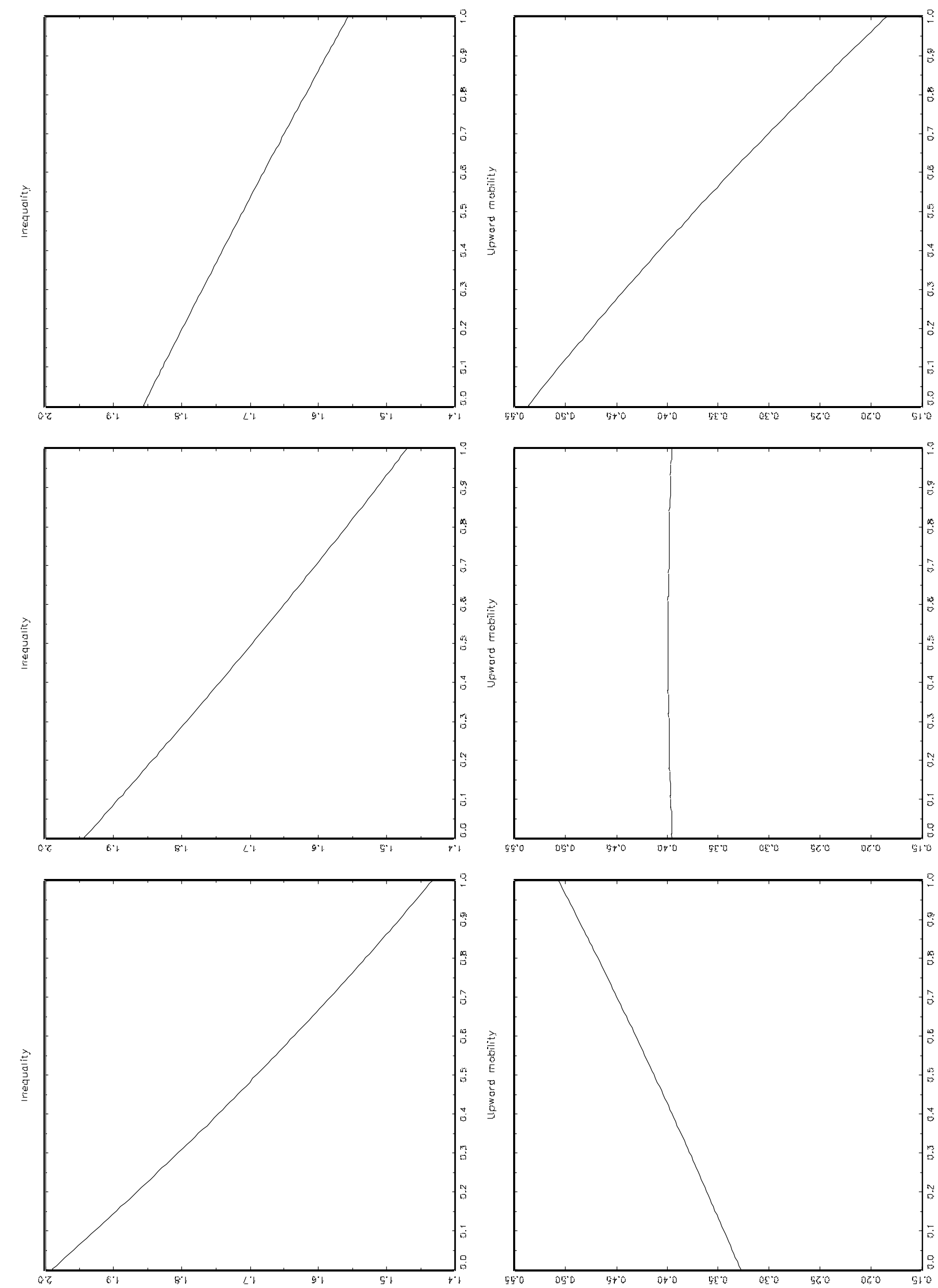

Figure 4: The relation between school subsidies and mobility under high, medium and low non-skilled tutoring efficiency.

4 Conclusion

Income inequality and intergenerational mobility are closely and intricately related. On the one hand, high inequality means that children of poor individuals have a strong incentive not to remain poor and hence, upward mobility might increase when inequality rises. On the other hand, high inequality means that children with rich parents could lose greatly from moving downward on the income ladder and hence, downward mobility might fall when inequality rises. Furthermore, high inequality may, in itself, produce a barrier to upward mobility, by reducing the ability of poor families to support their children's educational investments. In steady state, upward and downward flows must balance, but these flows are affected by inequality in a way making the relationship between inequality and mobility ambiguous.

From our analysis, we have learned that an important determinant of whether mobility and inequality are positively or negatively related, is to what extent children with skilled versus children with unskilled parents react differently to changes in educational subsidies and inequality. To model such differences and analyze their consequences has been the main focus of this paper.

Like many other papers, we assume that skilled parents can make it easier for their children to go to school. In any model where income differences are due to differences in educational attainments, such an intergenerational link is necessary to generate persistence of income across generations. There are many variations of this general assumption, however. In some models, such as Loury (1981) and Galor and Zeira (1993), parents affect their kids through transfers, since credit markets are imperfect, and transfers depend on income. In other models, e.g., Galor and Tsiddon (1997), parents affect their children's ability to become educated through non-pecuniary means, namely by giving direct help to them. Our model has both types of parental effects. First, there is no credit market for education and the only way of paying for education is by parental help or public support. In addition, we assume that skilled parents have an advantage in helping their children at school, since they are more educated themselves. In other words, the asset of having a skilled parent is complementary to public subsidies. Hence, resources, which are given to all parents, work better for skilled and affluent parents than for poor, unskilled parents. This assumption drives the main result of the paper, namely that public support to general education might reduce intergenerational social mobility.

Our model has shown that the relation between general public support to education and mobility may be non-monotonic. If inequality is very high, an increase in public support to education may have a strong negative effect on inequality. This reduces the incentives to become educated for everyone but also reduces the barriers to education for children from poor households. This may be the dominating effect, causing an increase in mobility, when public support is low and inequality is high. On the other hand, under the assumption that skilled parents can take better advantage of public educational support, an increase in such support also increases the difference between individuals due to different social backgrounds. The social handicap of coming from a non-skilled home may then actually increase as a result of an increase in public support to education. Sufficiently high public subsidies may then create an "Italian" situation – low mobility and low inequality.

Finally, we ask ourselves what we can learn from our results. First, they shed light on many empirical issues, which we believe to be important. Mostly, they help us in understanding differences between countries in degrees of inequality and degrees of mobility. Second, we claim that our results can affect our thinking on issues of policy, although we are fully aware that people with different views might reach different conclusions. One conclusion is clear though, namely that public general support to education, aimed at raising social mobility, might sometimes not work, instead reducing mobility. The question of what policy to adopt has a number of possible answers. Those who do not favor public support to education might use this as an excuse to reduce it. Those who favor public support to education might reach the conclusion that such support should not be given to all students equally, but should be targeted to children with unskilled parents. In other words, education should be subsidized to the poor rather than the rich.

But whatever policy implication our model might have, its main goal is to give us a better understanding of the relationship between mobility and inequality. This can help us understand differences in inequality and mobility, both across time and across countries. The main insight of the paper is that this relationship depends strongly on the different reactions of children with skilled and children with unskilled parents.

References

Bruno, M. (1976), 'Equality, completementarity and the incidence of public expenditures', *Journal of Public Economics* **6**(4), 395–407.

Card, D. and Lemieux, T. (1999), Can falling supply explain the rising return to college for younger men? a cohort-based analysis, Mimeo, University of California, Berkely.

Eriksson, R. and Goldthorpe, J. (1992), *The Constant Flux: A Study of Class Mobility in Industrial Societies*, Oxford Clarendon Press.

Fernandez, R. and Rogerson, R. (1996), 'Income distribution, communities, and the quality of public education', *Quarterly Journal of Economics* **111**(1), 135–64.

Galor, O. and Tsiddon, D. (1996), 'Income distribution and growth: The Kuznets hypothesis revisited', *Economica* **63**(250), S103–17.

Galor, O. and Tsiddon, D. (1997), 'The distribution of human capital and economic growth', *Journal of Economic Growth* **2**(1), 93–124.

Galor, O. and Zeira, J. (1993), 'Income distribution and macroeconomics', *Review of Economic Studies* **60**(1), 35–52.

Glomm, G. and Ravikumar, B. (1992), 'Public versus private investment in human capital endogenous growth and income inequality', *Journal of Political Economy* **100**(4), 813–34.

Goldin, C. and Katz, L. F. (1999), The returns to skill across the twentieth century United States, NBER Working Paper 7126.

Hassler, J. and Mora, J. V. R. (2000), 'Intelligence, social mobility and growth', *American Economic Review*. forthcoming.

Ichino, A., Checchi, D. and Rustichini, A. (1999), 'More equal but less mobile? education financing and intergenerational mobility in Italy and the U.S.', *Journal of Public Economy* **74**(3), 351–93.

Loury, G. C. (1981), 'Intergenerational transfers and the distribution of earnings', *Econometrica* **49**(4), 843–67.

Moav, O. and Maoz, Y. (1999), 'Intergenerational mobility and the process of development', *The Economic Journal* **109**, 677–697.

Owen, A. L. and Weil, D. N. (1998), 'Intergenerational earnings mobility, inequality and growth', *Journal of Monetary Economics* **41**(1), 71–104.

Solon, G. (1992), 'Intergenerational income mobility in the united states', *The American Economic Review* **82**(3), 393–408.

Zeira, J. (1999), Wage inequality, technology, and trade, Mimeo, Hebrew Univeristy, Jerusalem.

5 Appendix with proofs

5.1 Result 1

1. First, note that $\frac{dI}{da} = \frac{m^d S_a}{m_I^u - m_I^d S - m^d S_I} = \frac{I(1-F(e^s))(1-N)}{\sigma a\left(f(e^n)N + f(e^s)(1-N) + m^d(1-N)/\sigma\right)} > 0$, then, clearly $\frac{dm^u}{da} = m_I^u \frac{dI}{da} = \frac{f(e^n)}{I}\frac{dI}{da} > 0$.

2. Similarly, $\frac{dI}{dP^n} = -\frac{f(e^n)I}{f(e^n)+f(e^s)S+\frac{(1-F(e^s))}{\sigma N}} < 0$, $\frac{dm^u}{dP^n} = m_{P^n}^u + m_I^u \frac{dI}{dP^n}$ $= \frac{f(e^s)S+\frac{(1-F(e^s))}{\sigma N}}{f(e^n)+f(e^s)S+\frac{(1-F(e^s))}{\sigma N}} > 0$,

3. $\frac{dI}{dP^s} = -\frac{f(e^s)I}{f(e^n)+f(e^s)S+\frac{(1-F(e^s))}{\sigma N}} < 0$, and $\frac{dm^u}{dP^s} = m_I^u \frac{dI}{dP^s} =$ $-\frac{f(e^s)}{f(e^n)+f(e^s)S+\frac{(1-F(e^s))}{\sigma N}} < 0$.

5.2 Result 2

First, the derivative $\frac{\partial m^u(I,P;d^n)}{\partial I}$ equals $f(e^n)\frac{\partial e^n}{\partial I}$, where $\frac{\partial e^n}{\partial I} = [I - d^n g)]/I^2$, which is either positive for all $I \geq 1$, or first negative and then positive. Under assumption 1, $m^u(1,P;d^n) = 0$, since $e^n < \underline{e}$ when $I = 1$. Thus, if $\frac{\partial e^n}{\partial I} < 0$ for low values of I, mobility must be zero as long as $\frac{\partial e^n}{\partial I}$ remains negative. As for uniqueness of the steady state, since the slopes of the upward and downward mobility curves are of opposite signs, they cross once, at most.

5.3 Mobility and inequality driven by changes in P.

Using the definitions of e^n and e^s and the steady state condition (19),

$$
\begin{aligned}
m^u &= (1 - F[d^s(g - T + P) + \ln I])\, s(I) \\
m^u &= F[d^n(g - T + P) + \ln I]
\end{aligned}
$$

When $d^n > 0$, we can invert the previous expressions and solve for P, giving an implicit relation between m^u and I.

$$
\frac{F^{-1}(m^u)}{d^n} - \frac{F^{-1}(\left(1 - \frac{m^u}{s(I)}\right)}{d^s} = g\left(\frac{1}{I} - 1\right) + \left(\frac{1}{d^n} - \frac{1}{d^s}\right)\ln I. \qquad (29)
$$

First, note that the LHS of (29) always increases in m^u. Note also that the LHS can be written as $\frac{e^n}{d^n} - \frac{e^s}{d^s}$, with a minimum $\frac{\underline{e}}{d^n} - \frac{\bar{e}}{d^s}$, when $m^u = 0$. Furthermore, an increase in I rotates the LHS anti-clockwise. The RHS is independent of m^u and equal to zero at $I = 1$. If d^n is close to d^s, the RHS is decreasing in I over relevant ranges while it is increasing in I if d^n is sufficiently low.

Now, consider the case when d^n is close to d^s. Then, an increase in I reduces RHS and increases LHS of (29) if mobility is positive. Thus, for the equality to be satisfied, m^u must fall, producing a negative relation between inequality and mobility. Thus, if mobility is positive when $I = 1$, increases in inequality (reductions in P) reduces mobility. In a steady state without mobility, $F(e^n) = 0$ and all individuals are non-skilled ($s(I) = 0$) or all children with skilled parents become skilled ($F(e^s) = 1$). In both cases, $F(e^n) = (1 - F(e^s))\, s(I) = 0$.

For sufficiently low d^n, $\frac{\underline{e}}{d^n} - \frac{\bar{e}}{d^s}$ is positive implying zero mobility at $I = 1$. Increasing I shifts the RHS of (29) upward until it reaches $\frac{\underline{e}}{d^n} - \frac{\bar{e}}{d^s}$ (at A in Figure 5) and mobility starts increasing. Eventually, mobility starts falling again, due to the rotation of the LHS and when $I = a$, $s(I) = 0$ implying that the LHS is vertical at $m^u = 0$. Increases in inequality thus imply that mobility first increases (from A to B in 5) and then decreases (from B to C).

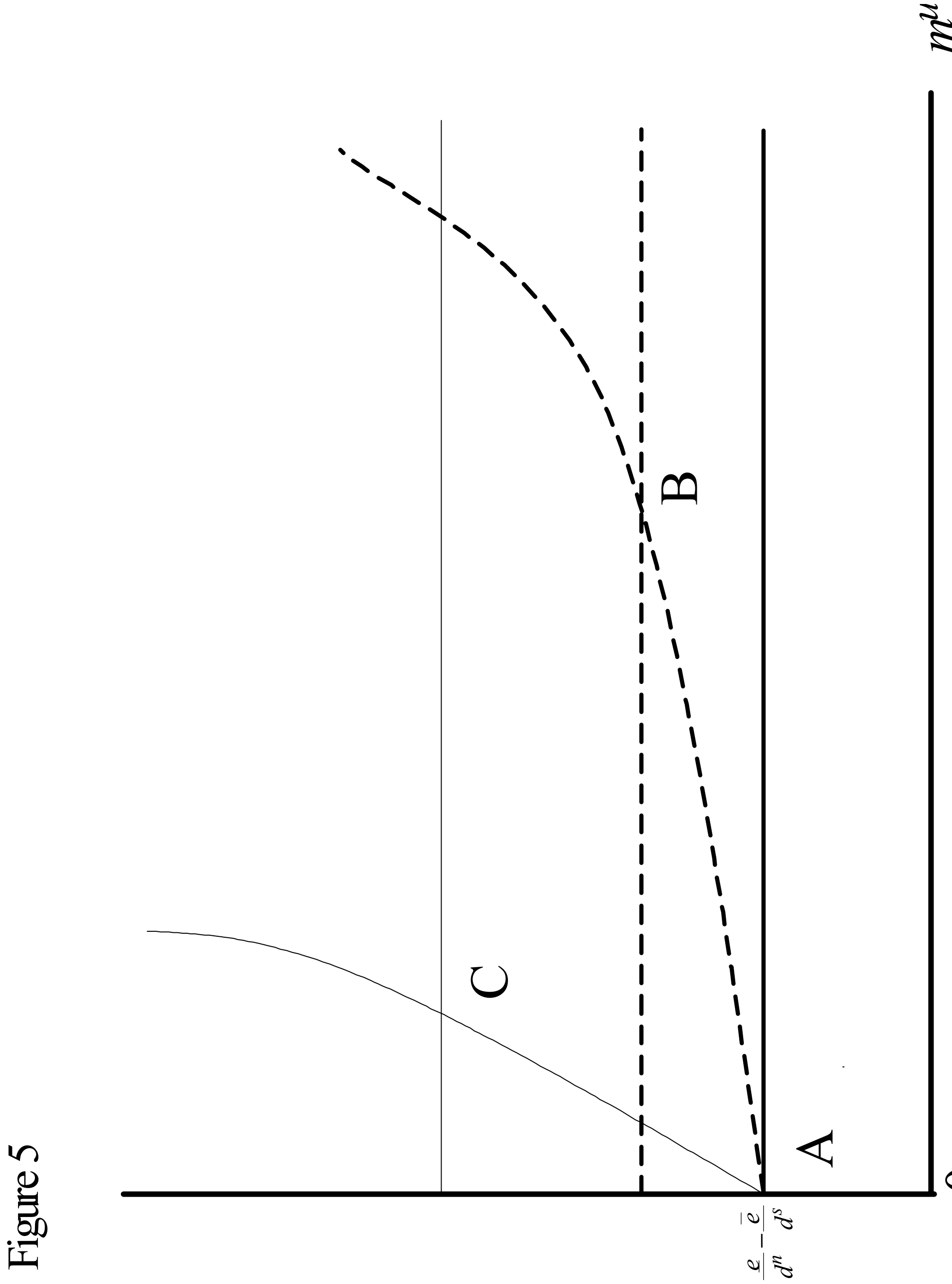

Figure 5: Equation 29