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## THE NONLINEAR DYNAMICS OF OUTPUT AND UNEMPLOYMENT IN THE US

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# THE NON-LINEAR DYNAMICS OF OUTPUT AND UNEMPLOYMENT IN THE US

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## ABSTRACT

# The Non-Linear Dynamics of Output and Unemployment in the US*

This Paper studies the joint dynamics of US output and unemployment rates in a non-linear VAR model. The non-linearity is introduced through a feedback variable that endogenously augments the output lags of the VAR in recessionary phases. Sufficient conditions for the ergodicity of the model, potentially applying to a larger class of threshold models, are provided. The linear specification is severely rejected in favour of our threshold VAR. However, in the estimation the feedback is found to be statistically significant only on unemployment, while it transmits to output through its cross-correlation. This feedback effect from recessions generates important asymmetries in the propagation of shocks, a possible key to interpret the divergence in the measures of persistence existing in the literature. The regime-dependent persistence also explains the finding that the feedback from recession exerts a positive effect on the long-run growth rate of the economy, an empirical validation for the Schumpeterian macroeconomic theories.

JEL Classification: C32, E32
Keywords: non-linearity, threshold VAR, ergodicity, impulse–response function, persistence, long-run effect of recessions

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# NON-TECHNICAL SUMMARY

The vector auto-regressive (*VAR*) system of output and unemployment rate is one of the most commonly studied in the macroeconometric tradition in order to analyse the propagation and the persistence of shocks in the real economy and the transmission mechanism between product and labour market. The common feature of those *VAR* applications is that the dynamics of the model are routinely assumed to be linear, meaning that the effects of the shocks perturbing the system are proportional to the size of the shocks itself. In light of the mounting empirical evidence on the non-linear properties of both time series, the validity of this assumption is questionable, as important features of the data might be concealed by imposing the linear specification.

The purpose of this work is to offer a thorough statistical investigation of the joint dynamics of output and unemployment rate, allowing for non-linear interactions between the two series. In order to accomplish this task, we specify a non-linear *VAR* model where the non-linearity arises from the inclusion in the system of a feedback variable measuring the depth of the current recession. Consequently, the model has a different linear structure according to the regime (expansionary or recessionary) the economy is undergoing.

Our framework is a multivariate model where the duration of the feedback from recession is optimally chosen in a model selection procedure, the feedback variable affects both the conditional mean and the conditional variance, and the definition of the expansionary or recessionary regimes is endogenously determined. When all these features are simultaneously included in the model there is a desirable gain in generality, but also a significant rise in the degree of complexity of the statistical analysis. Given the non-standard specification, great attention is posed in order to investigate the dynamic properties of the proposed model and the validity of the asymptotics results. Moreover, generally applied model selection procedures and standard tests of non-linearity prove to be inadequate in this context, so bootstrapping techniques are extensively used to cope with these problems in order to obtain the best possible specification to be estimated.

The first task we undertake with the estimated model is to formally test whether the non-linearity is statistically significant. The analysis performed severely rejects the linear structure. A deeper look at the estimates, though, shows that the feedback variable enters significantly only in the unemployment equation, while there is no evidence of a strong direct non-linear feedback in the time series for output. This is where our multivariate extension becomes relevant. Although univariate tests have often found clear signs of non-linearity and asymmetry in both series, our analysis suggests that

the non-linearity is directly present only in unemployment and transmits to output through its cross-correlation.

Second, we perform an impulse–response analysis on the model following the recently developed theory on non-linear impulse responses. In line with the existing applied work focusing on this class of univariate threshold model, we find that recessions have a positive feedback on the economic activity in the long run. Thus, in recession aggregate shocks are remarkably less persistent than they are in expansion and negative aggregate shocks display lower long-run persistence than positive ones. When we look at the propagation of reallocative shocks, we also find rich asymmetries in the impulse responses. This impulse–response analysis leads us to argue that the conflicting findings in the traditional literature on the persistence of shocks to output, relating the persistence coefficient to the order lag of the model, can be rationalized within our non-linear model.

The specification we choose for the non-linearity is particularly attractive when output is modelled with a stochastic trend, as we have done, because it establishes a potential link between aggregate fluctuations and long-run growth. This relationship is one of the cornerstones of the Schumpeterian approach. Schumpeter viewed recessions as times when the creative-destruction process was taking place with the highest intensity: old technologies would be swept away by new and more productive ones, thus opening the way to future growth. Various authors have recently revived the Schumpeterian idea of recessions as periods in which a cleansing process is activated in the economy through formal macroeconomic models whose main prediction is the existence of a positive feedback between recessions and long-run growth of output, similar to the one identified in the impulse–response analysis. Our model is a natural environment to uncover empirically this interaction between recessions and growth, while any linear model is inadequate since it would treat symmetrically both phases of the business cycle. Our main finding in this regard is that recessions do entail a positive feedback on growth, although quantitatively small.

# 1. Introduction

The bivariate system of output and unemployment rate is one of the most commonly studied in the $VAR$ tradition to analyze the propagation and the persistence of shocks in the real economy and the transmission mechanism between product and labor market. Notable examples include Blanchard and Quah (1989), Evans (1989), and Aoki and Fiorito (1993). The common feature of these $VAR$'s is that the dynamics of the model are routinely assumed to be linear.[1] In light of the mounting empirical evidence on the nonlinear structure of both series, the validity of this assumption is questionable, as important features of the data might be concealed by the linear specification.[2]

The purpose of this paper is to offer a thorough statistical investigation of the joint dynamics of output and unemployment rate, allowing for nonlinear interactions between the two series. In order to accomplish this task, we specify a *threshold VAR* model where the nonlinearity arises from the inclusion in the system of a feedback variable measuring the depth of the current recession. Consequently, the model has a different linear structure according to the regime (expansionary or recessionary) the economy is undergoing. The threshold growth rate of output which separates the two regimes is endogenously estimated.

This nonlinear specification was first introduced by Beaudry and Koop (1993) (hereafter, BK) in a univariate model of U.S. output where the feedback variable was allowed to enter the conditional mean with multiple lags, but the threshold parameter was fixed *ex-ante* and not estimated. Pesaran and Potter (1994) (hereafter, PP) generalize this model by allowing the threshold to be estimated and the feedback variable to enter the conditional variance as well, but fix deterministically the maximum lag at which the feedback variable can be active. Koop, Pesaran and Potter (1996) (hereafter, KPP) further extend the PP model to a multivariate framework.

Our framework is a multivariate model where the highest lag of the feedback variable is optimally chosen in a model selection procedure, the feedback variable affects both the con-

[1] An exception is the work of Koop, Pesaran and Potter (1996) where a threshold VAR of output and unemployment rate is used to illustrate some of the important issues in performing impulse response analysis in nonlinear multivariate models.

[2] The nonlinearity of US output data is discussed, among others, by DeLong and Summers (1986), Hamilton (1989), Potter (1995), and Pesaran and Potter (1997). Evidence on the nonlinearity of unemployment is found, among others, by Rothman (1991), and Montgomery, Zarnowitz, Tsay and Tiao (1996).

ditional mean and the conditional variance and the associated threshold parameter is endogenously estimated. When all these features are simultaneously included in the model there is a desirable gain in generality, but also a significant rise in the degree of complexity of the statistical analysis, compared with the aforementioned studies. Standard model selection procedures are inadequate because the competing models can be non-nested, and standard tests of nonlinearity cannot be implemented because the threshold parameter vanishes under the null of linearity. Bootstrapping techniques are extensively used to cope with these problems in order to obtain the best possible specification to be estimated.

Even though this type of threshold model is quite common in applied time-series, its stability properties and the asymptotics of its quasi-maximum likelihood estimator ($QMLE$) have never been properly investigated. We provide a set of sufficient conditions for ergodicity which are quite general and potentially apply to a larger class of threshold models. Our approach is based on their Markovian representation and on a modification of the $N$-step criteria proposed by Tjøstheim (1990). We also prove that the model can be written as a $SETAR$ with linear parameter restrictions, which leads us to argue that the consistency of the $QMLE$ can be proved by slightly extending the result in Chan's (1993) on the asymptotics of least squares estimators in $SETAR$ models.

The first task we undertake with the estimated model is to formally test whether the nonlinearity is statistically significant. The test performed accounting for the nuisance parameter problem severely rejects the linear structure. A deeper look at the estimates, though, shows that the feedback variable enters significantly only in the unemployment equation, while there is no evidence of a strong direct nonlinear feedback in the time series for output. This is where our multivariate extension becomes relevant. Although univariate tests have often found clear signs of nonlinearity and asymmetry in both series, our analysis suggests that the nonlinearity is directly present only in unemployment and transmits to output through its cross-correlation.

Second, we perform an impulse response analysis on the model following the recently developed theory on nonlinear impulse responses. In line with the existing applied work focusing on this class of univariate threshold model, we find that recessions have a positive feedback on the economic activity. Thus, in recession aggregate shocks are remarkably less persistent than they are in expansion, and negative aggregate shocks display lower long-run persistence than positive ones. When we look at the propagation of reallocative shocks, we also find rich asymmetries in the impulse responses. This impulse response analysis leads us to argue that the conflicting findings in the traditional literature on the persistence of shocks to $GNP$, relating the persistence coefficient to the order lag of the model, can be rationalized within our nonlinear model where the order lag is regime-dependent.

The specification we choose for the nonlinearity is particularly attractive when output is modeled with a stochastic trend, as we do, because it establishes a potential link between aggregate fluctuations and long-run growth. This relationship is one of the cornerstones of the Schumpeterian approach. Schumpeter viewed recessions as times when the creative-destruction process was taking place with the highest intensity: old technologies would be swept away by new and more productive ones, thus opening the way to future growth. Various authors have recently revived the Schumpeterian idea of recessions as periods in which a cleansing process is activated in the economy through formal macroeconomic models whose main prediction is the existence of a positive feedback between recessions and long-run growth of output.[3] Our model is a natural environment to uncover empirically this interaction between recessions and growth, while any linear model is inadequate since it would treat symmetrically both phases of the business cycle. Our main finding in this regard is that recessions do entail a positive feedback on growth, although quantitatively small: at most $3.5\%$ of output growth in the post-war U.S. economy can be accounted for by this channel.

The rest of the paper is organized as follows. Section 2 presents the statistical model. Section 3 lays out the conditions for its ergodicity, and discusses the asymptotic properties of the $QMLE$. Section 4 describes the results of the order selection procedure, the estimation, and the test against the null of linearity. In Section 5 we illustrate the impulse responses of the model to different types of innovations, and we discuss the issue of persistence of aggregate shocks on output. Section 6 tests empirically the creative-destruction hypothesis and quantifies the long-run positive feedback from recession. In Section 7 we make our concluding remarks. The Appendix contains the proof that the model admits a Markovian representation, the proof of ergodicity with an illustrative example, and a description of the bootstrap method

[3] See Aghion and Howitt (1994), Caballero and Hammour (1995), and Aghion and Saint-Paul (1998).

used in the paper.

## 2. A Threshold VAR of Output and Unemployment Rate

We will conduct our econometric analysis with a nonlinear bivariate model of changes in the log of real $GNP$ $(y)$ and in the unemployment rate $(u)$.[4] Following BK, KPP, and PP, the nonlinearity is introduced by constructing a feedback variable measuring the *current depth of the recession* $(CDR_t)$ which augments the linear vector autoregressive dynamics of $\Delta X_t \equiv (\Delta y_t, \Delta u_t)$. This feedback variable is defined as the gap between the current level of log-output and the economy's historical maximum level augmented by a threshold parameter $r$. More formally:

$$CDR_t\left(r, \tau\right) = y_t - \max\left\{y_t, y_{,t-1} + r, ..., y_{t-\tau} + r\right\}, \tag{1}$$

where $\tau$ is a finite integer. $CDR$ will be zero as long as output grows at least at a rate $r$. However, once a period with growth of output lower than $r$ switches on $CDR$, the latter will remain activated as long as the log of output is at a lower level compared to its previous maximum increased by $r$, i.e., as long as the economy has not fully recovered the gap of the recession. The threshold parameter $r$ will be endogenously estimated, thus duration and depth of recessions are endogenous as well.

After embedding the feedback variable into the linear $VAR$, the joint dynamics of $\Delta X_t$ follow the specification:

$$\Phi(L)\Delta X_t = \alpha + \Theta(L)CDR_{t-1}\left(r, \tau\right) + \varepsilon_t, \tag{2}$$

where $\alpha$ is a vector of constants, $p$ is the order of the polynomial $\Phi(L)$ and $(q-1)$ is the order of $\Theta(L)$. In the rest of the paper, we will refer to the situation in which the dynamics of the system are not under the effect of any feedback, i.e., $\sum_{i=1}^{q} CDR_{t-i} = 0$, as the *expansionary* regime, while we will speak of *recessionary* regime in the case in which at least one of the lags of the $CDR$ variable is non-zero.

Recessions are often perceived to be times of higher economic turbulence compared to

[4] The series are Citibase quarterly data from 1952.1 to 1990.4 of U.S. GNP at 1982 prices (GNP82) and total unemployment rate (LHURN). The series are scaled to percentage points.

expansions. French and Sichel (1993), among others, provided robust evidence to this common perception. Accordingly, we allow for regime-dependent heteroskedasticity and define $\varepsilon_t = V_t^{\frac{1}{2}} u_t$ with $u_t \overset{iid}{\sim} (0, I_2)$. We model the conditional variance $V_t$ as:

$$V_t = 1(\sum_{i=1}^{q} CDR_{t-i} = 0)(\Omega_e - \Omega_r) + \Omega_r, \tag{3}$$

where $1(\cdot)$ is the indicator function, and $\Omega_e$ (respectively, $\Omega_r$) is the covariance matrix in the expansionary (recessionary) regime.[5] It should be noticed that since $CDR$ depends on the threshold parameter $r$, the latter enters both the conditional mean and the conditional variance.

The model to be estimated is described by equations (1)-(3). To gain more intuition about the structure of the model, it is helpful to observe from (1) that $CDR$ can be either zero or a constant plus the sum of the $\Delta y$ terms from the date of the previous peak to the current time. Therefore, by substituting (1) into (2) we obtain a linear $VAR$ with time varying parameters and time varying number of lags determined by an endogenous deterministic threshold rule. It follows that when the economy is under the effect of the feedback, at each period the linear structure is modified with additional lags and different parameter values.

## 3. Ergodicity of the Model and Asymptotics of the QMLE

### 3.1 Ergodicity

In this section we aim at providing sufficient conditions for ergodicity for a large class of $k$-dimensional piecewise linear threshold processes $\{Z_t\}$ admitting the Markovian representation:

$$Z_t = \sum_{i=1}^{I} (C_i + A_i Z_{t-1}) 1(Z_{t-1} \in P_i) + \varepsilon_t, \quad P_i \in \mathcal{P} \tag{4}$$

where $P_i$, $i = 1, ..., I$ is a generic element of the partition $\mathcal{P}$ of the space of $Z_t$, and $\varepsilon_t$ is an independent zero mean random variable with positive density function and with $E\|\varepsilon_t\| < \infty$.

[5] This structure of the conditional variance is a form of Qualitative Threshold Autoregressive Conditional Heteroskedasticity (QTARCH), introduced by Gourieroux and Monfort (1992). This specification is a parsimonious way of allowing different volatilities in the two phases of the business cycle. An alternative specification of the conditional variance within the GARCH family would not be completely satisfactory due to the larger number of parameters and to the impossibility of clearly associating the regime for the conditional mean with the regime for the conditional variance. However, it will be clear later that the form of the conditional variance we adopt will induce some difficulties in the process of model specification (see Sections 3.2 and 4.2).

In the Appendix we show that, under the assumption of the finite memory of the feedback process (i.e. a finite $\tau$), our *threshold VAR* displays the above Markovian structure, where each partition of the space is constructed through the $CDR$ variable. Notice that (4) is a rather general specification which includes all the variants of the $SETAR$ family.

General conditions for the ergodic behavior of various specifications of nonlinear time series processes have been provided, among others, by Pham (1986), Tong (1990), and Tjøstheim (1990). Common to all is the use of Markov chain theory and in particular the construction of a test function to verify the existence of a central set in the space of the time series towards which the stochastic trajectories drift almost surely. Our proof of ergodicity for the class of models in (4) is based on the "$N$-step drifting condition" proposed by Tjøstheim (1990).

The strategy we follow is to characterize how the process $Z_t$ moves between each two elements of the partitions of the space of $Z_t$ and to verify under which conditions it drifts towards the center of the space within $N$-steps. Two remarks greatly simplify our task. First, the definition of the partitions imposes several constraints on the dynamics of the process and, consequently, it restricts the "adjacent" elements of the partition. Second, out of deterministic path realizations have probability decreasing with the dimension of $Z_t$. It follows that in verifying the drifting condition, without loss of generality we will confine our attention to those movements of $Z_t$ which occur within a restricted subset of the partition and are driven only by the deterministic part of the process.

Pursuing this logic, define an element $P_i$ of a given partition to be *deterministically adjacent to* $P_j$ if there is a $Z_{t-1} \in P_i$ such that $C_i + A_i Z_{t-1} \in P_j$. We also denote the set of all possible sequences of length $N$ of *deterministically adjacent* elements as $\widetilde{\sigma}(N)$. Finally, denote by $\widetilde{P}_i$ the union of the *deterministically adjacent* sets to $P_i$. We are now ready to state:

**Proposition 1:** Given the Markov process in (4), if:

**(A1)** there exists an $N > 0$ such that

$$\max_{\widetilde{\sigma}(N)} \left\{ \rho \left( \prod_{i=1}^{N} A_{h_i} \right) \right\} < 1,$$

where $\rho(A)$ is the maximum eigenvalue of the matrix $A$ in absolute value, and

**(A2)** for all $P_i$'s, as $\|Z_{-1}\| \to \infty$, for some $\epsilon > 0$ :

$$\Pr\left\{(Z \notin \widetilde{P}_i) \cap (Z_{-1} \in P_i)\right\} = O\left(\|Z_{-1}\|^{-\epsilon}\right),$$

then the process $Z_t$ is geometrically ergodic.

**Proof:** The proof and an illustrative example are provided in the Appendix.

Assumption (**A1**) is a condition on the eigenvalues of the $N$-step transition requiring the existence of a finite $N$ such that the deterministic dynamics of the process are "stable".[6] Assumption (**A2**) regulates the probability of dynamics not driven by the deterministic part of the process by imposing that, as $Z$ becomes large, this probability declines at a given rate. Since condition (**A1**) is in terms of eigenvalues, it does not easily allow us to characterize a region of the parameter space associated with the stability of the process. Rather, the logic of the proposition permits us to easily assess the ergodicity of the process associated to a given parametrization of the model. This assessment requires two steps: first, identify the deterministic adjacent sets and second, find the $N$-step horizon for which the eigenvalue condition is satisfied.

### 3.2 Asymptotics of the QMLE

The estimation of the parameters of the conditional mean and conditional variance of our model is performed by quasi-maximum likelihood under the nominal assumption of normality of the innovation. The algorithm we use is the two-step procedure suggested by Tong (1990) for $SETAR$ models, and implemented by PP as well. It consists of generating a finite grid of points over the domain of the threshold parameter $r$ and at each point on the grid maximizing the likelihood function with respect to the remaining parameters through a standard hill-climbing algorithm. The value of $r$ chosen in the second step is that point on the grid for which the log-likelihood attains its global maximum.[7]

Proving the consistency of this estimator is complicated by the discontinuity of the likelihood function with respect to the parameter $r$, the threshold coefficient. What generates the discontinuity is the conditional variance term changing discretely between regimes, whereas

[6] An important remark is that it is possible to have overall stable dynamics of the process associated with unstable dynamics (i.e. eigenvalues above one) in particular states, given that the relevant concept is stability in $N$-steps.

[7] The estimates have been performed with a grid of 400 point in the interval $(-.013, +.013)$ and at every point it has been allowed for 100 iterations over the likelihood function with a convergence criteria of $10^{-5}$ for each parameter. To reduce the already large computational burden, we do not estimate the parameter $\tau$ which is meant to capture the memory of the feedback process, but we assume it is larger than the length of our sample.

the conditional mean changes smoothly. Given the ergodicity of the process, consistency can be easily verified in two steps. First, as we show in the Appendix, our specification can be expressed as a $SETAR$ with a large number of regimes and linear restrictions among the parameters in the different regimes. Second, our model being a $SETAR$, the consistency theorem of Chan (1993) generalizes to our case because, under the correct specification of the model, the presence of linear restrictions among the parameters does not modify the logic of Chan's proof.[8]

Finally, assessing the asymptotic normality of the estimator would require proving the differentiability of the population likelihood with respect to the parameter vector, including $r$. We do not pursue this strategy, but more simply we observe that conditional on $r$, asymptotic normality descends from standard asymptotic theory. Hence, if the speed of convergence of $r$ is sufficiently fast, then the threshold value can be treated as known in performing inference on the autoregressive parameters and therefore the standard asymptotic theory would hold. This conjecture is based once again on the result in Chan (1993), where the superconsistency of $r$ is proved for a two-regime $SETAR$, and it is largely supported by a set of Monte Carlo experiments we performed with our model.

## 4. Model Specification, Estimation and Testing

### 4.1 Levels vs. First Differences

The first step in model specification is the choice of levels vs. first differences in the variables. Standard unit root testing suggested the existence of an autoregressive root in both series, but the presence of the nonlinearity might invalidate the asymptotic theory of the traditional tests. We therefore opted for the test proposed in Caner and Hansen (1997) which is robust with respect to a large class of threshold nonlinearities in the $DGP$, including ours. The bootstrap $p-values$ of the test statistics for the null of unit root on the level of the series are respectively $0.821$ for output and $0.153$ for unemployment, confirming the results of the standard Dickey-Fuller tests. An additional argument in favor of differencing unemployment

[8] We thank one Referee for suggesting this argument. An alternative proof of consistency can be constructed following Andrews (1987), as done in Altissimo and Violante (1998). There we show that, despite the discontinuity of the sample log-likelihood, its expectation is a smooth function of the parameters.

is related to our proof of ergodicity. When we specified and estimated the system with unemployment rate in levels, the maximum eigenvalue of the matrix driving the dynamics in the expansionary regime was found to be outside the unit circle thus precluding the possibility of finding a $N-$step sequence for which (**A1**) holds, whereas when we estimated the model in first differences of both series, condition (**A1**) was satisfied. Although (**A1**) is only a sufficient condition for ergodicity, it is not too restrictive. For example, it admits diverging dynamics (i.e. eigenvalues above one) in particular states associated with overall stable dynamics of the process, since the proof is based on a $N$-step drifting condition. Hence, it appears that violations of that condition should be taken as a strong signal of non-ergodicity.

These two findings have led us to choosing a specification in first differences for both variables as the benchmark model.[9] This choice differentiates sharply our work from the previous studies on $VAR$'s of output and unemployment which treated the unemployment rate as a stationary variable, although consistently recognizing that the evidence on this point was not unequivocal. To allow a more direct comparison with this literature, in Section 5.5 we study how sensitive are some of our conclusions to the choice of modeling unemployment as $I(0)$ or $I(1)$.

### 4.2 Optimal Lag Order

The next step is the choice for the lag order in the $\Phi(L)$ polynomial of the autoregressive linear part, and in the $\Theta(L)$ polynomial of the feedback variable. We started from a maximum lag of 8 for $\Delta X$ and 4 for $CDR$, and for each combination we computed Akaike and Schwartz information criteria. The results are presented in Table 5 at the end of the text. We conclude that, for any given lag of $CDR,$ the model with two lags of $\Delta X$ is a reasonable and conservative specification. However, models with different lag orders of $CDR$ are non nested as a result of the specification of the conditional variance, and for this reason the two information criteria are not useful to select among them. The choice of the best lag order of the $CDR$ has to rely upon a non-nested testing procedure in which models with two lags of $\Delta X$ and various

[9] The presence of a nonlinear cointegration relationship between output and unemployment has been investigated applying the procedures suggested by Balke and Fomby (1997) and by Corradi, Swanson and White (1997), but we found no evidence of cointegration.

lags of $CDR$ are contrasted among each other.[10]

To test the specification with $j$ lags of $CDR$ against the "true" one with $i$ lags, we used a Cox type statistics in its multivariate version, as proposed by Pesaran and Deaton (1978), given by:

$$\frac{LR_{j,i} - E_i(LR_{j,i})}{\sqrt{V_i(LR_{j,i})}},$$

where $LR_{j,i}$ is $2\left(\ell_j - \ell_i\right)$, $\ell_i$ and $\ell_j$ are the sample log-likelihoods of the two models, $E_i$ and $V_i$ are respectively the expectation and variance under the "true" specification. The discontinuity of the log-likelihood function prevents us from utilizing the asymptotic results proposed by Pesaran and Deaton and obliges us to resort to resampling techniques in computing the empirical distribution of the statistics.[11] The statistics and the bootstrap $p-values$ (in parenthesis) for the test are reported in the Table below.

**Table 1 - Non-Nested Test**

| $lag\, CDR$ | $j=1$ | $j=2$ | $j=3$ | $j=4$ |
|---|---|---|---|---|
| $i=1$ | ... | 4.704 (0.001) | 4.133 (0.001) | 4.598 (0.001) |
| $i=2$ | 0.010 (0.520) | ... | 0.777 (0.188) | 1.545 (0.085) |
| $i=3$ | 1.292 (0.096) | 1.722 (0.029) | ... | 2.518 (0.019) |
| $i=4$ | 0.974 (0.167) | 1.739 (0.030) | 0.722 (0.914) | ... |

The testing strategy is quite successful in identifying the proper lag of $CDR$, and it indicates that models with 2 or 4 lags dominate specifications with 1 or 3 lags. Since the test is inconclusive on the selection between 2 and 4 lags, we favor a parsimonious representation and choose the model with 2 lags of $CDR$.

### 4.3 Results of the Estimation

The results of the estimation of the $VAR$ with 2 lags of $(\Delta y, \Delta u)$ and 2 lags of $CDR$ are shown in the following Table.

[10] Beaudry and Koop compare models with different lags of $CDR$ only by looking at standard order selection criteria, correctly so as their models are homoskedastic.

[11] In Appendix, we describe the bootstrap methodology used throughout all the paper.

**Table 2 - Parameter Estimates**

| | $const.$ | $\triangle y_{-1}$ | $\triangle u_{-1}$ | $CDR_{-1}$ | $\triangle y_{-2}$ | $\triangle u_{-2}$ | $CDR_{-2}$ |
|---|---|---|---|---|---|---|---|
| $\triangle y$ | $0.501$ | $0.051$ | $-1.095$ | $-0.071$ | $0.193$ | $0.614$ | $-0.201$ |
| | $(3.524)$ | $(0.469)$ | $(-3.118)$ | $(-0.329)$ | $(1.908)$ | $(1.977)$ | $(-1.022)$ |
| $\triangle u$ | $0.107$ | $-0.051$ | $0.416$ | $-0.143$ | $-0.084$ | $-0.127$ | $0.199$ |
| | $(2.761)$ | $(-1.716)$ | $(4.036)$ | $(-2.027)$ | $(-3.063)$ | $(-1.461)$ | $(3.029)$ |

The $t$-values (in parenthesis) are based on the asymptotic standard errors conditional on the estimated value of the threshold parameter. We regard these values as good approximations of the unconditional standard errors, as argued in the previous section. The most striking result of the estimation is that none of the $CDR$ lags are significant in the output equation, while they are significant in the unemployment equation. This is an interesting finding for two reasons. First, this pattern is very different from what we found in the model with $1$ lag of $CDR$ (used for instance by KPP) as in the latter model $CDR$ is significant on output and not on the unemployment rate, leading to a potentially diverging interpretation of whether the feedback operates in the labor market or in the product market. Second, this finding suggests that estimating nonlinear univariate models of output can be misleading if the nonlinearity is originally present in other series (unemployment, in our case) and transmits to output purely through its cross-correlation.

The $CDR$ variable enters strongly in the unemployment equation and the signs of its coefficients reveal the dynamic effect of the nonlinearity on the series. $CDR$ enters with a negative sign in the first lag and with a positive sign (and a bigger coefficient) in the second. This combination of signs offers a very intuitive interpretation of the *direct* effect of the feedback on the economy. When the system enters a recession, the feedback initially accelerates the rise in unemployment, since the first lag of $CDR$ is larger than the second. On the contrary, when the economy is in the recovery process, the second lag of $CDR$ is larger than the first one, thus the nonlinearity reinforces the fall of unemployment, by making the recovery shorter and sharper. Through the strongly significant lags of unemployment in the output equation, such a nonlinear behavior transmits to output as well. Finally, notice that the long run coefficient on $CDR$ for both series implies a beneficial effect originating from recessions.

As expected, we find that recessions are times of stronger volatility. The standard deviation of innovations to output and unemployment is, respectively, 50% and 100% higher in contractionary phases of the cycle. The estimated value of $r$ is $-0.138$, which is not too far from zero (the value assumed in the univariate model of BK) and implies that 57 observations —approximately $1/3$ of the sample— fall into the recessionary regime.

Figure 1 shows the values for $CDR$ implied by the estimation together with the $NBER$ chronology for business cycles from the quarter following the peak to the quarter of the trough. Interestingly, in most cases the $CDR$ variable starts increasing at a date corresponding to the $NBER$ definition of peak and falls to zero in the quarter of the trough, or the following one. Therefore, its timing coincides strikingly well with the "conventional wisdom" about recessions and, in addition to the $NBER$ definition, the $CDR$ variable provides a measure of the depth of each recession.

### 4.4 Test for Nonlinearity

With the "best" specification of the nonlinear model in hand, one has to test the significance of the nonlinearity itself. This test is aimed at understanding whether a specification of the dynamics of $\Delta X_t$ with a nonlinear term but also with a possibly induced heteroskedasticity in the error term fits the data better than the best possible linear model. After the appropriate order selection procedure, we have chosen as null hypothesis an homoskedastic linear $VAR(2)$, which we have re-estimated on the same data. Testing the significance of the nonlinearity leads to a nuisance parameter problem because under the null of the test the threshold parameter $r$ vanishes.[12] Our test procedure can be summarized in three steps. First, for each value of the space of the nuisance parameter $r$, we compute the likelihood ratio $LR_{20}(r) = 2\left(\ell_2(r) - \ell_0\right)$, where $\ell_2$ is the log-likelihood of the nonlinear model with $2$ lags of $CDR$, and $\ell_0$ is the log-likelihood of the alternative linear model. Second, following Andrews and Ploberger (1994), we construct three different statistics: $supLR = \sup_r(LR(r))$, $expLR = \ln E_r \exp(LR(r))$ and $avgLR = E_r LR(r)$, where the expectation is taken with respect to the nuisance parameter. Third, the distributions of the statistics are computed by simulation.

[12] If $r$ is exogenously fixed, as in BK, this problem does not arise and the test for nonlinearity becomes a simple $F$ test with null hypothesis $H_0 : \Theta(L) = 0$.

**Table 3 - Nonlinearity Test**

| | $supLR$ | $expLR$ | $avgLR$ |
|---|---|---|---|
| $stat$ <br> $(p{-}value)$ | $57.959$ <br> (0.000) | $55.199$ <br> (0.002) | $48.530$ <br> (0.001) |

Table 3 reports the bootstrap $p - values$ for the three statistics and shows that all these tests largely rejected the linear model. Although we are aware to have chosen just one specification of nonlinearity among a variety of possibilities, the evidence against the linear specification is surprisingly strong. Even though the true unknown $DGP$ may not be a *threshold VAR*, it seems to be much closer to this nonlinear model than to a linear one. We reckon therefore that using a linear *VAR* in this context may induce misleading conclusions about the general dynamic behavior of the series, the pattern of the impulse responses and the persistence of the shocks. We examine all these issues in the next sections.

## 5. The Propagation and Persistence of Shocks

### 5.1 Nonlinear Impulse Responses

One of the main objectives of this paper is to understand the propagation mechanism and the persistence properties of the shocks perturbing our nonlinear system of output and unemployment rate. As explained in KPP, nonlinear Impulse Response (IR) functions in multivariate models are *history, shock* and *composition dependent*. Each one of these types of dependence poses methodological problems in the definition and the computation of IR functions which, as of today, do not have a unique established answer in the literature.[13] KPP define the Generalized Impulse Response $(GIR)$ function as:

$$GIR_{\Delta X}(T, \varepsilon_t, \mathcal{H}_{t-1}) = E(\Delta X_{t+T}|\varepsilon_t, \mathcal{H}_{t-1}) - E(\Delta X_{t+T}|\mathcal{H}_{t-1}),$$

where $T$ is the time-horizon, $\varepsilon_t$ is the vector of innovations at time $t$ and $\mathcal{H}_{t-1}$ is a generic history until time $(t-1)$.[14] The function above is a random variable with respect to the shock and the history with the property that its joint distribution contains all the information about propagation and persistence of the shocks in the system. Since we are mainly interested in the

[13] The theory of impulse responses in nonlinear multivariate models is fairly recent. See Gallant, Rossi and Tauchen (1993), and Koop, Pesaran and Potter (1996) for extensive discussions.

[14] In our model, all the relevant information at time $t-1$ is given by $\{CDR_{\mathrm{t-i}}, \Delta X_{\mathrm{t-i}}\}$ with $i = 1, 2$.

regime asymmetry of the IR function, we will focus on the expectation of the $GIR$ conditional on a restricted set of representative realizations of the shocks and on a particular regime. We therefore define our IR function as:

$$IR_{\Delta X}(T,\eta,R) = E\left[E(\Delta X_{t+T}|\varepsilon_t=\eta,\mathcal{H}_{t-1}) - E(\Delta X_{t+T}|\mathcal{H}_{t-1}) \mid \mathcal{H}_{t-1}\in R\right], \qquad (5)$$

where $\eta$ is a given realization of $\varepsilon_t$, and $R \equiv \{expansion,\ recession\}$ is an index of the regime of the economy at time $t-1$. This definition of IR is similar to that of Gallant, Rossi and Tauchen (1993), except for the different baseline forecast. They condition the baseline on a null realization of the current shock, while we average over all the possible realizations, as in KPP. The rationale for this choice comes directly from the meaning of the baseline, which should represent the average behavior of the system. Only in linear models the two baseline forecasts coincide.

From the representation in (2), and from the definition of IR in (5), it is easy to show that the IR for the levels of the variables cumulates to:

$$\begin{aligned} IR_X(T,\eta,R) = {} & \textstyle\sum_{k=0}^{T}\{\Psi_k\eta + \sum_{j=0}^{k} B_j[E(CDR_{t+k-1-j}|\varepsilon_t=\eta,R) \\ & -E(CDR_{t+k-1-j}|R)]\} \end{aligned} \qquad (6)$$

where $\Psi(L) = \Phi^{-1}(L)$ and $B(L) = \Phi^{-1}(L)\Theta(L)$. Therefore, not only does the persistence arise from $\Psi(1)$ as in the linear case, but also from the nonlinear structure. In particular, the difference in the realizations of $CDR_t$ between the shocked economy and the baseline forecast permanently affects the level of the variables.

Finally, we generalize the definition of persistence of Campbell and Mankiw (1987) to our nonlinear framework, and measure the persistence of a shock $\eta$ in regime $R$ on the variable $X_i$ as:

$$\pi(\eta,R) = \frac{IR_{X_i}(\infty,\eta,R)}{IR_{X_i}(0,\eta,R)}, \qquad (7)$$

which is that value the impulse response function converges to in the long-run, once it is normalized to one in the initial period.

### 5.2 Computation of Impulse Response Functions

Our computation of the IR functions follows closely the procedure of KPP who suggest a

Monte Carlo technique to numerically integrate the expectations in (6). The three main steps in the implementation of the procedure are the choice of the histories, $\mathcal{H}_{t-1}$, the calibration of the shocks $\varepsilon_t$, and the treatment of the future shocks.

We have used the histories in the observed sample, rather than generating new ones. Altogether, we have 97 sample paths ending up in the expansionary regime and 57 leading to the recessionary regime. To estimate the IR conditional on the regime, we should average over all the histories terminating in each regime. We have noticed though that a large number of realizations of the recessionary regime are very mild. Since we wanted to focus on those histories in which the economy is undoubtedly in a phase of recession, we decided to discard those observations in which both $CDR_{t-1}$ and $CDR_{t-2}$ are below $.1$.[15] Following this strategy, we are left with $25$ histories that are sharply different from those of the expansionary regime.

To calibrate the shocks, we have adopted the graphical method suggested by Gallant, Rossi and Tauchen (1993) consisting of scattering the residuals for the two time series and, by inspection, determining what could be regarded as typical shocks to the system. Shocks increasing (decreasing) the growth rate of output and decreasing (increasing) the change in the unemployment rate can be interpreted as *aggregate* shocks. The residual plot suggests to pick the pair $(1, -.2)$ for $(\Delta y, \Delta u)$ as a representative positive aggregate (P) shock. By changing the sign, we obtain a negative aggregate (N) shock and by doubling their magnitude (PP, and NN) we can explore the size-asymmetry of the IR.

Besides being hit by aggregate shocks, the economy is often perturbed by shocks which move output and unemployment in the same direction. These innovations can be interpreted as *reallocative* shocks, and the plot of the residuals indicates the pair $(.5, .1)$ (and its negative counterpart) as representative of this class of innovations. Since reallocative shocks occur more than $30\%$ of the times in our sample period, it is of great interest to study their propagation dynamics. The IR analysis will therefore be performed on both aggregate and reallocative shocks.

Once the initial shock is chosen from one of the cases above, the future sample paths of the system are generated by the bootstrap method described in detail in the Appendix. The

[15] Two entire (but very mild) recessionary phases of the NBER official calendar are eliminated through this procedure: the mini-recession of 1960 and that of 1970. See Figure 1 for the details.

regime-dependent empirical IR is therefore computed as:

$$\hat{I}R_X(T,\eta,R) = \frac{1}{N_R}\sum_{j=1}^{N_{\mathsf{R}}}\left\{\frac{1}{M}\sum_{i=1}^{M}\left[X_{t+T}^{i}\left(\eta,\mathcal{H}_j\right) - X_{t+T}^{i}\left(\varepsilon_t^i,\mathcal{H}_j\right)\right]\right\}, \tag{8}$$

where $N_R$ indicates the number of histories for the two regimes, $\mathcal{H}_j$ is the observed $j-th$ history for regime $R$ associated with the realizations until time $(t-1)$, and $M$ is the number of Monte Carlo replications. The Law of Large Numbers for $i.i.d.$ random variables ensures convergence of the sample mean within brackets in (8) —for each history— to the time invariant expectation characterizing the true IR conditional on the same history.

### 5.3 Results of the Impulse Response Analysis

The combination of regression coefficients on the various lags of the $CDR$ variable found in the estimation implies that, when the economy enters a downturn, with some lag it will benefit from an upward push proportional to the depth of the recession. This feedback effect is the key source of the asymmetry in the shape and the long-run persistence of the impulse responses of the model, plotted in Figures 2, 3 and 4.[16]

*Aggregate Shocks*

A striking regime-dependent pattern of long-run persistence stands out in our results: aggregate shocks are remarkably *less* persistent when they hit in a recession. Consider the dynamics of $GNP$ after the economy is perturbed by a negative aggregate shock (N), depicted in the lower-left panel of Figure 2. When the economy is in the recessionary regime, it responds through an initial drop in output, but only after two quarters the positive feedback from the $CDR$ variable sets in strongly and leads output well above its initial level. In the linear regime this feedback effect is weaker because the nonlinearity is likely to be activated less often —as it is evident from the smoother and delayed hump in the IR— so output levels off slightly below its initial value. In the IR for the unemployment rate (lower-right panel), the feedback effect from recession is even stronger as it appears from the sharp hump of the IR function in downturns. This is consistent with the larger and more significant coefficient of

[16] In all graphs, the impulse responses have been normalized so that their value at impact is 1. In this way, the persistence coefficient can be immediately calculated from the graphs.

the $CDR$ variable in the unemployment equation of the $VAR$. Turning to positive shocks (P), an even more pronounced regime-dependent pattern of persistence is found. To understand why this happens one has to recall that the IR is the difference between the response of the shocked economy and that of the baseline economy. In downturns, after the positive innovation, the shocked economy is pushed out of the recession quickly, while in the baseline economy the feedback is likely to stay activated for longer and its positive effect contributes to reduce the difference between the shocked and the baseline systems, reducing therefore the long-run persistence.

Interestingly, the observed pattern of the IR for unemployment is consistent with the predictions of the Schumpeterian model of Aghion and Howitt (1994). In their model, a positive aggregate shock is more beneficial to unemployment in expansion than in recession because the capitalization effect of more job creation dominates the creative-destruction effect for high growth rates. Moreover, as stressed by Postel-Vinay (1998), the Aghion and Howitt model implies a sign-asymmetry in the short-run dynamic response of unemployment to aggregate shocks. A negative shock makes unemployment jump up strongly at impact, while its response to a positive shock is much smoother, a qualitative pattern that we recognize in our pictures.

A comparison of Figures 2 and 3 shows a remarkable size-asymmetry in the IR functions. Positive shocks of double magnitude (PP) induce a much smoother IR and have larger persistence than the benchmark shocks of Figure 2 in both regimes. On the other hand, negative shocks of double magnitude (NN) display a more pronounced hump-shape because the upward push originating from recession is more intense, and consequently long-run persistence is lower. This finding suggests a pronounced sign-asymmetry in the IR functions. In the expansionary regime, positive aggregate shocks always show more persistence compared to negative ones, independently of their size. This is still a result of the feedback operating in recession: a negative innovation is much more likely than a positive one to precipitate the economy in the downturn regime, and consequently activate the feedback variable. In the recessionary regime, small positive shocks tend to be slightly less persistent than negative ones of comparable magnitude, but as the size increases the pattern is reversed as clear from Figure 3.

*Reallocative Shocks*

The responses of output and unemployment to a reallocative shock are pictured in Figure 4. As customary in the literature, we have defined a positive reallocation disturbance as one that has a positive net effect on the aggregate economic activity (hence, on output), but it also increases the variance of the shocks at the micro level, inducing a spur of labor reallocation and an increase in unemployment. When a positive reallocative innovation hits the economy in a recession, the positive shock to output is likely to push the system out of a recession and weaken the feedback effect. However, the baseline economy will fully exploit the positive feedback from the recession, hence the shock has barely any impact on output in the long-run. By the same token, the weak feedback effect is unable to counteract the impulse to unemployment, thus it provokes a sharp rise in the unemployment rate.

On the contrary, in expansions the reallocative shock only mildly increases unemployment, whereas it benefits output in the long-run. Finally, recessions are still associated with lower persistence in output, as with aggregate shocks, but the regime-asymmetry is completely reversed for unemployment.

*Comparison with the Literature*

In the existing literature, only KPP perform an impulse response analysis on a nonlinear $VAR$ of output and unemployment rate. There are a number of methodological differences with respect to our approach. First, they model the unemployment rate in levels. Second, they have a three-regime model that includes a ceiling regime when the economy "overheats". Third they explicitly say that they do not attempt a full estimation of the best nonlinear specification as the model is only used as an example to illustrate their approach to IR analysis in nonlinear multivariate models. Fourth, they compute "generalized" IR functions, and plot their entire distributions. Since they do not condition on the type of shock, their analysis does not document the sharp difference in the propagation of aggregate and reallocative shocks that we have reported above. Nevertheless, their findings on sign and regime asymmetry of shocks to output are quite similar to ours. The main discrepancies are the time-profile of the IR functions due to the higher lag structure they adopt and the long-run behavior of the IR for unemployment, due probably to the different stationarity assumptions of the series.[17]

### 5.4 The Persistence of Shocks to GNP

Following the influential paper of Nelson and Plosser (1982), a number of authors have engaged in the task of measuring the persistence of shocks in U.S. output, reaching conflicting conclusions. On one end of the range, Campbell and Mankiw (1987) using univariate parsimonious $ARMA$ models for $GNP$ find a persistence coefficient of about 1.5 and Cochrane's (1988) nonparametric approach provides estimates between 1.1 and 1.4, according to the window-size selected. On the opposite end, Watson's (1986) decomposition into stochastic trend and cycle, based on the assumption of orthogonality of the shocks to the two components, gives a measure of persistence between .36 and .57 and Evans (1989) using a bivariate *VAR* of $GNP$ and unemployment rate estimates the persistence factor between .26 and .55.

Evans reconciled his finding with the previous literature by arguing that his *VAR* specification implies a high order $ARMA$ process for $GNP$ including the dampening effect of higher lags, all entering with a negative sign and, as a result, his measure of persistence is lower. On the contrary, the low order $ARMA$ models as in Campbell and Mankiw miss this effect and overestimate persistence, producing measures above the random-walk mark.

Persistence factors of aggregate shocks for $GNP$ in our benchmark model, measured through $(7)$, are summarized below.

**Table 4 - Coefficients of Persistence for GNP**

| Expansion | | | | Recession | | | |
|---|---|---|---|---|---|---|---|
| P | PP | N | NN | P | PP | N | NN |
| 1.44 | 1.52 | -1.28 | -.81 | .67 | .92 | -.77 | -.49 |

As we already noted, persistence is extremely asymmetric across the two regimes. A striking feature of these measures is that the persistence coefficient is in the range .5-.9 for the recessionary regime and in the range .8-1.5 in the expansionary regime. These are roughly the two sets of numbers over which the debate in the literature has developed. Our model can reconcile the two different sets of persistence measures through the nonlinearity. Our nonlinear

[17] It is likely that their model would also predict different measures of persistence for output, although this is difficult to infer from their conditional distributions.

specification implies that there is not a unique optimal lag order to represent the dynamics of output and consequently there is not a unique coefficient of persistence. Persistence is strongly regime-dependent. Adapting the argument set forth by Evans to a nonlinear framework, one could say that during the phases of expansion output dynamics can be well approximated by a low order $ARMA$ process in which the persistence of the shocks is high. During downturns, to capture correctly the dynamics of the system, more lags of $GNP$ should enter the specification —and this is done in our model through the $CDR$ variable— with the effect of decreasing the persistence of the innovations.

Finally, notice that our persistence measures, although qualitatively similar to those reported by BK, present some differences. BK report a persistence factor for positive shocks of about 3, hence much larger than ours, while for negative shocks the two magnitudes are roughly comparable. We conjecture that Evans criticism on the lag order might apply to BK as well, since they work with a very parsimonious autoregressive representation for the linear regime.

### 5.5 Sensitivity of the IR Analysis to Modeling Unemployment as I(0)

Since the assumption of stationarity of the unemployment rate in levels rather than in first-differences is much more common in the (linear and nonlinear) literature, we have also explored how sensitive our conclusions are to modeling unemployment in levels. With unemployment in levels, the order selection indicates a specification with $3$ lags of the dependent variables and $2$ lags of $CDR$ which clearly nests our benchmark model in first-differences. The threshold parameter is estimated to $-.140$, which is remarkably close to the benchmark value. The lags of $CDR$ are still not significant in the output equation, while they are significant in the unemployment equation, with the same combination of signs as in the benchmark case.

From the IR analysis, summarized in Figure 5, the main conclusion emerging is that the IR for unemployment is extremely slow to level off at zero: after $60$ quarters it reaches a value still slightly above zero (.04). This represents, we reckon, further evidence (in addition to that already presented in Section 4.1) that the unit root model provides a satisfactory approximation for the unemployment rate.[18]

Regarding the short-run behavior of the IR functions, the regime asymmetry in the IR of unemployment is still visible as the response in the recessionary regime displays a sharper hump-shape, like in the benchmark case. Also the IR for output shows in the short-run roughly the same qualitative features of sign and regime asymmetry as in the benchmark case. However, a clear difference appears in the IR for the expansionary regime, where in the $I(0)$ case the response at impact is somewhat smaller and tends to revert to zero much more quickly.

The long-run behavior of the IR for output, albeit qualitatively similar, presents some quantitative differences with the benchmark case. Its measure of persistence ranges between .35 and .8, a factor of 1.5-2 lower than the benchmark case. Also the asymmetry in persistence across regimes is slightly reduced. These last two findings are not surprising: the nonlinearity enters strongly the output equation only through unemployment. Since unemployment has no persistence in the long run, obviously the nonlinear feedback on output will tend to vanish in the long run as well, softening the regime asymmetry and the overall level of persistence of the series.

We recognize that the issue of stationarity of unemployment rate is far from settled, nonetheless in the rest of the paper we will resume our benchmark specification. This section demonstrated that although the general qualitative behavior of the IR functions seems robust to this alternative specification, a cautionary remark should be raised on some of the quantitative conclusions.

### 5.6 Do Recessions Affect Long-Run Growth?

Given that our model features a stochastic trend in output and an asymmetry between the two phases of the business cycle, it provides a natural framework to analyze empirically the conjecture advanced by various authors following the Schumpeterian approach, that recessions foster long-run growth. The argument set forth by Aghion and Saint-Paul (1998) for example is that during recessions re-organizational activities will take place (such as restructuring, training, relocation of capital and labor, implementation of new technologies) that im-

[18] As suggested by a Referee, the $I(0)$ vs $I(1)$ dichotomy might be too rigid here. The best specification could lie in between, with a fractional integration model. We leave a rigorous analysis of this issue to future research.

prove productivity growth in the long-run.

The first test of this conjecture that we perform is a counterfactual experiment where we ask the following question: how much lower (or higher) would have been the average growth rate of output in the US, had recessions exerted no feedback effect onto the economy? To answer this question, we compare the actual time series of output in the data with the counterfactual series generated through the estimated model of equations (1)-(3) where we set to zero the polynomial $\Theta(L)$. The implicit assumption we make in the experiment is that the nonlinear feedback captured by $\Theta(L)$ is the reduced form of the cleansing effect of recessions conjectured in the Schumpeterian theories. Our finding is that the average yearly growth rate of output would have been $2.87\%$ compared to the actual growth rate of $2.96\%$. Hence, according to this calculation the feedback from recessions can explain only about $3.5\%$ of the overall growth of the post-war US economy. The intuition from this result is that when the feedback is turned off, then the model becomes a linear $VAR$ and displays the same levels of persistence for all shocks, while in the nonlinear model negative shocks are dampened more than positive shocks.[19]

A second experiment was inspired by a prediction of the Schumpeterian business cycle model in Aghion and Saint-Paul (1998). In their framework, a mean preserving spread in the distribution of shocks, under general conditions, would foster long-run growth. We have used our nonlinear $VAR$ to test this prediction, by simulating the time series first with the empirical distribution of the residuals, and then with mean preserving spreads of the latter. When the standard deviation was reduced by $25\%$, the economy entered the recessionary regime on average 49 times (57 in the baseline economy) and grew at $2.93\%$ per year, whilst when the standard deviation was increased by $25\%$, the economy entered the recessionary regime on average 67 times and grew at $3.0\%$. These results confirm the finding of the previous experiment and the conjecture in Aghion and Saint-Paul (1998), although the overall magnitude of the effects is fairly small.[20]

[19] Gali and Hammour (1991) find evidence of an interaction between recessions and growth in the fact that in their $VAR$ a *negative* aggregate shock has a long-run *positive* effect on productivity. Our view is slightly different: rather than being based on sign-reversion effects, it is based upon sign-asymmetry which is intrinsic in nonlinear structures.

[20] The details of these two experiment are provided in the Appendix.

## 6. Conclusions

The joint statistical analysis of output and unemployment rate is a classical exercise in the linear $VAR$ tradition. In this paper we have performed this same analysis allowing for nonlinear dynamics in the series. The nonlinearity is introduced, following a recent literature on univariate threshold models, through a lagged feedback variable that measures the depth of recessions. The resulting model is a $VAR$ with a fixed lag order when the economy is in expansion, and a time-varying lag-order when the economy is undergoing a recession.

In the first part of the paper we established a set of new theoretical results for this model. First, we identified a set of sufficient conditions for its ergodicity, which potentially apply to a larger class of nonlinear threshold frameworks. Second, we illustrated that our model belongs to the $SETAR$ family, hence consistency of the quasi-maximum likelihood estimator follows from Chan (1993).

In the second part of the paper, we applied the model to the empirical analysis of the data. First, we found that the best linear specification is severely rejected against our nonlinear model, but interestingly the nonlinearity is found to be statistically significant only in the equation for the unemployment rate, while it transmits to output via the tight cross-correlations between the series. Second, we found that the feedback effect captures one important feature of the data: the deeper is the recession the economy has gone through, the faster will be the recovery process in the following periods. Third, these forces that push the economy out of a recession generate important asymmetries in the propagation and the persistence of aggregate and reallocative innovations in the model. Fourth, we argued that asymmetry in persistence across different phases of the cycle helps in reconciling some of the conflicting findings of the traditional literature on output persistence. Finally, this same regime asymmetry explains our finding that recessions benefit the economy in the long-run, by fostering growth, as predicted by some recent Schumpeterian macroeconomic theories.

# 7. Appendix

## 7.1 Markov Representation

In this section we give a Markovian representation of the *threshold VAR* in equations (1)-(3) of the main text and show that the model can be interpreted as a $SETAR$ with a large number of regimes. This result will be crucial for the proof of ergodicity of the next section. Let $\{\Delta X_t\}$ be a sequence of $\Re^2$-valued random variables (r.v.'s) which are defined on the complete probability space $(S_{\Delta X}, F_{\Delta X}, \mu_{\Delta X})$ where $\mu_{\Delta X}$ is some $\sigma-$finite measure on $F_{\Delta X}$. Let also $Z_t = \{\Delta X_t, \Delta X_{t-1}, ..., \Delta X_{t-q-\tau+2}\}$ be a $(q+\tau-1)-tuple$ defined on the product space $(S_Z, F_Z, \mu_Z)$.

Define the feedback index $F_t$:

$$F_t(r,\tau) = \begin{cases} 0 & if \ \sum_{i=0}^{\tau-1} [1 - 1(X_{1t} - X_{1,t-\tau+i} > r)] = 0 \\ F_{t-1} + 1 & if \ \sum_{i=0}^{\tau-1} [1 - 1(X_{1t} - X_{1,t-\tau+i} > r)] > 0 \text{ and } F_{t-1} < \tau \\ \tau & if \ \sum_{i=0}^{\tau-1} [1 - 1(X_{1t} - X_{1,t-\tau+i} > r)] > 0 \text{ and } F_{t-1} = \tau \end{cases}, \quad (9)$$

which is zero when the current realization of $X_{1t}$ is higher than each of the past $\tau$ realizations augmented by $r$, the threshold growth rate, it is equal to $i$ when the economy has spent $i$ periods in recession for $i < \tau$ and it is equal to $\tau$ for $i \geq \tau$. It follows that:

$$CDR_t(r,\tau) = \begin{cases} 0 & if \ F_t = 0 \\ (\Delta X_{1t} - r) & if \ F_t > 0 \text{ and } F_{t-1} = 0 \\ (CDR_{t-1} + \Delta X_{1t}) & if \ F_t > 0 \text{ and } F_{t-1} > 0 \end{cases}. \quad (10)$$

The combination of (9) and (10) delivers the specification (1) in the text. We can generate $q$ different partitions $\wp^i$, $i = 1, ..., q$, of the space of $Z_{t-1}$ through the feedback indexes $F_{t-i}$, based on the different ways in which the $CDR_{t-i}$ variable can be activated. The partition $\wp^i$ is composed by $\tau + 1$ elements and the generic element $P^i(j) \equiv \{Z_{t-1} : F_{t-i} = j\}$. Let $\wp$ be the joint of the $\wp^i, i = 1, ..., q$, partitions.[21] This joint partition has maximum dimension $(1+\tau)^q$ and we can denote its generic element by $P(K) \equiv \cap_{i=1}^q P^i(k_i)$ where $P^i(k_i) \in \wp^i$, $k_i = 0, ..., \tau$ and $K = \{k_1, ..., k_q\}$. Hereafter, $P_0 \equiv \cap_{i=1}^q P^i(0)$ corresponds to that region of the space where the model is in the expansionary regime, i.e., the region where all the feedbacks are zeros. The joint partition $\wp$ divides the space of $Z_{t-1}$ into sub-regions $P(K)$

[21] Given two partitions $\mathcal{P}^1$and $\mathcal{P}^2$, $P^*$ is an element of the joint partition $\mathcal{P}^1 \cup \mathcal{P}^2$ if for some elements $P^1 \in \mathcal{P}^1$ and $P^2 \in \mathcal{P}^2$, $P^* \subseteq P^1 \cap P^2$ and there is no other element $P^{\emptyset}$ of the joint such that $P^{\emptyset} \subseteq P^1 \cap P^2$ and $P^* \subset P^{\emptyset}$.

in which the dynamics of the model are modified with respect to those of the expansionary regime through a matrix $\Lambda(K)$ . This matrix can be written as $\Lambda(K) = \sum_{i=1}^{q} \Lambda^i(k_i)$, where $\Lambda^i(k_i)$ is associated with the element $P^i(k_i) \in \wp^i$ and it is a square matrix of dimension $2(q+\tau-1)$ of zero elements, except at position $(1, 2(i+j)-1)$ for $j = 0, ..., k-1$ where elements are equal to $\Theta_{1i}$, and at position $(2, 2(i+j)-1)$ for $j = 0, ..., k-1$ where elements are equal to $\Theta_{2i}$. Recall that $\Theta_{1i}$ and $\Theta_{2i}$ are elements of the $(2 \times 1)$ vector $\Theta_i$ of coefficients of $CDR_{t-i}$ in the model. Using the definition of $Z_t$ and the partition $\wp$, the model in (1)-(3) can be rewritten in a vectorized form as:

$$Z_t = \begin{bmatrix} \alpha \\ 0 \end{bmatrix} + r\sum_{i=1}^{q} \Theta_i 1(Z_{t-1} \notin P^i(0)) + \begin{bmatrix} \Phi_1, ..., \Phi_p & 0 \\ I & 0 \end{bmatrix} Z_{t-1}$$
$$+ \sum_{K=1}^{(1+\tau)^q} \Lambda(K) Z_{t-1} 1(Z_{t-1} \in P(K)) + \varepsilon_t.$$

where $\varepsilon_t' = \{(V_t^{\frac{1}{2}} u_t)', 0, ..., 0\}$. This delivers the desired Markovian representation embedded in the more general structure of equation (4) in the main text. Moreover, this specification clearly is a $SETAR$ with $(1+\tau)^q$ regimes and linear constraints among the regimes since the $q$ matrices $\Theta_i$ are used to generate the $(1+\tau)^q$ matrices indexing the various regimes.

A final remark useful for the discussion on ergodicity is that even if the partition $\wp$ has $(1+\tau)^q$ elements, this does not imply that the process $Z_t$ can move among all elements of the partitions, due to the way the feedback variable has been constructed. Two elements, $P(K)$ and $P(K^*)$, of the partition $\mathcal{P}$ will be defined *deterministically adjacent* from $K$ to $K^*$ if it is feasible that $Z_t \in P(K)$ and $Z_{t+1} \in P(K^*)$ . This will be the case when $k_{i+1}^* = k_i$ for $i = 1, ..., (q-1)$ and, if $k_1 < \tau$, the difference in absolute value between $k_1^*$ and $k_1$ is not greater than one.

### 7.2 Proof of Proposition 1

The proof of geometric ergodicity is based upon the theory on stability of Markov chains developed in Nummelin (1984), and extended by Tjøstheim (1990). If the process $Z$ is $\mu_Z-$irreducible and aperiodic, then by proposition 5.21 of Nummelin (1984) the assessment of the geometric ergodicity essentially requires to verify whether a drift conditions for a given power function of $Z$ holds. For a non negative measurable function $g$ and $R > 1$, the $N-$step ahead drift conditions are:

$$(DC1): \quad RE(g(Z_{t+N})|Z_t = z) < g(z), \forall\, z \in \kappa^c$$
$$(DC2): \quad E(g(Z_{t+N})|Z_t = z) \leq M < \infty, \forall\, z \in \kappa$$

where $\kappa$ is a small set. In our framework piecewise linearity of the conditional mean function and the regularity conditions on the error term $\varepsilon_t$ ensure that every compact set on the space of $Z_t$ is a small set. For the formal definitions of irreducibility, small set, and aperiodicity, the reader can refer to Nummelin (1984), definitions 2.2, 2.3 and 2.4.

**Proof.** The aperiodicity and $\mu_Z-$irreducibility follow directly from definitions, under the assumptions on the marginal $pdf$ of the error term $\varepsilon_t$. Denote an element of the partition $P_i$ as ***adjacent to*** $P_j$ if there is a $Z_{t-1} \in P_i$ and an $\varepsilon_t$ such that $C_i + A_i Z_{t-1} + \varepsilon_t \in P_j$. The set $\sigma(N)$ will contain all the possible sequences of length $N$ of adjacent elements. A generic element of $\sigma(N)$ is $\{h_1, ..., h_N\}$ such that for $i = 1, ..., N-1$ $P_{h_i}$ is ***adjacent*** to $P_{h_{i+1}}$. To complete the notation, $\sigma(N)^c$ is the complement set of $\widetilde{\sigma}(N)$ (defined in the main text as the set containing all the possible sequences of length $N$ of ***deterministically adjacent*** elements) with respect to $\sigma(N)$. Define now the test function $g(\cdot)$ as a vector norm in $L_1$ and $\|A\|$ as the corresponding matrix norm.

From the Markov representation of $Z$, we obtain:

$$Z_{t+N} = \sum_{\widetilde{\sigma}} \left( \prod_{j=1}^N A_{i_j} Z_t + \sum_{l=1}^N \left[ \left( \prod_{j=l+1}^N A_{i_j} \right) (\varepsilon_{t+l} + C_{i_l}) \right] \right) 1(Z_{t+j} \in P_{i_j},\ j = 1, ..., N)$$

$$+ \sum_{\sigma^c} \left( \prod_{j=1}^N A_{i_j} Z_t + \sum_{l=1}^N \left[ \left( \prod_{j=l+1}^N A_{i_j} \right) (\varepsilon_{t+l} + C_{i_l}) \right] \right) 1(Z_{t+j} \in P_{i_j}, j = 1, ..., N).$$

Taking the expectation of the norm on both sides and rearranging terms, we have:

$$E\|Z_{t+N}\| \leq \|Z_t\| \sum_{\widetilde{\sigma}} \left\| \left( \prod_{j=1}^N A_{i_j} \right) \right\| \Pr(Z_{t+j} \in P_{i_j}\ j = 1...N)$$

$$+ \|Z_t\| \sum_{\sigma^c} \left( \left\| \prod_{j=1}^N A_{i_j} \right\| \right) \Pr(Z_{t+j} \in P_{i_j}\ j = 1...N)$$

$$+ \sum_{\sigma} \left\| \sum_{l=1}^N \left( \prod_{j=l+1}^N A_{i_j} \right) \right\| E\|\varepsilon_t\| + \sum_{\sigma} \left\| \sum_{l=1}^N \left( \prod_{j=l+1}^N A_{i_j} \right) C_{i_l} \right\|.$$

The third and fourth term are independent of $Z$ and by the assumptions on $\varepsilon_t$ there is a constant $\delta$ which bounds the sum of those terms. The second term, by the assumption (**A2**) on the transition probability, is an $O\left(\|Z\|^{1-\epsilon}\right)$ and furthermore, it is easy to see that by the eigenvalue assumption (**A1**) it exists an $\alpha < 1$ such that the following inequality holds:

$$E\left(\|Z_{t+N}\| \mid Z_t = z\right) \quad \leq \delta + \alpha \|Z\| + O\left(\|Z\|^{1-\epsilon}\right).$$

Note that for the norms in $L_1$, there is a norm such that $|\rho(A) - \|A\||$ is arbitrarily small. Hence, it is possible to find an $R > 1$ such that $R\alpha < 1$ and to rewrite that previous condition as

$$\begin{aligned} RE\left(\|Z_{t+N}\| \mid Z_t = z\right) \quad &\leq R\delta + R\alpha \|Z\| + O\left(\|Z\|^{1-\epsilon}\right) \\ &\leq \|Z\| + R\delta + (\alpha R - 1)\|Z\| + O\left(\|Z\|^{1-\epsilon}\right) \end{aligned}$$

Let us now define the small set $\kappa \equiv \{z : \|z\| \leq r\}$ and take $r > \frac{R\delta + O\left(\|Z\|^{1-\epsilon}\right)}{1-R\alpha}$. Then we obtain:

$$\begin{aligned} (DC1): \quad & RE(\|Z_{t+N}\| \, |Z_t = z) \quad < \|z\| \quad \text{on } \kappa^c \text{ and} \\ (DC2): \quad & E(\|Z_{t+N}\| \, |Z_t = z) \quad < M < \infty \quad \text{on } \kappa, \end{aligned}$$

where the second condition derives from the compactness of $\kappa$ and the continuity of the conditional mean of the process. Geometric ergodicity of the model follows by proposition $5.21$ in Nummelin (1984). ■

### 7.3 An example

To gain more intuition on the restrictions that this result on ergodicity puts on our specific model, let us consider the following example where $\Delta x$ is a scalar, $\tau = 1$, $q = 2$, $r = 0$ and $p = 1$, with $\varepsilon_t \sim NID(0, \sigma^2)$:

$$\Delta x_t = \quad \phi_1 \Delta x_{t-1} + \theta_1 \min(0, \Delta x_{t-1}) + \theta_2 \min(0, \Delta x_{t-2}) + \varepsilon_t \;. \tag{11}$$

The model is an $AR(1)$ in the expansionary regime, while under the effect of the feedback it changes slope and lag order. The above model can be interpreted as a $SETAR$ with four regimes and with linear cross equation restrictions, so:

$$\Delta x_t = \begin{cases} \phi_1 \Delta x_{t-1} + \varepsilon_t & \text{if } \Delta x_{t-1} \geq 0 \text{ and } \Delta x_{t-2} \geq 0 \\ (\phi_1 + \theta_1)\Delta x_{t-1} + \varepsilon_t & \text{if } \Delta x_{t-1} < 0 \text{ and } \Delta x_{t-2} \geq 0 \\ \phi_1 \Delta x_{t-1} + \theta_2 \Delta x_{t-2} + \varepsilon_t & \text{if } \Delta x_{t-1} \geq 0 \text{ and } \Delta x_{t-2} < 0 \\ (\phi_1 + \theta_1)\Delta x_{t-1} + \theta_2 \Delta x_{t-2} + \varepsilon_t & \text{if } \Delta x_{t-1} < 0 \text{ and } \Delta x_{t-2} < 0. \end{cases}$$

Using the same notation of the previous sections, we define $Z_t$ as $\{\Delta x_t, \Delta x_{t-1}\}$ and $F_{t-i}$,

$i = 1, 2$ as the feedback indexes. It follows that $F_{t-i} = 1$ whenever $Z_{1,t-i} \leq 0$ and $F_{t-i} = 0$ otherwise. The potential activation of the two lags of $CDR_t$ generates four possible states creating a partition $\wp \equiv \{P_0, P_1, P_2, P_3\}$ on the space of $Z_{t-1}$with elements:

$$P_0 = \{Z : F_{-1} = 0,\ F_{-2} = 0\}, \quad P_1 = \{Z : F_{-1} = 1,\ F_{-2} = 0\},$$
$$P_2 = \{Z : F_{-1} = 0,\ F_{-2} = 1\}, \quad P_3 = \{Z : F_{-1} = 1,\ F_{-2} = 1\}.$$

This partition is used to build the following Markovian representation of the model:

$$Z_t = \sum_{i=0}^{3} A_i Z_{t-1} 1(Z_{t-1} \in P_i) + \varepsilon_t$$

where the $A's$ $(2 \times 2)$ matrices associated to the four states are respectively:

$$A_0 = \begin{bmatrix} \phi_1 & 0 \\ 1 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} \phi_1 + \theta_1 & 0 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} \phi_1 & \theta_2 \\ 1 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} \phi_1 + \theta_1 & \theta_2 \\ 1 & 0 \end{bmatrix}.$$

The matrix $A_0$ describes the dynamics in the expansionary regime, while the other three matrices describe how these dynamics are modified through the activation of $CDR$. In the logic of Proposition 1, to asses the ergodicity of the system we need to start from some parameter values of the model. Assume for example that $0 \leq \phi_1 < 1$, $\theta_1 \leq -\phi_1$ and $\theta_2 < 0$. The partition generated by the $CDR$ variable imposes some constraints on the movements across states: $P_0$ is deterministically adjacent only to itself, $P_2$ to $P_0$, $P_1$ to $P_2$ and $P_3$ to $P_2$. For this particular parametrization, it is easy to verify that the system moves towards $P_0$, hence, given $0 \leq \phi_1 < 1$, there is an $N$ such that the eigenvalue condition is fulfilled and the model is geometrically ergodic. As a final remark, note that this parametrization allows the process to have unstable dynamics in all the elements of the partitions but $P_0$.

### 7.4 Description of the Bootstrap Technique

Due to the non standard asymptotics of the nonlinear model, we made extensive use of bootstrap techniques throughout all the paper. Recall that in our model the variance of the error term follows a qualitative threshold process of the form:

$$\varepsilon_t = V_t^{\frac{1}{2}} u_t, \text{ with } V_t = 1(\sum_{i=1}^{q} CDR_{t-i} = 0)(\Omega_e - \Omega_r) + \Omega_r.$$

We adapt the method of resampling proposed by Lamoureux and Lastrapes (1990) for models with conditional heteroskedastic errors. From the estimation procedure we obtain consistent estimators for $\widehat{\varepsilon}_t$ and for $\widehat{V}_t$ at every $t$, hence the adjusted-homoskedastic error $\widehat{u}_t$ can be computed as $\widehat{u}_t = \widehat{V}_t^{-\frac{1}{2}}\widehat{\varepsilon}_t$. For the errors $\{\widehat{u}_t\}$ we tested and could not reject the null hypothesis of no serial correlation. In bootstrapping, we used the following procedure. In every replication, we draw with replacement a new series $\left\{u_t^{(i)}\right\}_{i=1}^{N}$ from the homoskedastic and uncorrelated residuals. Given this new sample and assuming the original initial conditions, we generate the new sample $\left\{\Delta X_t^{(i)}\right\}_{i=1}^{N}$ by means of the recursive structure of the model specified in equations (1)-(3).

In the non-nested test for the order selection on the $CDR$ variable and in the test for linearity, we use this resampling strategy to compute the empirical distribution of the statistics under the null. For both tests, at each replication, we regenerate the data under the null of the test and then compute the $p - value$ of the statistics given the bootstrap sample. For each test, $2,000$ bootstrap replications were performed. A precautionary remark has to be raised concerning the non-nested testing procedure, as in the theoretical literature there are no results on the validity of bootstrap methods when the log-likelihood function is discontinuous. However, some Monte Carlo experiments support our testing strategy.

The same resampling methodology has also been used in the computation of the Impulse Responses. The maximum horizon of the IR has been set to $24$ quarters, which turns out to be sufficient for the long-run behavior to set in. For each history, we draw $1,000 \times 24$ realizations of the homoskedastic residuals for the baseline model and $1,000 \times 23$ for the shocked model, since the initial shock is fixed at the calibrated values. To induce a negative correlation between the sample estimates of the two expectations in (6) and reduce the experimental variance, we used the same set of random numbers in sampling the future innovations. For each of the $1,000$ replications, a future of length $24$ for $\Delta X$ and $CDR$ is recursively built both for the shocked and the baseline economies. The regime dependent Impulse Response is computed by averaging over all the selected histories in each regime. As explained in the text, we have used all $97$ actual histories for the linear regime, and $25$ out of $57$ histories in the recessionary regime.

The experiment of the mean preserving spread in the empirical distribution of residuals was performed using common random numbers with $1,000$ simulations of length equal to the sample size.

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**Table 5 - Order Selection**

| lag $CDR$ | lag $AR$ | $AIC$ | $SIC$ |
|---|---|---|---|
| 1 | 1 | 2.253 | 2.415 |
| 1 | 2 | **2.231** | **2.474** |
| 1 | 3 | 2.255 | 2.579 |
| 1 | 4 | 2.278 | 2.683 |
| 1 | 5 | 2.311 | 2.797 |
| 1 | 6 | 2.320 | 2.887 |
| 1 | 7 | 2.359 | 3.007 |
| 1 | 8 | 2.261 | 2.990 |
| 2 | 1 | 2.121 | **2.324** |
| 2 | 2 | **2.090** | 2.374 |
| 2 | 3 | 2.106 | 2.470 |
| 2 | 4 | 2.147 | 2.592 |
| 2 | 5 | 2.163 | 2.689 |
| 2 | 6 | 2.177 | 2.785 |
| 2 | 7 | 2.201 | 2.890 |
| 2 | 8 | 2.172 | 2.942 |
| 3 | 1 | 2.153 | **2.396** |
| 3 | 2 | **2.110** | 2.434 |
| 3 | 3 | 2.133 | 2.538 |
| 3 | 4 | 2.175 | 2.661 |
| 3 | 5 | 2.168 | 2.736 |
| 3 | 6 | 2.160 | 2.809 |
| 3 | 7 | 2.194 | 2.923 |
| 3 | 8 | 2.111 | 2.921 |
| 4 | 1 | 2.142 | **2.425** |
| 4 | 2 | **2.084** | 2.449 |
| 4 | 3 | 2.116 | 2.562 |
| 4 | 4 | 2.148 | 2.675 |
| 4 | 5 | 2.186 | 2.793 |
| 4 | 6 | 2.190 | 2.878 |
| 4 | 7 | 2.221 | 2.990 |
| 4 | 8 | 2.172 | 3.022 |

Akaike information criteria, $AIC = -2 \times \left(\frac{\ell oglike - \#(parameters)}{n}\right)$

Schwartz information criteria, $SIC = -2 \times \left(\frac{\ell oglike - 0.5 \times \ln n \times \#(parameters)}{n}\right)$

Number of observations, $n = 148$

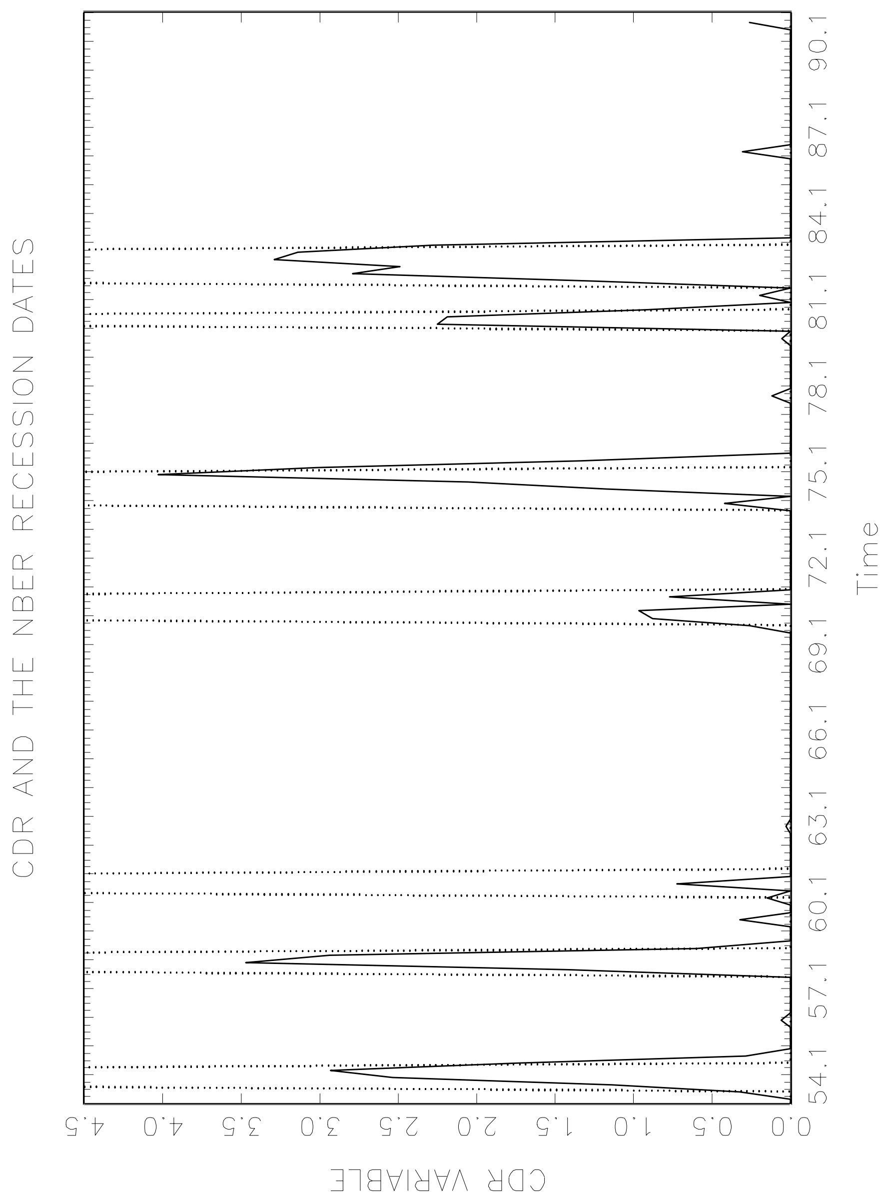


Figure 1: The variable CDR (—) compared with the NBER official business cycle chronology (· · ·)

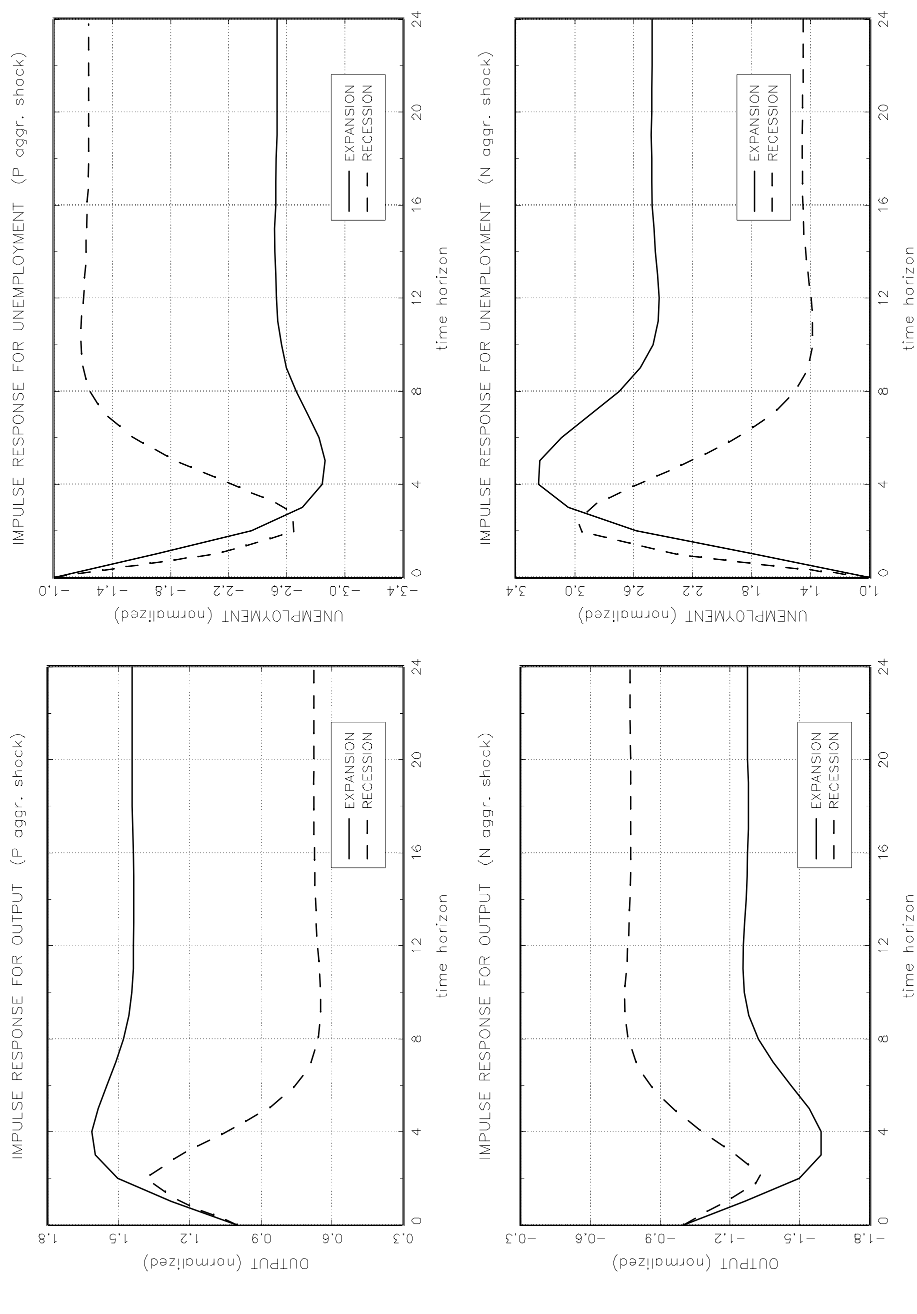


Figure 2: The Impulse Response Functions of the benchmark model to aggregate shocks

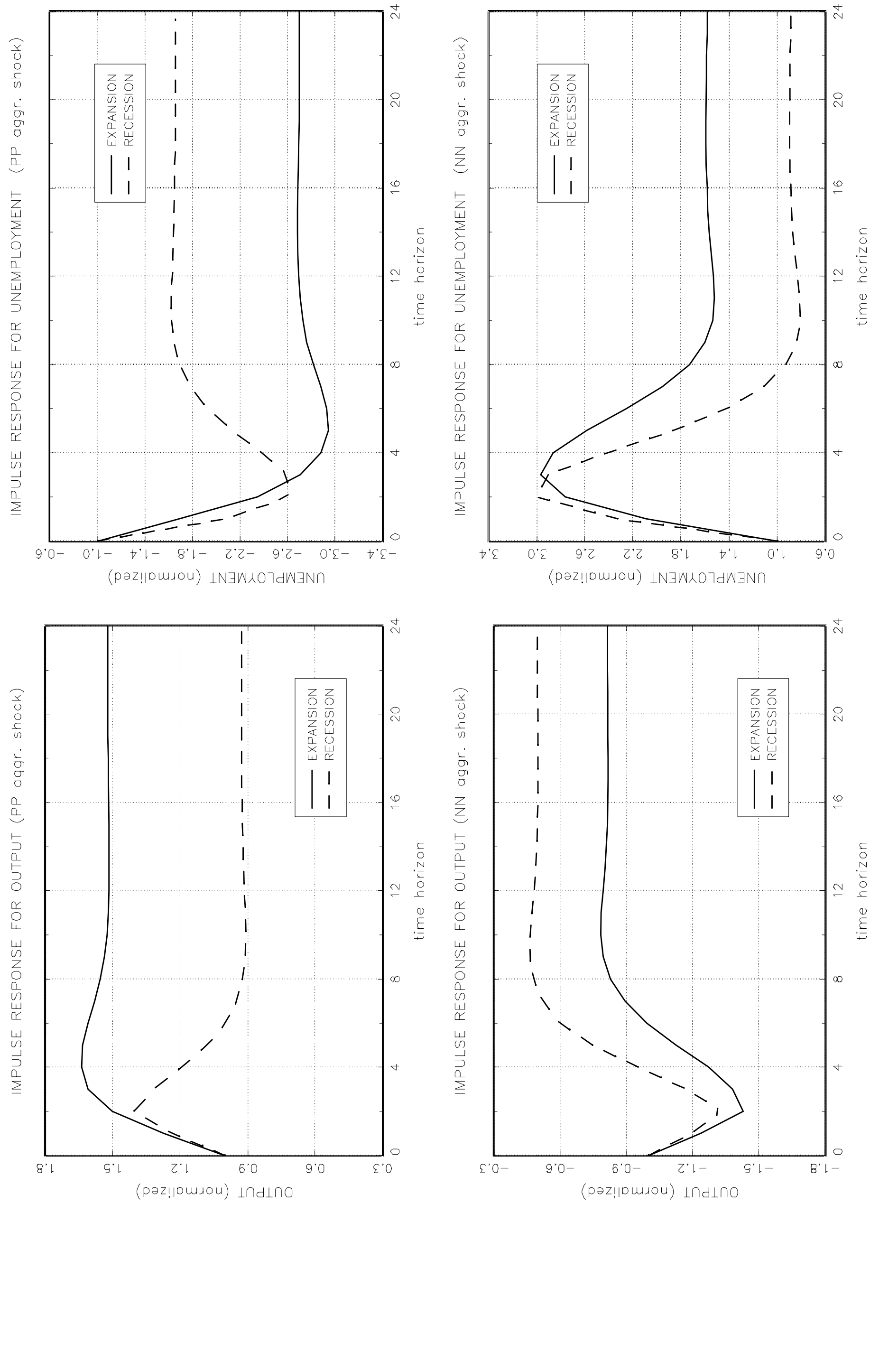


Figure 3: The Impulse Response Functions of the benchmark model to aggregate shocks of double magnitude

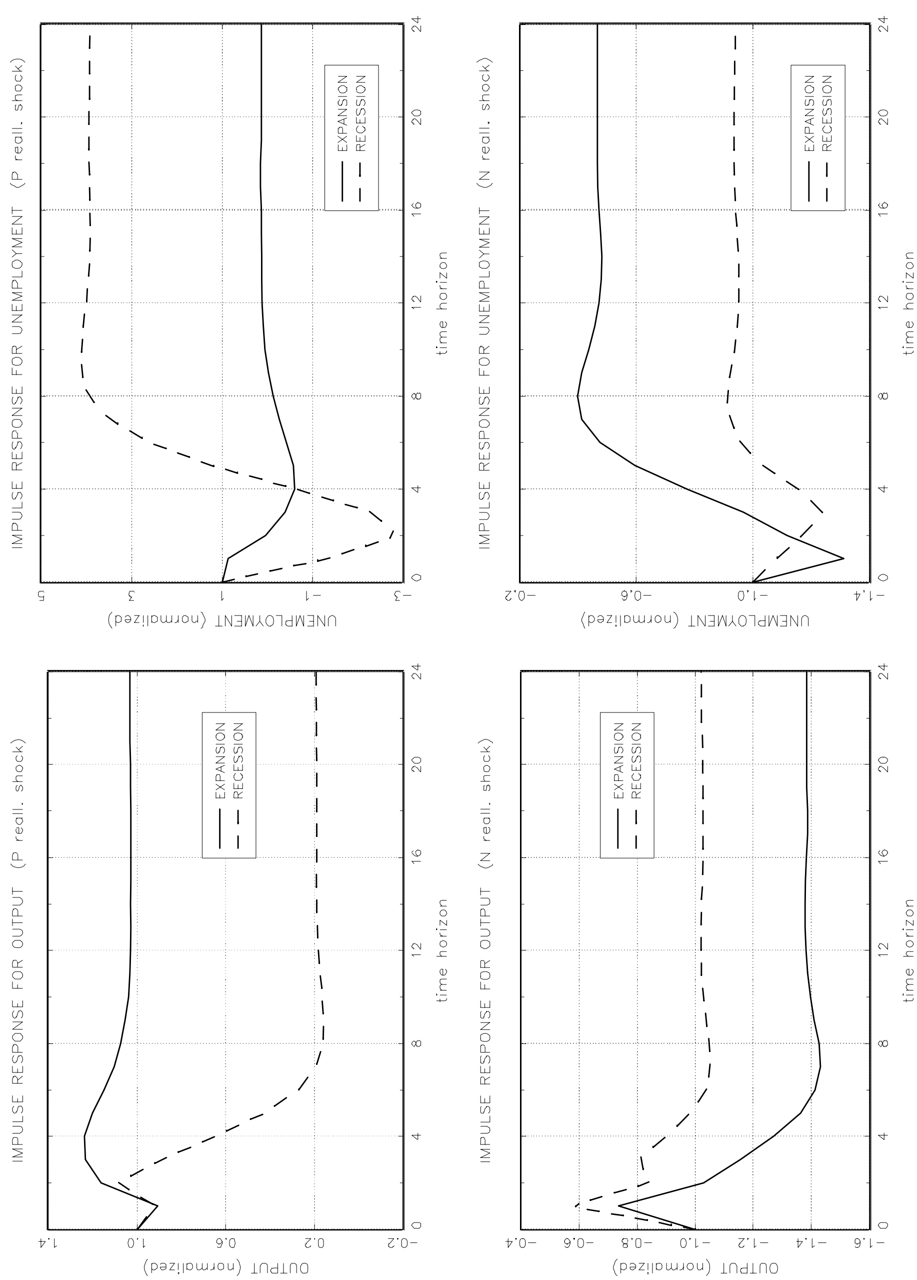


Figure 4: The Impulse Response Functions of the benchmark model to reallocative shocks

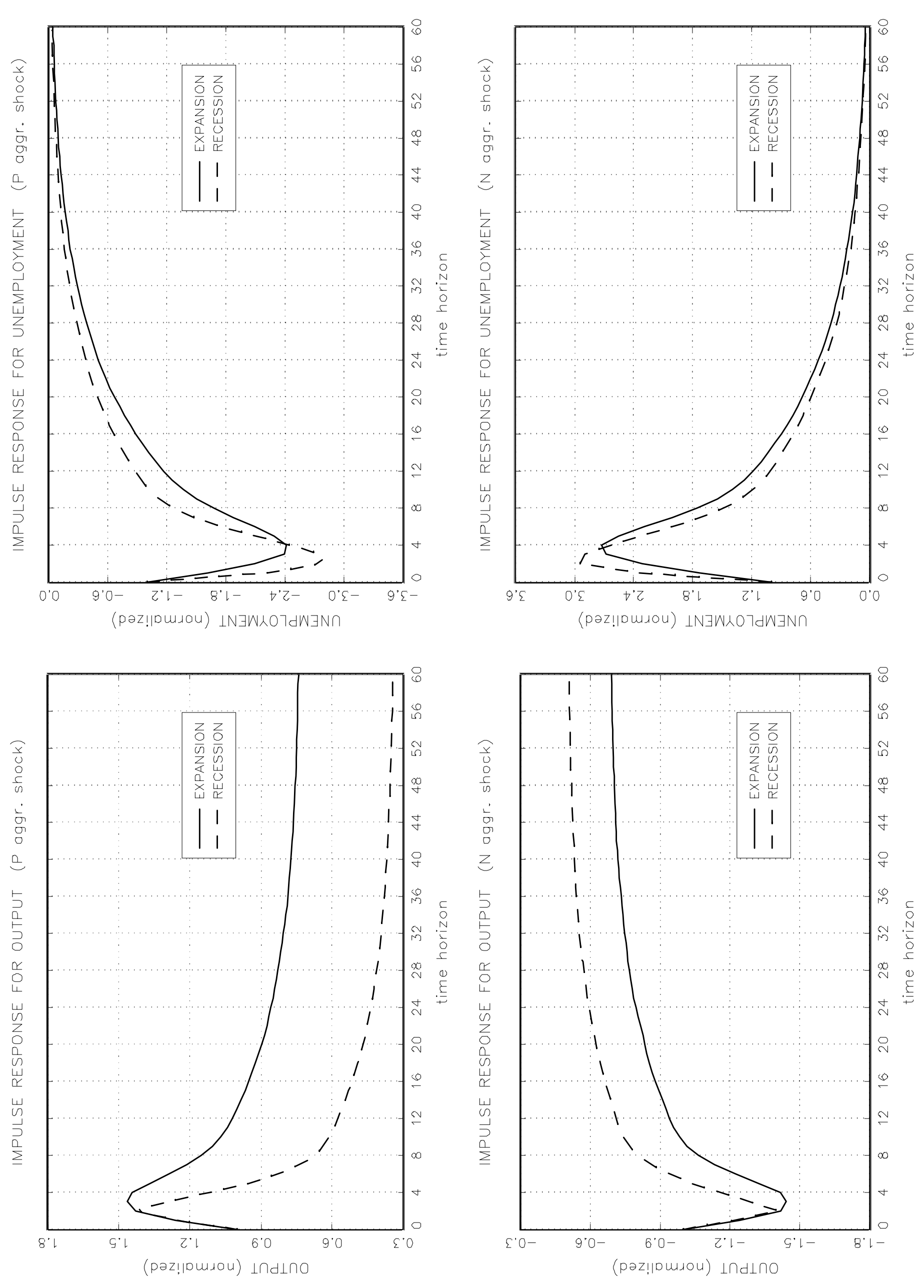


Figure 5: The Impulse Response Functions of the model with unemployment in levels to aggregate shocks.