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George-Marios Angeletos and Tryphon Kollintzas

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George-Marios Angeletos, Harvard University
Tryphon Kollintzas, Athens University of Economics and Business and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: http://www.cepr.org

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ABSTRACT

Rent Seeking/Corruption And Growth: A Simple Model*

The goal of this Paper is to propose a simple paradigm for understanding rent seeking and corruption in the growth context. We develop an endogenous growth model where entrepreneurs, as intermediate-good producers, may engage in rent-seeking activities. The latter are defined by the following properties: (i) their internal effect is positive; (ii) their external effect is negative; and (iii) they use real resources. Our formulation may be viewed as a parable for theft and fraud; organized crime; industrial espionage; lobbying and policy influence; misgovernance, institutional inefficiency, tax evasion, etc. The economy is shown to fall into a trap of high rent seeking/corruption and low growth. Agents' perceptions about the external effects of rent seeking, and the complementarity or substitutability of intermediate inputs, are crucial. Contrary to conventional wisdom, higher returns to capital and more competition can be detrimental for welfare and growth, as they induce more rent seeking/corruption. Finally, our paradigm yields insights into the relationship of R&D, politico-economic equilibrium, income distribution, and growth, as well as the design of tax/growth policies in the presence of rent seeking/corruption.

JEL Classification: E10, H20, O10
Keywords: rent seeking, corruption, growth, property rights

George-Marios Angeletos
Department of Economics
Harvard University
Littauer Center #200
Cambridge, MA 02143
USA
Tel: (1 617) 495 1205
Fax: (1 617) 495 7730
Email: angelet@fas.harvard.edu

Tryphon Kollintzas
Department of Economics
Athens University of Economics and Business
Patission 76
104 34 Athens
GREECE
Tel: (30 1) 8203388
Fax: (30 1) 8203301
Email: kollintzas@hol.gr

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NON-TECHNICAL SUMMARY

In this Paper, we seek to shed some light onto corruption and rent seeking questions like: how activities (e.g. theft and fraud; organized crime; industrial espionage; lobbying and policy influence; misgovernance, institutional inefficiency, tax evasion, etc.) that increase the income or profit of one that engages in them affect overall economic growth, income distribution and political outcomes. (i) Their internal effect is negative, and (ii) they use real resources.

In particular, we introduce corruption and rent-seeking activities into the neoclassical growth theory. Corruption or rent-seeking activities are defined by three properties: First, we model the internal effect or corruption/rent-seeking activities as being positive for those engaged in such activities; as being negative for everybody else. Second, we take corruption/rent seeking to be, at least eventually, a redistributive activity, so that if everyone is engaging in rent seeking/corruption, then none can profit from these activities. Third, we model corruption/rent-seeking activities as using up real resources. These properties constitute a 'rent-seeking or corruption technology', a 'black box' that characterizes the available opportunities for and the consequent benefits from engaging in rent seeking and corruption. This formulation can indeed be seen as a model of theft and fraud; or a model of industrial espionage; of lobbying and policy influence; of misgovernance and institutional inefficiency in the sale and provision of public goods; of tax evasion and tax-burden transfer; of patent imitation and property rights expropriation; of a broad class of activities that have a redistributive rather than productive effect. Notably, corruption/rent seeking in our model is not distinguished by whether or not it is lawful, morally correct, or socially acceptable.

The growth model we choose to work with is an endogenous growth model where certain agents, intermediate-good producers, may engage in rent-seeking activities. The latter are defined by the three properties mentioned above. The growth engine is human capital, but this is to be viewed only as a parable intended to capture the more fundamental relation that exists between the static allocation of real resources and the dynamics of growth. Our formulation is indeed in line with the idea of an endogenous allocation of entrepreneurship and talent between innovative and rent-seeking activities.

Despite the fact that this is a simple and highly stylized model, it accounts for several stylized facts of the relationship between rent seeking/corruption and growth. Moreover, it yields insights into the relationship of R&D, politico-economic equilibrium, income distribution, and growth, as well as the design of tax/growth policies in the presence of corruption.

The economy is shown to follow a path of high rent seeking/corruption and low growth. This is essentially a case of ‘the tragedy of commons’: Real resources, like labour time, entrepreneurship and talent, are allocated to unproductive corruption and rent seeking, rather than being employed in productive and growth-promoting activities. For instance, entrepreneurs, lawyers, lobbyists, chief executives, etc., invest time and resources in red tape and bribery, lobbying and political influence, spying, negotiations or court suits, and other wasteful corruption/rent-seeking activities.

Looking at the comparative statistics, the agents’ perceptions of the external effects of their corruption activities coupled with the degree of complementarity (or substitutability) between intermediate inputs, are found to be crucial determinants for growth. With strong complementarity, sophisticated perceptions result in less corruption and more growth, but with intense substitutability, myopia may be welfare-improving.

A novel and challenging implication of our theory is that the higher the intensity of intermediate-good inputs and final-good production, the higher the income share of capital, the higher the rents to be distributed through corruption/rent seeking, and therefore the higher is the demand for corruption. For that reason, and contrary to conventional wisdom, a higher capital share is coupled with lower investment and lower growth.

Further, our model allows for the possibility that more intense competition, or stronger substitutability between intermediate inputs, may be detrimental to growth, because it may induce more corruption. The driving force is the strategic interaction over the choice of the individually optimal level of rent seeking.

The equilibrium allocation in the presence of corruption/rent-seeking opportunities is of course inefficient, both in static and in dynamic terms. The normative case for an ‘anti-corruption tax’ is thus made. We consider in particular the second-best policy of a distortive tax on the rents of capital or the output of intermediate producers. We find that the second best tax rate is increasing in the level of corruption and, more interestingly, on the income share of capital. Indeed, while in the absence of rent seeking/corruption, a subsidy may be necessary in order to cure the market inefficiency due to monopolistic power in the intermediate sector, the second-best policy may instead call for a tax in order to discourage rent seeking/corruption.

We observe that higher corruption implies higher benefits for those employed in rent-seeking activities, and higher current wages, at the cost of a lower wage growth rate. This observation can provide us with a politico-economic explanation for the obstacles to the elimination of corruption. We discuss the related incentives of bureaucrats, politicians and lawmakers, lawyers and mediators, etc. We hence suggest a simple positive theory for the endogenous determination of the 'rules of the games' or the 'rent-seeking/corruption technology' in the spirit of Helpman and Grossman.

We finally discuss some positive implications for the relationship between growth and income distribution. A simple extension of our benchmark model can account for a negative correlation between growth and income inequality, or even for the Kuznets Curve observed in the data.

The plan of the Paper is as follows. In Section 2 we develop the basic model, characterize individual behaviour and determine the demand for rent seeking/corruption. In Section 3 we proceed to general equilibrium and characterize the steady state and the transition dynamics; we next compare with the first best; and we finally discuss the case of a second-best anti-corruption tax. In Section 4 we present four extensions of the model: (i) alternative engines of growth; (ii) endogenous determination of the corruption technology; (iii) the role of competition and anti-corruption coordination; and (iv) the relation between corruption/rent seeking, growth, and income distribution. Section 5 concludes the Paper.

"Competitive rent seeking results in a divergence between the private and social costs of certain activities... [and] leads to the operation of the economy inside its transformation curve." (Krueger, 1974, p. 291)

"The productive contribution of the society's entrepreneurial activities varies much more because of their allocation between productive activities such as innovation and largely unproductive activities such as rent seeking or organized crime." (Baumol, 1990, p. 893.)

1 Introduction

Although it is hard to define and almost impossible to measure, corruption and rent seeking are thought to have important effects on economic growth. The forms and the specifics of rent seeking may vary across time and space, but the phenomenon is always present and often pervasive. People living or doing business in Southern and Eastern Europe, Latin America, Africa, or India, know quite well that bribery and corruption is *the* rule of the game in dealing with the bureaucracy. But even in the US, rent seeking thrives in the political scene — lobbying at both the state and the federal level, or PACs and contributions to political candidates. William Baumol (1990, p. 915) also notes: "Today, unproductive entrepreneurship takes many forms. Rent seeking, often via activities such as litigation and take-overs, and tax evasion and avoidance efforts seem now to constitute the prime threat to productive entrepreneurship. ... Corporate executives devote much of their time and energy to legal suit and countersuit, and litigation is used to prevent excessive vigor in competition by rivals. ... Similarly taxes can serve to redirect entrepreneurial effort." And we all know that the steaks in such issues can be millions or billions of dollars.

However, the pertinent literature does not clearly agree on either the evidence or the theory regarding the effects of rent seeking and corruption. Bardhan (1997), in his extensive survey of the literature, cites several historical episodes where corruption and rent seeking activities are thought to have promoted growth. To the contrary, Mauro (1995), based on business survey data from seventy countries, finds significant negative correlation between the underlying corruption index, on the one side, and the investment rate or the growth rate, on the other side. And, Murphy et al. (1991) present evidence whereby countries with talented people engaging in corruption and rent seeking activities grow relatively slowly.

On the theoretical front, Leff (1964), Huntington (1968), and Lui (1985), among others, have emphasized the importance of bribes in circumventing business obstacles and speeding up things, in the presence of cumbersome regulations and unmotivated bureaucrats. But, Shleifer & Vishny (1993) have argued that the illegality of corruption and the need for secrecy make it much more distortionary and costly than the inefficiencies it removes, such as the harmful effects of taxation. In fact, Myrdal (1964) and Banerjee (1997) have pointed out that some of the red tape circumvented by corruption may be intentionally put in place just for extracting bribes. And, Murphy et al. (1993) have argued that increasing returns to scale in rent seeking activities take resources away from investment and create disincentives for innovative activities that promote growth.[1]

No doubt, these are empirical and theoretical observations that frequently appear to contradict each other. What is more, the reader of the pertinent literature is in danger to get himself lost in the specifics of the particular story or model in use, to focus on the tree and miss the forest. It is obvious that an explanation and clarification is in order. In this paper we are motivated by this quarry to investigate the relationship between corruption and rent seeking activities, on the one hand, and growth, on the other hand. The purpose of our paper is mostly theoretical: To propose a simple and clear paradigm, an abstract but illuminating conceptual framework for rent seeking and corruption in the growth context. We then turn more applied, to investigate the comparative statics of the level of rent seeking/corruption and explore some policy-oriented questions.[2]

In particular, we introduce corruption and rent seeking activities into the neoclassical growth theory. Corruption or rent-seeking activities are defined by three properties: First, following Leff (1964), Huntington (1968), and Lui (1985), we model the internal effect of corruption/rent seeking activities as being positive for those engaged in such activities; and, following Shleifer & Vishny (1993) and Murphy et al. (1993), as being negative for everybody else. Second, we take corruption/rent seeking to be, at least eventually, a redistributive activity, so that if everyone is engaging in rent seeking/corruption, then none can profit from these activities. Third, following Tullock (1967), Krueger (1974, 1978), and Murphy et al. (1993), we model corruption/rent-seeking activities as using up real resources. These properties constitute a 'rent-seeking or corruption technology', a 'black box' that characterizes the available opportunities for and the consequent benefits from engaging in rent seeking and corruption.[3] This formulation can indeed be seen as a model of theft and fraud; or a model of industrial

[1] Bardhan (1997) suggests the possibility of a nonlinear relationship between corruption and growth, with the relationship been positive at relatively low levels of development and turning into negative at relatively high levels of development. However, this is not supported by the data.

[2] Our companion paper (Angeletos & Kollintzas, 2000) goes to the empirical front, and finds new strong evidence for the adverse effect of corruption on growth, investment, and innovation.

[3] This black-box technology could be justified in terms of another strand of the literature on corruption, namely the principal–agent models that generate corruption as a mechanism for either allocating scarce public goods in the presence of market failure and imperfect monitoring (Banerjee, 1997); or securing the allocation of ex post benefits to investment, in situations where the enforcement of property rights is incomplete (Acemoglu & Verdier, 1996); or colluding within bureaucratic hierarchies Carrilo (1995a, 1995b).

espionage; of lobbying and policy influence; of misgonvernance and institutional inefficiency in the sale and provision of public goods; of tax evasion and tax-burden transfer; of patent imitation and property rights expropriation; of a broad class of activities that have a redistributive rather than productive effect. Notably, corruption/rent seeking in our model is not distinguished by whether or not it is lawful, morally correct, or socially acceptable.

The growth model we choose to work with is an endogenous growth model where certain agents, intermediate-good producers, may engage in rent-seeking activities. The latter are defined by the three properties mentioned above. The growth engine is human capital as in Lucas (1988), but this is to be viewed only as a parable intended to capture the more fundamental relation that exists between the static allocation of real resources and the dynamics of growth.[4] Our formulation is indeed in line with the idea of an endogenous allocation of entrepreneurship and talent between innovative and rent-seeking activities (Baumol, 1990; Murphy, Shleifer & Vishny, 1991; Acemoglu & Verdier 1996).

Despite the fact that this is a simple and highly stylized model, it accounts for several stylized facts of the relationship between rent seeking/corruption and growth. Moreover, it yields insights into the relationship of R&D, politicoeconomic equilibrium, income distribution, and growth, as well as the design of tax/growth policies in the presence of corruption.

The economy is shown to follow a path of high rent seeking/corruption and low growth. This is essentially a case of 'the tragedy of commons': Real resources, like labor time, entrepreneurship and talent, are allocated to unproductive corruption and rent seeking, rather than being employed in productive and growth-promoting activities. For instance, entrepreneurs, lawyers, lobbyists, chief executives, etc, invest time and resources on red tape and bribing, lobbying and political influence, spying, negotiations or court suits, and other wasteful corruption/rent-seeking activities.

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A novel and challenging implication of our theory is that the higher the intensity of final-good production in intermediate-good inputs, the higher the income share of capital, the higher the rents to be distributed through corruption/rent seeking, and therefore the higher is the demand for corruption. For that reason, and contrary to

[4]It is this interaction that is important in explaining that corruption may be beneficial to some in the short run, while it is detrimental to all in the long run. In subsection 5.1 we briefly extend our discussion to alternative sources of growth, like R&D and product innovation, or government investment.

conventional wisdom, a higher capital share is coupled with lower investment and lower growth.[5]

Further, our model allows for the possibility that more intense competition, or stronger substitutability between intermediate inputs, may be detrimental for growth, because it may induce more corruption. The driving force is the strategic interaction over the choice of the individually optimal level of rent seeking.

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The plan of the paper is as follows. In Section 2 we develop the basic model, characterize individual behavior, and determine the demand for rent seeking/corruption. In Section 3 we proceed to general equilibrium and characterize the steady state and the transition dynamics, we next compare with the first best, and we finally discuss the case of a second-best anti-corruption tax. In Section 4 we present four extensions of the model: (i) alternative engines of growth; (ii) endogenous determination of the corruption

[5] Conclusive evidence on this implication is hard to provide because of the lack or reliable (or any) data on the income share of capital. Nonetheless, the significant and large positive correlation between the capital share and measures of corruption and the significant and large negative correlation between the capital share and the growth rate in the OECD countries, as documented in our companion paper (Angeletos & Kollintzas, 2000), are quite striking.

[6] See, e.g., Barro (1999) for the evidence on the Kuznets curve. In Angeletos & Kollintzas (2000), we investigate empirically the effect of corruption on growth and inequality.

technology; (iii) the role of competition and anti-corruption coordination; and (iv) the relation between corruption/rent seeking, growth, and income distribution. Section 5 concludes the paper.

2 The Model

2.1 Households - Entrepreneurs

The economy is populated by a large number of identical dynasties. Preferences are standard time separable: $U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t)$, where $\beta \in (0,1)$ is the discount factor, $C_t \in \mathbb{R}_+$ is (per capita) consumption in period t, and $u : \mathbb{R}_+ \to \mathbb{R}$ is a neoclassical temporal utility function. Just to simplify, we consider the class of isoelastic (CEIS/CRRA) utility functions: $u(C) = \frac{C^{1-1/\theta}-1}{1-1/\theta}$, where $\theta > 0$ is the constant elasticity of intertemporal substitution.

Households operate also as 'entrepreneurs', who own the two factors of production, physical capital and labor time, and all firms in the economy. Capital evolves according to the standard fixed geometric depreciation pattern and each household (or entrepreneur) has one fixed unit of labor time (or talent, or entrepreneurship) that she employs in either productive/innovative activities or rent seeking. Thus, in maximizing (**??**), the representative household faces in every period the budget constraint, $C_t + I_t \leq r_t K_t + w_t(L_t^y + L_t^x) + D_t$; the law of motion for capital, $K_{t+1} = (1-\delta)K_t + I_t$; and the time constraint, $L_t^y + L_t^x \leq 1$; where K_t, I_t, D_t are capital endowment, fixed investment, and firm dividends, respectively, L_t^y and L_t^x is the fraction of labor, talent or entrepreneurship employed in productive/innovative activities and in rent seeking, respectively; w_t is the real wage, or the return to talent and entrepreneurship; and r_t the real rental price of capital.

Following Baumol (1990), we may think of L^y in a Schumpeterian context as 'productive entrepreneurship' that one way or the other drive innovation and growth. We instead think of L^x as 'unproductive entrepreneurship' that specializes in privately profitable but socially unproductive rent seeking. The meaning of this distinction will be made clearer in Sections 2.3 and 2.5.

The household, or entrepreneur, makes her consumption, saving, and employment decisions taking input returns as given. The usual Euler equation characterizes optimal savings and determines the equilibrium growth rate:[7]

$$\frac{C_{t+1}}{C_t} = \beta^\theta \left[1 + r_{t+1} - \delta\right]^\theta \tag{1}$$

[7] We presume that the parameters of the economy are such that unbounded utility is not feasible, the transversality condition is satisfied, and a maximum exists.

On the other hand, the two types of entrepreneurship receive in equilibrium the same return (wage), so that the entrepreneur is indifferent between her two alternative employment options, production or rent seeking — the allocation of entrepreneurship will be eventually determined by the demand side.[8]

2.2 Final-Good Producers

There is a large number of firms that produce a homogeneous final good, which can be either consumed or saved and invested. In doing so, firms employ, under constant returns to scale, labor services, talent or entrepreneurship, and a composite of intermediate inputs. We allow either a discrete number or a continuum of intermediates, depending on whether we want the intermediate firm to have a non-zero measure.[9] The representative final-good producer faces the following technology:

$$Y_t \leq F\left(H_t^y L_t^y, X_t\right) \equiv A\left[H_t^y L_t^y\right]^{1-\alpha} X_t^\alpha \tag{2}$$

$$X_t \equiv \begin{cases} \left[\int_0^N x_t(z)^\zeta dz\right]^{1/\zeta} & \text{if a continuum of intermediates} \\ \left[\sum_{z=1}^N x_t(z)^\zeta\right]^{1/\zeta} & \text{if countably many intermediates} \end{cases} \tag{3}$$

where $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is Cobb-Douglas; Y_t is final-good output; L_t^y is labor time, talent or entrepreneurship employed in the final-good sector, and H_t^y is its quality, or human capital specific to final-good production and innovation; z is an index for different intermediates; $x_t(z)$ is the services extracted from intermediate good z; X_t is the composite of all intermediates; $\alpha \in (0,1)$ measures the elasticity of production with respect to the composite intermediate (or with respect to capital, as we shall see); and $\zeta \in (0,+\infty)$ measures the substitutability between different intermediates. We finally allow for either a discrete number or a continuum of intermediates, depending on whether we want the intermediate firm (the player in the rent-seeking game) to have a non-zero measure.[10]

The production function in (2)-(3) belongs to the CES family, in that it exhibits constant elasticity of substitution across intermediates. The later is $\sigma = \frac{1}{1-\zeta}$, parameterized indeed by ζ. For $\zeta = \alpha$, the production function becomes that of Romer (1990), namely

[8] The model could be extended to allow for heterogeneous agents who differ with respect to their comparative advantage in each type of entrepreneurship. In this case, there would be a neat equilibrium shorting and not every entrepreneur but only the marginal one would be indifferent between the two types of employment. Heterogeneity is introduced, for instance, in Murphy, Shleifer & Vishny (1991, 1993). This heterogeneity, however, is not necessary in understanding the nature of rent seeking and its effect on growth, and may thus be omitted from the basic paradigm without loss of generality.

[9] That distinction will prove helpful in understanding the effect of competition on rent seeking, corruption, and growth.

[10] That distinction will prove useful in understanding the effect of competition on rent seeking, corruption, and growth.

$Y_t = A\left[H_t^y L_t^y\right]^{1-\alpha} \int_0^N x_t(z)^\alpha dz$, which has the property that the marginal productivity of each intermediate input is independent of the quantity of all other intermediates employed. Instead, $\zeta < \alpha$ ($\zeta > \alpha$) implies gross complementarity (substitutability) between the intermediates.[11]

At the beginning of any period t, the representative final-good firm takes all input prices as given, as well as the quality of human capital, and employs labor services (or talent and entrepreneurship) and intermediate-good inputs, so as to her maximize profits. Profits are simply $\Pi_t^y = Y_t - w_t L_t^Y - \int_0^N p_t(z)x_t(z)dz$ (continuum case) or $\Pi_t^y = Y_t - w_t L_t^Y - \sum_{z=1}^N p_t(z)x_t(z)$ (discrete case), where $p_t(z)$ denotes the real rental price of intermediate good z. The FOCs are both necessary and sufficient. The optimal input demands for the final-good sector are thus defined by:

$$w_t = (1-\alpha)\frac{Y_t}{L_t^y} = (1-\alpha)AH_t^y\left[H_t^y L_t^y\right]^{-\alpha} X_t^\alpha \tag{4}$$

$$p_t(z) = \alpha\frac{Y_t}{X_t}\left[\frac{x_t(z)}{X_t}\right]^{\zeta-1} = \alpha\left[H_t^y L_t^y\right]^{1-\alpha} X_t^{\alpha-\zeta} x_t(z)^{\zeta-1} \tag{5}$$

Notice that the demand for an intermediate $x_t(z)$ increases (decreases) with the total composite X_t, or with any other intermediate $x_t(\omega)\,\forall\omega \neq z$, if and only if $\zeta < \alpha$ ($\zeta > \alpha$); that is, if and only if intermediates are gross complements (substitutes).

2.3 Intermediate-Good Producers, and the Returns to Rent Seeking

Each intermediate-good firm is a quasi-monopolist, in that she sets the price of its own intermediate product. Intermediate goods are produced with capital and labor services, including 'unproductive entrepreneurship'. It is through the latter that we attempt to capture the role of corruption and rent-seeking activities. That is, each intermediate-good firm employs labor services, talents and skills that are not directly productive in the usual sense. The function of these corruption and rent seeking activities is to redistribute existing rents or net output among the various intermediate-good firms. That means that that their internal private effect and their external social effect fully cancel each other out. In particular, the effective labor services employed by any given intermediate-good firm in corruption and rent-seeking activities has a direct positive effect on her own output, and thereby on her profits and on the return of her capital, but it has a negative effect (as an externality) on the outputs and capital returns of all

[11] To verify the last assertion, notice that the marginal productivity of $x_t(z)$ is given by $\frac{\partial Y_t}{\partial x_t(z)} = \alpha\left[H_t^y L_t^y\right]^{1-\alpha} X_t^{\alpha-\zeta} x_t(z)^{\zeta-1}$; this is increasing (decreasing) in X_t, and thereby in $x_t(\omega)\forall\omega \neq z$, if and only if $\zeta < \alpha$ ($\zeta > \alpha$).

rival firms. It is precisely this negative externality or redistribution feature that provides us with a working definition for rent-seeking activities and corruption.

Technically, a 'rent-seeking/corruption technology' may be represented as follows. Each intermediate good z is produced using capital and labor services or entrepreneurship according to a technology of the form:

$$x_t(z) \leq \Phi\left(k_t(z), H_t^x l_t^x(z), H_t^x L_t^x\right) \tag{6}$$

where $\Phi : \mathbb{R}_+^3 \to \mathbb{R}_+$; $k_t(z)$ is capital in firm z; $l_t(z)$ is labor time, talent, or entrepreneurship employed by firm z and engaged in corruption, lobbying, bribing, or any form of rent seeking; H_t^x is the quality of this labor, or human capital specialized in rent-seeking activities; and, finally, $L_t^x = \int_0^N l_t(z)dz$ (continuum case) or $L_t^x = \sum_{z=1}^N l_t(z)$ (discrete case) is the total labor employed in the intermediate-good sector, or the aggregate level of rent seeking.[12]

For this technology, we assume the following properties: (i) Φ is homogeneous of degree one in $(k_t(z), H_t^x l_t^x(z), H_t^x L_t^x)$; (ii) Φ is homogeneous of degree zero in $(H_t^x l_t^x(z), H_t^x L_t^x)$; (iii) $\Phi_1(.) \equiv \frac{\partial \Phi(.)}{\partial k_t(z)} > 0$; (iv) $\Phi_2(.) \equiv \frac{\partial \Phi(.)}{\partial [H_t^x l_t^x(z)]} > 0$; and (v) $\Phi_3(.) \equiv \frac{\partial \Phi(.)}{\partial [H_t^x L_t^x]} < 0$.

Much of this structure can be relaxed, but it should be clear that these properties capture the fundamentals of our framework. Property (i) means that the sector as a whole exhibits constant returns to scale, and together with (ii) implies that the production is linear in own capital $k_t(z)$, so that:

$$x_t(z) = B_t(z) \cdot k_t(z) \tag{7}$$

where $B_t(z) \equiv \Phi\left(1, H_t^x l_t^x(z), H_t^x L_t^x\right) = \Phi_1(.) > 0$ is the return of capital in firm z, given as a function of the rent-seeking activities of the particular firm and of the sector as a whole.[13] Most important, conditions (iii), (iv), and (v) capture the properties of rent seeking discussed above. In particular, (ii) implies that if all agents increase their rent-seeking activities by the same factor, then the production level and capital returns for each of them will remain unaltered.

This does not mean that corruption is not a profitable economic activity for the individual firm or entrepreneur. To the contrary, own rent-seeking activities raise private output and capital returns: $\frac{\partial B_t(z)}{\partial [H_t^x l_t^x(z)]} = \Phi_2 > 0$ and $\frac{dB_t(z)}{d[H_t^x l_t^x(z)]} = \Phi_2 + \mu\Phi_3 \geq 0$,[14] where

[12] Nothing changes if we instead define the third term in $\Phi(.)$ to be the average level of corruption, or the sum of all rivals' rent seeking, rather than the aggregate level $H_t^z L_t^x$.

[13] The linearity in capital is not critical, but, as standard in endogenous-growth models, a kind of linearity in accumulatable factors is necessary to get a balanced-growth steady state. The linearity of the aggregate technology becomes transparent if we substitute $x_t(z) = B_t k_t = B_t K_t/N$ in (2)-(3) to get $Y_t = \tilde{A}[H_t^y L_t^y]^{1-\alpha}[K_t]^\alpha$ for $\tilde{A} \equiv AN^{(1-\zeta)\alpha/\zeta}B^\alpha$.

[14] From (i) and (ii), $\Phi_2(.)H_t^x l_t^x(z) + \Phi_3(.)H_t^x L_t^x = \Phi(.) - \Phi_1(.)k_t(z) = 0$, while $\Phi_2(.) > 0$ and $\Phi_3(.) < 0$. Thus, whenever $L_t^x > l_t^x(z)$, meaning not a single monopolist, we have $\Phi_2(.) + \Phi_3(.) > \Phi_2(.)l_t^x(z) + \Phi_3(.)L_t^x = 0$.

$\mu = 1$ in the discrete case and $\mu = 0$ in the continuum case — the first derivative ignores the external effect of individual rent seeking on the aggregate level, while the second incorporates it whenever the individual firm has a non-zero measure. On the other hand, $\frac{\partial B_t(z)}{\partial [H_t^x L_t^x]} = \Phi_3 < 0$ and $\frac{\partial B_t(\omega)}{\partial [H_t^x l_t^x(z)]} = \mu \Phi_3 \leq 0$, meaning that aggregate rent seeking decreases one's own rents.

So, the zero-degree homogeneity of $B_t(z)$ in $(H_t^x l_t^x(z), H_t^x L_t^x)$ simply means that corruption and rent seeking are *a zero-sum game with respect to output or capital returns*, consistent with the redistributive nature of corruption and rent seeking we have in mind. But, rent seeking activities are costly, and thus the game turns *negative-sum with respect to net profits*.

We now look at another issue, regarding strategic interaction: In the continuum case, the individual firm is infinitesimal and thus the effect of its own rent-seeking activities on the aggregate level is null. But in the discrete case, each firm has a positive measure and a non-zero effect on the aggregate level of rent seeking — and thereby on the rents and output of every rival firm. Whether the individual producer internalizes this external effect of her own rent-seeking activities on the aggregate level, is a matter of perception. To explore the consequences of different such perceptions for the strategic interaction of agents and thereby for the equilibrium level of rent seeking, we allow in the discrete case (where firms have a positive measure) for the following two alternatives: Either (a) agents are 'myopic' and ignore the external effect of their own rent seeking on the aggregate level and thereby on the output of their rivals; or (b) agents are 'sophisticated' and fully internalize the general-equilibrium effect of their activities.

Example A concrete example of a 'rent-seeking/corruption technology' satisfying all properties (i)-(v), has the following CES specification:

$$B_t(z) = \Phi\left(1, H_t^x l_t^x(z), H_t^x L_t^x\right) = \Gamma \left[\frac{H_t^x l_t(z)}{H_t^x L_t^x}\right]^{\phi}, \quad 0 < \phi < 1, \Gamma > 0 \qquad (8)$$

That is, rents for producer z depend on the ratio of z's own level over the aggregate level of rent-seeking activities. Under (8), the returns to rent-seeking are positive but diminishing ($\Phi_2 > 0$ but $\Phi_{22} < 0$) and that it pays more to be corrupt when there is a lot of corruption around ($\Phi_{23} > 0$).

Incidentally, the 'rent-seeking/corruption technology' we propose is notably consistent with what Baumol (1990) calls the 'rules of the game' — these are what "determine the relative payoff to different types of entrepreneurship." (p.899) This is also the precise function of the 'rent-seeking/corruption technology' in our formulation — to determine the pay off to rent seeking and unproductive entrepreneurship.[15]

[15] Baumol (1990) visits Ancient Rome, Medieval China, the Middle Ages, and later times, to ex-

2.4 The Demand for Rent Seeking/Corruption

Consider now the behavior of a typical intermediate-good firm. At the beginning of any period t, she takes all input prices as given and chooses her output and input levels and the price of her product so as to maximizes profits, $\Pi_t^x(z) = p_t(z)x_t(z) - r_t k_t(z) - w_t l_t(z)$, subject to the (inverse) market demand for her product, $p_t(z) = \alpha \left[H_t^y L_t^y\right]^{1-\alpha} X_t^{\alpha-\zeta} x_t(z)^{\zeta-1}$. Firm z chooses $l_t(z)$ and $k_t(z)$ taking as given $l_t(\omega)$ and $k_t(\omega)$ $\forall\omega \neq z$. That is to say, in every period intermediate-good firms play *a one-shot simultaneous-move Cournot game in rent seeking*.

The latter implies in the continuum case that firm z perceives the productivity $B_t(\omega)$ and output $x_t(\omega)$ of any rival $\omega \neq z$ as given and independent of her own rent seeking, which is of course negligible in the aggregate. The same perceptions are hold in the discrete case if agents are 'myopic', but these perceptions are now incorrect — the external effect is non-zero and is yet ignored. When instead agents are 'sophisticated', they identify the non-zero aggregate effect of their individual rent seeking and internalize the externality on rivals' output. The demand for rent seeking can therefore be critically affected by agents' perceptions and their ability to internalize the aggregate effect of their rent seeking.

We are now ready to characterize the optimal strategic behavior of the intermediate-good sector and thereby the demand for rent seeking:

Proposition 1 *Given any $w_t, r_t > 0$, there is a unique and symmetric Cournot-Nash equilibrium in the intermediate sector, with:*

$$x_t = BK_t/N \tag{9}$$

$$Y_t = \tilde{A}[H_t^y L_t^y]^{1-\alpha} K_t{}^{\alpha} \tag{10}$$

$$w_t = \mathcal{E}_\Phi \alpha\psi Y_t L_t^{x-1} \tag{11}$$

$$r_t = \alpha v Y_t K_t{}^{-1} \tag{12}$$

where $B = \Phi(1,1,N)$ and $\tilde{A} \equiv AN^{(1-\zeta)\alpha/\zeta} B^\alpha$; $v \equiv \zeta \left[1 + \frac{\alpha-\zeta}{\zeta}\frac{1}{N}\right]$ is the inverse of the mark-up rate; $\mathcal{E}_\Phi \equiv \frac{\tilde{\partial} \log B_t(z)}{\partial \log[H_t^x l_t(z)]}$ is the equilibrium perceived elasticity of capital returns, $B_t(z)$, with respect to own effective rent-seeking activities, $H_t^x l_t(z)$; $\mathcal{E}_\Phi = e_\Phi \equiv \frac{\Phi_2(.)+\Phi_3(.)}{\Phi(.)}$ and $\psi \equiv \zeta$ in the discrete-sophisticated case; $\mathcal{E}_\Phi = \varepsilon_\Phi \equiv \frac{\Phi_2(.)}{\Phi(.)}$ and $\psi \equiv v \equiv \zeta \left[1 + \frac{\alpha-\zeta}{\zeta}\frac{1}{N}\right]$ in the discrete-myopic or the continuum case.[16] *Besides, αv and $\alpha\mathcal{E}_\Phi\psi$ are the income*

plore how the 'rules of the game' or the 'corruption technology', meaning the returns to rent seeking, historically affected the allocation of entrepreneurship.

[16]For the example technology (8), $\varepsilon_\Phi(N) = \phi$ and $e_\Phi(N) = \phi - \phi/N$.

shares of capital and unproductive entrepreneurship, respectively.[17]

Notice that, as long as the perceived elasticity $\mathcal{E}_\Phi$ is positive, the equilibrium demand for unproductive entrepreneurship and rent seeking is positive as well. Hence, the economy falls in a trap of high corruption or rent seeking. If all producers could simultaneously seize their rent-seeking activities, then their capital returns and output level would remain the same, but they would save the rewards paid to unproductive entrepreneurship (lawyers, lobbyists, etc.), making everybody better off. But zero (or low) rent seeking/corruption is not an equilibrium, because each individual has an incentive to deviate and get involved in individual rent seeking/corruption. The situation is indeed an example of *the prisoner's dilemma*: It is a dominant but self-defeating strategy to be corrupt.

Some partial-equilibrium comparative statics are immediate from (11):

Corollary 1 *The demand for rent-seeking activities, L_t^z, is decreasing in the real wage, w_t, and increasing in the aggregate level of output, Y_t; increasing in the perceived elasticity or the returns to rent seeking, $\mathcal{E}_\Phi$; increasing in the income share of capital, αv; and increasing (decreasing) in the degree of intermediate-sector competitiveness (complementarity), or the elasticity of substitution, $\sigma = \frac{1}{1-\zeta}$.*

The result that the level of corruption and rent-seeking activities increases (ceteris paribus) with a higher aggregate demand for final goods or a higher share of income going to the intermediate sector, is straightforward: The higher $\alpha v Y_t$, the higher the steaks in the rent-seeking/corruption arena. The intuition for the effect of $\mathcal{E}_\Phi$ is immediate: The more effective rent seeking is in redistributing rents, the more the demand for it. What is less immediate is the fact that the elasticity $\mathcal{E}_\Phi$ is a sufficient statistic for the whatever rules of the game. That is, the specifics of the available opportunities for rent seeking and corruption, or the particular schemes of property rights and the 'technology' of redistributing rents, do not matter per se. All is captured by the perceived elasticity of capital rents with respect to own rent-seeking activities.

Notice next that, with discrete N, the myopic producers perceive a higher elasticity than the sophisticated ones, for simply the latter internalize the negative external effect of corruption. As a result, the aggregate demand for corruption tends to be higher with myopic agents than with sophisticated. That's not the end of the story, however, because the external effect of z's own rent seeking propagates itself through the aggregate supply of intermediate inputs and thereby feeds back to the demand for z's own product. If intermediate inputs are complementary (substitutes), then a decrease in rival's output

[17] An immaterial parameter restriction we need to impose in order to ensure $\Pi_t^x \geq 0$, is $N \geq \frac{\alpha-\zeta}{1-\zeta}$. See the Appendix for details.

and productivity due to an increase in z's rent seeking will decrease (increase) the demand for z's own product. Thus, sophistication decreases individual incentives for rent seeking if there is complementarity, but may exacerbate the problem if substitutability (and competition) is very intense:

Corollary 2 *With discrete N, suppose that the myopic producers make a lower profit rate than the sophisticated ones; then, and only then, the demand for corruption L_t^x is higher with myopic agents than with sophisticated. This is necessarily true for sufficiently strong complementarity (low ζ and σ) among intermediate goods. For the CES rent seeking/corruption technology (8), this is true always.*

We finally look at N, the number of firms in the intermediate sector. We can identify two partial effects of N on the demand for rent seeking, L_t^x: The one effect works indirectly through the 'rules of the game' or the 'rent-seeking/corruption technology' and the potential influence that N may have on the elasticity $\mathcal{E}_\Phi$; we call this the 'indirect elasticity effect'. The other is the 'direct' effect that N has on L_t^x keeping $\mathcal{E}_\Phi$ fixed. The direct effect indeed comes from the equilibrium mark-up and also the externality that one's rent-seeking activities have on her rivals.[18] We note that the direct effect is positive when substitutability is strong, meaning that more intense competition can increase rent seeking/competition and thereby be detrimental for welfare and growth. We return to this point when we characterize the general equilibrium.[19]

2.5 The Growth Engine: Entrepreneurship and Human Capital

Recall that there are two types of labor or entrepreneurship — the one, L^y, employed in final-good production and the other, L^x, engaged in rent-seeking activities. Correspondingly, we introduce two types of human capital or knowledge stock — the one, H^y, specialized in production and innovation and the other, H^x, specialized in corruption and all forms of rent seeking.

We think of L^y as labor, talent or entrepreneurship employed to directly productive or innovative activities that generate a net social surplus, and we correspondingly think of H^y as a parable for all forms of productive knowledge, formal education or learning-by-doing in productive activities, R&D and product innovation, technical progress, market innovation, etc. Then, almost by definition, H^y is the type of capital that drives economic growth.

[18] The direct effect of N on L_t^x is captured in the discrete-sophisticated case by $\psi'(N) = 0$, and otherwise by $\psi'(N) = \frac{\zeta-\alpha}{N^2}\begin{cases} <0 \text{ if } \zeta<\alpha \\ >0 \text{ if } \zeta>\alpha \end{cases}$.

[19] To give an example, consider the CES rent-seeking specification (8). We have $\varepsilon_\Phi(N) = \phi$ and $e_\Phi(N) = \phi - \phi\frac{1}{N}$, so that the indirect elasticity effect is always adverse. Combining with the direct effect, a higher N necessarily raises the demand for rent seeking when $\zeta > \alpha$.

On the other hand, we think of L^x as talent or entrepreneurship attracted by existing rents and imperfect property rights and thus employed in socially unproductive but privately profitable activities. The corresponding type of human capital, H^x, is a measure of skills in communicating and colluding with the bureaucracy, or in hiding and escaping law-enforcement, or a measure of expertise in lobbying, bribing, spying, conspiring, stealing, and all forms of rent seeking.[20] And again almost by definition, this type of human capital is irrelevant to innovation and growth.

As regards the accumulation of the productive or innovative type of human capital, H^y, we follow Lucas (1988) in assuming external learning by doing:

$$\frac{H_{t+1}^y}{H_t^y} = 1 - \delta_y + \mu_y L_t^y \tag{13}$$

where $\mu_y > \delta_y \in [0, 1]$. On the other hand, the accumulation of corruption-specific human capital is irrelevant and may be allowed to take any form.[21]

The particular engine of growth that we employ in our model is not critical — what is fundamental about (13) is that the growth rate is positively related with some measure of 'production' or 'surplus' in the economy; or with the portion of real resources that are oriented towards surplus-creating and growth-promoting activities, like production and learning, education, R&D and innovation, etc. Nor is the assumption of external accumulation critical — internalizing the learning-by-doing effect, or the whatever engine of growth, would simply change the magnitudes but would not alter the fundamental qualitative feature that it is the trade-off between the returns to productive activities and the returns to rent seeking that affect the allocation of entrepreneurship and talent and thereby the speed of innovation and growth. To repeat and clarify, what is critical is only that rent seeking and corruption absorb real resources that could have been alternatively employed in productive and innovative activities, thereby crowding out investment and innovation. Or, to quote Baumol (1990), "the allocation of entrepreneurship between productive and unproductive activities... can have a profound effect on the innovativeness of the economy and the degree of dissemination of its technological discoveries."

In overall, our formulation is capturing the idea that corruption and rent seeking opportunities affect the endogenous allocation of talent and entrepreneurship across activities: Profit opportunities always attract entrepreneurship, but the innovation is

[20] Baumol (1990, p. 897) suggests "innovations in rent-seeking procedures, for example, discovery of a previously unused legal gambit that is effective in diverting rents to those who are first in exploiting it."

[21] This would not be true if the aggregate stock of H^x could affect $\mathcal{E}_\Phi$ and thereby the attractiveness of rent seeking. The latter is precluded in our case because the internal and external effect of H^x exactly cancel each other.

not always the only or dominant profitable activity. Opportunities for exploiting and redistributing existing rents can be quite attractive in a world of imperfect property rights and fertile corruption. In all these we are consistent with a Schumpeterian view on the relation of rent seeking and innovation or growth, and in line with Baumol (1990), Murphy, Shleifer & Vishny (1991), and Acemoglu & Verdier (1996).

3 General Equilibrium, Steady State, and Welfare Analysis

3.1 The Quasi-Competitive Equilibrium Path

The definition of equilibrium is clear, and indeed analogous to that in Romer (1990). Recall that we assume perfect competition for the final-good, the labor and the capital markets, and imperfect competition a la Cournot-Nash for the intermediate-good market. To give this general equilibrium a name, we call it the 'quasi-competitive equilibrium' of the economy.

Proposition 2 *Given any initial $(K_0, H_0^y) > 0$, there exists a unique equilibrium.*[22] *Allocations $\{C_t, Y_t, L_t^y, L_t^x, K_{t+1}, H_{t+1}^y\}_{t=0}^{\infty}$ and prices $\{w_t, r_t, p_t\}_{t=0}^{\infty}$ are determined by:*

$$Y_t = \tilde{A}[H_t^y L_t^y]^{1-\alpha}[K_t]^\alpha \tag{14}$$

$$r_t = vBp_t = \alpha v Y_t / K_t \tag{15}$$

$$w_t = \mathcal{E}_\Phi \psi \alpha Y_t / L_t^x \tag{16}$$

$$w_t = (1-\alpha) Y_t / L_t^y \tag{17}$$

$$1 = L_t^y + L_t^x \tag{18}$$

$$K_{t+1} = Y_t + (1-\delta)K_t - C_t \tag{19}$$

$$\frac{C_{t+1}}{C_t} = \beta^\theta [1 + r_{t+1} - \delta]^\theta \tag{20}$$

$$\frac{H_{t+1}^y}{H_t^y} = 1 - \delta_y + \mu_y L_t^y \tag{21}$$

for all t, plus the transversality condition, $\lim_{t\to\infty} \beta^t K_{t+1} C_t^{-1/\theta} = 0$. $\tilde{A}, B, \mathcal{E}_\Phi, v, \psi$ are defined as in Proposition 1.

Notice that (14) gives a reduced form for the final-good technology, which is indeed of the AK class — linear in the bundle of accumulatable factors. The first equality in (15) is the optimal pricing for intermediate goods, and the second gives the intermediate sector's demand for capital. (16) is the demand for unproductive entrepreneurship, or

[22] Following up footnote 17, we presume $1 - \mathcal{E}_\Phi(N)\psi(N) + v(N) \geq 0$.

for corruption and rent seeking, while (17) is the demand of the final-good sector for labor and productive entrepreneurship.

A convenient result following from (16) and (17) is that the intersectorial allocation of entrepreneurship, as well as the income shares of various services, remain constant along the transition dynamics:

Lemma 1 *Along any equilibrium path, productive entrepreneurship is constant at* $L_t^y = L^y \equiv \frac{1}{1+\frac{\alpha}{1-\alpha}\mathcal{E}_\Phi\psi} \forall t$ *and rent seeking/corruption is constant at* $L_t^x = L^x \equiv \frac{\frac{\alpha}{1-\alpha}\mathcal{E}_\Phi\psi}{1+\frac{\alpha}{1-\alpha}\mathcal{E}_\Phi\psi} \forall t$*. The income share of capital is* αv*, that of productive entrepreneurship is* $(1-\alpha)$*, that of rent seeking is* $\mathcal{E}_\Phi\alpha\psi$*, and the residual* $\alpha[1-\mathcal{E}_\Phi\psi-v]$ *is the income share of profits/dividends.*

Observe that $L^y < 1$ and $L^x > 0$, so that the economy falls in a high rent seeking/corruption trap, for all $N > 0$ in the continuum and discrete-myopic cases, and for all $N \geq 2$ in the discrete-sophisticated case. The comparative statics for the equilibrium level of rent-seeking and corruption are straightforward, and the intuition is analogous to the one we had in the partial-equilibrium analysis of Section 2.4. We return to the comparative statics after we characterize the steady state.

3.2 Balanced-Growth Steady State

From Lemma 1, the intersectorial allocation of labor/entrepreneurship is constant along the transition path, and so is the growth rate of the productive human capital. An implication of the latter is that the transition dynamics of our economy resemble those of the neoclassical growth model with fixed exogenous technological change: Along the transition path, the ratios of capital stocks, H_t^y/K_t and H_t^x/K_t, adjust monotonically towards their steady-state values.

Proposition 3 *The economy exhibits a unique steady state, which is globally stable, and transition is monotonic in the capital stock. The steady state is characterized by a constant intersectorial allocation of entrepreneurship,* $\frac{L^x}{L^y} = \frac{\alpha\psi\mathcal{E}_\Phi}{1-\alpha}$*; a constant growth rate,* $\gamma = \mu_y L^y - \delta_y = \mu_y \left[\frac{\mathcal{E}_\Phi\alpha\psi+1-\alpha}{1-\alpha}\right]^{-1} - \delta_y$*; a constant interest rate,* $r = [1+\gamma]^\theta\beta^{-1} - 1 + \delta$*; a constant output-capital ratio,* $\frac{Y}{K} = \frac{r}{\alpha\zeta}\left[1 + \frac{\alpha-\zeta}{\zeta}\frac{1}{N}\right]^{-1}$*; and a constant human-physical capital ratio* $\frac{H^y}{K} = \tilde{A}^{-\frac{1}{1-\alpha}}\left[\frac{Y}{K}\right]^{\frac{1}{1-\alpha}} L_y^{-1}$*.*

Notice that, while the maximum feasible growth rate is $\mu_y - \delta_y > 0$, in equilibrium $L^y < 1$ and thus $\gamma < \mu_y - \delta_y$. The gap is indeed increasing in the level of rent seeking/corruption, and therefore the comparative statics for the steady-state growth rate are quite straight forward:

Corollary 3 *In steady state, an increase in* $\mathcal{E}_\Phi$ *raises rent seeking/corruption* L^x *and lowers growth* γ*, along with* $r, \frac{Y}{K}$*, and* $\frac{H^y}{K}$*. The growth rate,* γ*, and the rate of return,* r*, both decrease with capital intensity,* α*; and*[23] *with intermediate-sector competitiveness or substitutability,* ζ *or* σ*. Further, for sufficiently strong complementarity (low* ζ, σ*), both* γ *and* r *are lower when agents are myopic rather than sophisticated. Finally, the direct effect of* N *is detrimental if* $\zeta > \alpha$; *and, for the CES specification (8),* γ *and* r *unambiguously decrease with* N *if* $\zeta \geq \alpha$.

The intuition is as before. The effects on growth come simply via the allocation of resources (labor, entrepreneurship, or talent) between productive/innovative activities (L^y) and unproductive rent seeking (L^x). We then repeat that more competition, in the sense of a higher ζ and σ or a higher N, may well raise the aggregate level of rent seeking, and can thereby be detrimental for growth. Also detrimental for growth can be a higher capital intensity, as measured by α. This result goes against common wisdom and the prediction of standard growth models, because here a higher capital intensity raises the relative private returns to rent seeking and thereby induces more rent seeking at the expense of less innovation and growth. Finally, myopia and failure to account of the external effect of rent-seeking tends to raise corruption and slow down growth, at least when complementarity is strong.

3.3 Comparison with the First Best

The market or quasi-competitive equilibrium entails two distortions. The one is the monopolistic pricing of intermediate inputs, and the other is, apparently, the positive level of unproductive rent seeking. The social planner, instead, resolves both inefficiencies. The mark-up on intermediate input prices is zero and no resources are employed to rent seeking or corruption, the latter meaning that all labor, talent, and entrepreneurship is allocated to productive and innovative activities. As a consequence, productive human capital H^y_{t+1} grows at its maximum feasible rate and the growth path attains its optimum.

Proposition 4 *For any given* (K_0, H^y_0)*, the first-best path is uniquely determined by the system* $L^y_t = 1$, $L^x_t = 0$, $Y_t = \tilde{A}[H^y_t]^{1-\alpha}[K_t]^\alpha$, $H^y_{t+1}/H^y_t = 1 + \mu_y - \delta_y$, $K_{t+1} = (1-\delta)K_t + Y_t - C_t$, $r_t = \alpha\tilde{A}[H^y_t]^{1-\alpha}[K_t]^{\alpha-1}$, *and* $C_{t+1}/C_t = \beta^\theta[1 + r_{t+1} - \delta]^\theta$, *for all* t*, plus the transversality condition,* $\lim_{t\to\infty} \beta^t K_{t+1} C_t^{-1/\theta} = 0$. *Convergence is monotonic, and the first-best steady state has* $\gamma^* = \mu_y - \delta_y > 0$, $r^* = [1 + \mu_y - \delta_y]^{1/\theta}\beta^{-1} - 1 + \delta$, $\left[\frac{Y}{K}\right]^* = \frac{r^*}{\alpha}$, *and* $\left[\frac{H^y}{K}\right]^* = \tilde{A}^{-\frac{1}{1-\alpha}} \left[\frac{r^*}{\alpha}\right]^{\frac{1}{1-\alpha}}$.

[23] The effect of ζ is so in the discrete-sophisticated case for any N, and in the discrete-myopic or the continuum cases at least for N high enough.

The comparison of the quasi-competitive steady state with its first-best counterpart is immediate:

Corollary 4 *As long as $\mathcal{E}_\Phi > 0$, both the growth rate and the return to investment are always lower in the quasi-competitive steady state than in the first best: $\gamma < \gamma^*$ and $r < r^*$. Further, $L^y < L^{y*}$, $\frac{Y}{H^{y\,1-\alpha}K^a} < \left[\frac{Y}{H^{y\,1-\alpha}K^a}\right]^*$, and $\frac{Y}{K} < \left[\frac{Y}{K}\right]^*$ and/or $\frac{Y}{H^y} < \left[\frac{Y}{H^y}\right]^*$.*

Rent seeking and corruption involve both a *level* and a *growth* effect. The static level effect consists in that, at any point of time, the market equilibrium employs part of the labor in rent seeking and corruption activities rather than in direct production. As a result, the economy operates in the interior of its Pareto frontier: $Y_t < Y_t^* = \tilde{A}[H_t^y]^{1-\alpha}[K_t]^a$ since $L_t^y < 1$. Beyond that, the fact that a portion of entrepreneurship and talent is diverted away from productive and innovative activities has an adverse dynamic effect on learning by doing, innovation, and the accumulation of capital: $\frac{H_{t+1}^y}{H_t^y} < 1 + \mu_y - \delta_y$. This translates to an adverse growth effect both in steady state and along transition.[24]

The source of both effects lies on the negative externality that one's own rent-seeking activities creates on all other intermediate-good producers. Our model is indeed an example of the *tragedy of commons*, and in this sense the validity of our story is much broader: Intermediate-good producers compete with each other and with final-good producers over the pool of real resources — this pool is labor services, talent, or entrepreneurship, in our model. The intermediate-good producers' failure to internalize (even if they recognize) the adverse external effects of their own rent-seeking activities leads them to overdraw from the common pool of real resources, effectively crowding out productive/innovative activities. Rent seeking and corruption then come at the cost of decreased efficiency and lower growth.

3.4 A Second-Best Anti-Corruption Tax

To improve upon the market equilibrium, the social planner would like to deter rent seeking and shift resources to productive/innovative activities. If a differential tax[25] could be imposed on the alternative employments of the same factor (here, the two different types of labor or entrepreneurship), then the case would be simple: Just tax employment in corruption/rent seeking and/or subsidize employment in productive activities. If such

[24] We also observe that the first best has $r^* = \alpha\left[\frac{Y}{K}\right]^*$ while the quasi-competitive equilibrium has $r = \alpha v \frac{Y}{K}$, with $v < 1$. This discrepancy between the social and the market return to capital reflects the static inefficiency generated by monopolistic pricing of in market for intermediate inputs. The inverse of $v \equiv \zeta\left[1 + \frac{\alpha-\zeta}{\zeta}\frac{1}{N}\right]$ is simply the mark-up rate.

[25] A 'tax' can be interpreted as some detection and punishment mechanism intended to deter corruption/rent seeking.

a type-contingent factor taxation were feasible, the first-best could be achieved easily by imposing a prohibitive (100%) tax on the private returns of rent-seeking activities.[26] Such a prohibitive anti-corruption tax is Pigouvian, it cures the negative externality generated by rent seeking without bringing any new distortions, and restores the first best.

In real world, however, information asymmetries may prevent the government from detecting whether a type of labor service or entrepreneurship is truly productive or primarily aiming to rent seeking. Politicoeconomic reasons may also prevent taxes from being contingent on the type of employment. Under such circumstances, the above first-best restoring tax scheme will not be available. In a second-best environment, a tax aimed to cure the existing negative externality will cause some other distortion, and the costs and benefits will have to be weighed in order to determine the optimal (and clearly non-prohibitive) level of this second-best tax.

Consider in particular a tax imposed on the output (or the revenue, or the capital returns) of intermediate-good firms.[27] Since these firms employ labor and entrepreneurship for rent seeking and corruption, the benefit of taxing their production is precisely the indirect taxation of unproductive entrepreneurship: A higher tax on the intermediate inputs decreases the demand for rent seeking/corruption. But, there is now an associated cost: The tax reduces the demand for capital in the intermediate-goods sector and is partly transferred to final-good producers, via an increase in the intermediate input prices. This induces a substitution away from intermediate inputs towards labor services, which translates to a lower capital intensity — as necessary in order to compensate for the taxation and raise back the return to capital. The lower capital intensity finally translates to a negative level effect on output.

The trade off that characterizes the second-best tax scheme is thus clear: The higher the tax is, the lower the level of rent seeking and corruption, and the higher the growth rate of the economy, but also the stronger the savings distortion and the lower the level of the final-good production. What is more, there is a static inefficiency due to the monopolistic structure of the intermediate-good sector, an inefficiency that in the absence of corruption ($\mathcal{E}_\Phi = 0$ and $L^y = 1$) would call for a subsidy of the product of the intermediate sector, or equivalently a subsidy of the capital return.

In more detail, let τ be the (time-invariant and flat) tax rate imposed on intermediate-good output, so that profits of producer z are $\Pi_t^x(z) = (1-\tau)\cdot p_t(z)x_t(z) - r_t k_t(z) - w_t l_t(z)$. Taking the first-order conditions, we determine the demand of the intermediate sector

[26]This coupled of course with a subsidy of the intermediate-sector output intended to cure the inefficiency generated by monopolistic pricing.

[27]Regarding the tax revenues, assume that they are distributed back to consumers as lump-sum transfers.

for capital and labor services as follows:

$$r_t = (1-\tau) \cdot vBp_t = (1-\tau) \cdot v\alpha Y_t / K_t \tag{22}$$

$$w_t = (1-\tau) \cdot \mathcal{E}_\Phi \psi \alpha Y_t / L_t^x \tag{23}$$

The rest of the system (14)-(21) defining the equilibrium, remains as it was without the tax. We can thus conclude to the following steady-state results:

Lemma 2 *Set $\theta = \delta = \delta_y = 1$ (log utility and full depreciation). Given a tax rate τ, let $s = s(\tau)$, $\gamma = \gamma(\tau)$, and $\kappa = \kappa(\tau)$ be the steady-state saving rate, growth rate, and capital intensity $K_t/[H_t^y L_t^y]$, respectively. Along the transition path, the allocation of labor/entrepreneurship and the saving rate are constant. The path is given by $L_t^y = L^y = \frac{1}{1+(1-\tau)\frac{\alpha\psi}{1-\alpha}\mathcal{E}_\Phi}$, $K_{t+1} = sY_t$, $C_t = (1-s)Y_t$, $Y_t = \tilde{A}[H_t^y L_y]^{1-\alpha} K_t^\alpha$, $H_{t+1}^y = (1+\gamma)H_t^y$, at all t. And finally, $s = (1-\tau)\beta\alpha v$, $\gamma = \frac{\mu_y}{1+(1-\tau)\frac{\alpha\psi}{1-\alpha}\mathcal{E}_\Phi} - 1$, and $\kappa^{1-\alpha} = \frac{(1-\tau)\beta\alpha v\tilde{A}}{\mu_y}\left[1+(1-\tau)\frac{\alpha\psi}{1-\alpha}\mathcal{E}_\Phi\right]$. It follows that γ and L^y are increasing in τ, and κ is decreasing in τ. Social welfare is given by*

$$\begin{aligned} U_0 &= \log(1-s) + \tfrac{\alpha\beta}{1-\alpha\beta}\log s + \Lambda \log L^y = \\ &= \log[1-(1-\tau)\beta\alpha v] + \tfrac{\alpha\beta}{1-\alpha\beta}\log(1-\tau) - \Lambda \log\left[1+(1-\tau)\tfrac{\alpha\psi\mathcal{E}_\Phi}{1-\alpha}\right] \equiv U(\tau) \end{aligned} \tag{24}$$

for appropriate constant $\Lambda > 0$, and is single-peaked in τ.

In (24), the first term captures the negative effect of τ on the savings rate $s = (1-\tau)\beta\alpha v$ and thereby its positive effect on consumption levels. The second term results from the negative level effect of τ on savings and thereby on capital accumulation and capital intensity κ. And the last term combines the scale and growth effects of rent seeking/corruption.

In the absence of rent seeking/corruption ($\mathcal{E}_\Phi = 0, L^y = 1$) we can write $U_0 = \log(1-s) + \frac{\alpha\beta}{1-\alpha\beta}\log s$, which implies that the first-best saving rate is $s^* = \alpha\beta$. The market savings rate without a tax/subsidy ($\tau = 0$) is then only $s = \alpha v\beta$ and falls short of s^* since $v < 1$. This reflects indeed the static inefficiency generated by monopolistic pricing in the intermediate sector. The remedy would be to subsidize intermediate-good production at a rate $\tau^* = -\frac{1-v}{v} < 0$, so that $s = (1-\tau^*)\alpha v\beta = \alpha\beta = s^*$. The Pigouvian subsidy τ^* would implement the first best in the absence of rent seeking/corruption. But, what if rent seeking/corruption is present?

Proposition 5 *Set $\theta = \delta = \delta_y = 1$ and define $\tau^{sb} \equiv \arg\max_\tau U(\tau)$ as the second-best optimal tax, with $U(\tau)$ as in (24). Then $\mathcal{E}_\Phi > 0 \Rightarrow \tau^* < \tau^{sb} < 1$, where $\tau^* = -\frac{1-v}{v} < 0$ is first-best subsidy when $\mathcal{E}_\Phi = 0$. Further, letting $s^{sb} = (1-\tau^{sb})\alpha v\beta$ and γ^{sb} for the* *second-best saving and growth rates, respectively, and $s^* = \alpha\beta$ and γ^* for the first-best ones, $s^{sb} < s^*$ and $\gamma^{sb} < s^*$ whenever $\mathcal{E}_\Phi > 0$. Finally, τ^{sb} is increasing and γ^{sb} is decreasing in $\mathcal{E}_\Phi$ and α.*

Observe that, while $\tau^* < 0$ is the optimal subsidy in the absence of corruption, the presence of rent-seeking activities calls for a tax on intermediate production, so that $\tau^{sb} > \tau^*$ whenever $\mathcal{E}_\Phi > 0$ — for high enough corruption, $\tau^{sb} > 0$ and the subsidy turns to a fully fleshed tax. This is intended to deter employment of unproductive entrepreneurship, to which the intermediate sector is intensive relatively to the rest of the economy. And while in the absence of rent seeking/corruption the single subsidy could implement the first best, this is no more true in our second-best world. Our result is indeed reminiscent of Banerjee's (1997) point that the design of public policy may be substantially altered if the policy maker incorporates the existence of corruption.

Not surprisingly, a higher elasticity $\mathcal{E}_\Phi$, meaning higher relative returns to rent seeking and corruption, calls for stronger anti-corruption measures, in the form of a higher τ^{sb}. Finally, a higher income share of capital calls for a higher tax τ^{sb}, because, under our specification of the rent-seeking game, economies with high capital shares are more prone to rent seeking for any given $\mathcal{E}_\Phi$.

We finally note that our result that the second best involves a moderate only reduction (rather than full elimination) of rent seeking, is reminiscent of similar results by Banerjee (1996) or Acemoglu & Verdier (1996). In all cases, it is the presence of some other distortion that makes full elimination of rent seeking not desirable.[28]

4 Extensions–Discussion

4.1 Alternative Engines of Growth

In our model we assumed that the growth engine is a learning-by-doing process that is external to all individual agents.

Suppose we instead allow households/entrepreneurs to internalize the accumulation of the two types of human capital. This modification would not matter qualitatively. Each individual would simply internalize the learning-by-doing effect of his current employment and thus the two types of entrepreneurship would no more be perfect substitutes.[29] In equilibrium, the relative returns to rent seeking would still determine the intersectorial allocation of labor/entrepreneurship, and more corruption would still translate to less growth.

[28] If all other distortions could be resolved, if information asymmetries and moral hazard could be overcome, and if property rights could be perfectly defined and costlessly protected, then the first best could be trivially achieved with null rent seeking/corruption.

[29] There would be two wages, one for 'engineers' and different one for 'lawyers'. The wage differential would reflect the discounted value of learning by doing in each employment.

What if growth was driven by deliberate private R&D and a market mechanism for innovations? Suppose indeed that we allowed intermediate producers to engage in R&D á la Aghion & Howitt (1992) or Romer (1990). In such a case, the innovation rate would depend on the resources devoted to R&D and thus on the profits of the intermediate sector. From Lemma 1 we know that the income share of these profits is $[\alpha(1-v) - \mathcal{E}_\Phi \alpha\psi]$, which is decreasing in $\mathcal{E}_\Phi$, the perceived corruption elasticity, or in $\mathcal{E}_\Phi \alpha\psi$, the income share paid to rent-seeking services. Higher rent seeking/corruption would thus mean a lower rate of resources devoted to R&D and thereby a lower growth rate for the economy. This point has indeed been made in Romer (1994), who identifies corruption as a 'tax' on investment.

A similar point can be made if growth is driven by government investment, public goods, infrastructure, and the like — say, as in Barro (1990). If government services are financed with income taxes, the negative effect of rent seeking/corruption on income would translate to lower tax revenues, lower government investment, and thereby lower growth. This situation is similar to tax evasion: In our set up, rent seeking/corruption reduces the tax base, just as tax evasion, and there is some waste of resources involved in addition. Either way, the operation of the growth engine is distorted.

4.2 Competition and Anti-Corruption Cooperation

A striking results of our analysis has been that more competition may lead to more rent seeking/corruption and thereby may harm innovation and growth. In deriving this result, we assumed that the agents involved in rent seeking played no-cooperatively, in a Cournot-Nash fashion, ignoring reputation forces. If we extend our analysis to allow for cooperation, then an even stronger argument can be made against competition.

To see this, we just have to let the players exploit the infinite horizon of the game, introduce reputation, and invoke the folks theorems from game theory. If players are few and patient enough, then we know that they should be able to cooperate, get themselves out of the commons tragedy they are trapped in, and ensure the first-best outcome.

In particular, consider an economy as in our model, parameterized by N, the number of players, and β, the discount factor. For any N we may find a cut-off $\underline{\beta} = \underline{\beta}(N) \in (0,1)$ such that, whenever $\beta > \underline{\beta}$, the first-best allocation, with zero corruption and maximal growth, is self-sustained through reputation.[30] We may then show that $\partial\underline{\beta}/\partial N > 0$, meaning that anti-corruption cooperation is less likely the higher the number of players. Alternatively, for fixed (β, N), we may let $\underline{L}^x = \underline{L}^x(\beta, N)$ be the minimal level of rent seeking/corruption sustainable under cooperation/reputation, and $\bar{\gamma} = \bar{\gamma}(\beta, N)$ the corresponding maximal sustainable rate of growth. We may then show that $\partial\underline{L}^x/\partial N > 0$ and $\partial\bar{\gamma}/\partial N < 0$, meaning that more competition makes implicit cooperation harder and

[30] To make the proof easy, we may use reversion to the Cournot-Nash outcome as the credible threat.

therefore may raise rent seeking and lower growth.

Notably, our results are reminiscent of the more general observation that competition in dimensions other than prices may well be detrimental for economic efficiency and social welfare.

4.3 Endogenizing the Rent-Seeking Technology: 'Corruption for Sale'

Our analysis has treated 'rules of the game', or the opportunities for rent seeking/corruption and the ways via which rent-seeking activities are mapped to redistribution of capital rents, as exogenous. These all have been buried under our black-box 'rent-seeking technology'. Treating this 'technology' as exogenous is a legitimate first-level abstraction, but a broader and deeper understanding definitely calls us to endogenize the rules of the game.[31] Indeed, casual observation and history suggest that a complicated socio-politico-economic mechanism determines the 'parameters' of the rent-seeking technology.[32]

We of course acknowledge the importance of modeling the microfoundations of the 'rent-seeking technology', but we do not try to seriously pursue this direction here.[33] We instead follow the "Protection for Sale" work of Grossman & Helpman (1995) and sketch an extension of our model that could allow us to 'endogenize' the 'parameters' of the rent-seeking/corruption technology — by analogy, we name our extension "Corruption for Sale".

A neat implication of our specification is that all that matters out of the 'rules of the game' is a single parameter: $\mathcal{E}_\Phi$, the perceived elasticity of own private rents with respect to own rent-seeking activities. Clearly, a benevolent social planner would set $\mathcal{E}_\Phi = 0$. If however the choice of $\mathcal{E}_\Phi$ is somehow under the control agents who are directly or indirectly benefited from the existence of corruption, lobbying, political contributions,

[31] For a discussion on this point, see also Bardhan (1997).

[32] By such 'parameters' we mean elements like the legal system and law enforcement; the determination, allocation and protection of property rights; the political mechanism determining the production and provision of public goods, or the design of public policies; the constitutional rules; the opportunities for lobbying, or the schemes of political financing; the taxation system; the principal-agent relations, the inside monitoring, and the hierarchical structure within a government, a bureaucracy, or a firm; the detection and punishment of illegal rent-seeking activities; or even the very definition of what kind of profit-bearing activities are illegal; or even the social attitudes towards rent-seeking and corruption. To draw hereon an example from the contemporary US, the network of Washington lobbyists for various industries or interest groups, the workings of PACs (Political Action Committees) and the contributions to Congressional/Presidential candidates, inherently involve rent seeking; and their legal limits or their social acceptance are clearly endogenous.

[33] The question remains open, but useful insights can be gained from the principal-agent approach, including Banerjee (1997), Acemoglu & Verdier (1996), and Tirole (1996). See also footnote 3.

and the like,[34] then we should expect $\mathcal{E}_\Phi > 0$. In the spirit of Grossman and Helpman (1995), we may introduce an atomistic representative political agent whose function is to set $\mathcal{E}_\Phi$. This representative political entity, henceforth to be called 'the politicians', is a parable for some socio-politico-economic mechanism, a voting system, a governance system, etc.

We hence assume the preferences of 'politicians' to aggregate (in a weighted sum) social welfare with the private benefits they enjoy from rent seeking/corruption. As a measure of the private benefits from we may take the returns to rent seeking, or the wage paid to unproductive entrepreneurship. This hypothesis may reflect that bribes to bureaucrats or to lawmakers depend on the rents attained, or that fees to layers, mediators and lobbyists are proportional to the market returns of rent seeking. To adopt a particular specification, let the benefit enjoyed by a politician be a consumption flaw proportional to $w_t L_t^x$. That is, let $u\left(C_t + \omega \cdot w_t L_t^x\right)$ be the utility flow enjoyed by politicians in period t. The inclusion of consumption C_t captures social welfare, while $\omega > 0$ is effectively a weighing factor.[35]

A Stackelberg game sets in every period: Politicians move first, setting $\mathcal{E}_\Phi$, and the market moves second, responding with a quasi-competitive equilibrium along the lines of Propositions 2 and 3. Therefore, the politicians set $\mathcal{E}_\Phi$ so as to maximize

$$\Omega(\mathcal{E}_\Phi) = \sum_{t=0}^{\infty} \beta^t u\left(C_t + \omega \cdot w_t L_t^x\right) \tag{25}$$

subject to conditions (14) through (21). 'Corruption for Sale' then arises naturally, in the sense that the endogenous value for $\mathcal{E}_\Phi$ is strictly positive in all periods. To give a concrete example:

Proposition 6 *Set $\theta = \delta = \delta_y = 1$ and let $\mathcal{E}_\Phi \equiv \arg\max_{\mathcal{E}} \Omega(\mathcal{E})$. Then $\mathcal{E}_\Phi > 0 \forall \omega > 0$, and $\partial \mathcal{E}_\Phi / \partial \omega > 0$ and $\partial \mathcal{E}_\Phi / \partial \alpha > 0$ for ω sufficiently high. Even when $\partial \mathcal{E}_\Phi / \partial \alpha < 0$, it may still hold that $\partial L^x / \partial \alpha > 0$ and $\partial \gamma / \partial \alpha < 0$.*

Thus, our result that economies with high income shares of capital are more prone to corruption and rent seeking may remain robust to endogenizing $\mathcal{E}_\Phi$, the legal/governance system and the opportunities for corruption, or the 'rules of the game'. The case for $\partial \mathcal{E}_\Phi / \partial \alpha < 0$ arises because politicians internalize part of the detrimental social effects of corruption. Naturally, this can be the case only if the weight put on social welfare

[34] Such agents can be politicians, law-makers, bureaucrats, lobbyists, layers and mediators, holders of monopoly rights, private providers of public services or public-works contractors, etc.

[35] The weighting factor may reflect how extensive in society corruption is, how accessible it is and what portion of the population is engaged to it, what the political system is and whether there is a corrupt ruling elite, etc.

is very high (ω very low). Otherwise, the private-benefit incentives dominate, making $\partial \mathcal{E}^p_\Phi / \partial \alpha > 0$. But even when ω is low and $\partial \mathcal{E}^p_\Phi / \partial \alpha < 0$, the equilibrium level of corruption may still be increasing (and growth decreasing) in the capital share, because the direct effect α has on L^x.

4.4 Rent Seeking, Corruption, and Income Inequality

The relation between growth and income inequality has been a central theme in old and new research. A limited-participation story along the lines of our model can predict a negative relation between growth and income inequality, or even a Kuznets curve.[36]

Extend our model to allow for two groups of intermediate-good firms, and correspondingly two groups of entrepreneurs owning and operating these different firms. The one group is excluded from rent seeking/corruption and is limited to purely productive activities. The other group, instead, holds the privilege of access to the rent seeking/corruption technology, in the sense that they can engage in fruitful rent seeking at the expense of the rest of the economy. With this extension, the equilibrium level of rent seeking and the rate of growth now depend also on the relative size of the two groups, or the portion of the population that has access to rent seeking. Further, the privilege of access to rent seeking/corruption translates into a differential in rents, capital returns, output levels, and net profits. There is symmetry within each group, but the 'elite group' does much better than the 'disadvantaged laymen'. The effect of rent seeking is again null within the privileged group, but now there is a net transfer from the disadvantaged base to the privileged elite. Limited participation in rent seeking/corruption thus consists the source of inequality in our story.

Therefore, rent seeking/corruption is related both with inefficiently low growth and with social inequality. If variation in corruption levels, growth rates, and inequality is generated by variation in $\mathcal{E}_\Phi$, the underlying corruption/rent seeking opportunities, for given relative size of the two groups, then a negative relation results between growth an inequality: A higher $\mathcal{E}_\Phi$ motivates more rent seeking from all members of the elite group, and this in turn implies both more inequality and less growth.[37]

If on the other hand the source of variation is differences in the 'participation rate', or the relative size of the two groups, then a Kuznets curve results — a non-monotonic, inverted-U-shaped relation between growth and inequality. To see this, let $\mu \in [0, 1]$ be the relative size of the elite group and consider the comparative statics of varying μ for fixed $\mathcal{E}_\Phi$. When $\mu \approx 1$, then almost everybody engages in rent seeking, and we

[36] Along these lines, our companion paper (Angeletos & Kollintzas, 2000) finds empirical evidence for an adverse effect of corruption on both growth and inequality. For both evidence and a literature review on the Kuznets curve, see Barro (1999).

[37] It is the symmetry of the exogenous variation across groups that appears critical in predicting a negative relation between growth and inequality.

are back to our benchmark model. Growth is very low, because corruption is very high, but there is little inequality, because everybody has access to corruption. Indeed, $\mu = 1$ induces minimal growth coupled with minimal inequality. On the other extreme, when $\mu \approx 0$, almost nobody engages in rent seeking, and the market equilibrium is corruption-free with maximal growth[38] and minimal inequality. When μ takes intermediate values, there is a sizable elite group that exploits the rest of the economy, resulting to substantial inequality. Corruption is present, but not maximal, because participation to rent seeking is limited. The other side of the token, growth is low but not minimal. Therefore, intermediate values of μ are coupled with intermediate rates of growth and high levels of inequality — hence the Kuznets curve.

From a policy or empirical perspective, limited access to rent seeking and corruption may exacerbate inequality but may well promote growth. Is that probably a feature of East Asia?

5 Concluding Remarks

This paper sought to identify the fundamental nature of rent-seeking activities (including but not limited to corruption) and understand their effect on economic efficiency and growth. In essence, we took the relevant literature and summarized it in a simple coherent framework, by identifying rent seeking/corruption with the following three properties: (i) The internal effect of rent-seeking activities is positive; (ii) their external effect is negative; and (iii) they use real resources. We then incorporated this framework into a growth model and examined the simultaneous endogenous determination of the level of rent seeking/corruption and the rate of growth. 'The rules of the game', the opportunities for and the returns to rent seeking, affect the intersectorial allocation of entrepreneurship and talent. Thereby rent seeking has both a static and a dynamic efficiency cost — it reduces production and slows down innovation and growth.

The comparative statics suggested that a higher income share of capital (a higher proportion of income subject to redistribution through rent seeking) or stronger competition may result to a higher equilibrium level or rent seeking/corruption, to a lower level of productive entrepreneurship, and thereby to a lower growth rate. Turning focus to optimal anti-corruption policies, the second best involves partial only reduction of rent seeking/corruption; and a higher tax on capital rents may help in this direction. Our basic framework was also extended to give further insights on the relation between rent seeking/corruption and R&D, inequality, and politicoeconomic equilibrium.

We hope that we have provided a clear and useful framework for thinking about rent seeking and corruption in the growth context. We believe that this is a hot topic, and more research should be due.

[38] We bypass the inefficiency due to monopolistic pricing.

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7 Appendix: Proofs

Proof of Proposition 1: For the main results: □ We shall provide the proof for discrete N only, but the continuum case is immediate as well. Using (5), profits for an individual intermediate firm are:

$$\Pi_t^x(z) = p_t(z)x_t(z) - r_t k_t(z) - w_t l_t(z) = \alpha A \left[H_t^y L_t^y\right]^{1-\alpha} X_t^{\alpha-\zeta} x_t(z)^\zeta - r_t k_t(z) - w_t l_t(z) =$$
$$= \alpha A \left[H_t^y L_t^y\right]^{1-\alpha} \left[\sum_\omega B_t(\omega)^\zeta k_t(\omega)^\zeta\right]^{(\alpha-\zeta)/\zeta} B_t(z)^\zeta k_t(z)^\zeta - r_t k_t(z) - w_t l_t(z)$$

where $B_t(z) \equiv \Phi\left(1, H_t^x l_t^x(z), H_t^x L_t^x\right)$. We next take the FOCs, noticing that we need to differentiate the sum $\sum_\omega [.]$ as well. W.r.t. $l_t(z)$ for the myopic case we get:

$$w_t = \alpha\zeta A \left[H_t^y L_t^y\right]^{1-\alpha} \left[\sum_\omega B_t(\omega)^\zeta k_t(\omega)^\zeta\right]^{\frac{\alpha-\zeta}{\zeta}} B_t(z)^{\zeta-1} k_t(z)^\zeta \Phi_2(.) H_t^x +$$
$$+\alpha(\alpha-\zeta) A \left[H_t^y L_t^y\right]^{1-\alpha} \left[\sum_\omega B_t(\omega)^\zeta k_t(\omega)^\zeta\right]^{\frac{\alpha-\zeta}{\zeta}} B_t(z)^{2\zeta-1} k_t(z)^{2\zeta} \Phi_2(.) H_t^x$$

while for the sophisticated case:

$$w_t = \alpha\zeta \left[H_t^y L_t^y\right]^{1-\alpha} \left[\sum_\omega B_t(\omega)^\zeta k_t(\omega)^\zeta\right]^{\frac{\alpha-\zeta}{\zeta}} B_t(z)^{\zeta-1} k_t(z)^\zeta \left[\Phi_2(.) + \Phi_3(.)\right] H_t^x +$$
$$+\alpha(\alpha-\zeta) \left[H_t^y L_t^y\right]^{1-\alpha} \left[\sum_\omega B_t(\omega)^\zeta k_t(\omega)^\zeta\right]^{\frac{\alpha-\zeta}{\zeta}} B_t(z)^\zeta k_t(z)^\zeta \times$$
$$\times \left\{B_t(z)^{\zeta-1} k_t(z)^\zeta [\Phi_2(.) + \Phi_3(.)] H_t^x + \sum_{\omega\neq z}\left[B_t(\omega)^{\zeta-1} k_t(\omega)^\zeta \Phi_3(.) H_t^x\right]\right\}$$

where $\Phi_j(.) = \Phi_j\left(1, H_t^x l_t^x(z), H_t^x L_t^x\right)$. W.r.t. $k_t(z)$ in both cases we get:

$$r_t = \alpha\zeta A \left[H_t^y L_t^y\right]^{1-\alpha} \left[\sum_\omega B_t(\omega)^\zeta k_t(\omega)^\zeta\right]^{\frac{\alpha-\zeta}{\zeta}} B_t(z)^\zeta k_t(z)^{\zeta-1} +$$
$$+\alpha(\alpha-\zeta) A \left[H_t^y L_t^y\right]^{1-\alpha} \left[\sum_\omega B_t(\omega)^\zeta k_t(\omega)^\zeta\right]^{\frac{\alpha-\zeta}{\zeta}} B_t(z)^{2\zeta} k_t(z)^{2\zeta-1}$$

Given symmetry, and letting $\tilde{A} \equiv AN^{\alpha(1-\zeta)/\zeta}$, we get

$$r_t = \alpha\zeta \left[1 + \frac{\alpha-\zeta}{\zeta}\frac{1}{N}\right] \tilde{A} [H_t^y L_t^y]^{1-\alpha} B_t^\alpha [Nk_t]^{\alpha-1}$$

$$w_t = \begin{cases} \alpha\zeta \left[1 + \frac{\alpha-\zeta}{\zeta}\frac{1}{N}\right] \tilde{A} [H_t^y L_t^y]^{1-\alpha} [B_t N k_t]^{\alpha-1} B_t^{-1} \Phi_2(.) H_t^x & \text{(myopic)} \\ \alpha\zeta \tilde{A} [H_t^y L_t^y]^{1-\alpha} [B_t N k_t]^{\alpha-1} B_t^{-1} [\Phi_2(.) + \Phi_3(.)] H_t^x & \text{(shophisticated)} \end{cases}$$

and $\Pi_t^x = \alpha A [H_t^y L_t^y]^{1-\alpha} N^{\alpha(1-\zeta)/\zeta - 1} [B_t N k_t]^\alpha - r_t k_t - w_t l_t^x$. Also, $x_t = B_t k_t$, $X_t = N^{1/\zeta}$, $x_t = N^{(1-\zeta)/\zeta}[B_t N k_t]$ and $Y_t = \tilde{A}\left[H_t^y L_t^y\right]^{1-\alpha} [B_t N k_t]^\alpha$. By market clearing, $Nk_t = K_t$ and $Nl_t^x = L_t^x$. By homogeneity of $\Phi(.)$, $B_t = \Phi(1, H_t^x l_t^x, N H_t^x l_t^x) = \Phi(1, 1, N)$ and then, given the definition of ε_Φ and e_Φ, $\varepsilon_\Phi(.) = \frac{\Phi_2(.)}{B_t} H_t^x L_t^x$ and $e_\Phi(.) = \frac{\Phi_2(.) + \Phi_3(.)}{B_t} H_t^x L_t^x$. Combining all together, and defining $v(N)$ and $\psi(N)$ as in Proposition **1**, we get (9) through (12). Finally, $r_t K_t / Y_t = \alpha v(N)$ and $w_t L_t^x / Y_t = \alpha \mathcal{E}_\Phi(N) \psi(N)$ for the income shares. ■ Regarding now footnote 17: □ Individual profits are $\Pi_t^x = \alpha Y_t/N - r_t k_t - w_t l_t = \alpha Y_t[1 - \mathcal{E}_\Phi(N)\psi(N) - v(N)]/N$. For the proposed specification to consist an equilibrium, we need $\Pi_t^x \geq 0$. For this it is necessary that $v(N) \leq 1 \Leftrightarrow N \geq \frac{\alpha-\zeta}{1-\zeta}$. The latter is true for all $N > 0$ in case that $\zeta \geq \alpha$ (strong substitutability). But if $\zeta < \alpha$ (strong complementarity), there should be enough intermediate-good complements, $N \geq \frac{\alpha-\zeta}{1-\zeta}$, for individual production to be profitable. Further, with $v(N) \leq 1$ ensured, we still need $\mathcal{E}_\Phi(N) \leq \frac{1-v(N)}{\psi(N)}$ for $\Pi_t^x \geq 0$: if the perceived elasticity of own capital rents to own rent-seeking activities is too high, then too much money would be spent on rent-seeking, making overall production unprofitable. Fortunately, the restriction that $\mathcal{E}_\Phi$ is small enough is rather innocuous. If the 'rules of the game' are set endogenously by bureaucrats or politicians in some political-economy mechanism, like the 'corruption-for-sale' case we discuss later on, then $\mathcal{E}_\Phi$ will be forced to be small enough, in order precisely to implement an equilibrium. Finally, a free-entry equilibrium value for N could be found by solving the zero-profit condition $1 = \mathcal{E}_\Phi(N)\psi(N) + v(N)$. ■ **QED**

Proof of Corollary 1: Obvious from condition (11) and the definitions of $v(N)$ and $\psi(N)$, observing that $\alpha v(N) = r_t K_t/Y_t$ from (12). **QED**

Proof of Corollary 2: Using (11), for the sophisticated, $\Pi_t^x = [1 - e_\Phi(N)\zeta - v(N)]\alpha Y_t/N$ and $L_t^x = e_\Phi(N)\zeta\alpha Y_t/w_t$; for the myopics, $\Pi_t^x = [1-\varepsilon_\Phi(N)v(N)-v(N)]\alpha Y_t/N$ and $L_t^x = \varepsilon_\Phi(N)v(N)\alpha Y_t/w_t$. Thus, given Y_t, w_t, and N, $\Pi_t^x|_{myopic} < \Pi_t^x|_{soph} \Leftrightarrow \varepsilon_\Phi(N)v(N) > e_\Phi(N)\zeta \Leftrightarrow L_t^x|_{myopic} < L_t^x|_{soph}$. Further, given that $\varepsilon_\Phi(N) = \frac{\Phi_2}{\Phi} > \frac{\Phi_2+\Phi_3}{\Phi} = e_\Phi(N)$, a *sufficient* condition for $\varepsilon_\Phi(N)v(N) > e_\Phi(N)\zeta$ is $v(N) \equiv \zeta\left[1 + \frac{\alpha-\zeta}{\zeta}\frac{1}{N}\right] \geq \zeta \Leftrightarrow \frac{\alpha-\zeta}{\zeta}\frac{1}{N} \geq 0 \Leftrightarrow \zeta \leq \alpha \Leftrightarrow \sigma \leq \frac{1}{1-\alpha}$. When instead $\zeta > \alpha$, we need $\frac{v(N)}{\zeta} > \frac{e_\Phi(N)}{\varepsilon_\Phi(N)} \Leftrightarrow N > \frac{\zeta-a}{\zeta}\frac{\varepsilon_\Phi(N)}{\varepsilon_\Phi(N)-e_\Phi(N)}[> 0]$; notice that the right-hand-side depends on N, but this condition will hold for N high enough provided the gap $\varepsilon_\Phi - e_\Phi$ stays bounded above zero as $N \to \infty$. For the example technology (8), we have $\log B_t(z) = \log\Gamma + \phi\log[H_t^x l_t(z)] - \phi\log[H_t^x L_t^x]$ and thus $\varepsilon_\Phi(.) \equiv \frac{\partial \log B_t(z)}{\partial \log[H_t^x l_t(z)]} = \phi$ $e_\Phi(.) \equiv \frac{\partial \log B_t(z)}{\partial \log[H_t^x l_t(z)]} + \frac{\partial \log B_t(z)}{\partial \log[H_t^x L_t^x]}\frac{\partial \log[H_t^x L_t^x]}{\partial \log[H_t^x l_t(z)]} = \phi - \phi\frac{H_t^x l_t(z)}{H_t^x L_t^x} = \phi - \phi\frac{1}{N}$; it follows $\varepsilon_\Phi - e_\Phi = \frac{\phi}{N} \to 0$ as $N \to \infty$; nonetheless, $N > \frac{\zeta-a}{\zeta}\frac{\varepsilon_\Phi(N)}{\varepsilon_\Phi(N)-e_\Phi(N)} = \frac{\zeta-a}{\zeta}N$ and hence holds $\varepsilon_\Phi(N)v(N) > e_\Phi(N)\zeta$ for any $N > 0$, .and any ζ. **QED**

Proof of Proposition 2: It follows from Proposition 1, and the other results in Section 2. Conditions (14) through (16) follow from (9) through (12) and (5). (17) follows from (4); (20) repeats the Euler condition (1); (19) and (20) are the household's constraints; and (21) repeats (13). Existence and uniqueness are ensured provided that the parameters of the economy are such that the transversality condition is satisfied. In Proposition we indeed establish that the transitional dynamics are isomorphic to those of the neoclassical growth model, so that existence, uniqueness and characterization results can be drawn by direct analogy to the neoclassical growth model. **QED**

Proof of Lemma 1: □ From (16) and (17), $\mathcal{E}_\Phi\psi\frac{\alpha Y_t}{L_t^x} = w_t = \frac{(1-\alpha)Y_t}{L_t^y} \Rightarrow \frac{L_t^x}{L_t^y} = \frac{\alpha}{1-\alpha}\mathcal{E}_\Phi\psi$, a constant, $\forall t$. Combining with $L_t^x + L_t^y = 1$ gives the equilibrium L_t^x, L_t^y. Further, for all t, $r_t K_t/Y_t = \alpha v$ from (15); $w_t L_t^x/Y_t = \alpha\mathcal{E}_\Phi\psi$ from (16); $w_t L_t^y/Y_t = 1-\alpha$ from (17); and finally $N\Pi_t^x/Y_t = \alpha[1-\mathcal{E}_\Phi(N)\psi(N)-v(N)]$ either by direct calculation or as residual. **QED**

Proof of Proposition 3: Define $Z_t \equiv \tilde{A}^{\frac{1}{1-\alpha}}[H_t^y L_t^y]$ $\forall t$ and let $1 - \tau \equiv v(N)$. From Proposition 2 and (14) through (20) we get that, given a path $\{Z_t\}_{t=0}^\infty$, the transitional dynamics in $\{C_t, Y_t, K_{t+1}\}_{t=0}^\infty$

are given by the following system: $\frac{C_{t+1}}{C_t} = \beta^\theta \left[1 + (1-\tau)\alpha \left[\frac{Z_{t+1}}{K_{t+1}}\right]^{1-\alpha} - \delta\right]^\theta$, $Y_t = Z_t^{1-\alpha} K_t^\alpha$, and $K_{t+1} = Y_t + (1-\delta)K_t - C_t$, $\forall t$. Observe that this system is identical to the transitional dynamics of the neoclassical growth model, provided that we read Z_t as the 'exogenous' geometric productivity growth and $1 - v(N) \equiv \tau$ as 'tax rate'. From Lemma 1 and the definition of Z_t it follows that $\frac{Z_{t+1}}{Z_t} = \frac{H_{t+1}^y}{H_t^y} = 1 + \mu_y L^y - \delta_y$ with L^y constant along the transition; thus Z_t indeed follows a constant-growth geometric process. Therefore, technically the transitional dynamics are indeed isomorphic to those of the neoclassical growth model; the only (technically irrelevant but economically substantial) difference is that productivity Z_t grows at a constant but endogenous rate. Therefore, our economy exhibits a unique and globally stable steady state. Further, for any initial point, the ratio K_t/Z_t or, equivalently, K_t/H_t^y converge monotonically to its steady-state value; convergence is monotonic for the growth rate as well. Using Lemma 1 and evaluating (14) through (21) at the steady state, we conclude to the reported steady state values. **QED**

Proof of Corollary 3: Immediate from Proposition 3 and Corollary 1. **QED**

Proof of Proposition 4: Due to convexity, the social planner imposes symmetry across intermediate goods, so that $x_t(z) = x_t = B_t K_t / N$. Further, recognizing that L_t^x has negative level and growth effects, the social planner sets $L_t^x = 0$ and $L_t^y = 1$. Thus, $Y_t = \tilde{A}[H_t^y]^{1-\alpha}[K_t]^\alpha$.and $H_{t+1}^y / H_t^y = 1 + \mu_y - \delta_y$ for all t The rest then follow just as in the neoclassical growth model. **QED**

Proof of Corollary 4: We let a superscript * denote the first-best value. It is immediate from Propositions 3 and 4 that $L^x > 0, L^y < 1$ and thus $\gamma < \gamma^*, r < r^*$. Further $L^y < 1$ implies $\left[\frac{Y_t}{H_t^y}\right]^{1-\alpha} \left[\frac{Y_t}{K_t}\right]^\alpha \frac{Y_t}{H_t^{y\,1-\alpha} K_t^\alpha} = \tilde{A}[L^y]^{1-\alpha} < \tilde{A} = \left[\frac{Y_t}{H_t^{y\,1-\alpha} K_t^\alpha}\right]^* = \left[\frac{Y_t}{H_t^y}\right]^{*1-\alpha} \left[\frac{Y_t}{K_t}\right]^{*\alpha}$. It follows that $\frac{Y_t}{H_t^y} < \left[\frac{Y_t}{H_t^y}\right]^*$ and/or $\frac{Y_t}{K_t} < \left[\frac{Y_t}{K_t}\right]^*$ as well. **QED**

Proof of Lemma 2: □ Working as in Proposition 2 and Lemma 1 we get $L_t^y = L^y = \frac{1}{1+(1-\tau)\frac{\alpha\psi}{1-\alpha}\mathcal{E}_\Phi}$, implying $1 + \gamma = \mu_y L^y = \frac{\mu_y}{1+(1-\tau)\frac{\alpha\psi}{1-\alpha}\mathcal{E}_\Phi}$, since $\delta_y = 0$, and $H_{t+1}^y = (1+\gamma)H_t^y \forall t$. Evaluating the Euler condition $\frac{C_{t+1}}{C_t} = \beta r_{t+1} = \beta(1-\tau)\alpha v \frac{Y_{t+1}}{K_{t+1}} = \beta(1-\tau)\alpha v \left[\frac{H_{t+1}L^y}{K_{t+1}}\right]^{1-\alpha}$ in the steady state, gives $1 + \gamma = \beta(1-\tau)\alpha v \kappa^{-(1-\alpha)}$, whereby $\kappa^{1-\alpha} = \frac{\beta(1-\tau)\alpha v}{1+\gamma} = \frac{\beta(1-\tau)\alpha v}{\mu_y}\left[1 + (1-\tau)\frac{\alpha\psi}{1-\alpha}\mathcal{E}_\Phi\right]$ we get $1 + \gamma = \frac{C_{t+1}}{C_t} = \beta r_{t+1} = \beta(1-\tau)\alpha v \frac{Y_{t+1}}{K_{t+1}} = \beta(1-\tau)\alpha v \left[\frac{H_{t+1}L^y}{K_{t+1}}\right]^{1-\alpha}$. Further, in line with the neoclassical growth model and as we did in Proposition 3, given $\{H_t^y\}_{t=0}^\infty$ the dynamics in $\{C_t, K_{t+1}, Y_t\}_{t=0}^\infty$ are determined by the system: $Y_t = \tilde{A}[H_t^y L^y]^{1-\alpha} K_t^\alpha$, $K_{t+1} = Y_t - C_t$, $\frac{C_{t+1}}{C_t} = \beta(1-\tau)\alpha v \frac{Y_{t+1}}{K_{t+1}}, \forall t$. An educated guess for the solution to this system is $K_{t+1} = sY_t, C_t = (1-s)Y_t, \forall t$, for some $s \in (0,1)$. This guess is correct $\Leftrightarrow \frac{C_{t+1}}{C_t} = \frac{(1-s)Y_{t+1}}{(1-s)Y_t} = \frac{Y_{t+1}}{Y_t} = \beta(1-\tau)\alpha v \frac{Y_{t+1}}{K_{t+1}} \Leftrightarrow K_{t+1} = \beta(1-\tau)\alpha v Y_t \Leftrightarrow s = (1-\tau)\beta\alpha v$. Finally, $H_{t+1}^y / H_t^y = \mu_y L_y = 1 + \gamma, \forall t$. Thus far we have fully characterize the equilibrium for given τ. ■ Now, given τ and any initial $(K_0, H_0^y) > 0$, we compute the lifetime utility U_0 attained in equilibrium. □ Let $k_t \equiv \log K_t$. From $Y_t = \tilde{A}[H_t^y L^y]^{1-\alpha} K_t^\alpha$ and $H_{t+1}^y / H_t^y = 1 + \gamma \Rightarrow H_t^y = (1+\gamma)^t H_0^y$, we get $Y_t = \tilde{A}[H_0^y L^y]^{1-\alpha}(1+\gamma)^{(1-\alpha)t} K_t^\alpha$. W.l.o.g., normalize at $\tilde{A}[H_0^y]^{1-\alpha} = 1$, so that $Y_t = L^{y\,1-\alpha}(1+\gamma)^{(1-\alpha)t} K_t^\alpha$ Next, from $K_{t+1} = sY_t$, $k_{t+1} = \log[sY_t] = \log[sL^{y\,1-\alpha}] + \log[(1+\gamma)^{(1-\alpha)t}] + \alpha \log K_t \Rightarrow k_{t+1} = \Gamma + \Delta t + \alpha k_t$,where $\Gamma \equiv \log\left[sL^{y\,1-\alpha}\right]$ and $\Delta \equiv (1-\alpha)\log(1+\gamma)$. This gives a linear non-homogenous difference equation in k; the unique solution to $k_{t+1} = \Gamma + \Delta t + \alpha k_t$ can be shown to be

$k_t = \frac{1-\alpha^t}{1-\alpha}\Gamma + \frac{\alpha^t - 1 + (1-\alpha)t}{(1-\alpha)^2}\Delta + \alpha^t k_0\,,\ k_t \equiv \log K_t \qquad (A1)$

From $K_{t+1} = sY_t$ and $C_t = (1-s)Y_t$, it follows that $\log C_t = \log\frac{1-s}{s} + k_{t+1}$. Substituting k_{t+1} from $(A1)$ we get $\log C_t = \log\frac{1-s}{s} + \frac{1-\alpha^{t+1}}{1-\alpha}\Gamma + \frac{\alpha^{t+1}-1+(1-\alpha)(t+1)}{(1-\alpha)^2}\Delta + \alpha^{t+1}k_0$. W.l.o.g. set $K_0 = 1 \Rightarrow k_0 = 0$. Substituting $\Gamma \equiv \log\left[sL^{y\,1-\alpha}\right]$ and $\Delta \equiv (1-\alpha)\log(1+\gamma)$ we get

$\log C_t = \log\frac{1-s}{s} + \frac{1-\alpha^{t+1}}{1-\alpha}\log s + (1-\alpha^{t+1})\log L^y + \frac{\alpha^{t+1}-1+(1-\alpha)(t+1)}{1-\alpha}\log(1+\gamma) =$

$\quad = \log(1-s) + \frac{\alpha(1-\alpha^t)}{1-\alpha}\log s + (1-\alpha^{t+1})\log L^y + \left[t+1-\frac{1}{1-\alpha}(1-\alpha^{t+1})\right]\log(1+\gamma)$

and using $1+\gamma = \mu_y L^y$ we conclude

$\log C_t = \log(1-s) + f_2(t)\log s + f_3(t)\log L^y + f_0(t) \qquad (A2)$

where $f_2(t) \equiv \left[\alpha\frac{1-\alpha^t}{1-\alpha}\right] > 0$, $f_3(t) \equiv \left[(1-\alpha^{t+1}) + t + 1 - \frac{1}{1-\alpha}(1-\alpha^{t+1})\right] = [f_2(t)+t+1] > 0$ and $f_0(t) \equiv \left\{\left[t+1-\frac{1}{1-\alpha}(1-\alpha^{t+1})\right]\log\mu_y\right\}$. Notice that s and L^y are functions of the tax rate τ, while f_0, f_2, f_3 are not. Using $(A2)$ and we can compute lifetime utility as

$U_0 = \sum_{t=0}^{\infty}\beta^t\log C_t = F_0 + F_1\log(1-s) + F_2\log s + F_3\log L^y \qquad (A3)$

where $F_0 \equiv \sum_{t=0}^{\infty}\beta^t f_0(t)$, $F_1 \equiv \sum_{t=0}^{\infty}\beta^t = \frac{1}{1-\beta} > 0$, $F_2 \equiv \sum_{t=0}^{\infty}\beta^t f_2(t) = \frac{\alpha}{1-\alpha}\sum_{t=0}^{\infty}[\beta^t - (\beta\alpha)^t] = \frac{\alpha}{1-\alpha}\left[\frac{1}{1-\beta} - \frac{1}{1-\alpha\beta}\right] = \frac{\alpha\beta}{(1-\beta)(1-\alpha\beta)} < 0$, and $F_3 \equiv \sum_{t=0}^{\infty}\beta^t f_3(t) = F_2 + \sum_{t=0}^{\infty}\beta^t(t+1) = \frac{\alpha\beta}{(1-\beta)(1-\alpha\beta)} + \frac{\beta}{(1-\beta)^2} + \frac{1}{1-\beta}$. Next, dividing $(A3)$ by $F_1 = \frac{1}{1-\beta}$, defining $\Lambda \equiv F_3/F_1 = 1 + \frac{\alpha\beta}{1-\alpha\beta} + \frac{\beta}{1-\beta} > 0$ and applying a linear transformation to U_0, we get the first part (24). Finally, substituting $s = \alpha v(1-\tau)$ and $L^y = \left[1 + (1-\tau)\frac{\alpha\psi}{1-\alpha}\mathcal{E}_\Phi\right]^{-1}$ defines $U(\tau)$ as in the last part of (24). ■ We next prove that $U(\tau)$ is single-peaked in τ: □ It can be showed $U'(\tau) = 0$ is a quadratic equation in τ and has to distinct real solutions (τ_1, τ_2), such that $\tau_1 < -\frac{1-\alpha\beta v}{\alpha\beta v} < -\frac{1-v}{v} \leq \tau_2 < 1$, $U'(\tau_1) = U'(\tau_2) = 0$ and $U''(\tau_1) > 0 > U''(\tau_2)$, for any parameters $(\mathcal{E}_\Phi, \alpha, \beta, \psi)$. Thus, τ_1 is a local minimizer of U, and τ_2 is a local maximizer. Further, $s = (1-\tau)\alpha\beta v$ is bounded in $(0,1)$, and thus τ is bounded in $\left(-\frac{1-\alpha\beta v}{\alpha\beta v}, 1\right)$. It follows that τ_2, the larger root of $U'(\tau)$, is the global maximizer in the admissible range $\left(-\frac{1-\alpha\beta v}{\alpha\beta v}, 1\right)$, meaning that $U(\tau)$ is single peaked. ■ **QED**

Proof of Proposition 5: Given the arguments above, define $\tau^{sb} \equiv \arg\max U(\tau) = \{\tau \,|\, U'(\tau) = 0\}$ and $s^{sb} = s(\tau^{sb}) = (1-\tau^{sb})\alpha v\beta$. From (24),

$U'(\tau) = \frac{\alpha v\beta}{1-(1-\tau)\alpha v\beta} - \frac{\alpha\beta}{1-\alpha\beta}\frac{1}{(1-\tau)} + \Lambda\frac{\alpha\psi\mathcal{E}_\Phi}{1-\alpha}\left[1 + (1-\tau)\frac{\alpha\psi\mathcal{E}_\Phi}{1-\alpha}\right]^{-1} \qquad (A4)$

First consider $\mathcal{E}_\Phi = 0$; then $U'(\tau) = 0 \Leftrightarrow s(\tau) = \alpha\beta \equiv s^* \Leftrightarrow \tau = -\frac{1-v}{v} \equiv \tau^*$. Next consider $\mathcal{E}_\Phi = 0$; then $U'(\tau^*) = \Lambda\frac{\alpha\psi\mathcal{E}_\Phi}{1-\alpha}\left[1 + (1-\tau^*)\frac{\alpha\psi\mathcal{E}_\Phi}{1-\alpha}\right] > 0 = U'(\tau^{sb})$. By the single-peak property of U, it follows that U is concave around τ^* and τ^{sb}, and thus $U'(\tau^*) > U'(\tau^{sb})$ implies $\tau^* < \tau^{sb}$. Further, $\lim_{\tau\to 1} U'(\tau) = -\infty$, implying $\tau^{sb} < 1$. Next, applying the IFT we get $\partial\tau^{sb}/\partial\mathcal{E}_\Phi = -\frac{\partial U'(.)/\partial\mathcal{E}_\Phi}{U''(.)}$ and $\partial\tau^{sb}/\partial\alpha = -\frac{\partial U'(.)/\partial\alpha}{U''(.)}$; but $\partial U'(.)/\partial\mathcal{E}_\Phi > 0$ and $\partial U'(.)/\partial\alpha > 0$ by $(A4)$ and $U''(\tau^{sb}) < 0$ by the second order condition, implying $\partial\tau^{sb}/\partial\mathcal{E}_\Phi > 0$ and $\partial\tau^{sb}/\partial\alpha > 0$. Alternatively, an explicit (but long) solution for τ^{sb} can be provided, verifying our results. Finally, given the one-to-one correspondence τ and s or γ, the results for s^{sb} or γ^{sb} are immediate. **QED**

Proof of Proposition 6: The proof here is similar to that in Lemma 5 and Proposition 5**.** We can show that $C_t = (1-s)Y_t$ for $s = \alpha\beta v$; further, $w_t L_t^x = \alpha\psi\mathcal{E}_\Phi Y_t$ from (16). It follows that $\log[C_t + \omega w_t L_t^x] = \log[1 - \alpha\beta v + \omega\alpha\psi\mathcal{E}] + \log Y_t$ and that, up to a linear transformation: $\Omega(\mathcal{E}) =$

$\sum_{t=0}^{\infty} \beta^t u(C_t) = \log[1 - \alpha\beta v + \omega\alpha\psi\mathcal{E}] + \Lambda \log L^y = \log[1 - \alpha\beta v + \omega\alpha\psi\mathcal{E}] - \Lambda \log \left[1 + \frac{\alpha\psi\mathcal{E}}{1-\alpha}\right]$. Obviously, $\Omega(.)$ is strictly concave and single-peaked in $\mathcal{E}_\Phi$, with $\Omega'(\mathcal{E}) = \frac{\omega\alpha\psi}{1-\alpha\beta v+\omega\alpha\psi\mathcal{E}} - \Lambda\frac{\alpha\psi}{1-\alpha}\left[1 + \frac{\alpha\psi\mathcal{E}}{1-\alpha}\right]^{-1}$. The optimal $\mathcal{E}_\Phi$ is defined by $\Omega'(\mathcal{E}_\Phi) = 0$ and using the IFT we can show that $\partial\mathcal{E}_\Phi/\partial\omega > 0$ and $\partial\mathcal{E}_\Phi/\partial\alpha > 0$ for ω sufficiently high. Given that $L^x = \frac{\alpha\psi\mathcal{E}_\Phi}{1-\alpha}\left[1 + \frac{\alpha\psi\mathcal{E}_\Phi}{1-\alpha}\right]^{-1}$ and thus $\frac{\partial L^x}{\partial\alpha}\Big|_{(\mathcal{E}_\Phi \text{ fixed})} > 0$, the total $\partial L^x/\partial\alpha$ may be negative even when $\partial\mathcal{E}_\Phi/\partial\alpha < 0$. **QED**