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ABSTRACT

Liquidity Constraints, Production Costs And Output Decisions*

This Paper analyses the interaction of financing and output market decisions in an oligopolistic setting. We integrate two ideas that have been analysed separately in previous work: some authors argue that due to risk-shifting, debt (leverage) makes a firm 'aggressive' in its output market; others argue that a firm with debts tends to be 'soft', in order to avoid bankruptcy. Our model allows for both effects. Given the key role that debt plays in this analysis, we derive debt as an optimal contract. We find that an indebted firm produces less than an unleveraged firm. The extent to which a firm is financially constrained is measured by its net worth, which determines by how much the firm will reduce its output. We find that output is a non-monotonic function of net worth: while a moderately constrained firm reduces its output if its constraints become tighter, a more strongly constrained firm increases output. These results hold for a monopoly, but are more pronounced in a duopoly.

JEL Classification: G32, G33, L13
Keywords: liquidity constraints, debt contracts, product market competition

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NON-TECHNICAL SUMMARY

In the study of the interaction between firms' financing and output market decisions, two key ideas play a prominent role. One is that firms that incur debt behave more cautiously in their output market, to mitigate the risk of bankruptcy. The second idea is that debt leads to 'risk-shifting': leveraged firms may want to adopt risky strategies because they participate in the gains of such strategies, but are protected from losses by limited liability. This can lead them to compete more aggressively, i.e. to produce or invest more, or to set lower prices. These two ideas have so far been explored separately; little is therefore known about how they interact and whether one effect is likely to dominate the other. The purpose of this Paper is to address these questions.

In addition to focusing on either bankruptcy costs or risk-shifting, most analyses of financing and output market decisions assume that firms and their creditors write 'standard' debt contracts which lead to the liquidation of firms that default on their obligations. Such contracts, however, provide strong incentives for defaulting firms to divert their cash flows. When this happens, there is scope for renegotiation: creditors can offer some prospect of allowing a bankrupt firm to survive, in exchange for the firm's cash flow. Given the key role that 'debt' plays in this analysis, we consider a set-up in which debt emerges as an *optimal contract* in response to the unobservability of the borrower's earnings.

In our model, two firms compete in a Cournot market. One of these firms is financially constrained, i.e. has a low level of net worth, while the second is cash-rich. The constrained firm can borrow money to finance its production costs, which must be paid before the firm receives any revenue from the goods it produces. After the borrowing decision, both firms choose their output levels and then receive stochastic earnings. Failure of the first firm to repay its loan at the end of the game may result in liquidation. Otherwise, the firm stays in business and earns some non-transferable future benefits.

Contracting between firm and investor is constrained by two sources of moral hazard: first, the firm's earnings are unobservable to the investor, which captures the idea that the firm can easily divert or hide its cash flow. This basic information asymmetry gives rise to a debt-like contract. In addition, the firm's output cannot be observed either. This creates a risk-shifting possibility: after signing a debt contract, the firm may have an incentive to produce more than it would if the output could be fixed in the debt contract.

The optimal contract resembles a standard debt contract in two ways: it promises a constant repayment and default is followed by possible liquidation. But in contrast to a standard debt contract, default does not lead to liquidation

with certainty; instead, the probability of liquidation depends linearly on the extent of the default. The firm's future benefits are used as a hostage – securing these benefits gives the firm the incentive to repay its loan, or otherwise hand over its entire cash holdings. An important consequence of this contract is that debt does not induce any form of risk-shifting. On the contrary, once it has incurred debt, the firm has *first-best incentives* when choosing its output: the more the firm repays with money, the less it has to lose in terms of expected liquidation. Under the optimal contract, the costs of default exactly offset the risk-shifting incentives created by debt.

Next, we show that production costs are the critical link between the firm's financing and product-market decisions. If production is costly, the firm cannot produce more than it can finance using its retained and borrowed funds, even though it has first-best *incentives*. Hence, the firm produces as much as its funds allow, up to the Cournot quantity. In equilibrium, the financially constrained firm produces strictly less than it would if it were unconstrained, because the expected cost of inefficient liquidation leads to higher marginal costs of output expansion. This is not true if the marginal costs of production are zero, or subsumed in the firm's earnings, as is often assumed in the industrial organization literature: in that case, a firm always produces the Cournot output; i.e. the firm's financing and output decisions are unrelated.

We analyse *by how much* a financially constrained firm will cut back production, by characterizing the firm's output choice as a function of its net worth. This function turns out to be U-shaped: as long as the firm has some retained earnings available to finance production, a decrease in net worth increases the probability of liquidation for any given level of production, and induces the firm to produce less. This relationship may be reversed if net worth is negative, i.e. if the firm wants to pay down an earlier loan or has to incur large fixed costs. While the firm still has an incentive to be cautious, producing little would imply that a large fraction of the loan is used to refinance the negative net worth, while only a small fraction is used to finance production, which ultimately generates the earnings needed to repay the loan. By borrowing and producing more, the firm can raise its expected earnings and hence lower its 'average fixed costs', which serves to reduce the costs of borrowing. Thus, for very low net worth, optimal output and net worth are inversely related. This continues as long as the lender is willing to finance the firm and eventually output returns to the Cournot level.

Thus, in looking at the output-market effects of financial constraints, our results show that one has to distinguish between the *existence* of financial constraints and *changes* in the severity of those constraints. For example, it is often suggested that an increase in leverage leads a firm to produce more. If 'more' means, *compared to the previous output level*, this effect can occur in our model if net worth is strongly negative. On the other hand, the firm never produces *more than an unconstrained firm*.

Finally, we discuss issues relating to the duopoly context of our model. First, financial constraints weaken a firm's competitive position: it produces less than the Cournot output and in response its rival produces more. With differentiated goods, the constrained firm's resulting market price is higher than the rival's, but both firms' prices are higher than when the firms are unconstrained. Second, competition amplifies the effects of financial constraints: while even a monopolist's output has the characteristics described above, the effects are more pronounced in duopoly, because the rival's increase in output induces the constrained firm to reduce output even further. Thus, the output market effects of financial constraints are likely to be higher in industries in which competition is most intense. All these predictions are consistent with most empirical studies. Third, we discuss to what extent financial predation can occur in our model. The observation that a financially strong rival produces more and has a lower price than the unconstrained firm can be a normal consequence of duopoly interaction and is not in itself a sign of predation. On the other hand, if the rival gains from the firm's bankruptcy, the rival has an incentive to increase production beyond its Cournot response.

1 Introduction

In the study of the interaction between firms' financing and output market decisions, two key ideas play a prominent role. One is that firms that incur debt behave more cautiously in their output market, to mitigate the risk of bankruptcy (see e.g. Bolton and Scharfstein 1990). The second idea is that debt leads to "risk-shifting": leveraged firms may want to adopt risky strategies because they participate in the gains of such strategies, but are protected from losses by limited liability. This can lead them to compete more aggressively, i.e. to produce or invest more, or to set lower prices, as suggested by Brander and Lewis (1986).[1] These two ideas have so far been explored separately; little is therefore known about how they interact, and whether one effect is likely to dominate the other. The purpose of this paper is to address these questions.

In addition to focusing on either bankruptcy costs or risk–shifting, most analyses of financing and output market decisions assume that firms and their creditors write "standard" debt contracts which lead to the liquidation of firms that default on their obligations. Such contracts, however, provide strong incentives for defaulting firms to divert their cash flows. When this happens, there is scope for renegotiation: creditors can offer some prospect of allowing a bankrupt firm to survive, in exchange for the firm's cash flow. Given the key role that "debt" plays in this analysis, we consider a setup in which debt emerges as an *optimal contract* in response to the unobservability of the borrower's earnings. We find that this additional complication in fact greatly simplifies the analysis of a problem that otherwise quickly becomes intractable.

In our model, two firms compete in a Cournot market. One of these firms is financially constrained, i.e. has a low level of net worth, while the second is cash–rich. The constrained firm can borrow money to finance its production costs, which must be paid before the firm receives any revenue from the goods it produces. After the borrowing decision, both firms choose their output levels, and then receive stochastic earnings. Failure of the first firm to repay its loan at the end of the game may result in liquidation. Otherwise, the firm stays in business and earns some nontransferable future benefits.

[1] See Maksimovich (1995) for a survey of the literature.

Contracting between firm and investor is constrained by two sources of moral hazard: first, the firm's earnings are unobservable to the investor, which captures the idea that the firm can easily divert or hide its cash flow. As in Diamond (1984) and Bolton and Scharfstein (1990), this basic information asymmetry gives rise to a debt–like contract. In addition, and in contrast to those papers, the firm's output cannot be observed either. This creates a risk–shifting possibility as in Brander and Lewis (1986): after signing a debt contract, the firm may have an incentive to produce more than it would if the output could be fixed in the debt contract.

The equilibrium contract (Section 3.1) resembles a standard debt contract in two ways: it promises a constant repayment, and default is followed by possible liquidation. What is different from a standard debt contract is that default does not lead to liquidation with certainty; instead, the probability of liquidation depends linearly on the extent of the default. This is a consequence of renegotiation possibilities after a default, which are incorporated in the initial contract. The firm's future benefits are used as a hostage — securing these benefits gives the firm the incentive to repay its loan, or otherwise hand over its entire cash holdings. [2]

We find that debt does not induce any form of risk–shifting (Section 3.2). On the contrary, once it has incurred debt, the firm has *first–best incentives* when choosing its output. This follows from the structure of the optimal contract: while under a standard debt contract it makes no difference to the firm whether it defaults by 1% or 99%, in our case it does. The more the firm repays with money, the less it has to lose in terms of expected liquidation. Under the optimal contract, the costs of default exactly offset the risk–shifting incentives created by debt.

Next, we analyze the firms' output choices. Here, the question of whether marginal

[2] The contract is similar to the ones derived in Diamond (1984), Bolton and Scharfstein (1990) and Faure-Grimaud (1997). As we discuss in more detail in Section 3, the first two papers derive the contract in the simpler scenario in which the borrower has no (unobservable) investment decision to make. Faure-Grimaud explicitly considers a noncontractible output choice, but assumes that parties can fully reverse the output decision at a later stage if they wish to do so, such that the model is equivalent to one with contractible output choice.

costs are positive or zero becomes crucial. If production is costly, the firm cannot produce more than it can finance using its retained and borrowed funds, even though it has first–best *incentives*. Hence, the firm produces as much as its funds allow, up to the Cournot quantity. In equilibrium, the financially constrained firm produces strictly less than it would if it were unconstrained, because the expected cost of inefficient liquidation leads to higher marginal costs of output expansion (Section 4.2).

This is not true if the marginal costs of production are zero: in that case, a firm always produces the Cournot output, whether it has to incur debt (to finance fixed costs, say) or not. Although it would want to commit to a lower output level, it has no means to do so credibly. This sheds light on the role of the assumption that marginal costs are zero, or subsumed in a firm's earnings, which is common in the industrial organization literature that followed Brander and Lewis (1986): for a firm with a deep pocket, it does not matter whether production costs are paid out of currently available funds or later earnings. For a liquidity–constrained firm, however, this assumption matters. Whoever extends credit to pay for the production costs (banks, trade creditors, etc.) has to trust that the firm will repay the loan if its earnings are sufficient — in equilibrium, the parties will find it optimal to sign the debt–like contract derived here.

Production costs are thus the critical link between the firm's financing and product market decisions. If marginal costs are zero, the decisions are unrelated, and the firm always produces the Cournot quantity. In contrast, with positive marginal costs, the firm internalizes the costs of using outside finance and commits to a *lower* output level by borrowing less.

In Section 4.3, we ask *by how much* a financially constrained firm will cut back production, by characterizing the firm's output choice as a function of its net worth. This function turns out to be U-shaped: as long as the firm has some retained earnings available to finance production, the firm's output is increasing in its retained earnings. Here, a decrease in net worth increases the probability of liquidation for any given level of production, and induces the firm to produce less. With negative net worth this relationship may be reversed, and more severe financial constraints lead the firm to produce *more*: Suppose the firm wants to pay down an earlier loan, or that it has to pay for large fixed costs. As with positive net worth, the tradeoff between current profits and future benefits induces the firm to be cautious. Producing little, however, would imply that a large fraction of the loan is used to refinance negative net worth, while only a small fraction is used to finance production, which ultimately generates the earnings needed to repay the loan. By borrowing and producing more, the firm can raise its expected earnings and hence lower its 'average fixed costs', which serves to reduce the costs of borrowing. Thus, for very low net worth, optimal output and net worth are inversely related. This continues as long as the lender is willing to finance the firm. Eventually, when the firm reaches its debt capacity, output returns to the Cournot level.

Thus, in looking at the output market effects of financial constraints, our results show that one has to distinguish between the *existence* of financial constraints and *changes* in the severity of those constraints. For example, it is often suggested that an increase in leverage leads a firm to produce more. If "more" means, *compared to the previous output level*, this effect can occur in our model if net worth is strongly negative. On the other hand, the firm never produces *more than an unconstrained firm.*

A U-shaped relationship between a firm's investment and a financial variable (net worth or debt) has also been reported in Brander and Lewis (1988) and Aghion, Dewatripont and Rey (1998). The former paper relies on assumptions regarding the form of the debt contract and of bankruptcy costs, and the latter relies on verifiable types of investment. No such assumptions are made here. Instead, our U–shaped curve is generated only by the interaction of the firm's objective function and the lender's participation constraint. Moreover, in those papers, the firm's capital requirements are exogenous. Here, in contrast, the firm borrows to finance its scalable investment. This introduces a feedback from the output choice to the borrowing decision that is absent in the other papers.

In Section 5, we discuss issues relating to the duopoly context of our model. First, financial constraints weaken a firm's competitive position: it produces less than the Cournot output, and in response its rival produces more, while total industry output decreases. Under Cournot competition with differentiated goods, the constrained firm's resulting market price is higher than the rival's, but both firms' prices are higher than for the case with two unconstrained firms. Second, competition amplifies the effects of financial constraints: while even a monopolist's output has the characteristics described above, the effects are more pronounced in duopoly, because the rival's increase in output induces the constrained firm to reduce output even further. Thus, the output market effects of financial constraints are likely to be higher in industries in which competition is most intense. Third, we discuss to what extent financial predation can occur in our (static) model. The observation that a financially strong rival produces more and has a lower price than the unconstrained firm can be a normal consequence of duopoly interaction and is not in itself a sign of predation. On the other hand, if the rival gains from the firm's bankruptcy, the rival has an incentive to increase production beyond its Cournot response.

Our results are consistent with most empirical studies: Opler and Titman (1994), Chevalier (1995a), Phillips (1995, in three of four industries) and Kovenock and Phillips (1997) find that highly leveraged firms invest less and lose market share, in line with our underinvestment result. In addition, Chevalier (1995a) and Kovenock and Phillips find that for the less leveraged rivals of firms undergoing an LBO, both investments and share prices increase. Chevalier (1995b) finds that following an LBO, supermarkets charge higher prices if their rivals are also leveraged, but lower prices if the rivals are less leveraged and concentrated. The first effect is as predicted by our theory, the second possibly a result of predation. Phillips (1995) also finds that after LBOs, prices generally increase. Zingales (1998), in contrast, finds evidence of lower prices on part of overleveraged firms in the trucking industry.

2 The Model

Our model involves three risk-neutral players: two firms (1 and 2) and a lender L. The firms compete in quantities and produce q_1 and q_2, respectively, at marginal cost c. Firm i's earnings are $R^i(q_1, q_2, \theta)$, where θ is a random variable distributed with density $f(\theta)$ over some interval $[\underline{\theta}, \bar{\theta}]$. We make the following assumptions about R^i:

1. $R^1(0, q_2, \theta) = 0$ for all q_2 and θ.

2. R^1 and R^2 are twice differentiable in all arguments.

3. R^1_2 and R^1_{12} are both negative.

4. R^1 is strictly concave and has a unique maximum in q_1 for each q_2 and θ.

5. $R^1_{11} R^2_{22} > R^1_{12} R^2_{21}$ for all q_1, q_2, θ.

6. R^1 and R^2 are symmetric, i.e. $R^1(q, q', \theta) = R^2(q', q, \theta)$ for all q, q', θ. Thus, all assumptions above about R^1 hold *mutatis mutandis* for R^2.

The first five assumptions are standard in Cournot models. For convenience, they are stated more restrictive than necessary. Together with the symmetry of the R^i, they guarantee the existence and uniqueness of a symmetric Nash equilibrium in q_1 and q_2. That is, there exists q^* such that

$$q^* \; = \; \arg\max_{q_1} \int_{\underline{\theta}}^{\bar{\theta}} R^1(q_1, q^*, \theta) f(\theta) \mathrm{d}\theta - c q_1 \; = \; \arg\max_{q_2} \int_{\underline{\theta}}^{\bar{\theta}} R^2(q^*, q_2, \theta) f(\theta) \mathrm{d}\theta - c q_2 \quad (1)$$

We shall refer to q^* as the Cournot quantity, and to $\pi^* = E[R^1(q^*, q^*, \theta)] - c q^*$ as the Cournot profit. Finally, if $q_i^*(q_j)$ denotes firm i's best response to q_j as specified in (1), we assume that

7. The derivatives R^i_θ and $R^i_{q_i \theta}$ are both positive for any $q_i \leq q_i^*(q_j)$.

8. $R^i(q_1, q_2, \underline{\theta}) = 0$ for any q_1 and q_2.

Assumption 7 states that higher values of θ are "good" states of the world: they correspond to higher earnings and also a higher marginal return on output. A natural interpretation is to think of θ as the state of demand. The last assumption ensures that a firm that borrows will default with positive probability. Moreover, together with $R^i_\theta > 0$ this implies that the probability of default converges to zero as the amount borrowed goes to zero.

Our model embodies the assumption that production and sales are separated in time. In many industries, firms choose capacities and inputs (e.g. employees who must be paid) before they know the actual level of demand, and set or adjust prices when they learn about demand. Thus, we believe that in a model with stochastic demand, it is not an arbitrary modeling choice whether firms compete in quantities or prices: it seems much less common that firms commit to prices without knowledge of the level of demand and are unable to change them as information arrives.[3] To abstract from inventory building, we assume that products (or inputs) can be stored temporarily, but not beyond the current period. This assumption seems most appropriate for industries selling perishable goods, services, or durable goods with high market depreciation (e.g. cars).

We assume that Firm 1 is financially constrained, while Firm 2 is not. More precisely, suppose that firm 1 has retained earnings r_0 available and must finance both fixed costs F and variable costs cq. Let $w_0 = r_0 - F$ denote firm 1's "net worth", i.e. funds available to finance production, which can be positive or negative. Moreover, let $w^* := cq^*$ denote the cost of producing the Cournot output q^*. Then we say that firm 1 is *financially constrained* if its net worth does not suffice to produce the Cournot quantity, i.e. if $w_0 < w^*$.

To finance its desired level of production, firm 1 can raise funds from a lender L in a competitive capital market. In a first–best world, firm 1 would promise to produce q^*, and it would agree with L on some form of profit–sharing. However, we assume that neither q nor θ can be observed by L, and that firm 1 cannot be forced to repay more than it earned because of limited liability. Most forms of profit–sharing will be hard to implement under these circumstances, and the parties will have to resort to a second–best contract.

The timing of the game is as follows:

1. Firm 1 can offer a financial contract to L to borrow w_1. L can accept or reject. Firm 2 knows w_0 but cannot observe the contract between firm 1 and L.[4]

[3] Instances of commitment to prices before demand is known include prices quoted in annual catalogs, on books, or at restaurants. Showalter (1996) analyzes a model similar to that of Brander and Lewis (1986), but in which firms set prices instead of quantities. He shows that firms will use strategic debt to commit to higher prices if demand is uncertain, but that firms will not use strategic debt if costs are uncertain. Showalter (1999) presents evidence in line with these predictions.

[4] Thus, at the beginning of the game, the three players possess symmetric information. The assumption that firm 2 cannot observe the contract rules out that firm 1 can commit itself, at the contracting stage, to a certain behavior in the output market. In Section 4.5, we discuss this assumption in more detail, and describe how our results are affected if commitment is possible.

2. The firms produce q_1 and q_2, respectively, at constant marginal cost c. Firm 1's output is constrained by $cq_1 \leq w_0 + w_1$. L cannot observe either firm's quantity.

3. The firms receive revenue $R^i(q_1, q_2, \theta)$ $(i = 1, 2)$. While the distribution of θ is common knowledge, only firm 1, but not L, can observe θ and its earnings.

4. Firm 1 makes some payment to L. Depending on this payment and the provisions of the contract, the firm is either liquidated or allowed to continue. If firm 1 is allowed to continue, its owners earn an additional payoff π_2. If it is liquidated, there is no additional payoff.

3 Debt Contract and Output Choice Incentives

In this section, we characterize the optimal financial contract and its implications for firm 1's output choice. Our informational assumptions and the basic idea of the contract are similar to those in Diamond (1984) and Bolton and Scharfstein (1990): since earnings are not observable, the threat of liquidation, which leads to the loss of π_2, is necessary to induce the firm to repay any money. However, our analysis goes beyond the previous papers in an important way: while in those papers the entrepreneur has no strategic decision to make, here, firm 1 has to make a quantity choice that affects the distribution of its future earnings. This raises two questions: first, what is the structure of an optimal financial contract in the presence of this additional agency problem? Second, how does the optimal contract in turn affect firm 1's quantity choice?

3.1 Structure of the Optimal Debt Contract

If output were contractible, it would follow immediately from the analyses of Diamond (1984) and Bolton and Scharfstein (1990) that the optimal contract must have a debt–like structure. That such a contract remains optimal in our setting, in which firm 1 chooses its output after signing the contract, is less obvious. First, the terms of the contract may affect the firm's output choice and hence the distribution of its earnings, which in turn might affect the structure of the optimal contract. Second, borrowing and output choice are directly linked since the firm cannot produce more than it can finance, implying that in effect, the firm constrains itself in its output choice when deciding how much to borrow. In a companion paper (Povel and Raith 2000a), we show that with an unobservable investment decision, the optimal contract still has a simple, debt like structure:

Proposition 1 *(Structure of optimal financial contract) Firm 1 borrows* w_1 *from L and promises to repay* $D \leq \pi_2$. *If firm 1 repays D, it is allowed to continue. If it repays* $r < D$, *it is liquidated with probability* $1 - \beta(r)$, *where* $\beta(r) = 1 - (D - r)/\pi_2$.

Proof: see Povel and Raith (2000a).

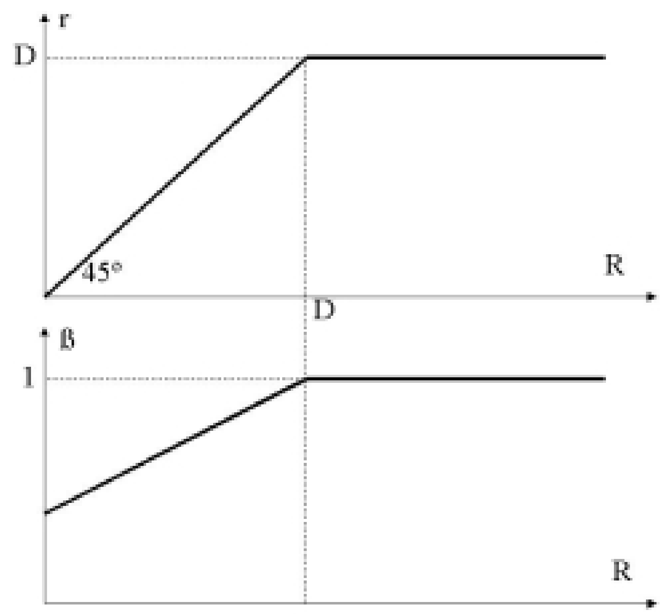

Figure 1: *Repayment and continuation probability as a function of earnings*

Firm 1's repayment and survival probability as functions of its earnings are depicted in Figure 1. The optimal contract resembles a standard debt contract in that firm 1 owes L a fixed amount D and faces the possibility of liquidation if it repays less. D, q_1 and q_2 implicitly define a "bankruptcy" state $\widehat{\theta}$:

$$D = R^1(q_1, q_2, \widehat{\theta}). \tag{2}$$

If the realized state is $\theta < \widehat{\theta}$, the firm is in default; if $\theta \geq \widehat{\theta}$, it can repay D in full.[5]

In contrast to a standard debt contract, our optimal contract has the feature that a defaulting firm is not liquidated with certainty, but with a probability that depends on the amount repaid: failing to repay 99% of a debt obligation is 'worse' than failing to repay 1%. To see why this is optimal, notice that the threat of liquidation is necessary to induce firm 1 to repay its loan. More precisely, repayment of D can be induced only if the probability of allowing the firm to continue, $\beta(r)$, satisfies the incentive constraint

$$R - D + \pi_2 \geq R - r + \pi_2 \beta(r) \tag{3}$$

for any $r < D$. The *optimal* contract then is the one that minimizes the expected cost of liquidation subject to (3). This is achieved by setting $\beta(r) = 1 - (D - r)/\pi_2$, such that (3) holds with equality for any $r < \min\{D, R\}$. Firm 1 is then indifferent between paying D and paying less but suffering a loss of future profits with some probability, and therefore weakly prefers to reveal its true earnings (Notice in Figure 1 that even if the repayment is zero, β may nevertheless be positive). Put differently, the contract uses π_2 as a "hostage" which firm 1 can buy back from L if its funds are sufficiently high. (This is why D must not be larger than π_2; if this were not the case, the optimal contract would be unchanged, but both w_1 and D would have to be smaller.)

Since a standard debt contract calls for liquidation of the firm whenever it fails to repay all it owes, we should expect that the parties renegotiate if the firm defaults: if the cash flow falls short of the promised repayment D, the firm can claim that it had zero earnings — this does not increase the punishment (certain liquidation), and allows it to keep the earnings. In this case, L can offer to allow the firm to continue in return for all or part of the firm's cash. With unobservable earnings, the situation in renegotiation is that of a seller who faces a buyer with unknown willingness (ability) to pay (for survival), and the optimal selling scheme is the optimal contract derived here. In contrast to a standard debt contract, therefore, the parties would not want to renegotiate this contract after the

[5] Proposition 1 characterizes the structure of repayment and liquidation as functions of D, but leaves open how D is determined. In Section 4, we close the model by assuming that L must break even on average, which allows us to determine D as a function of the anticipated duopoly equilibrium.

realization of R and *before* the firm's transfer of r. *After* the transfer of r, there is still no scope for renegotiation if the firm repays $r = \min\{D, R^1(.)\}$ as stipulated in the contract, since then the firm is either continued with certainty (if $r = D$), or has no money left to offer to L to avoid liquidation (if $r = R^1 < D$). Of course, renegotiation at the very end of the game can lead to problems: if the firm repays less than required ($r < \min\{R, D\}$), then there is scope for renegotiation if the firm has some bargaining power, since then the firm does have money to offer to L. On the other hand, if earnings are not observable and the firm's collateral is of no value to L, a lender would agree to finance the firm only if the threat of liquidation is credible, which we assume here. For a more detailed discussion of these issues, cf. Bolton and Scharfstein (1990).

Proposition 1 characterizes the optimal financial contract under the assumption that the firm's earnings are not verifiable. We believe that in many contexts, this contract is a more realistic description of actual debt relationships than a (non-renegotiated) standard debt contract (for a more detailed discussion, see Povel and Raith (2000a)). In the U.S., for example, insolvent firms can choose between Chapter 7 (a liquidation procedure) and Chapter 11 (a reorganisation procedure). Typically, debtors prefer Chapter 11, and creditors prefer Chapter 7; in practice, firms that end up in Chapter 7 are insolvent to a larger extent than firms in Chapter 11. A different kind of evidence comes from Kaplan and Strömberg's (1999) detailed study of financial contracts between venture capitalists and entrepreneurs in high-tech industries. One of their findings is that default does not trigger immediate liquidation, but gradually shifts control from entrepreneurs to venture capitalists, such that liquidation decisions depend on a firm's financial performance in a more continuous way.

3.2 Output Choice After Borrowing

We can now determine firm 1's second-stage choice of the production level q_1 *after* it has borrowed w_1 from L. The main result of this subsection is that firm 1 has first–best *incentives* at this stage, but is constrained by the funds borrowed:

Proposition 2 *Suppose that firm 1 has borrowed w_1 from L, signing a debt contract* *according to Proposition 1. Then firm 1 has the same* incentives *as a financially unconstrained firm, but its output may be constrained by its available funds. Specifically, if* $w_0+w_1 \geq w^*$*, firm 1 produces* q^**, while if* $w_0+w_1 < w^*$*, it produces* $q_1 = (w_0+w_1)/c < q^*$.

Proof: denote by $\widetilde{R}^1(q_1, q_2, \theta)$ firm 1's total cash holdings after realization of θ, which consist of $R^1(q_1, q_2, \theta)$ and any unspent money. Under the contract of Proposition 1, firm 1 weakly prefers to repay $\min\{D, \widetilde{R}^1(q_1, q_2, \theta)\}$. Then for $\theta \geq \widehat{\theta}$, firm 1 can repay D in full and is continued with certainty, whereas for $\theta < \widehat{\theta}$, it pays $\widetilde{R}^1$ to L and is continued with probability $1 - (D - \widetilde{R}^1)/\pi_2$. Hence, firm 1's expected payoff from choosing q is

$$\begin{aligned} E\pi(q_1, q_2) &= \int_{\underline{\theta}}^{\widehat{\theta}} \left[1 - \tfrac{D - \widetilde{R}^1(q_1,q_2,\theta)}{\pi_2}\right] \pi_2 f(\theta)\mathrm{d}\theta + \int_{\widehat{\theta}}^{\bar{\theta}} \left[\widetilde{R}^1(q_1, q_2, \theta) - D + \pi_2\right] f(\theta)\mathrm{d}\theta \\ &\qquad + w_0 + w_1 - cq_1 \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \widetilde{R}^1(q_1, q_2, \theta) f(\theta)\mathrm{d}\theta - D + w_0 + w_1 - cq_1 + \pi_2 \end{aligned} \tag{4}$$

This expression differs from the profit function in (1) only in the constant $w_1 - D$, once D is fixed in the contract. Firm 1 therefore has an objective function equivalent to that of an unconstrained firm, but it also faces the financing constraint $cq_1 \leq w_0 + w_1$. Then, if $w_0 + w_1 \geq w^*$, the standard Cournot equilibrium $q_1 = q_2 = q^*$ results. On the other hand, if $w_0 + w_1 < w^*$, then the second-stage equilibrium in quantities is given where firm 1 spends all its available funds on production. ■

The two cases of Proposition 2 are depicted in Figure 2, which shows the firms' reaction curves at the output choice stage. Firm 1's reaction curve is truncated at the highest output that it can pay for. In panel (a), firm 1 has borrowed more than it needs to produce the Cournot output: $w_1 > cq^* - w_0$. Here, firm 1's financing constraint $cq \leq w_0 + w_1$ is not binding: in the equilibrium of this subgame, both firms choose the Cournot output q^*. In panel (b), firm 1's own and borrowed funds are insufficient to produce q^*. Its reaction curve is truncated at a level below q^*, and the equilibrium is determined by the intersection of the two reaction curves; firm 1 produces less than q^*, firm 2 more.

Proposition 2 stands in sharp contrast to previous contributions in the literature in two ways. First, in our setup, in which a debt–like contract is optimal, debt has *no strategic effect* on the borrower's incentives when choosing an output level. This means

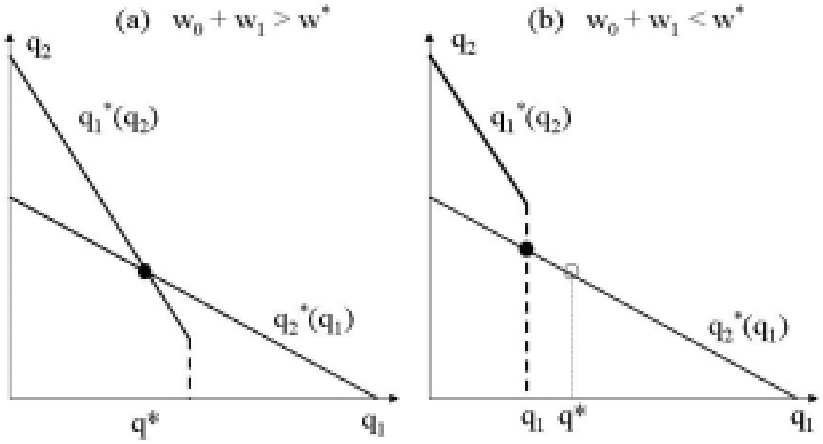

Figure 2: *Reaction curves at the output choice stage*

that the limited-liability effect that drives the results of Brander and Lewis (1986) and other papers hinges on the use of standard debt contracts. In our model, the problem of unobservable earnings makes it necessary to punish default with possible liquidation, with a probability that is increasing in the extent of the firm's default. This mitigates the risk–shifting incentives that are created by the limited liability effect. If the contract is not only incentive compatible but also optimal (cf. the discussion of Proposition 1) above), they are exactly offset: what the firm does not pay in money, it pays in expected liquidation loss. As a consequence, whatever the outcome, the firm loses a constant amount, and this makes it a residual claimant when choosing its output level.

Second, Proposition 2 demonstrates the significance of variable production costs. If they are zero, the firm may nevertheless have to borrow, e.g. to pay for fixed costs. In this case, the firm's financing and output decisions are unrelated: without a limited–liability effect, firm 1 just produces its Cournot output, regardless of its level of debt (with $c = 0$, we have $w^* = 0$, so that the first case in Proposition 2 obtains). In contrast, if production is costly and these costs are incurred before the firm sells its goods, the firm's financing and production decisions are linked directly: the firm cannot spend more than its available internal and borrowed funds. From the fact that the firm's incentives are undistorted it then follows that the firm either produces q^*, if $cq^* \leq w_0 + w_1$, or else as much as possible, i.e. $q_1 = (w_0 + w_1)/c$. In particular, a financially constrained firm always produces the same or a smaller quantity than an unconstrained firm, but never more. In Section 4, we make this statement more specific.

4 Borrowing and Output Choice in Equilibrium

We now derive the equilibrium of the full game between firm 1, firm 2 and L. First (4.1), we show how the equilibrium is determined. Then, we analyze the firms' equilibrium output choices (4.2), and see how these change as we vary firm 1's net worth (4.3). We illustrate our results with a simple example (4.4), and discuss additional issues (4.5).

4.1 Construction of the Equilibrium

If $w_0 + w_1 > w^*$, Proposition 2 implies that firm 1 spends only w^* on production and holds $\delta = w_0 + w_1 - w^*$ as cash. This part of the loan constitutes riskless debt: firm 1 neither gains nor loses anything from borrowing in excess of w^*. Therefore, we can without loss of generality assume that firm 1 does not borrow money it does not need. This establishes a one–to–one relationship between q_1 and w_1: firm 1 borrows exactly the amount needed to finance a desired level of q_1, after contributing its entire own funds: $w_1 = \max\{0, cq_1 - w_0\}$.

Firm 1 then determines its output level when it decides how much to borrow. On the other hand, since firm 2 cannot observe the contract between firm 1 and L, it is as if firms 1 and 2 and L play a simultaneous-moves game.[6] Formally, an equilibrium of the overall game is given by the q_1, q_2, D and $\widehat{\theta}$ such that q_1 and q_2 maximize firm 1's and 2's profit:

$$q_1 = \arg\max_{q_1'} \int_{\underline{\theta}}^{\bar{\theta}} R^1(q_1', q_2, \theta) f(\theta) \mathrm{d}\theta - D \quad \text{and} \tag{5}$$

$$q_2 = \arg\max_{q_2'} \int_{\underline{\theta}}^{\bar{\theta}} R^2(q_1, q_2', \theta) f(\theta) \mathrm{d}\theta - cq_2, \tag{6}$$

[6] See Section 4.5 for a discussion of this assumption.

subject to the lender's break–even constraint,

$$\int_{\underline{\theta}}^{\widehat{\theta}} R^1(q_1, q_2, \theta) f(\theta) \mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta}) D = c q_1 - w_0, \tag{7}$$

and (2), which defines $\widehat{\theta}$. Equation (7) states that the expected repayment to L, $\mathrm{Prob}(\theta \geq \widehat{\theta})D$ plus the expected value of firm 1's earnings if it cannot repay, must cover the amount $cq_1 - w_0$ that firm 1 borrows (in the Appendix, we show that the program above has a unique solution). According to Proposition 1, D must not exceed π_2. If necessary, w_1 (and hence q_1) must be reduced until both (7) and $D \leq \pi_2$ are satisfied. We abstract from this in the following by assuming that π_2 is sufficiently large.

4.2 Reduction of Output

Since L must break even, and since firm 1 has first–best incentives at the output choice stage, it fully internalizes the costs of possible liquidation and trades off the benefits (higher current earnings) and costs of debt finance when choosing its output. We can then show:

Proposition 3 *If firm 1 is financially constrained such that $w_0 \in (-\pi^*, w_0^*)$ and $c > 0$, it produces strictly less than q^*.*

Proof: see Appendix.

The first-order condition (20) derived in the proof can be equivalently expressed as

$$\int_{\underline{\theta}}^{\bar{\theta}} R_1^1(q_1, q_2, \theta) f(\theta) \mathrm{d}\theta - c + \frac{\int_{\underline{\theta}}^{\widehat{\theta}} [R_1^1(q_1, q_2, \theta) - c] f(\theta) \mathrm{d}\theta}{\mathrm{Prob}(\theta \geq \widehat{\theta})} = 0, \tag{8}$$

where R_1^1 is the derivative of R^1 with respect to q_1. Compared to an unconstrained firm, firm 1 places additional weight on the lower (default) states of demand, which are also states of lower marginal profit. It therefore produces less than the Cournot output. Put differently, since firm 1 loses future profits with some probability if it defaults, it has in incentive to reduce output below q^* in order to decrease the probability of default.

This result stands in contrast to the idea (due to Brander and Lewis 1986) that because of risk–shifting, a firm that takes on debt will increase its output. Given the reduced form of the game according to (5)-(7), Proposition 3 is quite intuitive. On the other hand, this result is not immediately obvious from the setup of the model in Section 2, as our model explicitly allows for risk–shifting effects. Three elements of our model are responsible for Proposition 3:

First, the debt contract is the optimal institutional response to the agency relationship between borrower and lender. Here, bankruptcy costs arise endogenously as inevitable agency costs. Due to these costs, a borrowing firm has an incentive (though not necessarily the means) to reduce its output. Second, as a by-product, optimality of the debt contract provides the firm with first-best incentives, instead of inducing risk-taking. Thus, even though it would be advantageous for the firm to commit to some output above the Cournot level (as in Brander-Lewis), such commitment is not possible. Third, positive production costs imply that the firm cannot produce more than it can finance. Thus, the firm effectively commits itself to a certain output level at the time when it borrows money, and bankruptcy costs lead it to choose an output below the Cournot level.

4.3 Net Worth and Output: Nonmonotonicity

In this Section, we look in more detail at how firm 1's output depends on its net worth w_0, which measures the tightness of its financial constraints. Denote by $q_1(w_0)$ firm 1's equilibrium output when its net worth is w_0.

Proposition 4 *Firm 1's equilibrium quantity $q_1(w_0)$ is U–shaped in the range $w_0 \in [-\pi^*, w^*]$. More precisely, $q_1(w^*) = q_1(-\pi^*) = q^*$, and $q_1(w_0)$ has a unique minimum at some $\widetilde{w} < 0$.*

Proof: See Appendix.

Proposition 4 is illustrated by the U-shaped curve in Figure 3 in Section 4.4 below. In the example shown, $q_1(w_0)$ is slightly concave over some range of $w_0 > \tilde{w}$. Thus, $q_1(w_0)$ is "U-shaped" as defined more precisely in the Proposition 4, but not convex throughout.

For $w_0 \geq w^*$, both firms produce the Cournot quantity q^*. If firm 1 is financially constrained, its output is smaller than q^*, and firm 2's is larger. At the most extreme level of financial constraint, beyond which firm 1's negative net worth cannot be refinanced any more, both firms produce the Cournot output again.

Firm 1's optimal output is determined by the marginal cost of debt-financed production, i.e. the increase in the required repayment given an increase in output, which we denote by $D'(q_1)$. Since L's expected profit is zero, the costs of borrowing are equal to the expected liquidation loss. With positive net worth, the effect of a decrease in net worth is very intuitive: since maintaining the same output level requires a higher repayment, and therefore liquidation with higher probability, the firm prefers to produce less. If net worth is negative, however, part of the money that the firm borrows is used to finance fixed costs, pay down old debt, etc. In this case, debt finance is feasible only if the firm also produces a sufficiently high output which generates earnings that allow the firm to repay its loan (otherwise, L cannot break even). As net worth decreases, the minimal output required to obtain finance increases, and eventually the marginal cost of borrowing $D'(q_1)$ decreases, inducing the firm to *increase* its output. More formally, $D'(q_1)$ is determined by differentiating L's break-even constraint (7) with respect to q_1:

$$\int_{\underline{\theta}}^{\widehat{\theta}} R_1^1(q_1, q_2, \theta) f(\theta) \mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta}) D'(q_1) = c. \tag{9}$$

That is, for L to break even, an increase in output (requiring in increase in borrowing by c) must be covered by the increase in D, times the probability of repayment, plus the increase in the firm's expected returns in bankruptcy states.

Holding q_1 fixed, a decrease in w_0 leads to an increase in $\widehat{\theta}$. Here, according to (9), an increase in output increases L's marginal payoff by $R_1^1(q_1, q_2, \widehat{\theta})$, and decreases it by D' times the probability of repayment. For (9) to hold, therefore, D' must be decreasing in w_0 (implying that output is increasing in w_0) whenever $D'(q_1) > R_1^1(q_1, q_2, \widehat{\theta})$. This is the case when w_0 is sufficiently large, i.e. when borrowing and $\widehat{\theta}$ are sufficiently low, because then $R_1^1(q_1, q_2, \widehat{\theta})$ is small. Eventually, however, as $\widehat{\theta}$ increases, this relationship will be reversed, because $D'(q_1)$ must always be smaller than $R_1^1(q_1, q_2, \bar{\theta})$ (otherwise (9) would not be consistent with underinvestment). Here, further decreases in w_0 lead to a decrease in D' and hence to an increase in q_1.

To see that the minimum of $q_1(w_0)$ is attained at some $\widetilde{w} < 0$, consider $w_0 = 0$. Here, output is entirely debt-financed, so that where L breaks even, *his* payoff must be decreasing in q_1 if $\widehat{\theta}$ is held fixed. Thus, for (7) to hold, $\widehat{\theta}$, and hence the probability of bankruptcy, must increase with q_1 at this point. Differentiation of (9) with respect to w_0 shows that this is the condition under which q_1 is increasing in w_0. Thus, since $q_1(w_0)$ has a unique minimum, it follows that this minimum must be attained at negative w_0.

4.4 An example

Consider a homogeneous-goods Cournot duopoly with inverse demand $p = \theta(1 - q_1 - q_2)$, where θ is uniformly distributed on $[0, 2]$. Then $R^i(q_1, q_2, \theta) = pq_i$, and let $c = 1/3$. A firm with a deep pocket hence maximizes

$$\int_0^2 \frac{\theta}{2}(1 - q_i - q_j)q_i \mathrm{d}\theta - \frac{q_i}{3} = \frac{1}{3}(2 - 3q_i - 3q_j)q_i$$

with respect to q_i. In the Cournot equilibrium between two unconstrained firms, we have $q_1 = q_2 = q^* := 2/9$.

If firm 1 has a net worth of $w_0 < \bar{w} = 2/27$ and decides to borrow, then for any q_1, it will borrow $w_1 = cq_1 - w_0$ and owes the lender D given by

$$\int_0^{\widehat{\theta}} \frac{\theta}{2}(1 - q_1 - q_2)q_1 \mathrm{d}\theta + \frac{2 - \widehat{\theta}}{2}D, \tag{10}$$

where the bankruptcy threshold $\widehat{\theta}$ is defined by

$$D = \widehat{\theta}(1 - q_1 - q_2)q_1. \tag{11}$$

Solving (10) and (11) leads to

$$D(q_1, q_2) = 2\left[q_1(1 - q_1 - q_2) - \sqrt{q_1(1 - q_1 - q_2)}\sqrt{(2 - 3q_1 - 3q_2)\frac{q_i}{3} + w_0}\right]. \tag{12}$$

Substituting this $D(q_1, q_2)$ into (5), we obtain, for any $w_0 < \bar{w}$, the Nash equilibrium of the game by (numerically) computing the q_1 and q_2 that solve

$$q_1 = \arg\max_{q_1'} -q_1'(1 - q_1' - q_2) + 2\sqrt{q_1(1 - q_1 - q_2)}\sqrt{(2 - 3q_1 - 3q_2)\frac{q_i}{3} + w_0} \tag{13}$$

$$q_2 = \arg\max_{q_2'} (2 - 3q_1 - 3q_2')\frac{q_2'}{3} \tag{14}$$

These are depicted in Figure 3, where the lower and upper curves are the equilibrium quantities of firms 1 and 2, respectively.

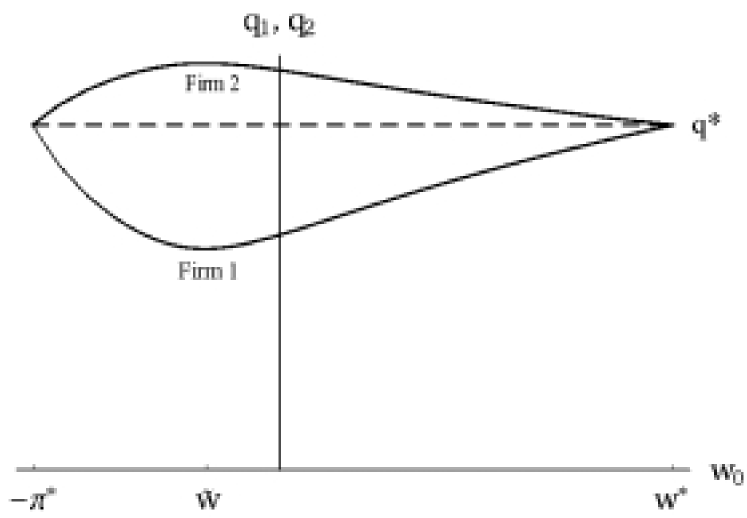

Figure 3: *Output as a function of firm 1's net worth*

4.5 Discussion

Financial distress: As w_0 decreases from w^* to $-\pi^*$, the probability of default increases monotonically from zero to one. Since at $\widetilde{w}$, we have $E[R_1^1(q_1, q_2, \theta)] = R_1^1(q_1, q_2, \widehat{\theta})$, the default probability is approximately 1/2 at $\widetilde{w}$ for symmetric distributions and revenue functions that are linear or multiplicative in θ (in the example above, it is near 0.4).

Firms with a high probability of bankruptcy are often described as being in "financial distress", a term which can have many interpretations. Some authors identify distress with the attempts of a firm to sell off assets (and similar actions) to generate cash, and avoid bankruptcy (see e.g. Asquith, Gertner, Scharfstein 1994, or Hendel 1996). In other words, financially distressed firms become "aggressive" and sharply increase their sales. The same is true for our model, but the reasons are different. We could interpret the range of very negative net worth levels as situations of financial distress: if the firm cannot refinance this net worth, it loses its future benefits for sure. If a lender is willing to refinance it, the debt contract will have some similarity with a junk bond: it carries a high risk premium (interest rate), and it is clear that the borrower will default with a high probability.

In this context, it is important to be precise about what it means to say that a firm becomes "more aggressive" as its financial situation worsens. While a decrease in net worth can lead a firm to produce more in our model, its output is still below the Cournot level. If the Cournot output is unknown, this prediction might be difficult to distinguish empirically from the prediction of other models that debt leads to an output *above* the Cournot level, if all that can be observed is the change in output in response to a change in net worth. As Propositions 3 and 4 make clear, however, whether a firm produces less than an unconstrained firm and whether it produces less if its net worth decreases are quite different questions.

Firm value and strategic debt: Can it ever pay for firm 1 to pay out cash to shareholders and thereby commit itself to be financially constrained? Such a move would be profitable if the resulting reduction in firm value is smaller than the reduction in net worth. There are two reasons why such an outcome might occur: First, for $w_0 > \widetilde{w}$, a decrease in w_0 would lead to a reduction not only of q_1, but also of industry output, and hence to an increase in (gross) industry profits. While firm 1 has the smaller market share, this might still be profitable. Second, for $w_0 < \widetilde{w}$, a decrease in w_0 leads to an increase in q_1 and a decrease in q_2. After all, a similar kind of Stackelberg effect is what leads to strategic debt in the model of Brander and Lewis (1986).

It turns out, however, that firm 1 would never want to strategically commit itself to be financially constrained in this way. Define the 'value' of an indebted firm as the sum of its expected profits the expected future benefits. Then we can show:

Proposition 5 *The value of Firm 1 is increasing in w_0 for all levels of w_0, with a slope of $1/Prob(\theta \geq \widehat{\theta}) > 1$ if $-\pi^* < w_0 < w^*$, and a slope of 1 if $w_0 \geq w^*$. The value of Firm 2 is decreasing in (firm 1's) w_0 if $w_0 > \widetilde{w}$, and increasing in w_0 if $w_0 < \widetilde{w}$.*

Proof: The value of firm 1 equals its equity value, because the debt value is zero according to (7). Then, the first part follows because the marginal increase of the equity value with respect to w_0 is given by the Lagrangian multiplier, which according to the proof of Proposition 3 equals $1/\text{Prob}(\theta \geq \widehat{\theta}) > 1$. The value of firm 2 depends on w_0 only indirectly through q_1. With strategic substitutes, the result follows immediately from Proposition 4. ■

While the first part of Proposition 5 shows that firm 1 would never take on strategic debt, the second part has important implications for predatory behavior on part of firm 2, cf. Section 5.3 below: if firm 1 is severely constrained, then weakening firm 1's financial position even further can in fact be disadvantageous for firm 2, since it will make firm 1 a more aggressive competitor.

Observability of the contract: By assuming that firm 2 cannot observe firm 1's contract, we rule out that firm 1 can commit to some output through a contract with L. An equivalent assumption is that firm 1 and L can publicly announce some contract but secretly renegotiate if it is in their interest, cf. Katz (1991). Either assumption seems plausible to us: competitors should find it hard to discover the terms of a lending agreement between a firm and its bank, say. Also, if the terms of the contract induce firm 1 to be aggressive in the output market, firm 1 and L have strong incentives to renegotiate once this threat has been communicated to firm 2. If firm 2 anticipates secret renegotiation, any commitment through the contract ceases to be credible, and the resulting equilibrium is the one derived above.

Suppose, in contrast, that the parties cannot renegotiate at any time. In principle, they can then write a contract in which the continuation probability β differs from that of Proposition 1, which would also affect firm 1's output choice. It turns out that our contract remains optimal even in this case: incentive compatibility requires that β have at least a slope of $1/\pi_2$ according to (3). A slope greater than $1/\pi_2$, on the other hand, is incentive compatible but not optimal. With such a contract, liquidation upon default would be more likely than with the contract of Proposition 1. A higher expected loss would make firm 1 more cautious, rather than more aggressive, than described in Propositions 3 and 4, which is never advantageous in a Cournot game.

While the structure of the optimal contract is the same with full commitment, the output market equilibrium is slightly different from the predictions of Propositions 3 and

4. Firm 1 in effect becomes a "Stackelberg leader" who chooses his output q_1 at stage 1, anticipating firm 2's response $q_2^*(q_1)$. On the other hand, Proposition 2 still holds, implying that firm 1 has first–best incentives when it chooses q_1. In particular, it would never want to produce more than q^* even if it has enough money. Thus, firm 1 maximizes $R^1(q_1, q_2^*(q_1))$ with the constraint $q_1 \leq q^*$, and as a result, it produces *exactly* q^* for w_0 in the vicinities of w_0^* and $-\pi^*$, as long as the costs of debt finance are not too high. This "constrained Stackelberg" effect is most pronounced when competition is intense, i.e. when firm 2 responds more strongly to changes in q_1. When the competition is relaxed, the gain from commitment decreases and the costs of debt finance become relatively more important, and the outcome gradually approaches the characterization of Proposition 4.

To conclude: our main results, underinvestment and nonmonotonicity, remain valid even when contractual arrangements between firm 1 and L are credible commitments, except that firm 1's output is *equal to* q^* over a range of values of w_0. In particular, a contractual commitment to be aggressive in the output market (as in Brander-Lewis (1986)) is not feasible. A harsh punishment upon default only has the opposite effect of making firm 1 more cautious, while more lenient treatment of a bankrupt firm undermines the firm's incentive to repay. This latter result is very similar to the tradeoff upon which Bolton and Scharfstein's theory of financial predation is based: there, a lender cannot completely protect a borrower from predation by a rival because the threat of liquidation is needed as an incentive for repayment. Here, the same requirement implies that a contract cannot make a financially constrained firm more aggressive than a firm with a deep pocket.

Related results in the literature: Brander and Lewis (1988) analyze a Cournot model in which firms take on debt, assuming that standard debt contracts are used, that there are no risk–shifting incentives, and that there are exogenously given bankruptcy costs, which are proportional to the extent of the firm's default. They obtain that output is a U-shaped function of debt. Technically, our model has similar features, but these arise endogenously. Another difference is that Brander and Lewis analyze output choice as a function of some strategically chosen *debt* level, with costless production. In our model, firms borrow (only) in order to finance costly production, which creates a feedback effect from output to financing needs.

In Aghion, Dewatripont and Rey (1998), an agent borrows money and subsequently incurs effort that improves his output market performance. Lenders are willing to finance the agent's project only if his effort is sufficiently large to generate the earnings that satisfy the lender's repayment requirements. The larger the lender's share of earnings is, however, the lower is the agent's incentive to exert effort. In this situation, the agent may have to make a verifiable investment that credibly lowers his cost of effort in order to obtain capital. This allows the agent to increase effort again as more capital is needed.

Our results are different from those in Aghion et al. in several ways. First, because of our optimal debt contract, firm 1 has first–best incentives ex post. Therefore, no contractible actions are needed to induce it to produce more if it decides to borrow more money: the lender *knows* that the firm will spend all money it has, up to $q_1 = q^*$. Second, the upward– and downward–sloping parts of the equilibrium effort in Aghion et al. correspond to two different regimes, a "shirking" regime and a "bonding" regime, where in the latter the agent can obtain outside finance only by making a verifiable investment to lower his costs of effort. In contrast, in our model, the entire U-curve is derived from the firm's first-order condition and the lender's break-even constraint (7). Finally, as in Brander and Lewis (1988), there is no feedback from output choice to financing needs in Aghion et al.; the latter are exogenous.[7]

5 Duopoly Interaction

In this section, we take a closer look at the duopoly interaction between the financially constrained firm 1 and its unconstrained rival, firm 2. Specifically, we discuss (1) equilibrium prices, (2) the role of the toughness of competition, and (3) financial predation.

[7] In Povel and Raith (2000a), we analyze the interaction between an entrepreneur's unobservable monetary investment in a project and his choice of effort. We show that if money and effort are substitutes, then a financially constrained entrepreneur who prefers to restrict his monetary investment may compensate by "overinvesting" in effort.

5.1 Prices

If firm 1 produces lower output because of liquidity constraints, firm 2's response is to produce a higher output, as depicted in Figure 3. As usual in Cournot models, total industry output decreases, because the slope of firm 2's reaction function is less than 1. Thus, with homogeneous goods, the corresponding market price is higher than if both firms are unconstrained.

If firms 1 and 2 sell differentiated products, then differences in the quantities produced also lead to differences in the resulting prices. For example, consider a differentiated Cournot duopoly with the inverse demand $p_i = \theta(1 - q_i - \sigma q_j)$, where $\sigma \in [0, 1]$ is the degree of product homogeneity. This is a generalized version of the example from above, and the resulting equilibrium quantities as a function of firm 1's net worth look as depicted in Figure 3 above. The corresponding average prices are shown in Figure 4 for $\sigma = 0.6$. Firm 1's price is a mirror image of its quantity function, i.e. a financially constrained firm

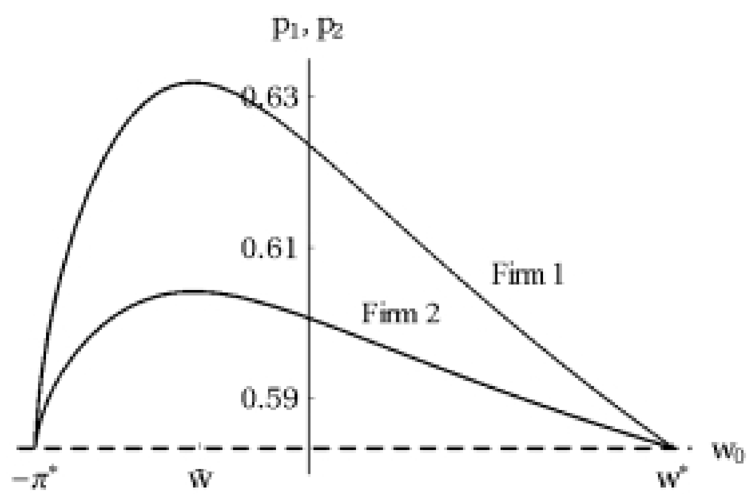

Figure 4: *Average prices in a differentiated Cournot market.*

charges a higher price than an unconstrained firm would. Firm 2 also charges more than it would without a financially constrained competitor (but less than firm 1), because a decrease of firm 1's output leads to an outward shift of firm 2's residual demand function and hence to an increase of both its quantity and its price.

Chevalier and Scharfstein (1996) offer a different explanation for why financially constrained firms might set higher prices: in the presence of switching costs, firms have an incentive to keep prices low in the long run, in order to attract new customers. In this sense, prices themselves are investments in market share. In the short run, however, firms in need of cash can increase profits by raising prices in order to exploit their locked-in customers. While this argument is very compelling for industries in which switching costs play a role, our theory shows that (1) switching costs are not a necessary assumption, and (2) prices need not be the firms' main strategic variable for high debt to result in higher prices. Rather, firms may charge higher prices simply because they previously chose a lower level of production.

Phillips (1995) and Chevalier (1995b) study price changes in industries following large increases in debt by some of the firms. Phillips finds that prices rise in the fiberglass, tractor trailer and polyethylene industries, but fall in the gypsum industry, in which several major competitors did not increase their leverage. Similarly, Chevalier finds that prices rise where the rivals of supermarkets undertaking an LBO are highly leveraged as well, but fall where the competitors are less leveraged, and concentrated. As Chevalier suggests, a price decrease could be a sign of predation (cf. also 5.3 below).

While in the example above prices are determined by the Cournot auctioneer, a more realistic setting would be one in which, upon observing the state of demand θ, the firms compete in prices, taking their previously determined production or capacity levels as given. In contrast to the problems that arise in Kreps and Scheinkman (1983), Maggi (1996) has shown that a short-run price equilibrium in such a game exists if the firms' products are sufficiently differentiated and if the capacity constraints are not strict, i.e. allow for production above capacity at a higher marginal cost. While we have not formally analyzed this kind of model, we conjecture that in its reduced form, the (first-stage) quantity game would have the same features as the model presented here, and that equilibrium prices would behave as in the model above.

5.2 Competition

If, in the example above, σ is decreased from 1 to 0, i.e. as one moves from homogeneous to differentiated to independent products, the effects of liquidity constraints on the output market become less pronounced. If firm 1 is an independent monopolist, $q_1(w_0)$ is still U-shaped, but its decrease in output below the monopoly level is smaller (in relative terms) than if a competitor is present. Thus, product market competition does not affect any of our results qualitatively, but rather amplifies them: if firm 1 reduces its output as a consequence of financial constraints, and firm 2 increases its output in response to this, then this second effect leads to further reduction of firm 1's output. Clearly, this additional effect depends on how competitive the market is.

Kovenock and Phillips (1997) find that debt has a significant effect on the product market only in relatively concentrated industries. Their explanation for this is that concentrated industries are less competitive, hence there is more scope for managers to spend cash flow in the output market in wasteful ways. In this case, debt is a useful disciplining device (cf. Jensen 1986). In the light of the previous discussion, an alternative explanation is that with free entry, high concentration may not be a sign of a lack of competition, but rather the result of *intense* competition, as emphasized by Sutton (1991). With this interpretation, Kovenock and Phillips' finding is as expected: the effects of debt on product market behavior are larger the more competitive the industry is.[8]

[8] While establishing this argument formally in the context of our model might be difficult, it can be illustrated using a simple circular road model (cf. Tirole 1988, pp. 282-284, for the basic model): suppose n firms are located at equal distances around a circle of circumference and consumer density one. If all firms have the same marginal cost c and fixed cost f, then, in the free-entry equilibrium, $n = \sqrt{t/f}$ firms will be in the market. How is firm i affected in this market if its marginal costs are higher than c, e.g. because of financial constraints? If its costs are c_i, then the optimal price in the short run, facing neighbors that charge p, is $p_i = p + (c - c_i)/t$, implying that its demand is $x_i = 1/(2n) + (c - c_i)/(4t)$. This means: if the market becomes more competitive (decrease in t), concentration increases (decrease in n), while at the same time, demand and profits of a firm with a cost disadvantage respond more sensitively to changes in own costs.

5.3 Predation

According to the "long purse" story of predation, a financially strong firm can drive a financially weak firm out of the market by inflicting short-term losses on it, even if the firms are otherwise similarly strong on the output market. Long regarded as suffering from inconsistencies, this theory was given a rigorous formal foundation by Bolton and Scharfstein (1990). The essential features of that model are also present in ours: even if firm 1 has a healthy position in the output market, the lender cannot completely protect it against predation: because of the agency problems in the lender-borrower relationship, the threat of liquidation is necessary.

In a sense, financial predation already occurs in our model, because firm 2 produces more than it would if firm 1 were not financially constrained. On the other hand, firm 2 is only a passive Cournot competitor: it increases its output *in response* to firm 1's reduction in output, which in turn is a consequence of firm 1's costs of borrowing. Technically, this situation does not seem much different from one in which firm 1 just has higher costs for some other reason. Thus, asymmetric market shares and prices are natural consequences of differences in financial status and not as such evidence of predation.

Firm 2 plays a passive role in our model because it does not benefit from firm 1's bankruptcy. Suppose instead that firm 2's future payoff if it becomes a monopolist (denote this by π_2^m) exceeds the profit it would get in duopoly (π_2^d). Firm 2's profit function now becomes

$$\int_{\underline{\theta}}^{\bar{\theta}} R_2(q_1, q_2, \theta) f(\theta) \mathrm{d}\theta - cq_2 + L_1(q_1, q_2)\pi_2^m + [1 - L_1(q_1, q_2)]\pi_2^d, \tag{15}$$

where

$$L_1(q_1, q_2) := \int_{\underline{\theta}}^{\widehat{\theta}} \frac{D(q_1) - R^1(q_1, q_2), \theta}{\pi_2} f(\theta) \mathrm{d}\theta$$

is the probability that firm 1 is liquidated (cf. Proposition 1). Firm 2's first-order condition then is:

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial R_2(q_1, q_2, \theta)}{\partial q_2} f(\theta) \mathrm{d}\theta - c - \frac{\pi_2^m - \pi_2^d}{\pi_2} \int_{\underline{\theta}}^{\widehat{\theta}} \frac{\partial R^1(q_1, q_2, \theta)}{\partial q_2} f(\theta) \mathrm{d}\theta,$$

Since the last term is positive, firm 2's reaction curve shifts out if firm 2 stands to gain from the exit of firm 1. The outcome now is closer to the intuitive meaning of "predation":

to increase the probability of firm 1's exit, firm 2 behaves more aggressively than it would if firm 1's presence in the market was certain. Notice, however, that firm 1 can never be driven out of the market right away if w_0 is positive, as firm 1 always has the option just to produce $q_1 = w_0/c$, i.e. not to borrow. Moreover, as before, driving the rival into certain bankruptcy does not ensure liquidation and exit from the market.

An interesting extension of our model would be one in which this period's net earnings are the retained earnings in the next period. This would expand firm 2's options: it could increase output, not to drive firm 1 out of the market quickly, but just to hold its earnings to a low level. This would increase firm 1's financing needs in the next period, lead to further reduction of its output, and possibly to bankruptcy in some later period. Recall from Proposition 4, though, that if firm 1 is already severely financially constrained, predatory behavior by firm 2 aimed at weakening firm 1's financial position even further could backfire, as it could lead firm 1 to produce *more* than before, reducing firm 2's profit (cf. Proposition 5).

Alternatively, if firm 1's constraints are serious, firm 2 could just decide to "wait and see": if this period's low output leads to low retained earnings and even lower output, firm 1 might, over time, be forced to exit the market without any "help" from rivals.

6 Conclusion

It is well known in the corporate finance literature that if the threat of liquidation is an inefficient but necessary element of a debt contract, higher costs of debt financing lead to underinvestment, i.e. "softer" output market behavior, *in the absence of additional agency problems* regarding the choice of the investment. Independently, the industrial organization literature has explored how debt affects firms' output market behavior by changing incentives *ex post*, i.e. by inducing risk–shifting. It is not well understood, however, how ex-ante and ex-post incentives interact. The study of this interaction is the first contribution of this paper. With standard debt contracts, models which allow for both bankruptcy costs and risk shifting quickly become analytically intractable. As it turns out, our optimal contracting approach does not lead to further complications, but in fact drastically simplifies the problem.

In our model, production costs are the critical link between a firm's financing and output decisions. With zero variable costs and an optimal debt contract, the firm's output is independent of its financial status. In contrast, if production is costly and these costs must be incurred before any revenue can be earned, they must not exceed the firm's internal and borrowed funds. In this situation, the firm's output choice is determined by the ex-ante costs of borrowing, and the result is softer output market behavior.

The second contribution of this paper is to characterize output market behavior as a function of financial status. While a financially constrained firm produces less than an unconstrained firm, its output is not increasing, but rather U-shaped, in the degree of its financial constraints, measured by its net worth. While for positive levels of net worth, output decreases as a firm's financial situation gets worse, output increases again if net worth is sufficiently negative: if not all borrowed funds are invested in production, output must be large enough to generate the earnings that allow the firm to repay its loan.

Finally, our results suggest that the effects of financial constraints on a firm's output or investment behavior are reinforced, but not fundamentally altered, by oligopolistic interaction, in contrast to what has been suggested elsewhere. The reinforcement effect of competition is a natural consequence of our Cournot setting, rather than the result of explicit predatory behavior on part of unconstrained competitors. This raises the issue of what "financial predation" means conceptually, what forms it can take both in a static and in a dynamic context, and how it can be measured and distinguished from more innocent forms of competition. We leave these questions for future research.

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Appendix: Proofs

Lemma 1 *The program defined by (5)-(7) and (2) has a unique solution.*

Proof: Since R^i is strictly concave in q_i, it follows that the program has a unique solution if D as defined by (7) is convex in q_1 for any given q_2. That is the case: Write (7) as

$$\int_{\underline{\theta}}^{\widehat{\theta}} R^1(\mathbf{q},\theta)f(\theta)\mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta})D = cq_1 - w_0,$$

where $\mathbf{q} = (q_1, q_2)$, and differentiate with respect to q_1:

$$\int_{\underline{\theta}}^{\widehat{\theta}} R_1^1(\mathbf{q},\theta)f(\theta)\mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta})\frac{\partial D}{\partial q_1} + \frac{\partial \widehat{\theta}}{\partial q_1}f(\widehat{\theta})[R^1(\mathbf{q},\widehat{\theta}) - D] = c,$$

where the last term on the l.h.s. vanishes because $R^1(\mathbf{q},\widehat{\theta}) = D$.Differentiate again with respect to q_1 to obtain

$$\int_{\underline{\theta}}^{\widehat{\theta}} R_{11}^1(\mathbf{q},\theta)f(\theta)\mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta})\frac{\partial^2 D}{\partial q_1^2} + \frac{\partial \widehat{\theta}}{\partial q_1}\left[R_1^1(\mathbf{q},\widehat{\theta}) - \frac{\partial D}{\partial q_1}\right] = 0, \tag{16}$$

From $R^1(\mathbf{q},\widehat{\theta}) = D(q)$, we have $R_1^1(\mathbf{q},\widehat{\theta}) + R_\theta(\mathbf{q},\widehat{\theta})\partial\widehat{\theta}/\partial q_1 = \partial D/\partial q_1$ and hence $\partial\widehat{\theta}/\partial q_1 = (\partial D/\partial q_1 - R_1^1)/R_\theta^1$. This means that the third term in (16) is negative. Since by assumption $R_{11}^1 < 0$, the first term in (16) is negative as well, which implies that $\partial^2 D/\partial q_1^2$ must be positive. ■

Proof of Proposition 3: Substituting $R^1(\mathbf{q},\widehat{\theta})$ for D into (5) and (7) and setting up a Langrangian for firm 1 leads to the first-order conditions

$$\begin{aligned}
E[R_1^1(\mathbf{q},\theta)] - R_1^1(\mathbf{q},\widehat{\theta}) + \lambda\left[\int_{\underline{\theta}}^{\widehat{\theta}} R_1^1(\mathbf{q},\theta)f(\theta)\mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta})R_1^1(\mathbf{q},\widehat{\theta}) - c\right] &= 0 \qquad (17)\\
-R_\theta(\mathbf{q},\widehat{\theta}) + \lambda\mathrm{Prob}(\theta \geq \widehat{\theta})R_\theta(\mathbf{q},\widehat{\theta}) &= 0 \qquad (18)\\
E[R_2^2(\mathbf{q},\theta)] - c &= 0 \qquad (19)
\end{aligned}$$

and (7). Using (18), eliminate $\lambda = 1/\mathrm{Prob}(\theta \geq \widehat{\theta})$ in (17), and the optimal $\mathbf{q}$ and $\widehat{\theta}$ are the solution to the system

$$\begin{aligned}
g(\mathbf{q},\widehat{\theta},w_0) &= \int_{\underline{\theta}}^{\widehat{\theta}} R_1^1(\mathbf{q},\theta)f(\theta)\mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta})E[R_1^1(\mathbf{q},\theta)] - c = 0 \qquad (20)\\
h(\mathbf{q},\widehat{\theta},w_0) &= \int_{\underline{\theta}}^{\widehat{\theta}} R^1(\mathbf{q},\theta)f(\theta)\mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta})R^1(\mathbf{q},\widehat{\theta}) - cq + w_0 = 0 \qquad (21)\\
k(\mathbf{q},\widehat{\theta},w_0) &= E[R_2^2(\mathbf{q},\theta)] - c = 0, \qquad (22)
\end{aligned}$$

For any w_0, the firm's optimal output is q_1 if there exists a $\widehat{\theta}$ such that q_1, q_2,w_0 and $\widehat{\theta}$ jointly solve (20)-(22). It is straightforward to establish that both $(w_0, \mathbf{q}, \widehat{\theta}) = (w^*, q^*, q^*, \underline{\theta})$ and $(w_0, \mathbf{q}, \widehat{\theta}) = (-\pi^*, q^*, q^*, \bar{\theta})$ are such solutions, since in both cases (20) reduces to the first-order condition of an unconstrained firm. If $w_0 \in (-\pi^*, w^*)$, in contrast, we have $\underline{\theta} < \widehat{\theta} < th\bar{e}ta$, and then (20) places relatively larger weight on the states $\theta < \widehat{\theta}$ with low R_1^1. Since $R_{12}^i < 0$, it then follows that the solution to (20) and (22) must satisfy $q_1 < q^* < q_2$. ■

Proof of Proposition 4: 1. From the proof of Proposition 3, we have $q_1 = q^*$ for both $w_0 = -\pi^*$ and $w_0 = w^*$, which fixes the endpoints of the function $q_1(w_0)$. Next, we determine the slope of $q_1(w_0)$. The partial derivatives of g, h and k with respect to q_1, q_2, $\widehat{\theta}$ and w_0 are (arguments omitted)

$$
\begin{aligned}
g_1 &= \text{Prob}(\theta \geq \widehat{\theta}) E[R_{11}^1(\mathbf{q}, \theta)] + \int_{\underline{\theta}}^{\widehat{\theta}} R_{11}^1(\mathbf{q}, \theta) f(\theta) \mathrm{d}\theta \\
g_2 &= \text{Prob}(\theta \geq \widehat{\theta}) E[R_{12}^1(\mathbf{q}, \theta)] + \int_{\underline{\theta}}^{\widehat{\theta}} R_{12}^1(\mathbf{q}, \theta) f(\theta) \mathrm{d}\theta \\
g_{\widehat{\theta}} &= -f(\widehat{\theta}) \{E[R_1^1(\mathbf{q}, \theta)] - R_1^1(\mathbf{q}, \widehat{\theta})\} \\
h_1 &= \int_{\underline{\theta}}^{\widehat{\theta}} R_1^1(\mathbf{q}, \theta) f(\theta) \mathrm{d}\theta + \text{Prob}(\theta \geq \widehat{\theta}) R_1^1(\mathbf{q}, \widehat{\theta}) - c \\
h_2 &= \int_{\underline{\theta}}^{\widehat{\theta}} R_2^1(\mathbf{q}, \theta) f(\theta) \mathrm{d}\theta + \text{Prob}(\theta \geq \widehat{\theta}) R_2^1(\mathbf{q}, \widehat{\theta}) \\
h_{\widehat{\theta}} &= \text{Prob}(\theta \geq \widehat{\theta}) R_\theta(\mathbf{q}, \widehat{\theta}) \\
k_1 &= \mathrm{E}[R_{12}^2] \\
k_2 &= \mathrm{E}[R_{12}^2] \\
g_w &= 0, \qquad h_w = 1 \qquad \text{and} \qquad k_w = 0
\end{aligned}
$$

Using $g = 0$, h_q can also be written as $-\text{Prob}(\theta \geq \widehat{\theta})\{E[R_1^1(\mathbf{q}, \theta)] - R_1^1(\mathbf{q}, \widehat{\theta})\}$. According to Cramer's rule, we have $dq_1/dw_0 = \det(\mathbf{M})/\det(\mathbf{M_1})$, where

$$
\mathbf{M} = \begin{pmatrix} g_1 & g_2 & g_{\widehat{\theta}} \\ h_1 & h_2 & h_{\widehat{\theta}} \\ k_1 & k_2 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{M_1} = \begin{pmatrix} 0 & g_2 & g_{\widehat{\theta}} \\ -1 & h_2 & h_{\widehat{\theta}} \\ 0 & k_2 & 0 \end{pmatrix}
$$

Since $\det(\mathbf{M_1}) = -k_2 g_{\widehat{\theta}}$ and $k_2 < 0$, $\det(\mathbf{M_1})$ has the same sign as $E[R_1^1(\mathbf{q}, \theta)] - R_1^1(\mathbf{q}, \widehat{\theta})$.

2. We now show that $q_{ww} > 0$ when $q_w = 0$, which implies that $q(w_0)$ has a unique extremal point, which is a minimum. Differentiate (20)-(22) twice with respect to w_0 to obtain

$$\mathbf{M}\begin{pmatrix} \frac{d^2 q_1}{dw_0^2} \\ \frac{d^2 q_2}{dw_0^2} \\ \frac{d^2 \widehat{\theta}}{dw_0^2} \end{pmatrix} = -\begin{pmatrix} \frac{dg_{\widehat{\theta}}}{dw_0}\widehat{\theta}_w + \frac{dg_1}{dw_0}\frac{dq_1}{dw_0} + \frac{dg_2}{dw_0}\frac{dq_2}{dw_0} \\ \frac{dh_{\widehat{\theta}}}{dw_0}\widehat{\theta}_w + \frac{dh_1}{dw_0}\frac{dq_1}{dw_0} + \frac{dh_2}{dw_0}\frac{dq_2}{dw_0} \\ \frac{dk_1}{dw_0}\frac{dq_1}{dw_0} + \frac{dk_2}{dw_0}\frac{dq_2}{dw_0} \end{pmatrix} \tag{23}$$

When $dq_1/dw_0 = 0$, we also have $dq_2/dw_0 = 0$, and $g_{\widehat{\theta}} = h_1 = 0$, according to Step 2. Then $\det(\mathbf{M})$ reduces to $-h_{\widehat{\theta}}(g_1 k_2 - g_2 k_1)$, which is negative because $R^1_{11}R^2_{22} > R^1_{12}R^2_{12}$. On the right-hand side of (23), all terms containing dq_i/dw_0 drop out, and then (again using Cramer's rule) we have $d^2q_1/dw_0^2 = k_2 h_{\widehat{\theta}}(dg_{\widehat{\theta}}/dw_0)\widehat{\theta}_w/\det(\mathbf{M})$, which has the same sign as $(dg_{\widehat{\theta}}/dw_0)\widehat{\theta}_w$. Here, we have

$$\frac{\mathrm{d}g_{\widehat{\theta}}}{\mathrm{d}w_0} = g_{1\widehat{\theta}}\frac{dq_1}{dw_0} + g_{\widehat{\theta}\widehat{\theta}}\widehat{\theta}_w + g_{\widehat{\theta}w} = g_{\widehat{\theta}\widehat{\theta}}\widehat{\theta}_w,$$

since the first and third terms vanish. Thus, d^2q_1/dw_0^2 has the same sign as $g_{\widehat{\theta}\widehat{\theta}}(\widehat{\theta}_w)^2$, where

$$g_{\widehat{\theta}\widehat{\theta}} = -f'(\widehat{\theta})\left[E[R^1_1(\mathbf{q},\theta)] - R^1_1(\mathbf{q},\widehat{\theta})\right] + f(\widehat{\theta})R_{q\theta}(\mathbf{q},\widehat{\theta}),$$

which in turn is positive because the term in [ ] vanishes when $dq_1/dw_0 = 0$.

3. Finally, we show that $\widetilde{w} < 0$ by proving that $q_1(w_0)$ must be increasing at $w_0 = 0$, from which the claim follows because $q_1(w_0)$ has a unique minimum. Define $\hat{h}(\mathbf{q})$ as L's profit as a function of q_1 and q_2 at $w_0 = 0$, holding $\widehat{\theta}$ fixed at the level where (7) is satisfied. That is,

$$\hat{h}(\mathbf{q}) = \int_{\underline{\theta}}^{\widehat{\theta}} R^1(\mathbf{q},\theta)f(\theta)\mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta})R^1(q_1,q_2,\widehat{\theta}) - cq_1$$

Since $\hat{h}(0,q_2) = 0$ and by construction $\hat{h}(q(0),q_2) = 0$, and since $\hat{h}$ is concave in q, it follows that

$$\frac{\partial h(q(0),q_2)}{\partial q_1} = \int_{\underline{\theta}}^{\widehat{\theta}} R^1_1(\mathbf{q},\theta)f(\theta)\mathrm{d}\theta + \mathrm{Prob}(\theta \geq \widehat{\theta})R^1_1(\mathbf{q},\widehat{\theta}) - 1 < 0.$$

But this derivative equals h_1 according to (23), and therefore equals $-\mathrm{Prob}(\theta \geq \widehat{\theta})\{E[R^1_1(\mathbf{q},\theta)] - R^1_1(\mathbf{q},\widehat{\theta})\}$. Thus, if $h_1 < 0$ at $w_0 = 0$, then we must have $\mathrm{E}[R^1_1(\mathbf{q},\theta)] > R^1_1(\mathbf{q},\widehat{\theta})$, implying that $q_1(w_0)$ must be upward-sloping at $w_0 = 0$. ■