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ABSTRACT

A Multi-Task Principal-Agent Approach to Organizational Form*

This Paper studies the choice of organizational forms in a multi-task principal-agent model. We compare a functional organization in which the firm is organized into functional departments such as marketing and R&D to a product-based organization in which the firm is organized into product lines. Managers' compensation can be based on noisy measures of product-line profits. Measures of a functional area's contribution to total profits are not available, however. This effect favours the product organization. However, if there are significant asymmetries between functional area contributions to organizational success and cross-product externalities within functions, organizing along functional lines may dominate the product organization. We also consider the effects of diseconomies of span of control and cross-functional complementarities. Diseconomies of span of control sometimes favours the product organization and sometimes favour the functional organization. Cross-functional complementaries tend to make the product organization relatively more profitable.

JEL Classification: D20, L00, L10, L20, M20
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NON-TECHNICAL SUMMARY

A significant part of the business community's thinking about the internal structure of firms revolves around designing and implementing 'organizational charts'. Such charts display three important dimensions of the firm's internal design: the number of levels in the hierarchy, the span of control of managers at each of these levels, and the main criterion according to which the organizational 'tree' is divided into branches. This Paper examines this last dimension more closely.

We compare the relative profitability of a firm when it organizes along product lines to that when it organizes along functional lines. To focus on the choice of the organization's 'organizing dimension', our model does not consider the other key elements of the organizational structure problem: the number of levels in the hierarchy and the span of control of managers. For this reason, this Paper does not provide a complete theory of endogenous organizational structure. Our objective instead is to identify economic forces that systematically favour organizing along a product dimension versus a functional dimension.

We find that the product-based organization has an inherent cost advantage over the functional organization due to the inability of the firm to measure functional area profit contributions accurately. This inability to observe functional area profits means that the reward of a manager in either organization must be tied to the profits of product lines and not functional areas. A functional manager's compensation must be tied to profit measures of all product lines in order to elicit positive effort on all tasks under the manager's control. On the other hand, a product manager's compensation may be tied to the profits of their own product line only in order to induce positive effort on all tasks necessary to the success of their product. Thus, holding effort levels fixed, managers bear more risk in a functional organization than in a product organization. This 'risk-bearing effect' would not arise if functional area profit contributions could be measured as accurately as product-line profitability. Moreover, the reward of a product division manager can be tied to the profits of other product divisions in order to provide the manager with some insurance, since the profits of the other divisions do not need to enter the compensation contract as an instrument to induce effort. Since the functional organization must use all profit measures simply to induce the desired level of effort in the managers' tasks, it has no such freedom to provide insurance through the compensation contract.

We then consider the effect of various asymmetries on the relative performance of the two organizations. We show that asymmetries between functions as well as asymmetries in the function/product mix improve the

relative performance of the functional form. On the other hand, asymmetries between product lines do not affect the relative performance of the two types of organization. Next, we introduce cross-functional externalities to capture the idea that effort expended on, say, R&D for one product line can also be useful for the other product line. Not surprisingly, such externalities improve the relative performance of the functional form. More interestingly, we can show that if cross-functional externalities are large enough, then functional asymmetries can actually make the functional organization more profitable than a product organization. Finally, we consider the possibility that there are functional area complementarities so that co-ordination of functional activities becomes important. We demonstrate that, under certain assumptions, as cross-functional co-ordination becomes more valuable, a product organization tends to become more profitable than a functional organization.

1 Introduction

A significant part of the business community's thinking about the internal structure of firms revolves around designing and implementing "organizational charts." These schematic representations are part of every consultant's toolbox and have found their ways into most annual reports and quite a few business school cases. Such charts display three important dimensions of the firm's internal design: the number of levels in the hierarchy, the span of control of managers at each of these levels, and the main criterion according to which the organizational "tree" is divided into branches. This paper examines this last dimension more closely.

We compare the relative profitability of a firm when it organizes along product lines to that when it organizes along functional lines. Our firm produces two products, each of which requires the contributions of two functional areas (e.g., marketing and manufacturing). Under either organizational form, a manager must supervise two activities. In a product-based organization, the head of a each product division must supervise the two functional activities required to ensure the success of her product line. In a functional organization, the head of each functional division must supervise the function's activities on behalf of each of the firm's two products. Each division head is modelled as a risk-averse agent who must be motivated to expand effort on supervising each activity under her control. The principal can induce such efforts by tying the manager's reward to the imperfectly measured profits of each product line.

To focus on the choice of the organization's "organizing dimension" (functional versus product), our model does not consider the other key elements of the organizational structure problem: the number of levels in the hierarchy and the span of control of managers. For this reason, this paper does not provide a complete theory of endogenous organizational structure. Our objective instead is to identify economic forces that systematically favor organizing along a product dimension versus a functional dimension.

The key findings of our analysis are as follows. When all products and all functions enter symmetrically into the profit and effort functions, the product-based organization involves a lower agency cost than the functional organization. The intuition is that tying a product manager's reward to the profits of her own product line is sufficient to induce symmetric levels of efforts in the supervision of each of the two activities under her control. By contrast, the pay of functional managers must be tied to all profit measures to elicit positive effort on all tasks. Thus, holding effort levels fixed, managers bear more risk in a functional organization than in a product organization. This "risk-bearing effect" would not arise if functional area profit contributions could be measured as accurately as product-line profitability. Moreover, the reward of a product division manager can be tied to the profits of other product division in order to provide her with some insurance. In a functional organization, all parameters of the compensation contract must be used to induce the desired levels of efforts so that such additional insurance cannot be offered. Hence, because of these two effects, throughout our analysis the product organization has an inherent advantage due to the inability of the firm to measure functional area profit contribution.

We then consider the effect of various asymmetries on the relative performance of the two organizations. We show that asymmetries between functions as well as asymmetries in the function/product mix improve the relative performance of the functional form. On the other hand, asymmetries between product lines do not affect the relative performance of the two types of organization. Next, we introduce cross-functional externalities to capture the idea that effort expended on, say, R&D for one product line can also be useful for the other product line. Not surprisingly, such externalities improve the relative performance of the functional form. More interestingly, we can show that if cross-functional externalities are large enough, then functional asymmetries can actually make the function-based organization more profitable than a product-based firm. Finally, we consider the possibility that there are functional area complementarities so that coordination of functional activities becomes important. We parameterize the degree of complementarities using a CES specification of the profit function and demonstrate that as cross-functional coordination becomes more valuable, a product-based organization tends to become more profitable than a functional organization.

The broad theme of this paper is that alternative organizational forms can affect profitability differently because they alter the nature of optimal incentive contracting. Given this, one might naturally wonder how important the incentive problem is for the choice of organizational form and why it worth constructing a model that focuses on the role played by incentive contracting in determining the effectiveness of an organizational form. Our response to this question is twofold. First, from an empirical perspective, the selection of organizational and incentive structures are often interrelated. Real world firms often make major changes in organizational form and incentive compensation structure at the same time. For example, when Citibank reorganized its corporate banking business in the mid 1990s, it dramatically changed both the structure of its organization and its incentive compensation program for senior managers.[1] The simultaneous choice of organizational structure and incentive systems suggests that these two elements of the firm's "organizational strategy" are often dependent on each other for success: the optimal incentive structure depends on organizational form and the benefits from changing organizational form can only be realized if incentives are adjusted.

Second, from a theoretical perspective, almost any study of organizational structure using

[1] Baron and Besanko (1998) studies this example in detail.

microeconomic theory runs into the difficulty that organizational structure would be irrelevant in the absence of some sort of hidden action or hidden information problems inside the organization. Without such problems, managers in the organization could be ordered make operating and resource allocation decisions to maximize corporate profit, and organizational structure would be irrelevant. Since incentive contracting models have become an important part of the economist's toolkit for analyzing solutions to hidden action and hidden information problems, it seems natural to explore whether incentive contracting models can deliver intutitively appealing insights about organizational structure. The purpose of this paper is push this inquiry forward in the context of an organization in which there are hidden action problems but no issues involving private information.

Our paper is only superficially related to the vast literature on the respective merits of the U-form and the M-form. Although the U-form is organized along functional lines and the M-form is often made of separate product or area divisions, these two organizations are also assumed to differ in their levels of centralization. Indeed, this greater centralization is at the heart of the usual comparisons between the two organizational structures[2]. While the U-form allows for better coordination across product lines, closer monitoring of top managers, and economies of scale in functional activities, the M-form allows decisions to be based on more precise information and is better designed to avoid informational bottlenecks. All of these effects are absent from our model. Our two organizational structures differ only in the way in which activities are grouped under the control of divisional managers. Otherwise they both involve the same number of hierarchical tiers, the same spans of control, and the same degree of decentralization. Moreover, in the absence of externalities, our framework explicitly avoids any kind of team effects.

This modelling approach is similar to that of Aghion and Tirole (1995) who, like us, consider two products and two functions leading to four activities that must be allocated pairwise between two managers. However, they do not consider monetary incentives, focusing instead on career concerns. Furthermore, in their model, the choice of organization depends on the trade-off between the greater economies of scale (in training) provided by the U-form and the lighter "overload" associated with the M-form. Both factors are absent in our model.

Since our two organizational forms only differ in the way activities are allocated between managers, it is related to the literature on "task assignment" (e.g. Holmstrom and Milgrom, 1991 and Itoh, 1991 and 1992). Indeed we will argue in Section 2 that our benchmark result for symmetric organizations has the same flavor as Holmstom and Milgrom's "task specialization" principle which states that each task ought to be the responsibility of a single agent.

[2]See, for example, Burton and Obel (1988), Holmstrom and Tirole (1989) or Williamson (1975).

The rest of the paper is organized as follows. Section 2 presents the model as well as our basic result for symmetric functions and products. Section 3 examines the effect of asymmetries on the relative performance of the two organizational forms. Section 4 studies the impact of cross-product externalities. Section 5 considers the effect of diseconomies of span. Section 6 considers the possibility of cross-functional complementarities. Section 7 briefly considers the economics of alternative organizational forms that could be analyzed in our framework. Section 8 summarizes and concludes. The proofs of all propositions are in Section 9, the Appendix.

2 The Model

2.1 Basic Model Set-Up

We consider a hypothetical firm that consists of a risk-neutral owner and two risk-averse agents, hereafter referred to as division managers. Our "firm" could either be a stand-alone firm or an autonomous business unit within a larger company. The question we address is how the owner of the firm would want to structure her firm to obtain the best possible performance from its division managers.

The firm sells two products, 1 and 2. Two functional areas, X and Y, contribute to the success of these products. One can, for example, think of these functional areas as R&D and marketing.

The profit π_i generated by product i depend on x_i and y_i, the efforts expended on functions X and Y for this product, and we assume that this relationship is linear

$$\pi_i = \beta_i x_i + \gamma_i y_i + \theta\beta_j x_j + \xi\gamma_j y_j \text{ for } i = 1, 2, i \neq j.$$

The quantities x_i and y_i have several possible interpretations. They could represent the effort that a division manager devotes to monitoring or motivating subordinates, determining the best mix of operating decisions within the areas under the manager's control, or identifying the best set of investment projects.[3] The parameters β_i and γ_i measure the marginal impact that functional efforts x_i and y_i have on the profit of product i, and the parameters $\theta, \xi \in [0, 1]$ indicate cross-product externalities within a particular functional area. These impact parameters capture the idea that marketing or R&D efforts expended on behalf of one product can often benefit the firm's other products as well.

We assume the firm's accounting system generates a verifiable signal $\widetilde{\pi}_i$ of the profitability of product i. This signal equals the actual product profit plus an additive, mean-zero measurement error $\widetilde{\varepsilon}_i$, i.e.,

[3] We ignore the problem of motivating the agents under the manager's control. One can, if one wishes, just think of these agents as infinitely risk-averse.

$$\widetilde{\pi}_i = \pi_i + \widetilde{\varepsilon}_i \text{ for } i = 1, 2.$$[4]

We assume that $(\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_2)$ is drawn from a bivariate normal distribution with a variance-covariance matrix Ω given by

$$\Omega = \begin{pmatrix} \sigma^2 & r\sigma^2 \\ r\sigma^2 & \sigma^2 \end{pmatrix},$$

where σ^2 is the variance of measured profit, and $r \in [-1, 1]$ is the correlation between measured product-line profits. Note that if $r = -1, Var(\widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2) = 0$, and thus total profit $\pi_1 + \pi_2$ can be measured without noise. We will use this interpretation below.

The owner of the firm must choose an organizational form. Once that organizational structure is chosen, the owner then chooses contracts to motivate managers within that structure. The organizational structure choice is a task assignment problem: which two agents are responsible for the four tasks x_1, y_1, x_2, y_2 that must be performed to make the organization profitable. To develop intuition about the economics of organizational forms employed by real firms, we focus on two specific allocations of tasks to agents: a functional organization and a product-based organization. In a *functional organization*, all activities related to the same function are under the control of one division manager. That is, the vector $v_X^T \equiv (x_1, x_2)$ of efforts of function X are determined by one division manager, while the vector $v_Y^T \equiv (y_1, y_2)$ of efforts of function Y are determined by another.[5] In a *product-based* organization, each division manager is responsible for the two functional activities relating to his product line. The division manager for product line i thus chooses a vector $z_i^T \equiv (x_i, y_i), i = 1, 2$, of functional area efforts on behalf of product i. Figures 1 and 2 depict these two organizational forms.

Two remarks are in order about the set of organizational structures we consider in this analysis:

- Throughout this analysis, we hold the firm's employment fixed at two agents. Dropping this assumption would allow us to consider other organizational structures. For example, the firm could disaggregate the choice of efforts even more by hiring four agents and assigning one agent to choose x_1, another agent to choose y_1, and so forth. By holding firm employment fixed at two agents, we focus attention on the trade-offs involved in the assignment of organizational tasks for a given amount of managerial authority. In the last section of the paper, we briefly consider the economics of a more disaggregated organization.

[4]Alternatively, we could interpret $\widetilde{\varepsilon}_i$ as the sum of measurement error and product market uncertainty.

[5]Throughout, the superscript "T"denotes a matrix transpose.

- Both organizational forms we study involve task specialization: i.e., one agent supplies all of the effort for two tasks (e.g., x_1 and y_1) while the other agent supplies all of the effort for the other two tasks (e.g., x_2 and y_2). We do not allow both agents to supply some effort to each of the four tasks. We justify this by assuming that prospective division managers have specialized know-how: the firm can hire product specialists or functional specialists, but it cannot find managers who can productively do all four tasks simultaneously.[6] In the final section of the paper, we return to the issue of specialization and compare the specialized structures considered here to the performance of a firm composed of task generalists.

The owner of the firm cannot directly monitor the effort levels of her divisional managers. We assume that the only variables that are observable and verifiable are the noisy signals $\tilde{\pi}_i$ of product-line profitability. In particular, signals of the profit contribution of any particular functional area are not available. The assumption that the firm can measure product-line profit contributions but not functional area profit contributions is meant to captures the generally accepted idea that in most firms it is *easier* to measure a product's contribution to total profits than it is to measure a functional area's contribution to total profits.[7]. The key difficulty in generating reliable product-line profit data is the assignment of costs to different products. This is not an easy task, but the problem can be minimized by grouping together products that are linked on the cost side and by using accounting techniques, such as activity-based costing, to assign costs to different products. By contrast, computing the "profit" of a functional division is a more daunting task because of the absence of market mechanism to determine the relevant revenues. Despite advances in accounting methods, few firms have developed reliable measures of functional area performance.

For simplicity we further restrict the compensation contracts to be linear functions of the product-line profit levels[8]. Thus, the compensation $\widetilde{W}_i$ received by the manager of division i is

$$\widetilde{W}_i \quad = \quad a_{i0} + \tilde{\pi}^T a_i \quad \text{for } i \in \{1, 2\},$$

[6]This eliminates the possibility that a manager can take hidden actions outside of his area of authority.

[7]See Tirole (1989), pp. 47-48 or Holmstrom and Tirole (1989), pp. 125-126 for a similar argument. For example, commenting on functional organizations Tirole (1989) states: "To control the functional divisions, the top management can use one of two methods: rewarding each functional division for good performance (i.e., basing incentive schemes on output) and supervising the divisions directly in order to assess individual contributions (i.e., to measure inputs). The first method clearly faces the accounting problem of separating the contributions of the various divisions, which may be hard to measure. This gives rise to an Alchian-Demsezt type team problem. The second method can be employed only if the firm is small."

[8]This can be justified by assuming that agents choose effort in continuous time to control the drift vector of a Brownian motion process and in which the agent can observe his accumuated performance before acting at any instant in time. Holmstrom and Milgrom (1987) show that in this context, the optimal wage contract for an agent with constant absolute risk aversion is a linear function of the final observable outcome.

$$\widetilde{W}_i = \alpha_{i0} + \widetilde{\pi}^T \alpha_i \quad \text{for } i \in \{X, Y\},$$

where $a_i^T \equiv (a_{i1}, a_{i2})$, $\alpha_X^T \equiv (\alpha_{X1}, \alpha_{X2})$, $\alpha_Y^T \equiv (\alpha_{Y1}, \alpha_{Y2})$, $\widetilde{\pi}^T \equiv (\widetilde{\pi}_1, \widetilde{\pi}_2)$, and a_{i0}, α_{i0} are scalars.

Division managers are risk averse and effort averse. The disutility of effort for a divisional manager is quadratic and is given by

$$\Delta_i \equiv \begin{cases} \frac{1}{2} z_i^T D z_i & \text{for } i \in \{1,2\} \\ \frac{1}{2} v_i^T D v_i & \text{for } i = \{X, Y\}. \end{cases}$$

where $D \equiv \begin{bmatrix} 1 & \delta \\ \delta & 1 \end{bmatrix}$, and $\delta \in [0, 1)$ measures the extent of *diseconomies of span*, i.e., the extra cost that results when a manager must split his time and attention between different tasks. In the limit, as $\delta \to 1$, we have a multi-task effort allocation model in which disutility depends on the total effort exerted by the division manager.

The expected utility function of a manager is:

$$EU_i \equiv E(\widetilde{W}_i) - \frac{\rho}{2} Var(\widetilde{W}_i) - \Delta_i \text{ for } i \in \{1, 2\} \text{ or for } i \in \{X, Y\},$$

where $\rho > 0$ measures the risk aversion of the managers.[9] Letting $Q_1 \equiv \begin{bmatrix} \beta_1 & \theta\beta_1 \\ \gamma_1 & \xi\gamma_1 \end{bmatrix}$, $Q_2 \equiv \begin{bmatrix} \theta\beta_2 & \beta_2 \\ \xi\gamma_2 & \gamma_2 \end{bmatrix}$, $R_X \equiv \begin{bmatrix} \beta_1 & \theta\beta_1 \\ \theta\beta_2 & \beta_2 \end{bmatrix}$, $R_Y \equiv \begin{bmatrix} \gamma_1 & \xi\gamma_1 \\ \xi\gamma_2 & \gamma_2 \end{bmatrix}$, we can express EU_i in matrix notation as follows

$$EU_i = \begin{cases} a_{i0} + \left(\sum_{j \in \{1,2\}} z_j^T Q_j \right) a_i - \frac{\rho}{2} a_i^T \Omega a_i - \frac{1}{2} z_i^T D z_i, & \text{for } i \in \{1, 2\}. \\ \alpha_{i0} + \left(\sum_{j \in \{X,Y\}} v_j^T R_j \right) \alpha_i - \frac{\rho}{2} \alpha_i^T \Omega \alpha_i - \frac{1}{2} v_i^T D v_i, & \text{for } i \in \{X, Y\}. \end{cases}$$

We normalize a manager's outside option to zero. The owner will choose the intercepts a_{i0} and α_{i0} of manager i's compensation so that $EU_i = 0$, which implies that the owner's objective is to maximize total surplus (profit minus risk premia minus effort disutilities), subject to incentive compatibility constraints on the managers' choice of efforts.

2.2 Optimal Contracting: Product Organization

In a product organization, the owner's maximization problem is

$$\max_{\{a_i, z_i\}} \Pi = \sum_{i \in \{1,2\}} \left\{ u^T Q_i^T z_i - \frac{\rho}{2} a_i^T \Omega a_i - \frac{1}{2} z_i^T D z_i \right\}, \tag{1}$$

[9] Although the assumption of managerial risk aversion is maintained throughout this analysis, it is not essential for our results. An alternative modeling assumption that generates the same results is that the managers receive private information *after* contracting with the firm's owner and that this *ex post* information affects their effort choices. Baker (1992) analyzes an agency model of this type where convex effort costs combined with *ex ante* uncertainty about effort-relevant private information work in exactly the same way as risk aversion in terms of their effect on optimal contracts. The results in this paper go through if we assume this specification.

$$\text{subject to: } Q_i a_i = D z_i,\ i \in \{1,2\}, \tag{2}$$

where $u^T \equiv (1,1)$. Equation (2) is the system of incentive compatibility constraints for the product division managers, and it can be solved for the vector of efforts

$$z_i = D^{-1} Q_i a_i,\ i \in \{1,2\}. \tag{3}$$

Substituting (3) into (1), we can restate the owner's optimization problem over the slopes a_i of the compensation schedule:

$$\max_{\{a_i\}} \ \Pi = \sum_{i \in \{1,2\}} \left\{ u^T \Theta_i^T a_i - \frac{\rho}{2} a_i^T \Omega a_i - \frac{1}{2} a_i^T \Theta_i a_i \right\}, \tag{4}$$

where

$$\Theta_i \equiv Q_i^T D^{-1} Q_i,\ i \in \{1,2\}.$$

The solution to this problem is

$$a_i^* = [\Theta_i + \rho\Omega]^{-1} \Theta_i u,\ i \in \{1,2\}. \tag{5}$$

Substituting (5) into (4) yields the expression Π^P for the maximal profit under a product organization:

$$\Pi^P = \frac{1}{2} \sum_{i \in \{1,2\}} u^T \Theta_i [\Theta_i + \rho\Omega]^{-1} \Theta_i u. \tag{6}$$

2.3 Optimal Contracting: Functional Organization

In a functional organization, the owner's maximization problem is

$$\max_{\{\alpha_i, v_i\}} \ \Pi = \sum_{i \in \{X,Y\}} \left\{ u^T R_i^T v_i - \frac{\rho}{2} \alpha_i^T \Omega \alpha_i - \frac{1}{2} v_i^T D v_i \right\}, \tag{7}$$

$$\text{subject to: } R_i \alpha_i = D v_i,\ i \in \{X,Y\}. \tag{8}$$

Equation (8) is the system of incentive compatibility constraints for the function division managers, and it can be solved for the vector of efforts

$$v_i = D^{-1} R_i \alpha_i,\ i \in \{X,Y\}. \tag{9}$$

Substituting (9) into (7) yields the modified maximization problem

$$\max_{\{\alpha_i\}} \ \Pi = \sum_{i \in \{X,Y\}} \left\{ u^T \Lambda_i \alpha_i - \frac{\rho}{2} \alpha_i^T \Omega \alpha_i - \frac{1}{2} \alpha_i^T \Lambda_i \alpha_i \right\}, \tag{10}$$

where

$$\Lambda_i \equiv R_i^T D^{-1} R_i,\ i \in \{X,Y\}.$$

The optimal solution to this problem is

$$\alpha_i^* = [\Lambda_i + \rho\Omega]^{-1} \Lambda_i u, \; i \in \{X, Y\}. \tag{11}$$

Substituting (11) into (10), we get the expression Π^F for the maximal profit under a functional organization:

$$\Pi^F = \frac{1}{2} \sum_{i \in \{X,Y\}} u^T \Lambda_i [\Lambda_i + \rho\Omega]^{-1} \Lambda_i u. \tag{12}$$

3 No Cross-Product Externalities

Conditions (6) and (12) provide closed form expressions for the profits of the two organizational forms that differ only in that the matrix Θ_i appears in (6) and Λ_i appears in (12). Despite their simple structure, though, these expressions are difficult to compare directly because several economic forces are at work concurrently. In particular, four key forces shape the comparison between Π^P and Π^F: (1) the fact that product-line profitability is easier to measure than functional area profit contribution; (2) asymmetries in marginal profitability across functions and products, which determines the relative incentive sensitivity of activities; (3) cross-product externalities; and (4) diseconomies of span. In order to isolate the effect of each of these forces, we analyze the model in four steps. In this section, we begin by considering the case in which there are no cross-product externalities ($\theta = \xi = 0$), all activities have the same marginal productivity ($\beta_1 = \beta_2 = \gamma_1 = \gamma_2$), and there are no diseconomies of span ($\delta = 0$). This analysis isolates the impact of (1). This section then continues with an analysis of the impact of asymmetric marginal productivities, compensating for the effect of (1). This analysis isolates the effect of (2). In Section 4, we let $\theta, \xi > 0$ to isolate the effect of (3). Finally, in Section 5, we let $\delta > 0$ to isolate the effect of (4).

3.1 Symmetric Activities

In order to derive our benchmark result, we initially assume complete symmetry; i.e., each of the four possible activities has the same coefficients in the profit and effort functions, $\beta_1 = \beta_2 = \gamma_1 = \gamma_2$. We also assume that there are no diseconomies of span, $\delta = 0$. Our first result is that under these conditions, the product-based organization dominates the functional organization.

Proposition 1 *Suppose (i) there are no cross-product externalities, $\theta = \xi = 0$; (ii) all activities have the same marginal profitability, $\beta_1 = \beta_2 = \gamma_1 = \gamma_2$;(iii) there are no diseconomies of span, $\delta = 0$. Then, as long as the correlation r of profit signals exceed -1, the product-based* *organization yields strictly higher profits than the functional organization, i.e.,* $\Pi^P > \Pi^F$*. When* $r = -1$*, the two organizational forms yield equal profit.*[10]

The intuition behind Proposition 1 stems directly from the absence of cross-product externalities and the fact that compensation can be tied to product-line profitability but not to functional area profit contributions. In a product organization, any desired symmetric level of efforts can be induced by tying a manager's reward to the performance of the division that he oversees; i.e., only a_{11} and a_{22} matter for achieving incentive compatibility. In a functional organization, on the other hand, the desired symmetric effort levels can only be obtained by linking the pay of each manager to the performance of each of the two products, i.e. $\alpha_{X1}, \alpha_{Y1}, \alpha_{X2}$ and α_{Y2} must all be used to achieve incentive compatibility. This makes the product organization more attractive in two ways. First, given levels of effort are less costly to induce in a product organization than in a functional organization because the head of a product division bears the risk associated with the noisy profit signal in his product line while the head of a functional division bears the risk associated with noisy profit signals in both product lines. We call this the *effort inducement effect*. Second, in a product organization, the owner can use a_{12} and a_{21} to link the compensation of a division manager (either positively or negatively depending on the sign of r) to the profits of the *other* division to provide him with insurance against compensation risk without affecting effort incentives. By contrast, in a functional organization desired effort levels can be obtained only by linking the pay of each manager to the profits of both products. Hence "free insurance" against compensation risk is unavailable, and the only insurance received by the managers is that which is naturally provided by the correlation between the measured product-line profits $\widetilde{\pi}_1$ and $\widetilde{\pi}_2$.[11]We call this the *insurance effect*. Since this effect is stronger for larger values of r, the relative performance of the product organization is better for larger r. For $r = -1$ the "natural" insurance provided by the functional organization is perfect and the risk premia under both types of organizations go to zero so that product and functional organizations perform equally well. This is stated formally in Corollary 1.

Corollary 1 *When all activities have the same marginal product, the relative performance of the functional organization compared to the product-based organization is best when* $r = -1$ *and decreases as* r *increases.*

When $r = -1$, total profit $\pi_1 + \pi_2$ can be measured without noise, no matter how noisy are the measures of individual product line profitability. Thus, one implication of Corollary 1 is

[10] Proposition 1 also holds for $\delta > 0$ and $\theta = \xi > 0$. A proof of this more general result is available from the authors.

[11] Note that free insurance *would be* available under a functional organization if functional area profit-contributions could be measured and managerial compensation were linked to them.

that if product line profit measures are unavailable but total profit can be measured essentially without noise (i.e., $\sigma \to \infty, r = 1$), then the owner will be indifferent among the two organizational forms considered here. This interpretation reinforces the intuition that the product organization has a built-in advantage when product line profit contributions can be measured better than functional area profit contributions.

One interpretation of Proposition 1 is that the firm should organize around what it can measure well. This is an old theme of the accounting literature, and it is one aspect of the traditional critique of the U-form (Williamson, 1975). However, it is important to note that the economics underlying Proposition 1 differ from the traditional analysis of the drawbacks of the U-form. That analysis emphasizes that the unobservability of functional area profitability gives rise to an "Alchian-Demsetz" type team problem (Holmstrom and Tirole, 1989, pp. 67 and 124). While the handicap of the functional organization captured in Proposition 1 stems from the unobservability of functional area profit contributions, we emphasize that Proposition 1 *is not* driven by a team problem. The reason is the following. First, since the profit functions are linear and there are no cross-product externalities, there is no direct "free rider" problem. Nor is there an "induced" team problem stemming from the incentive scheme.[12] In our setup, the owner could (if it wanted to) make the agent's marginal compensation equal to its marginal cost of effort and recover this expense through the intercept of the linear compensation.

Proposition 1 can also be interpreted in terms of task specialization. To do so, let us relabel the model as follows. Rather than considering how the four tasks x_i, y_i should be allocated pairwise between two agents, let there be only two tasks T_1 and T_2 and two measures of performance $\pi_1(T_1)$ and $\pi_2(T_2)$. Task i is running "product division i." Each of the two agents is able to shoulder a load equivalent to a full task. The choice of organizational form can then be framed as whether each task should be split equally between the two agents, as in the functional organization, or whether each agent should be solely responsible for a single task, as in the product-based organization. Proposition 1 implies that task specialization is optimal. Viewed in this way, Proposition 1 is similar to the task-specialization result of Holmstrom and Milgrom (1991) who find that if two agents must allocate their effort between a continuum of tasks it is never optimal for the two agents to be jointly responsible for any task. The key to their result as well as to ours is that there is a fixed cost of inducing an agent to exert effort on an additional task. However, the source of this fixed cost is somewhat different in the two papers. In Holmstrom and Milgrom, there is a continuum of tasks and, because the cost of effort is a function of the agent's total effort (i.e. $\delta = 1$), the marginal cost of expending effort

[12]By induced team problem, we mean the following. If $y = f(e_1, e_2)$ where y is output and e_i is agent i's effort, and if the marginal cost of each agent's effort is 1, then the owner of the firm would want to have $1 = f_{e_1} = f_{e_2}$. But, if the sharing rule is $s_1(y) = 1 - s_2(y)$, implementing this scheme is not possible.

on the shared task is positive at zero. In the present paper, the fixed cost comes from the combination of two factors. Firstly, our *effort inducement* and *insurance* effects imply that sharing tasks increases the risk premium demanded by the agents. Secondly, because we have discrete tasks and discrete sharing of tasks (i.e., tasks can only be split "in halves"), the weight attached to this additional risk premium is itself positive despite the fact that, with $\delta = 0$, the marginal cost of effort at zero is itself zero.[13]

3.2 Asymmetric Productivities

A theme of the modern literature on strategic management is that firms can be fruitfully thought of as collections of value-creating activities and that different activities may be more or less important in different economic environments (Porter, 1985). In consumer packaged goods companies, such as Procter and Gamble or Unilever, marketing and brand management activities are paramount, while in high tech firms, such as Hewlett Packard or 3M, R&D activity is paramount. In this section, we examine the organizational implications of asymmetries among value-creating activities by allowing the marginal profitability parameters β_i and γ_i to differ. We consider three cases: (1) the *dominant function case*, in which one function is unambiguously more important for both products than the other function; (2) the *dominant product* case in which one product is unambiguously more important for the firm than the other product; and (3) *privileged function/product couplings,* in which success in one product market is strongly dependent on one function, while success in the other product market depends strongly on the other function.

Throughout this subsection, we maintain the assumption that there are no cross-product externalities ($\theta = \xi = 0$) and no diseconomies of span ($\delta = 0$). Under these assumptions, it is straightforward to show that because of the two effects discussed above, the product organization continues to dominate the functional organization except when $r = -1$. Thus, to isolate the effect of activity asymmetries, we show how a change in each type of asymmetry affects the relative performance of the two organizational forms, *compensating for the two effects underlying Proposition 1.* To compensate for the insurance effect, we set $r = 0$ so that product division heads cannot be offered additional insurance by tying their pay to the profit of the other product division. To compensate for the effort inducement effect, we attribute to product division heads a coefficient of risk aversion ρ_P that is twice as large as the coefficient of risk aversion of the functional heads, i.e., $\rho_P = 2\rho$. This accounts for the fact that functional

[13]It is worth noting that the discreteness of tasks is not itself the crucial difference between the two papers. With their assumed cost function (i.e., $\delta = 1$), Holmstrom and Milgrom's result would go through even if they had discrete symmetric tasks and task-sharing rules. On the other hand, with $\delta = 0$ and in the absence of our two effects, Itoh (1991 and 1992) has shown that the principal would actually prefer discrete tasks to be shared between agents.

division heads must "bear two risk premia" to induce given amounts of effort while product division heads bear just one.

3.2.1 Dominant Function

In the *dominant function case,* $\beta_1 = \beta_2 \equiv \beta > \gamma \equiv \gamma_1 = \gamma_2$, i.e., function X is unambiguously more important for organizational success than function Y. In this, case we can prove

Proposition 2 *Suppose there are no cross-product externalities,* $\theta = \xi = 0$, *and there are no diseconomies of span,* $\delta = 0$. *Then compensating for the effects underlying Proposition 1, the profitability of the product-based organization relative to the functional organization,* $\frac{\Pi^P}{\Pi^F}$, *decreases as one function* X *becomes increasingly dominant, i.e., as* $\frac{\beta}{\gamma}$ *increases above 1.*

The intuition for this result is as follows. Given the higher marginal profitability of function X, the owner ideally wants to induce greater effort levels in that function than in function Y. The owner can achieve this with a functional form: it suffices to give the manager in charge of function X higher-power incentives than the manager in charge of function Y. In a product-based organization, however, the division manager for product i controls both x_i and y_i, so it is not possible to give higher-powered incentives for activity x_i than for activity y_i. As a result, the owner is forced to make an "incentive compromise."

Figure 3 illustrates this point by showing *effort supply functions* for activities x_i and y_i. The effort supply function shows the effort provided by a manager as a function of the slope of the manager's incentive contract. If function X has a higher marginal profitability than function Y, then the effort supply function for x_i will be flatter than the effort supply function for y_i; i.e., function X is more incentive sensitive than function Y.

In the absence of risk-aversion, the first-best solution would give managers incentive contracts of slope 1, making them the residual claimants. This solution is not optimal with risk-averse managers, but it illustrates that the induced effort level would be greater for the more incentive-sensitive activity. With risk aversion, the desired effort levels are smaller for each activity, and if the owner could do so, she would choose a larger contract slope for the more incentive-sensitive activity, as shown in the top panel of Figure 3.[14]

In the dominant function case, a functional organization groups activities according to their incentive sensitivity, so that the owner can indeed give function X and function Y activities different contract slopes. However, in the product-based organization, the same contract slope applies to both activities, as shown in the lower panel of Figure 3, so the owner is forced to make

[14]This is analogous to third-degree price discrimination in which a monopolist prefers to charge a lower price to consumers with more elastic demands.

an incentive compromise, which works to reduce her expected profitability. This illustrates what we will call the *Incentive Sensitivity Principle: It is better for a single general manager to have responsibility for activities which have approximately the same degree of "incentive sensitivity."* The reduction in profitability that arises from grouping two functional activities with different incentive sensitivities under the control of the same manager is not enough to overturn the inherent advantage of the product-based organization captured in Proposition 1, but it does diminish the product organization's advantage.

3.2.2 Dominant Product

In the *dominant product case* we have $\beta_1 = \gamma_1$ and $\beta_2 = \gamma_2$. In this case, one product is unambiguously more important for organizational success than the other. In this case we have

Proposition 3 *Suppose there are no cross-product externalities, $\theta = \xi = 0$, and there are no diseconomies of span, $\delta = 0$. Then compensating for the effects underlying Proposition 1, the profitability of the product-based organization relative to the functional organization, $\frac{\Pi^P}{\Pi^F}$, is unaffected by the extent of the asymmetry between products.*

In the dominant product case, the owner wants to provide higher-power incentives for activities x_1, y_1 and lower-power incentives for activities x_2, y_2. This can be readily achieved under both organizational forms: under a product-base organization, the owner can give the manager of product 1 a more steeply sloped incentive contract than the manager of product 2; under a functional organization, the owner can make the slope of both division managers' incentive contracts more sensitive to profits in product line 1 than in product line 2. Hence, this asymmetry does not affect the relative profitability of the two organizations.

3.2.3 Privileged Function/Product Coupling

Finally we consider the case where function X is especially important for product 1, while function Y is especially important for product 2, i.e. $\beta_1 = \gamma_2 > \beta_2 = \gamma_1$. This case is most likely to arise in firms which have a diversified portfolio of businesses operating in different industry environments.[15]

In this case, the owner would like to induce high levels of x_1 and y_2 and relatively low levels of x_2 and y_1. As in the case of a dominant function, such asymmetric effort levels cannot be induced in a product-based organization. They can, however, be obtained with a functional form by putting a higher weight on $\widetilde{\pi}_1$ than on $\widetilde{\pi}_2$ in the compensation scheme of the manager

[15] One could also argue that, to the extent that the relative importance of some functions such as R&D, manufacturing or marketing change over the lifecycle of a product, firms with product lines that are at different stages of their lifecycles are likely to be characterized by such product/function "couplings".

of function X and setting a lower weight on $\widetilde{\pi}_1$ than on $\widetilde{\pi}_2$ in the contract of the manager of function Y. Hence, like the presence of a dominant function, this asymmetry improves the relative performance of the functional form.

Proposition 4 *Suppose there are no cross-product externalities, $\theta = \xi = 0$, and there are no diseconomies of span, $\delta = 0$. Then compensating for the effects underlying Proposition 1, the profitability of the product-based organization relative to the functional organization, $\frac{\Pi^P}{\Pi^F}$, decreases as the extent of the privileged function/product asymmetry increases.*

4 Cross-Product Externalities

We now consider the possibility that functional area effort on behalf of one product can have a beneficial effect on the profitability of the firm's other product. Throughout, we continue to assume that there are no diseconomies of span, so that $\delta = 0$. This analysis in this section is divided into three parts: symmetric externalities and symmetric activities ($\theta = \xi$; $\beta_1 = \beta_2 = \gamma_1 = \gamma_2$); symmetric externalities and asymmetric activities; and asymmetric externalities ($\theta \neq \xi$).

4.1 Symmetric Externalities and Symmetric Marginal Profitabilities

We begin our analysis by returning to the case of activities with symmetric marginal profitability, i.e., $\beta_1 = \beta_2 = \gamma_1 = \gamma_2$. The main effect of cross-product externalities is to weaken the advantage of the product-based organization. Indeed, this advantage disappears when the externalities become perfect, i.e., when $\theta = \xi = 1$. The following argument sheds light on the mechanism involved.

The presence of symmetric externalities increases the desired levels of effort allocated to each of the activities. In a functional organization, the compensation of each divisional manager must still be tied to both profit measures, but incentives will be higher powered (i.e. the α_{ij}'s will be larger). Consider, now, a product-based organization, and for simplicity assume that $r = 0$. Without externalities, the compensation of a manager is only tied to the profits of his own product division (i.e., $a_{ij} = 0$, for $j \neq i$). However, with positive cross-product externalities, it will generally be desirable for the owner to tie the compensation of a product division manager to the profits of the other product division in order to provide better effort incentives. This diminishes both of the advantages of the product-based organization identified in Proposition 1, though (as Proposition 5 states below) the product-based organization still remains more profitable than the functional organization. But in the extreme case of "perfect externalities" where $\theta = \xi = 1$, the optimal way to induce extra effort in a product organization is to increase

a_{ij} up to the point where it is equal to a_{ii}. Therefore the optimal compensation scheme for a product-based organization facing perfect cross-product externalities is to tie the compensation of its divisional heads equally to both profit measures, just as in a functional organization. This neutralizes the insurance and effort inducement effects, and both organizational forms perform equally well. This intuition is formalized in Proposition 5.[16].

Proposition 5 *Suppose all activities have the same marginal profitability,* $\beta_1 = \beta_2 = \gamma_1 = \gamma_2$, *and there are no diseconomies of span,* $\delta = 0$. *If the cross-product externalities are positive but less than perfect,* $\theta = \xi \in (0,1)$, *the product based organization is more profitable than the functional organization, i.e.,* $\Pi^P > \Pi^F$. *But if the cross-product externalities are perfect,* $\theta = \xi = 1$, *the product-based organization and the functional organization yield the same level of expected profit, i.e.,* $\Pi^P = \Pi^F$.

4.2 Symmetric Externalities and Asymmetric Marginal Profitabilities

An analytical implication of Proposition 6 is that by studying the relative profitability of the two organizations in a neighborhood of $\theta = \xi = 1$, we can examine the interaction between externalities and asymmetries in the marginal profitability coefficients β_1, β_2, γ_1,and γ_2 without "interference" from the two effects underlying Proposition 1. As argued in Section 3, the existence of a "dominant function" makes the functional form relatively more attractive. Since perfect externalities neutralize the two "Proposition 1" advantages of the product-based organization, the compensated advantage identified in Proposition 2 now becomes an absolute advantage, and the owner will strictly prefer a functional organization to the product organization.

Proposition 6 *Suppose one of the two functions is unambiguously more important than the other,* $\beta_1 = \beta_2 \equiv \beta > \gamma \equiv \gamma_1 = \gamma_2$, *and there are no diseconomies of span,* $\delta = 0$. *Then if cross-product externalities are sufficiently large,* $\theta = \xi \approx 1$, *the functional organization is more profitable than the product-based organization, i.e.,* $\Pi^F > \Pi^P$.

We calculated numerical examples to see how large externalities and the asymmetries in the marginal profitability coefficients must be to make the functional form more profitable than the product-based form. Our calculations reveal that there can be large ranges of parameter values in which a functional organization is more profitable than a product-based organization. Figure 4 displays a representative set of results, showing how the relative profitability $\frac{\Pi^F}{\Pi^P}$ of

[16] Although it is stated and proved for the case in which $\delta = 0$, Proposition 5 can be proved for $\delta > 0$ as well.

the product organization varies with the level of (symmetric) externalities and the degree of functional dominance, $\frac{\beta}{\gamma}$.[17]

The result in Proposition 6 should be contrasted with the traditional intuition that a functional organization (U-form) allows the firm to exploit economies of scale within particular functions better than a product-based (M-form) organization does (Holmstrom and Tirole, 1989, p. 125). With linear profit functions, there are no economies of scale in functional activities in our model, so the traditional intuition does not apply. What happens instead is that by allowing for differentiated incentives across functions, the functional organization channels large amounts of managerial effort into the functional activity that is especially important to the firm's success and lesser amounts of effort into the activity which is not as important. This results in a more efficient pattern of effort across functional areas than what arises in a product-based organization. That is, when there is a dominant function, a functional organization can improve the specialization of managerial incentives and effort within the firm.

An implication of Proposition 7 and the numerical calculations summarized in Figure 4 is that if the firm has a strong competence in a particular functional area and there are functional area externalities across product lines, a functional organization may be desirable. Phrased in the language of strategic management, our model suggests that a firm should organize around its "core competences," even when it is hard to measure the profit contribution of those competences. The managerial logic of this is that when a firm has a core competence that resides in a function, then to maximize the impact of the competence on the firm's success, senior executives responsible for that function should receive higher power incentives than senior executives responsible for other functions. Organizing the firm along product lines gets in the way of providing differentiated incentives, which then implies that the firm will fail to exploit its competence as fully as it could.

An example that might be consistent with the results of Proposition 7 is IBM's U.S. computer business prior to about 1990.[18] The chief source of IBM's competitive advantage in the market for mainframe computers and peripheral equipment in the period between 1960 and 1990 was a brand reputation that enabled it to charge prices 20 to 30 percent higher than the prices charge for similar products sold by competitors.[19] The strength of IBM's brand came not so much from distinctive skills in either manufacturing, R&D, or new product development, but from its legendary customer service network. Consistent with the theory presented here, prior to 1990 IBM's U.S. computer business unit was organized functionally rather than by product.

[17] The parameter values for this example are $\rho = 2, \sigma^2 = 1, r = 0, \gamma_1 = \gamma_2 = \gamma = 1, \delta = 0$.

[18] This example draws from "IBM Corporation: Background Note," Harvard Business School case 9-180-034 (1979); "The Transformation of IBM," Harvard Business School case 9-792-105; and Carroll (1993).

[19] See, for example, Brock (1975) or Carroll (1993).

It consisted of two key functional divisions: the Marketing Group, which sold all of IBM's products to its corporate and government customers in the United States and was responsible for its customer service, and the Product Group, which was responsible for manufacturing and new product development. These two divisions, known as the Data Processing Complex, had joint profit responsibility for all of IBM's U.S. computer products.[20] The vice presidents of both the Product Group and the Marketing Group were subject to incentive compensation, with the compensation tied to the profitability of each of IBM's computer products. Bonuses for these individuals ranged from 25 to 50 percent of total compensation. Our theory would predict that the vice president of the Marketing Group would be faced with higher-power incentives than the vice president of the Product Group. Unfortunately, we do not have detailed evidence on how IBM's incentive compensation awards varied by individual during this period. Still, there is anecdotal evidence that suggests that success in the Marketing Group traditionally offered a clearer path to the ranks of senior management at IBM than did success in the Product Group.[21] For example, before Lou Gerstner became Chairman of IBM in 1993, the three previous chairmen of IBM (John Akers, John Opel, and Frank Cary) all had served as heads of IBM's Marketing Group.[22,23] This might represent the "footprints" of differentiated incentives for managers with career concerns.

The case of a "dominant product" is less straightforward than that of a "dominant product." Proposition 3 implies that in the absence of cross-product externalities, both organizational forms deal equally well with this asymmetry. With externalities, however, an additional concern arises: the weight given to $\widetilde{\pi}_1$ and $\widetilde{\pi}_2$ in a manager's compensation should be aligned with the marginal contribution of each of the manager's two activities to each of the two product lines. This issue appears most clearly in the case of perfect externalities. Since each activity contributes equally to each product line, $\widetilde{\pi}_1$ and $\widetilde{\pi}_2$ should be given equal weights in a manager's contract. This does not further constrain a product organization since the product asymmetry only requires that one manager be given higher-powered incentives than the other. However, the functional organization now faces a trade-off because it must offer contracts with asymmetric weights in order to accommodate the asymmetry of the two product lines. Hence, even in

[20]These products included mainframe computers, microcomputers, mass data storage devices, terminals, printers, software, semiconductors, and PBX systems.

A separate business unit within IBM was responsible for computer products sold outside the United States. IBM's office products, such as electric typewriters and photocopiers, were also in a separate business unit.

[21]See Carroll (1993).

[22]IBM's Chairman before Frank Cary was Thomas Watson, Jr., son and successor of the legendary Thomas Watson, Sr. Watson, Jr. also came out IBM's sales organization.

[23]Interestingly, poor performance also seemed to hit hardest in the sales and marketing organization. When, in 1991, IBM had its worst year since the mid-1940s, the highest profile IBM executive to lose his job was the head of the Marketing Group.

the limiting case of perfect externalities, where the "Proposition 1" advantages of the product organization vanish, the product-based organization remains strictly more profitable than the functional organization.

Proposition 7 *: Suppose one of the two products is unambiguously more important than the other,* $\beta_1 = \gamma_1 > \beta_2 = \gamma_2$*, and there are no diseconomies of span,* $\delta = 0$*. Then if cross-product externalities are sufficiently large,* $\theta = \xi \approx 1$*, the product-based organization is more profitable than the functional organization, i.e.,* $\Pi^P > \Pi^F$.

For less than perfect externalities, increasing $\theta = \xi$ above 0 has two opposing effects on the relative performance of the two types of organizations. On the one hand, it weakens the "Proposition 1" advantages of the product-based organization. On the other hand, it forces the functional organization into the incentive compromise just described. We calculated numerical examples to see which of the two effects is likely to dominate. For all the parameter values considered, the product-based organization was more profitable than the functional organization, but the relative performance of the product-based organization decreased as the externalities became more powerful. Figure 5 displays a typical set of results, showing the relative profitability $\frac{\Pi^F}{\Pi^P}$ of the product organization against the level of (symmetric) externalities and the degree of product dominance, $\frac{\beta_1}{\beta_2}$.[24]

Finally we turn to the case a privileged function/product coupling. We saw in Section 3 that this asymmetry favors the functional form. However, symmetric externalities make each of the four activities matter for *both* product lines, which weakens the impact of the asymmetry in the productivity parameters and tends to improve the relative performance of the product form. Strikingly, if $\theta = \xi = 1$, the impact of the initial asymmetry disappears altogether. Since the "Proposition 1" advantages of the product-based organization also vanish in the limit, the two organizational forms are then equally profitable!

Proposition 8 *Suppose there is a privileged function/product coupling,* $\beta_1 = \gamma_2 > \beta_2 = \gamma_1$*, and no diseconomies of span,* $\delta = 0$*. Then, if cross-product externalities are large,* $\theta = \xi \approx 1$*, the functional organization and the product-based organization are equally profitable.*

For less than perfect externalities, the decrease in the effective asymmetry of productivity parameters and the weakening of the "Proposition 1" advantages of the product form work in opposite directions. We calculated numerical examples to evaluate the net effect of an increase in the externality parameters. For all the parameter values considered, the product-based

[24]The parameter values for this example are $\rho = 2, \sigma^2 = 1, r = 0, \gamma_2 = \beta_2 = 1, \delta = 0$.

organization was more profitable, but its relative profitability worsened as externalities became stronger. Figure 6 displays a typical set of examples.[25]

We can summarize the implications of this section as follows:

- Symmetric cross-product externalities weaken the effort inducement and insurance advantages of the product organization identified in Proposition 1, and in the limit, as externalities become perfect, these advantages disappear.
- With sufficiently large symmetric cross-product externalities and a sufficiently dominant function, the functional organization dominates the product-based organization.
- With sufficiently large symmetric cross-product externalities and a sufficiently dominant product, the product-based organization dominates the functional organization.
- With sufficiently large symmetric cross-product externalities, a privileged function/product coupling makes the two organizational structures equally profitable.

4.3 Asymmetric Externalities

We now study a situation in which only one of the two functions generates significant cross-product externalities. Let us set ξ equal to zero so that externalities occur only within functional area X. To isolate the effect of this asymmetry, we assume that the marginal productivity coefficients are perfectly symmetric, i.e., $\beta_1 = \beta_2 = \gamma_1 = \gamma_2$.

The presence of an externality only in X affects the relative performance of the organizational forms in two ways. First, it makes it desirable to implement greater level of efforts in X then in Y. This creates an asymmetry of the same type as our "dominant function" case and, hence, favors the functional organization. Secondly, it becomes optimal for the product organization to link the compensation of its managers to the performance of both divisions and thus reduces the "Proposition 1" advantages of the product-based organization. Both effects lead to a better relative performance of the functional form. Indeed, for large enough values of θ, a functional organization leads to higher expected profits than a product-based organization.

Proposition 9 *Suppose the marginal productivity coefficients are perfectly symmetric, $\beta_1 = \beta_2 = \gamma_1 = \gamma_2$ and there are no diseconomies of span, $\delta = 0$. If only one of the two functions generates cross-product externalities, $\theta > 0, \xi = 0$, an increase in the strength of the externality improves the relative performance of the functional organization. If the externality is sufficiently large, the functional organization is more profitable than the product-based organization, i.e., $\Pi^F > \Pi^P$.*

[25] The parameter values for this example are $\rho = 2, \sigma^2 = 1, r = 0, \beta_2 = \gamma_1 = 1, \delta = 0$.

5 Diseconomies of Span

Up to now, we have maintained the assumption that there are no diseconomies of span, i.e., $\delta = 0$. When $\delta > 0$ there are two additional effects that influence the relative profitability of the two organizational structures. The first is an *effort disparity effect.* This effect arises because for given effort vectors, z_i or v_i, the difference in total effort cost $\sum_{i\in\{1,2\}} \Delta_i - \sum_{i\in\{X,Y\}} \Delta_i$ between a product organization and a functional organization is equal to $\delta(x_1 - y_2)(y_1 - x_2)$. This implies that when there is a dominant product, effort costs tend to be higher in a product organization, while if there is a dominant function, effort costs tend to be higher in a functional organization. This effect, then, moderates the impact of the incentive sensitivity principle discussed earlier. The second effect is the well known *multi-task effort allocation problem.* As δ increases, a manager has a tendency to devote large amounts of effort to the product (or function) with the higher marginal profitability. For $\delta \to 1$, this tendency becomes extreme, and the manager devotes no effort to the activity with the lower marginal profitability. This effect makes it difficult to induce effort supply under both organizational forms, and thus has no clear cut impact on the *relative profitability* of the two organizations. (It reduces the absolute profitability of both).

Table 1 illustrates the impact of diseconomies of scope on the relative profitability of the two organizational forms. In this example, product 1 is much more important to organizational success than product 2, while the function X is slightly more profitable than function Y. ($\beta_1 = 3.2, \gamma_1 = 3, \beta_2 = 1.2, \gamma_2 = 1$).[26] This configuration of parameter values creates a potential multi-tasking problem under both organizations, but that problem is much more severe under the functional organization because the marginal profitability of the two products are so different. On the other hand, the effort disparity problem works against the product organization. For values of δ close to 0, a product organization is more profitable (as our analysis from the previous section implies it should be), but as δ increases, the functional organization eventually becomes more profitable.[27] In this example (and in others that we do not report), the effort disparity effect seems to be an important determinant of how diseconomies of scope impact the relative profitability of the two organizational forms.

Table 1:
Comparison Between Product and Functional Organizations
as Diseconomies of Span Become More Severe

[26]The other parameter values for this example are $\theta = \xi = .5$, $\rho = 2, \sigma^2 = 1, r = 0$.

[27]And the functional organization continues to be more profitable even as $\delta \to 1$. For δ above about .28 in this example, we reach a corner solution under the functional organization where $x_2 = 0$ and $y_2 = 0$. Despite this, the functional organization still dominates the product organization.

δ	$\frac{\Pi^F}{\Pi^P} \times 100$	Best?
0	97.52	Product
.05	99.54	Product
.10	101.81	Function
.15	104.39	Function
.20	107.33	Function
.25	111.70	Function

One way to interpret the impact of the effort disparity effect on organizational form is in terms of task specialization. In the example in Table 1, a product organization entrusts two very important decisions (i.e., functional efforts, x_1 and y_1, on behalf of product 1) to a single agent. As δ increases, the diseconomies of span rapidly raise the marginal effort costs for both of these key decisions, resulting in a large drop in the level of amount of effort supplied to both. In this case, it is better for two agents to specialize along functional lines, with one agent choosing x_1 and x_2 and the other choosing y_1 and y_2. This specialization is not costless: for δ sufficiently large, the multi-tasking problem arises, and the manager of functional area X supplies no effort x_2 to the less important product 2 while the manager of Y supplies no effort y_2. Still, the functional organization is more profitable because it creates a greater effort disparity, with considerably more effort in both functional areas being devoted to product 1 than to product 2.

A key conclusion is that when there are diseconomies of span, some sets of activities may be too important to leave to the control of a single agent. An implication of this analysis is that firms with a dominant product line may be more profitably organized along functional lines than along product lines. Empirical evidence on the relationship between a firm's scope and its organizational structure is consistent with this prediction. Rumelt's (1974) study of organizational structure of large U.S. corporations over the period 1949 through 1969 indicated that firms with a dominant product line were significantly more likely to employ a functional organization than were firms with a more diversified product line.[28]

6 Complementarities and Functional Area Coordination

A traditional theme in the economics of organizations and management literatures is that organizing on a functional basis has the disadvantage of compromising coordination between the functional areas.[29] To study cross-functional coordination, we need cross-functional comple-

[28] Rumelt defined dominant businesses in terms of a specialization ratio: the proportion of a firm's revenues that come from its largest product market activity. Firms with dominant product lines in his study had specialization ratios greater than 70 percent.

[29] See, for example, Chandler's (1962, p. 91) discussion of the difficulties DuPont faced in coordinating purchasing, manufacturing, marketing activities as DuPont expanded its product line.

mentarities, i.e. the marginal profitability in one function must be increasing in the effort level of the other. To introduce complementarities, we make product-line profits a supermodular function of functional area activities. To parametrize the strength of cross-functional complementarities, we use a CES specification for the profit function:

$$\pi_i = \left[\beta_i x_i^\mu + \gamma_i y_i^\mu + \theta\beta_j x_j^\mu + \xi\gamma_j y_j^\mu\right]^{\frac{1}{\mu}} \text{ for } i = 1, 2, i \neq j,$$

where $\mu \in [-\infty, 1]$. As the parameter μ decreases from 1 to $-\infty$, functional area activities become increasingly complementary, and the value of coordinating the choice of functional area decisions x_i and y_i within a product line goes up. Analytical results are difficult to obtain with this specification, but our numerical calculations reveal that as μ falls, the product organization eventually dominates the functional organization, no matter how strong an advantage the functional organization might have had when $\mu = 1$. For example, Table 2 shows numerical calculations for the case in which $\beta_1 = \beta_2 = 150$, $\gamma_1 = \gamma_2 = 50, \xi = 0, \theta = .5$.[30] This is a case in which one function X is more important to organizational success than function Y, and there are strong cross-product externalities in function X. As discussed above, these are circumstances that favor a functional organization, as indicated by the fact that $\Pi^F > \Pi^P$ when $\mu = 1$. However, as μ decreases, the advantage of the functional organization diminishes, and for μ below about $-.1$, the product organization dominates. The intuition for this result is that as the functions become more complementary, each functional area requires increasingly similar incentive intensities, despite the difference between the marginal profitability parameters β and γ. The incentive specialization principle thus favors the functional organization less and less as μ falls, and the fact that product-line performance is easier to monitor than functional performance makes the product-based organization more attractive.

Table 2:
Comparison Between Product and Functional Organizations as Cross-Functional Coordination Becomes More Important

μ	$\frac{\Pi^F}{\Pi^P} \times 100$	Best?
1.0	123.85	Function
0.5	109.59	Function
-0.1	99.82	Product
-0.5	96.04	Product
-1.0	92.91	Product
-2.0	89.34	Product
-200	81.63	Product

[30] The other parameter values for this example are $\delta = 0, \rho = 2, \sigma^2 = 100, r = 0$.

7 Other Organizational Forms

As indicated in Section 2, there are other organizational structures that could be considered in this model. This section briefly discusses the economics of those alternative structures.

7.1 Disaggregated Organization

One way to organize our firm would be to assign responsibility for each of the product-function effort choices to an individual agent. In this structure, the firm would consist of *four* agents, one with responsibility for x_1, one with responsibility for y_1, and so forth. This structure disaggregates responsibility for decision making, so we refer to it as the *disaggregated organization.* The disaggregated organization has two advantage. First, by eliminating multi-tasking, this structure reduces the marginal cost of effort when there are diseconomies of span ($\delta > 0$). In addition, the disaggregated organization allows for differentiated incentives which, as discussed above, is valuable when activities have different marginal profitability. There are two key disadvantages of the disaggregated organization. First, the disaggregated organization entails creating incentives for four agents rather than two. This disadvantage can be seen most clearly when there are no cross-product externalities, i.e., $\theta = \xi = 0$. Then, to implement the same effort levels as are attained under the product organization, the owner pays a risk premium that is twice that which is paid in a product organization. Second, when there are cross-product externalities, managers in a disaggregated organization will fail to internalize them in their effort choices. This can make a functional organization perform better than a disaggregated organization even when there are diseconomies of span.

Although we have not been able to prove general results, we have computed numerical examples which reveal that these latter disadvantages often dominate. Representative calculations are summarized in Figure 7 which shows a disaggregated organization is dominated by either a product organization, a functional organization, or both unless diseconomies of span or asymmetries are sufficiently strong.

7.2 Generalists versus Specialists

We have assumed that division heads are specialists not generalists. That is, we have assumed that it is not possible to find an agent who can perform all four tasks productively. If we drop this assumption, then in our model there are circumstances in which the owner would want to have both agents do everything. To see why, consider the special case in which $\delta = 0$, there are no cross-product externalities, $\theta = \xi = 0$, and $r = 0$ so that the slopes of the product managers' compensation schedules in a product organization can be set independently of each other. In this case, hiring two "generalist" agents who can do everything has the same effect as doubling the number of specialist agents, and it is straightforward to show that under these conditions an organization of two generalists is twice as profitable as a product organization in which division heads are product specialists.[31] This implies that there is a positive "productivity effect" associated with hiring generalists.[32] This productivity effect is offset by diseconomies of span: as δ increases, a generalist's marginal cost of effort increases by a large enough amount that it eventually pays for the firm to employ specialists.

8 Summary and Conclusion

This paper has studied the choice among organizational forms in an environment in which agents must be motivated through incentive contracts to undertake multiple tasks on behalf of products. Two organizational forms are studied: a functional organization and a product-based organization. In a functional organization, the firm is divided into functional divisions, and a division manager has responsibility for a single function's activities on behalf of all products. In a product-based organization, the firm is organized into product divisions, and a division manager has responsibility for all functional activities on behalf of a single product.

Because our model abstracts from important issues, such as the number of levels in the organization's hierarchy and the degree to which decision making authority is centralized or decentralized, it does not provide a complete theory of endogenous organizational structure. Our objective instead has been to identify economic forces that systematically favor organizing along a product dimension versus a functional dimension in settings in which incentive contracting plays a significant role in a firm's internal structure. In particular, the model identifies five distinct forces that shape the relative profitability of functional and product-based organizations:

- *Noisiness of performance measures*: Measuring a product line's contribution to profitability is generally easier than measuring a functional area's contribution to profitability. This effect favors a product organization because it allows the owner to offer incentive contracts with better risk bearing properties (the "insurance" and "effort inducement" effects) than can be offered in a functional organization.

- *Disparities in the marginal profitability of functional areas*: Situations in which a particular function is more important to organizational success generally favors the functional

[31]Recall that a product organization is more profitable than a functional organization in this case so it suffices to use the product organization as the relevant comparison.

[32]Itoh's (1991, 1992) results can also be interpreted as implying generalists are preferred to specialists if $\delta = 0$.

organization. The functional organization allows the firm to better apply the incentive sensitivity principle: with a fixed number of agents it is better to group activities with similar degrees of incentive sensitivity together.

- *Cross-product externalities*: Situations in which there are cross-product externalities within a function favors a functional organization. This is because function managers automatically internalize these externalities in decision making whereas product managers must be given incentives to do so through the compensation scheme, which then erodes the risk bearing advantages of the product organization.

- *Diseconomies of span of control*: diseconomies of span generally favor the functional organization when one product is significantly more important to firm profitability than the other and favors the product organization when one function is significantly more important than the other.

- *Cross-functional coordination*: the presence of cross-functional complementarities generally favors a product organization.

While the role of cross-functional coordination and cross-product externalities has already been discussed in the literature about the U and M forms, we believe that the other three factors have not been analyzed before. Moreover, the complex interactions between cross-product externalities and various types of asymmetries have not been examined previously.

It is our hope that this paper illustrates how multi-task agency theory can fruitfully shed light on the economics a firm's organizational strategy. In future work, we hope to continue this line of research by showing how changes in the scope of a firm's activities (e.g., adding additional products or entering additional geographic markets) affect its choice of organizational structure. Baron and Besanko (1997) provide some insights into the relationship between firm scope and organizational structure, but their model does not study functional organizations, nor does it consider the roles of noisy observables and hidden action which are key parts of this paper. We would also like to endogenize not only the organizing dimension, but also the number of levels in the firm's hierarchy and the allocation of decision authority in the organization.

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9 Appendix[33]

Proof of Proposition 1:[34] Under the functional form, the optimal contract terms are $\alpha_{Xi} = \alpha_{Yi} \equiv \alpha^* = \frac{x^*}{\beta}$, where $x^* = x_i^* = y_i^*$ is the optimal (symmetric) effort level. In a product-based organization, the same effort levels can be induced by setting $a_{ii} = \frac{x^*}{\beta} = \alpha^*$. Note that effort incentives in the product organization are independent of $a_{ij},\ j \neq i$, so a_{ij} is available as a degree of freedom for risk sharing purposes. The expected wage bill of the functional organization is $E(\widetilde{W^F}) = 2[\Delta^* + 2\rho(\alpha^*)^2(1+r)]$. The expected wage bill of the product-based organization is $E(\widetilde{W^P}) = 2[\Delta^* + \rho(\alpha^*)^2(1 + 2ra_{ij}) + a_{ij}^2)]$. Since a_{ij} can be set to zero without affecting effort incentives, we have $E(\widetilde{W^F}) > E(\widetilde{W^P})$ and thus $\Pi^F < \Pi^P,\ \forall r > -1$. ■

Proof of Corollary 1: Expressions (5) and (11) imply $\alpha_{X1}^* = \alpha_{X2}^* = \alpha_{Y1}^* = \alpha_{Y2}^* = \frac{\beta^2}{\beta^2+\rho\sigma^2(1+r)}$ and $a_{ii}^* = \frac{2\beta^2}{2\beta^2+\rho\sigma^2(1-r^2)}$ with $a_{ij}^* = -ra_{ii}^*$. Substituting back into the expected profit functions we have

$$\Pi^P = \frac{2\beta^2}{1 + \frac{\rho\sigma^2}{2\beta^2}(1-r^2)},$$

and

$$\Pi^F = \frac{2\beta^2}{1 + \frac{\rho\sigma^2}{\beta^2}(1+r)}$$

so that $\Pi^P > \Pi^F\ \forall r > -1$ and $\Pi^P = \Pi^F$ for $r = -1$. Moreover

$$\frac{d(\frac{\Pi^P}{\Pi^F})}{dr} = \frac{2\beta^2 + \rho\sigma^2(1-r^2) + 2r(\beta^2 + \rho\sigma^2(1+r))}{[2\beta^2 + \rho\sigma^2(1-r^2)]^2}\rho\sigma^2 > 0\ \forall r \in (-1, 1).$$

■

Proof of Proposition 2

Solving expressions (6) and (12) we get $\Pi^P = \frac{(\beta^2+\gamma^2)^2}{\beta^2+\gamma^2+\rho_p\sigma^2(1-r^2)}$ and $\Pi^F = \frac{\beta^4}{\beta^2+\rho\sigma^2(1+r)} + \frac{\gamma^4}{\gamma^2+\rho\sigma^2(1+r)}$. Setting $r = 0$ and $\rho_p = 2\rho$ we get

$$\Pi^P = \frac{(\beta^2+\gamma^2)^2}{(\beta^2+\gamma^2) + 2\rho\sigma^2}. \tag{A.1}$$

$$\Pi^F = \frac{\beta^4}{\beta^2+\rho\sigma^2} + \frac{\gamma^4}{\gamma^2+\rho\sigma^2}. \tag{A.2}$$

[33] The proofs of the propositions rely on the solutions to (5)-(6) and (11)-(12), which are expressed in matrix form. Because the scalar solutions to (6) and (12) are extremely complicated, we omit them for brevity. They are, however, available from the authors upon request.

[34] Proposition 1 can also be proved for $\delta > 0$. A proof is available from the authors.

Define $Z \equiv \frac{\Pi^P}{\Pi^F}$ and consider a mean-preserving increase in the asymmetry. We have

$$\frac{\partial Z}{\partial \beta} - \frac{\partial Z}{\partial \gamma} = 2(\gamma - \beta)(\beta^2 + \gamma^2)\gamma\beta\rho\sigma^2 \frac{A}{\Delta}$$

where

$$\Delta \equiv (\beta^2 + \gamma^2 + \rho\sigma^2)^2(\beta^4\gamma^2 + \beta^4\rho\sigma^2 + \gamma^4\beta^2 + \gamma^4\rho\sigma^2)^2 > 0$$

and

$$\begin{aligned} A \quad \equiv \quad & \beta^6\rho\sigma^2 + 2\beta^5\gamma\rho\sigma^2 + +\gamma^3\beta^5 + 3\beta^4\gamma^2\rho\sigma^2 + 3\beta^4\rho^2\sigma^4 + \\ & 7\beta^3\gamma\rho^2\sigma^4 + 8\beta^3\gamma^3\rho\sigma^2 + \gamma^5\beta^3 + 2\beta^2\rho^3\sigma^6 + 6\beta^2\gamma^2\rho^2\sigma^4 + \\ & 3\beta^2\gamma^4\rho\sigma^2 + 2\beta\gamma^5\rho\sigma^2 + 4\beta\gamma\rho^3\sigma^6 + 7\beta\gamma^3\rho^2\sigma^4 + \\ & 3\gamma^4\rho^2\sigma^4 + \gamma^6\rho\sigma^2 + 2\gamma^2\rho^3\sigma^6 \\ > \quad & 0. \end{aligned}$$

Hence

$$(\frac{\partial Z}{\partial \beta} - \frac{\partial Z}{\partial \gamma}) \lesseqgtr 0 \text{ as } \beta \gtrless \gamma,$$

so that a mean preserving increase in asymmetry decreases Z. ■

Proof of Proposition 3

We have $\beta_1 = \gamma_1 \equiv \beta \neq \beta_2 = \gamma_2 \equiv \gamma$. Expressions (6) and (12) can be solved to obtain.

$$\Pi^P = \frac{\beta^4}{\beta^2 + \frac{\rho_P\sigma^2}{2}} + \frac{\gamma^4}{\gamma^2 + \frac{\rho_P\sigma^2}{2}}.$$

$$\Pi^F = \frac{\beta^4}{\beta^2 + \rho\sigma^2} + \frac{\gamma^4}{\gamma^2 + \rho\sigma^2}.$$

Setting $\rho_P = 2\rho$ we get $\Pi^P = \Pi^F$. ■

Proof of Proposition 4

We have $\beta_1 = \gamma_2 \equiv \beta \neq \beta_2 = \gamma_1 \equiv \gamma$. Expressions (6) and (12) can be solved to obtain

$$\Pi^P = \frac{(\beta^2 + \gamma^2)^2}{\beta^2 + \gamma^2 + \rho_P\sigma^2(1 - r^2)}. \tag{A.3}$$

$$\Pi^F = \frac{\beta^2[\beta^2(\gamma^2 + \rho\sigma^2) - r\gamma^2\rho\sigma^2] + \gamma^2[\gamma^2(\beta^2 + \rho\sigma^2) - r\beta^2\rho\sigma^2]}{(\beta^2 + \rho\sigma^2)(\gamma^2 + \rho\sigma^2) - r^2\rho^2\sigma^4}. \tag{A.4}$$

For $\rho_P = 2\rho$ and $r = 0$, the profit expressions in (A.3) and (A.4) reduce to those in (A.1) and (A.2), and the logic of the proof of Proposition 2 applies.

Proof of Proposition 5

Let us now assume $\xi = \theta, \delta = 0$, and $\beta_1 = \beta_2 = \gamma_1 = \gamma_2 \equiv \beta$. To keep notation as light as possible, set $\beta = 1$. Working through the matrix algebra of equation (12) we get

$$\Pi^F = \frac{2(1+\theta)^2}{1 + \frac{\rho\sigma^2(1+\delta)^2(1+r)}{(1+\theta)^2}}.$$

Similarly, solving (6) and taking limits for $\theta \to \xi$ yields[35]

$$\Pi^P = \frac{2(1+\theta)^2}{1 + \frac{\rho\sigma^2(1+\delta)^2(1-r^2)}{2(\theta^2-2\theta r+1)}}.$$

These two expressions are equal for $\theta = 1$. Also $\Pi^P > \Pi^F$ iff $\frac{1-r}{2(\theta^2-2\theta r+1)} < \frac{1}{(1+\theta)^2} \Leftrightarrow \frac{1-r}{2[(\theta-1)^2+2\theta(1-r)]} < \frac{1}{(1+\theta)^2} \Leftrightarrow -1 < r$, which proves Proposition 5. ■

Proof of Proposition 6

Assume $\xi = \theta$, $\delta = 0$ and $\beta_1 = \beta_2 \equiv \beta > \gamma_1 = \gamma_2 \equiv \gamma$. Working through the matrix algebra of (6) and (12) and letting $\theta \to 1$, we get

$$\Pi^F = 16[\frac{\beta^4}{4+\rho\sigma^2(1+r)} + \frac{\gamma^4}{4\gamma^2+\rho\sigma^2(1+r)}],$$

and

$$\Pi^P = \frac{8[\beta^2+\gamma^2]^2}{2(\beta^2+\gamma^2)+\rho\sigma^2(1+r)},$$

so that $\Pi^F = \Pi^P$ for $\gamma = \beta$. Moreover we have

$$\frac{\partial \Pi^F}{\partial \beta} = \frac{16\beta^3[2\beta^2+\rho\sigma^2(1+r)]}{[2\beta^2+\frac{\rho\sigma^2}{2}(1+r)]^2} > 0,$$

and

$$\frac{\partial \Pi^P}{\partial \beta} = \frac{8\beta(\beta^2+\gamma^2)[\beta^2+\gamma^2+\rho\sigma^2(1+r)]}{[\beta^2+\gamma^2+\frac{\rho\sigma^2}{2}(1+r)]^2} > 0,$$

so that

$$\frac{\partial E(\Pi_F^*)}{\partial \beta} \gtrless \frac{\partial E(\Pi_P^*)}{\partial \beta} \text{ as } \beta^2+\gamma^2 \lesseqgtr 2\beta^2 \text{ i.e., as } \beta \gtrless \gamma.$$

Hence $\Pi^F > \Pi^P$ for all $\beta \neq \gamma$. ■

Proof of Proposition 7

[35] Taking $\lim_{\xi\to\theta} \Pi^P$ requires numerous applications of L'Hospital's rule.

Assume that $\xi = \theta \to 1, \delta = 0$, and $\beta_1 = \gamma_1 \equiv \beta > \beta_2 = \gamma_2 \equiv \gamma$. Equations (6) and (12) can be solved to yield

$$\Pi^F = \frac{8[\beta^2 + \gamma^2]^2}{2(\beta^2 + \gamma^2) + \rho\sigma^2(1+r)},$$

and

$$\Pi^P = 16[\frac{\beta^4}{4 + \rho\sigma^2(1+r)} + \frac{\gamma^4}{4\gamma^2 + \rho\sigma^2(1+r)}],$$

which is just a reversal of the expressions in the proof of Proposition 6. ■

Proof of Proposition 8

Assume that $\xi = \theta \to 1, \delta = 0$, and $\beta_1 = \gamma_2 \equiv \beta > \beta_2 = \gamma_1 \equiv \gamma$. Equations (6) and (12) can be solved to get

$$\Pi^F = \frac{4(\beta^2 + \gamma^2)^2}{\beta^2 + \gamma^2 + \frac{\rho\sigma^2}{2}(1+r)},$$

and

$$\Pi^P = \frac{4(\beta^2 + \gamma^2)^2}{\beta^2 + \gamma^2 + \frac{\rho\sigma^2}{2}(1+r)}.$$

which are clearly identical. ■

Proof of Proposition 9

Let us set $\delta = r = \xi = 0$ and $\beta_1 = \beta_2 = \gamma_1 = \gamma_2 \equiv \beta$. Solving (6) and (12) and letting $\theta \to 1$, we get

$$\Pi^F = \frac{\beta^4(20\beta^2 + 17\rho\sigma^2)}{(\beta^2 + \rho\sigma^2)(4\beta^2 + \rho\sigma^2)},$$

$$\Pi^P = \beta^4 \frac{3(4\beta^2 + 3\rho\sigma^2)(\rho\sigma^2 + 3\beta^2)(\rho\sigma^2 + \beta^2) + (\beta^2 + 2\rho\sigma^2)(7\beta^4 + 10\beta^2\rho\sigma^2 + 2\rho^2\sigma^4)}{(\rho\sigma^2 + 3\beta^2)^2(\rho\sigma^2 + \beta^2)^2},$$

so that $\Pi^F > \Pi^P$ iff

$$\frac{20\beta^2 + 17\rho\sigma^2}{4\beta^2 + \rho\sigma^2} > \frac{3(4\beta^2 + 3\rho\sigma^2)(\rho\sigma^2 + 3\beta^2)(\rho\sigma^2 + \beta^2) + (\beta^2 + 2\rho\sigma^2)(7\beta^4 + 10\beta^2\rho\sigma^2 + 2\rho^2\sigma^4)}{(\rho\sigma^2 + 3\beta^2)^2(\rho\sigma^2 + \beta^2)}.$$

Defining $A \equiv \rho\sigma^2$ and simplifying, this is equivalent to

$$8\beta^8 + 14\beta^6 A + 16\beta^4 A^2 + 17\beta^2 A^3 + 4A^4 > 0,$$

which is indeed true. ■

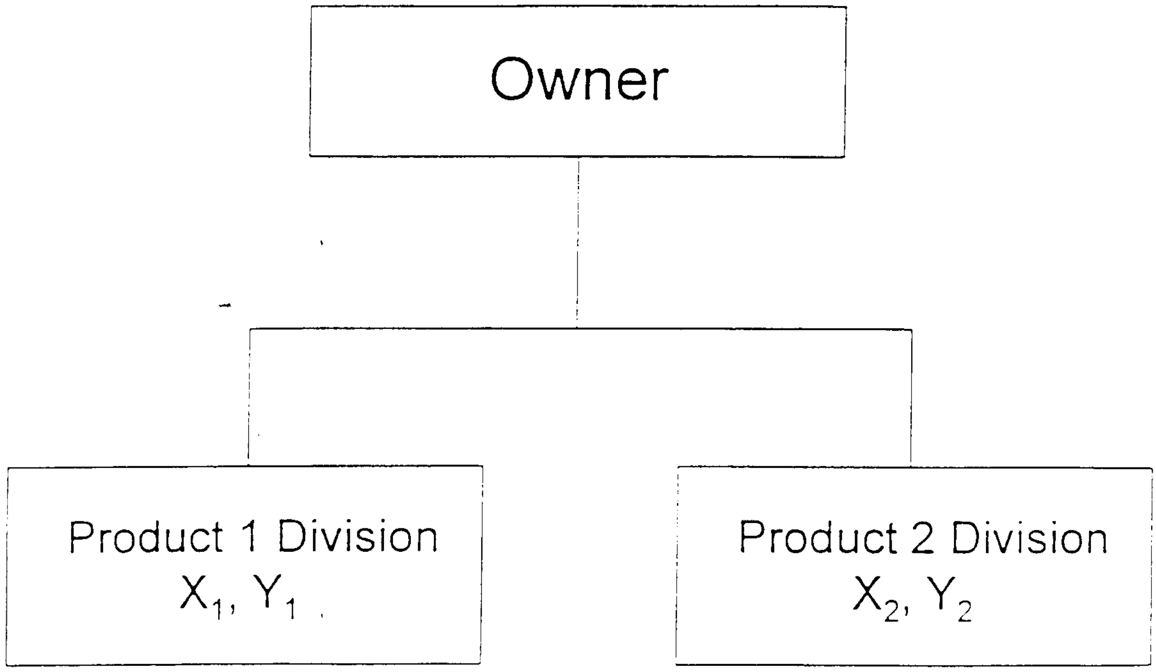

Figure 1: Product-Based Organization

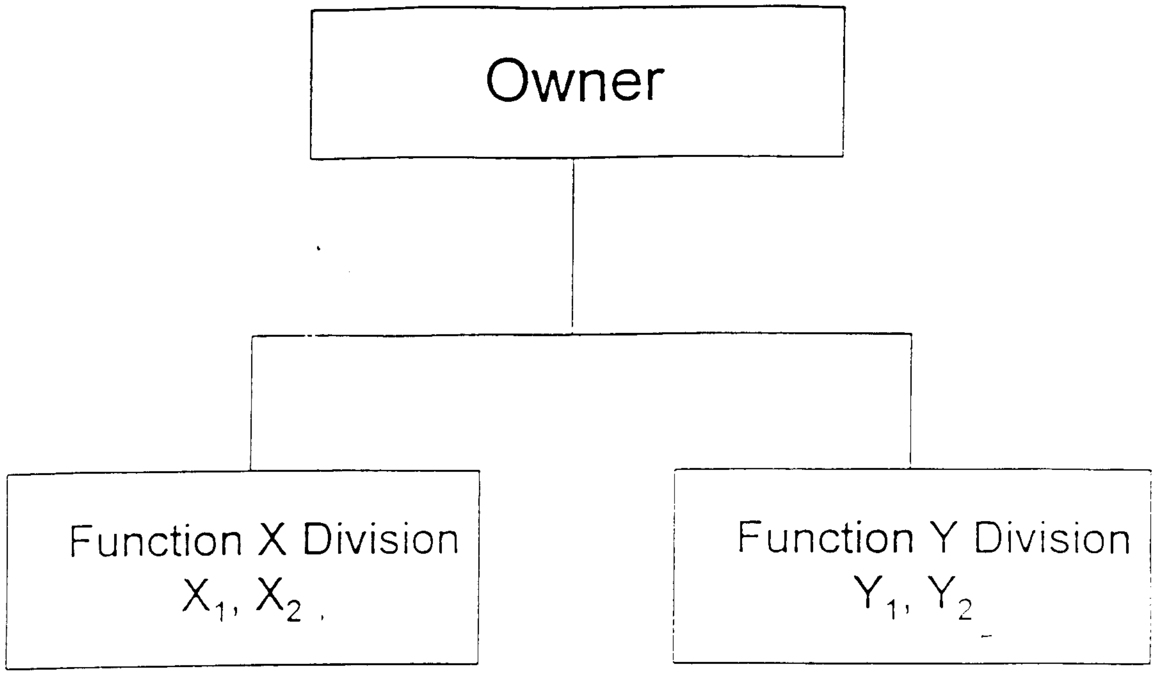

Figure 2: Functional Organization

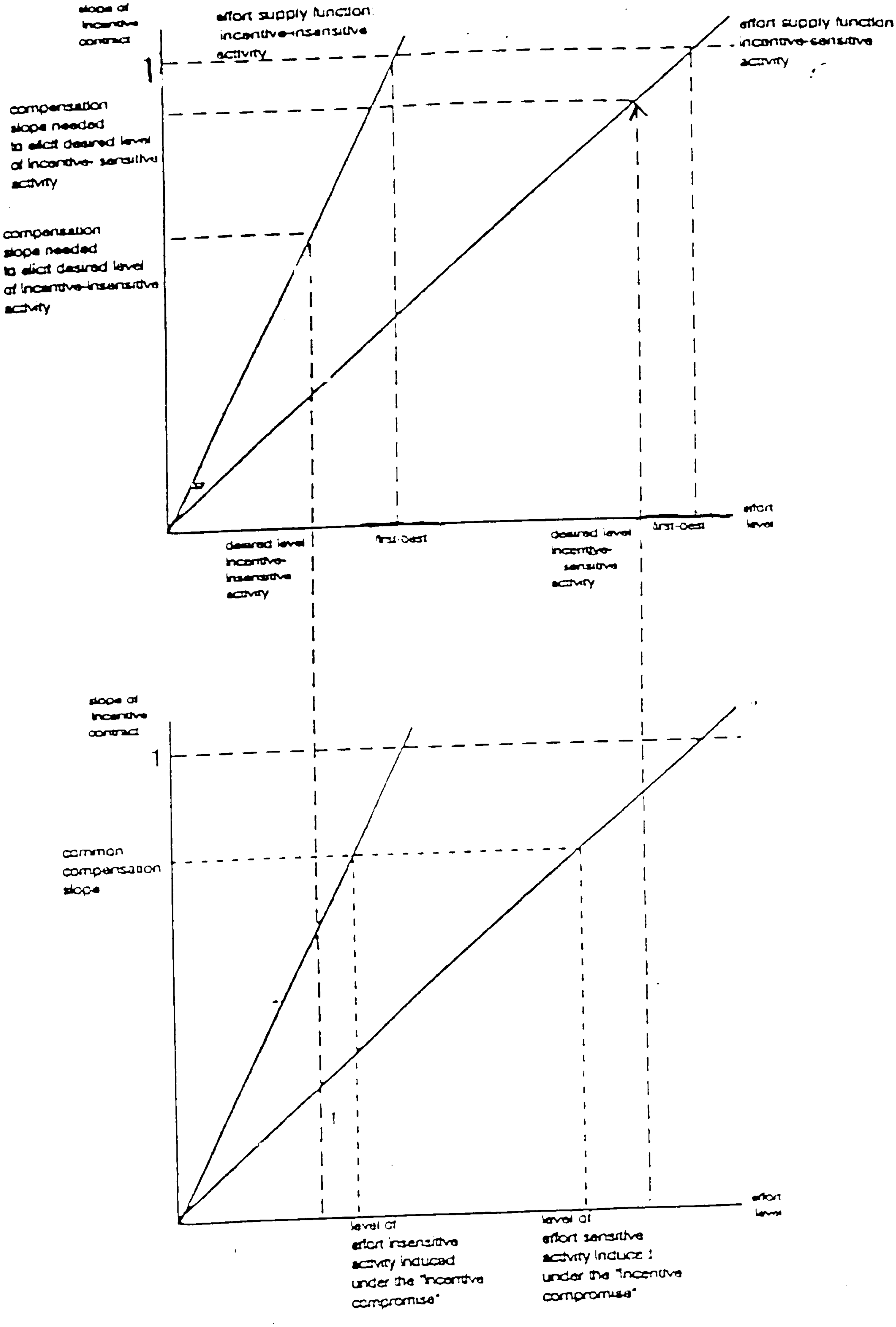

Figure 3: Effort Supply Functions

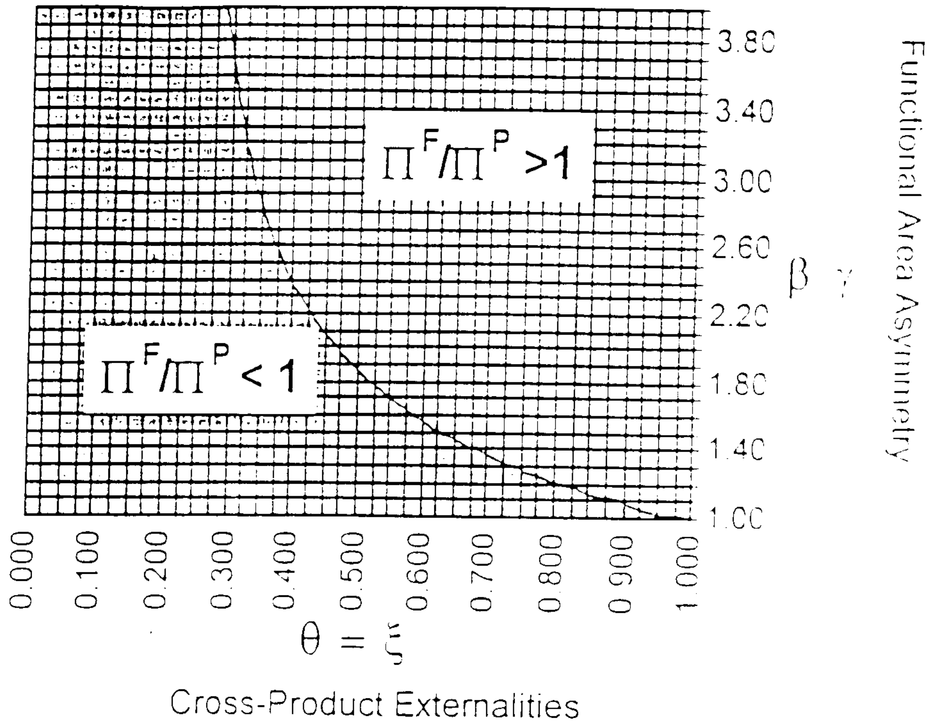

Figure 4: Relative Profitability of Functional and Product-Based Organizations with Cross-Product Externalities and Functional Area Asymmetries

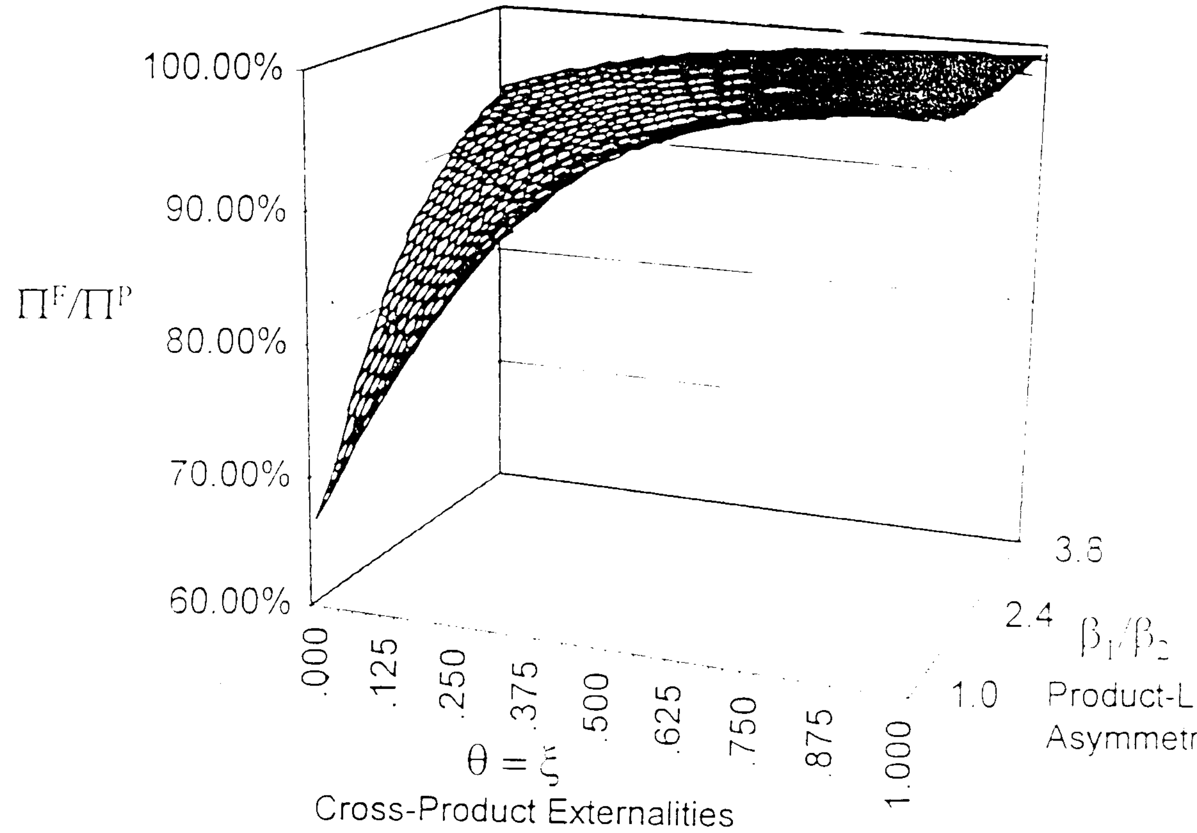

Figure 5: Relative Profitability of Functional and Product-Based Organizations with Cross-Product Externalities and Product Line Asymmetries

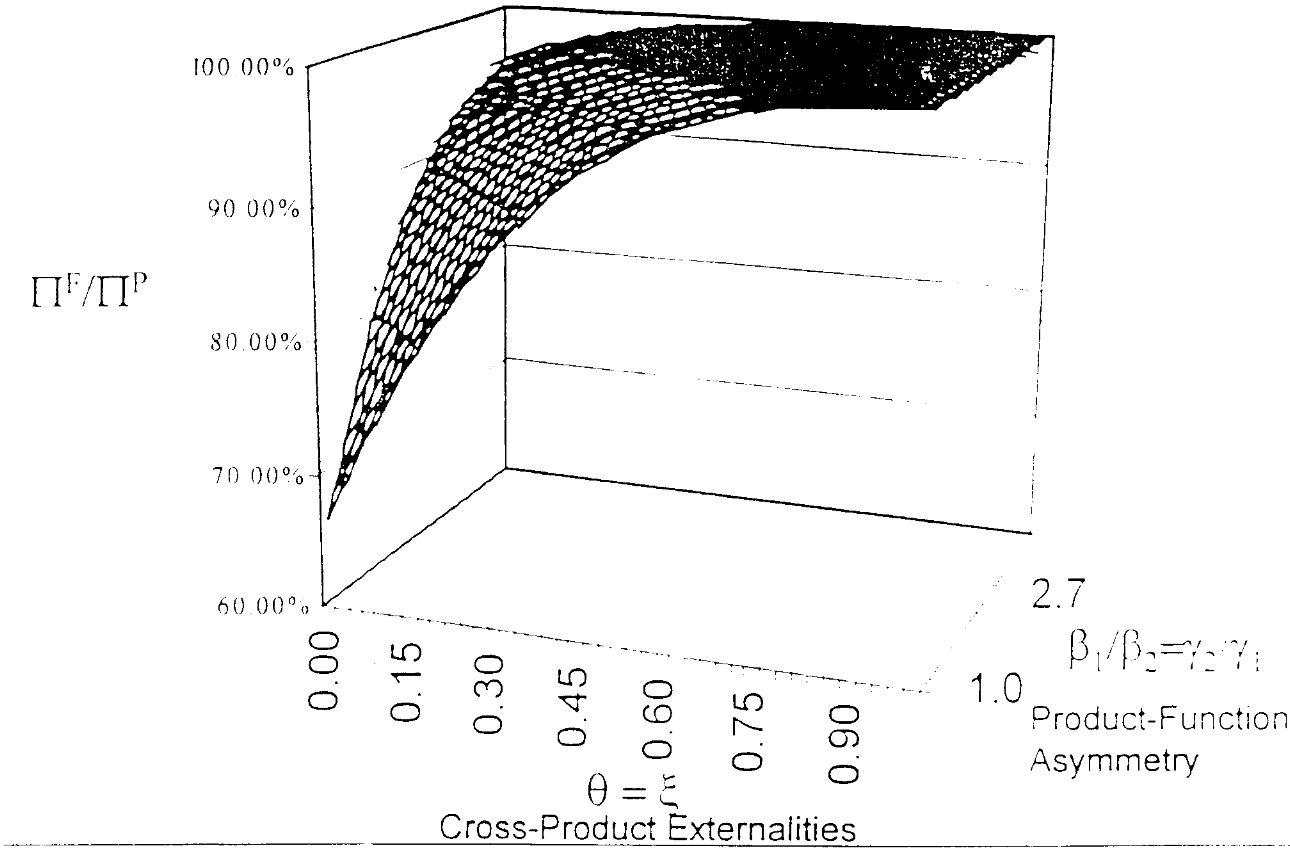

Figure 6: Relative Profitability of Functional and Product-Based Organizations with Privileged Product-Function Coupling ($\beta_1 = \gamma_2 > \beta_2 = \gamma_1$).

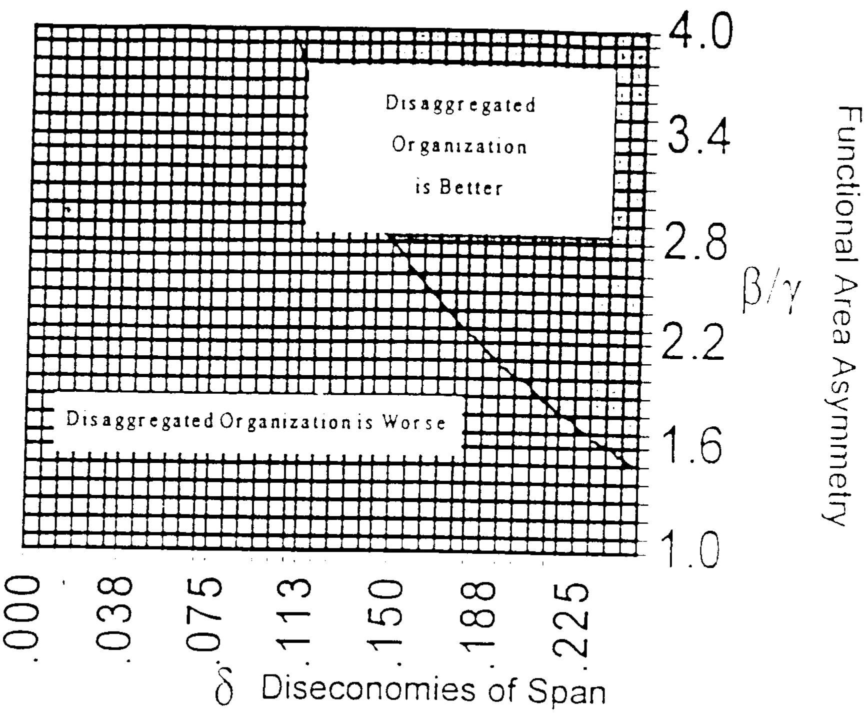

Figure 7: Comparison Between Disaggregated Organization and Functional and Product Organizations