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ABSTRACT

Looking Into The Black Box: A Survey Of The Matching Function*

We survey the microfoundations, empirical evidence and estimation issues underlying the aggregate matching function. Several microeconomic matching mechanisms have been suggested in the literature with some successes but none is generally accepted as superior to all others. Instead, an aggregate matching function with hires as a function of vacancies and unemployment has been successfully estimated for several countries. The Cobb-Douglas restrictions with constant returns to scale perform well. Recent work has utilized disaggregated data to go beyond aggregate estimates, with many refinements and suggestions for future research.

JEL Classification: E21, J60, J63, J64
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NON-TECHNICAL SUMMARY

Modern labour markets are characterized by large flows of jobs and workers between activity and inactivity. Every month a large number of workers join the unemployment pool because their jobs have ended and another large number leave the pool to take new jobs. Other workers move in and out of the labour force altogether, either permanently or temporarily. A large body of recent literature investigates the factors that influence unemployment in terms of their influence on these flows. A key building block that has attracted a lot of interest is the process that matches unemployed workers to available job vacancies.

The outcome of the matching process is influenced by frictions: heterogeneity in skills and locations, information imperfections, slow mobility of labour and capital, congestion from large numbers and other similar factors, and by the actions of firms and workers during recruitment and job search. A frequently used modelling tool that captures the outcome of this process is the aggregate matching function. It gives the number of job matches formed at each moment in time in terms of the inputs of firms and workers into search. In frictionless markets this number is the minimum of unemployed workers and job vacancies. In markets with frictions the number is less. The matching function captures the influence of frictions on market outcomes by introducing a well defined measure of frictions into otherwise competitive models. In this Paper we survey recent work on the existence and stability of the aggregate matching function, with emphasis on microfoundations and empirical findings.

Like most other aggregate functions in the macroeconomist’s tool kit, the matching function is a black box: we have good intuition about its existence and properties but only some tentative ideas about its microfoundations. Yet, those tentative ideas have not been rigorously tested. They have been used only to provide justification for the inclusion or exclusion of variables from the estimation of aggregate or regional matching functions, leaving it to the empirical specification to come up with a convincing functional form.

The most frequently used microeconomic matching mechanisms, which give rise to aggregate matching functions of the type that have been estimated, are based on coordination failures in the job application process and non-compatibility of the existing stocks of unemployed workers and job vacancies. When worker choices with respect to search intensity and job rejection are considered, both within these and other less specific mechanisms, it emerges that a large number of variables associated with the costs and rewards from employment and unemployment influence the aggregate matching rate.

The early aggregate studies estimated matching functions for the whole economy without imposing any specific restrictions on the underlying matching process. They converged on a Cobb-Douglas matching function with the flow of hires on the left-hand side and the stock of unemployment and job vacancies on the right-hand side, satisfying constant returns to scale, and with the coefficient on unemployment in the range 0.5–0.7. In some of the estimates that use total hires as a dependent variable (not only hires from unemployment) the coefficient on unemployment is lower, in the range 0.3–0.4, and the coefficient on vacancies correspondingly higher. But estimation of both Beveridge curves and aggregate matching function point also to other variables that influence the simple Cobb-Douglas relationship. Much of the estimation of matching functions in the last decade has looked for those other variables and for better empirical specifications. Micro studies suggest the age structure of the labour force, the geographical dispersion of job vacancies and unemployed workers, the incidence of long-term unemployment (exceeding one year), and unemployment insurance. Interestingly, however, although the other variables have been found significant where tested, unemployment insurance has not been identified as a significant influence on aggregate matching rates. We think that this may be related to measurement problems and the difficulty of getting a reliable time series for the generosity of unemployment insurance systems.

Recent empirical work has used disaggregate data and modelled the micro matching functions more carefully, paying attention to the issue of consistency between the timing of the flows and the timing of the stocks in the regressions, the regional spillovers in matching and the consistency between the flow and stock variables, given the observation that many matches involve either employed workers or workers classified as out of the labour force. The precision of the estimation has increased and the relation between hazard function estimation and aggregate matching function estimation has become clearer. It has been found that aggregation problems have played a role in some of the shifts in the aggregate matching function, though not to an extent that can render the aggregate function 'unstable'. Despite all the refinements and detailed tests, the findings of the first aggregate studies have not been challenged: the stable, constant returns aggregate function used in macroeconomic modelling finds strong support in the data of virtually all modern economies where tests have been conducted.

1. Introduction

Modern labor markets are characterized by large flows of jobs and workers between activity and inactivity.[1] Every month a large number of workers join the unemployment pool because their jobs have ended and another large number leave the pool to take new jobs. Other workers move in and out of the labor force altogether, either permanently or temporarily. A large body of recent literature investigates the factors that influence unemployment in terms of their influence on these flows. A key element of this structure is the process that matches unemployed workers to available job vacancies. The outcome of this process, in conjunction with the one that reallocates workers and jobs over time, is often shown graphically in vacancy-unemployment space by a convex-to-the-origin "Beveridge curve". The curve is further away from the origin in markets characterized by more reallocation or in markets where the process that matches firms and workers is less effective.[2]

The outcome of the matching process is influenced by frictions - heterogeneities in skills and locations, information imperfections, slow mobility of labor and capital, congestion from large numbers, and other similar factors, and by the actions of firms and workers during recruitment and job search. A frequently-used modeling tool that captures the outcome of this process is the aggregate matching function. It gives the number of job matches formed at each moment in time in terms of the inputs of firms and workers into search. In frictionless markets this number is the minimum of unemployed workers and job vacancies. In markets with frictions the number is less. The matching function captures the influence of frictions on market outcomes by introducing a well-defined measure of frictions into otherwise competitive models.

Frictions introduce monopoly rents in competitive markets which influence wages and the behavior of firms and workers. So even if empirically labor market outcomes are such that most unemployment is not matched by job vacancies, it does not follow that the reasons for this unemployment are unrelated to frictions. It generally does not make either theoretical or empirical sense to break up unemployment into "frictional" and other kinds. Understanding unemployment in this framework requires a formal model of equilibrium with frictions. The matching function is one of the modeling tools in this framework but it is one that plays a central role, as the frequently-used "search and matching" label for this literature testifies.

This paper surveys recent work on the existence and stability of the aggregate

[1] See Davis et al. (1996) on the importance of job flows and Blanchard and Diamond (1990a) on the importance of worker flows in the United States, and Burda and Wyplosz (1994) and Contini et al. (1995) on flows in Europe.

[2] See Pissarides (2000, chapter 1), Blanchard and Diamond (1989) and the many other references listed in the notes on the literature to chapters 1 and 2 of Pissarides (2000).

matching function, with emphasis on microfoundations and empirical findings. We do not discuss equilibrium models of the labor market, or address questions of policy, which have been surveyed elsewhere.[3] In section 2 we discuss the main ideas behind the matching function and we give some pertinent evidence. We then take a look at the theoretical foundations of the matching function and discuss some of the more important variables that are likely to be influential in empirical work (section 3). Section 4 discusses empirical results in the context of the methods most frequently adopted in the estimation of the matching function. Section 5 deals with the conceptual and measurement issues due to search on the job and to the transitions of workers from out of the labor force to employment. Aggregation problems across time and space are discussed in section 6. The main conclusions are brought together in section 7. The Appendix gives a brief historical overview of the literature on the role of labor market frictions, leading to the birth of the matching function.

2. The key idea and some evidence

The matching function summarizes a trading technology between agents who place advertisements, read newspapers and trade magazines, go to employment agencies and mobilize local networks that eventually bring them together into productive matches. The key idea is that this complicated exchange process is summarized by a well-behaved function that gives the number of jobs formed at any moment in time in terms of the number of workers looking for jobs, the number of firms looking for workers and a small number of other variables.

The matching function is a modeling device that occupies the same place in the macroeconomist's tool kit as other aggregate functions, such as the production function and the demand for money function. Like the other aggregate functions its usefulness depends on its empirical viability and on how successful it is in capturing the key implications of the heterogeneities and frictions. In this survey we will focus on the microfoundations underlying the matching function and on its empirical success but we will not discuss its modeling effectiveness.

The simplest form of the matching function is

$$M = m(U, V), \tag{2.1}$$

where M is the number of jobs formed during a given time interval, U is the number of unemployed workers looking for work and V the number of vacant jobs.

[3] Much of the equilibrium literature is surveyed by Mortensen and Pissarides (1999a,b). Equilibrium models up to 1984 are surveyed by Mortensen (1986) and empirical research in the context of search theory is surveyed by Devine and Kiefer (1991).

The matching function is assumed increasing in both its arguments and concave and usually homogeneous of degree 1. Testing for homogeneity, or constant returns to scale, has been one of the preoccupations of the empirical literature. Other restrictions usually imposed are $m(0, V) = m(U, 0) = 0$ and in discrete-time models where M is the flow of matches during an elementary period and U and V are the stocks at the beginning of the period, $m(U, V) \leq \min(U, V)$. In continuous time models, M is the instantaneous rate of job matching and U and V the instantaneous stocks of unemployment and vacancies. In the absence of frictions, $M = \min(U, V)$ in discrete-time formulations and $M \rightarrow \infty$ in continuous-time models. Under constant returns to scale, M, U and V are usually normalized by the labor force size, and denoted by lower-case letters.

On average an unemployed worker finds a job during a period of unit length with probability $m(U, V)/U$. Similarly, a vacant job is filled with probability $m(U, V)/V$. In a stationary environment, the inverse of each probability is the mean duration of unemployment and vacancies and respectively. Of course, if workers and jobs are heterogeneous, the transition probabilities (or hazard rates) will differ across the labor market, as will the mean durations. The aggregate matching function is a useful device for introducing heterogeneities across workers, by making the probability $m(U, V)/U$ depend on individual characteristics. This has been a theme of the empirical literature which estimates hazard functions for individual workers.

The dependence of the mean transition rates on the number of workers and firms engaged in search is an externality that has played an important role in the analysis of the efficiency of search equilibrium. The average time that it takes a firm to find a worker depends on what searching workers do before they meet the firm. Similarly, the probability that an unemployed worker finds a job depends on what hiring firms do, for example on whether they advertise or not and where they advertise. Generally, search equilibrium is inefficient because when firms and workers meet the costs of their search, which influence the transition probabilities, are sunk.

The returns to scale in the matching function play an important role in models with endogenous search effort. If there are increasing returns to matching, as the authors of some early models assumed (Diamond, 1982a, Howitt and McAffee, 1987) then there could be more than one equilibrium because of the complementarity between the search inputs of firms and workers: in one equilibrium firms and workers put more resources into search, pushing up the returns from search available to the other side, which justify the bigger inputs; in another they put less effort into search with lower returns form search, lower matching rate and higher unemployment. Increasing returns to scale can support the high and low activity equilibria even when there are increasing marginal costs to search effort, whereas constant returns cannot (although the complementarity between the actions of firms and workers is still present).

Evidence on the key matching-function idea comes form four sources. The first one uses aggregate data on stocks of unemployment and vacancies and estimates an equilibrium relation, the Beveridge (or UV) curve. The second uses aggregate data on flows out of unemployment and estimates the aggregate matching function, either for the whole economy or for a particular sector (usually manufacturing). The third uses data on local labor markets, which can be either a time-series or a panel, and estimates the matching function for each. The fourth uses data on individual transitions and estimates hazard functions for unemployed workers. We discuss each approach in some detail in subsequent sections. Here we summarize the main implications of the empirical research for the simple matching function in (2.1).

The Beveridge curve is an equilibrium relation that equates flows in with flows out of unemployment. In vacancy-unemployment space it slopes downward if the outflow from unemployment is given by the matching function in (2.1). Estimated Beveridge curves slope downwards but shift over time, especially in cases where there have been secular increases in unemployment, as in most European countries since the mid 1970s. So the matching function in (2.1) is not contradicted by the Beveridge curve evidence, but this evidence is indirect, it is consistent with other mechanisms and points to other variables that influence matchings too.

Direct estimates of the matching function give better information about the properties of (2.1). Most studies that estimate aggregate functions find that a log-linear approximation to (2.1) fits the data well. When aggregate data are used the estimated functions satisfy constant returns to scale but some estimates with manufacturing data show mildly increasing returns. The estimated elasticities with respect to unemployment and vacancies vary, depending on whether the dependent variable is the outflow from unemployment or the flow from unemployment to employment. When the dependent variable is the total outflow from unemployment the estimated elasticity on unemployment is about 0.7 and the elasticity on vacancies 0.3. Precise data on unemployment-to-employment transitions are rarely available but when an approximation for the matching rate is used the elasticity on unemployment drops, although not by much when other flows into employment are ignored. A plausible range for the empirical elasticity of unemployment is 0.5 to 0.7. There are good reasons for the drop in the elasticity estimates when flows from unemployment to non-employment are ignored, that we discuss when we discuss the estimates in more detail.

The aggregate estimates also find that there are other variables that influence matching in a systematic way. Disaggregate estimates have not contradicted the aggregate estimates but concentrated instead on finding out what are those other variables and whether aggregation introduces biases that can be estimated. With the number of estimates growing significantly in recent years, it is natural that there are estimates of both increasing and decreasing returns to scale. But such divergencies from constant returns are only mild and rare. The stylized fact that emerges from the empirical literature is that there is a stable aggregate matching function of a few variables that satisfies the Cobb-Douglas restrictions with constant returns to scale in vacancies and unemployment.

3. Microfoundations

What are the reasons for the existence of a well-behaved matching function and what are those other variables that influence the matching rate? In order to answer these questions we need to look at the microfoundations behind the aggregate matching function. The literature has done that; but although there are several microeconomic models that can be used to justify the existence of an aggregate matching function, none commands universal support and none convincingly says why the aggregate matching function should be of the Cobb-Douglas form. The literature has had more success, however, in suggesting what should be the other variables that influence the matching rate.

The other variables can be classified into two groups. The first group includes everything that individuals do during search, such as choosing how many applications to make, changing their advertising methods etc. The second includes shifts unrelated to individual search decisions. We take up the second group first. Most of the theoretical work on matching functions studies individual behavior and is discussed in the subsections that follow.

3.1. Mismatch

The shifts in the matching function that are unrelated to search decisions are due to technological advances in matching and to aggregation issues. Technological advances include reforms such as the computerization of employment offices, job advertising on the internet, an increase in the resources that governments put into matching, and other similar changes. Although changes of this type have been observed recently in most industrial countries (see OECD, 1994, ch. 6, 1999) and they have influenced the matching process to the extent that the OECD recommends them to its members as the most cost-effective "active" labor market policies, they have attracted little formal theoretical or empirical work.

Aggregation issues have attracted more attention from labor economists, often disguised under the label "mismatch". Mismatch is an empirical concept that measures the degree of heterogeneity in the labor market across a number of dimensions, usually restricted to skills, industrial sector and location. If mismatch in an economy were identically zero in all its dimensions, the matching function would not exist and jobs and workers would match instantaneously. It is because of the existence of some mismatch that meetings take place only after a search and application process. If there is an exogenous rise in mismatch, the rate of job matches at given inputs must fall, implying a shift in the aggregate matching function.

Of course, if empirically mismatch changes frequently in ways that cannot be accurately measured, the usefulness of the concept of the matching function is reduced. But this requirement is not different from the one on other aggregate functions in the macroeconomist's tool kit. Some of the early controversies in production theory (like the capital controversy of the two Cambridges) were about the question whether factors of production could be aggregated into two or three composites that enter a single-valued differentiable production function. Whether in practice aggregation problems are serious enough to question the usefulness of the matching function is an empirical question. The available evidence does not support serious aggregation problems that cannot be dealt with empirically.

In the empirical literature, mismatch bears some relationship to the frequently discussed "sectorial shifts hypothesis", and to the older view of "structural" unemployment, which was thought to be unemployment arising from fast structural change in the economy as a whole. For example, it has been argued that the oil, technology and other supply shocks of the 1970s and 1980s increased the speed with which unemployed workers needed to adapt to the changing requirements of employers. This led to increased mismatch between the skills possessed by workers and the skill requirements of employers, which increased the duration of unemployment (and hence the stock of unemployment) at given vacancies.

Lilien (1982) interprets mismatch as sectorial turbulence, and argues that the variance of the sectorial growth rates in employment should adequately account for fluctuations in employment due to mismatch. He finds that the mismatch hypothesis has some success in explaining US employment data but his findings have been effectively criticized (Abraham and Katz, 1986, Blanchard and Diamond, 1989).

Layard et al. (1991, chapter 6) follow a different approach and measure mismatch by the variance of sectorial unemployment rates. They show, however, that their measure of mismatch cannot account for the shifts in the aggregate matching function or the variance in UK unemployment. More recently, Manacorda and Petrongolo (1999) propose a measure of skill mismatch that makes use of information about the demand and the supply of skills, represented respectively by productivity parameters and labor force shares. This leads them to the conclusion that the unbalanced evolution of the demand and the supply of skills can explain some of the rise in unemployment in Britain and hence some of the shifts in the matching function but still not all.

On balance neither the sectorial shifts hypothesis nor mismatch has had much success in accounting for a large fraction of fluctuations in employment. So although empirical mismatch variables can account for some of the shifts in the aggregate matching function, we should look elsewhere for shift variables too. But some authors (see for example Entorf, 1998) argue that the measurement of mismatch in aggregate studies of matching functions still suffers from many problems and may be able to account for more of the unexplained variance in matchings than currently found in the literature.

If aggregation problems are not an issue, what can account for the matching function and what else can shift it?

3.2. Coordination failures

The first matching function owes its origins to a well-known problem analyzed by probability theorists, that of randomly placing balls in urns (Butters, 1977, Hall, 1979, Pissarides, 1979, Blanchard and Diamond, 1994). Firms play the role of urns and workers the role of balls. Even if there was exactly the same number of urns and balls, it is well-known that a random placing of the balls in the urns will not match all the pairs exactly, because of a coordination failure by those placing the balls in the urns. Some urns will end up with more than one ball and some with none. In the context of the labor market, if only one worker could occupy each job, an uncoordinated application process by workers will lead to overcrowding in some jobs and no applications to others. The imperfection that leads to unemployment here is the lack of information about other workers' actions, though simple extensions could enrich the source of frictions.

In the simplest version of this process U workers know exactly the location of V job vacancies and send one application each. If a vacancy receives one or more applications it selects an applicant at random and forms a match. The other applicants are returned to the pool of unemployed workers to apply again. The matching function is derived by writing down an expression for the number of vacancies that do not receive any applications. Given that each vacancy receives a worker's application with probability $1/V$, and there are U applicants, there is a probability $(1-1/V)^U$ that a given vacancy will not receive any applications at all. Therefore, the number of matches that take place at each application round is

$$M = V\left[1 - (1 - 1/V)^U\right]. \tag{3.1}$$

For a large V a good approximation to $(1-1/V)^U$ is the exponential $e^{-U/V}$, giving the matching function

$$M = V\left(1 - e^{-U/V}\right). \tag{3.2}$$

This matching function clearly satisfies the properties satisfied by the general function in (2.1) and in addition it satisfies constant returns to scale. It is, however, too restrictive to be empirically a good approximation to matching in real labor markets. For example, at a level of unemployment that is twice as large as the vacancy rate, the duration of unemployment is $U/M = 2.31$. Given that the length of the period is the time that it takes workers to send one application, the result is a mean duration of unemployment that is too short. Normally, in modern industrial societies, a level of unemployment that is twice as high as the level of vacancies is associated with a mean duration of unemployment of 3-6 months, giving an implausibly long lag between job applications. The function (3.2) also implies an implausible combination of levels and durations of unemployment. For example, if the level of unemployment and vacancies is the same, the mean duration of unemployment is 1.58 and if the level of unemployment is three times as high as that of vacancies, mean duration is 3.16. In actual labor markets duration would rise by more than the function (3.2) implies.

The introduction of small additional frictions to the urn-ball framework can enrich the matching function considerably. For example, suppose workers do not know the firms with the vacancies and choose at random one firm to apply. Then the probability that a vacancy receives no applications is $(1-1/(N+V))^U$, where N is the level of employment. If in addition the labor force size is L, $N = L - U$, and so the matching function becomes,

$$M = V\left(1 - e^{-U/(L-U+V)}\right). \tag{3.3}$$

This matching function exhibits increasing returns to scale in U and V and may even fail the assumption of diminishing returns to unemployment, though this would require that the level of vacancies exceeds that of employment. But it satisfies constant returns to L, U and V, so it avoids the counter-factual implication that larger countries should have lower equilibrium unemployment rates than otherwise identical smaller economies.

Also interesting is the extension that assumes fewer applications per period than the full number of workers. Suppose for example that a fraction $1 - s$ of the unemployed do not apply for a job in a given period. This fraction rotates, so each unemployed workers misses one application round out of every $1/(1 - s)$ rounds. Then, the probability that a given vacancy receives no applications during a given application round is $(1 - 1/V)^{sU}$, giving the matching function

$$M = V\left(1 - e^{-sU/V}\right). \tag{3.4}$$

This matching function satisfies all the properties of (2.1) for given s, but in addition opens up the possibility of modeling the frequency of applications s and so bringing the simple form (3.2) closer to the data. The mean duration of unemployment for this function is again U/M and so a lower application frequency s gives the longer mean durations for given vacancy-to-unemployment ratio that the data suggest. We take up the question of what might determine s next.

3.3. Worker heterogeneity: search intensity and reservation wages

The hazard rates (or unemployment durations) derived in the preceding section were for "representative" individuals, without much dependence on individual characteristics. Yet, in empirical estimates, it is found that individual characteristics play an important role in accounting for differences in hazard rates across individuals. In this sub-section and the next, we suggest two ways of introducing the influence of individual characteristics in the matching technology and show what this does to the aggregate matching function.

Worker heterogeneity is most conveniently introduced into the matching function by making the assumption that the intensity of search is a choice variable. We define intensity of search as the number of "units" of search supplied by a given individual. Units are defined as follows. If individual i supplies s_i units of search and individual j supplies s_j units, then in a small time interval individual i is s_i/s_j times more likely than individual j is to find a match. Search units are supplied at a cost, which is normally increasing, and they are chosen optimally to maximize the net returns from search (see Pissarides, 2000, chapter 5). Therefore, different individuals will choose different number of search units, depending on their search costs, the cost of unemployment and the expected returns from employment.

To derive the matching function implied by this extension, let s be the average number of search units supplied by an unemployed person.[4] Then, the total number of search units supplied is sU, and so the aggregate matching function is

$$M = m(sU, V), \tag{3.5}$$

a more general form of (3.4). Of course, varying intensity could also be introduced for job vacancies, in symmetric fashion. The hazard rate for an individual who supplies s_i units of search is $s_i m(sU, V)/sU$. The fact that this function depends on individual characteristics through the optimal choice of intensity of search justifies the econometric estimates of hazard functions that make use of individual survey data. On average, the representative individual will choose intensity s,

[4]This s bears a close resemblence to the s of the preceding section, which explains the use of a common symbol.

so the average transition rate for unemployed workers, which can be used in macro modeling, is $m(sU, V)/U$. Estimated matching functions can include all market parameters that influence search intensity, such as the level and duration of unemployment benefits, the mean elapsed unemployment duration and others.

Another channel through which heterogeneity can influence the matching function and market outcomes arises when there is a distribution of wage offers. The distribution may be due to either identical firms offering different wages, as in the model of Burdett and Mortensen (1998), or to match heterogeneity, as in the model of Jovanovic (1979). The individual chooses a reservation wage and rejects all wage offers below the reservation. In equilibrium models the reservation wage for each job that the worker encounters is such that neither the firm nor the worker will want to form a match if the wage is below reservation (see Pissarides, 2000, chapter 6). Of course, if individual characteristics differ, workers may choose different reservation wages.

Let now $m(U, V)$ be the technology that brings vacant jobs and unemployed workers together. When a pair meets it is faced with a wage offer w, which is assumed to be a drawing from a probability distribution $G(w)$. If the probability distribution is known to job seekers the optimal policy of individual i is characterized by a reservation wage R_i, such that the job is accepted if $w \geq R_i$, rejected otherwise. The hazard rate for this individual is $[1 - G(R_i)]m(U, V)/U$. Aggregation over all individuals gives the average transition rate and from there, multiplication by the unemployment rate gives the aggregate matching function. Clearly, given that in general the probability $G(R_i)$ is non-linear, the aggregate function takes a rather complicated form, but it is common in macroeconomic modeling to ignore this aggregation problem, define R as the average reservation wage and write the aggregate matching function as

$$M = [1 - G(R)], m(U, V). \tag{3.6}$$

As with the function derived for variable search intensity, (3.5), this function justifies the introduction of aggregate variables that influence individual decisions during search into estimated matching functions. The variables can be demographic variables that influence the intensity of search - for example, if youths search with lower intensity than adults the youth share in the population should be a shift variable. Or they can be variables that influence the cost of search and moving, such as unemployment insurance variables and housing transaction costs. The list of variables that can influence search intensity and reservation wages has been a fertile ground for searching for statistically significant shift variables in empirical matching functions, an issue discussed in the empirical sections that follow.

3.4. Ranking

Blanchard and Diamond (1994) consider the alternative assumption that firms receive many applications at a time and have preferences over job applicants. They rank applicants and offer the job to the person first in the rank. Their motivation for studying this process is a feature of European labor markets, that with the rise in unemployment durations, the long-term unemployed became "disenfranchised" and less good employees than those with more recent work experience.

The matching function used by Blanchard and Diamond (1994) is similar to the urn-ball function (3.2) but the implications of the ranking principle can be illustrated more generally. Suppose the unemployed are divided into two groups, the short-term unemployed and the long-term unemployed. Let the number of short-term unemployed be U_S and the number of long-term unemployed be U_L. Then, if a short-term and a long-term unemployed compete for the same job, the short-term unemployed always gets it. Therefore, the long-term unemployed do not cause congestion for the short-term unemployed during search and the long-term unemployed get only jobs for which there are no short-term applicants. The implication of the first claim is that the matching function for the short term unemployed is $m^S(U_S, V)$, where V are all the vacancies and the matching function satisfies all the properties of (2.1). If the long-term unemployed knew which vacancies are now being taken by the short-term unemployed, their matching function would be $m^L(U_L, V - M^S)$. But more generally, if there is a coordination failure between short-term and long-term unemployed, we write as usual $m(U^S + U^L, V)$ for total matches and then attribute the difference between M and M^S to matches involving long-term unemployed. That is, the aggregate matching function is

$$M = m(U^S + U^L, V) \tag{3.7}$$

but the hazard rate for the short-term unemployed is $m^S(U^S, V)/U^S$ and for the long-term unemployed $m(U^S + U^L, V)/U^L - m^S(U^S, V)/U^L$.[5] Simple calculations show that if the matching functions are identical the hazard rate of the short-term unemployed is always greater than the hazard rate of the long-term unemployed.

Blanchard and Diamond (1989) estimate a specification similar to (3.7) and impose that the short- and long-term unemployed are perfect substitutes up to a scale parameter. If the estimated value of this parameter is below one it is evidence in favor of the ranking hypothesis. Their point estimate of the scale parameter, however, slightly exceeds one, but is not significantly different from zero.

[5] Note that the expected duration of unemployement of the long-term unemployed is the inverse of their hazard rate but for the short-term unemployed account has to be taken of the fact that if they survive to long-term unemployment, their hazard rate will fall.

An alternative way to test for ranking in an aggregate matching function builds on the implication that the average matching rate should be higher the lower the incidence of long-term unemployment. This implies that equation (3.7) can be rewritten as

$$M = m\left(U^S + U^L, V, \frac{U^L}{U^S + U^L}\right) \tag{3.8}$$

where the last variable included should have a negative impact on the matching rate. This prediction is confirmed by Burgess (1993) for Britain, Mumford and Smith (1999) for Australia, and Bell (1997) for Britain, France, and Spain. Finding a significant long-term unemployment ratio, however, does not confirm the ranking hypothesis against the alternative that the long-term unemployed have lower search intensity. So although the evidence is consistent with the claim that the long-term unemployed are less successful applicants for available vacancies, the jury is still out about the reasons.

3.5. Stock-flow matching

The matching functions discussed so far were derived under the assumption that job seekers take a vacant job at random and apply for it. This assumption is convenient and realistic in many situations, given that there is an element of luck in hearing about job offers. But there is also a systematic element in search. This sub-section and the next discuss the derivation of an aggregate matching function from assumptions that go to the other extreme of no randomness in job applications.

Coles (1994) and Coles and Smith (1998) consider the implications of the assumption that job seekers have complete information about the available job vacancies and apply simultaneously to all the ones that they think are likely to be acceptable. Let this number be the entire universe of jobs on offer. But because of heterogeneity, not all job matches turn out to be acceptable. Let a constant α be the probability that a job match is unacceptable to the pair. A matching round then begins in a "marketplace". Job-worker pairs that made contact and are unacceptable are rejected. The remaining acceptable ones are sorted out so that no firm and worker who could form an acceptable match remain unmatched. Thus, unlike the urn-ball process of the preceding example, there is no coordination failure in this case. Those workers who remain unmatched do so because there are no vacancies that are suitable for them among the existing pool.

It follows that no job vacancy or unemployed worker who has been through one round of matching will attempt to match again with a pre-existing job seeker or vacancy. Of course, the assumption that the length of time when job seekers and vacant jobs get to know each other is one matching period is a simplifying one. Coles and Smith's assumption captures a realistic feature of search markets, that a job seeker scans a lot of advertisements before deciding where to apply and once an advertisement has been scanned and rejected, return to it is less likely than application to a new advertisement.

Under Coles and Smith's assumption there is a sharp distinction between the stocks of unemployed workers and vacant jobs and the new inflows. The stock of unemployed workers at the beginning of the period will not match with the stock of vacant jobs also at the beginning of the period, because they were both participants in the matching round in the previous period. The resulting matching process is therefore one where the unmatched stock of traders on one side of the market is trying to match with the flow of traders on the other side. This is often referred to as "stock-flow" matching.[6]

Let the stocks at the beginning of the period be U_0 and V_0. If the flow of new unemployed workers and new job vacancies into the respective pools during the period are U_1 and V_1, the U_0 initial workers match with the new inflow V_1 only, whereas the inflow U_1 matches with both V_0 and V_1. Coles and Smith consider a period of infinitesimal length and so ignore the probability of a newly unemployed worker matching with a newly created vacant job. In this case, the probability that a new vacancy is matched on entry is $1 - \alpha^{U_0}$, so the matches due to new vacancy creation are $V_1(1 - \alpha^{U_0})$. Recall that α is the probability that a random pairing is unacceptable. The probability that a new worker is matched on entry is $1 - \alpha^{V_0}$ and so the new matches due to the new entry of workers is $U_1(1 - \alpha^{V_0})$. Since there are no matches between old unemployed and old vacancies, the sum of the two matches gives the entire matching rate in the economy. That is, the matching function is

$$M = V_1(1 - \alpha^{U_0}) + U_1(1 - \alpha^{V_0}), \tag{3.9}$$

with $1 > \alpha > 0$.

The hazard rate for workers who are unemployed at the beginning of the period is $V_1(1 - \alpha^{U_0})/U_0$ and for the new inflow $1 - \alpha^{V_0}$. The latter is likely to be smaller because for the short period under analysis, the stock of jobs and workers is likely to be much bigger than the new flow, i.e., $V_1 \ll V_0$ and $V_1 \ll U_0$.

The matching function in (3.9) exhibits increasing returns to scale in the stocks and the flows, although it is not homogeneous. The reason is that job seekers apply to all the available job vacancies simultaneously. If we double the number of job vacancies and unemployed workers, the applications of each and every job seeker double. This contrasts with the matching function in (3.2), where each job seeker

[6] Coles (1999) discusses the turnover externalities implied by stock-flow matching.

applies only to one job and so doubling the number of jobs doubles the number of applications. Applying to more than one vacancy at a time is a realistic feature of the application process but it depends on a constant rejection probability α. When the rejection probability is endogenized, we would expect it to increase when the matching probability increases. Intuitively, the model captures the fact that in a large market job seekers have more options but not the fact that they would be more choosy as a result.

The model implies that the matching probability for the unemployment inflow does not suffer from congestion, whereas the pre-existing unemployed suffer congestion from each other. This result derives from the assumption that newcomers flow into the market individually, given the continuous time structure of the matching process that takes place across time periods of infinitesimal length. If instead we consider time periods of discrete length, a newly unemployed can match with a new vacancy, and at the same time all the newly unemployed can cause congestion to one another when trying to match with existing vacancies. The extra congestion externalities generated in this case are shown by Gregg and Petrongolo (1997) to rule out increasing returns to scale.

Stock-flow matching has received some empirical support. Coles and Smith (1998) argue that, due to stock-flow matching, exit rates are higher when traders first enter the labor market, and drop sharply thereafter. This suggests that traders who are unlucky at their first round of search need to wait and queue for new entrants in order to find a suitable match. There are, however, many other reasons for the fall in unemployment exit rates, which include ranking, discouragement and loss of skills during unemployment. But more detailed evidence on matching combinations among labor market traders shows that stock-flow matching plays a significant role in raising the matching probabilities of recently unemployed workers.

Coles and Smith estimate a log-linear matching function dividing the outflow from unemployment into duration classes. They find that both the stock and the inflow of vacancies increase the unemployment outflow at short durations of search but at longer durations only the inflow of new vacancies increases significantly the job finding rates of the unemployed. Qualitatively similar results are also found by Gregg and Petrongolo (1997), who estimate quasi-structural outflow equations for unemployment and vacancies derived from a stock-flow matching model in discrete time.

3.6. Aggregation over distinct markets

We finally discuss a derivation of the aggregate matching function that relies on the existence of disequilibrium in micro markets and limited mobility of labor.

The assumption is that the economy is divided into micro markets that do not suffer from frictions but suffer from a disequilibrium in the sense that the demand for labor in each market is not equal to the supply. There is no mobility of labor or capital between markets. This assumption can be interpreted as the source of the friction that gives rise to the aggregate matching function. It implies that markets with unemployment can co-exist with markets with job vacancies although no market has both. Aggregation over all markets gives an aggregate function that contains both vacancies and unemployment. With perfect mobility workers would move until the short side of the aggregate economy cleared and no aggregate matching function would exist.

A model of this form was first used by Hansen (1970) to derive the Beveridge curve. Other studies that follow this approach are Drèze and Bean (1990), Bentolila and Dolado (1991) and Frank (1991). Borrowing results discussed by Drèze and Bean (1990, p. 14), who credit Lambert (1988) for the derivations, suppose that the ratio of vacancies to unemployment in each micro market is lognormally distributed. Then, if the short side of each market clears and U and V are the aggregate quantities, there is a CES-type relationship that could be interpreted as an aggregate matching function

$$M = \left(U^{-\rho} + V^{-\rho}\right)^{-1/\rho}, \tag{3.10}$$

where $\rho > 0$ is related to the variance of the ratio of unemployment to vacancies across micro markets.

The derivation of this matching function needs the assumptions of exogenous distributions of unemployment and vacancies across space. Lagos (1997) derives instead optimal rules for the allocation of agents across space, under the assumption that there is uncertainly about the number of agents at each location. He shows that the resulting matching equilibrium is one where the short side of the market clears - but now the number of agents on one side is optimally selected (see also Lagos and Violante, 1999).

As far as we are aware, there are no tests of this microfoundation for the aggregate matching function. A key problem here is to define the unit of the micro market. If a micro market is infinitesimally small, and consists of at most one job each, the assumption is trivially correct. If it is large and equal to the economy as a whole, the assumption is incorrect, since at the aggregate level vacancies and unemployment co-exist. A travel-to-work area would appear to be the most appropriate disaggregation level but no tests have been conducted at this level. Another difficulty with the CES form is that it relies on distributional assumptions about unemployment and vacancies and a test of the CES restrictions (e.g. versus Cobb-Douglas) would need to test the validity of the distributional assumptions as well.

4. Empirical methods and findings

In the matching framework the equilibrium levels of unemployment and job vacancies that persist in steady state are the result of the intensity of the job reallocation process and of the matching effectiveness of the labor market. One way of making inferences about the empirical properties of the matching function is to estimate such a long-run vacancy-unemployment relationship, the UV or Beveridge curve. The advantage from taking this indirect route is that estimation of the Beveridge curve requires only data on stock variables, not flows, which are more readily available. The early literature on matching followed mainly this approach. But partly because of the difficulty of making accurate inferences about the matching function from estimated Beveridge curves (outlined below) and partly because the connection between the matching function and the Beveridge curve became better understood, most of the empirical literature since the late 1980s and early 1990s estimated directly the matching function. As more data became available estimated matching functions appeared in the literature making use of aggregate time-series for the whole economy or for some sector (most frequently manufacturing), panel data for regions or districts, and data on individual re-employment hazards. We review the main results of each approach with focus on the results not previously discussed.

4.1. Beveridge curves

A steady-state Beveridge (or UV) relationship between the unemployment rate and the vacancy rate can be derived from the simple matching function (2.1). Let U and V be the number of unemployed workers and job vacancies respectively, and N and L the level of employment and the labor force (so $L = N + U$). Define the unemployment rate $u = U/L$ and let the vacancy rate be $v = V/N$ (an inconsequential change from the alternative $v = V/L$). Assume also that the job separation rate is λ, so total separations are $S = \lambda N$. Then, imposing constant returns to scale on $m(.)$ and noting that in steady state the number of matches M equals the number of job separations S, we get the Beveridge curve,[7]

$$\lambda = m\left(\frac{U}{L}\frac{L}{N}, \frac{V}{N}\right) = m\left(\frac{u}{1-u}, v\right). \tag{4.1}$$

Given the separation rate λ, our assumptions on $m(.)$ imply a negative steady-state relationship between the unemployment rate and the vacancy rate.

[7] Note that constant returns in U and V are not needed here. Suppose for example that the matching function has constant returns in U, V and N, as in (3.3) but increasing returns in U and V. Then dividing through by N gives an expression with properties similar to (4.1).

An aggregate Beveridge curve of the form of equation (4.1) was estimated by a number of authors for the aggregate stocks of vacancies and unemployment (see for example Jackman and Roper 1987, Budd et al. 1988, Jackman et al. 1989 and Wall and Zoega 1997 for Britain, Abraham 1987 for the US, Franz 1991 for Germany, Edin and Holmlund 1991 for Sweden, Brunello 1991 for Japan and Jackman et al. 1990 for a multi-country study). The form preferred is usually log-linear, which implies a Cobb-Douglas matching function if the foundation for the Beveridge curve is the aggregate matching function. All studies establish the existence of a negative long-run relationship between the vacancy rate and the unemployment rate, as implied by (4.1). But virtually all studies also identify some shift variables, not present yet in (4.1).

Of course, (4.1) is consistent with many different micro frameworks, some perhaps unrelated to the matching framework. But if we posit that there is an aggregate matching function underlying (4.1), some lessons immediately emerge from the Beveridge curve studies about the properties of the matching function.

First, there is support for the restrictions on the simple two-variable matching function, including some tentative evidence for constant returns. The negative convex-to-the-origin shape predicted by the model fits the data well and in the cross-country regressions country size does not appear to be an influence on the position of the Beveridge curve, something that would be implied by some models of increasing or decreasing returns to scale. But no study conducts a careful test of increasing returns to scale by testing, for example, whether the matching rate improves when the total number of participants increases for given ratio of vacancies to unemployment, or whether there are increasing returns to U and V but constant returns to U, V and L, as implied for example by (3.3).

Second, there have been shifts in the relationship, especially in European countries. These shifts coincide with the secular rise in European unemployment, which started in the mid 1970s. The unemployment rate has increased despite the fact that the separation rate and the vacancy rate, λ and v in (4.1), have not shown any trend. The implication for the matching function is that there are variables besides u and v that have played an important role in matching in the last two decades and these variables contributed to a deterioration of the matching rate.

Reasons that have been suggested in the literature include mismatch (Jackman et al., 1989) - which as we have seen may explain some but not much of the shift - the growth in long-term unemployment, which reduces both the search intensity of the unemployed and their employability, through loss of skill (Budd et al. 1988), the generosity of the unemployment insurance system (Jackman et al. 1989) and active labor market policy (Jackman et al. 1990). Jackman and Roper (1987) have shown that in Britain the shifts in the regional Beveridge curves were of the same order of magnitude as the aggregate curve, casting more doubt on the power of regional mismatch to explain the shift in the aggregate curve. On a more positive note, Jackman et al. (1990) show that the different position of the estimated Beveridge curves in Europe is positively correlated with their spending on active labor market policies. Countries with more spending on policies that aid matching have Beveridge curves closer to the origin.

But on average no single or combination of variables can account for the deterioration of the matching rate since the mid 1970s, and the literature often attributes it to unmeasured elements of the unemployment insurance system and mismatch. It is interesting that measured components of the unemployment insurance system do not play a role in the deterioration of the matching rate. Unmeasured elements mentioned in the literature are usually statements about the leniency of the system and its coverage. In the estimation such measures are usually picked up by time trends, which could of course account for many other unobserved or unidentified influences on matching.

Estimation of log-linear UV curves, along the lines followed by most of the studies mentioned, suffers from some problems, connected with the assumption of flow equilibrium, the endogeneity of the separation rate and the fact that inferences about the micro process underlying matching cannot be easily made from such an aggregate framework. More recent studies estimate matching functions by making use of flow data, which are more disaggregated and do not have to rely on either a constant (or exogenous) job separation rate or flow equilibrium.

4.2. Aggregate studies

Table 1 gives a summary of the results of studies that have estimated aggregate matching functions. Pissarides (1986) estimates an aggregate matching function for Britain over the period 1967-1983. The specification adopted uses quarterly data with the average monthly outflow rate from male unemployment during the quarter as the dependent variable. The unemployment series used is for registered male unemployment and the series for vacancies is notified vacancies adjusted upwards for incomplete coverage. Results with both linear and log-linear specifications are reported. The estimated log-linear specification is

$$\begin{aligned} \ln\left(\frac{M}{U}\right)_t &= \alpha_0 + \alpha_1 \ln\left(\frac{V}{U}\right)_t + \alpha_2 t + \alpha_3 t^2 \\ &\quad + \text{lags} + \text{structural variables}, \end{aligned} \tag{4.2}$$

Both the linear and log-linear specification strongly support constant returns to scale in U and V. The estimated elasticity of matches with respect to vacancies is 0.3 with an implied elasticity of matching with respect to unemployment 0.7. No other variables were found to be significant except for the time trends, which indicate a large fall in the rate of job matches at given unemployment and vacancy rate during the sample period.

Later estimation of a similar regression by Layard et al. (1991, chapter 5) for 1968-1988 found similar elasticity estimates but also found that the rise in long-term unemployment reduces the matching rate at given unemployment rate. But the time trend remains significant in their regression. Also, the authors do not deal with the endogeneity of long-term unemployment but measure its impact by computing an index for duration effects which takes a weighted average of duration by applying fixed weights that are proportional to the outflow rates from each category in a base year. The fact that outflow rates fall with duration and duration increases during the sample gives an upward trend to the index, which is positively correlated with the trend in unemployment.

Blanchard and Diamond (1989, 1990b) estimate a matching function for the United States over the period 1968-1981. The estimated equation is a log-linear specification in levels

$$\ln M_t = \alpha_0 + \alpha_1 \ln U_t + \alpha_2 \ln V_t + \alpha_3 t, \tag{4.3}$$

where the log of monthly national hirings is used as the dependent variable, unemployment is interpreted as a proxy for all job seekers (including employed and out-of-the-labor force) and the vacancy series was constructed from the help-wanted index. The estimated elasticities of matches with respect to vacancies and unemployment are positive and significant, and the time trend generally comes in with a negative and significant coefficient (but smaller than in Britain or other large European countries), implying a deterioration in the matching effectiveness of the labor market since the late 1960s. They find clear evidence of the existence of an aggregate matching function with constant or mildly increasing returns to scale, unit elasticity of substitution, and weights of 0.4 and 0.6 on unemployment and vacancies respectively. But the weight 0.4 is found when the unemployment rate is used as a proxy for all job seekers, which may not be appropriate when the number of employed job seekers is pro-cyclical; when the left-hand side variable is restricted to include only job matches from unemployment, the weight on unemployment rises to 0.6.

The higher unemployment elasticity of matches found in the British studies can be the result of the different dependent variables used in each case. Pissarides (1986) and Layard et al. (1991) use the total outflow from unemployment whereas Blanchard and Diamond (1989, 1990b) construct a flow variable that approximates the total number of hires (including job-to-job moves and flows from inactivity directly into employment, a point that is not relevant here but addressed in the next section). Burda and Wyplosz (1994), who estimate log-linear matching functions for France, Germany, Spain and the United Kingdom by regressing total exits from unemployment on vacancy and unemployment stocks, also found high elasticities of matches with respect to unemployment, in the range $0.5 - 0.7$. To see the point that we are making more formally, let X denote total exits from unemployment and M denote total hires again from unemployment. Let also D denote exits from unemployment to out of the labor force, a combination of "discouraged" worker effects, early retirement and going back to school. Let M be a log-linear constant returns to scale function of the type estimated by Blanchard and Diamond and D depend on vacancies with elasticity $-\alpha$ and on unemployment with elasticity β. If the tightness of the market V/U is a good measure of the cycle (as it is likely to be under constant returns, see Pissarides, 2000), and movements from unemployment to inactivity depended only on the cycle, we would expect $\alpha = \beta$. But if the experience of unemployment has additional influences on retirement and dropping out, we would expect on a priori grounds $\beta \geq \alpha$. The function estimated by the European studies is (with constants omitted)

$$\begin{aligned} X &= M + D \\ &= U^{\eta} V^{1-\eta} + U^{\beta} V^{-\alpha}. \end{aligned} \tag{4.4}$$

Studies that use a measure of M in their regressions estimate η directly. Blanchard and Diamond's estimate for this number is 0.6 and similar estimates (in the range $0.55 - 0.70$) are found by van Ours (1995), Boeri and Burda (1996) and Burda and Profit (1996).

Studies that use X as dependent variable in a log-linear regression approximately estimate

$$\begin{aligned} \frac{\partial X}{\partial U}\frac{U}{X} &= \eta \frac{M}{X} + \beta \frac{D}{X} \\ &= \eta + (\beta - \eta)\frac{D}{X} \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} \frac{\partial X}{\partial V}\frac{V}{X} &= (1-\eta)\frac{M}{X} - \alpha \frac{D}{X} \\ &= 1 - \eta - (1 - \eta - \alpha)\frac{D}{X}. \end{aligned} \tag{4.6}$$

Thus, if $\beta > \eta$ and $\alpha < 1 - \eta$, the estimate obtained by studies that use the total exit as dependent variable should be higher on unemployment and lower on vacancies. Moreover, given that both sets of studies find constant returns to scale, the parameters must be such that

$$\beta = 1 - \alpha. \tag{4.7}$$

If we make use of Blanchard's and Diamond's and others estimate of 0.6 for η and the Pissarides-Layard et al. estimate of 0.7 for the unemployment elasticity of total exits, and a plausible mean value for the ratio D/X (the fraction of unemployment exits that leave the labor force) of 0.4[8] we get $\beta = 0.85$ and $\alpha = 0.15$. It would appear from these numbers that the experience of unemployment is a bigger influence on dropping out of the labor force than the cycle is.[9]

Blanchard and Diamond (1989, 1990b) also estimate equation (4.3) for the US manufacturing sector alone. The results that they obtain in this case are broadly consistent with the aggregate ones, with the important qualification that the manufacturing matching function displays increasing rather than constant returns to scale. The estimated sum of the elasticity of matches with respect to vacancy and unemployment is now 1.4. Estimates for US manufacturing are also reported by Warren (1996) who estimates a more flexible translog function for all manufacturing for 1969-1973, when a vacancy series for this sector was available. The translog specification gives a more accurate estimate of the returns to scale of a technology than the Cobb-Douglas form (see Guilkey et al. 1983). The dependent variable in Warren's study is total hires in manufacturing and the unemployment variable consists of all those currently unemployed who previously held jobs in the manufacturing sector. The correspondence of the flow variable that is used as dependent variable with the stock on the right hand side is poor but almost inevitable when hires in only one sector are used. He finds statistically significant increasing returns to scale with sum of coefficients on vacancies and unemployment of 1.33. Similar results are found by Yashiv (2000), on both a log-linear and a translog matching function for the whole Israeli economy over the period 1975-1989. The estimated returns to scale in his matching function lie in the range 1.20-1.36.

The other studies summarized in Table 1 generally confirm the results of the earlier studies discussed in this section for different countries and time periods.

4.3. Sectorial studies

The difficulty with making inferences about labor market matching from aggregate time series beyond the initial results of the studies discussed in the preceding section led many authors to switch to more disaggregate specifications, either in panel or single cross-sections. Table 2 summarizes results for a number of sectorial studies.

[8] This is obtained from the data on worker flows reported by Burda and Wyplosz, 1994.

[9] It should be borne in mind that the numbers computed here are derived from the difference between a point estimate of 0.6 and one of 0.7 and they are sensitive to small changes in these estimates.

Anderson and Burgess (1995) estimate a state-industry panel for the United States over the period 1978-1984, using a similar specification to Blanchard and Diamond's (1989) aggregate study. They also include variables for sex and age composition of the labor force and the degree of unionization, and distinguish new hires by origin, namely whether they come from employment or non-employment. Their results confirm a higher elasticity of matches with respect to vacancies than to unemployment (roughly 0.8 and 0.4 respectively), and find some evidence in favor of increasing returns to scale in the matching technology.

In an attempt to apply the matching function analysis to local labor markets, Coles and Smith (1996) and Bennet and Pinto (1994) both provide cross-section estimates of the matching function for local labor markets in Britain. Local labor markets are represented in Coles and Smith (1996) by travel-to-work areas. They use data for 257 areas in 1987 and estimate a regression for total hirings. As in the US studies they find an elasticity of 0.7 on vacancies and 0.3 on unemployment. Their study also shows the importance of the geographic density of unemployment and vacancies in the hiring process, with more concentrated labor markets having higher matching rates. The analysis of Bennet and Pinto (1994) uses instead data from Training and Enterprise Councils, estimating a time series for each (Britain is divided into about 100 such areas). They find that the parameters of the matching technology do not vary substantially across districts, the elasticities being within a narrow range of 0.5 for both unemployment and vacancies, and therefore confirm that there are no serious problems of aggregation.

The Coles and Smith and the Bennet and Pinto studies treat local labor markets as isolated marketplaces. Interactions among neighboring districts are modeled by Burda and Profit (1996), Burgess and Profit (1998), and Petrongolo and Wasmer (1999), who find evidence of matching spillovers across space but with smaller coefficients for neighboring districts. This finding highlights the importance of moving costs in matching and is consistent with Coles' and Smith's finding that population density matters in local matching rates. We take up the issue of spatial aggregation below.

4.4. Micro studies

The estimation of re-employment probabilities for unemployed individuals has the potential of distinguishing between the determinants of the probability of receiving a job offer and that of accepting it. The former depends on the set of characteristics that influence a worker's productivity (such as age, education and experience) and on local labor demand conditions, which is the effect captured by aggregate matching functions. The second probability depends on a worker's reservation wage, and therefore on the expected distribution of wages, the cost of search, unemployment income and the probability of receiving a job offer.

Structural studies (see Kiefer and Neumann 1979a,b, 1981; Flinn and Heckman 1982, Narendranathan and Nickell 1985; Wolpin 1987; and Eckstein and Wolpin 1995) identify separately an accepted wage equation and a wage offer equation, and so they can distinguish between the determinants of each of these probabilities. Reduced-form or hazard function studies estimate instead the factors affecting the product of the two probabilities, namely the transition of workers from unemployment to employment, and are therefore more directly comparable with matching function results.

Despite this connection, however, micro studies have not been used in the empirical search literature to make inferences about the properties of the aggregate matching function, although there are exceptions. Their contribution can be twofold. Micro studies control for a number of individual characteristics which can be aggregated to give shift variables in the aggregate matching function besides U and V. They can also be used to test for the effect of local labor market conditions on re-employment probabilities and from there aggregate to make inferences about the influence of local conditions on aggregate matching.

The early study by Lancaster (1979) uses a sample of British unskilled male workers to show that exit rates from unemployment are negatively affected by age, the duration of search, and the local unemployment rate. The age effect in the job-finding hazard implies that the age composition of the labor force should play a role in aggregate matching function estimates, with a younger pool of job-seekers delivering higher exit rates from unemployment (see for example Coles and Smith 1996 and Anderson and Burgess 1995). Negative duration dependence in job search implies that the incidence of long-term unemployment should reduce the unemployment outflow in aggregate specifications, which is confirmed by, among others, Layard et al. (1991) and Burgess (1993). Finally, the negative effect of unemployment captures the congestion effect of a larger pool of job-seekers on individual job-finding rates. This should translate into an aggregate elasticity of matches with respect to unemployment less than 1, which is the case in all aggregate studies.

Following Lancaster's application of duration models to re-employment probabilities, a large number of papers have studied the determinants of exits rates from unemployment, looking at a variety of specifications and control variables.[10] Perhaps surprisingly, a result that frequently appears in the micro studies but not in aggregate studies is the influence of the unemployment insurance system

[10] See Devine and Kiefer (1991) for a survey of hazard studies. Despite a rich literature on the study of unemployment exit rates, little work has been done so far on vacancy durations. Notable exceptions are van Ours and Ridder (1992, 1993) and, more recently, Burdett and Cunningham (1998).

(see Nickell, 1979 and Narendranathan et al. 1985). Although there are dissenting voices (e.g. Atkinson et al. 1984), on balance micro studies find a (small) influence of unemployment insurance on re-employment probabilities. Aggregate studies have failed to find a robust effect, perhaps because of the complexity of the system and the difficulty of measuring accurately its dimensions in a time series. For example, it has been claimed that the duration of unemployment benefits is the most important dimension of the system that influences matching. But because there is very little time-series variation in the duration of entitlements, only cross-country data can be used to test for this effect. Yet, in cross-country regressions variations in durations are also limited, with some countries having unlimited durations and some restricting it to six or twelve months (see OECD, 1994, Pissarides, 1999). Another dimension of the unemployment insurance system that has been emphasized in descriptive work is the leniency of the system. In a time series it is difficult to get a good measure of leniency.

In conditioning on the state of the local labor market, only a few micro studies (Nickell, 1979, Atkinson et al. 1984, Lindeboom et al. 1994 and Petrongolo, 2000) take into account the demand side of the labor market and employers' search, by controlling for the local vacancy-to-unemployment ratio. A higher labor market tightness, represented by the V/U ratio, significantly increases the job-finding hazard in these studies, confirming the results of aggregate studies. An explicit test of the influence of the size of the local market on re-employment probabilities was conducted by Petrongolo (2000). Re-employment probabilities are conditioned on the number of unemployed workers and vacancies within the travel-to-work area of each worker. The coefficients on $\ln U_t$ and $\ln V_t$ are estimated separately and found to be not significantly different from each other across a number of different specifications, which confirms constant returns to scale in matching.[11]

5. Search on the job and out of the labor force

A large number of job matches in modern labor markets are transitions from other jobs or directly from out of the labor force to employment. The former has an unambiguous theoretical interpretation: some employed workers are active job seekers. The latter is more vague. Since anyone without a job and actively searching for one is classified as unemployed, the workers who move directly from out of the labor force to employment are most likely the result of measuring problems, due for example to the length of time between survey points. A worker

[11] Mention should also be made of the study by Lindeboom et al. (1994), who make use of the the link between the aggregate matching function and hazard rate specifications for evaluating the relative effectiveness of alternative search channels.

previously out of the labor force may become an active searcher and get a job within a week, and so miss the classification of unemployment in a monthly survey. In countries where labor force surveys are quarterly this problem can lead to large inflows of workers from out of the labor force to employment.

In principle there is no difficulty introducing employed job seekers in the models underlying the matching function (in theoretical work on matching, those out of the labor force who transit to employment directly do not have a separate status from the unemployed, as the period of analysis can be made sufficiently short to ensure that all those who enter employment pass first from the pool of job seekers). The way in which employed job seekers enter the matching function depends on the assumptions that one makes about their search behavior and its relation to that of the unemployed job seekers (Burgess, 1993, Pissarides, 1994). For example, if employers prefer employed job seekers to the unemployed, a ranking model could be used to arrive at (3.7), but with the number of employed job seekers taking the place of the short-term unemployed in the expression and the total number of unemployed workers ranking below them in the application queue.[12] If, on the other hand, it is believed that the main difference between employed and unemployed job seekers is in the choice of search intensity or reservation wage, a function like (3.5) or (3.6) would be more appropriate. We derive the matching function (3.6) when there are employed job seekers as an illustration, under the reasonable assumption that employed job seekers have different (usually higher) reservation wage than unemployed job seekers.

Let R_E be the mean reservation wage of employed job seekers and R_U the mean reservation wage of the unemployed. The number of unemployed seekers is as before U and the number of employed job seekers E. The number of job vacancies is V and all workers qualify for all vacancies. If the unemployed search with intensity s and the employed with intensity normalized to unity, the contact technology is $m(sU + E, V)$ and the probability that an employed worker meets a job vacancy is $m(sU + E, V)/(sU + E)$. The probability that this vacancy is acceptable is $1-G(R_E)$, so the hazard rate for the employed is $[1-G(R_E)]m(sU+E, V)/(sU+E)$. The hazard rate for the unemployed satisfies a similar expression, $[1-G(R_U)]sm(sU+E, V)/(sU+E)$. Therefore, the aggregate matching function is

$$M = \frac{[1 - G(R_E)]E + [1 - G(R_U)]sU}{E + sU} m(sU + E, V). \tag{5.1}$$

The introduction of employed job seekers opens up two empirical challenges,

[12] This would be the most appropriate framework for the analysis of "vacancy chains", whereby the employed take the new and better vacancies first vacating jobs down the line and the unemployed get pushed to the bottom of the vacancy chain. See Contini and Revelli (1997) and Akerlof et al (1988).

which are also relevant to the group of workers who in the data move directly from out of the labor force to employment. The first is the need to ensure a good match between the flow variable on the left-hand side of the equation and the stock variable on the right-hand side. We have already encountered this problem when we considered the implications of the group who flow from unemployment to out of the labor force and a similar measurement problem arises for those who flow from employment and out of the labor force to employment. The second challenge is partly one of theory. It is the question whether one can regress, say, job matches from unemployment on the unemployment stock ignoring the employed job seekers and those out of the labor force. Are the estimates of the matching function elasticities obtained in this regression unbiased?

Before suggesting ways that the literature has dealt with these two questions we summarize some evidence on the relative importance of employment inflows that do not originate in recorded unemployment. Blanchard and Diamond (1989) construct a job-to-job flow series for the United States by making the assumption that these flows account for 40% of all job quits, the proportion estimated by Akerlof et al. (1988), and that the quit rate for the economy as a whole is the same as the quit rate in manufacturing. This procedure leads them to conclude that job-to-job movements account on average for 15% of total hires in the period 1968-1981. The remainder 85% is accounted for by hires from unemployment (45%) and hires from out of the labor force (40%).

Similar information for the United Kingdom can be derived from the Employment Audit which uses the quarterly Labour Force Survey data. Job-to-job moves in 1992 represented 51% of total hires, while flows from unemployment and inactivity represented 21% and 27% respectively. Due to the three-month gap between observations, these data tend to overstate the importance of job-to-job moves and moves from out of the labor force (and understate those from unemployment, as workers with less than three month unemployment durations are missed in the unemployment count). Even allowing for some correction, however, Pissarides (1994) suggests a lower bound for job-to-job moves of 40% of total hires. Elsewhere in Europe, job switches appear to be less frequent than in the United Kingdom. Burda and Wyplosz (1994) estimate that in Germany in 1987 job-to-job flows represented 16% of employment inflows, with the rest being shared in equal proportions by unemployment and inactivity flows. The picture for German worker flows is thus similar to the US picture. In France, 67% of the employment inflow was accounted for by unemployment outflows, with job-to-job flows accounting for a mere 10% and flows out of inactivity for 23%.

Thus both flows out of inactivity and job-to-job switches are large relative to the unemployment outflow. There is virtually no evidence on the properties of the flow from inactivity into jobs but some evidence on the properties of the job-to-job flow may shed light on its influence on the unemployment flow. What little evidence there is on the cyclical properties of flows in and out of inactivity gives mixed signals. Blanchard and Diamond (1989, 1990a) note that the flow of hires from out of the labor force is procyclical in the United Sates, while Burda and Wyplosz (1994) conclude that flows in and out of the labor force do not exhibit any particular cyclical pattern in Europe. A rich body of evidence, however, confirms that job-to-job flows are procyclical and closely linked to the quit rate.

Burgess (1993) builds a model of competition between employed and unemployed job seekers, and explains the procyclicality of job switches by modeling employed job search on the basis of a reservation wage rule. Employed workers whose wages fall below the (endogenous) reservation wage start searching for a better job. The reservation wage increases when the probability of receiving a job offer is higher, so, when the frequency of job offers rises in a boom, the employed have a stronger incentive to search, partially crowding out the unemployed from new jobs. In addition to this congestion effect, Pissarides (1994) argues that during a boom employers open vacancies that are more attractive to the employed, given that their proportion in the pool of job applicants rises, and destroy jobs that employed workers quit, which are now acceptable only to the unemployed. This further enhances the procyclicality of job-to-job flows. More recently, Boeri (1995) combines endogenous employed job search with the possibility that unemployed job search has different intensities at different durations.

Recent empirical work explicitly takes into account employed job search and sometimes out-of-labor-force job search. Blanchard and Diamond (1989) use alternative definitions of the relevant pool of searchers, allowing the unemployed and those classified as inactive to be perfect substitutes up to a scalar level. They find that inactive workers do not enter the matching function with a significant coefficient. Following a similar procedure, Boeri (1995) finds that the unemployment outflow in Germany, Spain and the United Kingdom is not affected by all employed job search but only by search of those on temporary jobs or jobs that are at risk.

For the United Kingdom, Burgess (1993) and Attfield and Burgess (1995) find evidence of endogenous job competition between employed and unemployed job seekers, obtaining an elasticity of the unemployment outflow with respect to total hires below 1. The standard matching function in U and V is then re-interpreted as a reduced-form relation for the unemployment outflow arising from the simultaneous determination of matching and job competition between employed and unemployed job-seekers, with on-the-job search being expressed as a function of the unemployment level. Similarly, Mumford and Smith (1999) use Australian data to extend the job search competition to workers who are out of labor force, and find evidence of inactive workers ranking below the unemployed, who in turn rank below the employed in the process of filling vacancies. No evidence of job competition is detected instead by van Ours (1995) for the Netherlands, finding that employed and unemployed workers mainly apply for different kinds of jobs.

It would appear from the literature discussed so far that data limitations make it difficult to ensure that the flow and stock variables in empirical matching functions refer to the same group of workers. The literature so far has not suggested a good alternative to getting the relevant data. What if the job-to-job flow is omitted in a regression of the unemployment matching rate, given the well-documented procyclicality of that flow? Of course, if employed job seekers did not cause congestion for the unemployed because they applied to different kinds of jobs, as van Ours's (1995) work seems to imply, that would cause no problems in the estimation of matching function for the unemployed. Suppose instead, for the sake of illustration, that the employed and unemployed apply to the same kinds of jobs and so congestion externalities are present. The simplest matching function in this case is $m(E+U, V)$, with the notation as before. The number of matches that go to unemployed workers is, on average, a fraction $U/(E+U)$ of the total, so the matching function for unemployed workers is

$$M_U = \frac{U}{E+U} m(E+U, V). \tag{5.2}$$

Let now (5.2) satisfy the Cobb-Douglas restrictions with constant returns to scale and the elasticity with respect to job seekers equal to η, a number between 0 and 1. Then (5.2) becomes, after some rearranging,

$$\ln \frac{M_U}{U} = (1-\eta)\ln V - (1-\eta)\ln(E+U), \tag{5.3}$$

where constants and other terms unrelated to U and V have been omitted.

This equation is, of course, simple to estimate, provided we have data for the stock of employed job seekers. Interestingly, we need such data even if our interest is only in the unemployment flow, because of the congestion that the employed cause for the unemployed. An increase in the number of employed job seekers reduces the transition rate of the unemployed into new jobs. Yet, although equation (5.3) is of the type estimated by several authors, the number of employed job seekers, E, is not normally included among the regressors. The closest approximation to (5.3) can be found in Burgess (1993) for the United Kingdom and Mumford and Smith (1999) for Australia. The specification estimated by Burgess regresses $\ln(M_U/U)$ on $\ln(M/L)$ and $\ln(U/L)$ (where M denotes total matches) and represents the reduced form equation stemming from a model of job competition between employed and unemployed job seekers, in which on-the-job search is a function of M/L and U/L. Mumford and Smith regress $\ln(M_U/U)$ on $\ln M$,

$\ln U$ and $\ln(E/U)$, in which E is proxied by the number of job quitters in the previous period.

To see the implications of the omission of E from the list of regressors, suppose that the cycle is measured by the ratio V/U, the tightness of the labor market, and let $E = \lambda(V/U)^{\alpha}$. Given the responsiveness of the number of employed job seekers to the cycle, the coefficient α is positive and likely to exceed 1. Then if a log-linear form of (5.3) is estimated with $\ln V$ and $\ln U$ as independent variables, the coefficients estimated are approximately the elasticities of the average transition rate M_U/U with respect to V and U evaluated at sample means. Let these be β_V and β_U. Differentiation of (5.3) gives,

$$\beta_V = (1-\eta)\left(1-\alpha\frac{E}{E+U}\frac{U}{V}\right), \tag{5.4}$$

$$\beta_U = -(1-\eta)\left(1-(1+\alpha)\frac{E}{E+U}\right), \tag{5.5}$$

with $E/(E+U)$ and U/V in these expressions evaluated at their sample means.

Two implications follow from these expressions. First, the regression that omits E will give too low an estimate of the effect of vacancies on matchings and too high an estimate of the effect of unemployment when compared with the underlying elasticity η. As a corollary, if the objective is to estimate the coefficient η, the estimate obtained from the estimated βs is biased upward. This is a direct implication of the pro-cyclicality of employed job search, since if $\alpha = 0$, β_V gives an unbiased estimate of η. Second, if the matching function satisfies constant returns, as assumed, then a test of constant returns by comparing the coefficients β_V and β_U, as normally done in the literature, will give constant returns only if

$$\alpha\frac{U}{V} = 1+\alpha. \tag{5.6}$$

Note that if $\alpha = 0$ this is never valid and the test will show increasing returns to scale. But if, say U, is on average twice as big as V, which is not unreasonable, a value of α close to 1, which is also not unreasonable, will confirm constant returns. So some of the tests that found constant returns in the literature despite omitting the number of employed job seekers may have done so by coincidence.[13]

[13] Of course, there is also the possibility that empirical matching functions with U and V as independent variables satisfy contant returns but the underlying matching function in $E+U$ and V does not.

6. Aggregation issues

6.1. Time aggregation

The matching function describes a process that takes place continually in spatially distinct locations. The use of discrete-time data for arbitrary regional divisions to estimate aggregate matching functions introduces both temporal and spatial aggregation problems.

Time aggregation problems arise when flow variables are estimated as functions of stock conditioning variables. This happens in the empirical production literature, where a production function is used to describe the flow of output from the stocks of inputs. Similarly, the matching function describes the flow of matches as a function of the stocks of unemployment and vacancies. In order to analyze the problems introduced by time aggregation in this case, we consider for convenience an explicit log-linear version of the matching function and introduce a well-behaved disturbance term ϵ_t

$$\ln M_t = \alpha_0 + \alpha_1 \ln V_t + \alpha_2 \ln U_t + \epsilon_t. \tag{6.1}$$

If M_t is measured as a flow over a time period, and U_t and V_t as stocks at some point during the period, U_t and V_t are depleted by matches M_t, and this generates a downward bias in the estimated coefficients α_1 and α_2. This problem is often dealt with by using beginning-of-period stocks U_{t-1} and V_{t-1} as conditioning variables or as instruments for U_t and V_t. If there is no serial correlation in the error term, lagged stocks U_{t-1} and V_{t-1} are uncorrelated with ϵ_t and are therefore good instruments.

But whatever stock variable is used on the right-hand side of the equation, the dependent variable is mismeasured, being the aggregated flow over a time interval during which the stocks change. The measured outflow over some time interval does not only include the outflow from the initial stocks, but also the outflow from the inflow over the same interval. For periods even as short as a quarter this can give rise to a situation in which the total outflow during the interval exceeds the initial stock. For vacancies, whose average completed duration is in most cases under a month, even monthly data would deliver exit rates above 1.

Here we discuss this problem more formally using an exponential probability distribution of duration, characterized by constant hazard with respect to duration during the measurement period. Assuming a hazard rate λ, the survivor probability of an unemployed worker is $S(t) = \exp(-\lambda t)$, with t denoting the elapsed duration of search. The probability of being matched (the outflow rate) over a time period of length t is therefore $F(t) = 1 - \exp(-\lambda t)$.

Let us consider a period of unit length. Assuming an initial stock of unemployment U_0, and a subsequent inflow $u(t)$, $t \in (0, 1]$, the unemployment outflow is given by

$$M = \left(1 - e^{-\lambda}\right) U_0 + \int_0^1 \left[1 - e^{-\lambda(1-t)}\right] u(t) dt \tag{6.2}$$

where the first term denotes the outflow from the initial stock and the second denotes the outflow from the inflow. A symmetric expression can be computed for vacancies. Estimating (6.1) on discrete data using beginning-of-period stocks as conditioning variables therefore omits the number of matches represented by the second term in (6.2).

Under the simplifying assumption of uniform inflow u during the whole period, we have

$$M = \left(1 - e^{-\lambda}\right) U_0 + \left[1 - \frac{1}{\lambda}\left(1 - e^{-\lambda}\right)\right] u. \tag{6.3}$$

It can be noted that the term in square brackets is bounded between zero and one, and therefore describes a plausible outflow rate from the inflow. Also the outflow rate from the inflow is lower than the one from the stock, for the reason that the inflow has, on average, less time available for a successful match.

In order to take into account the matches generated by inflows u and v, right-hand side variables in (6.1) should include the beginning-of-period stock, plus some proportion of the inflow. Given that each agent in U_0 has a matching probability which is $\left(1 - e^{-\lambda}\right)^{-1} - \frac{1}{\lambda}$ times the matching probability of each agent in u, the pool of unemployed job seekers between time 0 and time 1 can be expressed in homogeneous "search units" as

$$U_0 + \left[\left(1 - e^{-\lambda}\right)^{-1} - \frac{1}{\lambda}\right] u, \tag{6.4}$$

and similarly for vacancies. In order to compute the expression in (6.4), the hazard rate λ can be obtained by estimating equation (6.3) on stocks and flows. Alternatively, for small enough λ, the term in square brackets in (6.4) can be approximated by 1/2, using a second order Taylor expansion of $\exp(-\lambda)$ around $\lambda = 0$.

Gregg and Petrongolo (1997) follow this procedure in order to deal with the time aggregation problem in the estimation of an aggregate matching function for Britain for the period 1967-1995. Their analysis combines this treatment of time aggregation with a stock-flow matching mechanism (see Section 3.5). The resulting matching function estimates suggest that there has been no deterioration in the matching effectiveness of vacancies over the period considered. There seems instead to have been some fall in the matching effectiveness of the unemployed, although less severe than that implied by the conventional stock-based analysis of matching (namely, the influence of the time trend of aggregate studies is reduced).

Berman (1997) uses instead the sum of beginning-of-period stocks and subsequent flows to construct a proper instrument for U_t and V_t in estimating a log-linear referral function for Israel over the period 1978-1990. IV estimation delivers higher elasticities of referrals with respect to unemployment and vacancies than OLS estimation, detecting a downward (simultaneity) bias in OLS estimates.

An alternative way of ensuring matching probabilities strictly bounded between 0 and 1, proposed by den Haan et al. (1997), departs from the standard log-linear specification (6.1). They consider that matching takes place when a firm and a worker meet through a pair-specific channel. There are J_t channels in the economy, and each agent is randomly assigned to one of them. With this procedure, a worker locates a vacancy with probability V_t/J_t, and a firm locates a worker with probability U_t/J_t. Matches are given by $M_t = U_t V_t / J_t$. The properties of this matching function depend on the specification of J_t. The specification adopted by the authors is $J_t = \left(U_t^l + V_t^l\right)^{1/l}$, which restricts exit rates of unemployment and vacancies between 0 and 1, as l goes from 0 to ∞. den Haan et al. use these functions in a dynamic general equilibrium model with productivity shocks. The calibration of their model delivers a close match with data on labor market flows when the parameter l is set equal to 1.27.

Going back to empirics, Burdett et al. (1994) show that the use of beginning-of-period stocks as sole conditioning variables generates a bias in the resulting elasticities of M_t with respect to U_{t-1} and V_{t-1} that depends on the time series properties of the two stocks. Suppose that both U_{t-1} and V_{t-1} are mean-reverting series, an assumption which is implicit in a matching function where the number of matches is a positive function of U_{t-1} and V_{t-1}. In this case the average size of a stock over a time period tends to be negatively correlated with the size at the beginning of the period. This implies that, when unemployment (or the number of vacancies) is above the mean, the average size of the stock during the following period will be relatively small, depleting the number of aggregate matches during the period. On the other hand, when the initial stock is below the mean, its size tends to increase afterwards, generating a higher number of intra-period matches. This mechanism generates a downward bias in the estimated elasticities of M_t with respect to U_{t-1} and V_{t-1}.

It is shown however that, for a small enough measuring interval, the size of the bias can be approximated as a linear function of its length. Thus the size of the bias can be estimated by doubling the length of the measuring interval and comparing the obtained coefficients with those estimated using the original data frequency. This procedure, applied by Burdett et al. to the data used by Blanchard and Diamond, suggests that the bias is not important whenever the data frequency is monthly or higher and the cycle frequency is yearly or higher.

6.2. Spatial aggregation

The other issue that links aggregate production and matching technologies is aggregation across space. As in the empirical production literature, most authors of empirical matching functions aggregate the number of unemployed workers and job vacancies across space and use the aggregates to explain the flow of job matches in the same space. This practice treats the aggregate economy as a single labor market, ignoring the fact that it might be a collection of spatially distinct labor markets with possibly little interaction. The relevant issue is whether aggregating local labor market data biases the resulting estimates.

Coles and Smith (1996) argue that spatial aggregation might bias the results towards constant returns to scale in the matching function, while the matching process could display increasing returns instead. The underlying intuition is that replicating a marketplace of a given size and with a given number of searchers should double the number of matches if there is no interaction between the two marketplaces. But if there is interaction, the number of matches more than doubles, because cross-border matches can now be formed. So with interactions between markets, matches more than double when the number of searchers doubles within the original marketplace, implying increasing returns to scale. Since interactions are likely to be more common in more dense markets, Coles and Smith conclude that in estimation density is likely to be more important than market size, something for which they find evidence in their study. Indeed they find constant returns to scale on average but with more dense markets delivering higher matching rates for given size of the vacancy and unemployment pools.

Constant returns to scale are also not rejected in a similar study by Bennet and Pinto (1994), who estimate separate local matching functions over the period 1985-1991 for 104 areas of Training and Enterprise Councils that cover Britain. They find that most of the estimates for the returns to scale range between 0.7 and 1.15.

A further issue concerns the interaction between local matching conditions and regional migration or commuting behavior. The importance of job search considerations in worker migration is explicitly recognized in Jackman and Savouri (1992). They note that the direction of gross migration flows in Britain is consistent with a job search approach, in which migration is interpreted as the outcome of job matching. The magnitude of migration flows is best explained in time series regressions by the evolution of the total number of job-worker matches. Regional migration facts are instead difficult to reconcile with the predictions of competitive human capital theory, mainly on the grounds that high wage regions do not seem to attract significant migration flows.

The effects of regional migration and commuting on local matching conditions are analyzed by Burda and Profit (1996). They represent an aggregate economy as a two-dimensional space divided into a number of districts. Workers' decisions determine search intensity in all districts namely how many jobs to apply for in each district. This extension of the matching function to the spatial dimension relates job matching in a district to economic conditions everywhere in the economy, inducing a network of complex spillover effects between neighboring districts. Burda and Profit estimate a matching function that embodies regional spillovers for 76 Czech labor market districts, and find significant effects of neighboring unemployment on local matching. Constant returns to scale in the matching function cannot be rejected. This specification is also used by Burgess and Profit (1998) in order to study local matching and spillovers in 303 British travel-to-work areas. They find that more unemployed job-seekers (vacancies) in neighboring areas raise the local vacancy (unemployment) outflow but lower the local unemployment (vacancy) outflow.

Along similar lines, Petrongolo and Wasmer (1999) estimate a matching function for Britain (1986-1995) and France (1983-1994), using a regional panel for each country. Cross-regional spillovers are considered allowing each worker to search in her own or other regions with different search intensities. It is shown that search intensity is positive and significant in regions that are adjacent to the one where the worker resides, although it is only about 10% of the level of search intensity in the region of residence. Constant returns to scale in the matching function are not rejected by either the British or the French data.

To conclude, although the problem of spatial aggregation has only recently been discussed in the estimation of matching functions, the findings of those who explicitly embody a spatial dimension into the estimation do not invalidate earlier results on aggregate matching functions. Their analysis, however, sheds more light on the regional dimensions of job matching.

7. Conclusions

Like most other aggregate functions in the macroeconomist's tool kit, the matching function is a black box: we have good intuition about its existence and properties but only some tentative ideas about its microfoundations. Yet, those tentative ideas have not been rigorously tested. They have been used only to provide justification for the inclusion or exclusion of variables from the estimation of aggregate or regional matching functions, leaving it to the empirical specification to come up with a convincing functional form.

The early aggregate studies converged on a Cobb-Douglas matching function with the flow of hires on the left-hand side and the stock of unemployment and job vacancies on the right-hand side, satisfying constant returns to scale, and with the coefficient on unemployment in the range 0.5-0.7. In some of the estimates that use total hires as dependent variable (not only hires from unemployment) the coefficient on unemployment is lower, in the range 0.3-0.4, and the coefficient on vacancies correspondingly higher. But estimation of both Beveridge curves and aggregate matching functions points also to other variables that influence the simple Cobb-Douglas relationship. Much of the estimation of matching functions in the last decade has looked for those other variables and for better empirical specifications. Micro studies suggest the age structure of the labor force, the geographical dispersion of job vacancies and unemployed workers, the incidence of long-term unemployment (exceeding one year), and unemployment insurance; interestingly, however, although the other variables have been found significant where tested, unemployment insurance has not been identified as a significant influence on aggregate matching rates. We have argued that this may be related to measurement problems and the difficulty of getting a reliable time series for the generosity of unemployment insurance systems.

Recent empirical work has used disaggregate data and modeled the micro matching functions more carefully, paying attention to the issue of consistency between the timing of the flows and the timing of the stocks in the regressions, the regional spillovers in matching, and the consistency between the flow and stock variables, given the observation that many matches involve either employed workers or workers classified as out of the labor force. The precision of the estimation has increased and the relation between hazard function estimation and aggregate matching function estimation has become clearer. It has been found that aggregation problems have played a role in some of the shifts in the aggregate matching function, though not to an extent that can render the aggregate function "unstable". Despite all the refinements and detailed tests, the findings of the first aggregate studies have not been challenged: the stable, constant returns aggregate function used in macroeconomic modeling finds strong support in the data of virtually all modern economies where tests have been conducted.

Future work needs to elaborate a number of issues. First, on-the-job search and search out of the labor force need to be more carefully measured and their implications for unemployed search and matching studied. Second, although constant returns are strongly supported when the test is done on the numbers involved in matching, there have been no rigorous tests of the plausible property that the quality of matches is better in larger markets on the ground that participants have more choices. This may be more true in skilled labor markets, where skill heterogeneities are more likely to matter. Finally, the search for microfoundations needs to continue, partly because it aids the estimation but also because different microeconomic matching mechanisms may have different implications for wage determination and other types of behavior in markets with frictions.

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8. Appendix: Some history

What is the history of the matching function and how did labor economists deal with frictions before the recent vintage of models?

Early writers on the economics of labor markets were aware of the importance of frictions but were unable to bring them into their formal models. Hicks (1932) in the *Theory of Wages* devoted a chapter to unemployment. After introducing the "commonplace" definitions of unemployment, he made the claim that some kinds of unemployment induce wage changes and some do not; the ones that do not are "consistent with constant supply and demand for labor" and they make up "normal unemployment". An important reason for the existence of normal unemployment, which is close to Phelps's (1967) and Friedman's (1968) equilibrium or "natural" unemployment, is the fact that

> although the industry as a whole is stationary, some firms in it will be closing down or contracting their sphere of operations, others will be arising or expanding to take their place. Some firms, then, will be dismissing, others taking on labour; and when they are not situated close together, so that knowledge of opportunities is imperfect, and transference is attended by all the difficulties of finding housing accommodation, and the uprooting and transplanting of social ties, it is not surprising that an interval of time elapses between dismissal and re-engagement, during which the workman is unemployed. (Hicks, 1963, p.45).

Moreover, he claimed that these costs, the frictions, are important in determining equilibrium wages, because they imply a range of indeterminacy due to monopoly rents. But more importantly, frictions according to Hicks (1963, chapter 4), slow down the response of (real) wages to shocks and so are a major cause of short-run disequilibrium in the labor market.

Hutt (1939) also emphasized the importance of frictions in modern labor markets. In his *Theory of Idle Resources* he attempted to distinguish various supply-side reasons for unemployment, in the hope that they would be brought into the demand-side models of Keynes and others. Amongst them he included workers who are "*actively* searching for work" because they "judge that the search for a better opening is worth the risk of immediately foregone income". He then argued that such individuals should not be counted as unemployed because they are working on their own account and doing the job that an employment agency would do "if the course of politics had allowed such an institution to emerge in modern society" (Hutt, 1939, p. 60).

Hutt's plea to his contemporaries to take into account such causes of unemployment was ignored. The dominant view of unemployment that emerged out of the depression of the 1930s was Keynes's view that the unemployment that Hicks called "normal" could be ignored. Keynes (1936, p.6) defined some kinds of unemployment as compatible with "full employment" and uninteresting from his point of view, along similar lines to Hicks (1932) (though without crediting him). He called these kinds "frictional" - probably the first use of the term - and "between jobs", due to "various inexactness of adjustment which stand in the way of full employment". He also included "voluntary" unemployment to the kinds compatible with full employment. He credited Pigou (undated) for the best exposition of the "classical" view but criticized him for concentrating on real factors only and for claiming that only "frictional" unemployment will exist in equilibrium, and therefore "such unemployment as exists at any time is due wholly to the fact that changes in demand conditions are continually taking place and that frictional resistances prevent the appropriate wage adjustments from being made instantaneously" (Pigou's words, quoted by Keynes, 1936, p. 278). Thus, like Hicks, Pigou blamed frictions mainly for slow (real) wage adjustments, a point which Keynes considered irrelevant, if not erroneous (Keynes, 1936, p.278), to the point that he called the title of Pigou's book, *Theory of Unemployment*, "something of a misnomer" (p.275).

Keynes's followers replaced the slow real adjustment emphasized by Hicks and Pigou by slow nominal adjustment but did not attribute it to real frictions. Frictions re-appeared in the literature some time later, and only after Phelps (1967) and Friedman (1968) reiterated Hicks's claims that in equilibrium there is some "normal" unemployment, which is independent of nominal factors and which does not induce wage adjustments (see Phelps, 1968, Mortensen, 1970, and other contributions to Phelps et al. 1970). The frictions in Phelps's and Mortensen's models were summarized in a flow-of-labor function which depended on the firm's relative wage offer. (Of course, in competitive theory the flow of labor function to the individual firm is infinite.) The mechanism assumed was similar to one of the mechanisms in modern "efficiency wage" theory, and the more recent work of Phelps (1994) recasts that assumption more formally in an equilibrium framework with unemployment (see also Salop, 1979).

Early criticisms of this approach by Rothschild (1973) and others, who demonstrated that the optimizing actions of agents in these models could not support the assumed wage distribution, and also Diamond's (1971) demonstration that in sequential search price will converge to the monopoly price, led to attempts to find reasons for the persistence of wage differentials in equilibrium.[14] But the

[14] The equilibrium model of Lucas and Prescott (1974), although innovative, was difficult to merge with mainstream analysis. The debt that it owes to the ideas in the Phelps volume is

main impetus for new theoretical work came from the realization that there are large flows of jobs and workers in modern labor markets and that search models could shed light on them.[15]

The key concept in the new generation of models was the "matching function", which summarized in simple and easy-to-use form the frictions that slow down the mobility of workers from one job to another. Although something resembling it was present in the Phelps (1968) and Mortensen (1970) models, as well as in early mechanical models of the Phillips and Beveridge curves, it was not until the late 1970s that it explicitly appeared in equilibrium models. Butters (1977) described a process by which sellers let buyers know of their prices by posting ads at random in their mailboxes. Hall (1979) used this example to describe how recruiting firms select workers out of a homogenous unemployment pool, and derived an explicit functional form for the "job-finding rate." Pissarides (1979) derived the same functional form and combined it with a general constant-returns-to-scale "job matchings function" to describe the search and matching outcome in a model where employment agencies also match workers to jobs. Diamond and Maskin (1979) assumed that meetings in a frictional market are governed by a "search technology," which can be approximated by linear or quadratic functions. Bowden (1980) examined vacancy-unemployment dynamics in search markets by making use of an "engagements" function that is linear-homogenous in the participating vacancies and unemployed workers. Interestingly, he gave as example the Cobb-Douglas form, with the constant measuring the efficiency of matching.

The success of these early models, and more importantly the later models that considered issues of wage determination and labor market equilibrium (Diamond, 1982a,b, Mortensen, 1982a,b, and Pissarides, 1990) led to the equilibrium models that are used to study unemployment, employment fluctuations, job creation and destruction and flows in and out of employment and the labor force. The attraction of the matching function is that it enables the modeling of frictions in otherwise conventional models with the minimum of added complexity. It has retained its key role in these models as the representation of frictions that keep the labor market off its Walrasian equilibrium path.

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obvious, with its island equilibrium and the slow mobility across the islands, but its assumption that each island is in competitive equilibirum is very different from the "non-Walrasian" ideas in the Phelps volume. The subsequent empirical implementation by Lilien (1982) inspired a lot of work but eventually the framework used to test Lilien's "sectoral shifts" hypothesis became more akin to search and matching models.

[15] For early studies of empirical flows see Hall (1972), Feldstein (1973), Marston (1976) and Clark and Summers (1979). Later, the work of Leonard (1987), Dunne et al. (1989), Davis et al (1996), Blanchard and Diamond (1990a) and others provided new stimulus to theoretical developments.

Table 1: Aggregate matching function studies

Author	Country and coverage	Period and frequency	Dependent variable	Job seekers	Job vacancies	Other variables	Specification
Pissarides (1986)	UK, men	1967-1983 quarterly	male unempl. outflow rate	unemployed men	notified, adjusted	time trend mismatch replacement ratio.	linear; log -linear
Blanchard and Diamond (1989, 1990b)	US	1968-1981 monthly	all new hires	unemployed; laid-off; Out of LF; STU and LTU.	help wanted, adjusted	time trend	log-linear; CES
Layard, Nickell and Jackman (1991)	Britain	1968-1988 quarterly	unempl. outflow rate	unemployed	notified	time trend, search intensity index.	log-linear
van Ours (1991)	Netherlands	1961-1987 annual	unempl. duration	—	vacancy duration	time trend	log linear
Burgess (1993)	UK, men	1968-1985 quarterly	male unempl. outflow rate	male unempl. rate	—	male hires, replacement ratio, demographic variables, LTU/U.	log-linear
Burda and Wyplosz (1994)	France Germany Spain UK	1971-1993 1968-1991 1977-1992 1985-1993 (all monthly)	unemployment outflow	unemployed	notified	time trend	log-linear
Warren (1996)	US manufacturing	1969-1973 monthly	all new hires	unemployed (from manuf.)	vacancies (in manuf.)	—	translog
Feve and Langot (1996)	France	1971-1989 quarterly	—	—	notified	a general-equilibrium small open economy model is estimated, in which a log-linear matching function is included	

Table 1 (continued): Aggregate matching function studies

Author	Country and coverage	Period and frequency	Dependent variable	Job seekers	Job vacancies	Other variables	Specification
Berman (1997)	Israel	1978-1990 monthly	referrals	unemployed	notified	time trend	log-linear
Gross (1997)	Germany (West)	1972-1994 quarterly	all new hires	unemployed	notified	real wages real energy price	log-linear (with co-integration analysis)
Gregg and Petrongolo (1997)	Britain	1967-1996 quarterly	unempl. outflow; vacancy outflow	unemployed	notified	time dummies	non linear
Bell (1997)	France Britain Spain	1979-1994 1967-1985 1980-1995 (all quarterly)	unempl. outflow new hires new hires	unemployed	notified notified notified, adjusted	time trend, benefits, mismatch demographic variables, LTU/U.	log-linear (with co-integration analysis)
Coles and Smith (1998)	Britain	1987-1995 monthly	unempl. outflow by duration	U stock U inflow	V stock V inflow	time trend	log-linear
Mumford and Smith (1999)	Australia	1980-1991 quarterly	U outflow rate; outflow rate from out of LF	unemployed (from manuf.)	—	new hires, other groups of job seekers, LTU/U	log-linear
Yashiv (2000)	Israel	1975-1989 monthly	all new hires	unemployed	notified	structural breaks	log-linear; translog

Table 2: Sectorial matching function studies

Author	Country and coverage	Period and frequency	Level of disaggregation	Dependent variable	Job seekers	Job vacancies	Other variables	Specification
Bennet and Pinto (1994)	Britain, men	1967-1983 quarterly	104 local districts	unempl. outflow	unempl. rate	vacancy rate		log-linear
van Ours (1995)	Netherlands	1981-1983 annual	8 regions	hires from U; hires from N	unempl. + empl. seekers	all vacancies	reg. dummies	non-linear
Anderson and Burgess (1995)	US	1979-1984 quarterly	4 states × 20 industries	all new hires; hires from non-empl.; hires from empl.	unempl. rate	help wanted rate	Δ employm. in ind., replacement ratio, demographic variables	log-linear
Coles and Smith (1996)	England and Wales	1987	257 TTWAs	filled vacancies	unemployed	notified vacancies	wages, size of TTWA, demographic variables.	log-linear
Boeri and Burda (1996); Profit (1997)	Czech Republic	1992-1994 quarterly	76 district	hires from U	unemployed	notified	time dummies, area dummies, lagged dep. var.	log-linear
Burda and Profit (1996)	Czech Republic	1990-1994 monthly	76 district	hires from U	unemployed	notified	time dummies, spillover effects across areas	log-linear
Burgess and Profit (1998)	UK	1985-1995 monthly	303 TTWAs	unempl. outflow; filled vacancies	unemployed	notified	time trends, spillover effects across areas	log linear
Broesma and Van Ours (1998)	Netherlands	1988-1994 quarterly	6 industries	hires from U; filled vacancies	unemployed U + non-U seekers	notified	industry dummies	log-linear
Profit and Sperlich (1998)	Czech Republic	1992-1996 monthly	76 district	hires from U	STU and LTU	notified	area dummies, lagged dep. var.	log-linear; non parametric