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ABSTRACT

Political Risk And Irreversible Investment: Theory And An Application To Quebec*

Political risk is widely present in developing but also in developed countries, and stems from a variety of sources. The objective of this Paper is twofold. First, we develop a theoretical model to investigate the impact of political risk on irreversible investment. Second, we apply our model to an analysis of the risk of separation of the province of Quebec from the Canadian federation. We consider the investment decisions of a monopolistically competitive firm under uncertainty about demand and about the tax-adjusted price of investment goods. We develop a model of irreversible investment which incorporates learning and a regime switch with time-varying transition probabilities. If a given regime represents a riskier environment in terms of the state of demand or the state of investment price, then attaching a positive probability to a switch to that regime increases the marginal adjustment cost of investing, reduces the expected marginal value of capital and reduces irreversible investment. We use annual sectoral data for the Quebec economy for the period 1983–96 to match the behaviour of actual investment with simulated series from our model.

JEL Classification: D92, E22, O11, O16
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NON-TECHNICAL SUMMARY

It is widely accepted that political risk is present in developing countries. What is perhaps less readily recognized is that developed countries are also highly susceptible to political risk stemming from a variety of sources: expropriation, disruptions in market access, unfavourable government regulations, unsustainable exchange rates, debt crises, fiscal crises, policy reversals, risk of political disintegration, etc. Even when economic fundamentals are 'right', a subjective perception of unsustainability of the current policy regime may be strongly entrenched and may have deleterious consequences for investment. Rodrik (1991) emphasizes the importance of political risk (in his case the risk of policy reversal) for irreversible investment decisions and shows that political uncertainty amounts to levying a tax on investment. Several studies document the importance of political risk in the unsuccessful recovery of private investment following the adoption of IMF stabilization packages in various countries.

The industry-specific nature of most investment goods implies that investment decisions are, at least largely, irreversible. There is empirical evidence indicating that irreversibility is an important determinant of investment decisions. Some recent studies include Caballero, Engel and Haltiwanger (1995), and Caballero and Engel (1999).

There is also empirical evidence that uncertainty has a negative impact on investment. Ferderer (1993) and Huizinga (1993) examine the empirical impact of uncertainty on investment in the US manufacturing sector by making use of proxies such as the standard deviation of past inflation rates. Leahy and Whited (1996) use the variance of the firm's daily stock return for a panel of US manufacturing firms to construct an *ex ante* measure of risk. Their results support the view that uncertainty has negative consequences for investment. In their conclusions they point to 'irreversible investment as the most likely explanation for the observed [negative] correlation between investment and uncertainty'.

A number of studies have demonstrated a negative relationship between risk and irreversible investment, and uncertainty and irreversible investment. (Here we adopt the Knightian distinction between risk and uncertainty. That is, risk is defined as 'objective uncertainty' or randomness in the actual environment of agents whereas 'uncertainty' refers to 'subjective uncertainty', which includes the subjective beliefs of economic agents, their state of mind and expectations, their 'animal spirits'.) In the context of the irreversible investment model, these include Demers (1985, 1991), Pindyck (1988), Bertola (1989), Caballero (1991), Dixit and Pindyck (1994), Caballero and Pindyck (1996), Abel and Eberly (1996), and Altug, Demers and Demers (1999). This last paper demonstrates in particular that for some parameters the negative impact of uncertainty on investment is greater than that of risk, thus highlighting the importance of the learning behaviour of firms in understanding the determinants of investment.

While there is a considerable literature on the negative impact of risk and uncertainty on investment, there has been little work in quantifying the effect of political risk on investment decisions. In this Paper, we first develop a theoretical model to investigate the impact of political risk on irreversible investment. We then apply our model to an analysis of the risk of separation of the province of Quebec from the Canadian federation on Quebec's investment performance.

The case study of Quebec provides for a unique 'natural experiment' of the impact of political risk in a developed economy. The issue has existed for more than 30 years, the debate has been carried out in a climate almost free of violence and the data is of high quality. Two episodes of political risk are clearly identifiable. The first is that of the 1970s, characterized by the election in 1976 of a sovereignty party, the Parti Quebecois (PQ), for the first time in history and culminating in a referendum on sovereignty in 1980. The PQ lost the latter by a relatively large margin, thus putting the sovereignty issue at rest for the remainder of the decade. The second episode of political uncertainty is that of the 1990s, which was marked by the federal government's failed attempts at constitutional reconciliation of Quebec with the rest of Canada. This led to a rise in the popularity of the sovereignty option in Quebec, the subsequent election of the PQ in 1994 and another referendum on sovereignty very narrowly lost by the PQ in 1995.

In this Paper, we seek to determine the quantitative impact of the risk of separation on Quebec investment for the second episode of political risk, namely the 1990s. We consider the profit-maximisation problem of a risk neutral monopolistically competitive firm with irreversible investment and learning about the unknown state of demand and unknown costs of investment. In our model, the firm is uncertain about the permanent component of the state of demand and the tax-adjusted price of investment. It uses noisy observations on prices and tax rates, as well as observations on other informative variables, to make inferences about the permanent values using a Bayesian updating rule. As a way of modelling the presence of political risk, we also allow for regime switches between a 'good' regime and a 'bad' regime, where the former is characterised by more 'favourable' distributions for the state of demand or the costs of investing. While the firm knows in which regime it is currently residing, each period it must assess the transition probabilities on the basis of a vector of economic and political variables. We identify regime 0 (the 'good' regime) as the continuation of the Canadian federation and regime 1 as the separation of Quebec.

We use poll data as well as election and referendum results to estimate the probability of switching to the 'bad regime', i.e. the probability of the separation of Quebec from the Canadian federation. Using annual sectoral data for the Quebec economy, we match the actual investment-capital stock ratios during the second episode of political risk with their simulated counterparts from our model to assess the quantitative impact of the risk of separation and the impact of uncertainty and learning on investment behaviour.

We show that the increases in risk – which may arise from increases in the probability of transiting to a more unfavourable regime or from increases in the variability of profits in the 'bad' regime – can have quantitatively important effects on investment behaviour. The numerical results also show that the irreversible investment model under uncertainty and learning and regimes shifts is capable of reconciling the large drop in investment that occurred in Quebec during the episode of political risk of the 1990s.

1 Introduction

It is widely accepted that political risk is present in developing countries. What is perhaps less readily recognized is that developed countries are also highly susceptible to political risk stemming from a variety of sources: expropriation, disruptions in market access, unfavourable government regulations, unsustainable exchange rates, debt crises, fiscal crises, policy reversals, risk of political disintegration, etc... Even when economic fundamentals are "right", a subjective perception of unsustainability of the current policy regime may be strongly entrenched, and may have deleterious consequences for investment. Rodrik (1991) emphasizes the importance of political risk (in his case the risk of policy reversal) for irreversible investment decisions, and shows that political uncertainty amounts to levying a tax on investment. Several studies document the importance of political risk in the unsuccessful recovery of private investment following the adoption of IMF stabilization packages in various countries.[1][2] The success of structural adjustment programs typically require a positive response of investment. Yet as Faini and de Melo (1992) note, "uncertainty about the future course of an adjustment package will lead potential investors to adopt a wait and see attitude even if the crucial indicators for a decision, like real wages, are favorable." World Bank studies have shown that following a structural adjustment program, investment typically stagnates and falls, then stabilizes and responds favorably to the reform only after a considerable lag.

Due to the industry-specific nature of most investment goods, investment decisions are, at least largely, irreversible. There is empirical evidence indicating that irreversibility is an important de-

[1] See, for example, Solimano (1992), Serven and Solimano (1992, 1993) and Chhibber, Dailami and Shafik (1992) who study the experience of Latin American, Southeast-Asian and African countries. As another example, in a study of Ghana from 1983 to 1991, Hadjimichael, Nowak, Sharer and Tahari (1996) argue that policies had an impact on private investment mostly through the uncertainty that they generated.

[2] Another related literature examines how economic integration (and political stability) fosters trade. See John McCallum (1995) and John Helliwell (1995).

terminant of investment decisions. Using microeconomic data, Caballero, Engel and Haltiwanger (1995) find evidence in favour of irreversibility. Caballero and Engel (1999) develop a generalized (S, s) model which incorporates irreversibility and a fixed cost. Using macroeconomic data for equipment and structures in the US manufacturing sector for 1947-1992, they find further supporting evidence for the irreversibility effect, and demonstrate that microeoconomic nonlinearities are important at the aggregate level.[3]

There is also empirical evidence that uncertainty has a negative impact on investment. Ferderer (1993) and Huizinga (1993) examine the empirical impact of uncertainty on investment in the US manufacturing sector by making use of proxies such as the standard deviation of past inflation rates. Leahy and Whited (1996) use the variance of the firm's daily stock return for a panel of US manufacturing firms to construct an *ex ante* measure of risk. Their results support the view that uncertainty has negative consequences for investment. In their conclusions they point to "irreversible investment as the most likely explanation for the observed [negative] correlation between investment and uncertainty."

A number of studies have demonstrated a negative relationship between risk and irreversible investment, and uncertainty and irreversible investment. (Here we adopt the Knightian distinction between risk and uncertainty.)[4] In the context of the irreversible investment model, Demers (1985,1991) introduces the learning behaviour of a firm, and shows how output price uncertainty reduces the investment of a Bayesian firm, while Bertola (1989), Caballero (1991), Dixit and Pindyck (1994), Caballero and Pindyck (1996), Pindyck (1988) and Abel and Eberly (1996), working with Brownian motions without learning, show that the impact of output price risk on investment is neg-

[3]See also Caballero (1997), section 3.3.

[4]That is, risk is defined as "objective uncertainty" or randomness in the actual environment of agents whereas "uncertainty" refers to "subjective uncertainty" which includes the subjective beliefs of economic agents, their state of mind and expectations, their "animal spirits."

ative.[5] Altug, Demers and Demers (1999) examine both an uncertain and a risky tax-adjusted price of investment, and establish a negative relationship with irreversible investment in both cases. They show that for some parameters the negative impact of uncertainty on investment is greater than that of risk, thus highlighting the importance of the learning behaviour of firms in understanding the determinants of investment.[6]

While there is a considerable literature on the negative impact of risk and uncertainty on investment, there has been little work in quantifying the effect of political risk on investment decisions.

The objective of this paper is twofold. First, we develop a theoretical model to investigate the impact of political risk on irreversible investment. Second, we apply our model to an analysis of the risk of separation of the province of Quebec from the Canadian federation on Quebec's investment performance.

The case study of Quebec provides for a unique "natural experiment" of the impact of political risk in a developed economy. The issue has existed for more than thirty years, the debate has been carried out in a climate almost free of violence, and the data is of high quality. Two episodes of political risk are clearly identifiable. The first is that of the 1970s, characterized by the election in 1976 of a sovereigntist party, the Parti Québécois (PQ), for the first time in history and culminating in a referendum on sovereignty in 1980. The PQ lost the latter by a relatively large margin, thus

[5]See Caballero (1997) and Dixit and Pindyck (1994) for surveys. See also Abel and Eberly (1994) who develop a model with asymmetric costs of adjustment.

[6]All of the studies mentioned above (and almost all papers in the literature so far) identify a short-run effect, also termed a user cost of capital effect or hurdle rate effect, of risk or uncertainty in the presence of irreversibility. Recently, Abel and Eberly (1999) have analyzed the long-run effect, They examine the impact of output price risk on the expected long-run capital stock. In addition to the hurdle rate effect they also identify a hangover effect. Our work focuses on the short-run or hurdle rate effect in the presence of learning and regime shifts.

putting the sovereigntist issue on the backburner for the rest of the decade. The second episode of political uncertainty is that of the 1990s, marked by the federal government's failed attempts at constitutional reconciliation of Quebec with the rest of Canada, which led to a rise in the popularity of the sovereigntist option in Quebec, the subsequent election of the PQ in 1994, and another referendum on sovereignty very narrowly lost by the PQ in 1995.

In this paper, we seek to determine the quantitative impact of the risk of separation on Quebec investment for the second episode of political risk, namely the 1990s.

We consider the profit-maximization problem of a risk neutral monopolistically competitive firm with irreversible investment and learning about the unknown the state of demand and unknown costs of investment. Taxation is introduced in the manner of Hall and Jorgenson (1967), Abel (1982), and Hayashi (1982).

In our model, the firm is uncertain about the permanent component of the state of demand as well as of the tax-adjusted price of investment. It uses noisy observations on prices and tax rates, as well as observations on other informative variables, to make inferences about the permanent values using a Bayesian updating rule. As a way of modelling the presence of political risk, we also allow for regime switches between a "good" regime and a "bad" regime, where the former is characterized by more "favorable" distributions for the state of demand or the costs of investing. While the firm knows which regime it is currently residing in, each period it must assess the transition probabilities on the basis of a vector of economic and political variables. We identify regime 0 (the "good" regime) as the continuation of the Canadian federation and regime 1 as the separation of Quebec.

Following the approach in Demers (1991), we show that uncertainty about the future state of demand or the costs of investment unambiguously lowers investment. Uncertainty and irreversibility lead to a marginal adjustment cost which arises endogenously with the learning process of the firm.

The prospect of obtaining better future information increases the marginal adjustment cost and depresses current investment. Our framework allows a simple way of incorporating the effects of political risk. If a given regime represents a riskier environment in terms of the state of demand or the state of investment price, attaching a positive probability to a switch to that regime increases the marginal adjustment cost of investing, reduces the expected marginal value of capital, and reduces irreversible investment.

We use poll data as well as election and referendum results to estimate the probability of switching to the "bad regime," i.e., the probability of the separation of Quebec from the Canadian federation. Using annual sectoral data for the Quebec economy, we match the actual investment-capital stock ratios during the second episode of political risk with their simulated counterparts from our model to assess the quantitative impact of the risk of separation and the impact of uncertainty and learning on investment behavior. The simulation procedure is based on the approach in Altug, Demers, and Demers (1999). It combines numerical dynamic programming with a Monte Carlo simulation procedure to simulate the future expected valuation functions. This is similar to Keane and Wolpin (1994), who assume (as we do) that the exogenous state variables are drawn from continuous distributions. However, the simulation procedure used in this paper extends the approach in Keane and Wolpin (1994) to an environment with Bayesian learning and regime shifts. It also allows for trends in the underlying exogenous variables.

The remainder of this paper is organized as follows. Section 2 presents the case of Quebec. This section provides a historical discussion of the problem of political risk in Quebec and relates it to Quebec's investment performance. Section 3 describes the the theoretical framework and solves the firm's dynamic programming problem in the presence of trends, learning and regime shifts. Section 4 describes the solution and simulation procedure used in the paper. It also describes how to simulate the model with continuous random variables and under learning. Section 5 parameterizes the theoretical model, and presents the numerical results. Some concluding remarks are in Section 6.

2 Political risk: the case of Quebec

2.1 A brief historical perspective

In this section we provide a brief historical account of the political conflict opposing Quebec and the rest of Canada, and identify two major episodes of political risk that have had an important impact on the Quebec economy, and particularly, on Quebec investment.

Canada is a federal state composed of ten provinces. Of the nine provinces other than Quebec, eight have an overwhelming majority of English-speaking residents. Quebec, on the other hand, has a majority of residents whose mother tongue is French and a sizable minority of residents whose mother tongue is English.

Since the inception of the Canadian federation in 1867, the goal of every Premier of Quebec has been to obtain greater autonomy within the federation, but without necessarily pursuing independence. However, in 1968, a major political party, the Parti Québécois (PQ), dedicated to the pursuit of the independence of Quebec, was formed. Initially the aim of the PQ was to seek independence for Quebec by a vote of the Quebec legislature. Defeated in the 1970 and 1973 elections, it finally succeeded in taking power in 1976 (with 40% of the popular vote) on a promise to hold a referendum on Quebec sovereignty. This marks the beginning of a first episode of political risk in Quebec. When the PQ finally held its referendum in 1980, it was defeated, with 60% of the voters being opposed to giving a mandate to the Quebec government to negotiate "sovereignty-association"[7] with the rest of Canada. The defeat of the PQ in the referendum led to a period

[7]The referendum question did not ask directly for support on independence, but rather asked for support on

of political calm in the 1980's. As also evidenced by poll results (see Table 1), during the 1980s, Quebecers seemed to have lost interest in sovereignty. The PQ was re-elected in 1981, but only after shelving its sovereignty plans. Subsequently, it was defeated in December 1985 by its federalist opponent.

The second episode of political uncertainty began in 1990 when the federal government's attempt at addressing Quebec's constitutional demands failed to be ratified by two of the ten provincial legislatures. The failure of this accord (referred to as the "Meech Lake Accord") which aimed at constitutional reconciliation of Quebec with the rest of Canada, was viewed in Quebec as a rejection by the rest of Canada, and led to an unprecedented rise in the popularity of the sovereigntist option in this province, as also evidenced by poll results (see Table 1). A period of high uncertainty followed, with the election of the PQ in 1994 (its first come-back after its 1985 defeat), and another referendum on sovereignty held in 1995, but lost by the PQ by 0.8% of the vote.[8] The 1995 referendum was defeated, but by a margin so narrow that, contrary to the aftermath of the 1980 referendum when the issue was put to rest, the period of political uncertainty that started in 1990 still continues to this day.

"sovereignty" together with a form of association with the rest of Canada. What is notable is that the form of the association in question and the degree of autonomy that was sought were not clearly defined. The question was intentionally ambiguous to garner the maximum of support, but the PQ lost the referendum nevertheless.

[8]As in the 1980 referendum, again the referendum question did not ask directly for support on independence, but asked this time for support on a "partnership proposal" with the rest of Canada taking the European Union (EU) as an example of the proposed partnership. As in the case of the 1980 referendum, again there was no tangible evidence that the rest of Canada would agree to any form of partnership or association other than the currently existing federal form. See Demers and Demers (1995) for a discussion as to why the EU model is not a viable option for the case of Quebec-Canada. The referendum question also stipulated that if a partnership could not be negotiated, independence would be declared unilaterally.

2.2 Does independence constitute political risk?

Should it separate from the rest of Canada, would an independent Quebec take its place among Western nations in a seemless transition, without any disruptions in economic activity and without costs, or does the threat of separation constitute political risk? Evidence from a poll carried out one month prior to the 1995 referendum for Quebec's Business Council (QBC),[9] indicates that an overwhelming majority of business executives in Quebec perceive separation as very costly for the Quebec economy. Thus, according to the QBC poll results, 90% of Quebec executives believed that Quebec would incur substantial costs following a yes vote to sovereignty; 83% believed that Quebec's economy would be severely negatively affected in the five year period following a yes vote; 65% believed that Quebec's economy would suffer from a long-term negative impact; 93% believed that negotiations with the rest of Canada would be long and arduous; 84% believed that a long period of political and economic instability would follow. Finally, 81% believed that immigrants and investors would be deterred from coming to Quebec. These poll results clearly indicate that separation from the Canadian federation is perceived to carry with it both long and short term costs for the Quebec economy.

An analysis of the events that are likely to accompany a move towards independence also confirms that sovereignty could be very costly for Quebec. In the aftermath of a Yes vote to sovereignty, Quebec could well face a financial crisis similar to or even worse than Mexico's 1994 crisis, as is argued in Demers and Demers (1995, Chapter 10). First, like Mexico, Quebec would suffer from some weak fundamentals. For example, Quebec would suffer from a large current account deficit as did Mexico. In addition, unlike Mexico, it would also have a large debt problem amounting to over 120% of GDP in contrast to Mexico's 40%. Second, it would have to renegotiate

[9] A poll conducted in September 1995 for *le Conseil du Patronat du Québec*, by the firm CROP specialised in public opinion research.

simultaneously its membership in the Canadian economic union and in the North American Free Trade Area (NAFTA), its use of the Canadian dollar, the division of the federal debt, etc...[10] Third, Quebec might even have to face the risk of partition of its territory due to opposing claims by native groups and some federalist groups in Quebec wishing to remain within Canada. Fourth, financial capital being highly mobile in Quebec[11], Quebecers would not hesitate to transfer their assets out of Quebec, especially in the face of uncertainty as to whether Quebec would keep using the Canadian dollar or not.[12] Their actions would precipitate a financial crisis.

The above discussion together with the QBC poll results, point to the fact that shifting to a regime in which Quebec becomes an independent state is clearly perceived as a shift to a riskier regime and one where economic conditions are expected to be worse than the current regime.

2.3 Perception of political risk: poll results and bond spreads

What do Quebecers (and finacial markets at large) think about the likelihood of separation of Quebec from the Canadian federation? In this section we try to assess the popular perception of the likelihood of shifting to a "bad" regime. Firms investing in Quebec pay close attention to indicators of this perception as a means of measuring the probability of independence. While there is no direct observation of this perception, we look at some indicators such as opinion polls, election

[10]In contrast, Mexico benefited from the confidence-building element of joining the NAFTA. However, in counterpart, Mexico suffered from the instability due to a political transition as a new untested president was elected to succeed President Salinas in an election marred by political assasination.

[11]As indeed it was in Mexico. Better informed of the economic and political situation than foreign investors, domestic Mexican investors precipitated the financial crisis by moving their peso-denominated assets into US dollars, and shifting them out of the country.

[12]While the PQ has asserted that it would wish to keep the Canadian dollar as the monetary unit in an independent Quebec, the need to devalue in face of a financial crisis would make it a difficult and undesirable promise to keep. For a detailed discussion, see Demers and Demers (1995), Chapter 10.

and referendum results, as well as data on bond spreads.

Opinion polls report the voting "intentions" of Quebecers in a referendum on independence. As we discuss below, (and as is well perceived by Quebecers) some of these intentions do not actually materialize into actual "Yes" votes in a referendum, and hence overestimate the actual support for sovereignty. It is common knowledge in Quebec that poll results may systematically overstate the Yes vote for several reasons not the least of which is peer pressure. (See Kuran (1990) for arguments indicating that people lie about their voting intentions due to peer pressure.) In addition, the poll results for the Yes vote also capture the response of some nationalist (but not sovereigntist) Quebecers who untruthfully indicate an intention to vote for sovereignty, as a means of signalling their desire for greater autonomy.

As for election results, they are indicative in that they measure the support garnered by the party in favour of independence. However, since there are only two political parties, (one which happens to be federalist and the other sovereigntist), public favour may shift from one party to another for reasons (such as public policy issues) other than their views on sovereignty. (This is also true of that segment of the population which may be termed as "soft" nationalists, who favour greater autonomy and are not completely averse to sovereignty.) Hence, election results also overestimate the actual support for sovereignty.

In Table 1, we establish what we will refer to as the "raw" poll data (given in the second column of Table 1)[13] For years during which there was an election or a referendum, we give preference to these results over poll results. For years during which no survey of opinion poll was conducted, we use an approximate figure in view of the political events of the time (see Appendix A for a thorough discussion). Table 1 clearly reveals that there were two major periods of political risk (1976-80 and 1990-95) separated by a period of stability.

[13]We thank Pierre Drouilly for kindly providing us with the poll data.

For reasons mentioned above, the raw poll data given in Table 1 may overestimate the perceived probability of Quebec sovereignty. To derive a more accurate measure of the perception of the likelihood of shifting to the "bad" regime, the predicted values from a 5-year moving average applied to the Yes votes in the raw poll data may be used. A potential problem is that even the smoothed poll results (given in Table 1 and displayed in Figures 2 and 3 in the next subsection) indicate that support for separation has been over 50% for a number of years in our sample. Given this fact, one could ask why has separation not occurred if the support is as high as even the smoothed poll results indicate? One explanation is that the poll data even if smoothed still contain noise, and do not perfectly measure the fundamental support for separation for the same reasons as those that affect the raw poll data. Hence, they again overestimate the perceived probability of transiting to a "bad regime."[14]

As an additional indicator of expectations, we also consider data on bond spreads between Quebec and Ontario bonds. This information is in Figure 1, which shows the difference between the spreads on 10-year bonds for Quebec and Ontario relative to Canada and the spread between 10-year Canada and US bonds. In the top panel of Figure 1, the data are monthly data from March 1990 to October 1999 while in the bottom panel of Figure 1, the data are monthly between February 1980 and December 1999. The time plot in the top panel of Figure 1 shows that the probability of separation (as reflected by bond markets) increased in 1990 and early 1991 following the rejection of Meech Lake; then declined for the remainder of 1991, and for 1992 and 1993 as investors became convinced that fundamental support for separation was lower than 50 percent and that the (federalist) Premier of Quebec would work to diffuse tensions. The difference between

[14]Another possibility is to define the "bad" regime as "unfavourable business conditions" due to the increased risk of separation. The problem with this definition is that recessions also create a bad business environment, and that the two episodes of political risk coincide or overlap with the 1980-82 and the 1990-91 recessions in Canada.

the bond spreads rose again in 1994 and 1995 with the provincial election and the referendum but eventually fell after the referendum. The spread is large but not very high in 1995, reflecting the fact that markets were worried but ultimately did not believe that separation would really happen. (Note, however, that the monthly data hides substantial daily fluctuations especially in the last weeks prior to the referendum.)

The perception of political risk can also be observed by analyzing the movements in the Canada-US bond spreads. The bottom panel of Figure 1 shows that there are several episodes of very high spreads between long-term Canadian and US bonds. Focusing on the 1990-91 episode in particular, while the high bond spreads are partly ascribable to the the Bank of Canada's concern about preventing the Canada-US exchange rate from slipping, and partly (from 1991 on) to the radical inflation reduction strategy adopted by the Bank of Canada[15], an additional effect that came into play in 1990 is the Meech Lake Accord. One can see that the spread was fairly high during the entire Meech Lake episode (June 1990), that is, both before the rejection and after. The high spreads in 1995 can be directly ascribed to the effects of political risk. Specifically, the bottom panel of Figure 1 shows that the spread between long-term Canadian and US bonds peaked in June 1995 as the pre-referendum discussions began, fell somewhat in August as it seemed that the forces of the "no" were getting ahead, then rose again to 1.82 in October just before the referendum. It subsequently declined after the defeat of the "yes" at the end of October.

The data on Quebec-Ontario bond spreads also confirm our intuition that even smoothed poll results overstate the perceived probability of separation. While the spread increased in 1990 and early 1991 following the rejection of the Meech Lake Accord, it declined for the remainder of 1991,

[15] Inflation was targeted to be reduced to 2 percent in four years 2 (and in fact, the recession helping, the target was achieved even sooner). This strategy, (together with the efforts to maintain the Canada-US exchange rate) required very restrictive monetary policy which drove interest rates up.

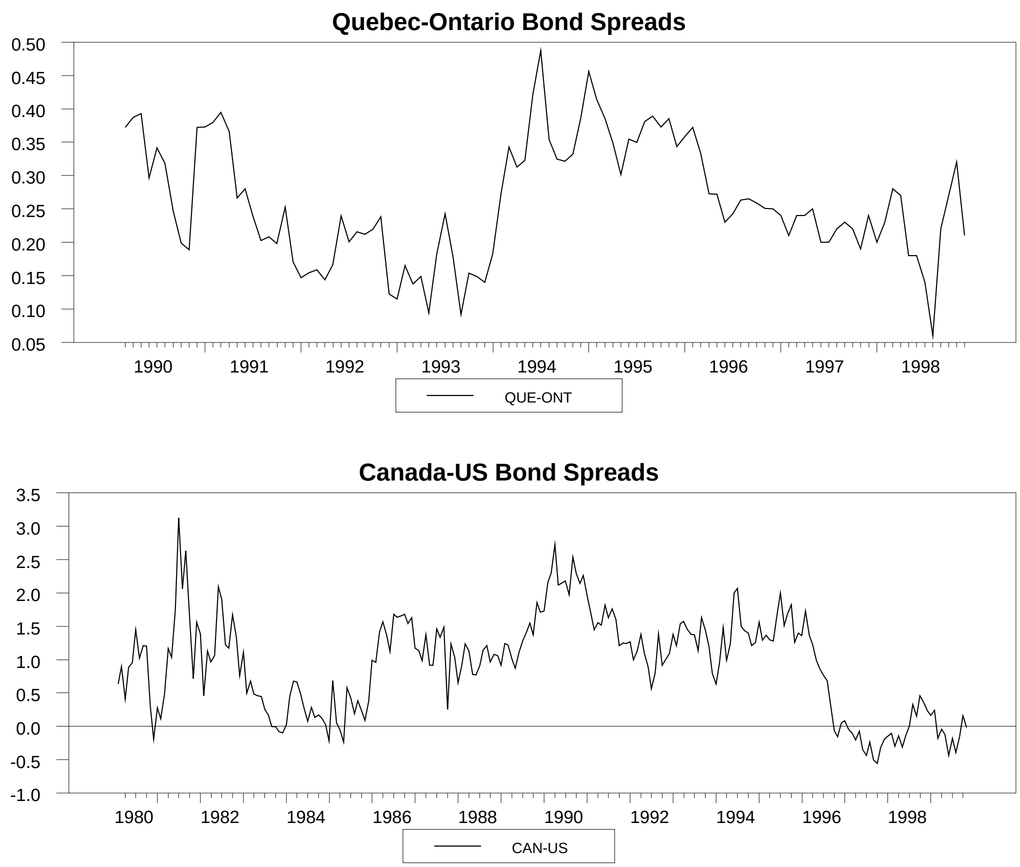

Figure 1: Bond Spreads

and for 1992 and 1993 as investors became convinced that the fundamental support for separation was lower than 50 percent, contrary to the smoothed poll results, which indicate that support for sovereignty was very high in this period.

All the indicators that we have considered in this section exhibit "nonstationarity". That is, there are breaks in the trend of the data depending on whether the economy is going through a period of political stability or one of political risk. Furthermore, as evidenced by the smoothed poll data, there is a difference in the trend even between the first episode of political risk and the second. To avoid problems of non-stationarity, we will only focus on the second episode of political risk, namely the 1990-96 period. In spite of the shortcomings mentioned above, and taking these caveats into account, we will also use the smoothed poll data as our measure of the perception of political risk in Quebec.

2.4 Investment and political risk: analyzing the data

We now relate our measure of political risk, namely, a five-year moving average of poll data (as described in the previous section), to investment behavior in Quebec. We also take the neighboring province of Ontario as a point of comparison and consider the relative investment performance in Quebec and Ontario. [16]

We use alternative measures of investment to examine investment performance by sector, by type of investment (public or private), and by type of the investment good (machinery and equipment or structures). Sectoral data on investment and capital stocks include both public and private investment expenditures.[17] To isolate the private response of investment to separation risk, we con-

[16]Taking Ontario's investment performance as a benchmark may be criticized on the grounds that political uncertainty relating to the Quebec issue has had an impact on the entire Canadian economy. However, its impact on Quebec has undoubtedly been far stronger. Hence the comparison is still relevant.

[17]The data that are used to calculate the investment series are described in detail in Appendix B.

sider investment expenditures in manufacturing industries (which includes nondurable and durable manufacturing industries). According to a study conducted by the firm Dun and Bradstreet, separatism has been responsible for the departure of more than 500 head offices from Montreal since 1976, and for the equally important departure of many strategic activities within corporate headquarters (though the extent of the latter is more difficult to quantify). To capture the effects of political risk on such headquarter or other coordinating activities, we also consider investment behavior in the broadly defined business sector (which includes manufacturing industries plus construction, transportation and storage, wholesale trade, retail trade, finance and insurance, real estate, business service industries, accomodation, food, and beverage services, and other service industries).

Figures 2 and 3 present time plots of the difference of the investment-capital stock ratios in Quebec and Ontario versus the results of the smoothed opinion polls (described in Section 2.3) for the period 1963-1998. The three panels in Figure 2 plot the difference of the investment-capital stock ratios for investment in machinery and equipment versus the poll results for each of the following sectors: the manufacturing sector, the more broadly defined business sector, and for total industries (including agriculture, natural resource industries, communications, and government and other public sector industries). Figure 3 provides the same information for investment in structures. One finding that emerges from Figure 2 is that while the investment-capital stock ratios in Quebec and Ontario are almost identical in 1963, a significant divergence has occurred by 1998, with the investment-capital stock ratios for investment in machinery and equipment across all three sectors in Quebec showing large declines relative to Ontario's after 1990. These declines coincide with the rise of political risk after the failure of the Meech Lake Accord in 1990 and the subsequent increase in support for separatism. As Figure 2 shows, the large decline in the investment-capital stock ratios after 1990 are accompanied by the equally large increases in the opinion polls favoring

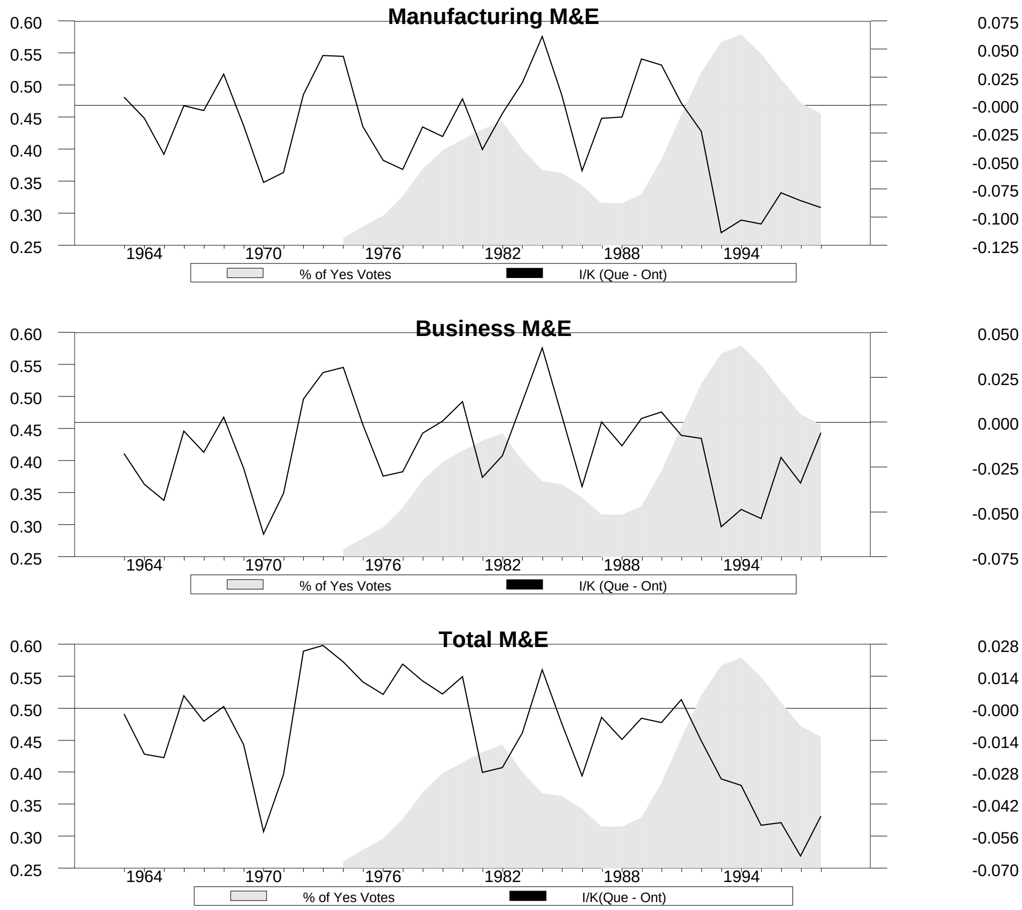

Figure 2: Poll Results versus Investment-Capital Stock Ratios – M&E

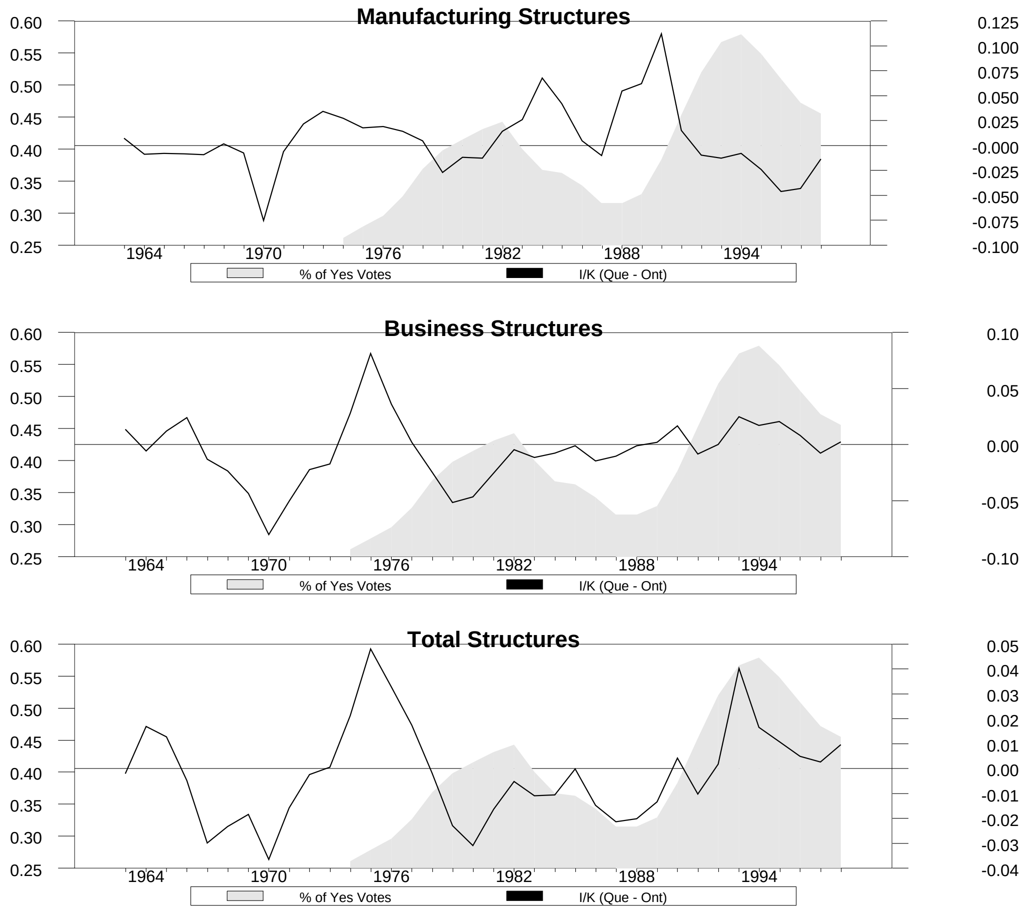

Figure 3: Poll Results versus Investment-Capital Stock Ratios – Structures

separation. As Figure 3 shows, similar declines in the investment-capital stock ratios are also observed for structures investment in Quebec after 1990. By contrast, investment as a fraction of the capital stock was higher in Quebec relative to Ontario for both machinery and equipment and structures in 1988.[18]

During the 1963-1998 period, we can identify four episodes: political stability (1962 -75); the first episode of political risk (1976-80); political stability (1981-89); and the second episode of political risk (1990-98). To test whether political risk had a significant effect in reducing the investment-capital stock ratio in Quebec relative to Ontario, we define the random variable $d_j \equiv [I/K_{Que} - I/K_{Ont}]^j$ as the difference between the investment-capital stock ratios for Quebec and Ontario, where $j = 0$ for the episode of political stability and $j = 1$ for the episode of political risk. Also define μ_j the population mean of d_j for $j = 0, 1$. As a simple test of the hypothesis that the divergence in the I/K values for Quebec versus Ontario are significantly related to the existence of political risk, we can test whether $\mu_0 = \mu_1$ versus the alternative that $\mu_0 > \mu_1$. We concentrate on the second episode of political risk because our simulation results pertain to this episode. Note that there was a recession in 1980-82 and another one in 1990-91. Thus by comparing the second episode of political stability 1981-1989 with the second episode of political risk, we can net out the effect of common Canadian factors that arise from the existence of the recessions in 1981 and 1990, respectively.[19]

[18] This is most likely due to a strong anticipatory reaction from Quebec to the 1988 Canada-US Free-Trade Agreement (FTA), which constituted a landmark in increasing its exports to the US market. The impact of the FTA on Ontario was also strong but was somewhat mitigated by industrial adjustments and by rationalization.

[19] There are two caveats to the above analysis. First, Ontario's economy (as all of Canada) has also been affected by political risk related to the Quebec issue, and second, Ontario's economy has been negatively affected by a social-democratic government in power from 1991 to 1995. However, with respect to the first caveat, the negative impact of political risk on Ontario's economy is substantially less than for Quebec. With respect to the second caveat, the

The sample values of the statistic for the test of the null hypothesis that $\mu_0 = \mu_1$ for the episodes of political stability 1981-1989 and political risk 1990-1998 are given by 2.71892, 1.79629, and 2.16126 for investment in machinery and equipment for manufacturing, business, and total industries.[20] Since the t-value for a one-sided test of the null hypothesis that $\mu_0 = \mu_1$ against the alternative hypothesis that $\mu_0 > \mu_1$ is equal to 1.746 at the 5% level of significance, we can reject the null hypothesis that the mean values of the variable $d_j \equiv [I/K_{Que} - I/K_{Ont}]^j$ were equal for $j = 0, 1$ in favor of the alternative that the mean value of $d_1 \equiv [I/K_{Que} - I/K_{Ont}]^1$ was lower than the mean value of $d_0 \equiv [I/K_{Que} - I/K_{Ont}]^0$. This indicates that I/K_{Que} fell significantly relative to I/K_{Ont} in the episode of political risk for investment in machinery and equipment in all three sectors described above. This result provides confirmation that the rise in political risk in 1990 led to a significant shortfall of the investment-capital stock ratio for Quebec relative to Ontario for investment in machinery and equipment, with the strongest effect occurring for the manufacturing sector.

Turning to the behavior of structures investment, the top panel of Figure 3 shows that $[I/K_{Que} - I/K_{Ont}]$ for manufacturing structures became slightly negative after 1990. The sample value of the test statistic that $\mu^0 = \mu^1$ versus the alternative that $\mu^0 > \mu^1$ for investment in structures in the manufacturing sector is 1.55988, which is significant at the 10% level but not at the 5 % level. Thus, there is some evidence for a decline in I/K_{Que} relative to I/K_{Ont} for structures

social democratic government in Ontario was defeated by a pro-business conservative government in 1996. As a result, there was a dramatic turnaround in Ontario's economy as early as 1994 as the defeat of the socio-democrats was widely anticipated.

[20] The sample statistic is calculated as

$$T = \sqrt{n_0 n_1/(n_0 + n_1)}\,\left(\bar{d}_0 - \bar{d}_1\right) / \sqrt{\left(\sum(d_{0t} - \bar{d}_0) + \sum(d_{1t} - \bar{d}_1)\right)/(n_0 + n_1 - 2)},$$

where n_0 and n_1 are the sample sizes for the period of political stability and political risk, respectively.

investment in the manufacturing sector during the episode of political risk. By contrast, when the broadly-defined business sector and total industries are considered, we find the opposite result, that $[I/K_{Que} - I/K_{Ont}]^1$ is larger than $[I/K_{Que} - I/K_{Ont}]^0$ for structures investment. The values of the relevant test statistics are given by -3.12284, and -3.93376, respectively. The second and third panels of Figure 3 show that while the value of $[I/K_{Que} - I/K_{Ont}]$ for investment in structures in the business sector and for total industries fell after 1993 or 1994, it was positive and larger on average during the episode of political risk than its corresponding average value in the episode of political stability. The reason for these results may be that structures investment responds with a time lag to increases in political risk due to a time-to-build feature in investment.[21]

These results show that the episode of political risk in the 1990's is associated with a significant decline in the investment-capital stock ratio for investment in machinery and equipment in Quebec relative to Ontario. This relationship is the strongest for investment in the manufacturing sector. Since the irreversibility constraint is likely to be the most binding for manufacturing industries, these findings provide preliminary support for considering a model of irreversible investment to study the impact of political risk and uncertainty on investment behavior.

3 The theoretical framework

In this section, we describe a model of irreversible investment with learning and regime shifts. We use this model to characterize the time series behavior of investment in Quebec and to determine the quantitative impact of political risk and other factors on Quebec investment. Section 3.1 describes the model, Section 3.2 describes the informational structure, and Section 3.3 characterizes the nature of the optimal solution.

[21] See, for example, Altug (1989).

3.1 The model

We consider a monopolistically competitive risk neutral firm. Each period, it makes variable input and investment decisions. At time t it produces output, Q_t, using its beginning-of-period capital stock, K_t, (which is predetermined at t), and a variable labour input, L_t. Output at date t is also affected by a stochastic technology shock A_t. We assume that the firm's production function has a Cobb-Douglas specification as follows

$$Q_t = F\left(K_t, L_t, A_t\right) = A_t K_t^{\eta_1} L_t^{\eta_2}, \tag{3.1}$$

where A_t is a stochastic shock to technology and $\eta_1 + \eta_2 \geq 1$ to allow for the possibility of increasing returns to scale.

Let p_t denote the stochastic output price. We assume a constant elasticity demand function. The inverse market demand function is given by:

$$p_t = \left(\alpha_t\right)^{-\frac{1}{\varepsilon}} \left(Q_t\right)^{\frac{1}{\varepsilon}}, \tag{3.2}$$

where $\varepsilon < -1$ is the price elasticity of demand and α_t is a stochastic parameter representing the state of demand.

Using the Cobb-Douglas specification of the production function and the constant elasticity demand function, and after carrying out static optimization with respect to the variable factors of production, the firm's short-run profit function has the form

$$\Pi\left(K_t, \alpha_t, A_t, w_t\right) \equiv \nu \alpha_t^{\mu_1} K_t^{\mu_2} A_t^{\mu_3} w_t^{\mu_4},$$

where w_t denotes the variable stochastic input price, and where $\mu_1 = 1/(\varsigma_2 - \varepsilon)$, $\mu_2 = -\varsigma_1/(\varsigma_2 - \varepsilon)$, $\mu_3 = -(1 + \varepsilon)/(\varsigma_2 - \varepsilon)$, and $\mu_4 = \varsigma_2/(\varsigma_2 - \varepsilon)$, with $\varsigma_1 = \eta_1(1 + \varepsilon)$ and $\varsigma_2 = \eta_2(1 + \varepsilon)$. We also have that $\nu = (\varepsilon/\varsigma_1)^{\varsigma_2/(\varsigma_2-\varepsilon)}[1-\varsigma_2/\varepsilon]$.[22] We assume that $\Pi(K_t, \alpha_t, w_t, A_t)$ is bounded for finite K_t, α_t, w_t and A_t.

The firm's after-tax cash flow at time t, R_t, is defined as

$$R_t = (1-\tau_t)\,\Pi\left(K_t, \alpha_t, w_t, A_t\right) + \tau_t \sum_{x=1}^{T} D_{x,t-x} p^k_{t-x} I_{t-x} - (1-\gamma_t)\, p^k_t I_t \tag{3.3}$$

where I_t is the firm's rate of gross investment measured in physical units, and p^k_t the purchase price of investment goods, τ_t is the corporate tax rate at time t, γ_t is the investment tax credit at time t as a percentage of the price of the investment good, $D_{x,t-x}$ is the depreciation allowance per dollar invested for tax purposes for capital equipment of age x on the basis of the tax law effective at time $t-x$, and T is the life of the equipment. Let r denote the real rate of interest. For future reference, define z_t as the present value of tax deductions on new investment and p^I_t as the tax-adjusted price of investment goods, where

$$z_t = \sum_{n=1}^{T} \tau_{t+n} D_{n,t} \left(1+r\right)^{-n} \tag{3.4}$$

and

$$p^I_t = (1-\gamma_t - z_t)\, p^k_t. \tag{3.5}$$

We will assume that the tax-adjusted price of investment $\widetilde{p}^I_t$ can be written in terms of an unknown permanent component denoted $\overline{p}^I_t$, and a transitory component denoted $\widetilde{\xi}^I_t$, where $\widetilde{\xi}^I_t$ is a random variable with a known distribution function. One possibility is that the price of capital is determined as a function of an unknown permanent component and a transitory component as $\widetilde{p}^k_t = \exp(\bar{p}^k)\xi^P_t$. In this case, the tax-adjusted price of investment $\widetilde{p}^I_t$ can be written as $\widetilde{p}^I_t =$

[22]It is straightforward to show that $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 > 0$, $\mu_4 < 0$, $\varsigma_1 < 0$, $\varsigma_2 < 0$ and $\nu > 0$ provided $\eta_2 < \varepsilon/(1+\varepsilon)$. Hence, Π_t is increasing in α_t, K_t, A_t and decreasing in w_t if $\eta_2 < \varepsilon/(1+\varepsilon)$. Finally, we have restrictions on the parameters that guarantee the concavity of the short-run profit function in K_t and α_t, namely, $\partial^2\Pi_t/\partial K_t^2 < 0$ if $(\eta_1+\eta_2) < \epsilon/(1+\epsilon)$ and $\partial^2\Pi_t/\partial\alpha_t^2 < 0$ if $\eta_2 < 1$.

$(1-\gamma_t - z_t)\,\widetilde{p}_t^k$, where $(1-\gamma_t - z_t)\exp(\bar{p}^k)$ is the permanent component and $\widetilde{\xi}_t^P$ is the transitory component.[23]. Similarly, the random variable denoting the state of demand is assumed to be written as $\widetilde{\alpha}_t = \exp(\bar{\alpha})\xi_t^{\alpha}$, where $\bar{\alpha}$ is the unknown permanent component and ξ_t^{α} is the transitory component.

Let δ be the deterministic depreciation rate, with $0 < \delta < 1$. The law of motion of the capital stock is

$$K_{t+1} = (1-\delta)\,K_t + I_t \tag{3.6}$$

We assume that investment is irreversible:

$$I_t \geq 0 \tag{3.7}$$

3.2 Informational structure

In this model, we analyze the impact of political risk when the firm must make irreversible investment decisions under uncertainty about demand and uncertainty about the tax-adjusted investment price. We assume that the true state of demand and the true price of the investment good are unknown but constant. However, firms can use noisy realizations for these variables to learn about their true values using a Bayesian updating scheme.[24] Before turning to the learning scheme used by firms, we discuss the process governing regime shifts.

[23] Alternatively, it may be the case that the price of capital p_t^k is known but the variable $(1-\gamma_t - z_t)$ showing the effects of the investment tax credit and the present value of depreciation allowances is comprised of an unknown permanent component and a transitory component.

[24] In a different context, F.S. Demers (1985) analyses the role of Bayesian learning as a propagation mechanism in a model of cyclical fluctuations at the aggregate level.

3.2.1 Regimes shifts

A key feature of the informational structure of the model is that the firm's environment is characterized by two possible regimes, the current regime, regime 0 (which is favourable) and another less favourable regime, regime 1, a transition to which may occur with positive probability. Thus, while firms can observe the current regime (regime 0), they must take into account the possibility of a regime shift in the future. In what follows, we define the "bad" regime as a regime as the separation of Quebec from the Canadian federation.

The regime shift is governed by a two-state Markov chain with time-varying transition probabilities. Each period, firms assess the transition probabilities on the basis of a vector $\mathbf{x_t}$ of economic and political variables that may affect the transition probability to next period's regime.[25] Thus, economic indicators, indicators of political stability, etc. could be elements of the vector x_t that would be considered by firms when assessing the probability of transiting to regime 1.

To describe the two-state Markov process governing the regime shift, let $\boldsymbol{P_t}$ denote the matrix of transition probabilities at time t where

$$\boldsymbol{P_t} = \begin{bmatrix} \chi_{t,00} & \chi_{t,01} \\ \chi_{t,10} & \chi_{t,11} \end{bmatrix} \tag{3.8}$$

where

$$\chi_{t,00} = Pr(s_{t+1} = 0 \mid s_t = 0, \boldsymbol{x_t}) \tag{3.9}$$

$$\chi_{t,11} = Pr(s_{t+1} = 1 \mid s_t = 1, \boldsymbol{x_t}), \tag{3.10}$$

where $\mathbf{x_t}$ is a vector of economic and political variables observed at time t. To simplify the notation,

[25] Another example of a situation where the probability of a regime shift depends on economic and political variables is the case of an economy which is currently functioning under a policy of trade liberalization, but where a deterioration of economic and political conditions could provoke a shift to protectionism.

we supress the dependence of the transition probabilities on the vector $\boldsymbol{x_t}$ (that is, instead of $\chi_{t,00}(\boldsymbol{x_t})$ we simply write $\chi_{t,00}$; note, however, that χ is a time-independent function of $\boldsymbol{x_t}$.)[26] In addition, we specify

$$\chi_{t,01} = Pr(s_{t+1} = 1 \mid s_t = 0, \boldsymbol{x_t}) = (1 - \chi_{t,00})$$

as the probability of a regime shift in period $t+1$ given that regime 0 is in effect at time t, and we similarly let:

$$\chi_{t,10} = Pr(s_{t+1} = 0 \mid s_t = 1, \boldsymbol{x_t}) = (1 - \chi_{t,11})$$

be the probability of shifting to regime 0, given that regime 1 is in effect at time t (if the policy regime shift has occured in some previous period $s < t$). This specification with $\chi_{t,10} > 0$, admits the possibility that should a policy shift to regime 1 occur at some time $s < t$, there is a possibility of returning to the status quo ante. (In other words, the regime shift is not perceived as being itself totally irreversible.) Even though firms observe the realization of $\mathbf{x_t}$ at time t, they do not know the future realizations of this random variable which is governed by a stationary first-order Markov process and has the conditional density $f_{\boldsymbol{x}}(\boldsymbol{x_t} | \boldsymbol{x_{t-1}})$.

This specification of a regime shift with time-varying transition probabilities is similar to the scheme in Diebold, Lee and Weinbach (1995) and in Filardo (1994). However, whereas in these models the current regime cannot be directly observed, we are here concerned with a situation where the current regime is observed by firms, but the future regime is not known with certainty.

[26] In principle, any functional form that maps the vector $\boldsymbol{x_t}$ into the unit interval can be used to obtain the transition probabilities. Of these, the logit and probit among others yield a well-defined log-likelihood function and are compatible with maximum likelihood estimation. See Filardo (1994).

3.2.2 Learning

Political risk will influence the distributions of the state of demand and of the purchase price of capital faced by firms. We assume that the firm knows the *possible* states of demand and of investment prices but does not know with certainty which one is the true state of demand or the true state for the after-tax purchase price of capital goods. Even though the firm does not know which is the true state of demand, it is assumed to know that the probability of high values of the state of demand in the "good regime" (here, the status quo) will be higher than the probability of high values of the state of demand in the "bad regime" (if a policy regime change should occur).

Let $\overline{A} \equiv \{\overline{\alpha}_1, ..., \overline{\alpha}_{n_1}\}$ denote the set of possible states of demand, and let $\overline{P} \equiv \{\overline{p}_1^I, \ldots, \overline{p}_{n_2}^I\}$ be the set of possible states for the price of investment goods. Let us define $\Omega \equiv \{\overline{\omega}_1, ..., \overline{\omega}_n)$ as the set of possible pairs of demand and purchase price states where $\overline{\omega} \equiv (\overline{\alpha}, \overline{p}^I)$ and $n = n_1 \cdot n_2$. The firm has a prior probability distribution about the true state $\overline{\omega}$, denoted by Ψ^{0s}, where

$$\Psi^{0s} = [\psi_1^{0s}, \ldots, \psi_n^{0s}], \qquad \psi_i^{0s} > 0, \;\; i = 1, \ldots, n, s = 0, 1 \;\text{ and }\; \sum_{i=1}^{n} \psi_i^{0s} = 1;$$

ψ_i^{0s} is the prior probability that $\overline{\omega} = \overline{\omega}_i, i = 1, \ldots, n$ conditional on the current regime being $s = 0, 1$. We will refer to the $\overline{\omega}$'s as the state (or true state) and to $s_t = 0, 1$ as the regime at t.

Each period the firm observes the realization of $\widetilde{\alpha}_t$ and $\widetilde{p}_t^I$ and the realization of a non-price signal $\widetilde{b}_t$ (which can be a vector) which is informative about $\overline{\alpha}$ and $\overline{p}^I$ and which may consist of various pieces of information such as the monetary growth rate, financial surveys, etc.. We denote by H the set of all possible signals, $h_t \equiv (\alpha_t, p_t^I, b_t)$. We define the likelihood of observing h_t, given that the true state is $\overline{\omega}_i$ and that the current regime is $s_t = 0, 1$ as

$$\lambda(h_t \mid \overline{\omega}_i, s_t) \equiv f_\alpha^s(\alpha_t \mid \overline{\omega}_i) f_p^s(p_t^I \mid \overline{\omega}_i) f_b^s(b_t \mid \overline{\omega}_i). \tag{3.11}$$

where $f_j^s(\widetilde{z}_t \mid \overline{\omega}_i)$, $j = \alpha, p, b$, $s = 0, 1$ is the density $\widetilde{z}_t = \widetilde{\alpha}_t, \widetilde{p}_t^I, \widetilde{b}_t$, conditional on $\overline{\omega}_i$ being the true state given regime s.The firm revises its prior probability distribution Ψ^{0s} by applying Bayes' law, so that its state of information at $t = 1, 2, ...$ after observing the current regime $s_t = i, i = 0, 1$, and the realization of h_t, will be given by the vector whose k^{th} element is as follows:

$$\psi_k^{ti} = \frac{\lambda(h_t \mid \overline{\omega}_k, s_t = i)\psi_k^{t-1,i}}{\sum_{m=1}^{n} \lambda(h_t \mid \overline{\omega}_m, s_t = i)\psi_m^{t-1,i}}, \qquad h_t \in H,\ k = 1, \ldots, n,\ i = 0, 1, \tag{3.12}$$

$$\text{and} \qquad \psi_k^{tj} = \psi_k^{t-1,j}, \qquad k = 1, \ldots, n,\ j = 0, 1,\ \text{for } s_t = i \neq j,\ i = 0, 1. \tag{3.13}$$

for $t = 1, 2, \ldots$ When the current regime is (say) $s_t = i = 0$, then the observation h_t is drawn from the distribution that pertains to regime 0. Therefore, the probability vector $\Psi^{t0}(h_t)$ has as its k^{th} element the expression given in the first equation above, where the prior probability $\psi_k^{t-1,0}$ is updated, whereas, as indicated in the second equation, the prior probability $\psi_k^{t-1,1}$ will not be revised this period, since the observation pertains to a different regime. Hence, when $s_t = i = 0$, $\Psi^{tj} = \Psi^{t-1,j}$ for $j = 1 \neq i$. Then, $(\Psi^{ti}(h_t), \Psi^{tj}(h_t)) = ([(\psi_1^{ti}(h_t), \ldots, \psi_n^{ti}(h_t)], [\psi_1^{tj}(h_t), \ldots, \psi_n^{tj}(h_t)])$ represents the firm's state of information at time t, conditional on regime i occurring at time t, and where $\psi_k^{ti}, \psi_k^{tj}, k = 1, \ldots n$ are given by (3.12)-(3.13). For all t and s, $\Psi^{ts} \in D(\Omega)$, the set of all probability distributions on Ω. The evolution of the information state can be expressed as

$$\Psi^{ti}(h_t, s_t = i) = g^i\left(\Psi^{t-1,i}, h_t\right) \tag{3.14}$$

$$\Psi^{tj}(h_t, s_t = i) = \widehat{g}^i(\Psi^{t-1,j}), \quad j \neq i. \tag{3.15}$$

When the regime at time t is $s_t = i$, the function g^i denotes updating according to Bayes' law, while the function $\widehat{g}^i$ is the identity function. Note that $g^i, \widehat{g}^i$ do not have a time subscript since the revision process is time-invariant. Furthermore, the evolution of the information state has the Markov property since the past information states $\Psi^{1s}, \ldots, \Psi^{t-2,s}$ are irrelevant to the revision process as long as $\Psi^{t-1,s}$ and the current regime are known. In other words, $\Psi^{t-1,s}$ completely describes the state of information at time $t - 1$ conditional on regime s.

The predictive density of h_{t+1} conditional on regime $s_{t+1} = i$ and on the information vector (Ψ^{ti}, Ψ^{tj}) is given by

$$\Theta^i(h_{t+1} \mid \Psi^{ti}) = \sum_{m=1}^{n} \lambda(h_{t+1} \mid \overline{\omega}_m, s_{t+1} = i)\psi_m^{ti}, \;\; i = 0, 1, \tag{3.16}$$

where the i superscript denotes the regime at time $t + 1$.

3.3 Characterizing the optimal solution

We can express the firm's problem recursively using a dynamic programming approach. The state variables of the problem consist of the current period capital stock K_t, the information vector Ψ^{ti}, the current realization of the vector of economic variables $\boldsymbol{x_t}$, and the current regime s_t. Letting the discount factor $\beta = (1 + r)^{-1}$ where r is the real rate of interest, we can express the firm's problem as

$$\begin{aligned} V(K_t, \Psi^{ti}, \boldsymbol{x_t}, s_t = i) = \max\nolimits_{I_t}\{(1 - \tau_t)\,\Pi\,(K_t, \alpha_t, w_t, A_t) - p_t^I I_t \\ +\beta \textstyle\sum_{j=0}^{1} \int_{H\times X} \chi_{t,ij} V(K_{t+1}, \Psi^{t+1,j}\,(h_{t+1})\,, \boldsymbol{x_{t+1}}, s_{t+1} = j)\Theta^j(h_{t+1}|\Psi^{ti}) f_{\boldsymbol{x}}(\boldsymbol{x_{t+1}}|\boldsymbol{x_t}) d\boldsymbol{x_{t+1}} dh_{t+1}\} \end{aligned} \tag{3.17}$$

subject to (3.6), (3.7), (3.14), (3.15), $K_t, \Psi^{ti}, \Psi^{tj}$ given, and where $V(K_t, \Psi^{ti}, \boldsymbol{x_t}, s_t = i)$ denotes the conditional value function, conditional on regime i at time t. The value of undertaking additional investment at date t is the sum of the current return from the investment and the expected future value, conditional on the regime at date $t+1$. In particular, the possibility of a regime shift at date $t + 1$ affects the way in which the firm evaluates uncertainty about the unknown state of demand and the tax-adjusted price of capital.

Let $Y \equiv [0, \overline{K}] \times D[\Omega] \times X \times \{0, 1\}$. Define $C(Y)$ as the space of continuous and bounded functions. For any $V \in C(Y)$, define the mapping $(TV)(K_t, \Psi^t, \boldsymbol{x_t}, s_t)$ from the right-side of (3.17).

Proposition 1 *There exists a unique continuous bounded solution V to the functional equation $TV = V$. Given any $V^0 \in C(Y), \lim_{m\to\infty} T^m V^0 = V$ where convergence is uniform. The value function V is concave in K_t, is continuously differentiable in K_t almost everywhere, and is uniquely attained by the single valued investment policy function $I(K_t, \Psi^t, \boldsymbol{x_t}, s_t)$ which is almost everywhere continuously differentiable in its arguments.*

Proofs of all propositions are provided in Appendix C. Notice that the conditional valuation function in (3.17) differs from the irreversible model of investment studied by Demers (1991) and Altuğ, Demers and Demers (1999) due to the existence of regime shifts.

Let V_K denote the partial derivative of V with respect to K. The first-order necessary and sufficient conditions for the optimization problem at time t are

$$\begin{aligned} -p_t^I + \beta E_t V_K(K_{t+1}, \Psi^{t+1,j}(h_{t+1}), \boldsymbol{x_{t+1}}, s_{t+1} = j) \leq 0 \quad &\text{if } I_t^* = 0 \\ = 0 \qquad &\text{if } I_t^* > 0. \end{aligned} \tag{3.18}$$

where E_t indicates that expectations are taken conditional on information available at time t in accordance with the distributions described above. Using the envelope theorem, we find

$$\begin{aligned} V_K(K_{t+1}, \Psi^{t+1,i}(h_{t+1}), \boldsymbol{x_{t+1}}, s_{t+1} = i) &= (1-\tau_{t+1})\,\Pi_K(K_{t+1}, \alpha_{t+1}, w_{t+1}, A_{t+1}) \\ &+ (1-\delta)\min\left[p_{t+1}^I, \beta E_t V_K(K_{t+1}, \Psi^{t+1,i}, \boldsymbol{x_{t+1}}, s_{t+1} = i)\right] \end{aligned} \tag{3.19}$$

where $V_K(K_{t+1}, \Psi^{t+1,i}(h_{t+1}), \boldsymbol{x_{t+1}}, s_{t+1} = i)$ is the shadow value of capital and Π_K is the partial derivative of Π with respect to K_{t+1}. This expression shows that the future marginal product of capital (or its shadow price) at time $t+1$ equals next period's marginal value of capital plus the minimum of next period's price of investment goods (if an interior solution prevails at $t+1$) and of the expected marginal value of capital from $t+2$ onwards evaluated at the undepreciated capital stock $(1-\delta)K_{t+1}$ (if a corner solution occurs at $t+1$).

Lemma 1 *V_K is an increasing and concave function of α_t and p^I_t, and it is a concave function of Ψ^t.*

The first-order condition (3.18) for time t can be rearranged after substituting for the shadow price and for $\beta = (1+r)^{-1}$ as

$$(1-\tau_{t+1})E_t\Pi_K\left(K_{t+1}, \widetilde{\alpha}_{t+1}, \widetilde{w}_{t+1}, \widetilde{A}_{t+1}\right) = c_t + (1-\delta)\, E_t\{\widetilde{p}^I_{t+1} -$$

$$\min\left[\widetilde{p}^I_{t+1}, \beta E_{t+1} V_K\left((1-\delta)\, K_{t+1}, g^{s_{t+2}}\left(\Psi^{t+1}, \widetilde{h}_{t+2}\right), \widetilde{\boldsymbol{x}}_{t+2}, \widetilde{s}_{t+2}\right)\right]\} \tag{3.20}$$

where $c_t = p^I_t\,(r+\delta) - (1-\delta)\,(E_t\widetilde{p}^I_{t+1} - p^I_t)$ is the firm's cost of capital in the sense of Jorgenson (1963).[27] That is, the cost of one unit of capital is expressed as net of expected capital gains on the undepreciated portion of the unit. The second term on the right-side of (3.20) represents a positive marginal cost arising from the irreversibility of investment. If investment were reversible, equation (3.20) would reduce to

$$(1-\tau_{t+1})E_t\Pi_K\left(K_{t+1}, \widetilde{\alpha}_{t+1}, \widetilde{w}_{t+1}, \widetilde{A}_{t+1}\right) = c_t \tag{3.21}$$

The firm's problem would then be purely static. The second-term on the right-side of (3.20) represents a risk premium that the firm requires for the loss of flexibility that it incurs when investment is irreversible. This risk premium arises because the firm cannot disinvest if the state of demand of the price of investment should turn out to be less favorable than expected. It plays an analogous role to the marginal adjustment cost in cost of adjustment models of investment, but is endogenously determined and depends specifically on the uncertainty and risk faced by the firm. We note from (3.20) that political risk adversely affects the expected marginal value of capital and the cost of adjustment term. In fact, as in Rodrik (1991), the presence of political risk is akin to a

[27] See also Nickell (1978), Chapters 2, 8 and 9.

tax on capital accumulation.[28]

Proposition 2 *If the bad regime is riskier than the good regime in the sense of first order stochastic dominance (FSD) or in the sense of a mean preserving spread (MPS), political risk increases the marginal adjustment cost, reduces the expected marginal value of capital, and reduces irreversible investment. Furthermore, the anticipation of learning the true state of demand and price reduces current investment as the firm adopts a wait-and-see attitude.*

If demand is stochastically lower and/or the investment price is stochastically higher in the bad regime or, if demand and the investment price are expected to be more variable in the bad regime, then even a small probability that a regime shift may occur will increase the uncertainty that the firm must face when making irreversible investement decisions. This will be the case even if the bad regime has never been observed to date, as is true of our application to the Quebec economy. Furthermore, since investors do not yet know, but are in the process of learning the true state of demand and price of investment, they wait for better information before undertaking investments that are irreversible, especially if they do not have high confidence in their current (prior) beliefs, or alternatively if they are pessimistic about future economic prospects. As a result, investment is depressed. Hence, in our model, the psychology of investors or the "investment climate" plays a central role in the determination of investment.

In the long-run, once learning is complete the firm will reach its steady-state. (See Appendix B.) The firm's steady-state (or desired) capital stock corresponds to a situation where it has learned the true state of demand and price of investment goods but where it still faces risk (due to the randomness in the objective distributions of these economic variables) including the possibility of

[28] See Rodrik (1991) who shows in his model that uncertainty regarding the duration of a reform (or the probability of a policy reversal) is equivalent to raising a tax on capital investment.

a regime shift. Hence, as long as the economy continues to be subject to political risk, investment decisions will be negatively affected.

We have assumed that the passage from the good regime to the bad regime is not irreversible. That is, we view investors as ascribing a small probability of Quebec remaining in the federation even after a victory of a Yes vote on a referendum question. This would be the case if once the process of separation began, it were to come to a halt because of all the difficulties of implementation. If, on the other hand, the bad regime were an absorbing state (in this case a disaster state from which there is no return) and were accurately perceived as such by investors, the risk and uncertainty faced by the firm would substantially increase. As a result, due to the increased risk irreversible investment would fall even more.[29]

3.4 Accounting for trends

The model we have just described cannot be directly applied to the Quebec economy for our chosen sample period due to the existence of trends. The data on real wages, the corporate tax rate, the tax-adjusted price of investment, and the measure of technology shocks that we use contain significant trends over the sample period. Hence, we must modify the dynamic programming model in order to account for these trends in the data. As a simplifying device, we assume that the technology shock $\widetilde{A}_t$, the real wage $\widetilde{w}_t$, the tax-adjusted price of investment $\widetilde{p}_t^I$, and one minus the corporate tax rate $1-\widetilde{\tau}_t$ evolve as stationary processes around a deterministic trend as

$$\widetilde{A}_{t+1} = (1+n_A)^{t+1} A_0 \widetilde{\xi}_{t+1}^A, \tag{3.22}$$

[29]Proposition 2 examines the short-run effect of political risk on irreversible investment. If the bad regime is realized and persists for a number of years, firms may find themselves with too much capital in (or what Abel and Eberly refer to as a hangover effect.) We do not consider this issue here and leave it for future work.

$$\widetilde{w}_{t+1} = (1+n_W)^{t+1} w_0 \widetilde{\xi}^W_{t+1}, \tag{3.23}$$

$$\widetilde{p}^I_{t+1} = (1+n_P)^{t+1} p^I_0 \widetilde{\xi}^P_{t+1}, \tag{3.24}$$

$$(1-\widetilde{\tau}_{t+1}) = (1+n_\tau)^{t+1}(1-\tau_0)\widetilde{\xi}^\tau_{t+1}, \tag{3.25}$$

where $\widetilde{\xi}^A_{t+1}$, $\widetilde{\xi}^W_{t+1}$, $\widetilde{\xi}^P_{t+1}$, and $\widetilde{\xi}^\tau_{t+1}$ are the innovations to $\widetilde{A}_{t+1}$, $\widetilde{w}_{t+1}$, $\widetilde{p}^I_{t+1}$, and $1-\widetilde{\tau}_{t+1}$, respectively. We further assume that $\widetilde{\xi}^A_{t+1}$, $\widetilde{\xi}^W_{t+1}$, $\widetilde{\xi}^P_{t+1}$, and $\widetilde{\xi}^\tau_{t+1}$ are lognormally distributed, or equivalently, that $\ln(\widetilde{\xi}^A_{t+1}) \sim N(\bar{A}, \sigma^2_A)$, $\ln(\widetilde{\xi}^W_{t+1}) \sim N(\bar{W}, \sigma^2_W)$, $\ln(\widetilde{\xi}^P_{t+1}) \sim N(\bar{p}^I, \sigma^2_P)$, and $\ln(\widetilde{\xi}^\tau_{t+1}) \sim N(1-\bar{\tau}, \sigma^2_\tau)$.

It is straightforward to show that the desired capital stock K^*_{t+1} in a static version of the firm's problem evolves as

$$K^*_{t+1} = (1+n)(\xi^K_{t+1}/\xi^K_t)K_t, \tag{3.26}$$

where

$$(1+n) = \left[\frac{(1+n_\tau)(1+n_A)^{\mu_3}(1+n_W)^{\mu_4}}{1+n_P}\right]^{1/(1-\mu_2)},$$

$$\xi^K_{t+1} = \left\{\frac{E_t\left[(\widetilde{\xi}^\tau_{t+1})(\widetilde{\alpha}_{t+1})^{\mu_1}(\widetilde{\xi}^A_{t+1})^{\mu_3}(\widetilde{\xi}^W_{t+1})^{\mu_4}\right]}{\xi^c_t}\right\}^{1/(1-\mu_2)},$$

$$\xi^c_t = (r+\delta)\xi^P_t - (1-\delta)\left[(1+n_P)E_t(\widetilde{\xi}^P_{t+1}) - \xi^P_t\right],$$

given the initial conditions K_0, τ_0, A_0, w_0, and p^I_0. Given our assumptions on the underlying variables, ξ^K_{t+1} is a stationary variable. Thus, the desired capital stock evolves randomly around an endogenous deterministic trend. If $\{\widetilde{\xi}^\tau_t\}$, $\{\widetilde{\xi}^A_t\}$, $\{\widetilde{\xi}^W_t\}$, and $\{\widetilde{\alpha}_t\}$ are independently and identically distributed over time and there is no learning in the firm's problem, then the conditional expectations of future values of innovations to taxes, technology, demand, and real wages are zero and the ratio (ξ^K_{t+1}/ξ^K_t) depends only on the innovations to the standard user cost of capital. If there is learning, however, this does not occur because the conditional expectation of future values of $\tilde{\xi}_t^P$ and $\tilde{\alpha}_t$ are evaluated using the predictive density for these variables, which depends on the current realizations of the shocks.

We can derive an alternative expression for the endogenous trend n as

$$1 + n = \frac{(1+n_A)^{-(1+\epsilon)/[(\eta_1+\eta_2)(1+\epsilon)-\epsilon]}(1+n_W)^{\eta_2(1+\epsilon)/[(\eta_1+\eta_2)(1+\epsilon)-\epsilon]}}{[(1+n_P)/(1+n^\tau)]^{[\eta_2(1+\epsilon)-\epsilon]/[(\eta_1+\eta_2)(1+\epsilon)-\epsilon]}}.$$

Since $\epsilon < -1$ and $\eta_1 + \eta_2 \geq 1$, a positive trend in the technology shock implies a positive trend in the desired stock of capital. By contrast, with $0 < \eta_2 < 1$, we have that $\eta_2(1+\epsilon) < 0$, which implies that a positive trend in real wages tends to reduce the growth of the desired capital stock. Finally, since $\eta_2(1+\epsilon) - \epsilon > 0$, a positive trend in one minus the corporate tax rate ($n_\tau > 0$) or a negative trend in the price of capital ($n_P < 0$) tends to increase the growth of the desired capital stock.

We now show how to derive sufficient conditions for the existence of a solution to the firm's problem in the irreversible investment model.[30] We then show how this approach can be extended to the model with regime shifts. For any finite initial conditions K_0, τ_0, A_0, w_0, and p_0^I, the firm's expected discounted cash flows evaluated at the optimal solution for K_{t+1}^* are given by

$$V(K_0, \tau_0, A_0, w_0, p_0^I) = \max_{\{K_{t+1}/K_t\}_{t=0}^{\infty}} \Lambda_0 K_0^{\mu_2} E_0 \left\{ \nu\alpha_0^{\mu_1} - \frac{p_0^I K_0^{1-\mu_2}}{\Lambda_0} \left[\frac{K_1}{K_0} - (1-\delta) \right] \right.$$

$$\left. + \sum_{t=1}^{\infty} \bar{\beta}^t \left(\frac{K_t}{K_{t-1}} \cdots \frac{K_1}{K_0} \right)^{\mu_2} \left[\xi_t^\tau \alpha_t^{\mu_1} (\nu \xi_t^A)^{\mu_3} (\xi_t^W)^{\mu_4} - \frac{p_t^I K_t^{1-\mu_2} [K_{t+1}/K_t - (1-\delta)]}{\Lambda_0 (1+n_\tau)^t (1+n_A)^{t\mu_3} (1+n_W)^{t\mu_4}} \right] \right\}, \tag{3.27}$$

where $\Lambda_0 = (1-\tau_0) A_0^{\mu_3} w_0^{\mu_4}$ and $\bar{\beta} = \beta(1+n_\tau)(1+n_A)^{\mu_3}(1+n_W)^{\mu_4}$.

[30] The results for the model without irreversibility follows directly from those for the irreversible investment model.

First, for $V(K_0, A_0, w_0, p_0^k)$ to be finite, it must be the case that

$$\beta(1+n_\tau)(1+n_A)^{\mu_3}(1+n_W)^{\mu_4}(K_{t+1}/K_t)^{\mu_2} < 1 \text{ for } t = 0, 1, 2, \ldots. \tag{3.28}$$

Notice that this condition restricts the maximum growth rate of the capital stock as $K_{t+1}/K_t < (1+\kappa)$, where $(1+\kappa) = [\beta(1+n_\tau)(1+n_A)^{\mu_3}(1+n_W)^{\mu_4}]^{-1/\mu_2}$. Thus, we can define the feasible set of points for the optimal capital stock ratio as $\bar{K} = [(1-\delta), (1+\kappa)]$. Second, we require that the terms involving the initial conditions be finite as

$$\nu\alpha_0^{\mu_1} < M1, \quad \text{and} \quad \nu\alpha_0^{\mu_1} - \frac{p_0^I K_0^{1-\mu_2}\,[(1+\kappa)-(1-\delta)]}{(1-\tau_0)A_0^{\mu_3} w_0^{\mu_4}} > -M_2, \tag{3.29}$$

respectively, where M_1 and M_2 are finite, positive constants. Third, we require that expected cash flows be bounded from above and below for all dates $t = 0, 1, 2, \ldots$. Thus,

$$E_0\left\{\nu\xi_t^\tau \alpha_t^{\mu_1}(\xi_t^A)^{\mu_3}(\xi_t^W)^{\mu_4}\right\} < M_3 < \infty \tag{3.30}$$

and

$$E_0\left\{\nu\xi_t^\tau \alpha_t^{\mu_1}(\xi_t^A)^{\mu_3}(\xi_t^W)^{\mu_4} - \frac{p_t^I K_t^{1-\mu_2}\,[(1+\kappa)-(1-\delta)]}{\Lambda_0(1+n_\tau)^t(1+n_A)^{t\mu_3}(1+n_W)^{t\mu_4}}\right\} > -M_4, \tag{3.31}$$

respectively, where M_3 and M_4 are finite, positive constants. The conditions in (3.30) and (3.31) restrict the means and variances of the lognormally distributed random variables $\xi_t^\tau, \xi_t^A, \xi_t^W$, and ξ_t^P. Since p_t^I has the deterministic trend n_P and K_t grows at a rate less than κ, a necessary condition for expected cash flows to be bounded from below is that

$$\frac{(1+n_P)(1+\kappa)^{1-\mu_2}}{(1+n_\tau)(1+n_A)^{\mu_3}(1+n_W)^{\mu_4}} \leq 1. \tag{3.32}$$

Next, we allow for the effects of regime shifts. Define $\widehat{\mathbf{z}}_t$ as the exogenous state variables for the firm at date t, namely, $\widehat{\mathbf{z}}_t = (\tau_t, A_t, p_t^I, w_t)$. Using the notation of Section 3.3, the value function for the irreversible investment model with regime shifts satisfies

$$V(K_t, \Psi^{ti}, \mathbf{x}_t, \widehat{\mathbf{z}}_t, s_t = i) = \max_{I_t} \Big\{ (1-\tau_t)\Pi(K_t, A_t, w_t, \alpha_t) - p_t^I I_t +$$

$$\beta E_t \Big[\chi_{t,ii} V(K_{t+1}, \Psi^{t+1,i}, \mathbf{x}_{t+1}, \widehat{\mathbf{z}}_{t+1}, s_{t+1} = i) + (1-\chi_{t,ii}) V(K_{t+1}, \Psi^{t+1,j}, \mathbf{x}_{t+1}, \widehat{\mathbf{z}}_{t+1}, s_{t+1} = j) \Big] \Big\}.$$

We can proceed as before and re-write the firm's value function as

$$\frac{V(K_t, \Psi^{ti}, \mathbf{x}_t, \widehat{\mathbf{z}}_t, s_t = i)}{\Lambda_0 (1+n_\tau)^t (1+n_A)^{t\mu_3} (1+n_W)^{t\mu_4} K_t^{\mu_2}} =$$

$$\max_{K_{t+1}/K_t \in \bar{K}} \left\{ \nu \xi_t^\tau \alpha_t^{\mu_1} (\xi_t^A)^{\mu_3} (\xi_t^W)^{\mu_4} - \frac{p_t^I K_t^{1-\mu_2} \left[K_{t+1}/K_t - (1-\delta) \right]}{\Lambda_0 (1+n_\tau)^t (1+n_A)^{t\mu_3} (1+n_W)^{t\mu_4}} + \right.$$

$$\left. \bar{\beta} \left(\frac{K_{t+1}}{K_t} \right)^{\mu_2} E_t \left[\frac{\chi_{t,ii} V(K_{t+1}, \Psi^{t+1,i}, \mathbf{x}_{t+1}, \widehat{\mathbf{z}}_{t+1}, s_{t+1} = i) + (1-\chi_{t,ii}) V(K_{t+1}, \Psi^{t+1,j}, \mathbf{x}_{t+1}, \widehat{\mathbf{z}}_{t+1}, s_{t+1} = j)}{\Lambda_0 (1+n_\tau)^{t+1} (1+n_A)^{(t+1)\mu_3} (1+n_W)^{(t+1)\mu_4} K_{t+1}^{\mu_2}} \right] \right\} \tag{3.33}$$

Define the function $v(K_t, \Psi^{ti}, \mathbf{x}_t, \widehat{\mathbf{z}}_t, s_t)$ as

$$v(K_t, \Psi^{ti}, \mathbf{x}_t, \widehat{\mathbf{z}}_t, s_t = i) \equiv \frac{V(K_t, \Psi^{ti}, \mathbf{x}_t, \widehat{\mathbf{z}}_t, s_t = i)}{\Lambda_0 (1+n_\tau)^t (1+n_A)^{(t)\mu_3} (1+n_W)^{(t)\mu_4} K_t^{\mu_2}}.$$

Let $\widehat{Y} \equiv [0, \overline{K}] \times D[\Omega] \times X \times \widehat{Z} \times \{0,1\}$. Define $C(\widehat{Y})$ as the space of continuous and bounded functions whose elements v_i satisfy $\|v_i\| = \sup \left| v(K_t, \Psi^{ti}, \mathbf{x}_t, \widehat{\mathbf{z}}_t, s_t = i) \right|$ for $i = 0, 1$. For any $v \in C(widehatY)$, define the mapping $(\overline{T}v)(K_t, \Psi^t, \mathbf{x}_t, \widehat{\mathbf{z}}_t, s_t)$ from the right-side of (3.33) subject to (3.14) (3.15), given K_t, Ψ^{ti}, $\mathbf{x}_t$ and $\widehat{\mathbf{z}}_t$.

Proposition 3 *Provided conditions (3.29-3.32) hold, there exists a unique continuous solution v to the functional equation defined by equation (3.33).*

4 Solving and simulating the model

The simulation procedure we use to solve and simulate the model of irreversible investment with learning is based on the analysis in Altug, Demers, and Demers (1999).[31] It combines numerical dynamic programming with Monte Carlo simulation.

We solve for the optimal investment policy function using numerical dynamic programming with value iteration. (See Bertsekas, 1976 or Judd, 1998.) The solution to the model involves finding the optimal policy function for all points in the discretized state space. To see how value function iteration is implemented, for any $v_i \in C(\widehat{Y})$, define the mapping $(\overline{T}v_i)(K_t, \Psi^{ti}, \mathbf{x}_t, \widehat{\mathbf{z}}_t, s_t = i)$ from the right-side of (3.33) subject to (3.14), (3.15), given K_t, Ψ^{ti}, and $\widehat{\mathbf{z}}_t$. Since the mapping $\overline{T}$ satisfies the conditions for the weighted contraction mapping theorem to be a contraction, iterations of the form $\overline{T}^n v_i^0$ where $v_i^0 \in C(\widehat{Y})$ will converge to the true value function v_i^* as n goes to infinity. Since evaluation of the mapping that defines the valuation function v_i involves maximization with respect to the desired capital stock ratio, the optimal investment-capital stock ratio is found as a by-product of determining the function v_i. In practice, the grid search is done by choosing values of K_{t+1}/K_t in the set $\mathcal{K}$ defined as $\mathcal{K} = [(1-\delta), (1+\kappa)]$, where κ denotes the maximum sustainable growth rate of the capital stock.[32]

An additional computational issue arises when the shocks have a continuous distribution. In this case, it is necessary to approximate the expectation of the future valuation function appearing on the right-side of (3.33) as part of the value function iteration used to obtain the solution of the model. In what follows, we use simple Monte Carlo integration for this purpose as described, for example, by Keane and Wolpin (1994). When there is learning, however, firms form their

[31] See Miller (1984) for an early application of solving and estimating a dynamic discrete choice problem under uncertainty with learning.

[32] In an earlier contribution, Sargent (1980) also analyses an irreversible investment model with numerical methods.

expectation of $\widetilde{\widetilde{\alpha}}_{t+1} = \ln(\widetilde{\alpha}_{t+1})$ and $\widetilde{\widetilde{p}}^I_{t+1} = \ln(\widetilde{p}^I_{t+1})$ by using the predictive density for these variables. Using the notation of Section 3, this can be expressed as

$$\Theta^i(\widehat{\alpha}_{t+1}, \hat{\xi}^P_{t+1} \mid \Psi^{ti}) = \sum_{m=1}^{n} f^i_\alpha(\widehat{\alpha}_{t+1}|\bar{\alpha}_m, \sigma^i_\alpha) f^i_P(\widehat{\xi}^P_{t+1}|\bar{p}^I_m, \sigma^i_P)\psi^{ti}_m,$$

where $f^i_\alpha(\widehat{\alpha}_t|\bar{\alpha}, \sigma^i_\alpha)$ and $f^i_P(\widehat{\xi}^P_t|\bar{p}^I, \sigma^i_P)$ denote the normal density functions, conditional on the regime $s_t = i$. Thus, to simulate the expected future valuation function, we need to use draws from the predictive density for $(\widetilde{\widetilde{\alpha}}_{t+1}, \widetilde{\widetilde{\xi}}^P_{t+1})$. However, the predictive density is a mixture of normals and is not normal.

Our approach to simulating the future expected valuation functions is based on the approach in Altuğ, Demers, and Demers (1999). This procedure extends the analysis in Keane and Wolpin (1994) to allow for learning and regime shifts. Specifically, given an estimate of

$$v^n_{t+1,ji} \equiv E[v^n(K_{t+2}, \Psi^{t+2,j}, \mathbf{x}_{t+2}, \widehat{\mathbf{z}}_{t+2}, s_{t+2} = j)|\Psi^{t+1,i}]$$

for $i, j = 0, 1$ from the previous iteration, we draw D draws on the future random variables $\widetilde{\xi}^\tau_{t+1}$, $\widetilde{\xi}^A_{t+1}$, $\widetilde{\xi}^W_{t+1}$, $\widetilde{\alpha}_{t+1}$, and $\widetilde{\xi}^P_{t+1}$ and calculate the value function associated with each set of the simulated random variables as

$$\max_{K_{t+2}/K_{t+1} \in \bar{K}} \Bigg\{ \nu \xi^\tau_d \alpha^{\mu_1}_d (\xi^A_d)^{\mu_2} (\xi^W_d)^{\mu_3} - \frac{p^I_d K^{1-\mu_2}_{t+1} \left[K_{t+2}/K_{t+1} - (1-\delta)\right]}{\Lambda_0 (1+n_\tau)^{t+1} (1+n_A)(t+1)\mu_3 (1+n_W)^{(t+1)\mu_4}} +$$

$$\bar{\beta} E_{t+1} \left[\chi_{t+1,ii} v^n_{t+1,ii} + (1-\chi_{t+1,ii}) v^n_{t+1,ji}\right] \Bigg\},$$

for $d = 1, \ldots, D$. We simulate the future values of $\widetilde{\tau}_{t+1}$, $\widetilde{A}_{t+1}$ and $\widetilde{w}_{t+1}$ using the objective distributions for these variables. To simulate $\widetilde{\widetilde{\alpha}}_{t+1}$ and $\widetilde{\widetilde{\xi}}^P_{t+1}$, we draw a fraction $\psi^{t+1,i}_m$ of the D draws on $\widetilde{\widetilde{\xi}}^P_{t+1}$ and $\widetilde{\widetilde{\alpha}}_{t+1}$ from the normal distributions with parameters $\bar{p}^I_m$, σ^i_P and $\bar{\alpha}_m$, σ^i_α, respectively, for $m = 1, \ldots, 9$. Averaging the resulting maximum function over the D draws yields the simulated value of $E[v^{n+1}(K_{t+1}, \Psi^{t+1,j}, \mathbf{x}_{t+1}, \widehat{\mathbf{z}}_{t+1}, s_{t+1} = j)|\Psi^{t,i}]$ for $i, j = 0, 1$ that is used at the next iteration.

5 Numerical results

In this section, we present numerical solutions of the model that illustrate the quantitative impact of irreversibility, political risk learning and uncertainty on investment behavior. In Section 5.1, we parameterize the model for the Quebec economy. In Section 5.2, we present the model simulations. The results in Section 2.4 suggest that investment in machinery and equipment for the manufacturing sector has been affected most by the existence of political risk. Thus, we focus on machinery and equipment investment in manufacturing. We choose the sample period 1983-1996 because there exist consistent data for all the variables of interest during this period, and it comprises the second episode of political risk in Quebec which is the focus of our analysis.

5.1 Parameterizing the model

We first consider the issue of parameterizing the stochastic processes for the series that are exogenous or given from the point of view of the firm's decisions. These include real wages, the price of capital, and the various tax variables. The nominal wage rate is constructed from annual data on average weekly earnings, including overtime.[33] The price of capital is constructed as the ratio of nominal investment expenditures and real investment expenditures by sector and by type of capital stock from the sectoral investment and capital stock data.[34] An initial analysis of the data revealed that real wages display a positive trend for the sample period, although there is considerable disparity in the levels and growth rates of real wages across sectors. Likewise, there appears

[33]Specifically, we obtain a measure of hourly earnings, measured as an average per year, by dividing average weekly earnings by 50. The real wage rate w_t is obtained as the ratio of the nominal hourly wage rate and the implicit price deflator for final domestic demand.

[34]The real price of capital p_t^K is obtained from the ratio of the nominal price of capital and the implicit price deflator for final domestic demand.

to be significant negative trends in the price of machinery and equipment and for structures in manufacturing, a broadly defined business sector, and for all industries.

Turning to the behavior of the tax variables, the theoretical framework in Section 3 shows that the corporate tax rate has two effects. First, it affects after-tax profits and second, it affects the fraction of depreciation allowances that firms are able to deduct when determining after-tax cash flows. By contrast, the investment tax credit γ_t and the present value of depreciation allowances z_t only affect the tax-adjusted price of investment. Table B.2 in the appendix shows that there has been a decline in the corporate tax rate over the sample period. To determine whether this decline is significant, we regressed the logarithm of one minus the corporate tax rate, $1-\tau_t$, against a constant and a linear trend. We find that the coefficient on the trend term is 0.0055, with a standard error of 3.714, confirming the existence of a significant negative trend in the corporate tax rate. Thus, in what follows, we allow for a deterministic trend in the corporate tax rate. Table B.2 also shows that the investment tax credit γ_t has shown little variation over the sample period and that the variation in the behavior z_t is largely attributed to variation in τ_t. In what follows, we test for trends in real wages. To allow for the combined effects of the trends in p_t^K and z_t, we also test for trends in the tax-adjusted price of investment defined as $p_t^I = p_t^K(1-\gamma_t - z_t)$.

Parts (a) and (b) of Table 2 show the estimated trends of real wages and the tax-adjusted price of investment for the sample period 1983 to 1996, respectively.[35] Part (a) of this table shows that aside from real wages in the construction industry, all of the real wage series have significant positive trends over the sample period, with the growth in real wages being largest for business

[35]The acronyms HWMN, HWFIN, HWCONS, HWRET, HWW, HWB denote real wages for manufacturing, finance, insurance, and real estate, construction, retail trade, wholesale trade, and business service industries, respectively. Similarly, IPQMM, IPQBM, and IPQTM denote the tax-adjusted price of investment for machinery and equipment in manufacturing industries, the business sector, and total industries while IPQMS, IPQBS, and IPQTS are the corresponding acronyms for the tax-adjusted price of investment for structures in these sectors.

service industries followed by finance, insurance, and real estate, manufacturing, and wholesale trade industries. By contrast, the tax-adjusted price of machinery and equipment and structures all display negative trends over the sample period, with these trends being significant except for the price of structures in manufacturing industries. The results in part (b) show that the largest and most significant price declines have occurred in the price of machinery and equipment for the broadly defined business sector as well as for total industries.

An observable series on technology shocks is obtained as the Solow residual from an empirical production function for Quebec manufacturing. Specifically, $\ln(A_t)$ is measured as

$$\ln(A_t) = \ln(Q_t) - \eta_1 \ln(K_t) - \eta_2 \ln(L_t),$$

where Q_t is Quebec gross domestic product, K_t is the end-of-year capital stock computed with geometric depreciation, and L_t is the number of employees in firms of all sizes in Quebec manufacturing times the total annual hours worked per worker.[36]

Finally, data on changes in manufacturing shipments is used to measure $\widetilde{\alpha}_t$, which is the noisy indicator about the underlying true state of demand $\bar{\alpha}$. Unlike the remaining exogenous variables affecting firms' optimization problem, it is not possible to reject the null hypothesis that $\{\widetilde{\alpha}_t\}$ is a stationary stochastic process. We assume that $\widetilde{\alpha}_t$ is lognormally distributed, or equivalently, that $\ln(\widetilde{\alpha}_t) \sim N(\bar{\alpha}, \sigma_\alpha^2)$.

The remaining parameters ε, η_1, η_2, and β must be chosen to guarantee the concavity of the short-run profit function in K_t and to satisfy conditions (3.29) through (3.32) restricting the maximum growth rate of the capital stock κ. The average growth rate of the capital stock for the sample period 1983-1996 is 0.0376. However, this growth rate varies from a maximum of 0.1575

[36] Since we do not have a consistent series on total annual hours worked per worker, we assume this to equal the constant value of 1700. Another potential problem with this measure of technology shocks is that it is based on aggregate Quebec GDP as opposed to sectoral output for the manufacturing sector in Quebec.

in 1989 to a minimum of -0.0889 in 1993. To capture such variation, we consider the version of our model with increasing returns to scale and set $\eta_1 = 0.37$ and $\eta_2 = 0.83$. These values imply that the degree of the returns to scale is 1.2, which is consistent with the estimates that Morrison (1992, 1994) and Robidoux and Lester (1992) report.[37] Next, we let $\varepsilon = -5$, which implies a markup of 1.25. Morrison (1992, 1994) reports estimates of the markup of price over marginal cost for Canadian manufacturing firms between 1960 and 1982. The average markup reported in the first study is equal to 1.1358, with a standard deviation of 0.0435 while the average of the markup for 1962, 1967, 1972, 1977. and 1982 reported in the second study is 1.1942, with a standard deviation of 0.07766.[38] While the implied demand elasticity is somewhat smaller and the markup larger than those reported by Morrison (1992,1994) for the Canadian manufacturing sector, these parameter values are consistent with the conditions that guarantee the concavity of the short-run

[37] Morrison (1992, 1994) and Robidoux and Lester (1992) have estimated the degree of returns to scale in Canadian manufacturing. Morrison (1992, 1994) estimates the degree of returns to scale, the magnitude of markups, and various capacity utilization measures based on the appropriate elasticities from an indirect generalized Leontief cost function. She reports evidence for increasing returns to scale at the industry level for Canada between 1960 and 1982, with the economies of scale tending to increase over the sample period. The mean and standard deviation of the returns to scale estimates are 1.0959 and 0.0359, respectively.

Robidoux and Lester (1992) use establishment data on 7610 plants in Canadian manufacturing industries for the year 1979 to provide estimates of the returns to scale. They report that there are increasing, constant, and decreasing returns to scale in 68%, 25%, and 7% of the industries, respectively. They interpret this finding as suggesting that "there are increasing returns in a majority of industries, but also that a significant fraction of industries are characterized by constant returns."

[38] To find the implied elasticity of demand, we note that in a static version of the firm's problem, the markup of price over marginal cost can be expressed as $\mathrm{MRKP} = \varepsilon/(1+\varepsilon)$, where the markup is defined as $\mathrm{MRKP} = p_t/\mathrm{MC}_t$ and MC_t is the marginal cost of producing an additional unit of output. Thus, the implied elasticities of demand corresponding to the average markups of 1.1358 and 1.1942 are -8.3624 and -6.1493, respectively, which suggest that demand is fairly elastic for the manufacturing industries as a whole.

profit function in K_t and α_t, namely, $(\eta_1 + \eta_2) < \varepsilon/(1+\varepsilon)$ and $\eta_2 < 1$. Finally, we set $\beta = 0.85$ to yield a maximum growth rate of the capital stock equal to 0.179 and to satisfy condition (3.32) restricting the exogenous trends in τ_t, A_t, p_t^I, and w_t jointly with the endogenous trend in K_t. Table 3 summarizes the parameter values used in the simulations.

5.2 Simulations

According to our informational assumptions, the firm is assumed to know the current realizations of the corporate tax rate τ_t, the demand shock α_t, the technology shock A_t, the real wage w_t, and the tax-adjusted price of investment p_t^I as well as the form of the distributions generating all the random variables. However, it does not know the permanent component of the demand shock and the tax-adjusted price of investment. It uses noisy indicators of these variables to learn about the true underlying values. We assume that the detrended tax-adjusted price of investment $\widetilde{p}_t^I/(1+n_P)^t$ provides information about the true state of costs while changes in manufacturing shipments $\widetilde{\alpha}_t$ provide information about the true state of demand. To simplify our calculations, we also assume that the shocks $\widetilde{\alpha}_t$, $\widetilde{\xi}_t^A$, $\widetilde{\xi}_t^P$, $\widetilde{\xi}_t^W$, and $\widetilde{\xi}_t^\tau$ are independent over time and mutually independent.

The logarithm of the detrended tax-adjusted price of investment $\ln(\widetilde{p}_t^I) - \ln(p_0^I) - t\ln(1+n_P)$ and the logarithm of the change in shipments $\ln(\widetilde{\alpha}_t)$ are assumed to follow normal distributions with means $\bar{p}^I$, $\bar{\alpha}$ and variances σ_P^2 and σ_α^2. While the firm knows the form of this distribution, it does not know the true value of $\bar{p}^I$ or $\bar{\alpha}$. It has a prior probability distribution about the true state denoted by Ψ^0, where $\Psi^0 = [\psi_1^0, \ldots, \psi_n^0]$, where ψ_m^0 is the prior probability that $\bar{p}^I = \bar{p}_m^I$ and $\bar{\alpha} = \alpha^m$, $m = 1, \ldots, n$. In the simulations, we assume that $\bar{p}^I$ and $\bar{\alpha}$ each take on three values, defined as the sample means of $\ln(\widetilde{p}_t^I) - \ln(p_0^I) - t\ln(1+n_P)$ and $\ln(\widetilde{\alpha}_t)$ and values that are ± 0.06 different, respectively. Using the results from Table 1, the set of possible values for $\bar{p}^I$ and $\bar{\alpha}$ denoted $\bar{P}$ and $\bar{A}$ are defined as $\bar{P} \equiv \{\bar{p}_1^I, \bar{p}_2^I, \bar{p}_3^I\} = \{0.0479, 0.1079, 0.1679\}$ and $\bar{A} = \{-0.016, 0.044, 0.104\}$.

Similarly, σ_P^2 and σ_α^2 are set equal to the sample variance of $\ln(\tilde{p}_t^I) - \ln(p_0^I) - t\ln(1+n_P)$ and $\ln(\tilde{\alpha}_t)$, respectively.

There are different approaches to matching the implications of the model that we presented in Section 3.4 with actual data. One method that has been used in real business-cycle literature is to match the various moments of the sample series with those implied by the data. (See Cooley and Prescott (1995) for an articulation of this approach.) However, given the fact that we are interested in explaining a specific incident of political risk and its impact on investment behavior, this approach does not seem useful for our purposes. Another alternative is to generate the entire time series for the optimal investment-capital stock ratios, taking as given the observed sequences of the exogenous variables. Since our interest lies in determining whether a model of irreversible investment and regime shifts can capture the response of firms to the large apparent increase in political risk in recent Quebec history, we generate investment-capital stock ratios for each point in time as the optimal solution from the model, taking as given the actual capital stocks and the historical values of the exogenous variables at that point in time. This approach allows us to examine the factors determining firms' investment responses at each date without necessarily imposing what may be counterfactual restrictions for the evolution of their responses across the different dates.[39]

Our approach to simulating the model is as follows. We solve for the desired capital stock ratio K_{t+1}/K_t (or equivalently, I_t/K_t) as a function of the observed values of $K_t, \tau_t, \alpha_t, A_t, w_t$, and p_t^I. We use the detrended value function defined in equation (3.33) and allow for 100 possible values for next period's capital stock ratio defined on the interval $[(1-\delta),(1+\kappa)]$, where δ is measured as the

[39] Another alternative is to use nonparametric or semiparametric estimation methods to provide a characterization of a subset of restrictions implied by the model. For an application of this approach, see the analysis in Altuğ and Miller (1998). To implement this approach, a panel data set on firms is required.

actual rate of depreciation of the observed capital stock at each date. During the value function iteration, we draw $D = 150$ draws on each of the exogenous random variables $\widetilde{\xi}^{\tau}_{t+1}, \widetilde{\alpha}_{t+1}, \widetilde{\xi}^{A}_{t+1}, \widetilde{\xi}^{P}_{t+1}$, and $\widetilde{\xi}^{W}_{t+1}$ to simulate the future expected value function at each iteration of the value function iteration.

In Table 4, we present the actual and simulated values for investment and the investment-capital stock ratios based on simulations of the one-step ahead capital stock ratios over the 1992-96 period. We choose this period because we want to focus on the second episode of political risk in Quebec.[40] During these years the investment/capital stock ratios attain their lowest values in the entire sample period. The simulations reported in Table 4 allow for irreversibility and learning but no regime shifts.[41] The results in Table 4 are generated by assuming uniform prior distributions over values of $(\bar{p}^I, \bar{\alpha})$, and one-step Bayesian updating of these priors based on sample information available at date t. Since we use actual data on the capital stock in our simulations, the value of the demand shock must be scaled by a parameter that we denote by $\bar{a}$ to yield a plausible value for the level of profits. In the simulations reported in Table 4, we assume that $\bar{a} = 175$.[42]

The simulation results show that the model without regime shifts overpredicts the investment-capital stock ratios for 1992-1996. Since we have allowed time-variation in all the other exogenous variables affecting the firm's problem using actual data from the Quebec economy for the period in question, these results suggest that the model without regime shifts and political risk does not adequately predict the investment response in Quebec across different years.[43]

[40] We leave out 1990 and 1991 because of the deep recession that affected the entire Canadian economy and which could bias the results.

[41] We do not present results for the standard reversible investment model because the goal of this paper is not to demonstrate the impact of irreversibility *per se*.

[42] Alternatively, this value could be calibrated from the data given actual series on the output price for firms.

[43] An alternative explanation is that there is misspecification in some of the exogenous series. For example, it may be that changes in shipments do not fully capture the variations in the level of demand across the different years in

Next, we consider the predictions of the model with uncertainty and learning and with regime shifts for 1992-96. We estimate the time-varying transition probabilities, $\chi_{t,01}$, of switching to the "bad" regime using the smoothed poll results given in Table 1 and discussed in Section 2.3. In terms of the time-varying transition probabilities modelled in Section 3.2, this implies that we assume that the poll-referendum-election results constitute in fact a mapping from an underlying vector of political and economic indicators into the unit interval. As we have already noted in Section 2.3, Table 1 and the graphs in Figures 2 and 3 show that the average support for separation increased significantly in the second episode of political risk experienced in the 1990's relative to the period of political stability in the 1980's and relative to the first episode of political risk experienced in the 1970's. This suggests that the smoothed poll data and hence the process we use to characterize the transition probabilities may not be stationary. In terms of our model, this implies that either distribution of the vector of underlying economic and political variables $f(\mathbf{x_t})$ has not been invariant over time or that the mapping of $\mathbf{x_t}$ into the zero-one interval that determines time-varying transition probabilities, $\chi_{t,01}$ itself is nonstationary. Thus, strictly speaking, our model should account for this nonstationarity if it is to apply to the Quebec economy from 1976-96, thus covering two episodes of political risk and one of political stability in between. While it is possible to account for such a phenomenon in our theoretical framework[44], we focus specifically on only one episode of political risk (the post-1990 period) for which the transition probabilities do not appear to exhibit nonstationary behavior. Thus, the model as given in Section 3 remains valid for this

our sample and that a better measure of demand could improve the simulated investment levels in the absence of regime shifts. A similar problem may exist in relation to the measurement of technology shocks.

[44]In this regard, two avenues are possible. One could model an additional regime shift to account for shifts in $f(\mathbf{x}_t)$. Alternatively, one could model $\chi_{t,01}$ as being a nonstationary process directly, and modify our convergence theorems to take this nonstationarity into account. (See Hinderer (1970) for proofs of convergence in dynamic programming problems involving non-stationary distributions.)

period.

To distinguish between the "good" and the "bad" regimes, we assume that the"good" regime is characterized by more favorable distributions for the demand shock or the tax-adjusted price of investment compared to the "bad" regime. One way of modeling differences in the distributions across the "good" and "bad" regimes is to assume that the variances of the demand shock and the innovation to the tax-adjusted price of investment are larger in the "bad" regime than in the "good" regime. Initial simulations showed that differences in variances of the distributions across the "good" and "bad" regimes were insufficient to account for the large differences in the investment response across the different years. For this reason, we assume that the unknown state of demand in the "good" regime is "uniformly better" than the unknown state of demand in the "bad" regime. Specifically, we assume that the value of the parameter measuring the level of demand in the "good" regime is higher than the value in the "bad" regime. Denote the former by $\bar{a}_0$ and the latter by $\bar{a}_1$. Thus, the simulated values of the demand variable in each regime are defined as $\bar{a}_i \exp(\bar{\alpha}_j + 0.5\sigma_{\alpha}^2)$ for $i = 0, 1$ and $j = 1, 2, 3$. In these expressions, i denotes the regime and j denotes the possible values that $\bar{\alpha}$ can take on in each regime. Thus, the distribution of the demand variable in the "good" regime dominates the one in the "bad" regime by first order stochastic dominance.

We further assume that the value of $\bar{a}_1$ is time-varying, and that it decreases whenever the estimated probability of transiting to the "bad" regime increases, and vice versa. This parameterization captures the notion that firms may alter their beliefs about the mean of the distribution that is expected to prevail in the "bad" regime if they feel that current events tend to make separation more likely.[45] Using the process for the smoothed transition probabilities, we see that the transition probabilities increased by 7% between 1991 and 1992, by 5% between 1992 and 1993, and by 1% between 1993 and 1994. After 1994, the probability attached to separation (or the unfavorable

[45] Alternatively, the parameterization for $\bar{a}_1$ could be linked to increases or decreases in bond spreads.

business climate ensuing from it) declined by 3% between 1994 and 1995, and by 4% between 1995 and 1996. Thus, we choose $\bar{a}_1$ to equal 165 in 1992, 150 in 1993, 145 in 1994, 150 in 1995, and 165 in 1996. (We recall that a value of 175 had been assumed for $\overline{a}$ in the case without regime shifts.)

Table 5 reports the simulation results with regime shifts. Taking as given the observed values of the other exogenous variables, the results in Table 5 show that regime shifts provide one way of rationalizing the large observed decline in investment and the investment-capital stock ratios that occurred in Quebec in the1990s. The model with regime shifts is, on the whole, quite successful in explaining the investment responses for firms in the 1992-1996 period. The model underpredicts the investment response of firms in 1992 and to a lesser extent, in 1994. One reason for the former result may be that the estimate of the transition probability for 1992 based on the smoothed poll-referendum-election results tends to overestimate the perceived probability of a transition to the "bad" regime. For example, in 1992 investors may have perceived the actual probability of a switch to the "bad" regime to be less than 52%, as is implied by the moving average estimates. That is, investors may have discounted part of the large support for separation on the basis that it would not transform into actual support in an election or a referendum. Furthermore, this perception may have been strengthened by the fact that a federalist party was going to be in power for another two more years in Quebec. The bond spread data given in Table 1 also confirm that this may indeed have been the case: the Quebec-Ontario bond spreads started to decline from mid-1991 to 1993 following their initial increase in 1990-91.

In Table 6, we focus on a specific year and ask what would have happened to firms' investment if there were further changes in investors' prior beliefs, the variances of the distributions for the alternative regimes, and the magnitude of the probability of a transition to the "bad" regime. We generate these results for 1993 because investment fell significantly in that year.[46] Table 6 reports

[46]It is worth noting that the magnitude of the responses for a given year cannot be attributed as the expected

the results of these simulations, assuming the same paramerization for $\bar{a}_1$ as in Table 5. Case I in Table 6 assumes uniform prior distributions over values of $(\bar{p}^I, \bar{\alpha})$. The other cases allow for changes in priors. Unless otherwise noted, all simulations assume that the variances of the noisy observations on the unknown level of demand and the unknown price of investment are equal across the "good" regime and the "bad" regime. For use of exposition, part (I-a) in Table 6 includes the result for 1993 from Table 6 as the baseline case.

First, we consider a mean-preserving increase for the distributions of the noisy signals in the "bad" regime compared to the "good" regime. Specifically, we assume that the variances of the shocks to demand and the price of investment in the "bad" regime are three times as large as their corresponding values in the "good" regime. Given a non-zero probability of a shift to the "bad" regime next period, our theoretical framework predicts that the level of current investment should fall. Part I-(b) shows that this is indeed the case: the level of investment falls to 501 units, and the simulated investment-capital stock ratio to 0.0442. This result shows that increases in the variability of payoffs associated with the "bad" regime can be a potentially important reason for the decline in investment during the second episode of political risk in Quebec.

Next, we consider how changes in the magnitude of the transition probabilities affect invesment behavior. Based on the results of the smoothed survey opinion polls, the probability of transiting to the "bad" regime, conditional on being in the "good" regime, for 1993 is 0.56718. (We use this for the baseline case reported in part I-(a) as well as all the other cases that do not involve changes in the transition probability.) Parts I-(c) and I-(d) of Table 6 show the effects of changing this probability by 25% in either direction. Specifically, if investors believe that the probability of transiting to the "bad" regime, conditional on being in the "good" regime, declines to 0.4254, then

magniture of the responses for all years. The reason is that the predicted responses in a given year are conditional on the values of the state variables and the realizations of the exogenous variables for that year.

they choose to invest 2240 units. Put differently, a 25% decrease in the probability of transiting to the "bad" regime, conditional on being in the "good" regime, implies about a 24% increase in investment compared to the baseline case reported in part I-(a). By contrast, if investors believe that the probability of transiting to the "bad" regime increases to 0.7090, then they choose to cut their investment expenditures to 994 units in the current peiod, implying that a 25% increase in the probability of transiting to the "bad" regime persisting leads to about a 45% decline in investment. In either case, we see that a given percentage change in the probability of transiting to the "bad" regime elicits a significant change in investment expenditures in the opposite direction. These findings provide another indication of the importance of regime shifts that is consistent with our theoretical framework. Since increases in the probability of transiting to the "bad" regime is a simple way of modelling increases in political risk, these findings also support the popularly held view that increases in political risk have been an important determinant of the large decline in investment in Quebec.

Another exercise that we can perform is to allow for different prior beliefs by investors. In part II, we consider the case of the optimistic firm, which systematically overestimates the level of demand in both the "good" and "bad" regimes while in part III, we consider the case of the pessimistic firm that does the opposite. Part II of Table 6 shows that irrespective of the probability of regime shifts, if investors are optimistic about the level of demand that will prevail in both the "good" and the "bad" regime, then they will prefer to invest as much as they would in a world without regime shifts. Conversely, if investors are pessimistic about the level of demand in both the "good" and the "bad" regimes, then investment falls to zero units. These results are in line with our earlier results reported in Altug, Demers, and Demers (1999), and show that investors' prior beliefs can also be an important determinant of investment expenditures in a model with irreversible investment and learning. In part IV of Table 6, we consider the case of an informative prior for the unknown state of demand, which puts the largest prior probability on the average value of the unknown state of demand for both the "good" and "bad" regimes. According to part IV-(a), if investors believe that the unknown state of demand will equal its (regime-specific) average value with a probability of 0.8 as opposed to being abnormally "low" or abnormally "high" with probabilities of 0.1 each for both the "good' and the "bad" regimes, then they prefer to invest nothing.

To understand the results in part IV of Table 6, we need to examine the joint posterior distributions for $(\bar{\alpha}, \bar{p}^I)$ implied by the different priors for $\bar{\alpha}$. Since we consider three values each for $\bar{\alpha}$ and $\bar{p}^I$, the relevant posterior distributions have the from:

$$\begin{bmatrix} \Pr(\text{low } \bar{\alpha}, \text{ high } \bar{p}^I) & \Pr(\text{low } \bar{\alpha}, \text{ mid } \bar{p}^I) & \Pr(\text{low } \bar{\alpha}, \text{ low } \bar{p}^I) \\ \Pr(\text{mid } \bar{\alpha}, \text{ high } \bar{p}^I) & \Pr(\text{mid } \bar{\alpha}, \text{ mid } \bar{p}^I) & \Pr(\text{mid } \bar{\alpha}, \text{ low } \bar{p}^I) \\ \Pr(\text{high } \bar{\alpha}, \text{ high } \bar{p}^I) & \Pr(\text{high } \bar{\alpha}, \text{ mid } \bar{p}^I) & \Pr(\text{high } \bar{\alpha}, \text{ low } \bar{p}^I) \end{bmatrix}.$$

While it is difficult to rank multivariate distribitions in terms of first-order stochastic dominance (FSD) or second-order stochastic dominance (SSD), it is nevertheless possible to make some observations by focusing on the states that can be unambigously ranked, namely, "good state" (high $\bar{\alpha}$, low $\bar{p}^I$), the "medium state" (mid $\bar{\alpha}$, mid $\bar{p}^I$), and the "bad state" (low $\bar{\alpha}$, high $\bar{p}^I$). The joint posterior distributions are presented in Table 7.

Returning to the results in Table 6, we can see that the firm invests with uniform priors; in this case, the "good state" receives a substantial posterior weight (0.2492, which is the highest probability for all the states in the uniform case) and the firm invests 1681 units, which is less than in the optimistic case but more than the cases with pessimistic and informative priors. By contrast, the firm invests the same amount with both informative and pessimistic priors. To explain this result, we need to consider the probability placed on each of the possible states for the case with an informative prior. The posterior distributions in Table 7 show that even though the probability of the worst state in each regime has dropped to 0 (so that essentially downside risk is very low) and the probabilitiy of the medium state has risen compared to the uniform, optimistic, and pessimistic priors, these are not enough to have the firm invest. The firm does not invest, because with a prior that is concentrated around the medium state, it knows that the probability of obtaining the high value of demand (jointly with the low value of the price of investment) is very small in either regime. Furthermore, the medium state of demand in the "bad" regime is rather low (in fact it is lower than the low state in the "good" regime). Thus, the fact that the posterior distributions are more concentrated around the medium state is not viewed by the firm as being such a favourable event especially when there is a positive probability of shifting to a bad regime.

In summary, the numerical results in this paper show that such features as the increases in risk, which may arise from increases in the probability of transiting to a more unfavorable regime or from increases in the variability of profits in the "bad" regime, can have quantitatively important effects on investment behavior. The numerical results also show that the irreversible investment model under uncertainty and learning and regimes shifts is capable of reconciling the large drop in investment that occurred in Quebec during the episode of political risk of the 1990's.

6 Conclusion

In this paper, we have presented a consistent theoretical framework for examining the effects of political risk on investment behavior. Our framework incorporates irreversibility in investment, learning, and regime shifts. It also allows for trends and random variation in the exogenous variables that affect firms' investment decisions, including wages, tax rates, the price of capital, demand shocks, and technology shocks. The model is used to examine the quantitative impact of political risk on investment using sectoral data on the Quebec economy for the period 1983 to 1996. The results of survey opinion polls are used to estimate the probability of a shift to a "bad" regime, defined here as the separation of the province of Quebec from the Canadian federation. Taking as given the observed values of the exogenous variables affecting firms' problems, we find that the irreversible investment model with regime shifts provides a useful framework for analysing the reasons behind the large drop in investment that occurred in the 1990's in Quebec.

Unlike the standard neoclassical investment model, the irreversible investment model under uncertainty with learning predicts the smooth response of investment to changes in the exogenous variables. Furthermore, it predicts a negative relationship between uncertainty and investment, and risk and investment, and introduces an important role for investors' beliefs about such variables as the unknown state of demand and the unknown costs of investing. While the lack of data on investors' beliefs makes it difficult to disentangle the separate effects of investors' prior beliefs versus their beliefs about the nature of objective distributions across the different regimes, our framework nevertheless allows for a quantitative analysis of the effects of changes in subjective beliefs on investment. The model that is augmented with regime shifts shows that even a small probability of transiting to an unfavorable regime together with investors' beliefs about the unknown demand and cost-of-investment conditions in future regimes can be an important determinant of current investment behavior.

There is a large literature that studies the negative effect of political risk on investment behavior. In contrast to most of this literature, however, our analysis is based on a fully specified structural model of investment that incorporates many features useful for explaining investment behavior, and that is amenable to policy analysis. In the current paper, we have used a simulation approach combined with actual data from the Quebec economy for a specific historical period. We believe that our approach has wide applicability, and can be used in other contexts where political risk is present, and where there exists a subjective perception of unsustainability of the current policy regime. One example of a possible application among many is related to a further analysis of the Quebec economy.[47] Another application relates to the success of a structural adjustment and stabilization package. Since the nature of political risk can be quite diverse, and may be stemming from expropriation, disruptions in market access, unfavourable government regulations, unsustainable exchange rates, debt crises, fiscal crises, policy reversals, risk of political disintegration, etc., we believe that there is a rich choice of applications for our approach.[48]

[47] Recently, Matthews (1998) and Migué (1998) have questioned Quebec's economic performance in many areas, including private investment. In particular, Migué has criticized Quebec's reliance on state intervention and corporatism which has resulted from the Quiet Revolution in the 1960s. Furthermore, there has been a debate on the Quebec's capital income tax which was raised in the early 80s (coincidentally with a deepening of Quebec's recession.) Our model could be extended to address these instances of regime shifts and their impact on investment.

[48] See, for example, Altug, Demers and Demers (2000) for an analysis of tax policy changes on investment.

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Appendix A

In this appendix, we discuss the raw poll data given in Table 1. For years during which there was an election or a referendum, we give preference to these results over poll results. For years during which no survey of opinion poll was conducted, we use an approximate figure in view of the political events of the time. We isolated poll results pertaining to the question: "If there were a referendum on sovereignty today would you vote Yes (OUI), No (NON) or are you Undecided (NSP or IND)?"

All polls are scientific polls based on sound statistical principles with a standard margin of error (ranging between 3% and 5%). A study of Quebec voting behaviour (Drouilly, 1996) reveals an empirical regularity, namely, that a sizeable portion of those declaring themselves as undecided with respect to their voting intentions are in reality reluctant to reveal their true colors due to social pressures in their professional and/or private environment. Furthermore, a larger proportion of the undecided end up voting no. In accordance with Pierre Drouilly's study, we apportioned the undecided between the no and the yes votes, with three quarters being added to the no and one quarter to the yes.

According to the poll data, support for political parties favouring independence was about 10% in the 1960s (not shown in Table 1). In 1970, it climbed to 23% as evidenced by the PQ's popular vote in the provincial election, and we assume that it remained steady in 1971 and 1972 (no poll data being available for these years). In 1973, support for sovereignty climbed to 30% (the PQ's vote in the provincial election). For 1974, 1975 and 1976 we assume that support for sovereignty was steady at 32% (the result of a 1974 opinion poll). For 1977, 1978 and 1979 we use an average of several opinion poll results taken during each of these years. For 1980, we use the outcome of the referendum: 40% yes and 60% no. For 1981 and 1982 we again use poll data. For 1983 and 1984 and from 1986 to 1988, no polls were taken but noticeably support for sovereignty had fallen, so we assign 30% of yes and 70% of no on the basis of several studies which have concluded that the hard core for sovereignty is approximately 30%. For 1985 we use the support for the PQ in provincial elections: 37.75% of yes and 62.25% no. From 1989 to 1994 we use poll data. In 1995 we use the outcome of the referendum: 49.6% of yes and 50.4% of no. Finally, from 1996 to 1998, we also use poll data which indicate that support for sovereignty fell somewhat to 45%.

Appendix B

This appendix describes the data that are used to construct measures of real investment expenditures, real capital stocks, real wage rates, the price of capital goods and other determinants of the user cost of capital.

There exist alternative measures of investment at the provincial level. One measure can be obtained from sectoral data on gross investment expenditures, discards, capital consumption allowances, and the corresponding end-of-period capital stocks computed under alternative assumptions about depreciation for the 2-digit SIC code industries. (Source: Statistics Canada, Investment and Capital Stock Division, National Wealth and Capital Stock Section.) The sectoral data are annual for the period 1963 to 1998 and they contain information on both private and public investment expenditures (and the corresponding capital stocks) for machinery and equipment, road repairs, non-residential structures, and the total capital stock for these industries. The Statistics Canada publication *Fixed Capital Flows and Stocks: Methodology* describes how these data were constructed.

Uisng these data, investment expenditures for machinery and equipment and nonresidential structures are measured as gross capital stock formation at annual rates for various sectors, including the manufacturing sector and a broadly defined business sector. The capital stock in period t is measured as the end-of-period net capital stock, which is equal to the cumulated value of gross capital formation minus capital consumption allowances from some initial data up to period t. The (time-varying) depreciation rates for machinery and equipment and nonresidential structures are obtained as the ratio of capital consumption allowances to the end-of-period net capital stock.

The nominal price of output is measured using the implicit price deflator for final domestic demand. The sectoral nominal hourly wage rate, measured as an average over the year, is constructed by dividing annual data on average weekly earnings, including overtime, by 50. Average weekly earnings are defined as wages and salaries for all employees in firms of all sizes. (Source: Statistics Canada, Employment Section, Labour Division.) The hourly real wage rate is obtained by taking the ratio of the hourly nominal wage rate and the implicit price deflator for final domestic demand.

The price of capital is constructed as the ratio of nominal investment expenditures and real investment expenditures by sector and by type of capital stock from the sectoral investment and capital stock data. The real price of capital p_{Kt} is obtained from the ratio of the nominal price of capital and the implicit price deflator for final domestic demand.

Since a typical firm in the province of Quebec is taxed at both the federal and provincial rates, the corporate tax rate is defined to be the weighted average of the federal corporate income tax rate τ^F, where the weighting is done to reflect the different tax rates applied to general, manufacturing and processing, and small business income, respectively, plus the Quebec corporate income tax rate applied to large corporations.[49] To account for the surtax that was used to raise additional tax revenue at the federal level without raising federal tax rates, the federal corporate tax rate is multiplied by the surtax rate τ^S to yield the combined corporate tax rate $\tau^F \tau^S + \tau^P$. The time series of the weighted average federal tax rate and the Quebec corporation tax rate are reported in Table B.1.

Table B.1 also shows the time series of the federal investment tax credit applied to machinery and equipment and nonresidential structures as well as the present value of $1 of future capital cost allowances for investment in machinery and equipment and for non-residential structures. The latter series satisfy the formula for z_t given in Section 3.1 by assuming that the future corporate tax rates are constant and using the nominal interest rate to discount the future nominal cash flows.[50]

[49] See Table 11.1 in Iqbal (1997).

[50] These data were kindly provided to us by John Sargent from the Department of Finance.

Appendix C

Proposition 1: The proof is identical to the proof of Proposition 1 in Demers (1991), p.338.

Lemma 1: The proof of the concavity of V_K in Ψ^t follows along the lines of Lemma 2 in Demers (1991). The proof of V_K being increasing and concave in p_t^I follows from Lemmas 4, 5 and 6 in Altuğ, Demers, and Demers (1999a).The proof of V_K being increasing in α_t follows since Π_K is increasing in α_t provided $\eta_2 < \varepsilon/(1+\varepsilon)$. Finally, the proof of V_K being concave in α_t follows since Π_K is concave in α_t if $\eta_2 < 1$.

Proposition 2: The proof follows by Lemma 1.

Proposition 3: The proof follows by applying the Weighted Contraction Mapping Theorem.See Boyd (1988) or Altug and Labadie (1994).)

Convergence to the desired stock of capital

The firm's desired stock of capital is the capital stock achieved by the firm when learning is complete and the firm's state of information has converged to a vector Ψ^* which assigns probability one to the true state $\bar{p}^I$ and $\overline{\alpha}$. In view of the irreducible purchase price risk $\widetilde{u}_{It}$, the desired capital stock is stochastic. Thus, the ergodic set is $\Gamma \equiv \left\{(K, \Psi, \mathbf{x}, s) \mid K \in \left[K^L, K^H\right], \Psi = \Psi^*, \mathbf{x} \in X, s \in \{0,1\}\right\}$ where K^L and K^H solve

$$\bar{p}^I + \bar{u}_I = \beta E[V_k\left((1-\delta)\,K + I(K, \Psi^*, \, \bar{p}^I + \bar{u}_I), \Psi^*, \mathbf{x}', s'\right)|\mathbf{x}, s]$$

$$\bar{p}^I + \underline{u}_I = \beta E[V_k\left((1-\delta)\,K + I(K, \Psi^*, \, \bar{p}^I + \underline{u}_I), \Psi^*, \mathbf{x}', s'\right)|\mathbf{x}, s]$$

where E denotes that expectation is taken with respect to knowledge of the true distribution of $\bar{p}^I$ and $\overline{\alpha}$ and where $\bar{u}_I$ and $\underline{u}_I$ denote the highest and lowest values of the random component of the investment price.[51]

Let $y = [K, \Psi, \mathbf{x}, s]$ denote the current state of the firm, with $y \in Y$, where $Y \equiv \left[0, \overline{K}\right] \times D[\Omega] \times X \times \{0,1\}$, and where $\mathcal{Y} = B(Y)$ is its σ-algebra and $\overline{K}$ denotes an upperbound for K. Hence, $(Y, \mathcal{Y})$ is the state space. The measurable space of events is $(H \times X \times \{0,1\}, B(H \times X \times \{0,1\}))$, where $B(H \times X \times \{0,1\})$ is the σ-algebra of $H \times X \times \{0,1\}$. Define a stochastic kernel

$Q : Y \times B(H \times X \times \{0,1\}) \to [0,1]$ as $Q(y, B) = \int_B \zeta\left(h', \mathbf{x}', s' | y\right) dh' d\mathbf{x}'$ where $\zeta(h', \mathbf{x}', s' | y) \equiv \chi_{ss'} \theta^i(h' | \psi^i) f_x(\mathbf{x}' | \mathbf{x})$ is the predictive density of $[h', \mathbf{x}', s']$ conditional on the state y and where $\chi_{ss'}$ is the probability of shifting to regime s',given regime s today.

The evolution of the firm can be described as follows. If the current state of the firm is $y \in Y$ an event $[h', \mathbf{x}', s'] \in H \times X \times \{0,1\}$ is realized according to the stochastic kernel $Q(y, \cdot)$. Next period's state is then determined by the laws of motion for $y' \equiv [K', \Psi', \mathbf{x}', s']$. Hence, we can express the dynamic evolution of the firm's state as: $y' = \Phi\left(y, [h', \mathbf{x}', s']\right)$ where $\Phi : Y \times H \times X \times \{0,1\} \to Y$. It can be shown that $\Phi\left(\cdot, [h', \mathbf{x}', s']\right)$ is continuous in y for all $[h', \mathbf{x}', s'] \in (H \times \overline{X} \times \{0,1\})$. and that $\Phi(y, \cdot)$ is measurable with respect to $B(H \times X \times \{0,1\})$ for all $y \in Y$.

As a result, we have

Lemma 1 *Define* $\Phi^{-1}(M)_y \equiv \left\{[h', \mathbf{x}', s'] \in (H \times X \times \{0,1\}) \mid y' \in M\right\}, M \in S$, *and* $P : Y \times \mathcal{Y} \to (0,1)$ *by*

$$P(y, M) = \int_M \zeta\left(\Phi^{-1}(M)_y \mid y\right) dy' \tag{0.1}$$

Then $P(y, M)$ *is a transition probability on the state space* $(Y, \mathcal{Y})$.

Define $C(Y)$ as the set of all continuous and bounded $\mathcal{Y}$ measurable real valued functions on Y. $C(Y)$ is a Banach space with the sup norm $|m| \equiv \sup_{y \in Y} |m(y)|$. As a result of lemma 1, a

[51] We simplify the notation and let the variables without time subscript (or time superscript) denote the current period's values and primes denote next period's values.

continuous linear transformation $T: C(Y) \to C(Y)$ can be defined by $(Tm)(y) = \int m(z)P(y,dz)$. T is a Markov operator associated with the transition probability P. The adjoint T^* of the Markov operator T is defined by $(T^*\mu)(L) = \int P(z,L)(dz)$ where μ is a bounded countably additive set function. $T^* : ca(y) \to ca(y)$ where $ca(y)$ is the Banach space of bounded countably additive set functions on $(Y, \mathcal{Y})$ and is a dual space to $C(Y)$. The space $ca(y)$ has as its norm the total variation norm, defined as $|\mu| = \sup \sum_{i=1}^{n} |\mu(N_i)|$ where the supremum is taken over all finite partitions of Y into disjoint subsets $\{N_i\}$. If $\mu(N)$ is the probability that the firm's state is in the set N at time t then $(T^*)(N)$ is the probability that the state is in N at time $t+1$.

The steady state of the stochastic dynamic process defined by T is an invariant probability measure μ^* such that $T^*\mu^* = \mu^*$.

Theorem 1 *There exists a unique invariant probability measure μ^* satisfying $T^*\mu^* = \mu^*$.*

Theorem 2 *The sequence $\{\sum_{i=0}^{n-1} T^{*i}\mu^*/n\}$ converges weakly to the invariant probability measure μ^*. The convergence is uniform.*

The proofs of these theorems are similar to the proofs of Theorems 6 and 7 in Altug, Demers and Demers (1999). These last two theorems guarantee the existence of a unique steady state and the convergence of the firm's initial information state and capital stock. Thus, in the long run once learning is complete, and the true state $\bar{p}^I$ and $\overline{\alpha}$. is known with certainty, the firm's capital stock reaches the desired level.

Table 1

Year	YES† (%)	NO† (%)	Comments
1970	23	77	Provincial election results (1970)
1971	23	77	”
1972	23	77	”
1973	30	70	Provincial election results (1973)
1974	31.7	68.3	Opinion poll results (1974)
1975	31.7	68.3	”
1976	31.7	68.3	”
1977	38.17	61.37	Opinion poll results (1977)
1978	51.19	48.81	Opinion poll results (1978)
1979	46.46	53.54	Opinion poll results (1979)
1980	40	60	Referendum result (1980)
1981	40	60	”
1982	43.75	56.25	Opinion poll results (1982)
1983	30	70	Estimate (see text)
1984	30	70	”
1985	37.75	62.25	Provincial election results (1985)
1986	30	70	Estimate (see text)
1987	30	70	”
1988	30	70	”
1989	37	63	Opinion poll results (1989)
1990	64.9	35.1	Opinion poll results (1990)
1991	64.76	35.24	Opinion poll results (1991)
1992	63.48	36.53	Opinion poll results (1992)
1993	53.46	46.54	Opinion poll results (1993)
1994	43.08	56.95	Opinion poll results (1994)
1995	49.75	50.25	Referendum result (1995)
1996	45	55	Opinion poll results (1996)
1997	45	55	Opinion poll results (1997)
1998	45	55	Opinion poll results (1998)

† After apportioning the undecided vote and after averaging over polls taken in the same year.

Table 2

(a)			(b)		
Variable	Constant†	Trend†	Variable	Constant†	Trend†
HWMN	2.234 (99.86)	0.008 (10.80)	IPQMM	-0.117 (-2.091)	-0.006 (-3.347)
HWFIN	1.974 (32.84)	0.015 (7.45)	IPQMS	-0.293 (-3.951)	-0.002 (-0.672)
HWCONS	2.680 (84.71)	-0.002 (-2.36)	IPQBM	0.312 (6.501)	-0.020 (-12.642)
HWRET	1.773 (80.62)	0.003 (3.89)	IPQBS	-0.159 (-1.971)	-0.006 (-2.131)
HWW	2.091 (58.83)	0.008 (7.08)	IPQTM	0.303 (6.370)	-0.020 (-12.571)
HWB	1.728 (26.87)	0.021 (9.72)	IPQTS	-0.183 (-2.346)	-0.005 (-1.940)

† t-statistics in parentheses

Table 3

Parameter	**Value**	**Parameter**	**Value**	**Parameter**	**Value**
β	0.85	n_W	0.0070	σ_W^2	0.0003
η_1	0.37	n_τ	0.0055	σ_τ^2	0.0005
η_2	0.83	n_A	0.0109	σ_A^2	0.0033
ϵ	-5	n_P	-0.0062	σ_P^2	0.0008
σ_α^2	0.0028	-	-	-	

Table 4

Simulation Results: No Regime Shifts

Year	Actual $I_t^{\dagger}$	Simulated I_t	Actual I_t/K_t	Simulated I_t/K_t
1992	2748.4	4508.0	0.2210	0.3624
1993	1992.7	4325.6	0.1758	0.3816
1994	2568.6	4154.5	0.2355	0.3809
1995	2725.3	4034.9	0.2557	0.3786
1996	2922.6	4008.5	0.2756	0.3780

† Millions of 1992 Canadian dollars

Table 5

Simulation Results: Regime Shifts

Year	Actual $I_t^{\dagger}$	Simulated I_t	Actual I_t/K_t	Simulated I_t/K_t
1992	2748.4	481	0.2210	0.0387
1993	1992.7	1807	0.1758	0.1594
1994	2568.6	1455	0.2355	0.1334
1995	2725.3	2383	0.2557	0.2236
1996	2922.6	2283	0.2756	0.2153

† Millions of 1992 Canadian dollars

Table 6

Simulation Results for 1993

	Simulated $I_t^\dagger$	**Simulated I_t/K_t**
I. Uniform prior for $\widetilde{\alpha}_t$		
(a) Baseline case	1807	0.1594
(b) $\sigma^2_{1,j} = 3\sigma^2_{0,j},\ j = \alpha, p$	501	0.0442
(c) $\chi_{01} = 0.4254$	2240	0.1976
(d) $\chi_{01} = 0.7090$	994	0.0877
II. Optimistic prior for $\widetilde{\alpha}_t$		
$\psi_1 = 0.1, \psi_2 = 0.1, \psi_3 = 0.8$	4326	0.3816
III. Pessimistic prior for $\widetilde{\alpha}_t$		
$\psi_1 = 0.8, \psi_2 = 0.1, \psi_3 = 0.1$	0.0	0.0
IV. Informative prior for $\widetilde{\alpha}_t$		
$\psi_1 = 0.1, \psi_2 = 0.8, \psi_3 = 0.1$	0.0	0.0

† Millions of 1992 Canadian dollars

Table 7

Posterior Distributions

(i) Uniform prior	0.0050	0.0510	0.0636
	0.0020	0.1921	0.2395
	0.0021	0.2000	0.2492
(ii) Optimistic prior	0.0000	0.0025	0.0251
	0.0001	0.0095	0.0947
	0.0008	0.0790	0.7882
(iii) Pessimistic prior	0.0180	0.2191	0.2731
	0.0085	0.1031	0.1285
	0.0088	0.1073	0.1337
(iv) Informative prior	0.0000	0.0247	0.0038
	0.0010	0.7429	0.1157
	0.0001	0.0966	0.0151

Table B.1

	τ^f	τ_t^P	τ_t^S	γ_t †		z_t	
Year				M&E	S	M&E	S
1970	0.3477	0.12	1.0	0.0	0.0	0.366	0.196
1971	0.3353	0.12	1.0	0.0	0.0	0.390	0.174
1972	0.3278	0.12	1.0	0.0	0.0	0.370	0.164
1973	0.3219	0.12	1.0	0.0	0.0	0.355	0.144
1974	0.3077	0.12	1.033	0.0	0.0	0.344	0.136
1975	0.3034	0.12	1.017	0.00415	0.00197	0.343	0.136
1976	0.3038	0.12	1.0	0.00832	0.00381	0.350	0.144
1977	0.3055	0.12	1.0	0.00914	0.00457	0.347	0.141
1978	0.3061	0.12	1.0	0.00956	0.00471	0.338	0.132
1979	0.3011	0.12	1.0	0.0140	0.00664	0.343	0.122
1980	0.3007	0.13	1.05	0.01444	0.00756	0.330	0.107
1981	0.3070	0.13	1.05	0.01436	0.00788	0.295	0.100
1982	0.2974	0.13	1.039	0.0138	0.00777	0.313	0.116
1983	0.2988	0.13	1.02	0.0126	0.00735	0.304	0.110
1984	0.3003	0.13	1.0	0.01239	0.00763	0.318	0.123
1985	0.2969	0.13	1.025	0.01239	0.00763	0.323	0.133
1986	0.2838	0.13	1.05	0.01239	0.00763	0.318	0.129
1987	0.2762	0.1394	1.03	0.01239	0.00763	0.287	0.100
1988	0.2442	0.1394	1.03	0.01239	0.00763	0.285	0.100
1989	0.2334	0.1456	1.03	0.01239	0.00763	0.279	0.092
1990	0.2301	0.1495	1.03	0.01239	0.00763	0.276	0.096
1991	0.2282	0.1394	1.03	0.01239	0.00763	0.298	0.109
1992	0.2272	0.1625	1.03	0.01239	0.00763	0.304	0.118
1993	0.2254	0.1625	1.03	0.01239	0.00763	0.299	0.112
1994	0.2235	0.1625	1.03	0.01239	0.00763	0.301	0.116
1995	0.2235	0.1625	1.03	0.01239	0.00763	0.311	0.127
1996	0.2235	0.1670	1.03	0.01239	0.00763	0.319	0.141
1997	0.2235	0.1670	1.03	0.01239	0.00763	-	-

† Source: Department of Finance

‡ Source: *The Conference Board of Canada*